Preference Cloud Theory: Imprecise Preferences and Preference Reversals

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This paper presents a new theory, called Preference Cloud Theory, of decision-making under uncertainty. This new theory provides an explanation for empirically-observed Preference reversals. Central to the theory is the incorporation of *preference imprecision* which arises because of individuals' vague understanding of numerical probabilities. We combine this concept with the use of the Alpha model (which builds on Hurwicz's criterion) and construct a simple model which helps us to understand various anomalies discovered in the experimental economics literature that standard models cannot explain.

1. Introduction

Suppose you are asked to state your subjective value for a lottery ticket which gives 10 dollars with a probability of 0.3 and zero otherwise: Can you pin your value down to a single precise number or do you end up with a range of values? Our preference imprecision argument states that individuals most likely end up having an interval; preferences are not as precise as standard theory presumes (for evidence, see: Butler and Loomes, 1988, 2007, 2011; Dubourg *et al*, 1994, 1996; and Morrison, 1998). Such imprecision might explain observed anomalies of standard theory (Butler and Loomes 2011). Most importantly, imprecision might be the explanation of the anomalies, despite researchers' efforts, in the last four decades, focusing on precise but non-standard preferences such as loss aversion.

Suppose now you are assigned to be a buyer. You are most likely to state a value close to the lower bound of this range; the converse is true when you are assigned to be a seller. The model presented in this paper answers two key questions: what is the psychological mechanism behind the formation of this range?; how do individuals select one value from this range? It also shows how the incorporation of imprecision can provide an explanation for preference reversals.

For the first question our theory proposes that imprecision arises due to the decision-maker's vague understanding of the probabilities involved. The empirical support for this assertion comes from the psychophysics literature; see Budescu *et al* (1988). In an experiment reported by them subjects were asked to state bids for lotteries; in the lotteries the probabilities were represented numerically, graphically or verbally. The results suggested that bids and attractiveness ratings are almost identical under the different representations (See Budescu and Wallsten (1990) and Bisantz *et a*l (2005) for further evidence). Wallsten and Budescu (1995) explain that the similarity of behavior under different representation modes is due to similarities in the vague understanding of probabilities. We therefore argue that a numerical, objective, probability corresponds to a range of probabilities and subjects use this range in their calculations[1]. There is also implicit evidence from Plott and Zeiler (2005) and Isoni *et al* (2011) who find that the endowment effect is observed only for the lottery tickets, but not for ordinary market goods such as mugs and candies.

Zimmer (1984) introduced a useful insight from an evolutionary perspective: he noted that the probability concept in a numerical sense is a relatively new concept, appearing as recently as the 17th century. However, people were communicating uncertainty *via* verbal expressions long before probability was codified in mathematical terms. Zimmer further suggested that people process uncertainty in a verbal manner and make their decisions based on this processed information, not on the numerical information. We therefore assume that decision makers map any given objective probability into an interval. This implies that people end up with a *range* of expected utilities (EUs) and they do not have prior knowledge about their "true" EU from this range. For the second question, pinning down this range to a single value can be modelled as decision problem under ambiguity. We use the Alpha Model (embodying Hurwicz's criterion) to provide a valuation of the

[1] Verbal expressions include "rare", "very likely" etc. Each expression can be interpreted as a range of probabilities that may vary from individual to individual.

prospect, given as the weighted average of the worst and the best possible EU. The next section formalises these ideas.

2. Theory

In order to give the intuition of the theory we focus on two-outcome lotteries. Let S be a finite state space, with elements s_1 and s_2; let x_1 and x_2 be the corresponding consequences. We assume that $u(x_1) \geq u(x_2)$, where $u(.)$ is the individual's Neumann-Morgenstern utility function. For each state there is an objective probability; call them p and $1-p$. The crucial point of our theory is that individuals perceive these probabilities vaguely, and hence p is mapped to a range: $[p-\beta, p+\beta]$. As far as the individual is concerned, the worst situation is when probability $p-\beta$ is allocated to state s_1 (and probability $1-p+\beta$ to state s_2, and the best situation is when probability $p+\beta$ is allocated to state s_1 (and probability $1-p-\beta$ to state s_2). So the worst and best expected utilities are $(p-\beta)u(x_1)+(1-p+\beta)u(x_2)$ and $(p+\beta)u(x_1)+(1-p-\beta)u(x_2)$ respectively.

At this point PCT uses the Alpha Max-Min criterion to evaluate the lottery. This gives

$$\alpha EU = \alpha\left[(p-\beta)u(x_1)+(1-p+\beta)u(x_2)\right]+(1-p)\left[(p+\beta)u(x_1)+(1-p-\beta)u(x_2)\right] \quad \textbf{(1)}$$

One can think of such a decision-maker as a mixture of a pessimist and an optimist: α of the time he or she assumes the worst, and $1-\alpha$ of the time the best.

