Workshop on Diophantine approximation and related fields: York 2017

University of York : 26 — 30 June 2017

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Abstracts of Talks

June 16, 2017

Shabnam Akhtari (University of Oregon)

The difficulty of proving effective results for norm form equations

Abstract. Let $\omega_1, \ldots, \omega_n$ be algebraic numbers that are linearly independent over the rationals, and let $k = \mathbb{Q}(\omega_1, \ldots, \omega_n)$. We define a homogeneous polynomial in n variables $\omega_1, \ldots, \omega_n$ by

$$N(\mathbf{x}) = \mathrm{Norm}_{k/\mathbb{Q}}(x_1\omega_1 + \cdots + x_n\omega_n).$$

We are interested in the points in $\mathbb{Z}^n$ that satisfy the equation $N(\mathbf{x}) = b$, for a fixed nonzero $b \in \mathbb{Q}$. In his breakthrough work on norm form equation, Wolfgang Schmidt used his celebrated Subspace theorem to prove that norm form equations have finitely many families of solutions. Schmidt also obtained some bounds on the number of families of integer solutions to norm form equations. We are interested in finding effective bounds for the height of the representatives of integral solutions of norm form equations. I will try to explain the difficulty of obtaining such results and will report on some partial progress that we have made in this direction. This is a joint work in progress with Jeffery Vaaler.

Demi Allen (University of York)

A mass transference principle for systems of linear forms and some applications

Abstract. In Diophantine approximation we are often interested in the Lebesgue and Hausdorff measures of certain lim sup sets. In 2006, Beresnevich and Velani proved a remarkable result — the Mass Transference Principle — which allows for the transference of Lebesgue measure theoretic statements for lim sup sets arising from sequences of balls in $\mathbb{R}^k$ to Hausdorff measure theoretic statements. Subsequently, they extended this Mass Transference Principle to the more general situation in which the lim sup sets arise from sequences of neighbourhoods of "approximating" planes. In this talk I will discuss a recent strengthening (joint with V. Beresnevich) of this latter result in which some potentially restrictive conditions have been removed from the original statement. This improvement gives rise to some very general statements which allow for the immediate transference of more-or-less any Lebesgue measure Khintchine–Groshev type theorem to its corresponding Hausdorff measure analogue.

Jinpeng An (Peking University)

Gaps between values of functions at lattice points

Abstract. Kleinbock and Weiss proved that for an indefinite binary quadratic form Q, the

set of two-dimensional unimodular lattices L such that the values $Q(L \setminus \{0\})$ have a gap at a given real number is hyperplane absolute winning. In this talk, we will discuss extensions of this result to functions of more variables. The proof will be based on investigations of nondense orbits of nonquasiunipotent homogeneous flows. This is an ongoing joint work with Lifan Guan and Dmitry Kleinbock.

Jozsef Beck (Rutgers)

Uniformity of cube surface lines and related equidistribution results

Abstract. With my Ph.D. student, Michael Donders, we proved that every irrational slope geodesic on the cube surface is uniformly distributed. In the general case we have a "soft" qualitative result, because we use Birkhoff's ergodic theorem (which does not have any error term). However, for some "Diophantine slopes" we have sharp quantitative results describing the order of the error term in equidistribution. The error term turns out to be unexpectedly large, substantially larger than the usual square-root size "random fluctuation" of hyperbolic dynamical systems. This indicates a new kind of chaotic behavior for the equidistribution of 2-dimensional flat dynamical systems. There are some far-reaching generalizations in the 2-dimensional case, and, if I have time, I want to mention some first equidistribution results for 3-dimensional flat dynamical systems (where, as far as I know, there was no result at all).

Natalia Budarina (Maynooth University)

On the rate of convergence to zero of the measure of extremal sets

Abstract. We investigate the problem on the rate of convergence to zero of the measure of the set $x \in \mathbb{R}$ for which the inequality $|P(x)| < Q^{-w}$ for $w > n$ has a solution in the integer polynomials of degree n and height bounded by $Q \in \mathbb{N}$. The question on an effective estimate for this rate of convergence to zero will be discussed in the talk.

