# Tensorial Description of non-Minkowskian manifolds for understanding deflection of electromagnetic radiation : A mathematical approach

A Einstein

## Introduction

The special theory of relativity presents an alternative to traditional Newtonian approaches to mechanics by assuming a Minkowskian space-time geometry, where time is represented by an imaginary axis. Crucially, this approach assumes a limiting, frame dependent, velocity for all particles of

$$v_{\lim} = c = \sqrt{\frac{1}{\epsilon_0 \mu_0}}$$

However, it is clear that such an approach is limited to applications within inertial reference frames. This paper will extend this assumption to non-inertial frames and investigate solutions of the resulting tensorial equations with regards to in-vacuo propagation of radiation in regions of high energy-density

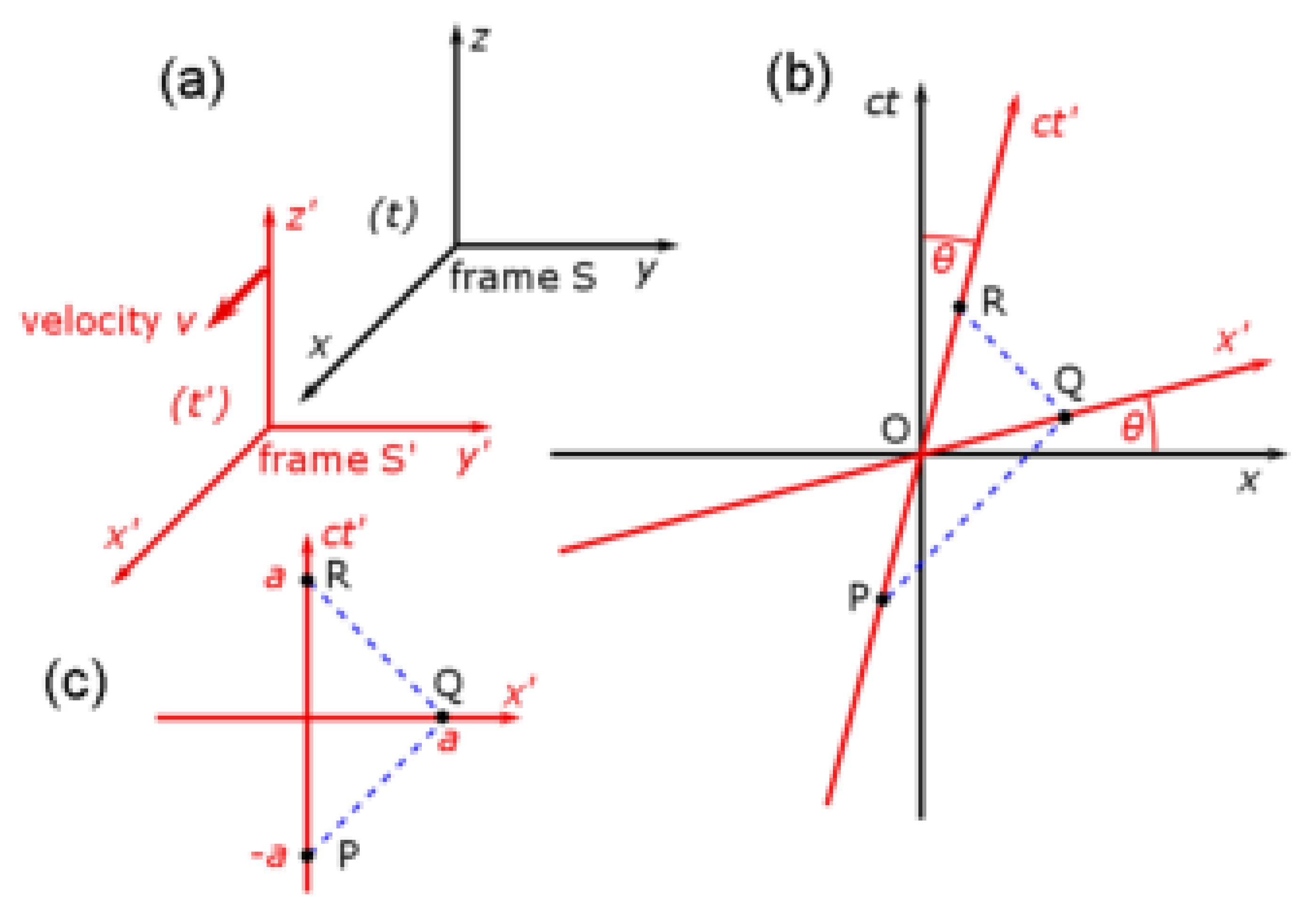



Figure 1 : An example of a 1+1d Minkowskian space-time geometry

## Methodology

- Assume a geodesic of the form

$$ds^2 = a^2(\tau)[-(1+2\phi)d\tau^2 + (1-2\phi)\gamma^{-2}\delta_{ij}\,dx^i dx^j] \,,$$