The parameter β we call the *imprecision level*; our theory postulates that it is a function of p (the objective probability) and ψ (the individual-specific sophistication level). A relatively unsophisticated individual would display a relatively high imprecision β. For example, stock brokers and gamblers who are expected to be more familiar with the concept of probability exhibit lower imprecision than the ordinary man.

As far as the shape of $\beta(\psi, p)$ is concerned we assume that individuals exhibit no imprecision if the probability is 0 or 1 since the events occurring with these probabilities are not probabilistic events in daily language, that is, the event either never happens or always happens. Secondly, imprecision reaches a maximum at 0.5 because it implies the event is neither likely nor unlikely, this ‘incommensurability’ makes it difficult to derive a meaning from this probability. Finally, for simplicity we assume that $\beta(\psi, p)$ is symmetric around $p = 0.5$.

3. Explaining Preference Reversals

We show how preference reversals can arise with our model. We illustrate using the CRRA utility function for $u(.)$

$$u(z) = z^a \tag{2}$$

For $a < 0$, the function is concave, implying risk aversion. For simplicity, we focus on P and \$ bets that give either a positive payoff or zero: P-bet $= (x, p; 0)$ and \$-bet $= (y, q; 0)$ with $y > x > 0$ and $1 > p > q > 0$ where p and q are the winning probabilities, and x and y are the winning prizes of the P-bet and \$-bet, respectively.

If the individual prefers the P-bet over the \$-bet in the choice task, we can write:

$$\alpha EU(P - bet) \geq \alpha EU(\$ - bet) \tag{2}$$

$$\alpha\left[(p-\beta)x^a\right] + (1-\alpha)\left[(p+\beta)x^a\right] \geq \alpha\left[(q-\beta)y^a\right] + (1-\alpha)[p+\beta]y^a \tag{3}$$

Thus the critical value for α in determining whether the P-bet preferred to the \$-bet is:

$$\alpha^* = \frac{y^a(q+\beta) - x^a(p+\beta)}{2\beta\left(y^a - x^a\right)} \tag{4}$$

If the actual α is greater than α^* the individual chooses the P-bet; if lower the \$-bet.

When we come to valuations we need to tell a different story. Let us consider willingness-to-accept. Take the P-bet. Imagine that the individual owns the P-bet and is asked the minimum amount for which he or she would sell it – the WTA. If it is sold the individual has WTA (presumably an amount less than x), and the worst thing that can ‘happen’ to the individual is that the P-bet would have paid out x, and the best thing that can ‘happen’ to the individual is that the P-bet would have paid out 0.

So the pessimist attaches weight $p+\beta$ to the possibility of getting x, while the optimist attaches weight $p-\beta$. So we get

$$u^{-1}\left(WTA_{P-bet}\right)=\alpha\left[\left(p+\beta\right)x^{a}\right]+\left(1-\alpha\right)\left[\left(p-\beta\right)x^{a}\right] \tag{5}$$

Similarly,

$$u^{-1}\left(WTA_{\$-bet}\right)=\alpha\left[\left(q+\beta\right)y^{a}\right]+\left(1-\alpha\right)\left[\left(q-\beta\right)y^{a}\right] \tag{6}$$

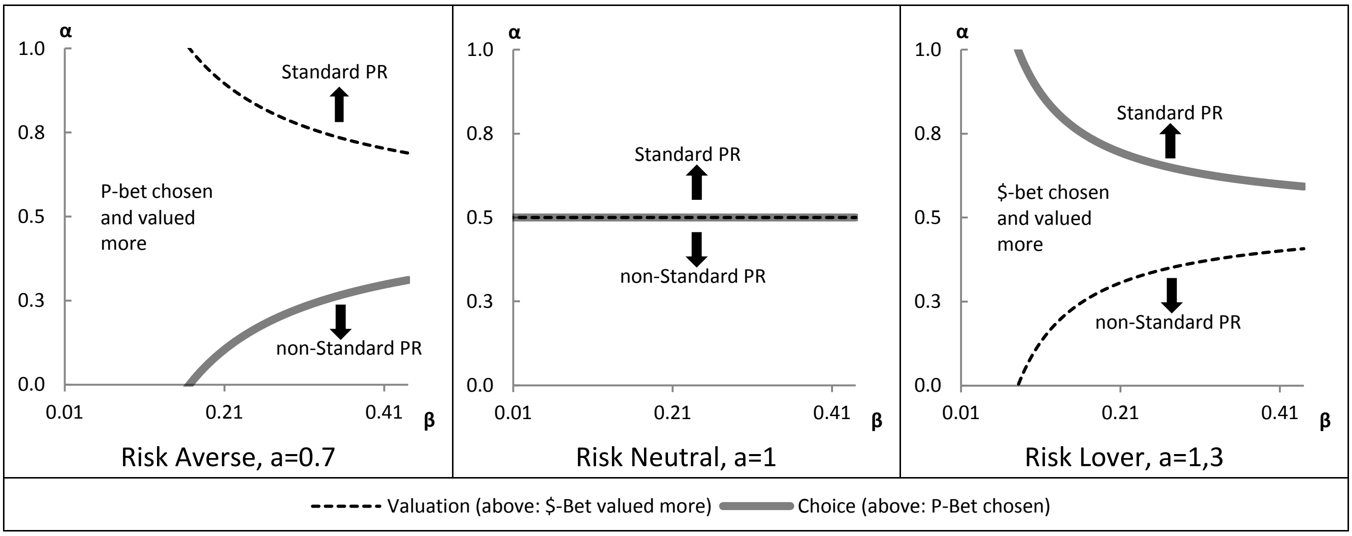
The critical value for α is:

$$\alpha^{**}=\frac{x^{a}\left(p-\beta\right)-y^{a}\left(q-\beta\right)}{2\beta\left(y^{a}-x^{a}\right)} \tag{7}$$

If α is greater α^{**} the individual values the \$-bet higher than the P-bet; if it is lower the P-bet is valued higher.

We explore the parameters of PCT in three cases: risk neutral, risk averse and a risk loving. Consider Figure 2, where r set to 1; P-bet$=(1.25,0.8;0)$, \$-bet$=(5,0.2;0)$[2]

Figure 1

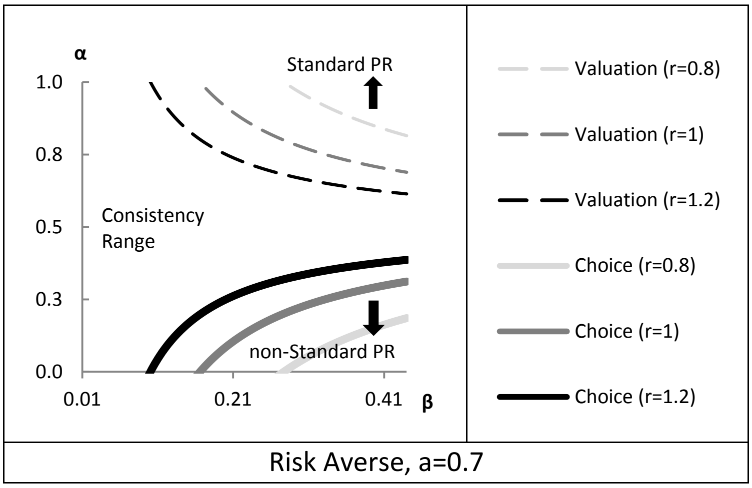


The dashed line shows the α^{**} boundary and the solid line is the α^{*} boundary. Above the dashed line the \$-bet is valued higher and above the solid gray line the P-bet is chosen; the region between

[2] For the imprecision level, we use $\beta\left(\psi,p\right)=\psi p\left(1-p\right)$, there is no particular reason behind choosing this, except it is simple and satisfies the assumptions of the theory. We normalize the expected value (EV) of the P bet by setting its payoff equal to $1/p$; r is the EV of the \$ bet as a ratio of the EV of the P-bet. Therefore the winning prize of the \$ bet equals r/p.

the two lines is called the *consistency range* where the chosen bet is valued more. For a risk-neutral individual, in the case of imprecision $(\beta > 0)$, a standard Preference reversal occurs if $\alpha > 0.5$, when it is lower than 0.5, the model predicts a non-standard Preference reversal. For a risk-averse individual, in the consistency range individual chooses the P-Bet and values it higher; whereas for the risk loving case the $-Bet is chosen and valued more. This makes sense: the P-bet would be more attractive for a risk-averse individual. Overall, in the case of imprecision $(\beta > 0)$, a sufficiently high level of pessimism results in a standard Preference Reversal while optimism implies a non-standard Preference Reversal. Next we consider the case in which the winning probabilities remain the same, but the winning prize of the $-Bet varies. Figure 3 shows the critical bounds for three cases:

Figure 2



The dashed lines show the valuation boundary, and the solid lines show the choice boundary, for three levels of r (0.8,1,1.2). For a risk-averse decision-maker, the consistency range shrinks as r increases up to a certain level. The parameter values to induce standard and non-standard preference reversals converge to the risk neutrality baseline case. However, above this critical level of r, the consistency range favors the $-bet and it expands as r increases. Even if we increase the relative attractiveness of the $-bet to extreme values, the model predicts that both standard and non-standard preference reversals can be observed.

Conclusion

We demonstrated the intuition of the Preference Cloud Theory and explained the possible preference reversals. In the same manner other anomalies of EUT can be explained such as Endowment Effect and Allais Paradox. To best of our knowledge, this is the only theory that can explain anomalies in Expected Utility theory without incorporating the loss aversion notion of the Prospect Theory.

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Appendix A

Figure 4