Nicolas Chevallier (LMIA)

Hausdorff dimension and uniform exponents in dimension two

Abstract. We will study the Hausdorff dimension of the set of non degenerate singular two-dimensional vectors with uniform exponent $\mu \in (1/2, 1)$. It is a joint work with Y. Bugeaud and Y. Cheung. We improve some previous estimates of R. Baker, and of Y. Bugeaud and M. Laurent. We prove that the Haudorff dimension is $2(1-\mu)$ when $\mu \geq \sqrt{2}/2$, whereas

for $\mu < \sqrt{2}/2$, we have only a lower bound and an upper bound. We also establish that this dimension tends to 4/3 (which is the dimension of the set of singular two-dimensional vectors) when μ tends to 1/2.

Yitwah Cheung (San Francisco State University)

A Game Approach to Littlewood Conjecture

Abstract. In this talk I will describe a single player game in which the player chooses a nested sequence of rectangles with the goal of optimizing a certain parameter describing the state of the game. The player wins if the sequence of nested rectangles can be chosen to keep the parameter uniformly bounded. The state parameter is explicitly computable in terms of the rational endpoints determining the corners of the rectangles. The game is designed so that a counterexample to the Littlewood Conjecture would result if a winning strategy can be found.

Sam Chow (University of York)

Bohr sets and multiplicative Diophantine approximation

Abstract. In two dimensions, Gallagher's theorem is a strengthening of the Littlewood conjecture that holds for almost all pairs of real numbers. We prove an inhomogeneous fibre version of Gallagher's theorem, sharpening and making unconditional a result recently obtained conditionally by Beresnevich, Haynes and Velani. The idea is to find large generalised arithmetic progressions within inhomogeneous Bohr sets, extending a construction given by Tao. This precise structure enables us to verify the hypotheses of the Duffin–Schaeffer theorem for the problem at hand, via the geometry of numbers.

Arturas Dubickas (Vilnius University)

On the size of algebraic integers

Abstract. We study a classical construction of lattices from number fields and obtain a series of new results about their minimum distance and other characteristics by introducing a new measure of algebraic numbers. In particular, we show that when a number field has few complex embeddings, the minimum distance of its lattice can be computed exactly: it equals 1. For an algebraic number α with s real conjugates $\alpha_1, \ldots, \alpha_s$ and $2t$ complex conjugates $\alpha_{s+1}, \overline{\alpha_{s+1}}, \ldots, \alpha_{s+t}, \overline{\alpha_{s+t}}$ this measure is defined as

$$m(\alpha) = \frac{\sum_{j=1}^{s+t} |\alpha_j|^2}{s+t}.$$

In fact, for any nonzero algebraic integer α this quantity $m(\alpha)$ (which we call the absolute normalised square size of α) is greater than $(e \log 2)/2 = 0.942084\ldots$, where e is the base of the natural logarithm. On the other hand, the smallest value among algebraic integers of degree at most 6 is

$$m(\zeta) = \frac{\zeta^2 + \zeta^{-1}}{2} = 0.946467\ldots,$$

where $\zeta = 0.826031\ldots$ is the root of $x^6 + x^2 - 1 = 0$. Part of the results are joint with Min Sha and Igor Shparlinski (Sydney).

Manfred Einsiedler (ETH)

Measure Rigidity of diagonal actions and Diophantine approximation

Abstract. We will review some of the previous applications of measure rigidity of diagonal actions to Diophantine approximation and discuss a new case of measure rigidity as well as a new application. This is joint work with Elon Lindenstrauss.

Kenneth Falconer (University of St Andrews)

Fractal Projection Theorems - Old and New

Abstract. Marstrands theorems relating to the Hausdorff dimension of projections of sets date back over 60 years, but continue to motivate a great deal of research. We will review a range of developments, such as projections of specific classes of set, exceptional projections, box-dimension analogues, etc.