$$\gamma = 1 + \frac{k}{4}\,|x|^2 \,,$$

- Assuming solutions to Poisson’s equation

$$(\nabla^2 + 3k)\phi = \tfrac{1}{2}\rho_b a^2 \delta \,,$$

- Solving –

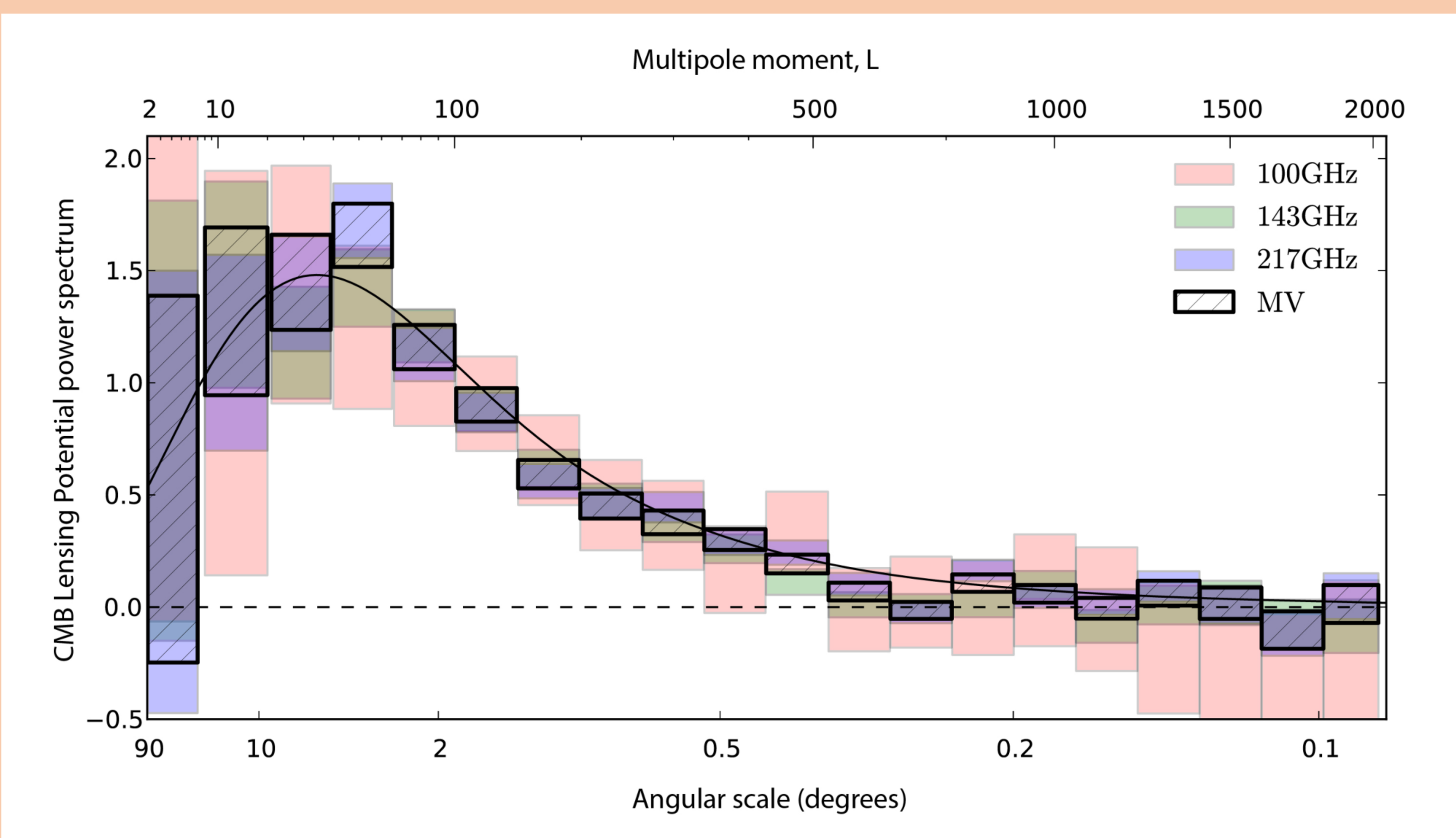
$$\sigma^2(\theta) = \frac{72\Omega^2}{\pi}\int_0^\infty \frac{dk}{k}\int_0^1 d\lambda \left[\frac{W(\lambda)}{a}\right]^2 \frac{P(a,\,k)}{1-(1-\Omega)\lambda^2} \times \left[1 - J_0 + \frac{1}{2}\sin^2\theta J_0 - \sin^2\frac{\theta}{2}J_2\right],$$

$$\xi(\theta) = \sigma^2 + \frac{36\Omega^2}{\pi}\int_0^\infty \frac{dk}{k} \times \int_0^1 d\lambda \left[\frac{W(\lambda)}{a}\right]^2 \frac{P(a,\,k)}{1-(1-\Omega)\lambda^2} \times [(\cos\theta - 3)(1 - J_0 - J_2) - \sin^2\theta J_0] \,,$$

- This clearly implies a deviation of electromagnetic radiation in the vicinity of a massive object equal to

$$\vec{\alpha}(\vec{\xi}) = \frac{4G}{c^2}\int d^2\xi' \int dz\rho(\vec{\xi}',z)\frac{\vec{b}}{|\vec{b}|^2},\ \vec{b} \equiv \vec{\xi} - \vec{\xi'}$$

## Application to CMB




The amplitude of the gravitational potential is shown as a function of different angular scales on the sky, starting at ninety degrees on the left side of the graph, through to the smallest scales on the right hand side. The multipole moments corresponding to the various angular scales are indicated at the top of the graph.

This graph was compiled by analysing the tiny distortions imprinted on the photons of the CMB by the gravitational lensing effect of massive cosmic structures

# How can we avoid the ultraviolet catastrophe?

M Plank

## Summary

Classical models of radiation fail to explain the distribution of radiation from a black body

We develop a quantized model of radiation

This model correctly predicts the distribution of radiation, suggesting a quantum theory is required

## Classical Theory of Radiation

- The classical theory of radiation defines a black body as an idealized material which absorbs and emits all frequencies of electromagnetic radiation
- As materials are heated they are seen to glow first red, and then white hot, with a spectrum characteristic of its temperature
- A good model is a cavity with a small hole, through which any frequency of radiation can be transmitted

## Ultraviolet Catastrophe

- The classical theory of radiation predicts that the emitted radiation from a blackbody scales as –

$$B(\omega) \propto \omega^2 kT d\omega$$

- This means that a black body at a temperature T will emit ever larger quantities of radiation at larger frequency, $\omega$ (see figure 1)
- This leads to the so called “Ultraviolet catastrophe”; a phenomenon which is not experimentally observed

