The Strategic Power of Signal Acquisition

A. Menichini*and P. Simmons†

October 30, 2007

Abstract

A firm wants to borrow funds for a project whose resulting revenues are private information to the firm unless the investor undertakes costly monitoring. However the firm can buy a costly and publicly observed signal which reduces the variance of revenues (it is positively correlated with revenues). It can do this either before or after proposing a loan contract to the investors. We consider the case of commitment by the investors to their monitoring strategy so there is actually truthful reporting. We find that it is never optimal for the firm to acquire the signal before the contract is agreed but it may be optimal to acquire the signal either for sure or randomly after the contract is agreed. When the signal is acquired for sure there is only any monitoring following a good signal and never following a bad signal. When the signal is randomly acquired, then again low reports following a good signal are surely monitored and those following a bad signal never are. But if the randomisation leads to no signal then low reports are either monitored for sure (if high state revenues are high) or are never monitored. The precise form of the optimal signal and monitoring policy depends on the relative signal and monitoring costs, the interim stage profitability of the project in particular signal situations and the correlation between the signal and revenues.

Keywords: financial contracts, signalling, information gathering, monitoring.

JEL Nos: D82, D83

Introduction

A project owned by an agent needs external finance, the project has random returns but the realised return is only observed by the agent not by the lender. Both parties know the cost of the project and the probability distribution of returns. If there is a positive chance that the project will not meet the investment cost then in the absence of information revealing devices, the lender will refuse the loan since the agent will always cheat in revenue reports. This happens even if both parties are risk neutral and the expected return of the project is positive so that it is in the social interest for it to be undertaken. The CSV approach (and in reality auditing) overcomes this market failure to get a second best solution where (under commitment to audit) the lender can force the agent to truthfully reveal project revenue through the threat of audit. To be credible some audit of low revenue agent reports must actually happen and the expected cost of this is a deadweight loss which makes the outcome second best. In this commitment scenario the optimal audit policy is stochastic-the lender randomises his audit strategy to save on audit costs. What happens then is that the agent always truthfully reveals revenues and, if they are high, makes a high repayment; if they are low makes a low repayment report which is sometimes checked through audit and a low repayment return, and overall the lender gets a fair expected return.

There is a contemporary debate about the desirability of random audits centering about the issue of commitment. If the lender cannot commit to an audit policy, then at the interim stage, once realised revenues are private information to the agent, the audit strategy can be adjusted. Krasa-Villamil (2004) allow for a renegotiation between the two parties at this stage. For the original contract to be renegotiation proof at the interim stage there must be no incentive for any type of agent to recontract and in particular no incentive for any type of agent to reveal information to the lender about their type. This requires that the original contract should involve repayments that are constant across types: a pure debt contract which then takes us back to the

*Università di Salerno and CSEF. E-mail: amenichini@unisa.it

†Department of Economics and Related Studies, University of York. E-mail: ps1@york.ac.uk

market failure. There are other noncooperative ways in which the audit strategy can be adjusted: in Khalil-Parigi (1997) there is a noncooperative game in cheating by the firm and audit by the lender which results in random auditing.

But audit is not the only information revelation device. It occurs after the project has been financed, the revenues have been realised and an agent report on these has been made to the lender. An alternative would be an ex-ante signal or test of the project designed to improve both parties view of the likelihood of success (and hence the expected value) of the project. The signal can reveal information to all parties directly (it is public and credible) or can reveal information indirectly, when the signal is private, through the behaviour of the signal acquirer. The literature considers that latter case. Generally here the only reason for the agent to acquire a signal is to increase his strategic power relative to the lender and there is no information sharing. But in some circumstances, e.g. if an ex-ante experiment can show that low revenues are very likely and they are so low as to make the project socially undesirable, the experiment can be designed so that the agent abandons such projects and they are never undertaken (Cremer). In these cases generally the lender can force the agent to undertake the ex-ante experiment which then, if bad, makes the agent voluntarily renounce the project. But this is an extreme case.