Jonathan Fraser (University of St Andrews)

(Almost) arithmetic progressions and dimension

Abstract. The celebrated Erdos-Turan conjecture on arithmetic progressions is that any set of positive integers whose reciprocals form a divergent series should contain arbitrarily long arithmetic progressions. I will relate the existence of (almost) arithmetic progressions to dimension and discuss a weak version of Erdos-Turan. This is joint work with Han Yu (St Andrews).

Oleg German (Moscow State University)

On the spectrum of lattice exponents

Abstract. Let Λ be a lattice of rank d in $\mathbb{R}^d$. For each $\boldsymbol{v} = (v_1 \dots v_d)$ let us denote

$$\Pi(\boldsymbol{v}) = |v_1 \dots v_d|^{1/d}.$$

If Λ is algebraic, then $\Pi(\boldsymbol{v})$ is bounded away from zero at nonzero lattice points. The same holds if Λ is the image of an algebraic lattice under the action of a non-degenerate diagonal matrix. It is an open question whether other such lattices exist for $d \geqslant 3$. A negative answer to this question is known to imply the Littlewood conjecture.

We turn our attention to a more general situation, when $\Pi(\boldsymbol{v})$ can attain however small values. In this case it is reasonable to talk about the rate of tending $\Pi(\boldsymbol{v})$ to zero over a sequence of lattice points, which leads to the notion of a Diophantine exponent of Λ defined as

$$\omega(\Lambda) = \sup \left\{ \gamma \in \mathbb{R} \;\middle|\; \Pi(\boldsymbol{v}) < |\boldsymbol{v}|^{-\gamma} \text{ admits } \infty \text{ solutions in } \boldsymbol{v} \in \Lambda \right\}.$$

In our talk we discuss all that is currently known about Diophantine exponents of lattices and pay special attention to the spectrum of this quantity.

Anish Ghosh (Tata Institute of Fundamental Research)

Values of quadratic forms at integer points

Abstract. I will discuss Oppenheim's conjecture (Margulis' theorem) and variations on this theme. I will emphasize recent progress on effective versions of these results.

Alexander Gorodnik (University of Bristol)

Discrepancy in Diophantine problems

Abstract. We discuss some recent results describing the distribution of solutions of Diophantine inequalities.

Ben Green (University of Oxford)

Monochromatic solutions to $x + y = z^2$

Abstract. Suppose that the natural numbers greater than 2 are coloured red and blue. Then there are x, y and z, all of the same colour, with $x + y = z^2$. However, this is false if we allow a third colour. I will describe the proof of this result, focussing on the ingredients which have a flavour of Diophantine approximation. Joint work with Sofia Lindqvist.

Anna Gusakova (University of Bielefeld)

On the distribution of points with conjugate algebraic coordinates near smooth curves

Abstract. This is a joint work with V. Bernik and F. Götze. Let $J_0 \subset \mathbb{R}$ be a finite open interval and let $f : J_0 \to \mathbb{R}$ be a continuously differentiable function. It is very interesting and difficult question to calculate the number of integer and rational points in domains of $\mathbb{R}^k$. As a domain one can consider some neighbourhood of the curve parametrized by f. In this case we define the following set:

$$N_f(Q, \gamma, J) := \left\{ \left(\frac{p_1}{q}, \frac{p_2}{q} \right) \in \mathbb{Q}^2 : 0 < q \leq Q, \, \frac{p_1}{q} \in J, \left| f\left(\frac{p_1}{q}\right) - \frac{p_2}{q} \right| < Q^{-\gamma} \right\},$$

where $J \subset J_0$, $0 \leq \gamma < 2$ and $Q > 1$ some natural number. The problem is to determine the behaviour of the quantity $\# N_f(Q, \gamma, J)$ when Q tends to infinity. During the last 12 years this problem was investigating by M. Huxley, V. Bernik, V. Beresnevich, S. Velani, D. Dickinson, E. Zorin and R. Vaughan and the following upper and lower estimates of the same order where obtained:

$$c_1 \, Q^{3-\gamma} \leq \# N_f(Q, \gamma, J) \leq c_2 \, Q^{3-\gamma}.$$

This talk discusses the analogouse question for the set

$$M_f^n(Q, \gamma, J) := \left\{ (\alpha_1, \alpha_2) \in \mathbb{A}_n^2(Q) : \, \alpha_1 \in J, |f(\alpha_1) - \alpha_2| \leq Q^{-\gamma} \right\},$$

where $0 < \gamma < 1$, $n \geq 2$ and $\mathbb{A}_n^2(Q)$ denotes the set of points $(\alpha_1, \alpha_2) \in \mathbb{R}^2$ with algebraically conjugate coordinates of degree $\leq n$ and height $\leq Q$. In this case we have

$$c_3 \, Q^{n+1-\gamma} \leq \# M_f(Q, \gamma, J) \leq c_4 \, Q^{n+1-\gamma}.$$

The result of the same type was obtained for the points with integer algebraically conjugate coordinates.

References

[1] M.N. Huxley, *Area, lattice points, and exponential sums*, London Mathematical Society Monographs, New Series, Vol. 13, Oxford University Press, New York, 1996.

[2] V. Beresnevich, D. Dickinson and S. Velani, *Diophantine approximation on planar curves and the distribution of rational points*, Ann. of Math. (2), **166**:2 (2007), pp. 367 — 426. (With an appendix "'Sums of two squares near perfect squares"' by R. C. Vaughan.)

[3] V. Beresnevich, *Rational points near manifolds and metric Diophantine approximation*, Ann. of Math. (2), **175**:1 (2012), pp. 187 — 235.

[4] V. Bernik, F. Götze, A. Gusakova, *On distribution of points with algebraically conjugate coordinates in neighborhood of smooth curves*, Zapiski POMI, 448 (2016), pp. 14 — 47.

Jin-Jin Huang (University of Nevada)

Rational points near manifolds and Diophantine approximations

Abstract. We will discuss various types of questions regarding the interplay of the two objects mentioned in the title.

Shirali Kadyrov (Nazarbayev University)

Bernstein-Walsh Inequalities in Higher Dimensions over Exponential Curves

Abstract. For any positive integer d, consider an exponential curve Γ in complex $d+1$-space given by $f(z) = (e^z, e^{x_1 z}, \dots, e^{x_d z})$ where x_j's are real numbers. When the vector $(x_1, x_2, \dots, x_d)$ is Diophantine, we obtain a sharp Bernstein-Walsh type inequality, which estimates the growth of a polynomial in complex $d+1$-space in terms of its degree and sup norm on the closed unit disc in Γ. Moreover, we show that the set of vectors $(x_1, x_2, \dots, x_d)$ for which the growth is faster, has Hausdorff dimension $d-1$. This is a joint work with Mark Lawrence.

References:

[1] S. Kadyrov, M. Lawrence. "BernsteinWalsh Inequalities in Higher Dimensions over Exponential Curves" Constructive Approximation 44.3, 327-338 (2016).

[2] D. Coman and E.A. Poletsky. “Polynomial estimates, exponential curves and Diophantine approximation” Math. Res. Lett. 17(6), 1125-1136 (2010).

Dmitry Kleinbock (Brandeis University)

Shrinking targets on homogeneous spaces and improving Dirichlet's Theorem

Abstract. Optimal results on the metric theory for improvements to Dirichlet's Theorem are obtained in the one-dimensional case. For simultaneous approximation the problem is open. I will describe reduction of the problem to dynamics both in one-dimensional case (via continued fractions) and for higher dimensions (via diagonal flows on the space of lattices). If time allows I'll speak about an inhomogeneous version which happens to be easier than the homogeneous one. Joint work with Nick Wadleigh.