## Quantisation

- We attempt to address the shortfalls in the classical theory by assuming that light is composed of distinct packets of energy which we will call “quanta”
- This model allows us to calculate the spectrum of radiation for a quantum system as being

$$B(\omega) \propto \omega^3 \frac{1}{e^{\frac{\hbar\omega}{kT}} - 1} d\omega$$

## Results

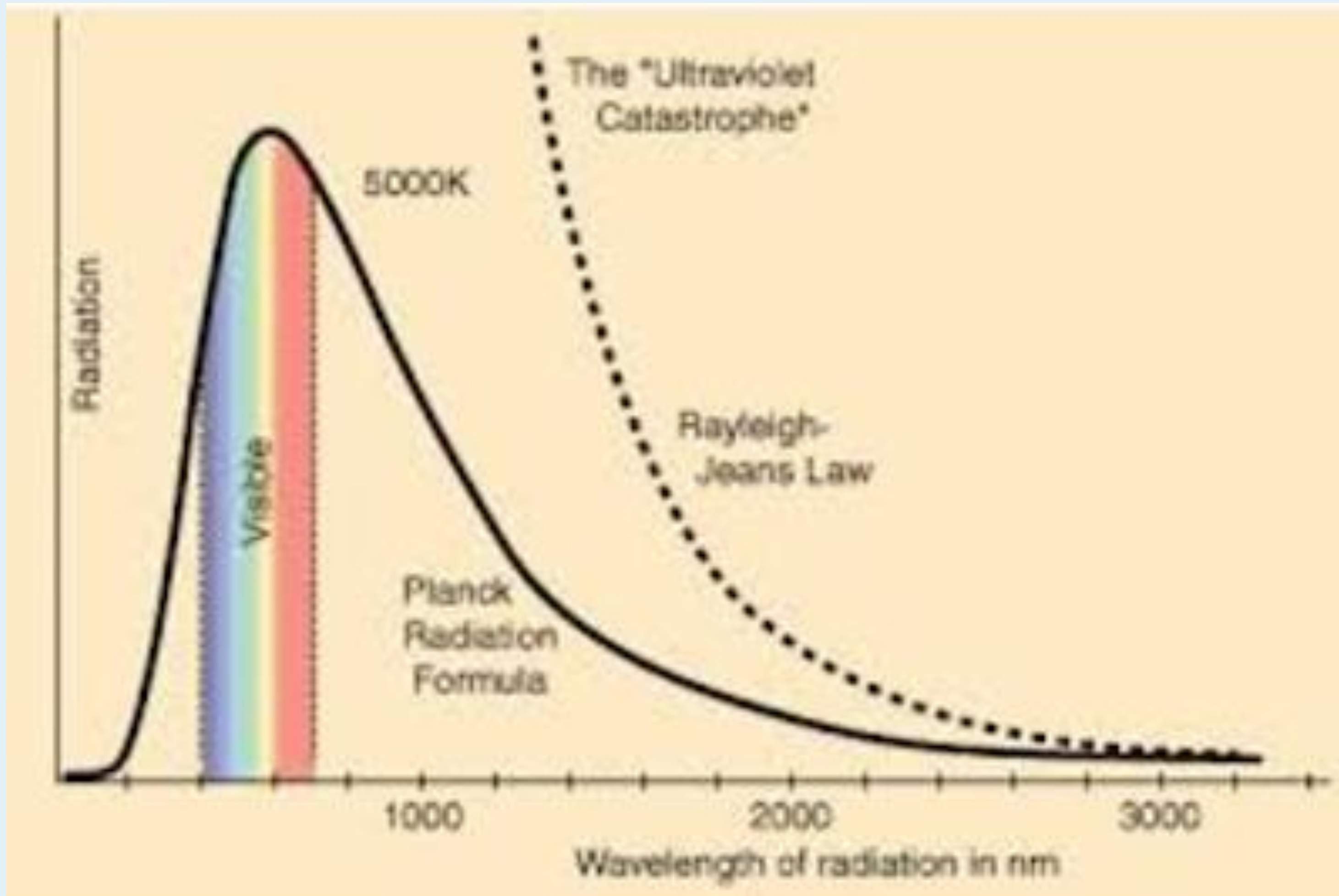



**Figure 1 :** The experimental and classical results for the emission spectrum of a black body.

- Shown above are the classical and experimental results for black body radiation
- The quantized result is in agreement with the experimental curve, and critically, does not exhibit an ultraviolet catastrophe
- This suggests that light may indeed by quantized

## Conclusions

- The classical model of radiation fails to explain the behavior of black bodies
- By developing a model of radiation based on quantization of energy we are able to explain the experimental spectrum
- This suggests that light is in fact quantized, requiring a radical reformulation of modern physics

**References**

[1] Rayleigh and Jeans, “Power radiated by a black body”, Phys. Rev. 21,483; 22,409 (1905)

[2] Plank, “A quantized model of radiation”, Phys. Rev. 7, 362, (1900)

# Generation of artificial radiation by alpha particle bombardment of Aluminium

I. Curie-Joliot

## Background

- In 1986 Henri Becquerel discovered naturally occurring radiation being emitted from Uranium
- Marie and Pierre Curie discovered further elements such as polonium and radium by closely observing the radioactive processes in uranium ore
- This radiation was later realized to be a product of the Uranium nucleus splitting, or "fissioning", producing showers of sub-atomic particles and electromagnetic radiation
- These naturally occurring radioactive materials have found widespread use in products from dentures to watch faces (see figure 1)

Figure 1 : An example of glassware made luminescent by the addition of uranium salts.

