An Open-Economy Macro-Finance Model of International Interdependence: The OECD, US and the UK

Zhuoshi Liu and Peter Spencer*

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Department of Economics and Related Studies

The University of York, York, U.K., YO10 5DD

zl104@york.ac.uk; ps35@york.ac.uk

*Corresponding author, Peter Spencer, Department of Economics and Related Studies; ps35@york.ac.uk. We are grateful to Gerald Dwyer, Martin Ellison, John Hutton, Marcus Miller, Gucin Ozkan, Neil Rankin, Peter Smith, Dagfin Rime, Mike Wickens and participants in the Royal Economic Society 2008 Annual Conference, Warwick University and Infiniti 2008 Conference, Trinity College Dublin for helpful comments.

Abstract

This paper develops a multi-country macro-finance model to study international economic and financial linkages. This approach models the economy and financial markets jointly in a closed economy framework and uses both economic and financial data to throw light on such issues. The world economy is modelled using observable data for the US and aggregate OECD economies as well as the US Treasury bond market, with two latent variables representing underlying inflation and real interest rate factors. We find surprisingly strong evidence of open economy effects on the US, calling into question the standard closed economy macro-finance specification. We identify a common non-stationary 'world' factor driving OECD inflation and US inflation & interest rates, which also impinges upon the UK. We also find that the UK economy is influenced by the world interest rate factor and macroeconomic spillovers from the OECD and the US, helping to explain the comovement of yields in the US and UK Treasury bond markets.

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1 Introduction

As recent developments in credit and commodity markets demonstrate, the global economy is becoming increasingly integrated through trade and financial linkages. There is a long-standing concern about spillovers from the United States to smaller economies (and in particular the UK). Today the US is still the dominant global economy, accounting for 20 percent of global imports. It also has the world's deepest, most sophisticated financial markets. Event studies show that U.S. recessions usually coincide with significant reductions in global growth. Empirical studies based on panel growth regression analysis (e.g. Arora and Vamvakidis (2006)) also find evidence of large spillovers from the US to other economies.

Comovement in economic activity across countries may also reflect global common shocks, such as changes in oil prices or sharp movements in asset prices in the major financial centers. Kose, Otrok, and Whiteman (2003) show that the 'global factor' generally plays an important role in explaining business cycles for the industrial countries. A related empirical study of the G-7 countries (Kose, Otrok, and Whiteman (2005)) finds that the 'common factor' among these countries explains a large share of output fluctuations. Both of these studies are based on so-called dynamic factor models which extract the latent variables representing these global or common factors from the comovement between observable macroeconomic time series in different countries.

This paper develops a multi-country macro-finance modelling framework that studies the effects of spillovers and common shocks simultaneously. The macro-finance model was pioneered by Ang and Piazzesi (2003). As the name suggests, this allows bond yields to reflect macroeconomic variables as well as latent financial market factors. Unlike earlier Vector Auto-regression (VAR) studies, bond yields are

fitted using an arbitrage-free restriction rather than as part of the vector of variables under study. In turn, the behavior of bond yields helps inform the specification of the macroeconomy, yielding new insights into its behavior. In particular, early macro-finance studies showed that although macroeconomic variables provide a good description of the behavior of short rates they do not provide an adequate description of long term yields. This observation has prompted the use of Kalman filters in these models, reflecting changes in long run inflation expectations and other 'financial' factors. This model is thus well-suited to a global setting in which there are both observable macroeconomic and latent factors. However, macro-finance models have so far focused on the US assuming that it is a closed economy. Macro-finance models of countries such as the UK (Joyce, Lildholdt, and Sorensen (2008), Spencer (2006)) have also been modelled in this way despite their open trade and financial structures. But is this realistic?

This paper adapts the macro-finance framework in a way that allows us to address this type of question. Our initial objective was to anneals the effect of global variables on the UK's economy and bond market. The model is based on the 'small country assumption' which means that we only consider the effect of the world on the UK, assuming that the reverse effects are negligibly small. One of the advantages of the macro-finance model over other models (like dynamic factor models) lies in the fact that the estimated latent common factors are informed by both macroeconomic and financial market information. This allows global factors to affect the UK financial market and business cycle simultaneously. The model also allows us to study the effects of common factors on various economies and financial markets.

The econometric specification is based on the 'central bank model' (CBM) developed by Svensson (1999), Rudebusch (2002), Smets (1999) and others, which represents the behavior of the macroeconomy in terms of the output gap, inflation

and the short term interest rate. It consists of two sub-systems that are originally estimated independently. The first represents the world economy and is modelled using a reduced form specification with both OECD and US variables. This allows the two-way linkages between the US and the rest of the OECD to be studied. We identify a common non-stationary 'world' factor driving OECD inflation and US inflation interest rates. This is modelled using a latent variable, with another representing real interest rate movements. Although this was not the initial focus of our attention, this model reveals remarkably strong open-economy effects on the US. We then develop a version of the model that also explains yield data for the US Treasury market using the usual arbitrage-free approach.

The second sub-system is a standard closed economy model of the UK economy and Treasury bond market. This also includes two UK specific latent variables representing inflation and real interest rate trends. Finally we estimate the world and UK systems jointly, allowing for global effects on the latter. We distinguish here between common factors and spillover effects. The global inflation and real interest rate trends are called global common factors since they are latent variables that could represent influences like oil and commodity prices which affect all economies simultaneously. The one-way effects of US and OECD macro variables on the UK are labelled spillover effects.

The rest of the paper is organized as follows. Section 2 specifies the theoretical macro-finance model. Section 3 describes the data and estimation method and reports the results of the specification tests. Section 4 discusses the empirical results for the preferred model. Section 5 provides a summary of the key findings and offers our concluding remarks.

2 The model framework

As noted, the econometric specification is based on the 'central bank model' which represents the behavior of the macroeconomy in terms of the output gap, inflation and the short term interest rate. This is often specified as a simple VAR, designed to reflect the broad reduced from empirical relationships between these variables, rather than structural linkages. This model has been modified by Kozicki and Tinsley (2001), Kozicki and Tinsley (2005) and Dewachter and Lyrio (2006) add Kalman filters to allow for 'inflation asymptotes' or 'stochastic trends' that shift the equilibrium values of nominal variables like interest and inflation rates.[1] This VAR approach uses a closed economy model, developed originally for the US, but we adapt by allowing world factors, output, inflation and interest rates to affect the UK economy. This modification is based on the 'small country assumption', which means we allow the world economy to affect the UK, but assume that any reverse feedback effects are negligibly small.

2.1 The macro models

The world economy is represented by the OECD and the US, which still represents about a quarter of OECD GDP and effectively acts as the fulcrum for world real interest rates given the importance of its financial sector and the dollar[2]. Our macro system includes 8 observable macro variables: the aggregate OECD output gap g_t^{**} and inflation π_t^{**}; the US output gap g_t^*, inflation π_t^* and interest rates r_t^*; and the UK output gap g_t, inflation π_t and interest r_t rates. OECD variables represent the world economy and are denoted by (**)−superscripts, while US variables are

[1] These are also known as 'variable end-points,' and are non-stationary latent variables (i.e. integrated of order one: I(1)). They are modelled by the Kalman filter and designed to capture common trends that cause the associated nominal variables to be 'co-integrated'. This means that although these nominal varaibles are non-stationary, there is a linear relationship between them that is stationary.

[2] Although our OECD aggregate measures include the UK, the weight of the UK is small (5.55%).

denoted by (*)−superscripts and UK 'home country' variables are unsubscripted[3]. The estimation period of 1979-2007 was determined by the availability of discount bond equivalent data for the UK Treasury market (Section 3) as well as evidence of a structural break in the UK when the Thatcher government came into power (Hendry and Mizon (1998), Clements and Hendry (1996)).

We started by estimating two separate closed economy macro-only KVARs, one for the 'world'(modeling g_t^{**} , π_t^{**}, g_t^*, π_t^* and r_t^*) and one for the UK (modeling g_t, π_t and r_t). Preliminary empirical analysis (see Table 2) suggested that inflation and interest rates were all non-stationary. Remarkably, we find (Table 3) that there is a nonstationary common trend driving OECD and US inflation rates with the cointegrating vector [1,-1], meaning that the OECD and US inflation asymptotes move on a one–for-one basis. This suggested the use of a single nominal stochastic trend to represent this common nominal trend in the world model: f_t^*. This was augmented by a stationary (i.e. mean-reverting or I(0)) latent variable (z_t^*) to represent the effect of a real factor, temporarily affecting the real interest rate. Similarly, previous work on a stand alone macro-finance model for the UK (Spencer (2006)) suggested the use of two UK-specific factors: f_t is an I(1) stochastic trend representing the non-stationary trend in the nominal variables and z_t is a stationary variable representing real interest effects. Table 1 summarizes the order of integration of these variables.

Table 1: Stationarity of variables

	OECD-US variables	UK variables
Non-stationary: I(1)	$f_t^*, \pi_t^{**}, \pi_t^*, r_t^*$	f_t, π_t, r_t
Stationary: I(0)	g_t^*, g_t^{**}, z_t^*	g_t, z_t

[3] In the interests of simplicity we did not model the (average) OECD interest rate, relying instead upon the theory that in equilibrium this should be approximated by $(r_t^* + \pi_t^{**} - \pi_t^*)$.

2.2 The OECD-US 'world' macro KVAR

We use a KVAR(N^*) process to describe the joint OECD-US or 'world' macroeconomic dynamics under the real world or state density probability measure $\mathcal{P}$, where N^* is the order of the lag length. The small country assumption implies that this world (OECD-US) sub-model is *not* influenced by the UK

$$\begin{aligned} \mathbf{x}_t^* &= \boldsymbol{\kappa}^* + \boldsymbol{\phi}_z^* z_t^* + \boldsymbol{\phi}_f^* f_t^* + \sum_{j=1}^{N*} \boldsymbol{\Phi}_j^* \mathbf{x}_{t-j}^* + \mathbf{w}_t^* \\ \mathbf{w}_t^* &= \mathbf{G}^* \mathbf{D}^{x*} \boldsymbol{\epsilon}_t^{x*} \end{aligned} \tag{1}$$

where $\mathbf{x}_t^* = [g_t^{**}, g_t^*, \pi_t^{**}, \pi_t^*, r_t^*]'$ is the observed world macro vector; $\mathbf{w}_t^*$ is a 5 by 1 error vector; $\boldsymbol{\epsilon}_t^{x*}$ is an orthogonal error vector that applies to the normal distribution $N(\mathbf{0}, \mathbf{I})$[4]; $\mathbf{D}^{x*}$ is a 5 by 5 diagonal matrix with positive diagonal elements; $\mathbf{G}^*$ is a 5 $\times$ 5 lower triangular matrix with unit diagonal. The BIC test indicated that a first order difference system was appropriate in this case ($N^* = 1$)[5]. The macro vector is affected by its lagged value $\mathbf{x}_{t-1}^*$, the real factor z_t^* and the nominal factor f_t^*. The real factor z_t^* and the US specific factor f_t^* follow I(0) and I(1) processes respectively:

$$\begin{pmatrix} z_t^* \\ f_t^* \end{pmatrix} = \begin{pmatrix} \xi^* z_t^* \\ f_{t-1}^* \end{pmatrix} + \begin{pmatrix} v_t^* \\ u_t^* \end{pmatrix}; \quad where: \quad \begin{pmatrix} v_t^* \\ u_t^* \end{pmatrix} = \begin{pmatrix} \delta^{z*} & 0 \\ 0 & \delta^{f*} \end{pmatrix} \begin{pmatrix} \epsilon_t^{z*} \\ \epsilon_t^{f*} \end{pmatrix} \tag{2}$$

ϵ_t^{z*} and ϵ_t^{f*} are independent standard normal errors and ξ^* represents a mean reversion parameter that is less than unity in absolute value.

[4] We use bold font $\mathbf{0}_{i,j}$ to denote an i by j zero matrix; bold font $\mathbf{0}$ to denote a zero matrix with appropriate dimension; bold font $\mathbf{I}_i$ to denote an i by i identity matrix.; bold font $\mathbf{I}$ to denote an identity matrix with appropriate dimension.

[5] BIC tests are carried out using OLS results.

BIC Tests			
Lags	1	2	3
The OECD US macro VAR	-61.1803	-60.9239	-60.4136
The UK macro VAR	-37.0716	-37.1694	-37.0849

As noted, the preliminary test results (Table 3) suggested that OECD inflation was cointegrated with the US inflation with the restricted cointegrating vector [1, -1]. The macro-finance literature draws the distinction between the long run expectation or 'asymptote' $\bar{r}_t^*$ of a variable like r_t^* and its conditional expectation $\tilde{r}_t^*$ (or temporary equilibrium) given a particular value of a stationary factor like z_t^*, which is known as the 'central tendency'. Thus the US inflation asymptote is simply $\bar{\pi}_t^* = E_t\left(\lim_{m\to\infty} \pi_{t+m}^*\right) = f_t^* + \varphi_\pi^*$ and the OECD inflation asymptote is $\bar{\pi}_t^{**} = E_t\left(\lim_{m\to\infty} \pi_{t+m}^{**}\right) = f_t^* + \varphi_\pi^{**}$ where φ_π^* and φ_π^{**} are constants (to be estimated). The central tendencies of the US and OECD inflation rates are equal to their asymptotes since they just depend on f_t^* which has a unit root process: $\tilde{\pi}_t^* = \bar{\pi}_t^*$, and $\tilde{\pi}_t^{**} = \bar{\pi}_t^{**}$. However, the central tendency of the US *real* interest rate ρ_t^* depends upon the stationary factor z_t^*, which reverts to an equilibrium value of zero. Thus: $\tilde{\rho}_t^* = z_t^* + \varphi_\rho^*$ but $\bar{\rho}_t^* = E_t\left(\lim_{m\to\infty} \rho_{t+m}^*\right) = \varphi_\rho^*$, where φ_ρ^* is another intercept term. Accordingly, the US *nominal* interest rate asymptote equals $\tilde{r}_t^* = \tilde{\pi}_t + \tilde{\rho}_t^* = z_t^* + f_t^* + \varphi_\pi^* + \varphi_\rho^*$. The OECD and US output gaps are mean-reverting variables with zero asymptotes/central tendencies: $\bar{g}_t^{**} = \tilde{g}_t^{**} = \bar{g}_t^* = \tilde{g}_t^* = 0$. Therefore the central tendency of the macro vector $\tilde{x}_t^*$ is:

$$\tilde{x}_t^* = (\tilde{g}_t^{**}, \tilde{g}_t^*, \tilde{\pi}_t^{**}, \tilde{\pi}_t^*, \tilde{r}_t^*)' = \mathbf{R}^* \cdot \begin{pmatrix} z_t^* \\ f_t^* \end{pmatrix} + \boldsymbol{\varphi}^*, \tag{3}$$

$$\text{where : } \boldsymbol{\varphi}^* = \begin{pmatrix} 0 \\ 0 \\ \varphi_\pi^{**} \\ \varphi_\pi^* \\ \varphi_\pi^* + \varphi_\rho^* \end{pmatrix}, \mathbf{R}^* = \begin{pmatrix} 0\,0 \\ 0\,0 \\ 0\,1 \\ 0\,1 \\ 1\,1 \end{pmatrix}.$$

The macro model (1) also implies the following central tendency:

$$\tilde{x}_t^* = (\mathbf{I} - \mathbf{\Phi}_1^*)^{-1} \left(\boldsymbol{\kappa}^* + \boldsymbol{\phi}_z^* z_t^* + \boldsymbol{\phi}_f^* f_t^* \right) \tag{4}$$

A set of restrictions are imposed on $\boldsymbol{\kappa}^*$, $\boldsymbol{\phi}_z^*$, $\boldsymbol{\phi}_f^*$ to ensure the equivalence of (3) and (4):

$$\boldsymbol{\kappa}^* = (\mathbf{I} - \mathbf{\Phi}_1^*)\, \boldsymbol{\varphi}^*$$

$$\boldsymbol{\phi}_f^* = (\mathbf{I} - \mathbf{\Phi}_1^*)\, \mathbf{R}^{2*}$$

$$\boldsymbol{\phi}_z^* = (\mathbf{I} - \mathbf{\Phi}_1^*)\, \mathbf{R}^{1*}$$

where $\mathbf{R}^{1*}$ is the first column and $\mathbf{R}^{2*}$ the second column of[6] $\mathbf{R}^*$. We stack (1) and (2) and write the model in the the companion or state space form:

$$\mathbf{X}_t^* = \mathbf{K}^* + \mathbf{\Phi}^* \mathbf{X}_{t-1}^* + \mathbf{W}_t^* \tag{5}$$

$$\mathbf{W}_t^* = \mathbf{C}^* \mathbf{D}^* \boldsymbol{\epsilon}_t^*, \mathbf{W}_t^* \sim N(\mathbf{0}, \mathbf{\Omega}^*)$$

$$\text{where : } \mathbf{\Omega}^* = \mathbf{C}^* \mathbf{D}^* \mathbf{D}^{*\prime} \mathbf{C}^{*\prime}$$

where $\mathbf{X}_t^* = (z_t^*, f_t^*, \mathbf{x}_t^{*\prime})'$; and $\mathbf{K}^*$, $\mathbf{\Phi}^*$, $\boldsymbol{\epsilon}_t^*$, $\mathbf{C}^*$, $\mathbf{D}^*$ are defined in Appendix A.1.

The macro model is estimated under the measure $\mathcal{P}$ showing the state density (i.e. the actual probability of any state). Term structure models are usually developed under a risk-neutral probability measure such as $\mathcal{Q}^*$, which has the effect of adjusting these probabilities so that all assets have the same expected dollar rate of return. This adjustment is made by changing the parameters of (5) to get a congruent first-order

[6] Assuming that the two latent variables have unit effects on US inflation and real interest rates respectively determines their scale and does not restrict the model in the sense that it can reduce the likelihood. The other 8 entries in $\mathbf{R}^*$ are restrictions in this sense but were found acceptable on the basis of the standard χ^2 likelihood ratio test.

process:

$$\mathbf{X}_t^* = \mathbf{K}^{*Q^*} + \mathbf{\Phi}^{*Q^*}\mathbf{X}_{t-1}^* + \mathbf{W}_t^{*Q^*} \tag{6}$$

$$\mathbf{W}_t^{*Q^*} \sim N(\mathbf{0}, \mathbf{\Omega}^*).$$

where:

$$\mathbf{K}^{*Q*} = \mathbf{K}^* - \mathbf{C}^*\mathbf{D}^*\mathbf{D}^{*\prime}\mathbf{\Lambda}_1^{**} \tag{7a}$$

$$\mathbf{\Phi}^{*Q*} = \mathbf{\Phi}^* - \mathbf{C}^*\mathbf{\Lambda}_2^*. \tag{7b}$$

The parameters $\mathbf{\Lambda}_1^{**}$, and $\mathbf{\Lambda}_2^*$ are specified in Appendix A.2.

