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THE UNIVERSITY *of York*

Degree Examination 2004

ENVIRONMENT DEPARTMENT

BSc in Environment, Economics and Ecology, Part 1a

QUANTITATIVE METHODS FOR ENVIRONMENTAL ECONOMICS AND MANAGEMENT

Time allowed: **one and a half hours**

Answer **ONE** question from **SECTION A** and **ONE** question from **SECTION B**

Standard University calculators, graph paper and statistical tables will be provided.
Short-cuts to the calculus of derivatives are attached to the back of the paper.

Pay adequate attention to spelling, punctuation and grammar, so that your answers can be readily understood

SECTION A

Question 1

The cost of controlling emissions at a firm increases rapidly as the amount of emissions reduced increases. Here is a possible model:

$$C(S, L) = 2,000 + 200S^2 + 100L^2$$

where S is the reduction in sulphur emissions, L is the reduction in lead emissions (in kg of pollutant per day) and C is the daily cost to the firm (in UK sterling) of this reduction.

Government clean-air subsidies amount to £100 per kg of sulphur and £500 per kg of lead removed.

a) How many kg of the pollutants, S and L, should the firm remove each day to minimise net cost NC (cost minus subsidy) (**20 marks**)?

b) How much is the minimum net cost (**10 marks**)?

c) Verify that the second order sufficient conditions for a minimum net cost are satisfied (**20 marks**).

Question 2

A water company provides both water (W) and sewage (S) services so that its profit function is

$$\pi = 100W^2 + 50S^2$$

and the water and sewage production constraint is

$$50W + 40S = 456$$

You are required to:

a) write the Lagrangian function (**10 marks**);

b) find the first order necessary conditions (**20 marks**);

c) find the optimum levels of W and S (and λ) and the maximum value of the profit function (**20 marks**)

Section B

Question 3.

a) An environmental scientist has measured the biological oxygen demand (BOD) of water collected from a river from the same site in 10 different years. The observations made are listed below (measured in mg/l):

3.37 3.84 4.49 3.69 4.02 2.98 3.13 3.13 3.54 3.78

The level of BOD acceptable for the river to be classified as good quality is 4.0 mg/l. Calculate the mean and standard deviation of the sample in order to carry out a hypothesis test to ascertain whether or not the river sampled has an average BOD significantly different to the specified acceptable value. Perform the test at a level of significance of 0.05 (statistical tables required for this question are available on page 41 of the tables provided). **[17 marks]**

b) The table below shows the results of an ecologist's field work. The ecologist visited 100 survey sites and recorded whether the bird species, golden plover, was present or absent at the site. The vegetation type at the site was also recorded, all sites were either heather moorland or grass moorland. The cells in the table give the observed counts of sites containing the different combinations of bird presence/absence and moorland type.

	Heather moorland	Grass moorland
Presence	35	10
Absence	23	32

The ecologist is interested in finding out if there is any evidence that the species might be associated with the vegetation type. Perform a hypothesis test that may provide such evidence. Perform the test at a 0.05 significance level (statistical tables required for this question are available on page 42 of the tables provided). **[16 marks]**

c) An environmental economist is interested in the level of fish catches from three different potential fishing areas, each exhibiting different property right regimes. The weight of fish landings from each area is recorded in tonnes on each of four days. The economist decides to carry out an analysis of variance to see if there is any evidence that the fishing areas might yield different average fish landings. The economist has calculated that the sum of squares total for this analysis is 47.285. The sum of squares between fishing areas is 11.247.

i. Write down the hypothesis that the economist is testing **[3 marks]**
ii. Construct an ANOVA table for this ANOVA. **[10 marks]**
iii. Interpret the calculated F statistic if the ANOVA is being carried out at a significance level of 0.05 (statistical tables required for this question are available on page 44 of the tables provided). **[4 marks]**

Question 4.

a) An ecologist suspects that the birds of a migratory species found at a study site are from a well studied population of the species in the country from which they have migrated. This population is distinguishable from populations in other areas by wing span measurement. The ecologist takes a sample of 50 birds and calculates a mean wing span for the sample of 21.2 cm. The standard deviation of the sample is 2.3 cm . Given that the mean wing span for the well studied population is 22.8 cm, perform a hypothesis test to provide the ecologist with evidence as to whether or not the sample is likely to come from the population. Perform the test at a 0.05 significance level (statistical tables required for this question are available on page 34 of the tables provided). **[16 marks]**

b) Ten soil samples were taken and exchangeable calcium (Ca) and sodium (Na) were measured from the samples (in cmol (+) kg^{-1}). The measurements from the samples are given below.

Ca	38	35.6	38.7	41.8	35.6	32.7	36.6	39.6	38.8	34.3
Na	2	1.8	1.9	2.1	1.7	1.9	2.2	2.1	2	2.2

i. Produce a scatter plot of these data on the graph paper provided. **[7 marks]**

ii. Given the formulae for calculating covariance of two variables (x and y) and standard deviation for a variable (x) calculate a Pearson's correlation coefficient between calcium and sodium. **[7 marks]**

$$COV = \frac{\sum (x-\bar{x})(y-\bar{y})}{n-1} \qquad s_x = \frac{\sum (x-\bar{x})^2}{n-1}$$

iii. What does your correlation coefficient suggest about the two variables? **[3 marks]**

c) A soil scientist performed an ordinary least squares regression using total nitrogen (TOTN) concentration of soil samples (%) as the dependent variable and organic carbon (ORGC) concentration (%) as the independent variable. The SPSS output from the regression is given below.