Our interest is in less clear cut and hence more delicate cases. We have an ex-ante device (a signal) which is costly but which will improve the information of both parties about the distribution of revenues. In contrast to the literature, the signal is public and, whilst positively correlated with the agent type, only imperfectly reveals his type. The agent knows that the lender will always demand an ex-ante fair expected return. The agent knows that there is also a costly, credible audit mechanism to stop him cheating. Then should the agent choose to get the signal and improve the information of both parties? And if so when? Before or after an agreement has been reached with the lender which guarantees the latter a fair return and truthtelling? If the agent does get the signal, what is the effect on the optimal audit strategy? Since the signal is generally costly, it is only worth getting if it reduces the expected costs of audit. But which audits will be reduced? Those following good signals which make both parties think that the chance of high revenues has increased, or bad signals? And should the agent get the signal for sure or randomly? The possibility of signalling at different stages gives the agent some strategic power.

Our results show that:

1) it is better for the agent to agree terms with the lender before revealing his signal strategy and the results of any signal acquisition. This is partly because repayments and audit can then be conditioned on the subsequent signal which gives the agent greater strategic freedom to play off one state-signal combination against another. And partly it allows the agent to meet the overall exante fair return to the lender by cross subsidising between different state-signal combinations.

(2) Whenever it is optimal to acquire the signal (either for sure or randomly), low revenue reports following a bad signal are never audited, but low revenue reports following a good signal are audited for sure. It is optimal to acquire the signal if it's cost is not too high and if it is sufficiently informative. The improved information provided by the signal allows audit to be specialised into those state-signal combinations where the incentive to cheat is highest. If the signal is sometimes acquired but only randomly, then following a no signal outcome, the audit decision after a low revenue report is deterministic. In these circumstances with no signal if the expected return after a good signal is positive, then low reports are never audited, but if it is negative low reports are always audited. But when the signal is too costly or too uninformative it is never acquired and then there is strictly random monitoring of low state reports.

(3) When acquiring the signal is costly, there is a single critical variable which determines whether it is optimal to get the signal at all: this variable combines effects of the relative costs of signal acquisition and audit and the informativeness of the signal. It can be seen as the regret of wastefully monitoring high state reports as against wastefully paying for the signal.

When the signal completely reveals the state, it is worth acquiring the signal if its cost is less than the expected audit cost. If the signal is completely uninformative it is never acquired. For positive but imperfect correlation beween the signal and state, the relative cost of signal acquisition and monitoring are moderated by the imperfection with which the signal reveals the state.

(4) if the signal is free then it is always worth getting the signal.

(5) We find that in nearly all cases when it is optimal to acquire the signal The increased precision that it gives allows zero audit of low reports following a bad signal outcome and that when the signal is randomly acquired, if the randomisation results in no signal, then again sometimes there is zero audit. In this sense the

costly signal substitutes for costly monitoring. But, put better, the signal reduces the variance of revenues and increases the precision of which state will occur, which allows precise targeting of audit on signal report combinations where cheating is most likely.

Why does all this matter? At a theoretical level we have shown that signalling is socially useful-it does not just change the distribution of surplus between parties but allows both savings in overall information revelation costs (signalling+audit costs) and allows a wider range of socially desirable projects to be undertaken.

Some empirical examples of the type of scenario we have in mind might be:

(i) an entrepreneur wants funding to develop a new product. Success of the product depends partly on its technical characteristics and partly on consumer demand response. Truly realised revenues are private to the entrepreneur. The entrepreneur can build a prototype which is publicly observable to both the potential lender and himself. The realised prototype has qualities that are positively correlated with whether it will take off with consumers. Alternatively he could choose not to build the prototype but still seek funding.

(ii) a football team is interested in getting funding to buy a new player, they may choose to take him on trial. The performance of the player in games during the trial period is publicly observable but imperfectly correlated with the revenues of the team after the player has been purchased. The team could either just outright buy the player without trial or go through the trial.

(iii) a component is available and the entrepreneurs project involves using it in a novel product. The entrepreneur could commission a scientific study of the properties and cost of the component whose results would be publicly available or could go ahead without the study. In a related context the entrepreneur could pay for quality certification of the component from experts, partly here there is the idea that using an outside expert with a public certification of quality removes incentives for the agent to misreport signals.

(iv) A tax payer may have labour and interest income-low labour income signals low interest income. The taxpayer could hire an accountant to certify labour income, then make a report of total income to the tax authorities. The decision to audit depends on the "matching" between the taxpayer report and the presumed income calculated by the tax authorities on the basis of the signal provided by the taxpayer.

(v) A firm has to decide whether to get a rating company to produce a report on its financial situation. The actual revenues are the managers private information, but the shareholders decision to send auditors may be influenced by the rating company assessment.