2.3 The UK macro KVAR

Under $\mathcal{P}$, the general form of the UK macro model is given by the VAR(N) process:

$$\mathbf{x}_t = \boldsymbol{\kappa} + \boldsymbol{\phi}_f f_t + \boldsymbol{\phi}_z z_t + \boldsymbol{\theta}\mathbf{X}_t^* + \sum_{j=1}^{N} \mathbf{\Phi}_j \mathbf{x}_{t-j} + \mathbf{w}_t \tag{8}$$

$$\text{where : } \mathbf{w}_t = \mathbf{G}\mathbf{D}^x\boldsymbol{\epsilon}_t^x, \boldsymbol{\epsilon}_t^x \sim N(\mathbf{0}, \mathbf{I})$$

where $\mathbf{x}_t = [g_t, \pi_t, r_t]'$ is the observed UK macro vector. The BIC test (see previous footnotes) indicated that a second order difference system was necessary in this case ($N = 2$). The 3×7 matrix $\boldsymbol{\theta}$ parameterizes the various spillover effects from the state vector $\mathbf{X}_t^*$ of the world sub-model to the UK economy. This is specified in section 2.5 below. The error term $\mathbf{w}_t$ is a 3×1 vector; $\mathbf{D}^x$ is a diagonal matrix with positive diagonal elements; $\mathbf{G}$ is a lower triangular matrix with unit diagonal. The model allows $\mathbf{x}_t$ to be affected by its lagged values, the real factor z_t, the nominal

factor f_t, and the US-OECD state vector $\mathbf{X}_t^*$. The factor dynamics resemble (2):

$$\begin{pmatrix} z_t \\ f_t \end{pmatrix} = \begin{pmatrix} \xi z_t \\ f_{t-1} \end{pmatrix} + \begin{pmatrix} v_t \\ u_t \end{pmatrix} \text{ where : } \begin{pmatrix} v_t \\ u_t \end{pmatrix} = \begin{pmatrix} \delta^z & 0 \\ 0 & \delta^f \end{pmatrix} \begin{pmatrix} \epsilon_t^z \\ \epsilon_t^f \end{pmatrix} \tag{9}$$

and where ϵ_t^z and ϵ_t^f are independent standard normal errors and ξ represents a mean reversion parameter that is less than unity in absolute value.

In the closed economy version of the UK model (i.e. model M0, where $\boldsymbol{\theta} = \mathbf{0}$), the central tendency of $\mathbf{x}_t$ is driven by the factors z_t and f_t. The UK inflation asymptote (and tendency) $\bar{\pi}_t$ is: $\bar{\pi}_t = E_t\left(\lim_{m\to\infty} \pi_{t+m}\right) = f_t + \varphi_\pi$ where φ_π is to be estimated. The central tendency of the inflation rate $\tilde{\pi}_t$ equals its asymptote $\bar{\pi}_t$. The central tendency of the nominal interest rate asymptote is $\tilde{r}_t = z_t + f_t + \varphi_\pi + \varphi_\rho$. The UK output gap is another mean reverting variable with zero asymptote and central tendency: $\bar{g}_t = \tilde{g}_t = 0$. Thus the central tendency of the UK macro vector is:

$$\tilde{x}_t = (\tilde{g}_t, \tilde{\pi}_t, \tilde{r}_t)' = \mathbf{R} \begin{pmatrix} z_t \\ f_t \end{pmatrix} + \boldsymbol{\varphi} \tag{10}$$

where:

$$\boldsymbol{\varphi} = \begin{pmatrix} 0 \\ \varphi_\pi \\ \varphi_\pi + \varphi_\rho \end{pmatrix}, \mathbf{R} = \begin{pmatrix} 0\,0 \\ 0\,1 \\ 1\,1 \end{pmatrix},$$

The M0 UK macro model (8) has the central tendency:

$$\tilde{x}_t = \left(\mathbf{I} - \sum_{j=1}^{N} \boldsymbol{\Phi}_j\right)^{-1} \left(\boldsymbol{\kappa} + \boldsymbol{\phi}_z z_t + \boldsymbol{\phi}_f f_t\right) \tag{11}$$

Equations (10) and (11) give the restrictions:

$$\boldsymbol{\phi}_z = \left(\mathbf{I} - \sum_{j=1}^{N} \boldsymbol{\Phi}_j\right) \mathbf{R}^1, \boldsymbol{\phi}_f = \left(\mathbf{I} - \sum_{j=1}^{N} \boldsymbol{\Phi}_j\right) \mathbf{R}^2, \boldsymbol{\kappa} = \left(\mathbf{I} - \sum_{j=1}^{N} \boldsymbol{\Phi}_j\right) \boldsymbol{\varphi}$$

where $\mathbf{R}^1$ is the first column and $\mathbf{R}^2$ the second column of $\mathbf{R}$. The M1 model (where $\boldsymbol{\theta} \neq \mathbf{0}$) allows for global influences and in this case the inflation and interest rate asymptotes are affected by f_t^*. The scale of this latent variable is however fixed by the $\mathbf{R}^*$ restrictions of the world model and its effect on UK inflation and interest rate asymptotes is not restricted. As explained in section 2.5, we apply restrictions similar to these on $\mathbf{R}$ to ensure that it has the same asymptotic effect on interest and inflation rates with no asymptotic effect on the output gap. (Also see the impulse responses reported in the next section).

With $N = 2$, the companion form of the UK macro model is:

$$\begin{aligned} \mathbf{X}_t &= \mathbf{K} + \boldsymbol{\Theta}\mathbf{X}_{t-1}^* + \boldsymbol{\Phi}\mathbf{X}_{t-1} + \mathbf{W}_t \\ \mathbf{W}_t &= \mathbf{A}\mathbf{D}^*\boldsymbol{\epsilon}_t^* + \mathbf{C}\mathbf{D}\boldsymbol{\epsilon}_t, \mathbf{W}_t \sim N(\mathbf{0}, \boldsymbol{\Omega}) \\ \boldsymbol{\Omega} &= \mathbf{A}\mathbf{D}^*\mathbf{D}^{*\prime}\mathbf{A}' + \mathbf{C}\mathbf{D}\mathbf{D}'\mathbf{C}' \end{aligned} \tag{12}$$

where $\mathbf{X}_t = \left(z_t, f_t, \mathbf{x}_t', \mathbf{x}_{t-1}'\right)'$ and $\mathbf{K}, \boldsymbol{\Theta}, \boldsymbol{\Phi}, A, \mathbf{D}^*, \boldsymbol{\epsilon}_t^*, \mathbf{C}, \mathbf{D}, \boldsymbol{\epsilon}_t$ are defined in Appendix A.1. Similarly the dynamics of $\mathbf{X}_t^*$ and $\mathbf{X}_t$ under the sterling risk-neutral measure $\mathcal{Q}$ (which equates sterling expected returns) are given by:

$$\begin{aligned} \mathbf{X}_t^* &= \mathbf{K}^{*Q} + \boldsymbol{\Phi}^{*Q}\mathbf{X}_{t-1}^* + \mathbf{W}_t^{*Q} \\ \mathbf{W}_t^{*Q} &\sim N(\mathbf{0}, \boldsymbol{\Omega}^*). \end{aligned} \tag{13}$$

$$\mathbf{X}_t = \mathbf{K}^Q + \mathbf{\Theta}^Q \mathbf{X}^*_{t-1} + \mathbf{\Phi}^Q \mathbf{X}_{t-1} + \mathbf{W}^Q_t \tag{14}$$

$$\mathbf{W}^Q_t \sim N(\mathbf{0}, \mathbf{\Omega})$$

where:

$$\mathbf{K}^{*Q} = \mathbf{K}^* - \mathbf{C}^*\mathbf{D}^*\mathbf{D}^{*\prime}\mathbf{\Lambda}^*_1; \mathbf{K}^Q = \mathbf{K} - \mathbf{A}\mathbf{D}^*\mathbf{D}^{*\prime}\mathbf{\Lambda}^*_1 - \mathbf{C}\mathbf{D}\mathbf{D}'\mathbf{\Lambda}_1; \tag{15a}$$

$$\mathbf{\Phi}^{*Q} = \mathbf{\Phi}^* - \mathbf{C}^*\mathbf{\Lambda}^*_2; \mathbf{\Phi}^Q = \mathbf{\Phi} - \mathbf{C}\mathbf{\Lambda}_2; \mathbf{\Theta}^Q = \mathbf{\Theta} - \mathbf{A}\mathbf{\Lambda}^*_2. \tag{15b}$$

The parameters $\mathbf{\Lambda}^*_1$, $\mathbf{\Lambda}_1$, $\mathbf{\Lambda}^*_2$ and $\mathbf{\Lambda}_2$ are specified in Appendix A.2.

2.4 Bond price models

Appendix A.3 uses the standard approach to show that under these Gaussian assumptions US dollar and UK sterling discount (or zero coupon) bond prices are log-linear in the state variables. In the first case, the price $P^*_{\tau,t}$ at time t of a dollar payment at time $t+\tau$ can be represented as:

$$-\ln P^*_{\tau,t} = \eta^*_\tau + \mathbf{\Psi}^*_\tau \mathbf{X}^*_t \tag{16}$$

The coefficients η^*_τ, $\mathbf{\Psi}^*_\tau$ are defined by the recursion:

$$\eta^*_\tau = \eta^*_{\tau-1} + \mathbf{\Psi}^*_{\tau-1}\mathbf{K}^{*Q*} - \frac{1}{2}\mathbf{\Psi}^*_{\tau-1}\mathbf{\Omega}^*\mathbf{\Psi}^{*\prime}_{\tau-1} \tag{17}$$

$$\mathbf{\Psi}^*_\tau = \mathbf{J}^*_r + \mathbf{\Psi}^*_{\tau-1}\mathbf{\Phi}^{*Q*} = \mathbf{J}^*_r \left[\mathbf{I} - \left(\mathbf{\Phi}^{*Q*}\right)^\tau\right]\left(\mathbf{I} - \mathbf{\Phi}^{*Q*}\right)^{-1}$$

with initial conditions:

$$\eta^*_1 = 0, \mathbf{\Psi}^*_1 = \mathbf{J}^*_r$$

where $\mathbf{J}^*_r$ is a selection vector (with zero-one elements) that extracts r^*_t from $\mathbf{X}^*_t$:

$r_t^* = \mathbf{J}_r^* \mathbf{X}_t^*$.

The UK bond price is affected by both the US state vector $\mathbf{X}_t^*$ and the UK state vector $\mathbf{X}_t$ due to the spillover effects:

$$-\ln P_{\tau,t} = \eta_\tau + \mathbf{\Psi}_\tau \mathbf{X}_t^* + \mathbf{\Upsilon}_\tau \mathbf{X}_t \tag{18}$$

where $\eta_\tau, \Upsilon_\tau, \mathbf{\Psi}_\tau$ are defined by the recursion relationships:

$$\begin{aligned}
\eta_\tau &= \eta_{\tau-1} + \mathbf{\Upsilon}_{\tau-1}\mathbf{K}^Q + \mathbf{\Psi}_{\tau-1}\mathbf{K}^{*Q} - \mathbf{\Upsilon}_{\tau-1}\mathbf{A}\mathbf{D}^*\mathbf{D}^{*\prime}\mathbf{C}\mathbf{\Psi}_{\tau-1}{}' \\
&\quad -\frac{1}{2}\mathbf{\Psi}_{\tau-1}\mathbf{\Omega}^*\mathbf{\Psi}'_{\tau-1} - \frac{1}{2}\mathbf{\Upsilon}_{\tau-1}\mathbf{\Omega}\mathbf{\Upsilon}'_{\tau-1} \\
\mathbf{\Psi}_\tau &= \mathbf{\Psi}_{\tau-1}\mathbf{\Phi}^{*Q} + \mathbf{\Upsilon}_{\tau-1}\mathbf{\Theta}^Q \\
\mathbf{\Upsilon}_\tau &= \mathbf{J}_r + \mathbf{\Upsilon}_{\tau-1}\mathbf{\Phi}^Q
\end{aligned} \tag{19}$$

with:

$$\eta_1 = 0, \mathbf{\Psi}_1 = \mathbf{0}, \mathbf{\Upsilon}_1 = \mathbf{J}_r.$$

where $\mathbf{J}_r$ is a selection vector that extracts r_t from $\mathbf{X}_t : r_t = \mathbf{J}_r \mathbf{X}_t$.

The log bond models are obtained by substituting (7a), (7b), (15a), (15b) and the recursive solutions (17) and (19) into (16) & (18). Dividing by maturity and adding an error term gives the two empirical yield models:

$$y_{\tau,t}^* = -\ln P_{\tau,t}^*/\tau = \alpha_\tau^* + \mathbf{B}_\tau^* \mathbf{X}_t^* + e_{\tau,t}^* \tag{20}$$

$$y_{\tau,t} = -\ln P_{\tau,t}/\tau = \alpha_\tau + \mathbf{B}_\tau \mathbf{X}_t^* + \mathbf{\Gamma}_\tau \mathbf{X}_t + e_{\tau,t} \tag{21}$$

where :

$$\alpha_\tau^* = \eta_\tau^*/\tau, \mathbf{B}_\tau^* = \mathbf{\Psi}_\tau^*/\tau, \alpha_\tau = \eta_\tau/\tau, \mathbf{B}_\tau = \mathbf{\Psi}_\tau/\tau, \mathbf{\Gamma}_\tau = \mathbf{\Upsilon}_\tau/\tau$$

$$e_{\tau,t}^* \sim N(0, \sigma_\tau^{*2}), e_{\tau,t} \sim N(0, \sigma_\tau^2); \sigma_\tau^* > 0, \sigma_\tau > 0$$

where $y^*_{\tau,t}$ and $y_{\tau,t}$ are the yields of the τ-period US and UK discount bonds respectively at time t; $e^*_{\tau,t}$ and $e_{\tau,t}$ are measurement errors. The slope coefficients $\mathbf{B}^*_\tau, \mathbf{B}_\tau$ and $\mathbf{\Gamma}_\tau$ are known as the 'factor loadings' in this literature.

Stacking the two yield models for M different maturities:

$$\mathbf{y}^*_t = \boldsymbol{\alpha}^* + \mathbf{B}^*\mathbf{X}^*_t + \mathbf{e}^*_t, \mathbf{e}^*_t \sim N(\mathbf{0}, \mathbf{\Sigma}^*) \tag{22}$$

$$\mathbf{y}_t = \boldsymbol{\alpha} + \mathbf{B}\mathbf{X}^*_t + \mathbf{\Gamma}\mathbf{X}_t + \mathbf{e}_t, \mathbf{e}_t \sim N(\mathbf{0}, \mathbf{\Sigma}) \tag{23}$$

where $\mathbf{y}^*_t = [y^*_{m_1,t}, y^*_{m_2,t} ... y^*_{m_M,t}]'$; $\mathbf{y}_t = [y_{m_1,t}, y_{m_2,t} ... y_{m_M,t}]'$. We use 1, 2, 3, 5, 7, 10 and 15 year maturity rates for both US and UK bonds, so that we have $\{m_1, m_2, m_3, ..., m_M\} = \{4, 8, 12, 20, ..., 60\}$ measure in calendar quarters. $\boldsymbol{\alpha}^*$, $\mathbf{B}^*$, $\mathbf{e}^*_t$, $\mathbf{\Sigma}^*$, $\boldsymbol{\alpha}$, $\mathbf{B}$, $\mathbf{\Gamma}$, $\mathbf{e}_t$, $\mathbf{\Sigma}$ are defined as

$$\boldsymbol{\alpha}^* = [\alpha^*_4, \alpha^*_8, ...\alpha^*_{60}]', \boldsymbol{\alpha} = [\alpha_4, \alpha_8, ...\alpha_{60}]'$$

$$\mathbf{B}^* = [\mathbf{B}^{*\prime}_4, \mathbf{B}^{*\prime}_8, ...\mathbf{B}^{*\prime}_{60}]', \mathbf{B} = [\mathbf{B}'_4, \mathbf{B}'_8, ...\mathbf{B}'_{60}]'$$

$$\mathbf{\Gamma} = [\mathbf{\Gamma}'_4, \mathbf{\Gamma}'_8, ...\mathbf{\Gamma}'_{60}]'$$

$$\mathbf{e}^*_t = diag\left(e^*_{4,t}, e^*_{8,t}, ..., e^*_{60,t}\right), \mathbf{e}_t = diag\left(e_{4,t}, e_{8,t}, ..., e_{60,t}\right)$$

$$\mathbf{\Sigma}^* = diag\left(\sigma^{*2}_4, \sigma^{*2}_8, ..., \sigma^{*2}_{60}\right), \mathbf{\Sigma} = diag\left(\sigma^2_4, \sigma^2_8, ..., \sigma^2_{60}\right)$$

2.5 Modeling global effects on the UK

One of the most important ways that macroeconomic impulses can be transmitted between different countries is through their bilateral exchange rates. However, it can be shown that if the terminal (or equilibrium) value of the real exchange rate is constant, the arbitrage-free complete-market assumptions of the macro-finance approach imply that exchange rates can be expressed as log-linear functions of the

state variables of the model. So in the specific case of the UK, exchange rate variables are redundant because their effects should be picked up by domestic and overseas variables already included in the model, in the same way that the effects of long yields on the economy are picked up by model variables in a single country complete market setting. In this framework, the exchange rate should not exert any additional effect on a country like the UK. This is the basic approach used in this paper.

The main objective of this study is to investigate various global effects on the UK economy. However we started with a conventional macro-finance model in which the world economy did not affect the UK. This was estimated as two separate world and UK models. In each case we started with a macro-only model, adding nominal and real factors, which gave the dynamics under $\mathcal{P}$. The yield data and associated prices of risk were then added to each model. This gave the baseline model M0 with $\boldsymbol{\theta} = \mathbf{0}_{3,7}$, to which we then added external effects. Recall that there are 7 variables in $\mathbf{X}_t^*$ ($z_t^*, f_t^*,\ g_t^{**}, g_t^*, \pi_t^{**}, \pi_t^*$ and r_t^*). Since the first two of these are latent variables, we call them 'common factors' in line with the literature cited in the introduction. Since the last five are observable variables we describe their influence as 'spillover' effects. In principle, all of these variables could affect the UK. This gives model M3 in which $\boldsymbol{\theta}$ in (8) is an unrestricted 3×7 coefficient matrix.