## The Nucleus and Radiation

- Atoms consist of a nucleus (made of protons and neutrons), surrounded by electrons. Each element has a unique number of protons in the nucleus.
- Natural radiation typically occurs when large, unstable nuclei 'split' into 2 smaller parts - a process called fission. This produces two atoms of a different type to the 'parent' (see figure 2).
- Any process which alters the number of protons in a nucleus not only changes the type of atom, but typically produces "by-products" such as nuclear fragments (e.g neutrons or alpha particles), electrons (beta particles), or excess energy in the form of gamma rays.

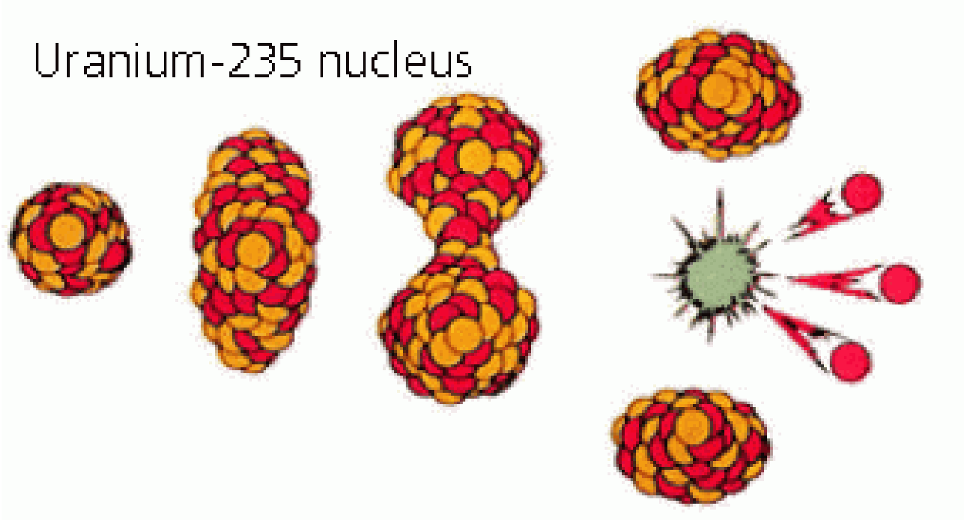



Figure 2 : The fission of a uranium atom, forming to new atomic nuclei

## Methodology

- In this work we bombard an aluminum target (which is normally radioactively stable) with alpha particles (Helium nuclei)
- We assume that these alpha particles 'react' with the Al nucleus to produce products which conserve the total number of protons and neutrons, e.g.

$$^{27}_{13}Al +^{4}_{2} He =^{30}_{15} P +^{1}_{0} n$$

- In this case the phosphorous isotope produced would be unstable (possessing too few neutrons), and so will most likely decay
- If radioactive phosphorous is observed in an initially pure Al sample, artificial radioactivity has been produced

## Results

- On bombardment both neutrons and positrons are observed being emitted from the sample
- In addition, chemical analysis shows the presence of both phosphorous and silicon in the Aluminum
- These results can be explained if radioactive phosphorous has been artificially produced, as described above, and then naturally decays to silicon by converting a proton to a neutron, and emitting a positron (so called $\beta^+$ decay) –

$$^{30}_{15}P =^{30}_{14} Si + e^{+}$$

- Such artificial isotopes may find use in medicine due to their unique radioactive properties

## Conculsions

- Artificial radioactive isotopes can be created by bombarding stable nuclei with alpha particles
- These man made isotopes may further decay via natural processes to generate stable nuclei
- Such processed may ultimately generate isotopes with uses in medicine

## References

[1] H Becquerel, "The discovery of radiation", Science, 23432, (1896)

[2] M Curie and J Curie, "Radium: A new element", Physical Review Letters, 138493, (1898)

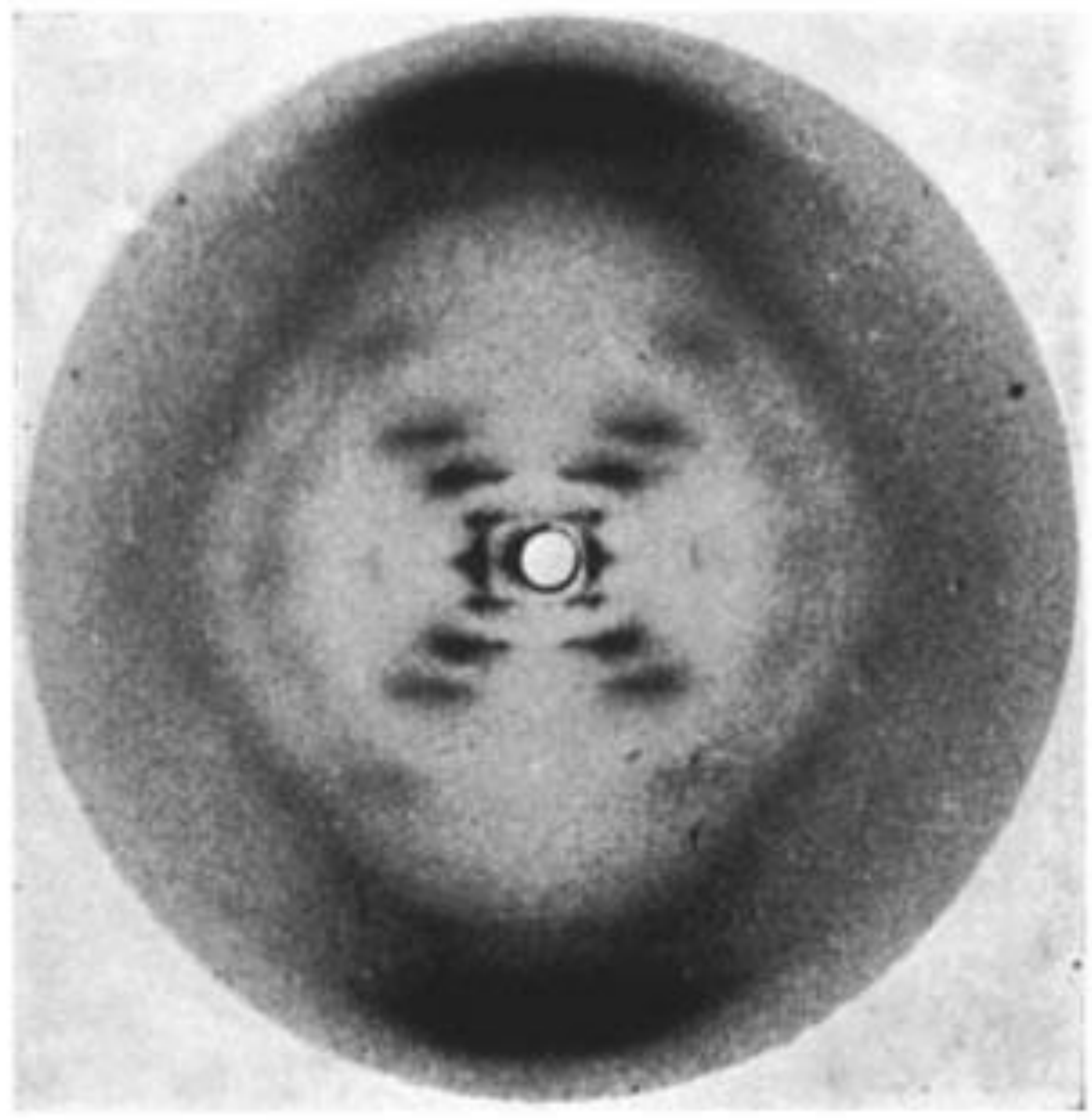
[3] I Joliot-Curie, "Artificial radiation generation", Nature, 9382, (1934)