(vi) the signal could be a mini 100% audit that the agent finances e.g. I'll pay for you to examine all my incomes exactly for a month then if I report they are bad you do not need to audit me. (Sansing, Accounting Review)

(vii) In Italy, firms have to periodically liquidate VAT on purchases and sales. Then, at the end of the year, the firm makes an overall income report which is correlated with the VAT report. IRS matches VAT liquidation with income report and decides whether to audit. If VAT liquidation is inconsistent with income report, say VAT liquidation indicates value of sales much higher than value of purchases, and reported end-of-year income is low, then IRS monitors for sure. If no liquidation is made periodically (no signal), but the firm has an income report, then this may also trigger monitoring, as IRS suspects the firm to evade VAT.

Other examples: references, testimonials, exams

When the agent is making the signal acquisition decision, he knows that he will be compelled to truthfully reveal the state through threat of audit and knows that in ex ante terms he has to make expected repayments that will meet the cost of the project. He then writes the contract (which includes the audit strategy since we in a commitment world) taking into account the effects of his signal acquisition strategy on the audit behaviour of the lender. In this sense he is using the strategic power on signal acquisition to offer a contract which the lender will just accept and which manipulates the audit behaviour of the lender.

The plan of the paper is to outline the assumptions and framework in Section 2, to analyse the case in which no signal is collected before or after the contract offer in section 3 and 4 respectively, the generic properties of the optimal contract is section 5, the optimal timing of information acquisition in section 6, and the otimal signal strategy in section 7. The last section concludes.

1 The Model Assumptions

An agent has a project costing I and giving a random return f_s, $s \in \{H, L\}$, with $f_H > f_L \geq 0$, with probability π_L and π_H respectively, $\pi_L = 1 - \pi_H$. To finance the project, the agent needs to raise funding from a risk neutral principal who cannot observe the realisation of the state, but can ex post audit the firm's report at a cost c_m. The return from the project is only realised when it is actually undertaken, which is necessarily after the loan has been made and so after the loan contract has been agreed. The agent learns the revenues of the project for free on their realisation, the principal can only discover them by audit. Audit reveals revenues with certainty and the result of it is verifiable by third parties.

The necessary investment cost exceeds the low state revenues:

$$f_L < I \tag{1}$$

which means that repayment of the loan to ensure a nonnegative expected return for the principal requires repayments to vary with state.

At any time the agent can acquire a costly signal σ which is positively but imperfectly correlated with the true state of nature, say $\sigma \in \{G, B\}$. Then π_{ij} represents the joint probability that $s = i$ and $\sigma = j$, and $\pi_{j|i} = \Pr(\sigma = j | s = i)$ is the conditional probability that signal $\sigma = j$ is received, given that state $s = i$ subsequently realises. $1 > \pi_{ii}, \pi_{jj} > \pi_{ij}, \pi_{ji} > 0$ defines the requirement of positive correlation between state and signal. Any signal that is acquired immediately becomes public information to both the agent and the lender. The cost of acquiring the signal is initially borne by the agent and is c_a. In contract problems between two parties, one party writes the contract subject to a participation constraint of the second party. This means that on the one hand who bears the direct cost of monitoring or auditing does not matter much but on the other hand the contract writer has the greater strategic power.

The joint probability distribution of the signal and revenues from the project is:

$s \backslash \sigma$	G	B	Σrow
H	π_{HG}	π_{HB}	π_H
L	π_{LG}	π_{LB}	π_L
Σcol	π_G	π_B	1

Remark 1 *Notice that* $1 > \pi_{ii}, \pi_{jj} > \pi_{ij}, \pi_{ji} > 0$ *imply the following:*
$\pi_{H|G} > \pi_H > \pi_{H|B}$
$\pi_{L|B} > \pi_L > \pi_{L|G}$

In the sequel the sign of the correlation between the state and the signal is important and is set by

$$\rho = \pi_{HG}\pi_{LB} - \pi_{HB}\pi_{LG}$$

The signal is fully informative if $\pi_{H|G} = 1$ and $\pi_{L|B} = 1$ which implies that the probability matrix is diagonal. In this case $r = 1$.[1] Conversely the signal is completely uninformative if $\pi_{H|G} = \pi_{H|B}$ and $\pi_{L|G} = \pi_{L|B}$ which implies $\pi_{HG} = \pi_{HB}$ and $\pi_{LG} = \pi_{LB}$ and then $r = \pi = 0$. Under our assumptions $\rho > 0$. So a good signal improves the chance that the state is actually high and vice versa. The signal reduces the amount of uncertainty over the state. Since for any random variable f, $var(f) = E_\sigma(var(f|\sigma)) + var(E_\sigma(f|\sigma))$, at the ex ante stage uncertainty is reduced in the presence of the signal.

