6

2510122

THE UNIVERSITY of York

Degree Examination 2005

ENVIRONMENT DEPARTMENT

BSc in Environment, Economics and Ecology, Part 1a

QUANTITATIVE METHODS FOR ENVIRONMENTAL ECONOMICS AND MANAGEMENT

Time allowed: **two hours**

Please answer TWO questions:

ONE from **SECTION A** and **ONE** from **SECTION B**

University Calculators, graph paper and statistical tables will be provided.

Pay adequate attention to spelling, punctuation and grammar, so that your answers can be readily understood

SECTION A

Question 1

The cost of controlling emissions at a firm increases rapidly as the amount of emissions reduced increases. Here is a possible model:

$$C(S,L) = 1{,}580 + 120S^2 + 70S + 100L^2 + 30L$$

where S is the reduction in sulphur emissions, L is the reduction in lead emissions (in kg of pollutant per day) and C is the daily cost to the firm (in UK sterling) of this reduction.

Government clean-air subsidies amount to pounds 130 per kg of sulphur and pounds 80 per kg of lead removed.

a) How many kg of the pollutants, S and L, should the firm remove each day to minimise net cost (i.e. NC=cost minus subsidy)? **[20 marks]**

b) How much is the minimum net cost? **[10 marks]**

c) Verify that the second order sufficient conditions for a minimum net cost are satisfied. **[20 marks]**

Question 2

A water company provides both water (W) and sewage (S) services so that its profit function is

$$\pi = 8W^2 + 30W + 20S^2 + 12S$$

and the water and sewage production constraint is

$$10W + 30S = 120$$

You are required to:

a) write the Lagrangian function **[5 marks]**

b) find the first order necessary conditions **[15 marks]**

c) find the optimum levels of W and S (and λ) and the maximum value of the profit function **[15 marks]**

d) indicate what the effects of relaxing/tightening the water and sewage production constraint are on the levels of W and S (and λ) and the maximum value of the profit function **[15 marks]**

SECTION B

Question 3

a) An environmental scientist has made the following observations of monthly rainfall in mm at a number of sites:

Site	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Rainfall	91	92	105	103	102	100	95	107	98	99	106	98	98	103	95

i) What is the mean value of monthly rainfall across the sites? **[5 marks]**

ii) What is the standard deviation of monthly rainfall across the sites? **[5 marks]**

iii) Construct a histogram of the rainfall data using a bin width of 5 mm. **[5 marks]**

iv) How would you describe the shape of the distribution shown in the histogram? **[5 marks]**

v) Assuming that the rainfall observations are taken from a normal distribution, what is the probability of obtaining a rainfall value of between 94 and 102 mm? **[5 marks]**

vi) The environmental scientist knows that the mean rainfall for sites in another area that has been studied is 107mm. Perform a hypothesis test that will indicate whether or not the observations above are part of the same population of values observed in the other area. Perform the test at a significance level of 0.05. **[5 marks]**

c) An ecologist has measured the diameter at breast height (dbh) of naturally regenerating tree species at four woodland regeneration sites that have each been under a separate regime of management practices. The ecologist is performing an analysis of variance to establish whether or not the trees at the different sites are growing at different rate. The ecologist has partially completed the ANOVA table below:

	Sum of Squares	df	Mean Square	F
Between Sites	52.164			
Within Sites				
Total	543.507	23		

i) Copy the table into your exam script and complete the table. **[12 marks]**

ii) At a significance level of 0.05, does the ANOVA suggest that one or more of the sites are different? Explain your answer. **[8 marks]**

Question 4

a) An environmental manager has recorded the annual cost (in thousands of £s) of running 15 different nature reserves in a number of countries.

reserve	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
cost	2	2	28	21	18	10	4	46	8	10	39	8	7	20	4

i) What are the median, upper and lower quartile costs? **[8 marks]**
ii) Construct a box plot to illustrate the cost variable. **[5 marks]**
iii) How would you describe the shape of the cost variable given the box plot? **[5 marks]**

b) The manager knows that, globally, 30% of such nature reserves are paid for by funds from the state. Assuming that the 15 reserves studied have been randomly sampled, what is the probability that between 4 and 7 (inclusive) of the 15 reserves are paid for by the state? **[5 marks]**

c) In another study, the environmental manager has also recorded how many people are employed to run each of 100 different reserves, as well as the areas of the reserves in km^2. Both variables appear to be normally distributed. The standard deviation of reserve area is 257.3 and the mean of reserve area is 487.7. Calculate a 95% confidence interval for the mean. **[7 marks]**

d) The following is the output from a simple linear regression, performed by the manager, using area as the independent variable and staff as the dependent variable.