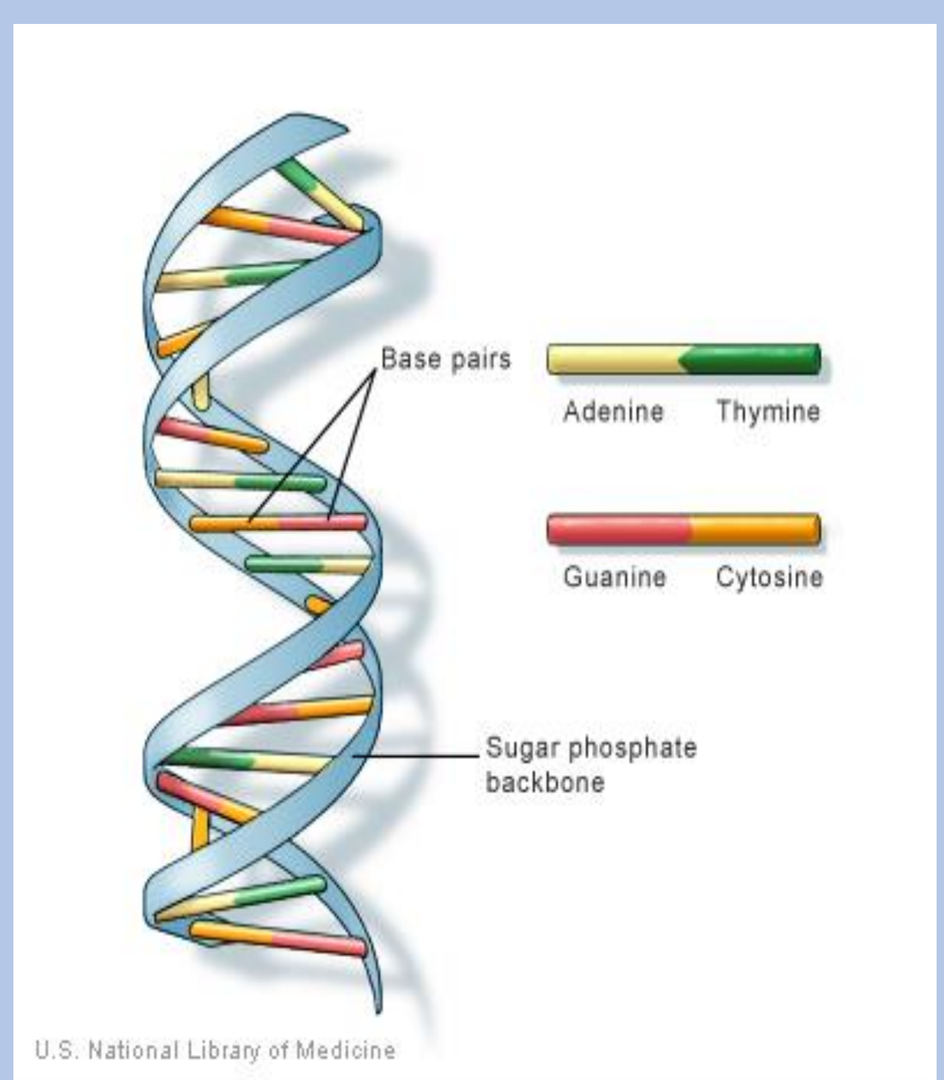
# You'll never believe what structure this polymeric nucleic acid chain has!