These external effects were investigated using the specification.

$$\boldsymbol{\theta} = (\mathbf{I} - \boldsymbol{\Phi}_1 - \boldsymbol{\Phi}_2)\, \boldsymbol{\Xi}$$

This allows for 7 different types of spillover, which affect the UK dynamics through the parameters of the 3×7 matrix $\boldsymbol{\Xi}$. This matrix shows the long run effects, which can be restricted (as in $\mathbf{R}$). The 3×7 matrix $(\mathbf{I} - \boldsymbol{\Phi}_1 - \boldsymbol{\Phi}_2)\, \boldsymbol{\Xi}$ shows the short run impacts. In model M3 these are all unrestricted. However in model M2 the long run

effects of the nominal variables (f_t^*, π_t^{**}, π_t^* and r_t^*) are constrained in a way that ensures that the long run effect of the world inflation factor (and the other nominal variables which are anchored to this) only affects the UK's inflation rate, leaving its output gap and real interest rate unchanged. First, the UK output gap is isolated from nominal external varaibles using the restrictions: $\chi_{12} = \chi_{15} = \chi_{16} = \chi_{17} = 0$, where χ_{ij} is the element in the i-th row and j-th column of the matrix $\mathbf{\Xi}$. Second, a rise in the real world interest rate (a rise in r_t^*, with other world variables held constant) only affects UK nominal (and real) interest rates ($\chi_{27} = 0$). Third, a rise in all nominal world variables has the same long-run effect on UK inflation and nominal interest rates, leaving real rates unchanged. This combined effect is given by the parameter ν_{22} and the restriction is enforced by defining $\chi_{22} = \nu_{22} - \chi_{25} - \chi_{26}$ and $\chi_{23} = \nu_{22} - \chi_{35} - \chi_{36} - \chi_{37}$. This gives model M2, with the identification and parameter matrices:

$$\mathbf{\Xi} = \begin{pmatrix} \chi_{11} & 0 & \chi_{13} & \chi_{14} & 0 & 0 & 0 \\ \chi_{21} & (\nu_{22} - \chi_{25} - \chi_{26}) & \chi_{23} & \chi_{24} & \chi_{25} & \chi_{26} & 0 \\ \chi_{31} & (\nu_{22} - \chi_{35} - \chi_{36} - \chi_{37}) & \chi_{33} & \chi_{34} & \chi_{35} & \chi_{36} & \chi_{37} \end{pmatrix}. \tag{24}$$

These constraints on $\mathbf{\Xi}$ ensure that the side-effects of the I(1) variables (f_t^*, π_t^{**}, π_t^* and r_t^*) preserve the zero asymptote for g_t and have identical effects on π_t and r_t. Thus model M2 has 15 more parameters than M0, but 6 fewer then M3.

3 Data and estimation

3.1 Data

These models were estimated and tested using quarterly time series of the macro variables and yields from 1979 Q1 to 2007 Q1. The three output gaps (g_t^{**}, g_t^*, g_t) are measured as the difference between actual and estimated potential GDP, in volume

terms and as a % of potential GDP and are obtained from the OECD website, vintage December 2007. Inflation rates are measured as the annual logarithmic change in the consumer price index. The OECD inflation rate π_t^{**} is calculated using the aggregate OECD consumer price index (excluding high inflation countries). OECD aggregate measures are weighted averages of individual OECD countries where the weight for the UK is 5.55%, and 35.94% for the US. The US inflation rate π_t^* is derived from the all items CPI (Source: US Bureau of Labor Statistics). The UK inflation rate π_t^* is derived from the RPIX, which excludes mortgage interest payments (Source: Office for National Statistics). The US short interest rate r_t^* is the 3-month Treasury Bill rate and similarly r_t is the 3-month UK Treasury Bill rate. The US and UK macroeconomic data are shown in Figures 1a and 1b. The US yield data are continuous compounded zero coupon rates taken from McCulloch and Kwon (1991), updated by the New York Federal Reserve Bank. The UK yield data are continuous compounded zero coupon rates taken from the Bank of England website and as noted are only available from 1979. These yield data are shown in Figures 2a and 2b. Data summary statistics of both macro and financial data are given in table 2.

3.2 Estimation

We estimate the companion form of the world and UK macro KVARs ((5) and (12)), the US and UK yield models ((22), (23)) simultaneously using the Kalman Filter and Maximum Loglikelihood Estimation method (appendix A.5). The loglikelihood is optimized numerically using the Nelder-Mead Simplex algorithm to solve the optimization problem (implemented in the MatLab *fminsearch* function).

The baseline closed economy model M0 has 146 parameters[7]. The M2 and M3

[7] These are: (ξ^*, ξ, δ^{z*}, δ^{f*}, δ^z, δ^f, $\boldsymbol{\varphi}(2)$, $\boldsymbol{\varphi}^*(3)$, $\boldsymbol{\Phi}_1^*(25)$, $\boldsymbol{\Phi}_1(9)$, $\boldsymbol{\Phi}_2(9)$, $\boldsymbol{\Xi}(0)$, $\mathbf{G}^*(10)$, $\mathbf{G}(3)$, $\mathbf{D}^{x*}(5)$, $\mathbf{D}^x(3)$, λ_1^{z**}, λ_1^{f**}, $\boldsymbol{\lambda}_1^{x**}(5)$, λ_1^{z*}, λ_1^{f*}, $\boldsymbol{\lambda}_1^{x*}(5)$, λ_1^z, λ_1^f, $\boldsymbol{\lambda}_1^x(3)$, λ_2^{z*}, λ_2^{f*}, λ_2^z, λ_2^f, $\boldsymbol{\Lambda}_2^{x*}(25)$, $\boldsymbol{\Lambda}_2^x(9)$, $\boldsymbol{\Sigma}^*(7)$, $\boldsymbol{\Sigma}(7)$).

open economy models have 161 and 167 parameters respectively. Table 4 reports the optimized loglikelihood of each model. The LR tests in this table provide the basic result of this paper, showing that the closed economy model M0 is strongly rejected in favour of the open economy models. This table shows the result of a LR test of M2 against M3, showing that the restrictions embodied in (24) are accepted at the conventional 95% level. Further work on this model showed that many of the price of risk parameters were poorly determined. Sequential elimination of risk parameters with a $t-$value of less than unity gave the model reported as M1.

4 Empirical results

In view of these results we now focus on the estimates obtained from the M1 model. Estimates for the other models are available upon request from the authors. Table 5a, 5b and 5c report the parameter estimates and t-values. The fitted values of the macro and yield variables are given in Figures 1a, 1b, 2a, and 2b. The estimated common and country specific factors are shown in Figure 3. In addition, Figure 4 shows the central tendencies of the US and UK inflation, real and nominal interest rates.

The macro models are best understood in terms of their impulse responses, which show the dynamic effects of innovations in the macroeconomic variables on the system. Because these innovations are correlated empirically, we work with orthogonalized innovations (i.e. $\boldsymbol{\epsilon}^*_{t+1}$ and $\boldsymbol{\epsilon}_{t+1}$) using the triangular factorization defined in (5) and (12). The orthogonalized impulse responses show the effect on the macroeconomic system of increasing each of these shocks by one unit for just one period by using the Wold representation of the system as described in Hamilton (1994). The OECD-US responses are shown in Figure 5 and Figure 7a & 7b are those for the UK. We also use Analysis of Variance (ANOVA) techniques to show the contri-

bution of different shocks to the volatility of each macro variable. Figures 10a and 10b show the share of total variance attributable to the orthogonal innovations at different lags lengths using the Wold representation of the system. They indicate the contribution each innovation would make to the volatility of each model variable if the error process was suddenly started (having been dormant previously).

The yield models are described in terms of their factor loadings, impulse responses, ANOVA results and risk premia. The factor loadings reveal the effect of the state variables on the yield at each maturity. Figures 9a and 9b show the loadings of the US and UK yield models as functions of maturity, measured in calendar quarters. Combining these with the macro impulse responses gives the responses of the yield curves to various macro shocks: 6 for the US and 8a and 8b for the UK. Figures 11a and 11b show the results of ANOVA variance decomposition for these yield curves. Figure 12 shows the respective risk premia. We now ask what these results tell us about the respective economies and bond markets, and the relationships between them using our preferred model: M1.

4.1 The US and the world economy

The novelty here is the use of a single KVAR to model the world economy. This specification allows for the cross-country effects between the US and other OECD countries by introducing OECD variables into a standard closed economy US KVAR. The finding of a common nominal trend and the first order lag structure keeps this model compact. Figure 3 reveals a sharp downward movement in the nominal factor as oil prices fell in 1986 followed by a gradual downtrend, which this model identifies as a common global trend. The factor z^* representing the central tendency of the world real interest clearly coincides with movements in US monetary policy, increasing sharply during the Volker deflation of 1979-80 and falling sharply under

the Greenspan stimuli of 1991-2 and 2001-2.

The effect of US macroeconomic shocks on the US economy revealed by Figure 5 are in line with previous KVAR results such as Dewachter and Lyrio (2006). As in the model of Kozicki and Tinsley (2005), the use of Kalman filters to pick up the effect of unobservable expectational influences helps to solve the notorious price puzzle - the tendency for increases in policy interest rates to anticipate inflationary developments and apparently cause inflation. In this model, an increase in US interest rates apparently has an initial upward effect on OECD inflation but otherwise depresses the output and inflation variables. However OECD-wide shocks have a very significant effect on the US - note in particular the large short-run effect of g^{**} on g^{*} and π^{**} on π^{*}. The values of G^{*}_{21}, G^{*}_{51} and G^{*}_{43} reported in table 5b are significantly greater than unity, suggesting the US could be more susceptible to world output and inflation shocks than other OECD economies. While this result may exaggerate the effect of global influences on the US, it raises doubts about the validity of the standard closed economy macro-finance specification of the US.

These results are reflected in Figure 10a, which report the results of the Analysis of Variance (ANOVA) exercise. The top panel of the figure shows the effect of shocks to the three real variables z^{*}, g^{**} and g^{*} on the varaince of the five macro variables. The lfirst of these only appears to influence the US interest rate. The second panel shows the effect of shocks to the four nominal variables f^{*}, π^{**},π^{*} and r^{*}. The effect of the underlying inflation factor f^{*} is small to start with but eventually dominates given cointegration. US inflation and interest rates appear to be affected by world rather than US inflation.

4.2 The US Treasury market

The behavior of the yield curve model is determined by the factor loadings in (20). These depend upon the dynamics of the system under the risk neutral measure $\mathcal{Q}$ equation (17), which in turn depends upon the risk adjustments as well as the dynamic behavior under $\mathcal{P}$. The empirical factor loadings for this market are depicted in Figure 9a. These are fairly standard except that the effects of the factors z^* and f^*can be interpreted as global rather than just US factors. The loadings on the macro variables are relatively small. As in any macro-finance model, the spot rate provides the link between the macroeconomic model and the term structure. Since it *is* the 3 month yield, this variable has a unit coefficient at a maturity of one quarter and other factors have a zero loading. The spot rate loadings decline over the next few years, reflecting the adjustment of the spot rate towards the central tendency dictated by z^* and f^*. The spot rate thus determines the slope of the short-term yield curve, while three to five year maturity yields are strongly influenced by the behavior of the real rate factor. The loading on this factor then fades gradually over the longer maturities, allowing this to act as a 'curvature' factor. In contrast, f^* is very persistent and the loading on this moves up to unity quickly and stays relatively high, so it acts as a 'level' factor. These results are reflected in the results of ANOVA tests shown in Figure 11a. Fluctuations in the real rate factor z^* are influential in the 1-5 year area over the medium term, but have very little impact on the volatility of the 15 year yield, which is dominated by f^*. This factor has an effect on all yields, which builds up over time.

As (39) shows, the risk premia depend upon the log price loadings and the factor risk premia. The orthogonality and admissibility assumptions mean that the prices

of risk[8] associated with each latent variable just depends upon that varaible. Empirically, the parameter describing the varaibility price of risk associated with the real interest rate filter (z^*) was insignificant in M2 and was set to zero in M1 ($\lambda_2^{z*} = 0$). This risk is thus constant, determined by λ_1^{z**}, reported in table 5c. The parameters determining the factor risk premium ($-(\lambda_1^{f**} + \lambda_2^{f*} f^*)$) associated with exposure to the underlying inflation trend f^* are both highly significant. As in Dewachter and Lyrio (2006) we find $\lambda_2^{f*} > 0$, which has the implication that this factor risk premium is negatively related to the underlying rate itself.

The factor risk premia for the macro variables (29) depend linearly upon this premium and those associated with the macro shocks. The latter are determined by the coefficient vectors $\boldsymbol{\lambda}_1^{x*}; \boldsymbol{\lambda}_1^{x**}; \boldsymbol{\Lambda}_2^{x*}; \boldsymbol{\Lambda}_2^{x}$ in (33) and (34). However, many of these coefficients were poorly determined and eliminated in M1. The remaining coefficients are reported in Table 5c. One notable feature of these results is that changes in the nominal interest rate have a very significant effect on the prices of risk associated with the three nominal world variables ($\boldsymbol{\Lambda}_{2,i5}^{x*}, i = 3, 4, 5$). Similarly, the price of OECD inflation risk is not affected by π^* ($\boldsymbol{\Lambda}_{2,34}^{x*} = 0$), but is strongly affected by the other world macro variables ($\boldsymbol{\Lambda}_{2,3j}^{x*\prime} j = 1, 2, 3, 5$). The risk premia on for 1, 5 and 15 year US Treasury bonds are shown in Figure 12. They closely resemble estimates from earlier studies such as (Dewachter, Lyrio, and Maes (2006)).

4.3 The UK and the world economy

The effect of UK macroeconomic shocks on the UK economy revealed by Figure 7b are broadly in line with the closed economy literature cited in the introduction. As in the world model, the use of Kalman filters seems to remove the price puzzle. The

[8] Since this is a complete financial market, it is possible to form portfolios that track each of the shocks and state variables, being uniquely exposed to the associated risk. The 'price of risk' associated with any variable is the factor risk premium or expected excess return on its tracker portfolio, divided by its standard deviation, a measure of the associated risk-reward ratio.

novelty here is the allowance for common global factors and spillovers from world macro variables, which was the basic motivation for this study and as noted is very significant statistically. In this model the long run nominal asymptotes are dictated by f^* as well as the UK specific factor f , with a unit weight on the latter. The restrictions in (24) mean that the overall effect on the underlying rate of inflation in the UK of a unit increase f^* (and other world nominal variables that are cointegrated with it) is given by ν_{22}, which takes an empirical value of 0.7681. This effect is highly significant, with a $t-$value of 6.76. Thus the underlying rate of inflation in the UK varies with: $f + 0.7681f^*$.

While the decline in f^* seems to be gradual, the decline in f is more step-like, with strong downward movements occurring around the time of the 1982 Falklands War and the 1983 election (which gave Mrs. Thatcher a second term in office) and again after the election of Mr. Blair in 1997 when the Bank of England gained its independence. In contrast, the adoption of the $2\frac{1}{2}\%$ RPIX inflation target in 1992 appears to have had only a temporary effect, leaving the underlying rate of inflation some way above this target.

Figure 7a shows the effect of the world economy on the UK. The world real interest rate factor z^* has a significant short-run impact on the UK spot rate, which has the effect of depressing output and inflation temporarily. Changes in the US interest rate have a similar qualitative effect. Inflation and interest rates respond quickly to the global inflation trend which has a permanent effect. Spillovers from OECD and US macro shocks are also significant, in particular π^{**} and π^* both seem to influence π positively in the short run. Curiously the effect of g^{**} seems to be felt in UK inflation rather than output, perhaps reflecting the weakness of the UK export sector and the effect of the world business cycle on commodity import prices. Nevertheless US output (g^*) has a remarkably strong impact on UK output, consistent with the old

saying that 'if the US sneezes the UK catches a cold'. These effects are reflected in the UK ANOVA exercise reported in Figure 10b. As expected, the long run volatility of UK interest and inflation rates is dominated by f^* and f .

4.4 The UK Treasury market

The factor loadings for this market are depicted in Figure 9b. As in the US Treasury model the UK spot rate determines the slope of the short-term yield curve, while 1 - 5 year maturity yields are strongly influenced by the behavior of the real rate factors z and z^*. Similarly the long yields are determined by both of the nominal factors: f and f^*. The US spot rate also has some impact in the 1-2 year area while the influence of the other world macro varaibles seems to be more important than the UK output gap and inflation. The results of ANOVA tests are shown in Figure 11b. The UK spot rate is only important as a short term influence for short maturities. Fluctuations in z are influential in the 1-5 year area over the medium term, but reflecting the US result have very little impact on the volatility of the 15 year yield, which is dominated by f. This factor has an effect on all yields, which builds up over time, as does the effect of f^*.

Now consider the factor risk premia. The price of risk coefficients are reported in Table 5c.The sterling price of risk associated with z^* is zero ($\lambda_1^{z*} = \lambda_2^{z*} = 0$ in Table 5c).The price of risk associated with the UK real interest rate filter (z) was insignificant in M2 and was set to zero in M1 ($\lambda_1^z = \lambda_2^z = 0$). In other words, this risk is not priced: a tracker portfolio structured to have an overall weight of unity on z but no exposure to other variables would be expected to earn a rate equal to the spot rate r. Risks associated with exposure to the underlying inflation trends f^* and f are again very significant (see λ_2^{f*}, λ_1^f and λ_2^f in Table 5c). The price associated with the latter (-($\lambda_{f1} + \lambda_{f2} f$)) is positively related to the underlying rate itself (since

$\lambda_{f2} < 0$). The the price of risk associated with the UK output gap error is significant but constant ($\lambda^{x}_{2,2j} = 0, j = 1, 2, 3$). Those associated with the other macro errors are significant and time varying.

The risk premia on the UK Treasury bonds depend upon all 12 risks that are priced in the sterling markets and it is hard to distinguish factors that are consistently dominant. However the time series reported in Figure 12 can be keyed in with the political and economic developments during this period. In particular, the effect of the two deep recessions seen in 1980-81 and 1990-92 (as well as the world recession in 2001-2002) seem to have reduced these premia. The sharp increase in the 15 year premium in 1993 is probably associated with a loss of confidence in the Major government's economic competence, which was clearly reflected in the opinion polls and is picked up by the underlying inflation trend f. This trend also accounts for the sharp reversal in the 15 year premium in 1997-98 is probably associated with the May 1997 election. This brought in a new Labour government which handed over operational control of monetary policy to the Bank of England.

5 Conclusion

This paper develops a multi-country macro-finance model to study international economic and financial linkages. This specification first allows for cross-country effects between the US and other OECD countries by introducing aggregate OECD output and inflation variables into a standard closed economy US KVAR. We find that although US monetary policy influences the rest of the OECD, US macroeconomic variables have little effect. Surprisingly, we find strong evidence of that OECD output and inflation variables have a significant impact on the US, suggesting that it is inappropriate to model the US using the standard closed economy macro-finance specification. Consistent with earlier work on international economic linkages, we

find strong evidence of a common inflation trend and real business cycle, as well as real interest rate spillovers. The OECD and US inflation trends are cointegrated, moving together on a one-for-one basis, in lock step as it were. Neither OECD output nor inflation has much impact in the US Treasury market, but nor do US output and inflation. Consequently this bond market sub-model is similar to that emerging from previous macro-finance studies.

We then look at the one-way effects that these world economic variables have in a macro-finance model of the UK economy and Treasury bond market. The results confirm our prior view that it is inappropriate to model countries such as the UK using a closed economy specification. We find that the global inflation factor affects the UK inflation, although its effect is less than one-for-one. The global real interest rate factor also plays a role. These linkages help explain the comovement of the bond yields of the US and UK financial markets. Moreover, although the US business cycle does not have much of an effect on the rest of the OECD it has a significant effect on the UK. OECD output variables have a particularly strong effect on UK inflation. This model also provides insights into the working of the UK monetary system and the macroeconomy. In particular, the significance of the latent variables strongly suggests that to understand the behavior of the economy over this period it is important to allow for changes in inflationary expectations . The fall in the inflation trend when Mrs. Thatcher won a second term in office in May 1983 and when and the new monetary arrangements were introduced in May 1997 provides a good example of this kind of change.