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	55.951	39.410		1.420	.159
	AREA	.962	.072	.805	13.451	.000

a. Dependent Variable: STAFF

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.805[a]	.649	.645	183.1471

a. Predictors: (Constant), AREA

i) Sketch a graph of the regression line produced by the analysis. **[5 marks]**

ii) What does the R square value reported tell us about the relationship between the two variables? **[5 marks]**

iii) Explain and interpret the t -test associated with the constant. **[5 marks]**

iv) Explain and interpret the t-test associated with the coefficient for the independent variable. **[5 marks]**

Short-cuts to the calculus of derivatives

The power rule
If $f(x) = x^n$ then $f'(x) = nx^{n-1}$ Eg. $f(x) = x^2$ and $f'(x) = 2x^{2-1} = 2x$
Derivative of a constant (C)
If $f(x) = C$ then $f'(x) = 0$ Eg. $f(x) = 3$ and $f'(x) = 0$
Derivative of a constant (C) times a function
If $f(x) = Cx^n$ then $f'(x) = Cnx^{n-1}$ Eg. $f(x) = 3x^4$ and $f'(x) = 3(4)x^{4-1} = 12x^3$
Derivative of sums/differences
If $f(x) = [g(x) \pm h(x)]$ then $f'(x) = g'(x) \pm h'(x)$ Eg. $f(x) = 3x^4 + 2x^2$ and $f'(x) = 12x^3 + 4x$
The Product Rule
If $f(x) = g(x)h(x)$ then $f'(x) = g'(x)h(x) + g(x)h'(x)$ Eg. $f(x) = x^2(3x-1)$ and $f'(x) = 2x(3x-1) + x^2(3) = 9x^2 - 2x$
The Quotient Rule
If $f(x) = \frac{g(x)}{h(x)}$ then $f'(x) = \frac{g'(x)h(x) - g(x)h'(x)}{[h(x)]^2}$ Eg. $f(x) = \frac{x^3}{x^2+1}$ and $f'(x) = \frac{3x^2(x^2+1) - x^3(2x)}{(x^2+1)^2}$
The Chain Rule
If $f(x) = g[h(x)]$ then $f'(x) = g'(h)h'(x)$ Eg. $f(x) = (2x^2 + x)^3$ and $f'(x) = 3(2x^2 + x)^2(4x + 1)$
Derivative of exponential functions
If $f(x) = a^{g(x)}$ then $f'(x) = \ln(\text{a})g'(x)a^{g(x)}$ Eg. $f(x) = 2^{(x^3)}$ and $f'(x) = \ln(2)(3x^2)[2^{(x^3)}]$
If $f(x) = e^{g(x)}$ then $f'(x) = g'(x)e^{g(x)}$ Eg. $f(x) = e^{x^2}$ and $f'(x) = 2xe^{x^2}$
Derivative of logarithmic functions
If $f(x) = \log_a[g(x)]$ then $f'(x) = \frac{1}{\ln a}\frac{g'(x)}{g(x)}$ Eg. $f(x) = \log_3[2x^2 + 1]$ and $f'(x) = \frac{1}{\ln 3}\frac{4x}{[2x^2+1]}$
If $f(x) = \ln[g(x)]$ then $f'(x) = \frac{g'(x)}{g(x)}$ Eg. $f(x) = \ln(6x^2)$ then $f'(x) = \frac{12x}{6x^2} = \frac{2}{x}$

The properties of exponents

PROPERTY	EXAMPLE
$a^x a^y = a^{x+y}$	$2^2 2^3 = 2^5 = 32$
$\frac{a^x}{a^y} = a^{x-y}$	$\frac{4^3}{4^2} = 4^{3-2} = 4^1 = 4$
$\frac{1}{a^x} = a^{-x}$	$\frac{1}{3^{-2}} = 3^2 = 9$
$a^0 = 1$	$3^0 = 1$
$(a^x)^y = a^{xy}$	$(3^2)^3 = 3^6 = 729$
$(ab)^x = a^x b^x$	$(4 * 2)^2 = 4^2 2^2 = 16 * 4 = 64$
$\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$	$\left(\frac{4}{2}\right)^2 = \frac{4^2}{2^2} = \frac{16}{4} = 4$