Henna Koivusalo (Universität Wien)

Relationship between cut and project sets and Diophantine approximation

Abstract. Cut and project sets are, in many senses of the word, regular, but still aperiodic point patterns obtained by projecting a part of the integer lattice to a subspace. Recently, a flexible formalism was discovered for translating information on multi-dimensional Diophantine approximation to regularity properties of cut and project sets. In this talk I explain recent developments of the theory: how to quantify the translation of Diophantine properties to regularity properties of cut and project sets, and how discrepancy theory can be used to gain more detailed information on speed of convergence to asymptotics, of frequences of patterns in cut and project sets. The talk is based on joined work with Alan Haynes, Antoine Julien and Jamie Walton.

Cong Ling (Imperial College London)

Diophantus meets Shannon: Some aspects of Diophantine approximation in coding theory

Abstract. This talk presents a couple of problems in coding theory where Diophantine approximation arises in a constructive or destructive manner. The first problem, known as compute-and-forward in channel coding, is to decode a linear combination of codewords

in a wireless relay network. On block fading channels, the problem boils down to approximating channel coefficients by algebraic integers. The second problem is concerned with Shannon sampling in source coding, where Meyer sets (quasicrystals) yield universal sampling patterns for analogue signals. Again, Diophantine approximation and algebraic number theory play a crucial role.

Stephen McGuire (Maynooth University)

On a Lemma of Bernik

Abstract. I will discuss a generalization of a lemma of Bernik that demonstrates that a polynomial cannot be simultaneously small at three distinct points as well as having simultaneously small derivatives at those three points.

Nikolay Moshchevitin (Moscow State University)

Oscillation theorems for irrationality measure functions

Abstract. For a real θ we consider the irrationality measure function

$$\psi_\theta(t) = \min_{q \in \mathbb{Z}_+,\ q \le t} ||\theta q||,$$

where $||\cdot||$ stands for the distance to the nearest integer. In 2010 I. Kan and N. Moshchevitin proved that the difference $\psi_\alpha(t) - \psi_\beta(t)$ oscillates when $t \to \infty$, provided $\alpha \pm \beta \notin \mathbb{Z}$. In our lecture we will speak about possible generalizations of this result.

Felipe Ramirez (Wesleyan University)

A characterization of bad approximability using inhomogeneous approximations

Abstract. Recall that Kurzweils Theorem gives a characterization of badly approximable vectors using inhomogeneous approximations. I will explain and discuss an alternate characterization that shows that if we try to restrict Kurzweils Theorem to singletons in the inhomogeneous part, we find that the opposite of Kurzweils Theorem is true. Specifically, badly approximable vectors are exactly those that cannot, for any fixed inhomogeneous parameter, be inhomogeneously approximated at every monotone divergent rate.

Damien Roy (University of Ottawa)

Parametric geometry of numbers

Abstract. This theory, recently created by Wolfgang Schmidt and Leonard Summerer, is the study of the n successive minima of a varying family of symmetric convex bodies in Euclidean n-space with respect to a fixed lattice (or of a fixed symmetric convex body with respect to a varying family of lattices). For the standard one-parameter family attached to an arbitrary nonzero point of the space, we possess a complete description of all resulting maps up to bounded multiplicative factors. In this talk, we present this result, a strategy for its proof, and some of its applications.

Johannes Schleischitz (University of Ottawa)

Classical exponents of Diophantine approximation

Abstract. The talk aims to present recent results on classical exponents of Diophantine approximation. Let $n \geq 1$ denote an integer and ζ be a real number. In 1932, Mahler introduced the exponent $w_n(\zeta)$ as the supremum of η such that

$$0 < |a_0 + a_1\zeta + \cdots + a_n\zeta^n| \leq (\max_{0\leq j\leq n} |a_j|)^{-\eta}$$