Compared to the mainstream finance models of the bond market (Duffie and Kan (1996), Dai and Singleton (2000), and Duffee (2002)), the macro-finance model can use a relatively large number of factors (7 for the US and effectively 15 for the UK) because the parameters of the model are informed by macroeconomic as

well as yield data[9]. It can also use an unrestricted specification of the price of risk, with a large number of parameters. However, the relative adjustment speeds mean that the behavior of the yield curve is largely dictated by three factors: the inflation asymptote, the real interest rate factor and the spot rate. The model is consistent with the traditional three-factor finance specification in this sense, but links these factors to the behavior of the macroeconomy. This research opens the way to new open-economy studies of monetary policy and a much richer bond market specification, incorporating the best features of the international interdependence and macro-finance models.

References

Ahn, D. H. (2004): "Common Factors and Local Factors: Implications for Term Structures and Exchange Rates," *Journal of Financial and Quantitative Analysis*, 39.

Ang, A., and M. Piazzesi (2003): "A No-Arbitrage Vector Autoregression of Term Structure Dynamics with Macroeconomic and Latent Variables," *Journal of Monetary Economics*, 43, 75–87.

Arora, V., and A. Vamvakidis (2006): "The Impact of U.S. Growth on the Rest of the World: How Much Does It Matter," *Journal of Economic Integration*, 21, 21–39.

Clements, M. P., and D. Hendry (1996): "Intercept Corrections and Structural Change," *Journal of Applied Econometrics*, 11, 475–496.

Dai, Q., and K. Singleton (2000): "Specification Analysis of Affine Term Structure Models," *Journal of Finance*, 55, 415–41.

[9] 117 quarterly observations on 8 macro variables and 14 bond yields gives a total of 2574 data points.

Dewachter, H., and M. Lyrio (2006): "Macro Factors and the Term Structure of Interest Rates," *Journal of Money, Credit and Banking*, 38, 119–140.

Dewachter, H., M. Lyrio, and K. Maes (2006): "A Joint Model for the Term Structure of Interest Rates and the Macroeconomy," *Journal of Applied Econometrics*, 21, 439–462.

Duffee, G. (2002): "Term Premia and Interest Rate Forecasts in Affine Models," *Journal of Finance*, 62, 405–443.

Duffie, D., and R. Kan (1996): "A Yield-Factor Model of Interest Rates," *Mathematical Finance*, 6, 379–406.

Hamilton, J. (1994): *Time Series Analysis*. Princeton University Press, Princeton.

Hendry, D. F., and G. F. Mizon (1998): "Exogeneity, Causality, and Co-Breaking in Economic Policy Analysis of a Small Econometric Model of Money in the UK," *Empirical Economics*, 23, 267–294.

Joyce, M., P. Lildholdt, and S. Sorensen (2008): "Extracting inflation expectations and inflation risk premia from the term structure: a joint model of the UK nominal and real yield curves," *Bank of England Working Paper*.

Kaminska, I. (2008): "A No-Arbitrage Structural Vector Autoregressive Model of the UK Yield Curve," *Working Paper, Bank of England*.

Kose, M. A., C. Otrok, and C. H. Whiteman (2003): "International Business Cycles: World, Region, and Country-Specific Factors," *American Economic Review*, 93.

——— (2005): "Understanding the Evolution of World Business Cycles," *IMF Working Paper*, 05-211.

Kozicki, S., and P. Tinsley (2001): "Shifting Endpoints in the Term Structure of Interest Rates," *Journal of Monetary Economics*, 47, 613–652.

——— (2005): "Permanent and Transitory Policy Shocks in a Macro Model with

Asymmetric Information," *Journal of Economic Dynamics and Control*, 29, 1985–2015.

McCulloch, J., and H.-C. Kwon (1991): "Us Term Structure Data," *Manuscript, Ohio State University.*

Rudebusch, G. (2002): "Assessing Nominal Income Rules for Monetary Policy with Model and Data Uncertainty," *The Economic Journal*, 112, 402–32.

Smets, F. (1999): "Output Gap Uncertainty: Does It Matter for the Taylor Rule," in *Monetary Policy under Uncertainty*, ed. by B. Hunt, and A. Orr, pp. 10–29, Wellington. Reserve Bank of New Zealand.

Spencer, P. D. (2006): "UK Macroeconomic Volatility and the Term Structure of Interest Rates," *University of York Discussion Paper Series.*

——— (2008): "Stochastic Volatility in a Macro-Finance Model of the US Term Structure of Interest Rates 1961-2004," *Journal of Money, Credit, and Banking*, 40, 1177–1215.

Svensson, L. (1999): "Inflation Targetting as a Monetary Policy Rule," *Journal of Monetary Economics*, 43, 607–54.

Vasicek, O. (1977): "An Equilibrium Characterisation of the Term Structure," *Journal of Financial Economics*, 5, 177–188.

A Appendix

A.1 The companion form of the macro models

Stacking (1) and (2) gives the companion form (5) where: $\mathbf{W}_t^* = \mathbf{C}^*\mathbf{D}^*\boldsymbol{\epsilon}_t^*$, $\mathbf{K}^* = (0, 0, \boldsymbol{\kappa}^{*\prime})'$ and $\boldsymbol{\Phi}^*$, $\boldsymbol{\Omega}^*$, $\mathbf{C}^*$,$\mathbf{D}^*$ are defined as:

$$\boldsymbol{\Phi}^* = \begin{pmatrix} \xi^* & 0 & \mathbf{0}_{1,5} \\ 0 & 1 & \mathbf{0}_{1,5} \\ \xi^*\boldsymbol{\phi}_z^* & \boldsymbol{\phi}_f^* & \boldsymbol{\Phi}_1^* \end{pmatrix}; \mathbf{C}^* = \begin{pmatrix} 1 & 0 & \mathbf{0}_{1,5} \\ 0 & 1 & \mathbf{0}_{1,5} \\ \boldsymbol{\phi}_z^* & \boldsymbol{\phi}_f^* & \mathbf{G}^* \end{pmatrix}; \boldsymbol{\epsilon}_t^* = \begin{pmatrix} \epsilon_t^{z*} \\ \epsilon_t^{f*} \\ \boldsymbol{\epsilon}_t^{x*} \end{pmatrix}, \mathbf{D}^* = \begin{pmatrix} \delta^{z*} & 0 & \mathbf{0}_{1,5} \\ 0 & \delta^{f*} & \mathbf{0}_{1,5} \\ \mathbf{0}_{5,1} & \mathbf{0}_{5,1} & \mathbf{D}^{x*} \end{pmatrix}$$

Similarly, stacking (8) and (9) gives the companion form of the UK macro model (12), where $\mathbf{K}, \boldsymbol{\Theta}, \boldsymbol{\Phi}, A, \mathbf{C}, \mathbf{D}, \boldsymbol{\epsilon}_t$ are defined as:

$$\mathbf{K} = \begin{pmatrix} 0 \\ 0 \\ \boldsymbol{\kappa} + \boldsymbol{\theta}\mathbf{K}^* \\ \mathbf{0}_{1,3} \end{pmatrix}, \boldsymbol{\Theta} = \begin{pmatrix} \mathbf{0}_{1,7} \\ \mathbf{0}_{1,7} \\ \boldsymbol{\theta}\boldsymbol{\Phi}^* \\ \mathbf{0}_{1,3} \end{pmatrix}, \boldsymbol{\epsilon}_t = \begin{pmatrix} \epsilon_t^z \\ \epsilon_t^f \\ \boldsymbol{\epsilon}_t^x \\ \mathbf{0}_{1,3} \end{pmatrix}, \mathbf{A} = \begin{pmatrix} \mathbf{0}_{1,7} \\ \mathbf{0}_{1,7} \\ \boldsymbol{\theta}\mathbf{C}^* \\ \mathbf{0}_{3,7} \end{pmatrix}$$

$$\boldsymbol{\Phi} = \begin{pmatrix} \xi & 0 & \mathbf{0}_{1,3} & \mathbf{0}_{1,3} \\ 0 & 1 & \mathbf{0}_{1,3} & \mathbf{0}_{1,3} \\ \xi\boldsymbol{\phi}_z & \boldsymbol{\phi}_f & \boldsymbol{\Phi}_1 & \boldsymbol{\Phi}_2 \\ \mathbf{0}_{3,1} & \mathbf{0}_{3,1} & \mathbf{I}_3 & \mathbf{0}_{3,3} \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 1 & 0 & \mathbf{0}_{1,3} & \mathbf{0}_{1,3} \\ 0 & 1 & \mathbf{0}_{1,3} & \mathbf{0}_{1,3} \\ \boldsymbol{\phi}_z & \boldsymbol{\phi}_f & \mathbf{G} & \mathbf{0}_{3,3} \\ \mathbf{0}_{3,1} & \mathbf{0}_{3,1} & \mathbf{0}_{3,3} & \mathbf{0}_{3,3} \end{pmatrix}, \mathbf{D} = \begin{pmatrix} \delta^z & 0 & \mathbf{0}_{1,3} & \mathbf{0}_{1,3} \\ 0 & \delta^f & \mathbf{0}_{1,3} & \mathbf{0}_{1,3} \\ \mathbf{0}_{3,1} & \mathbf{0}_{3,1} & \mathbf{D}^x & \mathbf{0}_{3,3} \\ \mathbf{0}_{3,1} & \mathbf{0}_{3,1} & \mathbf{0}_{3,3} & \mathbf{0}_{3,3} \end{pmatrix}$$

A.2 Modeling the price of risk

The shift from the state price density $\mathcal{P}$ to any risk neutral measure is implemented by multiplying the state probability density by an appropriate state-dependent non-negative utility weight (called the Radon-Nikodym derivative). For the US market this is M_{t+1}^* and for the UK it is M_{t+1}. The expected values of a variable like X_{t+1}^*

or $P^*_{\tau-1,t+1}$ under $\mathcal{P}$ and $\mathcal{Q}^*$ are thus related by:

$$E_t^{Q^*}\left[X^*_{t+1}\right] = E_t\left[M^*_{t+1}X^*_{t+1}\right] \tag{25}$$

Similarly for X_{t+1} or $P_{\tau-1,t+1}$:

$$E_t^{Q}\left[X_{t+1}\right] = E_t\left[M_{t+1}X_{t+1}\right] \tag{26}$$

In the essentially affine specification of Duffee (2002) these weights are loglinear in the state variables:

$$-\ln M^*_{t+1} = \frac{1}{2}\mathbf{\Lambda}_t^{**\prime}\mathbf{\Lambda}_t^{**} + \mathbf{\Lambda}_t^{**\prime}\boldsymbol{\epsilon}^*_{t+1} \tag{27}$$

$$-\ln M_{t+1} = \frac{1}{2}\mathbf{\Lambda}_t^{*\prime}\mathbf{\Lambda}_t^{*} + \frac{1}{2}\mathbf{\Lambda}_t^{\prime}\mathbf{\Lambda}_t + \mathbf{\Lambda}_t^{*\prime}\boldsymbol{\epsilon}^*_{t+1} + \mathbf{\Lambda}_t^{\prime}\boldsymbol{\epsilon}_{t+1} \tag{28}$$

where $\mathbf{\Lambda}_t^{**}$ and $\mathbf{\Lambda}_t^{*}$ are price vectors of the risks which are associated with shocks to the US state vector X^*_{t+1} under $\mathcal{Q}^*$ and $\mathcal{Q}$ respectively; $\mathbf{\Lambda}_t$ is the price risk vector associated with shocks to X_{t+1} under $\mathcal{Q}$. It can be shown that the expectations under $\mathcal{P}$,$\mathcal{Q}$ and $\mathcal{Q}^*$ are related by:

$$E_t^{Q^*}\left(\mathbf{X}^*_{t+1}\right) = E_t\left(\mathbf{X}^*_{t+1}\right) - \mathbf{C}^*\mathbf{D}^*\mathbf{\Lambda}_t^{**} \tag{29}$$

and

$$E_t^{Q}\begin{pmatrix}\mathbf{X}^*_{t+1}\\ \mathbf{X}_{t+1}\end{pmatrix} = E_t\begin{pmatrix}\mathbf{X}^*_{t+1}\\ \mathbf{X}_{t+1}\end{pmatrix} - \begin{pmatrix}\mathbf{C}^*\mathbf{D}^* & \mathbf{0}\\ \mathbf{AD}^* & \mathbf{CD}\end{pmatrix}\begin{pmatrix}\mathbf{\Lambda}_t^*\\ \mathbf{\Lambda}_t\end{pmatrix} \tag{30}$$

These equations explain the factor risk premia in terms of the prices of risk. For example (29) takes $\mathbf{\Lambda}_t^{**}$, the vector showing the excess return to standard deviation ratios associated with $\boldsymbol{\epsilon}^*_{t+1}$; multiplies these by the standard deviations $\mathbf{D}^*$ to get their

differential drift and then maps these into the factor risk premia (the differential drift in $\mathbf{X}^*_{t+1}$) using $\mathbf{C}^*$. Following Duffee (2002), these prices of risk are affine in the state vector. For the US dollar investors, they price market risks as below:

$$\mathbf{\Lambda}^{**}_t = \mathbf{D}^*\mathbf{\Lambda}^{**}_1 + \mathbf{D}^{*-1}\mathbf{\Lambda}^{**}_2\mathbf{X}^*_t \tag{31}$$

where $\mathbf{D}^*$ is defined in (5). For the UK investors, the prices of risk are:

$$\begin{pmatrix} \mathbf{\Lambda}^*_t \\ \mathbf{\Lambda}_t \end{pmatrix} = \begin{pmatrix} \mathbf{D}^* & \mathbf{0} \\ \mathbf{0} & \mathbf{D} \end{pmatrix} \begin{pmatrix} \mathbf{\Lambda}^*_1 \\ \mathbf{\Lambda}_1 \end{pmatrix} + \begin{pmatrix} \mathbf{D}^* & \mathbf{0} \\ \mathbf{0} & \mathbf{D} \end{pmatrix}^{-1} \begin{pmatrix} \mathbf{\Lambda}^*_2 & \mathbf{0} \\ \mathbf{0} & \mathbf{\Lambda}_2 \end{pmatrix} \begin{pmatrix} \mathbf{X}^*_t \\ \mathbf{X}_t \end{pmatrix} \tag{32}$$

$\mathbf{D}$ is defined in (12). We assume that the price of risk vectors $\mathbf{\Lambda}^{**}_t$ and $\mathbf{\Lambda}^*_t$ only differ in the constant terms, so we have $\mathbf{\Lambda}^{**}_2 = \mathbf{\Lambda}^*_2$. The parameters $\mathbf{\Lambda}^{**}_1$, $\mathbf{\Lambda}^{**}_2$, $\mathbf{\Lambda}^*_1$, $\mathbf{\Lambda}_1$, $\mathbf{\Lambda}^*_2$ and $\mathbf{\Lambda}_2$ are specified as below:

$$\mathbf{\Lambda}^{**}_1 = \begin{pmatrix} \lambda^{z**}_1 \\ \lambda^{f**}_1 \\ \boldsymbol{\lambda}^{x**}_1 \end{pmatrix}, \mathbf{\Lambda}^*_1 = \begin{pmatrix} \lambda^{z*}_1 \\ \lambda^{f*}_1 \\ \boldsymbol{\lambda}^{x*}_1 \end{pmatrix}, \mathbf{\Lambda}^{**}_2 = \mathbf{\Lambda}^*_2 = \begin{pmatrix} \lambda^{z*}_2 & 0 & \mathbf{0}_{1,5} \\ 0 & \lambda^{f*}_2 & \mathbf{0}_{1,5} \\ \mathbf{0}_{5,1} & \mathbf{0}_{5,1} & \mathbf{\Lambda}^{x*}_2 \end{pmatrix}, \tag{33}$$

$$\mathbf{\Lambda}_1 = \begin{pmatrix} \lambda^z_1 \\ \lambda^f_1 \\ \boldsymbol{\lambda}^x_1 \\ \mathbf{0}_{3,1} \end{pmatrix}, \mathbf{\Lambda}_2 = \begin{pmatrix} \lambda^z_2 & 0 & \mathbf{0}_{1,3} & \mathbf{0}_{1,3} \\ 0 & \lambda^f_2 & \mathbf{0}_{1,3} & \mathbf{0}_{1,3} \\ \mathbf{0}_{3,1} & \mathbf{0}_{3,1} & \mathbf{\Lambda}^x_2 & \mathbf{0}_{3,3} \\ \mathbf{0}_{3,1} & \mathbf{0}_{3,1} & \mathbf{0}_{3,3} & \mathbf{0}_{3,3} \end{pmatrix}, \tag{34}$$

where $\boldsymbol{\lambda}^{x**}_1, \boldsymbol{\lambda}^{x*}_1$ are 5 by 1 column vectors; $\boldsymbol{\lambda}^x_1$ is a 3 by 1 column vector; $\mathbf{\Lambda}^{x*}_2$ is a 5 by 5 matrix and $\mathbf{\Lambda}^x_2$ is a 3 by 3 matrix; rest of the parameters are scalars.

Substituting (5), (6), (12), (14), (31), and (32) into (29) and (30) gives (7a), (7b), (15a) and (15b).

A.3 Arbitrage free yield curve restrictions

Absent arbitrage, the prices of the $\tau-$period US and UK discount bonds at time t are given by the time-discounted values of the risk-neutral expectations of their prices in the next period:

$$P^*_{\tau,t} = \exp\left(-r^*_t\right) E^{Q*}_t \left[P^*_{\tau-1,t+1}\right] \tag{35}$$

$$P_{\tau,t} = \exp\left(-r_t\right) E^{Q}_t \left[P_{\tau-1,t+1}\right] \tag{36}$$

where r^*_t, r_t are the one-period risk-free US and UK interest rates generated by (6) and (14). The operator E^M_t denotes the conditional expectation at time t under measure $\mathcal{M}$. Similarly V_t is the variance operator (which is the same under all measures). Since the state variables are normally distributed, these prices are lognormally distributed, with expected values given by the well known formula for the expectation of a lognormal variable:

$$P^*_{\tau,t} = \exp\{-r^*_t + E^{Q*}_t \left[\ln P^*_{\tau-1,t+1}\right] + \frac{1}{2} V_t \left[\ln P^*_{\tau-1,t+1}\right]\} \tag{37}$$

$$P_{\tau,t} = \exp\{-r_t + E^{Q}_t \left[\ln P_{\tau-1,t+1}\right] + \frac{1}{2} V_t \left[\ln P_{\tau-1,t+1}\right]\} \tag{38}$$

Taking logs of both sides of (37), substituting (6) and (16) and equating coefficients on the state variables with those in (16) gives the recursion restrictions (17). Similarly, taking logs of (38) substituting (18) gives the recursion (19).

