1 Maximum Likelihood estimation under the isolation and Wenting Zhou hypotheses

We will use the data from Hey *Experimental Economics* 2001 in which each subject was asked 100 pairwise choice questions on 5 separate occasions. There were just 4 outcomes. Suppose a subject has been given a sequence of choices $n = 1, ..., N$. Suppose that in question number n the probabilities of the four outcomes x_1, x_2, x_3, x_4 (which are listed in *increasing* order of magnitude) are $(p_{1n}, p_{2n}, p_{3n}, p_{4n})$ and $(q_{1n}, q_{2n}, q_{3n}, q_{4n})$. Let us denote the decision of the subjects on question n by d_n, where d_n takes the values 0 or 1 depending on whether the subject chose Left or Right.

1.1 Estimation under the isolation hypothesis

Let us suppose that the preference function is $V(\mathbf{r})$ where r are the probabilities. To save writing, let us write this in vector form. So let $\mathbf{r}$ denote the vector (r_1, r_2, r_3, r_4) and $V(\mathbf{r})$ denote $V(r_1, r_2, r_3, r_4)$. Suppose we add in an error term ϵ which we presume for the moment is normally distributed with mean 0 and precision s^2. (This is the inverse of the variance) Then we have the following if the subject makes no errors:

$$d_n = 0(1) \; if \; V(\mathbf{p}) > (<) V(\mathbf{q})$$

That is

$$d_n = 0(1) \; if \; V(\mathbf{p}) - V(\mathbf{q}) > (<) 0$$

Suppose now we add in an error term ϵ which we presume for the moment is normally distributed with mean 0 and precision s^2 (the inverse of the variance) σ^2 for the *difference* between the valuations.

Then we have

$$d_n = 0(1) \; if \; V(\mathbf{p}) - V(\mathbf{q}) + \epsilon > (<) 0$$

Or

$$d_n = 0(1) \; if \; \epsilon > (<) V(\mathbf{q}) - V(\mathbf{p})$$

$$d_n = 0(1) \; if \; \epsilon s > (<) [V(\mathbf{q}) - V(\mathbf{p})] s$$

Now let $F(.)$ denote the c.d.f of a unit normal. Then the probability of observing $d = 0$ is

$$1 - F\{[V(\mathbf{q}) - V(\mathbf{p})]s\} = F\{[V[(\mathbf{p}) - V(\mathbf{q})]s\}$$

And the probability of observing $d = 1$ is

$$F\{[V(\mathbf{q}) - V(\mathbf{p})]s\}$$

So the contribution to the log-likelihood of any observation is

$$d \log(F\{[V(\mathbf{q}) - V(\mathbf{p})]s\}) + (1-d) \log(F\{[V(\mathbf{p}) - V(\mathbf{q})]s\})$$

(note that the second term has the p's and q's interchanged.)

Or with the subscript n added:

$$d_n \log(F\{[V(\mathbf{q}_n) - V(\mathbf{p}_n)]s\}) + (1-d_n) \log(F\{[V(\mathbf{p}_n) - V(\mathbf{q}_n)]s\})$$

1.2 Estimation under the Wenting hypothesis

For the first observation/problem f (I am not using 1 as there were 5 repetitions in that experiment) in any session the contribution to the log-likelihood is

$$d_f \log(F\{[V(\mathbf{q}_f) - V(\mathbf{p}_f)]s\}) + (1-d_f) \log(F\{[V(\mathbf{p}_f) - V(\mathbf{q}_f)]s\})$$

For subsequent observations/problems we need some additional material.

If $d_{n-1} = 0$, that is the subject had chosen the p's in problem $(n-1)$ and the subject, in answering problem n, mixes that problem with his/her answer to problem $(n-1)$, then the choice (assuming reduction) is perceived as between

$$[(\mathbf{p}_{n-1}+\mathbf{p}_n)/2] \text{ and } [(\mathbf{p}_{n-1}+\mathbf{q}_n)/2]$$

So the contribution to the log-likelihood for this observation is

$$d_n \log(F\{[V[(\mathbf{p}_{n-1}+\mathbf{q}_n)/2]-V[(\mathbf{p}_{n-1}+\mathbf{p}_n)/2]]s\})+(1-d_n) \log(F\{[V[(\mathbf{p}_{n-1}+\mathbf{p}_n)/2]-V[(\mathbf{p}_{n-1}+\mathbf{q}_n)/2]]s\})$$

Contrariwise if $d_{n-1} = 1$ (that is the subject had chosen the q's) the contribution is

$$d_n \log(F\{[V[(\mathbf{q}_{n-1}+\mathbf{q}_n)/2]-V[(\mathbf{q}_{n-1}+\mathbf{p}_n)/2]]s\})+(1-d_n)\log(F\{[V[(\mathbf{q}_{n-1}+\mathbf{p}_n)/2]-V[(\mathbf{q}_{n-1}+\mathbf{q}_n)/2]]s\})$$

Hence the contribution of observation $n(\neq f)$ is

$$\begin{aligned}
&(1-d_{n-1})d_n \log(F\{[V[(\mathbf{p}_{n-1}+\mathbf{q}_n)/2] - V[(\mathbf{p}_{n-1}+\mathbf{p}_n)/2]]s\}) + \\
&(1-d_{n-1})(1-d_n)\log(F\{[V[(\mathbf{p}_{n-1}+\mathbf{p}_n)/2] - V[(\mathbf{p}_{n-1}+\mathbf{q}_n)/2]]s\}\}) + \\
&d_{n-1}\{d_n \log(F\{[V[(\mathbf{q}_{n-1}+\mathbf{q}_n)/2] - V[(\mathbf{q}_{n-1}+\mathbf{p}_n)/2]]s\}) + \\
&d_{n-1}(1-d_n)\log(F\{[V[(\mathbf{q}_{n-1}+\mathbf{p}_n)/2] - V[(\mathbf{q}_{n-1}+\mathbf{q}_n)/2]]s\}\})
\end{aligned}$$

1.3 Preference Functions

We normalise so that $u(x_1) = 0, u(x_2) = u, u(x_3) = v$ and $u(x_4) = 1$.
Under EU the preference function is simply

$$V(\mathbf{r}) = ur_2 + vr_3 + r_4$$

Under RDEU it is

$$V(\mathbf{r}) = uw(r_2 + r_3 + r_4) + (v-u)w(r_3 + r_4) + (1-v)w(r_4)$$

which can be written as

$$V(\mathbf{r}) = u[w(r_2 + r_3 + r_{4)} - w(r_3 + r_4)] + v[w(r_3 + r_4) - w(r_4)] + w(r_4)$$

Note that if $w(r) = r$ then this reduces to the EU form.

1.4 Utility and Weighting Functions

I propose that, with just four outcomes, we do not specify a particular functional form for the utility function and that we just estimate u and v.

For the weighting function, I propose in the first instance to use Quiggin's

$$w(r) = \frac{r^g}{[r^g + (1-r)^g]^{1/g}}$$

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