R. Franklin

*Fibre diffraction was employed utilising 20keV x-rays. Fibre diffraction does not usually provide good quality images because of the thinness of the fibres and therefore a very small mass to scatter the radiation. Nevertheless, the fibres' remarkable uniformity when wetted allowed us to manipulate them into a bundle and mount them on a wire frame to obtain x-ray diffraction images.*

*Air scattering also affected the quality of the image. The solution was to pass hydrogen through the camera and control the relative humidity of the sample. With this in place, the resulting images were much sharper and showed a clear crystalline diffraction pattern.*





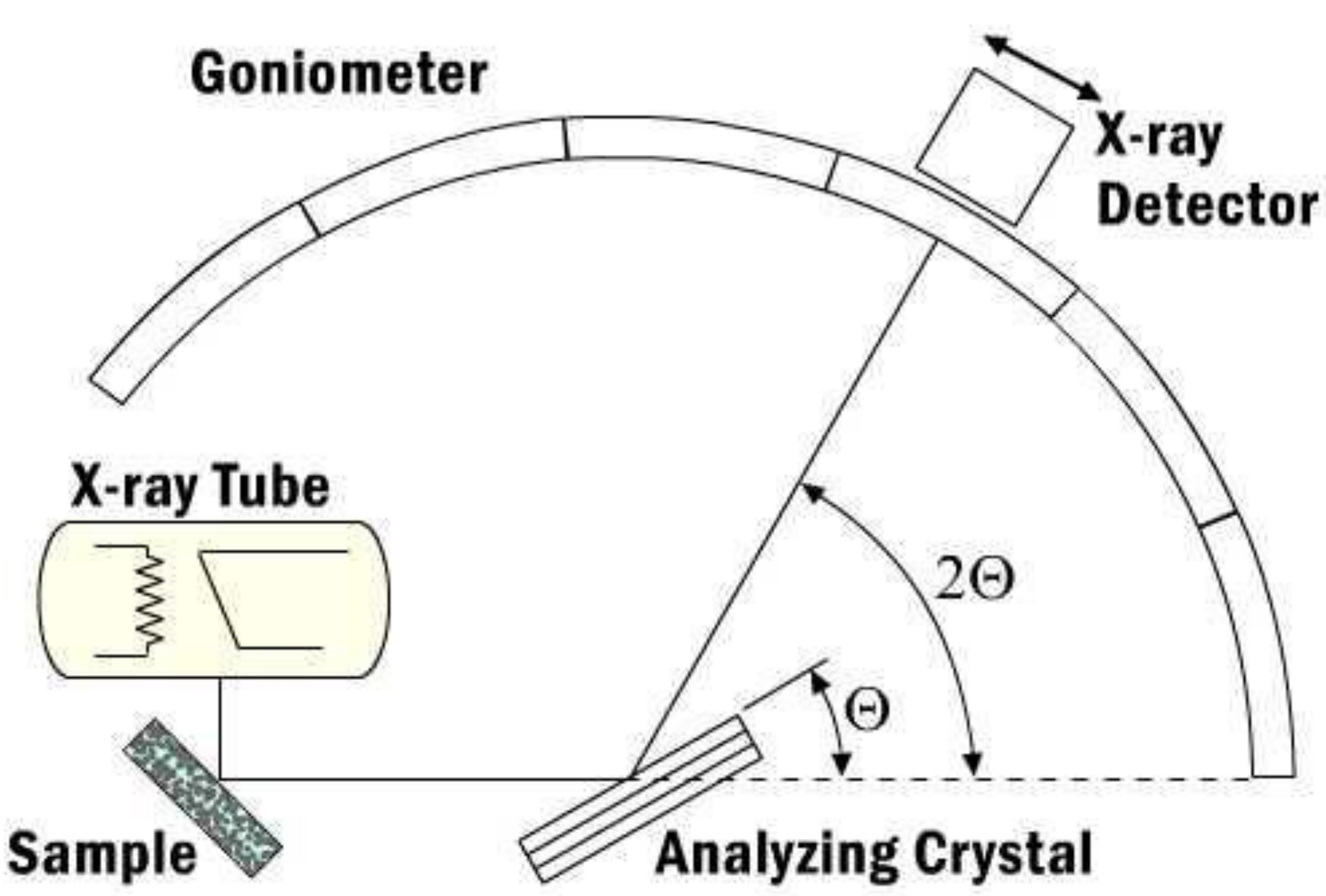

The lack of scattering along the direction of the fibre suggests a helical structure for the nucleic acids. Furthermore, the resultant diffraction pattern perpendicular to the fibres exhibits two clear arms, indicting a double helical structure with two intertwined helices.

The 10 spots in each arm suggests a 10 base pairs exist in each turn of the helix, given a pitch angle of 36º. Furthermore, the missing spot indicates an offset between the two helical strands, leading to the formation of major and minor grooves.

It should be noted that this diffraction corresponds to the hydrated, β form of DNA which is believed to be important in-vivo. Further work also investigated the α form, which was found to be prevalent in less hydrated environments.

**DNA, or deoxyribonucleic acid, seems to exist in humans and almost all other organisms. Nearly every cell in a person's body has the same DNA. Most DNA is located in the cell nucleus (where it is called nuclear DNA), but a small amount of DNA can also be found in the mitochondria (where it is called mitochondrial DNA or mtDNA).**

**This suggests that DNA may contain the genetic information needed to allow for the existance of life. But how does the DNA store this genetic information? In this work we use X-ray diffraction to determine the structure of DNA.**