has infinitely many integral solution vectors $(a_0, \ldots, a_n)$. In 2005, Bugeaud and Laurent defined exponents $\lambda_n(\zeta)$ concerning simultaneous rational approximation to $(\zeta, \zeta^2, \ldots, \zeta^n)$. Mahler partitioned the transcendental real numbers with respect to the growth of the sequence $(w_n(\zeta))_{n\geq 1}$ in S-numbers, T-numbers and U-numbers. A recent result presented in the talk is that, rather surprisingly, the Mahler classification is induced as well by imposing some natural decay porperties on the sequence of exponents $(\lambda_n(\zeta))_{n\geq 1}$. The method has several applications. For example, metric results on the Hausdorff dimension of the sets $\{\zeta \in \mathbb{R} : \lambda_n(\zeta) \geq a\}$, corresponding to the set of vectors on the Veronese curve approximable to a given degree $a > 1/n$ by rational vectors, are deduced.

Nimish A. Shah (Ohio State University)

Equidistribution of dilations of curves on nilmanifolds

Abstract. We indicate a proof of the following result: Let G be a k-step Nilpotent group with Lie algebra $\mathfrak{g}$ and a lattice Γ. Let $\phi : [0,1] \to \mathfrak{g}$ be such that $\phi^{(k)}(t)$ exists for almost all $t \in [0,1]$, and that for any nontrivial continuous character χ on the torus $G/[G,G]\Gamma$, $\phi^{(1)}(t) \notin \ker d\chi$ for almost all $t \in [0,1]$. Let $x_0 \in G/\Gamma$. For $T > 0$, let μ_T be the parameter measure on the curve $\{\exp(T\phi(t))x_0 : t \in [0,1]\}$. Then as $T \to \infty$, μ_T converges to the

G-invariant probability measure on G/Γ with respect to the weak* topology. Actually we prove a more applicable result about equidistribution of limits of dilating translates of locally shrinking pieces of the curve. This is a joint work with B. Kra and W. Sun.

David Simmons (University of York)

A variational principle in the parametric geometry of numbers, with applications to metric Diophantine approximation

Abstract. The parametric geometry of numbers, introduced by Schmidt and Summerer, is a framework for analyzing the Diophantine properties of a vector in terms of the successive minima of a certain one-parameter family of convex regions (or equivalently of a certain family of lattices) defined in terms of that vector. We generalize this framework to the setting of matrix approximation, and we calculate the Hausdorff and packing dimensions of certain sets defined in terms of the parametric geometry of numbers. One of the many applications of our theorem is a proof of the conjecture of Kadyrov, Kleinbock, Lindenstrauss, and Margulis stating that the Hausdorff dimension of the set of singular $m \times n$ matrices is equal to $mn(1 - \frac{1}{m+n-1})$. This work is joint with Tushar Das, Lior Fishman, and Mariusz Urbański.

Topi Törmä (University of Oulu)

On irrationality exponents in generalized continued fraction Cantor sets

Abstract. In a joint work with Kalle Leppälä we study which asymptotic irrationality exponents are possible for numbers in a *generalized continued fraction Cantor set*

$$E_{\mathcal{B}}^{\mathcal{A}} = \left\{ \mathop{\mathrm{K}}_{k=1}^{\infty} \frac{a_k}{b_k} \colon a_k \in \mathcal{A}, b_k \in \mathcal{B} \right\},$$

where $\mathcal{A}$ and $\mathcal{B}$ are some given finite sets of positive integers. I will give a brief overview of what is known on the subject and what kind of new results we have obtained.