Since the parties are risk neutral, acquiring the signal has no risk sharing gain, but the reduction in uncertainty may have incentive effects (generally in costly state verification models, if the variance of revenues falls, the amount of monitoring required to implement truthtelling also falls). In this sense we may expect to find that costly signal acquisition can substitute for some costly monitoring. Moreover it may allow some socially profitable projects to be undertaken which would not otherwise be possible.

We impose conditions which insure that the ex ante social benefits of the project cover the combined cost of signal acquisition and monitoring:[2]

$$\pi_H f_H + \pi_L f_L - I - \pi_L c_m - c_a > 0 \tag{2}$$

[1] The correlation coefficient is $r = \frac{[\pi_{HG}\pi_{LB} - \pi_{HB}\pi_{LG}]}{[(\pi_{LG}+\pi_{LB})(1-\pi_{LG}-\pi_{LB})(\pi_{HG}+\pi_{LG})(1-\pi_{HG}-\pi_{LG})]^{0.5}}$. So when $\pi_{HG} + \pi_{LB} = 1$, $r =$ $\pi_{HG}\pi_{LB}/[\ \pi_{LB}\ \pi_{HG}\ \pi_{HG}\ \pi_{LB}]^{0.5} = 1$

[2] Notice that assuming a fixed cost in signal acquisition and audit gives a bias against small projects which have low cost and low revenues.

and that, even after receiving a bad signal, having sunk the signal collection cost, the interim social benefits of the project are still positive:

$$\pi_{H|B}f_H + \pi_{L|B}f_L - I - \pi_{L|B}c_m > 0 \tag{3}$$

In itself what does this imply about the good signal state? Since we are in a commitment setting in which truthtelling will be induced, monitoring will only occur following low reports so in anticipation there is only a monitoring cost in low states. (3) means that even if the principal could remove funding after a bad signal has become public but before the state report, there is no incentive to do so.

Notice that (3) implies[3]

$$\pi_{H|G}f_H + \pi_{L|G}f_L - I - \pi_{L|G}c_m > 0 \tag{4}$$

and also that

$$\pi_{HB}f_H + \pi_{LB}(f_L - c_m) > (\pi_{HB} + \pi_{LB})I \tag{5}$$

which we can rewrite as

$$\pi_{HB}(f_H - f_L) - \pi_{LB}c_m > (\pi_{HB} + \pi_{LB})(I - f_L) > 0$$

so

$$\pi_{HB}(f_H - f_L) > \pi_{LB}c_m$$

from which

$$\begin{aligned} \pi_{HG}(f_H - f_L) &> \pi_{HB}c_m \\ (\pi_{HB} + \pi_{HG})(f_H - f_L) &> (\pi_{HB} + \pi_{LB})c_m \end{aligned}$$

and also

$$\pi_{HB}(f_H - f_L) > \pi_{HB}c_m \Rightarrow (f_H - f_L) > c_m$$

From a welfare view, the importance of acquiring the signal is to screen the projects more accurately, in particular with a good signal some projects will be undertaken which previously were socially unprofitable, given costly observation.

The contract date is fixed: at this time the agent must propose a contract detailing an audit strategy and repayments which must be made after the realisation of the returns of the project in different circumstances. The agent has limited liability in that ex post he can never be asked to repay more than the revenues of the project in any state. The agent has the opportunity of gathering a signal at any time. If he does so, the outcome of the information acquisition process is publicly revealed. If the signal is gathered before the contract is written, the principal knows the signal outcome and so contracted repayments and audit probabilities are conditioned on the signal outcome (G, B). If it is not gathered before the contract is written, the principal knows this as well (otherwise the principal would have observed the signal outcome) and also that the agent may subsequently decide to collect the signal. Consequently in this case the contract must include repayments conditional on each possible signal gathering decision and outcome. In terms of a general game tree we have:

[3] Notice that $\pi_{H|G} - \pi_{H|B} = \frac{\rho}{\pi_G \pi_B} = \pi_{L|B} - \pi_{L|G}$. Hence

$$\begin{aligned} & \pi_{H|G}f_H + \pi_{L|G}f_L - I - \pi_{L|G}c_m - (\pi_{H|B}f_H + \pi_{L|B}f_L - I - \pi_{L|B}c_m) \\ = \ & (\pi_{H|G} - \pi_{H|B})f_H + (\pi_{L|G} - \pi_{L|B})(f_L - c_m) \\ = \ & \frac{\rho}{\pi_G \pi_B}(f_H - f_L + c_m) > 0 \end{aligned}$$

which implies that $\pi_{H|G}f_H + \pi_{L|G}f_L - I - \pi_{L|G}c_m > (\pi_{H|B}f_H + \pi_{L|B}f_L - I - \pi_{L|B}c_m) > 0$

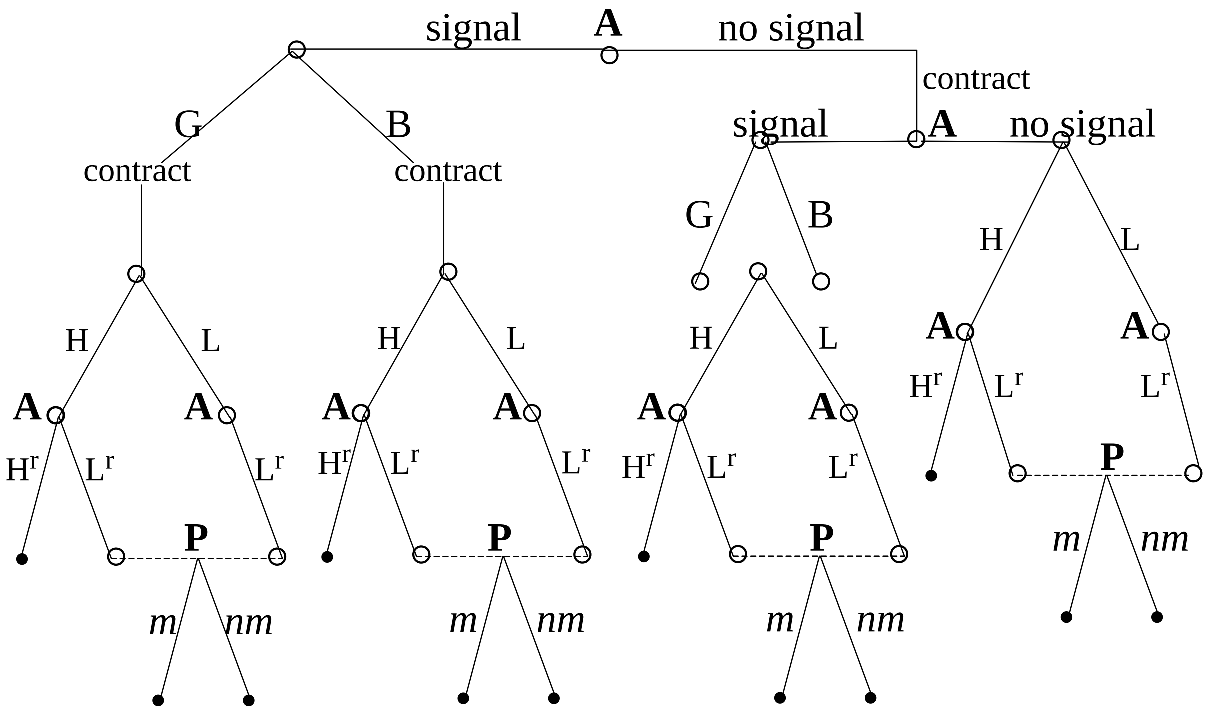


Figure 1

We refer to the two top branches of Fig. 1 as the left hand branch (the agent initially chooses to acquire the signal before offering a contract) and the right hand branch (the agent first offers the contract and then decides whether or not to get the signal). In the left hand branch initially the agent either gets a signal or does not-the principal knows this action since the signal is public. With a signal whose result is known to the principal, the agent then offers a contract conditional on the signal outcome. This is written on terms which the principal will accept. Then the true state occurs and the agent reports either H^r or L^r. Following the agents report and with knowledge of the value of the signal, the principal chooses whether to incur the costs of monitoring. Then, dependent on all these actions and outcomes, repayments are made. Conversely in the right hand branch the agent chooses to gather no signal but offers a contract to the principal (again on terms which the principal will accept) which allows for subsequent decisions by the agent to acquire the signal and by the principal to monitor. After the contract has been agreed the agent chooses the signal acquisition strategy. Next the true revenue state emerges and the agent makes a report of the revenue state to the principal. Using his information at that stage the principal decides whether or not to monitor and finally repayments are made given all earlier actions and events.