A.4 Risk premia

The risk premium on a $\tau-$period bond is the expected return, less the spot rate, which depends upon the difference between the measures $\mathcal{P}$ and $\mathcal{Q}^*$ (or for the UK: $\mathcal{Q}$). The gross expected rate of return on a US Treasury bond is the expected payoff

$E_t[P^*_{\tau-1,t+1}] = \exp\{E_t\left[\ln P^*_{\tau-1,t+1}\right] + \frac{1}{2}V_t\left[\ln P^*_{\tau-1,t+1}\right]\}$ divided by the current price, which is given by a similar form (37). Taking the natural logarithm expresses this as a net percentage return and subtracting the spot rate r^*_t then gives the risk premium:

$$\begin{aligned}\rho^*_{\tau,t} &= E_t\left[\ln P^*_{\tau-1,t+1}\right] - E^{Q*}_t\left[\ln P^*_{\tau-1,t+1}\right] \\ &= \Psi^*_{\tau-1}\{E_t\left[\mathbf{X}^*_t\right] - E^{Q*}_t\left[\mathbf{X}^*_t\right]\} \\ &= -\Psi^*_{\tau-1}\mathbf{C}^*\mathbf{D}^*\mathbf{\Lambda}^{**}_t.\end{aligned} \tag{39}$$

using (16) and (29). Recall that the vector $\mathbf{C}^*\mathbf{D}^*\mathbf{\Lambda}^{**}_t$ in the third line shows the factor risk premia. The log price factor loading $\Psi^*_{\tau-1}$ then maps these into the excess return on each bond. Substituting (31) we see that this is affine in the world state variables:

$$\begin{aligned}\rho^*_{\tau,t} &= -\Psi^*_{\tau-1}\{\mathbf{C}^*\mathbf{D}^*\mathbf{D}^{*\prime}\mathbf{\Lambda}^{**}_1 + \mathbf{C}^*\mathbf{\Lambda}^*_{\mathbf{2}}\mathbf{X}^*_t\}. \\ &= \Psi^*_{\tau-1}\{\mathbf{K}^{*Q^*} - \mathbf{K}^* + (\mathbf{\Phi}^{*\mathbf{Q}^*} - \mathbf{\Phi}^*)\mathbf{X}^*_t\}.\end{aligned} \tag{40}$$

The second expression is obtained using (7a) and (7b) and clearly shows the differential drift effect. Similar steps for the UK model using (18), (30), (32) and finally (15a) and (15b) give equivalent expressions for the UK Treasury premia:

$$\begin{aligned}\rho_{\tau,t} &= E_t\left[\ln P_{\tau-1,t+1}\right] - E^Q_t\left[\ln P_{\tau-1,t+1}\right] \\ &= \Psi_{\tau-1}\{E_t\left[\mathbf{X}^*_t\right] - E^Q_t\left[\mathbf{X}^*_t\right]\} + \Upsilon_{\tau-1}\{E_t\left[\mathbf{X}_t\right] - E^Q_t\left[\mathbf{X}_t\right]\} \\ &= -\Psi_{\tau-1}\mathbf{C}^*\mathbf{D}^*\mathbf{\Lambda}^*_t - \Upsilon_{\tau-1}\{\mathbf{A}\mathbf{D}^*\mathbf{\Lambda}^*_t + \mathbf{C}\mathbf{D}\mathbf{\Lambda}_t\} \\ &= -\Psi_{\tau-1}\{\mathbf{C}^*\mathbf{D}^*\mathbf{D}^{*\prime}\mathbf{\Lambda}^*_1 + (\mathbf{C}^*\mathbf{\Lambda}^*_{\mathbf{2}})\mathbf{X}^*_t\} - \Upsilon_{\tau-1}\{(\mathbf{A}\mathbf{D}^*\mathbf{D}^{*\prime}\mathbf{\Lambda}^*_{\mathbf{1}} + \mathbf{C}\mathbf{D}\mathbf{D}'\mathbf{\Lambda}_{\mathbf{1}}) + \mathbf{A}\mathbf{\Lambda}^*_2\mathbf{X}^*_t + \mathbf{C}\mathbf{\Lambda}_2\mathbf{X}_t\}. \\ &= \Psi_{\tau-1}\{\mathbf{K}^{*Q} - \mathbf{K}^* + (\mathbf{\Phi}^{*\mathbf{Q}} - \mathbf{\Phi}^*)\mathbf{X}^*_t\} + \Upsilon_{\tau-1}\{\mathbf{K}^Q - \mathbf{K} + (\mathbf{\Theta}^Q - \mathbf{\Theta})\mathbf{X}^*_t + (\mathbf{\Phi}^Q - \mathbf{\Phi})\mathbf{X}_t\},\end{aligned} \tag{41}$$

A.5 Kalman Filter Estimation

The model is estimated using Kalman filter techniques. We rewrite the OECD-US-UK macro and yield models in the following state-space system:

$$\mathbf{Z}_t = \mathbf{M} + \mathbf{N}\mathbf{Z}_{t-1} + \mathbf{V}_t, \mathbf{V}_t \sim N(\mathbf{0}, \mathbf{\Pi}) \tag{42}$$

$$\mathbf{Y}_t = \mathbf{L} + \mathbf{H}\mathbf{Z}_t + \mathbf{E}_t, \mathbf{E}_t \sim N(\mathbf{0}, \mathbf{Q}) \tag{43}$$

The transition equation (42) is the stacked OECD-US-UK macro model that consists of model (5) and (12). The measurement equation (43) consists of empirical yield models (22), and (23).

The 15×1 state vector $\mathbf{Z}_t$ consists of the world vector $\mathbf{X}_t^*$ and UK vector $\mathbf{X}_t$:

$$\mathbf{Z}_t' = [\mathbf{X}_t^{*\prime}, \mathbf{X}_t']$$

where the parameters in the transition equation are defined as below:

$$\mathbf{N} = \begin{pmatrix} \mathbf{\Phi}^* & \mathbf{0} \\ \mathbf{\Theta} & \mathbf{\Phi} \end{pmatrix}, \mathbf{M} = \begin{pmatrix} \mathbf{K}^* \\ \mathbf{K} \end{pmatrix}$$

$$\mathbf{\Pi} = \begin{pmatrix} \mathbf{C}^* & \mathbf{0} \\ \mathbf{A} & \mathbf{C} \end{pmatrix} \begin{pmatrix} \mathbf{D}^*\mathbf{D}^{*\prime} & \mathbf{0} \\ \mathbf{0} & \mathbf{D}\mathbf{D}' \end{pmatrix} \begin{pmatrix} \mathbf{C}^* & \mathbf{0} \\ \mathbf{A} & \mathbf{C} \end{pmatrix}'$$

$$\mathbf{V}_t = \begin{pmatrix} \mathbf{C}^* & \mathbf{0} \\ \mathbf{A} & \mathbf{C} \end{pmatrix} \begin{pmatrix} \mathbf{D}^* & \mathbf{0} \\ \mathbf{0} & \mathbf{D} \end{pmatrix} \begin{pmatrix} \boldsymbol{\epsilon}_t^* \\ \boldsymbol{\epsilon}_t \end{pmatrix}.$$

The 22×1 observation vector $\mathbf{Y}_t$ includes observed macro and yield data.

$$\mathbf{Y}_t' = [\mathbf{y}_t^{*\prime}, \mathbf{x}_t^{*\prime}, \mathbf{y}_t', \mathbf{x}_t']$$

where 7×1 vector $\mathbf{y}_t^*$ is the US yield vector, 5×1 vector $\mathbf{x}_t^*$ the US observed macro vector, 7×1 vector $\mathbf{y}_t$ the UK yield vector, 3×1 vector $\mathbf{x}_t$ the observed UK macro vector. The parameters in the measurement equation are defined as below:

$$\mathbf{L} = \begin{pmatrix} \boldsymbol{\alpha}^* \\ \mathbf{0} \\ \boldsymbol{\alpha} \\ \mathbf{0} \end{pmatrix}, \mathbf{H} = \begin{pmatrix} \mathbf{B}^* & \mathbf{0} \\ [\mathbf{0}, \mathbf{I}_3, \mathbf{0}] & \mathbf{0} \\ \mathbf{B} & \boldsymbol{\Gamma} \\ \mathbf{0} & [\mathbf{0}, \mathbf{I}_3, \mathbf{0}] \end{pmatrix}, \mathbf{Q} = \begin{pmatrix} \boldsymbol{\Sigma}^* & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \boldsymbol{\Sigma} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix}$$

After specify the state space system, we are now in the position to derive the Kalman filter.

The predicting equations are,

$$\hat{\mathbf{Z}}_{t|t-1} = \mathbf{M} + \mathbf{N}\hat{\mathbf{Z}}_{t-1} \tag{44a}$$

$$\mathbf{P}_{t|t-1} = \mathbf{N}\mathbf{P}_{t-1}\mathbf{N}' + \boldsymbol{\Pi} \tag{44b}$$

The updating equations are,

$$\hat{\mathbf{Z}}_t = \hat{\mathbf{Z}}_{t|t-1} + \mathbf{P}_{t|t-1}\mathbf{H}'\mathbf{F}_t^I \left(\mathbf{Y}_t - \mathbf{L} - \mathbf{H}\hat{\mathbf{Z}}_{t|t-1}\right) \tag{45a}$$

$$\mathbf{P}_t = \mathbf{P}_{t|t-1} - \mathbf{P}_{t|t-1}\mathbf{H}'\mathbf{F}_t^I\mathbf{H}\mathbf{P}_{t|t-1} \tag{45b}$$

$$\mathbf{F}_t = \mathbf{H}\mathbf{P}_{t|t-1}\mathbf{H}' + \mathbf{Q} \tag{45c}$$

$$\mathbf{F}_t^I = \mathbf{F}_t^{-1} \tag{45d}$$

$$\mathbf{F}_t^D = |\mathbf{F}_t| \tag{45e}$$

The conditional mean for $\mathbf{Y}_t$ at time $(t-1)$ is

$$\hat{\mathbf{Y}}_{t|t-1} = E_{t-1}(\mathbf{Y}_t) = \mathbf{L} + \mathbf{H}\hat{\mathbf{Z}}_{t|t-1} \tag{46}$$

So the prediction error of $\mathbf{Y}_t$ at one period ahead is,

$$\mathbf{u}_t = \mathbf{Y}_t - \hat{\mathbf{Y}}_{t|t-1} = \mathbf{Y}_t - \left(\mathbf{L} + \mathbf{H}\hat{\mathbf{Z}}_{t|t-1}\right) \tag{47}$$

As long as we set the initial conditions for the state vector $\mathbf{Z}_t$ and the covariance matrix of the estimation error, $\mathbf{P}_t$, we could derive $\hat{\mathbf{Z}}_t$ of each period using Kalman Filter.

We maximize the log-likelihood function below to estimate the unknown parameter set $\mathbf{P}$,

$$\begin{aligned}\ln L\left(\mathbf{P}|\mathbf{Y}_{t=1,\ldots,T}\right) &= \sum_{t=1}^{T} \ln f\left(\mathbf{Y}_t|\mathbf{Y}_{t-1}, \mathbf{P}\right) \\ &= c - \frac{1}{2}\sum_{t=1}^{T} \ln \mathbf{F}_t^D - \frac{1}{2}\sum_{t=1}^{T} \mathbf{u}_t' \mathbf{F}_t^I \mathbf{u}_t\end{aligned}$$

where $f\left(\mathbf{Y}_t|\mathbf{Y}_{t-1}\right)$ is the probability distribution function of $\mathbf{Y}_t$ conditional on $\mathbf{Y}_{t-1}$; $\mathbf{F}_t^D, \mathbf{F}_t^I$ are defined in (45); $\mathbf{u}_t$ is defined in (47); c is a constant term (the value of c has no impacts on the final estimate results).

B Tables

Table 2: Data summary statistics: 1979Q1-2007Q1

Macro Variables								
	g^{**}	π^{**}	g^{*}	π^{*}	r^{*}	g	π	r
Mean	-0.553	3.872	-0.763	3.998	6.089	-1.188	4.817	8.142
Std.	1.510	2.614	1.951	2.728	3.273	2.505	3.605	3.484
Skew.	-0.259	1.732	-0.896	2.050	0.819	-0.305	2.027	0.499
Excess Kurt.	0.493	2.288	1.675	3.718	0.571	-0.155	4.038	-0.977
ADF	-2.967*	-1.749	-3.367*	-2.248	-1.804	-2.999*	-1.867	-1.624
The US Yield (subscript is the number of month to maturity)								
	y_4^*	y_8^*	y_{12}^*	y_{20}^*	y_{28}^*	y_{40}^*	y_{60}^*	
Mean	6.535	6.813	7.005	7.278	7.469	7.635	7.955	
Std.	3.300	3.214	3.101	2.965	2.861	2.731	2.565	
Skew.	0.699	0.658	0.658	0.701	0.702	0.701	0.682	
Excess Kurt.	0.316	0.121	0.012	-0.081	-0.226	-0.290	-0.302	
ADF	-1.528	-1.389	-1.538	-1.344	-1.198	-1.131	-1.117	
The UK Yield (subscript is the number of month to maturity)								
	y_4	y_8	y_{12}	y_{20}	y_{28}	y_{40}	y_{60}	
Mean	8.140	8.197	8.249	8.334	8.391	8.384	8.169	
Std.	3.213	3.081	3.027	3.013	3.042	3.054	2.925	
Skew.	0.319	0.263	0.247	0.217	0.195	0.195	0.221	
Excess Kurt.	-1.131	-1.066	-0.986	-0.972	-1.014	-1.001	-0.864	
ADF	-1.454	-1.454	-1.474	-1.42	-1.299	-1.146	-1.078	

Note:
1. The data are annualized percentage rate.
2. 5% significance level for ADF test is -2.89; the lags of ADF test is determined by AIC.
3. Output gaps are from OECD website; CPI/RPIX inflation and 3 month Treasury bill rates are from Datastream. Yields are discount bond equivalent data compiled by the New York Fed and Bank of England. Mean denotes sample arithmetic mean expressed as percentage p.a.; Std. standard deviation and Skew.& Excess Kurt are standard measures of skewness (third moment) and excess kurtosis (fourth moment). ADF is the Adjusted Dickey-Fuller statistic for the null of non-stationarity. The 5% significance level is (-)2.877.

Table 3: Cointegration tests

Dependent Variable	π_t^*
Regressor	π_t^{**}
Coefficient (β)	$\beta = 0.998$
Residual (u_t)	$u_t = \pi_t^* - \beta * \pi_t^{**}$
ADF tests	
H0: u_t follows unit root	-4.385*
LR tests	
H0: $\beta = 1$	0.004

Note:
* The hypothesis is rejected at 5% significant level.
ADF Test reject u_t follows unit root process, which indicates the cointegration between π_t^* and π_t^{**}.
LR tests accept unit coefficient hypothesis, implying [1,-1] cointegrating vector between π_t^* and π_t^{**}.

Table 4: Estimated Model Log Likelihood and LR tests

Model	Parameters			Loglikelihood		
Specification	k(M)	k(3)-k(M)	k(2)-k(M)	L(M)	2(L(3)-L(M))	2(L(2)-L(M))
M0	146	21	15	274.00	279.80*	272.40*
Closed economy					(0.00)	(0.00)
				H0:	M0	M0
				H1:	M3	M2
M1:	134	33	27	402.00	23.80	16.40
Preferred					(0.88)	(0.94)
				H0:	M1	M1
				H1:	M3	M2
M2:	161	6		410.20	7.40	
Restricted					(0.29)	
open economy				H0:	M2	
				H1:	M3	
M3:						
Unrestricted	167			413.90		
open economy						

* The hypothesis is rejected at 5% significant level.
The number in brackets is the probability that the hypothesis is true.

Table 5a: Estimated parameters for Model M1

Parameters	Estimates	t-value	Parameters	Estimates	t-value
$\Phi^*_{1,11}$	0.8252	15.24	$\Phi_{1,11}$	0.6607	6.87
$\Phi^*_{1,12}$	0.1172	2.89	$\Phi_{1,12}$	-0.1827	-2.18
$\Phi^*_{1,13}$	-0.2157	-3.27	$\Phi_{1,13}$	-0.0824	-0.93
$\Phi^*_{1,14}$	0.1128	2.01	$\Phi_{1,21}$	0.5244	7.23
$\Phi^*_{1,15}$	-0.0094	-0.35	$\Phi_{1,22}$	1.1082	13.76
$\Phi^*_{1,21}$	-0.1243	-1.32	$\Phi_{1,23}$	0.0606	0.71
$\Phi^*_{1,22}$	1.0094	14.10	$\Phi_{1,31}$	0.0596	1.55
$\Phi^*_{1,23}$	-0.3394	-2.81	$\Phi_{1,32}$	0.0655	1.50
$\Phi^*_{1,24}$	0.1601	1.59	$\Phi_{1,33}$	0.5748	11.04
$\Phi^*_{1,25}$	0.0001	0.00	$\Phi_{2,11}$	0.2990	3.01
$\Phi^*_{1,31}$	0.0001	0.00	$\Phi_{2,12}$	-0.0892	-1.20
$\Phi^*_{1,32}$	0.0309	0.86	$\Phi_{2,13}$	0.0710	0.92
$\Phi^*_{1,33}$	0.6583	9.84	$\Phi_{2,21}$	-0.4082	-5.27
$\Phi^*_{1,34}$	0.2563	4.53	$\Phi_{2,22}$	-0.2484	-3.77
$\Phi^*_{1,35}$	0.0332	1.20	$\Phi_{2,23}$	-0.1628	-2.21
$\Phi^*_{1,41}$	-0.0992	-1.07	$\Phi_{2,31}$	0.1401	3.42
$\Phi^*_{1,42}$	0.2412	3.95	$\Phi_{2,32}$	-0.0412	-1.29
$\Phi^*_{1,43}$	0.0878	0.75	$\Phi_{2,33}$	0.0978	2.25
$\Phi^*_{1,44}$	0.8304	8.00	ξ^*	0.8611	69.46
$\Phi^*_{1,45}$	-0.0054	-0.10	ξ	0.9423	93.06
$\Phi^*_{1,51}$	0.0055	0.04	φ^{**}_π	0.0113	6.77
$\Phi^*_{1,52}$	0.2316	2.38	φ^*_π	0.0119	6.86
$\Phi^*_{1,53}$	0.3355	1.98	φ^*_ρ	0.0062	5.64
$\Phi^*_{1,54}$	-0.0041	-0.03	φ_π	0.0102	5.68
$\Phi^*_{1,55}$	0.2627	4.92	φ_ρ	0.0096	2.86