The properties of logarithms

PROPERTY	EXAMPLE
$\log_a(xy) = \log_a x + \log_a y$	$\log_2(16 = 8 * 2) = \log_2 8 + \log_2 2$
$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$	$\log_2\left(\frac{5}{3}\right) = \log_2 5 - \log_2 3$
$\log_a(x^b) = b \log_a x$	$\log_2(6^5) = 5 \log_2 6$
$\log_a a = 1$	$\log_2 2 = 1$
$\log_a 1 = 0$	$\log_2 1 = 0$
$\log_a 0 = +\infty$	$\log_2 0 = +\infty$
$\log_a\left(\frac{1}{x}\right) = -\log_a x$	$\log_2\left(\frac{1}{3}\right) = -\log_2 3$
$\log_a x = \frac{\log_b x}{\log_b a}$	$\log_2 5 = \frac{\log_{10} 5}{\log_{10} 2}$
$\ln e^x = x$	$\ln e^3 = 3$
$e^{\ln x} = x$	$e^{\ln 3} = 3$
$\log_a x = (\log_a e)(\ln x)$	$\log_2 3 = (\log_2 e)(\ln 3)$

General Solution of Quadratic Equations of the form $ax^2 + bx + c = 0$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Rules of integration

The power rule
$\int x^n dx = \frac{1}{n+1}x^{n+1} + C \qquad (n \neq -1)$ Eg. $\int x^2 dx = \frac{1}{2+1}x^{2+1} + C = \frac{1}{3}x^3 + C$ $\int x^{-1} dx = \ln x + C \qquad (n = -1)$
Integral of a constant (k) times a function
$\int k\ f(x)dx = k \int f(x)dx$ Eg. $\int 5x^3 dx = 5\int x^3 dx = 5\frac{1}{4}x^4 + C$
Integral of sums/differences
$\int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$ Eg. $\int (x^2+1)dx = \int x^2 dx + \int 1dx = \frac{1}{3}x^3 + \frac{1}{2}x^2 + C$
Integral of exponential functions
$\int f'(x)e^{f(x)}dx = e^{f(x)} + C$ $\int e^x dx = e^x + C$
Integral of logarithmic functions
$\int \frac{f'(x)}{f(x)}dx = \ln f(x) + C \quad [f(x) > 0]$ $\int \frac{1}{x}dx = \ln x + C$
Integration by substitution
$\int f(y(x))\frac{dy}{dx}dx = \int f(y)dy = F(y) + C$ Eg. $\int x^2(x^3+1)^2 dx = \int x^2(y)^2 dy =$ $\int \frac{1}{3}(y)^2 dy = \frac{1}{9}(y)^3 + C = \frac{1}{9}(x^3+1)^3 + C$
Integration by part
$\int ydz = yz - \int zdy$ Eg. $\int 4x(x+1)^3 dx = 4x\left[\frac{1}{4}(x+1)^4\right] - \int \left[\frac{1}{4}(x+1)^4\right]4dx$ $= x(x+1)^4 - \int (x+1)^4 dx = x(x+1)^4 - \frac{1}{5}(x+1)^5 + C$

Conditions for critical point(s):

$$y = f(x) \tag{1}$$

Conditions	Maximum	Minimum	Inflection point
FOCs	$\frac{\partial y}{\partial x} = 0$	$\frac{\partial y}{\partial x} = 0$	$\frac{\partial y}{\partial x} = 0$
SOCs	$\frac{\partial^2 y}{\partial x^2} < 0$	$\frac{\partial^2 y}{\partial x^2} > 0$	–

$$z = f(x, y) \tag{2}$$

Conditions	Maximum	Minimum	Saddle
FOCs	$f_x = f_y = 0$	$f_x = f_y = 0$	$f_x = f_y = 0$
SOCs	$f_{xx}, f_{yy} < 0$	$f_{xx}, f_{yy} > 0$	–
	$f_{xx}f_{yy} - f_{xy}^2 > 0$	$f_{xx}f_{yy} - f_{xy}^2 > 0$	$f_{xx}f_{yy} - f_{xy}^2 < 0$