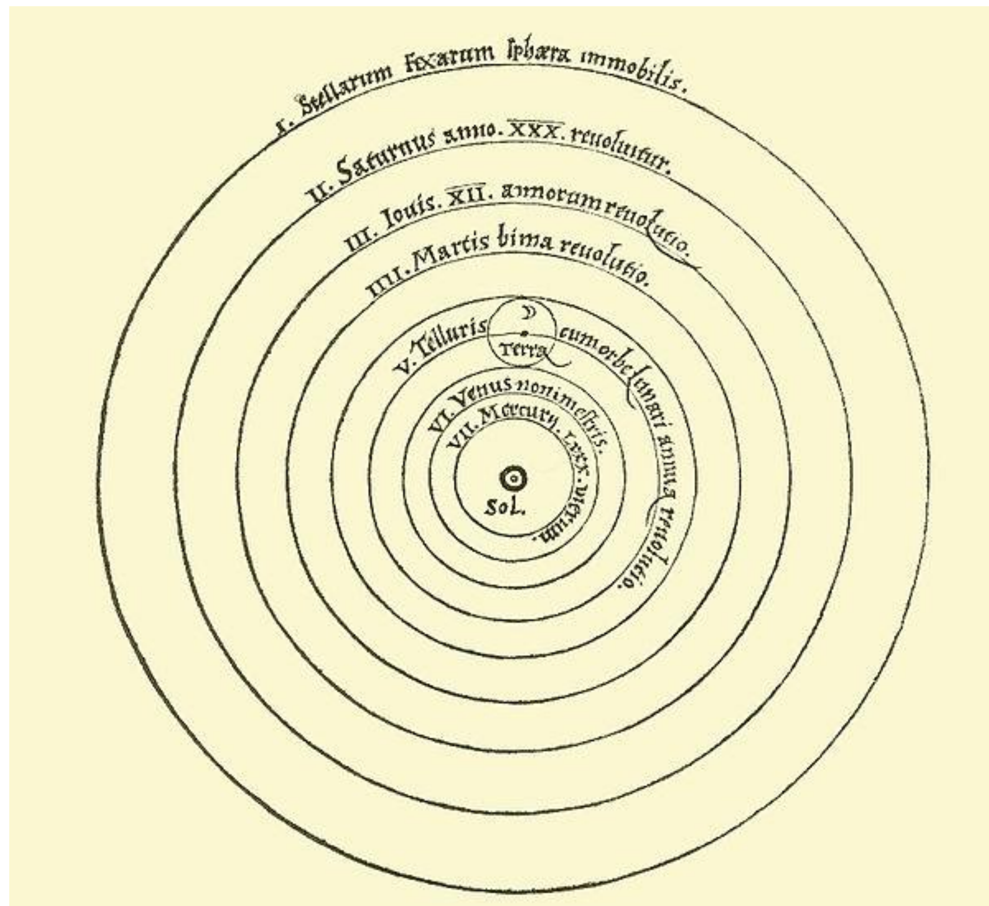

References: Bragg, 1912; Franklin, 1953

# Putting Earth in its place : A heliocentric model of the solar system

## N. Copernicus

While the sphericity of the Earth was widely recognized in Greco-Roman astronomy from at least the 3rd century BC, the Earth's daily rotation and yearly orbit around the Sun was never universally accepted until the Copernican Revolution.  While a moving Earth was proposed at least from the 4th century BC in Pythagoreanism, and a fully developed heliocentric model was developed by Aristarchus of Samos in the 3rd century BC, these ideas were not successful in replacing the view of a static spherical Earth, and from the 2nd century AD the predominant model, which would be inherited by medieval astronomy, was the geocentric model described in Ptolemy's Almagest.  The Ptolemaic system was a sophisticated astronomical system that managed to calculate the positions for the planets to a fair degree of accuracy. Ptolemy himself, in his Almagest, points out that any model for describing the motions of the planets is merely a mathematical device, and since there is no actual way to know which is true, the simplest model that gets the right numbers should be used. However, he rejected the idea of a spinning earth as absurd as he believed it would create huge winds. His planetary hypotheses were sufficiently real that the distances of moon, sun, planets and stars could be determined by treating orbits' celestial spheres as contiguous realities. This made the stars' distance less than 20 Astronomical Units, a regression, since Aristarchus of Samos's heliocentric scheme had centuries earlier necessarily placed the stars at least two orders of magnitude more distant.  Problems with Ptolemy's system were well recognized in medieval astronomy, and an increasing effort to criticize and improve it in the late medieval period eventually led to the Copernican heliocentrism developed in Renaissance astronomy.

Nicolaus Copernicus in his De revolutionibus orbium coelestium ("On the revolution of heavenly spheres", first printed in 1543 in Nuremberg), presented a discussion of a heliocentric model of the universe in much the same way as Ptolemy in the 2nd century had presented his geocentric model in his Almagest. Copernicus discussed the philosophical implications of his proposed system, elaborated it in geometrical detail, used selected astronomical observations to derive the parameters of his model, and wrote astronomical tables which enabled one to compute the past and future positions of the stars and planets. In doing so, Copernicus moved Heliocentrism from philosophical speculation to predictive geometrical astronomy. In reality, Copernicus's system did not predict the planets' positions any better than the Ptolemaic system. This theory resolved the issue of planetary retrograde motion by arguing that such motion was only perceived and apparent, rather than real: it was a parallax effect, as an object that one is passing seems to move backwards against the horizon.




This issue was also resolved in the geocentric Tychonic system; the latter, however, while eliminating the major epicycles, retained as a physical reality the irregular back-and-forth motion of the planets, which Kepler characterized as a "pretzel".Copernicus cited Aristarchus in an early (unpublished) manuscript of De Revolutionibus (which still survives), stating: "Philolaus believed in the mobility of the earth, and some even say that Aristarchus of Samos was of that opinion." However, in the published version he restricts himself to noting that in works by Cicero he had found an account of the theories of Hicetas and that Plutarch had provided him with an account of the Pythagoreans Heraclides Ponticus, Philolaus, and Ecphantus. These authors had proposed a moving earth, which did not, however, revolve around a central sun.

Copernican heliocentrism is the name given to the astronomical model developed by Nicolaus Copernicus and published in 1543. It positioned the Sun near the center of the Universe, motionless, with Earth and the other planets rotating around it in circular paths modified by epicycles and at uniform speeds. The Copernican model departed from the Ptolemaic system that prevailed in Western culture for centuries, placing Earth at the center of the Universe, and is often regarded as the launching point to modern astronomy and the Scientific Revolution.