Table 5b: Estimated price of risk parameters for Model M1

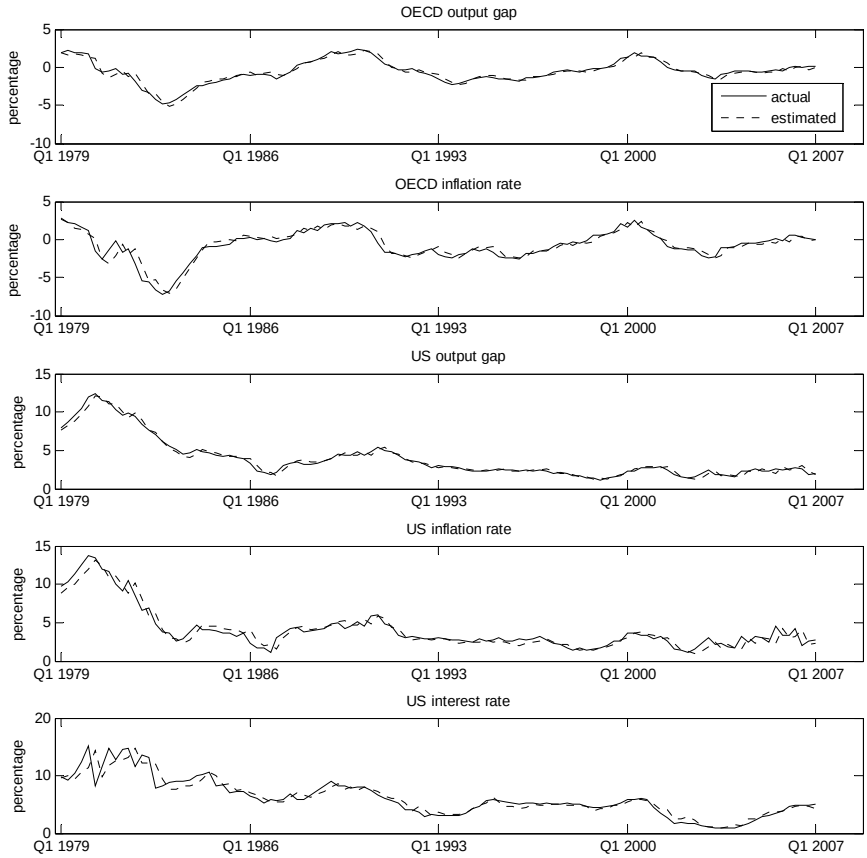
Parameters	Estimates	t-value	Parameters	Estimates	t-value
ν_{22}	0.7681	6.76	G^*_{21}	1.4431	17.53
χ_{11}	0.3208	0.46	G^*_{31}	0.1587	1.76
χ_{21}	-0.2225	-1.61	G^*_{41}	0.6076	3.38
χ_{31}	0.3381	0.72	G^*_{51}	1.1749	5.93
χ_{13}	-2.9839	-1.26	G^*_{32}	0.0251	0.27
χ_{23}	-0.3107	-0.62	G^*_{42}	0.0435	0.26
χ_{33}	-1.6924	-1.10	G^*_{52}	-0.0680	-0.43
χ_{14}	1.8400	1.50	G^*_{43}	1.6646	15.9
χ_{24}	0.4924	1.88	G^*_{53}	0.5861	2.90
χ_{34}	1.1820	1.48	G^*_{54}	0.2996	1.63
χ_{25}	0.4154	1.67	G_{21}	-0.3406	-4.58
χ_{35}	0.7830	3.07	G_{31}	0.1901	2.81
χ_{26}	0.2404	1.08	G_{32}	0.4411	5.44
χ_{36}	-0.7615	-3.09	σ^*_4	0.00032	36.46
χ_{37}	0.4233	2.59	σ^*_8	0.00014	23.22
δ^{z*}	0.0018	14.02	σ^*_{12}	0.00007	27.45
δ^{f*}	0.0014	11.16	σ^*_{20}	0.00013	63.63
Δ^{x*}_{11}	0.0009	23.45	σ^*_{28}	0.00006	24.98
Δ^{x*}_{22}	0.0008	24.66	σ^*_{40}	0.00019	18.92
Δ^{x*}_{33}	0.0009	23.77	σ^*_{60}	0.00043	30.75
Δ^{x*}_{44}	0.0010	22.72	σ_4	0.00045	1.79
Δ^{x*}_{55}	0.0017	18.29	σ_8	0.00020	44.81
δ^z	0.0022	13.15	σ_{12}	0.00001	0.00
δ^f	0.0009	10.13	σ_{20}	0.00012	40.11
Δ^x_{11}	0.0015	19.93	σ_{28}	0.00001	0.01
Δ^x_{22}	0.0016	19.58	σ_{40}	0.00027	33.72
Δ^x_{33}	0.0014	19.81	σ_{60}	0.00068	3.52

Table 5c: Estimated parameters for Model M1: continue

Parameters	Estimates	t-value	Parameters	Estimates	t-value
λ_1^{z**}	902.13	1.23	$\lambda_{2,51}^{x*}$	0	-
λ_1^{f**}	-121.19	-3.42	$\lambda_{2,52}^{x*}$	-0.0926	-1.04
$\lambda_{1,1}^{x**}$	0	-	$\lambda_{2,53}^{x*}$	0.2677	1.29
$\lambda_{1,2}^{x**}$	0	-	$\lambda_{2,54}^{x*}$	-0.2066	-1.25
$\lambda_{1,3}^{x**}$	40189	1.04	$\lambda_{2,55}^{x*}$	-0.3760	-3.20
$\lambda_{1,4}^{x**}$	-38935	-1.02	λ_1^{z*}	0	-
$\lambda_{1,5}^{x**}$	-2475	-0.66	λ_1^{f*}	0	-
λ_2^{z*}	0	-	$\lambda_{1,1}^{x*}$	0	-
λ_2^{f*}	0.0071	4.42	$\lambda_{1,2}^{x*}$	0	-
$\lambda_{2,11}^{x*}$	0.1272	-1.00	$\lambda_{1,3}^{x*}$	-1315.1000	-3.11
$\lambda_{2,12}^{x*}$	0.1618	2.79	$\lambda_{1,4}^{x*}$	0	-
$\lambda_{2,13}^{x*}$	0	-	$\lambda_{1,5}^{x*}$	0	-
$\lambda_{2,14}^{x*}$	0	-	λ_1^{z}	0	-
$\lambda_{2,15}^{x*}$	0	-	λ_1^{f}	-163.0900	-1.94
$\lambda_{2,21}^{x*}$	0.9470	1.78	$\lambda_{1,1}^{x}$	0	-
$\lambda_{2,22}^{x*}$	-0.3067	-1.62	$\lambda_{1,2}^{x}$	-374.5900	-2.24
$\lambda_{2,23}^{x*}$	0	-	$\lambda_{1,3}^{x}$	0	-
$\lambda_{2,24}^{x*}$	-0.7204	-2.15	λ_2^{z}	0	-
$\lambda_{2,25}^{x*}$	0	-	λ_2^{f}	-0.0140	-3.14
$\lambda_{2,31}^{x*}$	-0.3162	-2.20	$\lambda_{2,11}^{x}$	-0.3883	-4.63
$\lambda_{2,32}^{x*}$	0.1926	2.91	$\lambda_{2,12}^{x}$	0.5579	3.37
$\lambda_{2,33}^{x*}$	0.4940	2.51	$\lambda_{2,13}^{x}$	-0.0949	-1.90
$\lambda_{2,34}^{x*}$	0	-	$\lambda_{2,21}^{x}$	0	-
$\lambda_{2,35}^{x*}$	0.3654	2.92	$\lambda_{2,22}^{x}$	0	-
$\lambda_{2,41}^{x*}$	0	-	$\lambda_{2,23}^{x}$	0	-
$\lambda_{2,42}^{x*}$	0	-	$\lambda_{2,31}^{x}$	0.2518	15.68
$\lambda_{2,43}^{x*}$	-0.3769	-1.86	$\lambda_{2,32}^{x}$	-0.0329	-1.00
$\lambda_{2,44}^{x*}$	0	-	$\lambda_{2,33}^{x}$	0	-
$\lambda_{2,45}^{x*}$	-0.4364	-2.50			

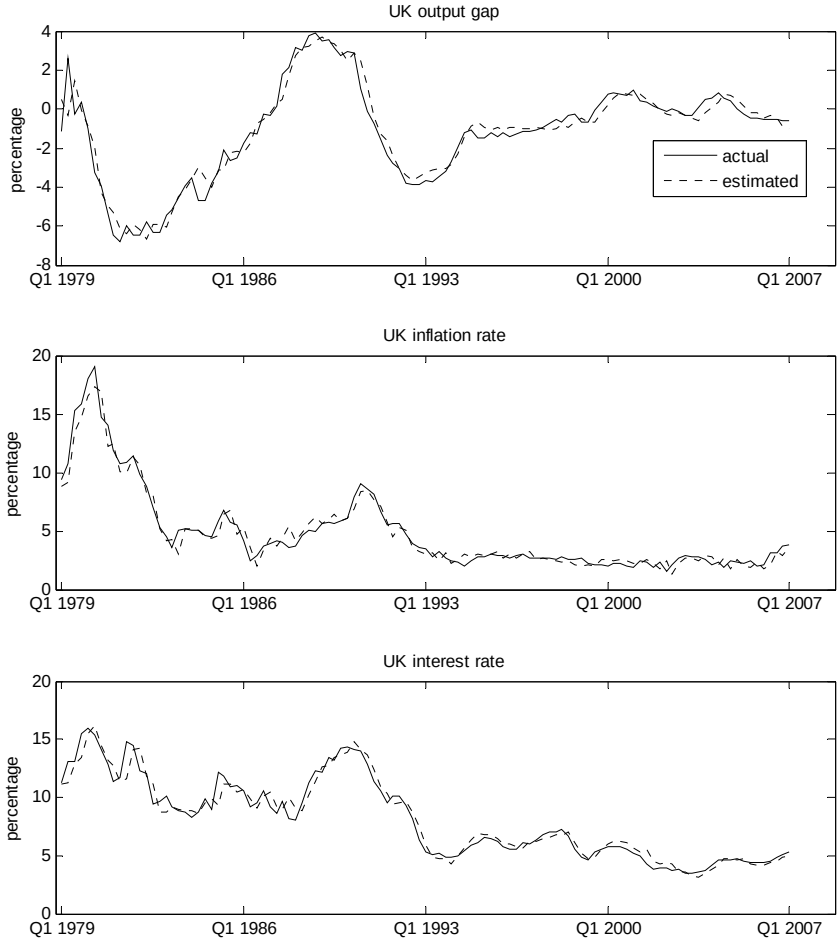
C Figures

Figure 1a: OECD US macro data



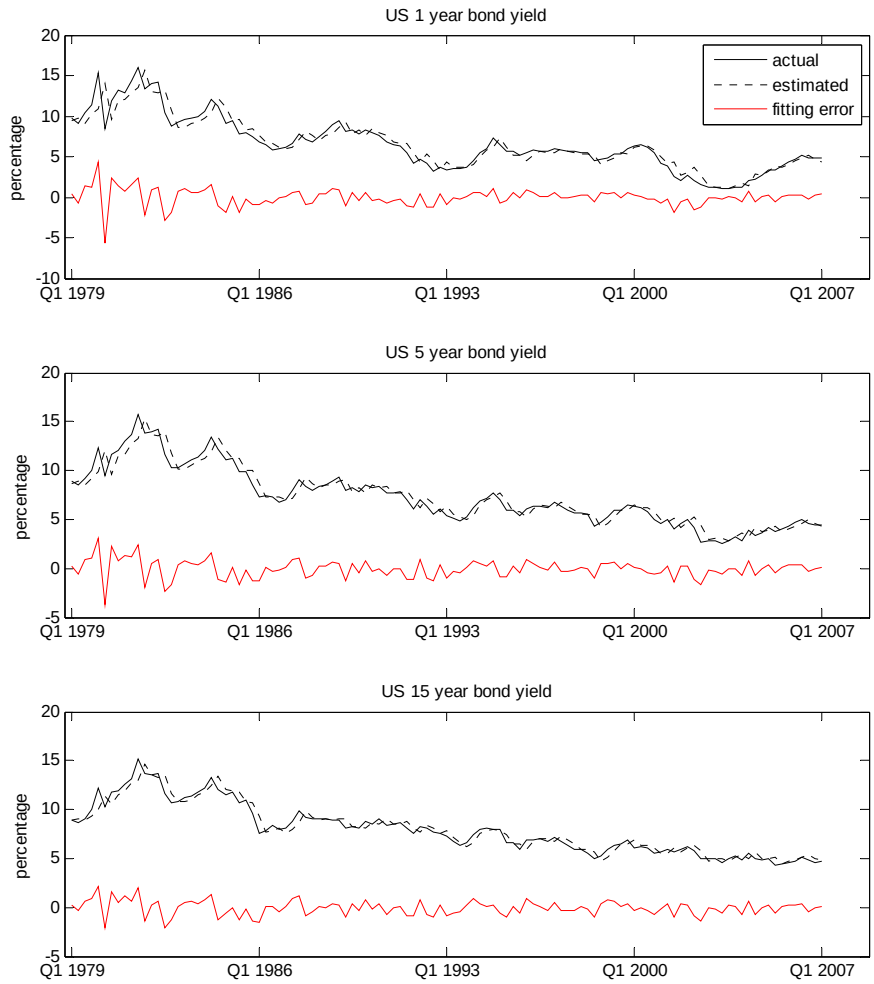
See notes to Table 2.