Bao-Wei Wang (Huazhong University of Science and Technology)

A dimension transference principle for dynamical Diophantine approximations

Abstract. Dynamical Diophantine approximation concerns the Diophantine properties of the orbit in a dynamical system. More precisely, let (X, T) be a dynamical system with a metric $|\cdot|$. One concerns the size of the following limsup set defined via a dynamical system:

$$W(\psi) := \Big\{x \in X : |T^n x - y| < \psi(n, x),\ \text{i.o.}, n \in \mathbb{N}\Big\}.$$

Following Hill & Velani's pioneer work [Invent. Math. 95'], there have been many works done in concrete dynamical systems. We hope to find a general principle about the dimension of $W(\psi)$ in a general framework. By introducing a *dynamical ubiquity property*, it is shown that in an expanding exact topological dynamical system, when $\psi(n, x) = e^{-(f(x)+\cdots+f(T^{n-1}x))}$, the dimension of $W(\psi)$ is given by the solution to the pressure function

$$\mathtt{P}(-s(\log|T'| + f)) = 0,$$

while the dimension of the phase space X is given by the solution to

$$\mathtt{P}(-s\log|T'|) = 0.$$

In other words, from the dimension of X to that of $W(\psi)$, one needs only transfer the potential in the pressure equation. For this partial analogy with the mass transference principle in classic Diophantine approximation [Beresvenich & Velani, Ann. of Math. 06'], we call the above phenomenon as a *dynamical dimension transference principle.* This is a joint with Guahua Zhang.

Barak Weiss (Tel Aviv University)

Random walks on homogeneous spaces and Diophantine approximation on fractals

Abstract. We extend results of Y. Benoist and J.-F. Quint concerning random walks on homogeneous spaces of simple Lie groups to the case where the measure defining the random walk generates a semigroup which is not necessarily Zariski dense, but satisfies some expansion properties for the adjoint action. Using these dynamical results, we study Diophantine properties of typical points on some self-similar fractals. Joint work with David Simmons

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Ben Worrell (University of Oxford)

On the Zero Problem for Exponential Polynomials

Abstract. Many fundamental algorithmic problems on continuous linear dynamical systems reduce to determining the existence of zeros of exponential polynomials (equivalently, functions satisfying linear differential equations). In this talk we consider the decision problem of determining whether such a function has a zero (the Zero Problem), the problem of determining whether it has infinitely many zeros (the Infinite Zeros Problem), and the problem of determining whether it has a zero in a given bounded interval (the Bounded Zero Problem). Decidability of all of these problems is open.

Using Baker's theorem on linear forms in logarithms, we show decidability of the Infinite Zeros Problem for functions satisfying linear differential equations or order at most 8. We show moreover that decidability at order 9 would entail substantial progress in Diophantine approximation—namely computability of the Lagrange constants of all real algebraic numbers. For the Zero Problem we likewise show decidability at order 8 and "hardness" at order 9. The latter decidability result relies on Schanuel's Conjecture. In fact we show that the Bounded Zero Problem is decidable at all orders, subject to Schanuel's Conjecture, and that the Zero Problem can be reduced to the Bounded Zero Problem at order at most 8.

This is joint work with Ventsislav Chonev and Joel Ouaknine.

Lei Yang (Hebrew University)

Badly approximable points on curves and unipotent orbits in homogeneous spaces

Abstract. We will study n-dimensional badly approximable points on curves. Given an analytic non-degenerate curve in $\mathbb{R}^n$, we will show that any countable intersection of the sets of weighted badly approximable points on the curve has full Hausdorff dimension. This strengthens a previous result of Beresnevich by removing the condition on weights. Compared with the work of Beresnevich, we study the problem through homogeneous dynamics. It turns out that the problem is closely related to the study of distribution of long pieces of unipotent orbits in homogeneous spaces.

Agamemnon Zafeiropoulos (University of York)

Inhomogeneous Diophantine approximation with restricted denominators

Abstract. We formulate and prove a Khintchine-type law for inhomogeneous Diophantine approximation. The denominators form a lacunary sequence; the size of the set of well approximable numbers is given in terms of a probability measure with Fourier coefficients with a prescribed logarithmic decay rate.

Wadim Zudilin (University of Newcastle)

Some hypergeometry inspired by irrationality questions

Abstract. I report on new hypergeometric constructions of rational approximations to $\log 2$, Catalan's constant and π^2, their connection with already known ones and underlying 'permutation group' structures. The talk is based on joint work in progress with Christian Krattenthaler (Vienna).
