3 Complex Numbers Solutions

1) If \( z_1 = 2 + 3j \) and \( z_2 = 4 - j \), then:

\[
\begin{align*}
\text{z}_1 + \text{z}_2 &= 2 + 3j + 4 - j = 6 + 2j \\
\text{z}_1 - \text{z}_2 &= 2 + 3j - 4 + j = -2 + 4j \\
\text{z}_1 \cdot \text{z}_2 &= (2 + 3j)(4 - j) = 8 + 12j - 2j - 3j^2 = 8 + 12j - 2j + 3 = 11 + 10j \\
\text{z}_1 / \text{z}_2 &= \frac{2 + 3j}{4 - j} = \frac{2 + 3j}{4 - j} \cdot \frac{4 + j}{4 + j} = \frac{8 + 2j + 12j + 3j^2}{16 - j^2} = \frac{8 + 2j + 12j - 3}{16 - j^2} = \frac{5 + 14j}{17}
\end{align*}
\]

2) Evaluate \( 1 + \exp(j \pi) \)

Zero. \( \exp(j \pi) = \cos(\pi) + j \sin(\pi) = -1 \), so \( 1 + \exp(j \pi) = 0 \).

3) Express \( \sqrt{2} \exp\left(-j \frac{5\pi}{4}\right) \) in terms of its real and imaginary components.

\[
\sqrt{2} \exp\left(-j \frac{5\pi}{4}\right) = \sqrt{2} \cos\left(-\frac{5\pi}{4}\right) + j \sqrt{2} \sin\left(-\frac{5\pi}{4}\right)
\]

\[
= \sqrt{2} \left(\frac{-1}{\sqrt{2}} + j \frac{1}{\sqrt{2}}\right)
\]

\[
= -1 + j
\]

4) Starting from Euler’s formula, prove that:

\[
\cos(\theta) = \frac{\exp(j\theta) + \exp(-j\theta)}{2}
\]

\[
\sin(\theta) = \frac{\exp(j\theta) - \exp(-j\theta)}{2j}
\]

Euler’s formula states that:

\[
\exp(j\theta) = \cos(\theta) + j \sin(\theta)
\]

Therefore:

\[
\exp(-j\theta) = \cos(-\theta) + j \sin(-\theta)
\]

\[
= \cos(\theta) - j \sin(\theta)
\]
since \( \cos(x) \) is an even function, so \( \cos(-x) = \cos(x) \), and sine is an odd function, so \( \sin(-x) = -\sin(x) \).

Then by adding and subtracting these two equations, we get:

\[
\exp(j\theta) + \exp(-j\theta) = 2\cos(\theta)
\]

\[
\exp(j\theta) - \exp(-j\theta) = 2j\sin(\theta)
\]

and the desired results follow.

5) What would you get if you add a cosine wave of amplitude \( \sqrt{10} \), frequency 10 Hz and phase 18.4 degrees to a cosine wave of amplitude \( \sqrt{2} \), frequency 10 Hz and phase 315 degrees? (Hint – express both in terms of a complex number, and then add the real and imaginary parts of the two complex numbers.)

The amplitude and phase of the first cosine wave can be expressed as the complex number:

\[z_1 = \sqrt{10} \exp\left(j \frac{18.4}{180} \pi \right) = 3 + j\]

\[z_2 = \sqrt{2} \exp\left(j \frac{315}{180} \pi \right) = 1 - j\]

Therefore,

\[z_1 + z_2 = 3 + j + 1 - j = 4\]

So the result is a cosine wave of amplitude four, and phase zero.