Figure 1b: UK macro data



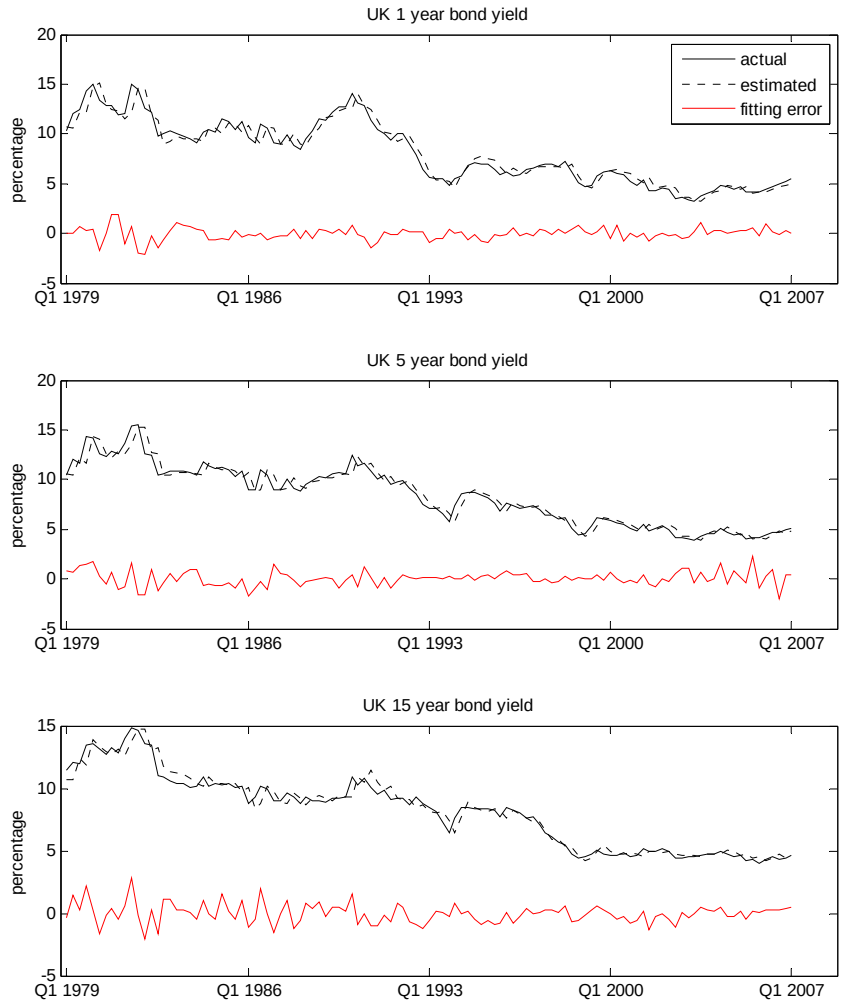
See notes to Table 2

Figure 2a: US yield data



See notes to Table 2

Figure 2b: UK yield data



See notes to Table 2

Figure 3: The estimated latent factors

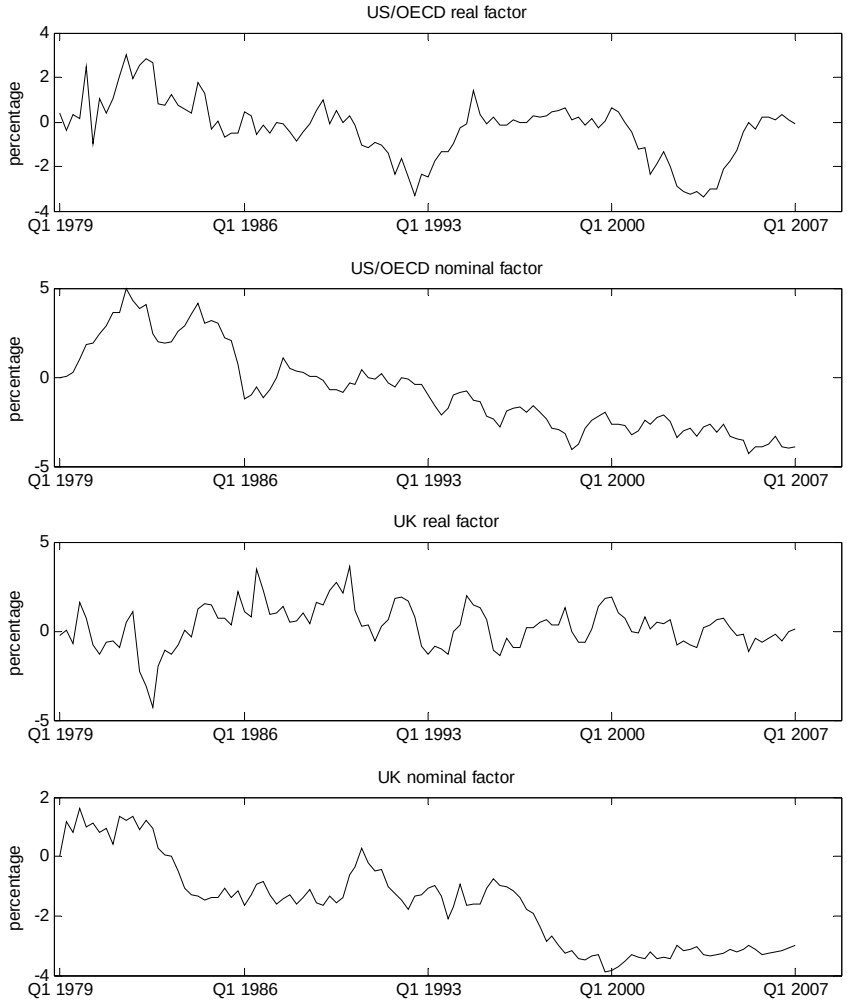


Figure 4: The estimated inflation/interest central tendencies

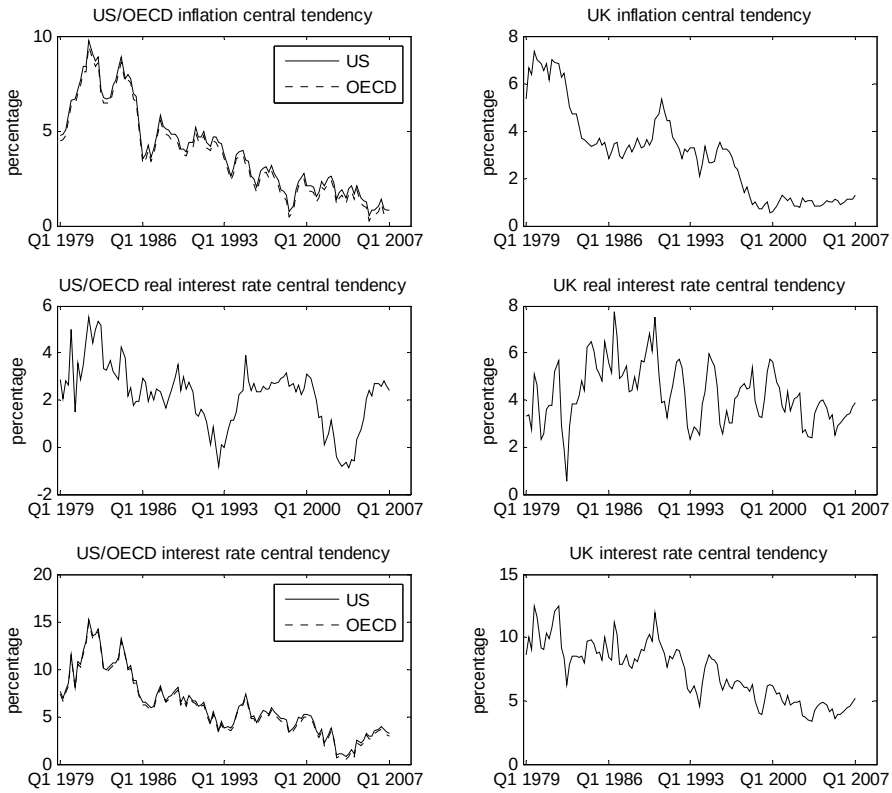
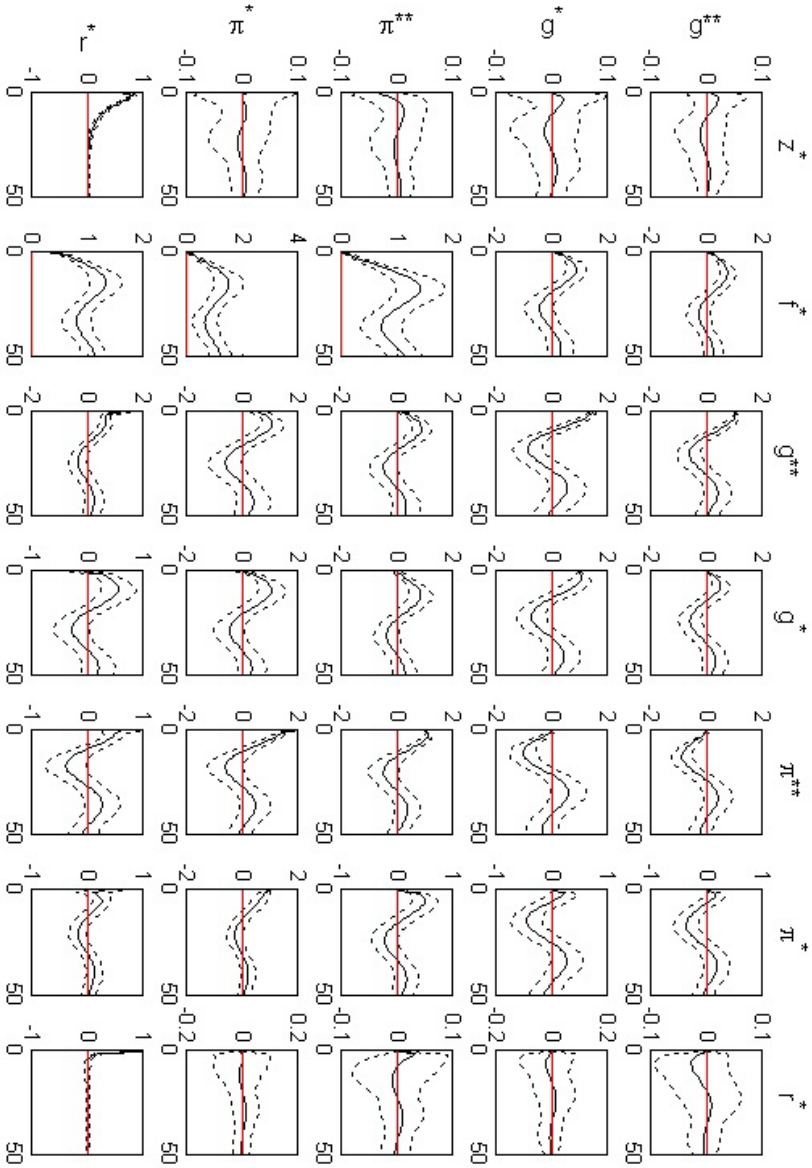
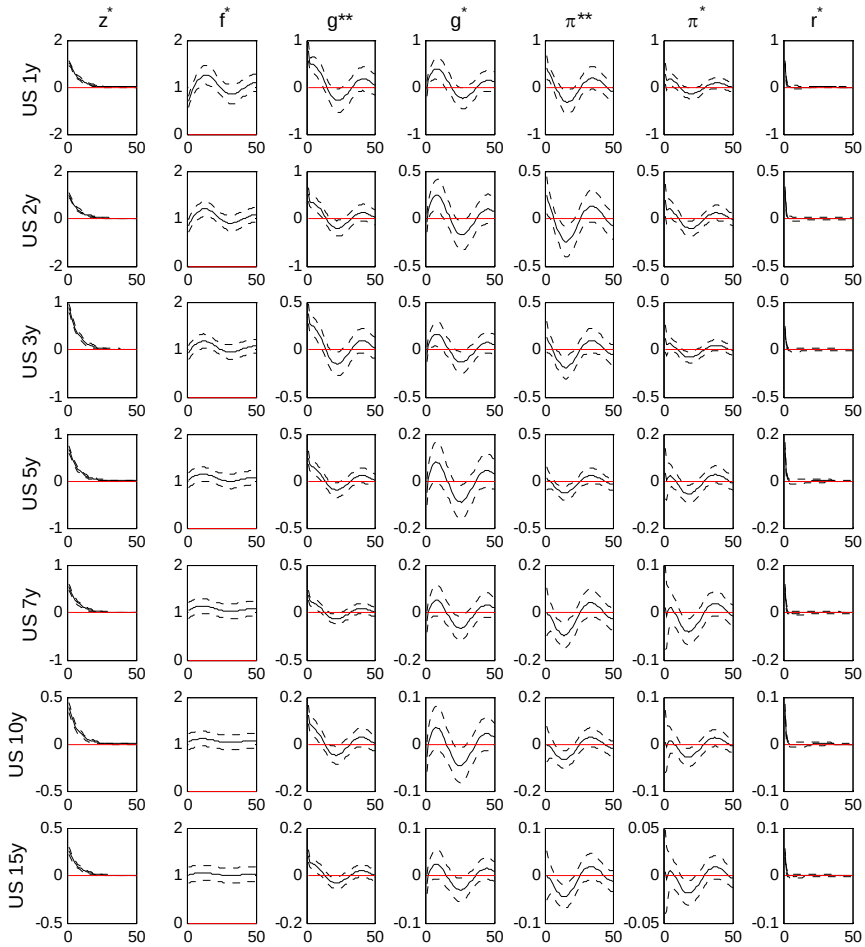


Figure 5: OECD-US macro impulse responses -with confidence intervals of 95 percentage



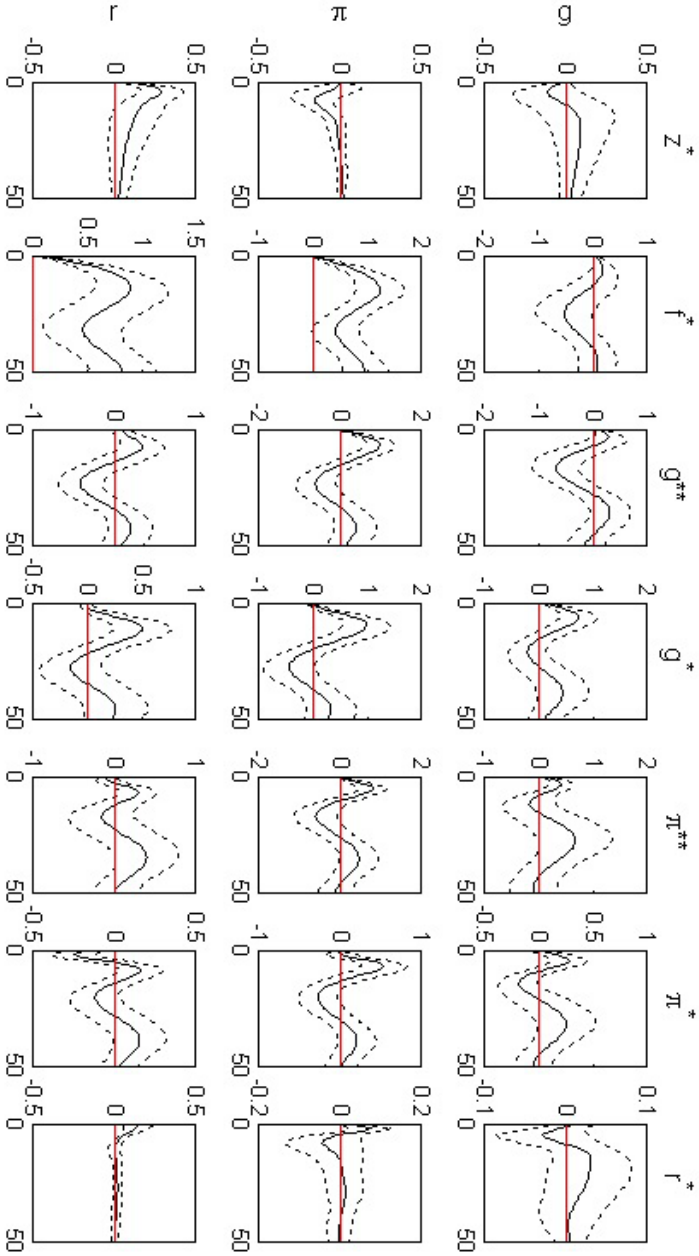
Each panel shows the effect of a shock to one the seven orthogonal innovations (ϵ) shown in (1) and (2). These shocks increase each of the driving variables in turn by one percentage point compared to its historical value for just one period. Since f^* is a martingale, the second shock (ϵ^{f^*}) has a permanent effect on inflation and interest rates, while other shocks are transient. Elapsed time is measured in calendar quarters.

Figure 6: US yield impulse responses -with confidence intervals of 95 percentage



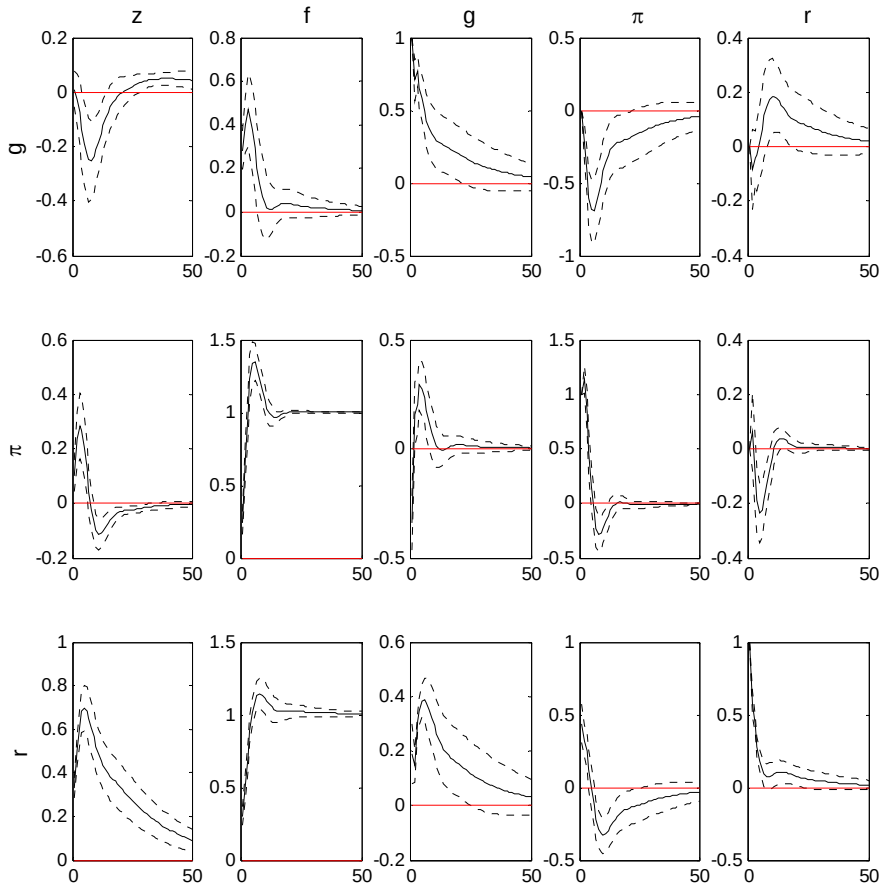
Each panel shows the effect of a shock to one the seven orthogonal innovations (ϵ) shown in (1) and (2). These shocks increase each of the driving variables in turn by one percentage point compared to its historical value for just one period. Since f^* is a martingale, the second shock (ϵ^{f^*}) has a permanent effect onyields, while other shocks are transient. Elapsed time is measured in calendar quarters.

Figure 7a: UK macro impulse responses 1 -with confidence intervals of 95 percentage



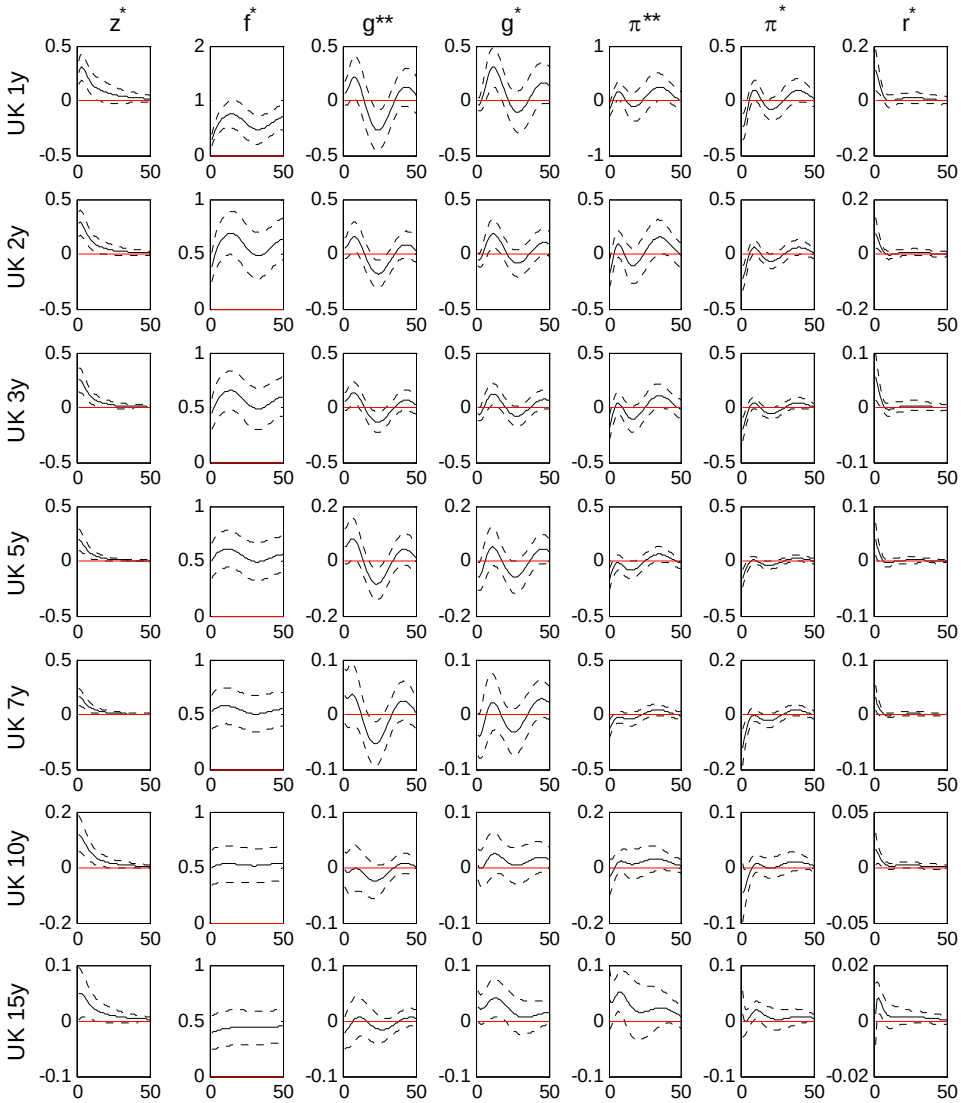
Each panel shows the effect on the UK economy of a shock to one the seven orthogonal global innovations (ϵ) shown in (1) and (2). These shocks increase each of the driving variables in turn by one percentage point compared to its historical value for just one period. Since f is a martingale, the second shock (ϵ^f) has a permanent effect on inflation and interest rates, while other shocks are transient. Elapsed time is measured in calendar quarters.

Figure 7b: UK macro impulse responses 2: -with confidence intervals of 95 percentage



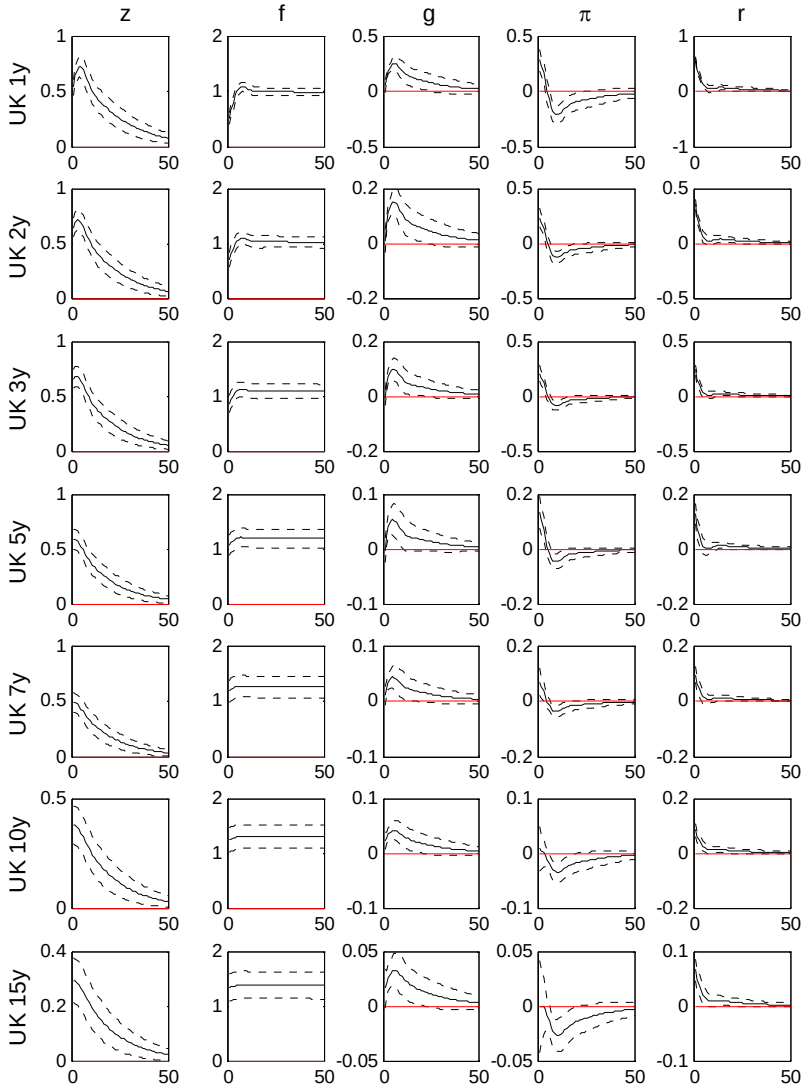
Each panel shows the effect of a shock to one the five orthogonal innovations (ϵ) shown in (7) and (8). These shocks increase each of the driving variables in turn by one percentage point compared to its historical value for just one period. Since f is a martingale, the second shock (ϵ^f) has a permanent effect on inflation and interest rates, while other shocks are transient. Elapsed time is measured in calendar quarters.