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.975[a]	.950	.949	6.325E-03

a. Predictors: (Constant), ORGC

ANOVA[b]

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	2.879E-02	1	2.879E-02	719.534	.000[a]
	Residual	1.520E-03	38	4.001E-05		
	Total	3.031E-02	39			

a. Predictors: (Constant), ORGC

b. Dependent Variable: TOTN

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	8.742E-03	.003		2.638	.012
	ORGC	7.034E-02	.003	.975	26.824	.000

a. Dependent Variable: TOTN

i. Write down the regression equation that the soil scientist has calculated. **[3 marks]**
ii. Comment on the R Square value in the output. **[4 marks]**
iii. Interpret the t statistics and their significance values from the coefficients table. **[6 marks]**
iv. From the ANOVA table, how many observations has the soil scientist used? **[4 marks]**

end of paper

Short-cuts to the calculus of derivatives

The power rule
If $f(x) = x^n$ **then** $f'(x) = nx^{n-1}$
Eg. $f(x) = x^2$ **and** $f'(x) = 2x^{2-1} = 2x$

Derivative of a constant (C)
If $f(x) = C$ then $f'(x) = 0$
Eg. $f(x) = 3$ and $f'(x) = 0$

Derivative of a constant (C) times a function
If $f(x) = Cx^n$ then $f'(x) = Cnx^{n-1}$
Eg. $f(x) = 3x^4$ and $f'(x) = 3(4)x^{4-1} = 12x^3$

Derivative of sums/differences
If $f(x) = [g(x) \pm h(x)]$ then $f'(x) = g'(x) \pm h'(x)$
Eg. $f(x) = 3x^4 + 2x^2$ and $f'(x) = 12x^3 + 4x$

The Product Rule
If $f(x) = g(x)h(x)$ then $f'(x) = g'(x)h(x) + g(x)h'(x)$
Eg. $f(x) = x^2(3x-1)$ and $f'(x) = 2x(3x-1) + x^2(3) = 9x^2 - 2x$

The Quotient Rule
If $f(x) = \frac{g(x)}{h(x)}$ then $f'(x) = \frac{g'(x)h(x) - g(x)h'(x)}{(h(x))^2}$
Eg. $f(x) = \frac{x^3}{x^2+1}$ and $f'(x) = \frac{3x^2(x^2+1) - x^3(2x)}{(x^2+1)^2}$

The Chain Rule
If $f(x) = g[h(x)]$ then $f'(x) = g'(h)h'(x)$
Eg. $f(x) = (2x^2 + x)^3$ and $f'(x) = 3(2x^2 + x)^2(4x + 1)$

Derivative of exponential functions	
If $f(x) = a^{g(x)}$ then $f'(x) = \ln(a)g'(x)a^{g(x)}$	If $f(x) = e^{g(x)}$ then $f'(x) = g'(x)e^{g(x)}$
Eg. $f(x) = 2^{(x^3)}$ and $f'(x) = \ln(2)(3x^2)[2^{(x^3)}]$	Eg. $f(x) = e^{x^2}$ and $f'(x) = 2xe^{x^2}$

Derivative of logarithmic functions	
If $f(x) = \log_a[g(x)]$ then $f'(x) = \frac{1}{\ln a}\frac{g'(x)}{g(x)}$	If $f(x) = \ln[g(x)]$ then $f'(x) = \frac{g'(x)}{g(x)}$
Eg. $f(x) = \log_3[2x^2 + 1]$ and $f'(x) = \frac{1}{\ln 3}\frac{4x}{[2x^2+1]}$	Eg. $f(x) = \ln(6x^2)$ then $f'(x) = \frac{12x}{6x^2} = \frac{2}{x}$

Optimisation

$$y = f(x) \tag{1}$$

Conditions	Maximum	Minimum	Inflection point
FOCs	$\frac{\partial y}{\partial x} = 0$	$\frac{\partial y}{\partial x} = 0$	$\frac{\partial y}{\partial x} = 0$
SOCs	$\frac{\partial^2 y}{\partial x^2} < 0$	$\frac{\partial^2 y}{\partial x^2} > 0$	–

$$z = f(x, y) \tag{2}$$

Conditions	Maximum	Minimum	Saddle
FOCs	$f_x = f_y = 0$	$f_x = f_y = 0$	$f_x = f_y = 0$
SOCs	$f_{xx}, f_{yy} < 0$	$f_{xx}, f_{yy} > 0$	$-$
	$f_{xx}f_{yy} - f_{xy}^2 > 0$	$f_{xx}f_{yy} - f_{xy}^2 > 0$	$f_{xx}f_{yy} - f_{xy}^2 < 0$