From a modern point of view, the Copernican model has a number of advantages. It accurately predicts the relative distances of the planets from the Sun, although this meant abandoning the cherished Aristotelian idea that there is no empty space between the planetary spheres. Copernicus also gave a clear account of the cause of the seasons: that the Earth's axis is not perpendicular to the plane of its orbit. In addition, Copernicus's theory provided a strikingly simple explanation for the apparent retrograde motions of the planets—namely as parallactic displacements resulting from the Earth's motion around the Sun—an important consideration in Johannes Kepler's conviction that the theory was substantially correct.

# On the information content of black holes

S. Hawking

## Vacuum Fluctuations

- Quantum mechanics allow for the existence of vacum fluctuations – the temporary creation of particle/antiparticle pear in vacuum.
- Such pairs can only exist in accordance with Heisenberg's Energy-Time uncertainly principle

$$\Delta E \Delta t \geq \frac{\hbar}{2}$$

- A pair of energies E will therefore annihilate within a time Δt

## Black holes

- Black holes is theoretical astrophysicla objects which are left behind when starts come to the end of there life
- They are surrounded by an "event-horison", which is a boundart from which even light cannot escape
- This boundary lies at a radius [1]

$$r = \frac{2GM}{c^2}$$

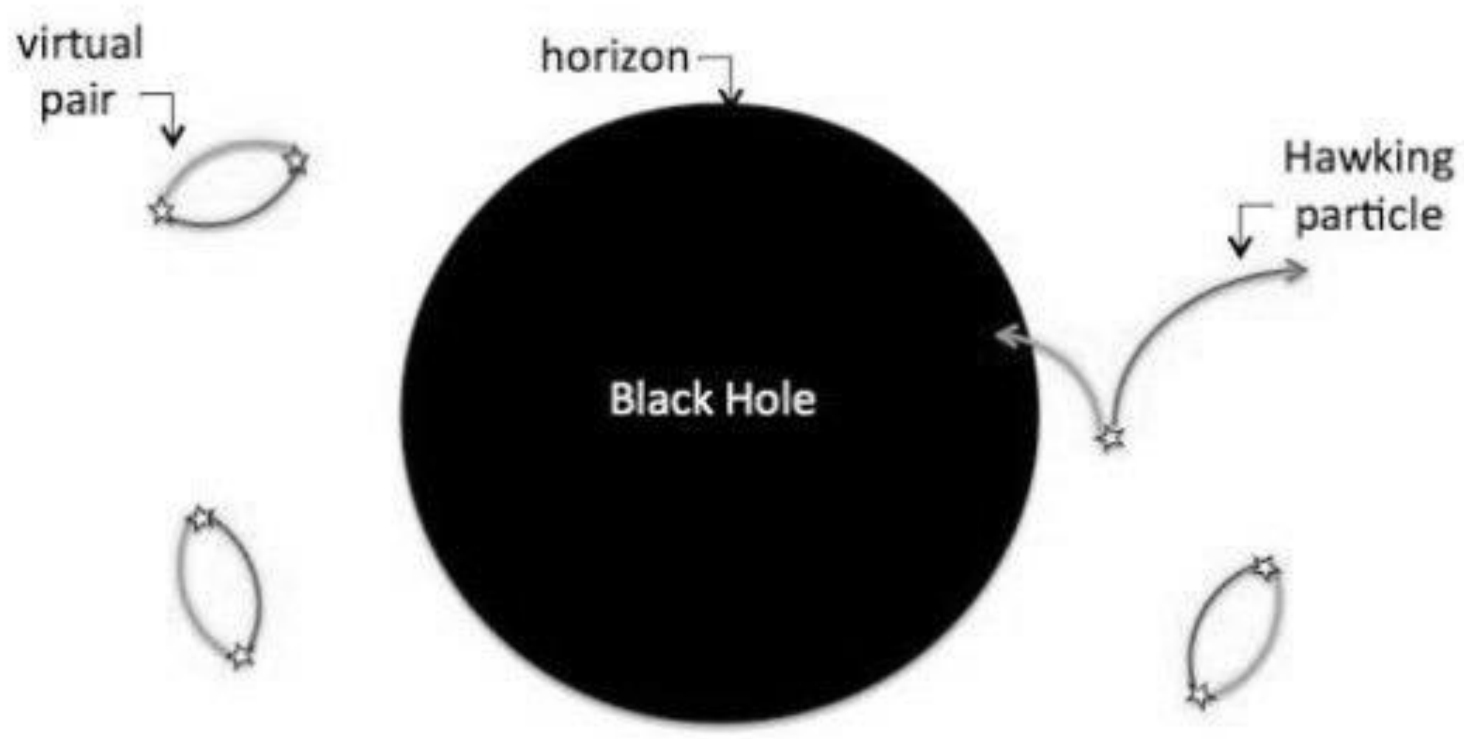



Figure 1 : A schematic of Hawking radiation

## Hawking Radiation

- Now assume that a vacuum fluctuation happens near the event horizon of a blak hole.
- One of the two particles falls into the black hole, and so can never anialate with it's partner
- This partner particle will escape from the black hole as a 'real particle'
- Since energy cannot be created or detroyed, the energy this particle possess must have come from the black hole
- This leads to the black hole 'evaporating' as enegy leaves in the form of this "Hawking radition" [2]

## Implications for information

- Infomration about the material falling into a black hole is typically lost
- The Hawking radiation produced by a black hole may in fact recover this lost inforamtion, which would resolve the so called "information paradox"

## References

[1] Schwartzchild, Nature (1916)
[2] Hawking, Science (1974)