

THE UNIVERSITY *of York*

**Degree Examination 2007**

**ENVIRONMENT DEPARTMENT**

**BSc in Environment, Economics and Ecology  
Part 1a**

**QUANTITATIVE METHODS FOR STUDYING THE ENVIRONMENT  
(ENVIRONMENT, ECONOMICS & ECOLOGY)**

Time allowed: **one and a half hours**

Answer **ONE** question from **SECTION A** and **ONE** question from **SECTION B**

Statistical tables, calculator and graph paper will be provided

*Pay adequate attention to spelling, punctuation and grammar, so that your answers  
can be readily understood*

## SECTION A

### Question 1

The cost of controlling emissions at a firm increases as the amount of emissions reduced increases. Here is a possible model:

$$C(S,N) = 750 + 15S^2 + 7S + 10N^2 + 18N$$

where S is the reduction in SO<sub>2</sub> emissions, N is the reduction in NO<sub>2</sub> emissions (in Kg of pollutant per day) and C is the daily cost to the firm (in UK£) of this reduction.

Government clean-air subsidies amount to £52 per kg of SO<sub>2</sub> and £80 per Kg of NO<sub>2</sub> removed.

- How many Kg of the pollutants, S and N, should the firm remove each day to minimise net cost (i.e. NC=cost minus subsidy)? [20 marks]
- How much is the minimum net cost? [10 marks]
- Verify that the second order sufficient conditions for a minimum net cost are satisfied. [20 marks]

### Question 2

A waste company provides both recycling (R) and treatment (T) services so that its profit function is

$$\pi = 16R^2 + 60R + 21T^2 + 24T$$

and the water and sewage production constraint is

$$20R + 60T = 240$$

You are required to:

- write the Lagrangian function [5 marks]
- find the first order necessary conditions [15 marks]
- find the optimum levels of R\* and T\* (and  $\lambda^*$ ) and the maximum value of the profit function [10 marks]
- indicate what the effects of relaxing/tightening the recycling and treatment production constraint are on the optimal levels of R\*, T\* and  $\pi^*$  [20 marks]

## SECTION B

### Question 1.

- a. Marine biologists have sampled 20 fish of a particular species in a marine protected area where no fishing is allowed. Recorded below are the lengths of the fish sampled in metres.

0.91	1	0.97	0.96	0.81	0.82	0.97	1.16	0.98	0.97
0.97	0.88	1.14	0.91	0.96	0.82	0.85	1.11	0.86	0.97

- Using the graph paper provided construct a histogram of the length variable. [7 marks]
  - Calculate the median and the interquartile range of the length variable. [7 marks]
  - From the histogram and the statistics you have calculated, describe the shape, central tendency and dispersion of the data. [6 marks]
  - In the region where the marine protected area now is, historical records suggest that the average fish length prior to protection was 80 cm. Perform a hypothesis test that investigates whether or not the mean value for the marine protected area is significantly different to the historical mean. Use a significance level of 0.05 for your test. [12 marks]
  - Assuming the population from which the marine protected area sample is taken is normally distributed. Construct a 95% confidence interval for the mean. [10 marks]
- b. Research on a campus has shown that 20% of students don't support a plastic recycling scheme in operation at the university. A student is carrying out a project to investigate reasons why these students don't approve of the scheme. The student has a list of 12 randomly selected students from which there is time during the project to interview 7 in depth about their views. What is the probability that all seven selected students don't support the recycling scheme. [8 marks]

## Question 2.

- a. 30-year average minimum temperature records for January (in °C) are available for five locations in each of three river catchments. A researcher studying water quality in the catchments wishes to know if the catchments are comparable in terms of their January temperatures. Below is a partially completed ANOVA table, produced by the researcher.

	Sum of Squares	df	Mean Square	F
Between catchments	4.94			
Within catchments				
Total	6.53			

- Copy the ANOVA table to your answer booklet and complete the missing parts of the table. [14 marks]
  - State the null and alternative hypotheses that would be tested by such an ANOVA. [3 marks]
  - At a 0.05 significance level, would you reject or accept the null hypothesis on the basis of the completed ANOVA table? Explain why you have made your decision. [3 marks]
- b. In the table below are the average minimum temperature values from part a) above. Also presented are the altitudes (in metres above sea-level) of the locations in the catchments where the records were made.

Temperature	Altitude
1.8	50
1.7	66
2.11	29
0.85	46
1.58	38
0.51	282
0.29	264
0.39	312
0.71	316
-0.11	306
1.44	50
1.68	56
1.18	50
1.44	56
1.96	52

- On the graph paper provided draw a scatterplot of the two variables with the temperature variable on the Y-axis and altitude on the X-axis. [5 marks]
- Calculate the mean and standard deviation of the temperature variable. [5 marks]
- The tables below represent the output from a simple least squares regression where temperature is the dependent variable and altitude is the independent variable.
  - Write out the regression equation [5 marks]

- 2) Draw the line represented by this equation on the scatterplot. [5 marks]
- 3) How much of the variation in average minimum temperature appears to be explained by the altitude variable? [5 marks]
- 4) Is the altitude variable significant in the regression? Explain how you came to this answer. [5 marks]

**Model Summary**

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.865 <sup>a</sup>	.748	.729	.35550

a. Predictors: (Constant), ALTITUDE

**Coefficients<sup>a</sup>**

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	1.810	.138		13.111	.000
	ALTITUDE	-4.87E-03	.001	-.865	-6.219	.000

a. Dependent Variable: TEMPERAT