Figure 8a: UK yield impulse responses 1: -with confidence intervals of 95 percentage



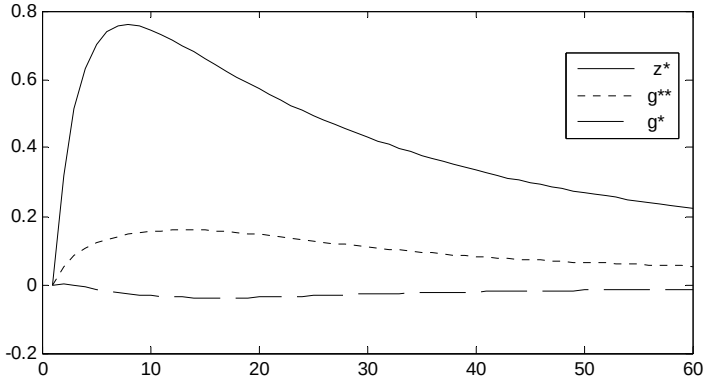
Each panel shows the effect on selected UK yields of a shock to one the seven orthogonal global innovations (ϵ) shown in (1) and (2). These shocks increase each of the driving variables in turn by one percentage point compared to its historical value for just one period. Since f^* is a martingale, the second shock (ϵ^{f^*}) has a permanent effect on yields, while other shocks are transient. Elapsed time is measured in calendar quarters.

Figure 8b: UK yield impulse responses 2: -with confidence intervals of 95 percentage



Each panel shows the effect of a shock to one the five orthogonal innovations (ϵ) shown in (7) and (8). These shocks increase each of the driving variables in turn by one percentage point compared to its historical value for just one period. Since f is a martingale, the second shock (ϵ^f) has a permanent effect on inflation and interest rates, while other shocks are transient. Elapsed time is measured in calendar quarters.

Figure 9a: Factor loadings of the US yield model



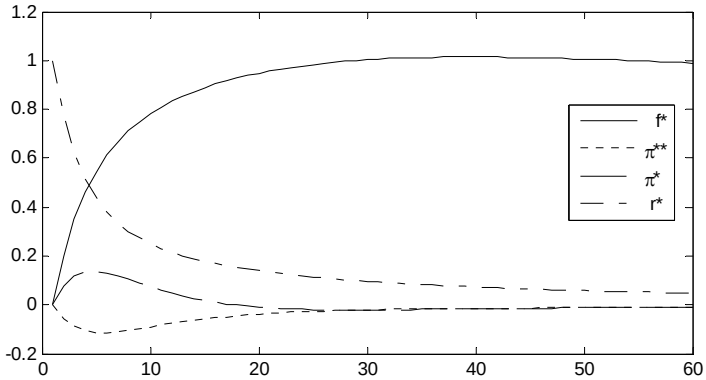


Figure 9b: Factor loadings of the UK yield model

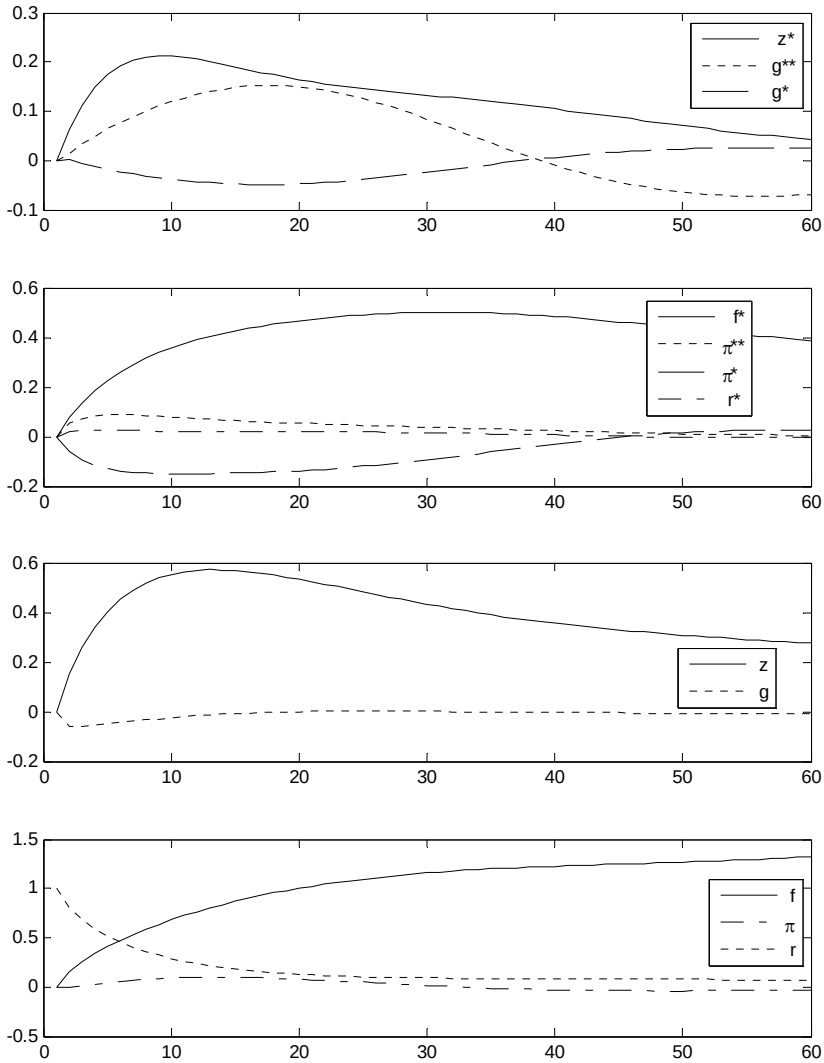
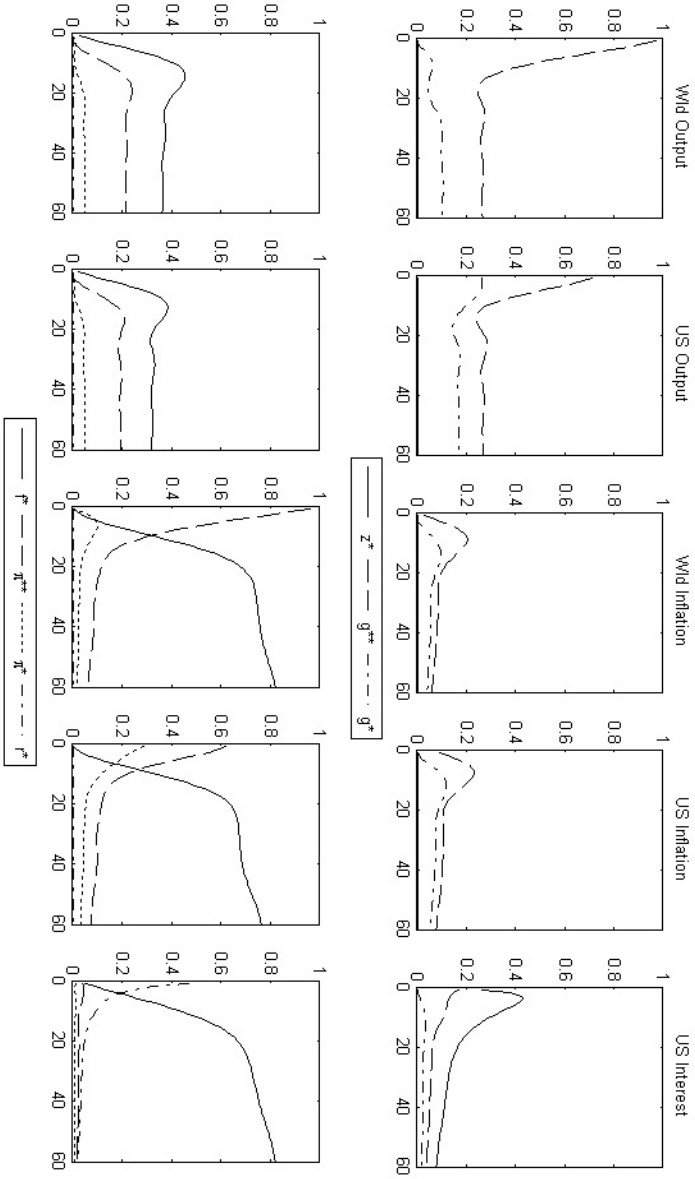
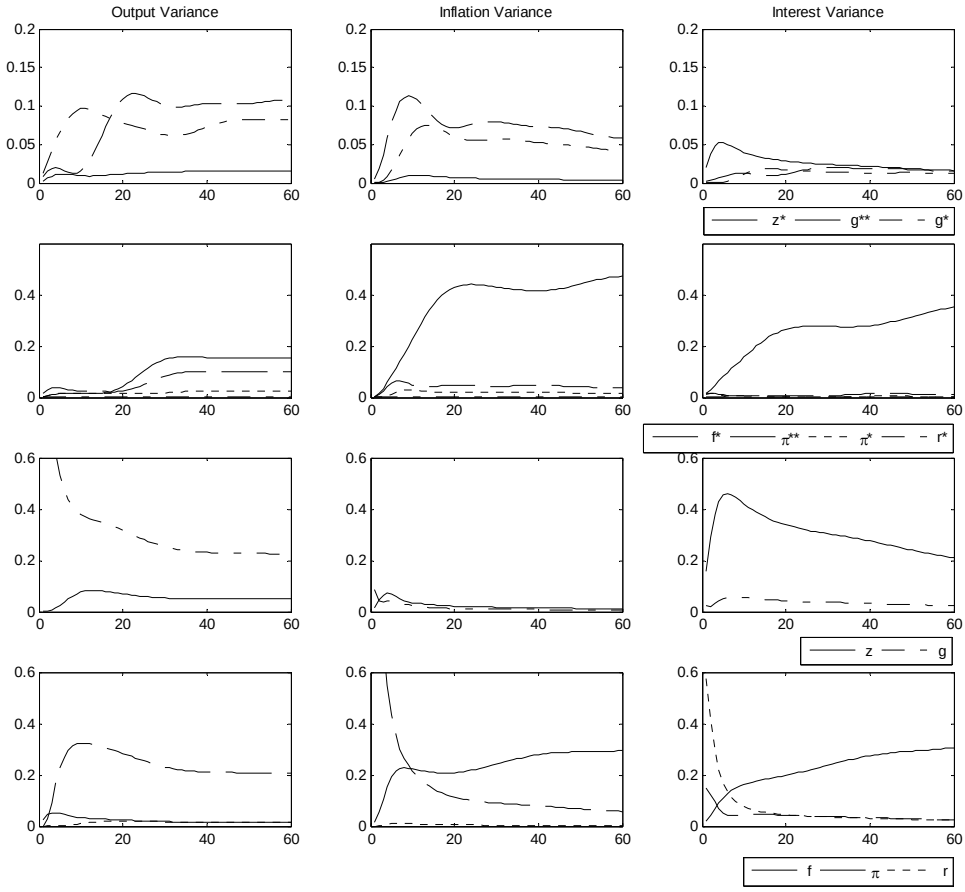


Figure 10a: Variance Decomposition: OECD-US macro variables



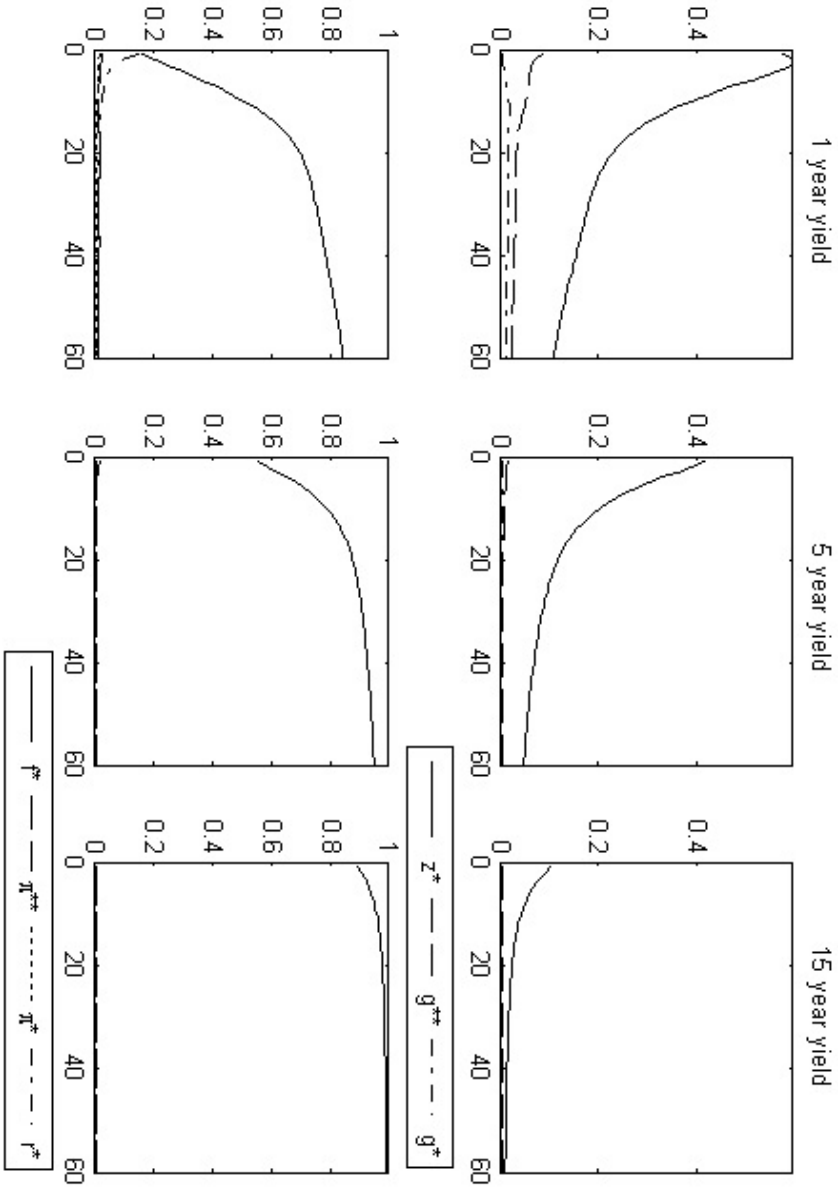
These panels show the proportion of the total variance of each variable explained by shocks to the various driving variables. Elapsed time is measured in quarters.

Figure 10b: Variance Decomposition: UK macro variables



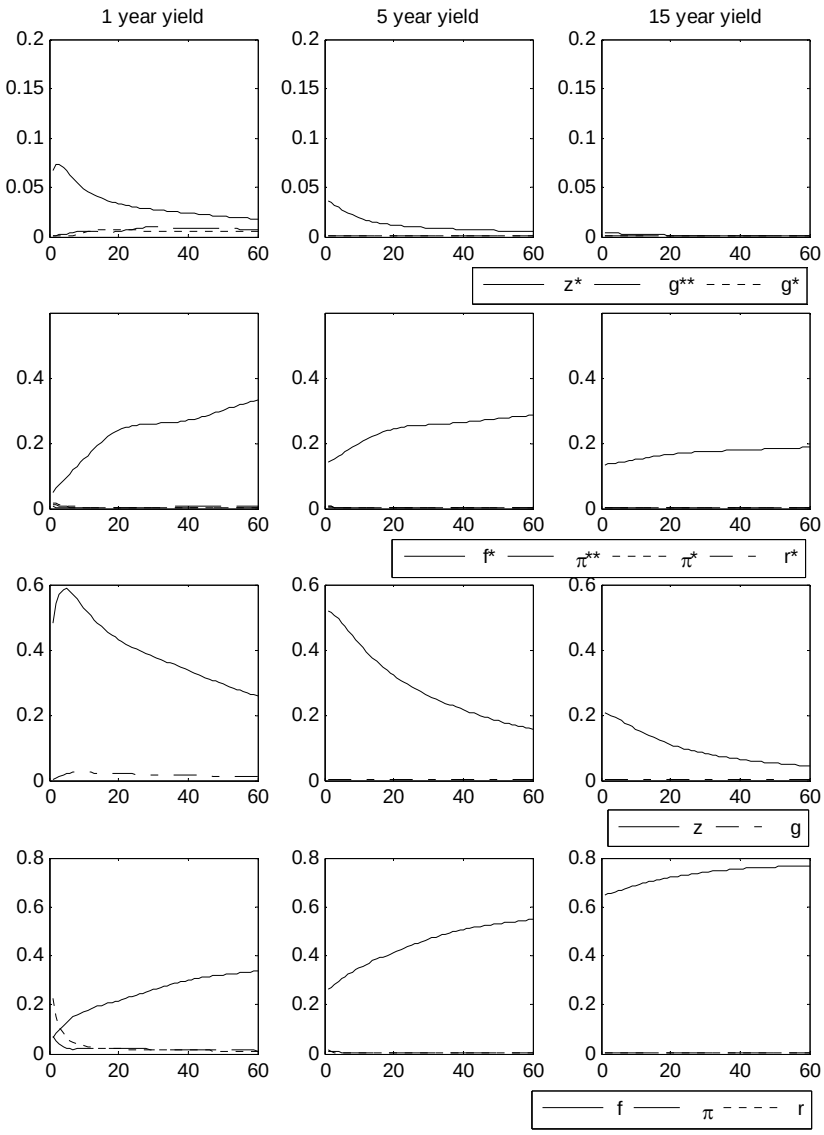
These panels show the proportion of the total variance of each variable explained by shocks to the various driving variables. Elapsed time is measured in quarters.

Figure 11a: Variance Decomposition: US yields



The plot shows the proportion of the total variance of the1, 5 and 15 year US yield explained by shocks to the various driving variables. Elapsed time is measured in quarter.

Figure 11b: Variance Decomposition: UK yields



The plot shows the proportion of the total variance of the 1, 5 and 15 year UK yield explained by shocks to the various driving variables. Elapsed time is measured in quarter.

Figure 12: Bond risk premium