In each contract the variables written into the contract can depend on subsequent observable events and actions prior to the repayments actually being made. Hence $R_{sr\sigma}$ is the repayment due following a report r, a signal value $\sigma \in \{G, B\}$ and an audit which reveals that the state is s. R_{srN} is the analogous repayment with no signal acquisition. $R_{r\sigma}$ is the repayment with report r and signal σ, but with no audit. Similarly, R_{rN} is the analogous repayment with no signal acquisition. Notice that in the contracts offered in the left hand branch the contract variables are much more restricted than in the right hand branch - within one of the contracts, they cannot depend on the signal acquisition strategy since that has already been revealed to the principal at the contract writing stage.

We can solve the problem by backward induction, first following the right hand branch of Fig. 1 where no signal has been acquired at the contract date, calculate the optimal contract and signal acquisition plan in this branch. Then move to the left-hand branch of Fig. 1 and compute the optimal contracts given that a signal has been acquired before the contract date. Finally, by comparing these, we can see whether the agent will choose to acquire the signal before the contract date.

2 No signal acquired before the contract has been agreed

Following the right hand branch of Figure 1, a contract between principal and agent specifies repayments, audit probabilities and the probability with which information will be gathered. Since information gathering occurs prior to any report, repayments and audit probabilities can be conditioned on the realisation of the signal, if it is collected, and the state report. It is equivalent to define the optimal contract either by simultaneous choice of all of the contract variables or sequentially, first choosing the information gathering probability and then subsequently selecting repayments and audit probabilities within truthtelling constraints and a sequential rationality constraint ensuring that the selected information gathering probability will be used. Since there are three basic forms of information gathering (always, never or randomly gather information), we can find the overall optimal contract by successively computing contracts that are optimal for each of these forms of information gathering and then compare them.

The agent's payoff is

$$E\Pi_A = \alpha \sum_{\sigma} \pi_\sigma U_\sigma + (1-\alpha) U_N \tag{6}$$

where

$$\begin{aligned} U_N &= \pi_H (f_H - R_{HN}) + \pi_L [f_L - (1-m_N) R_{LN} - m_N R_{LLN}] && (7) \\ U_\sigma &= \pi_{H|\sigma} (f_H - R_{H\sigma}) + \pi_{L|\sigma} [f_L - (1-m_\sigma) R_{L\sigma} - m_\sigma R_{LL\sigma}] - c_a, \qquad \sigma = G, B && (8) \end{aligned}$$

(6) is the expected return of the agent with expectations being taken over the revenues of the project using the relevant probability distribution corresponding to the optimal signal strategy and over the probability of audit using the optimal audit strategy.

The principal's payoff is

$$E\Pi_P = \alpha \sum_{\sigma} \pi_\sigma PC_\sigma + (1-\alpha) PC_N - I \tag{9}$$

where

$$\begin{aligned} PC_N &: \pi_H R_{HN} + \pi_L [(1-m_N) R_{LN} + m_N (R_{LLN} - c_m)] && (10) \\ PC_\sigma &: \pi_{H|\sigma} R_{H\sigma} + \pi_{L|\sigma} [(1-m_\sigma) R_{L\sigma} + m_\sigma (R_{LL\sigma} - c_m)] && (11) \end{aligned}$$

stating that the expected return to the lender net of the expected audit cost must pay the investment cost.

To induce truthful reporting, repayments following a truthful high state report must not exceed repayments following a false low state report which can be audited with probability m:

$$\begin{aligned} R_{HN} &\leq m_N R_{LHN} + (1-m_N) R_{LN} && (12) \\ R_{H\sigma} &\leq m_\sigma R_{LH\sigma} + (1-m_\sigma) R_{L\sigma} && (13) \end{aligned}$$

Last, the agent has limited liability. Having zero wealth, this implies that repayments can never exceed realised income net of any signal acquisition cost c_a:

$$\begin{gathered} f_H - c_a \geq R_{H\sigma}, R_{LH\sigma} \\ f_H \geq R_{HN}, R_{LHN} \\ f_L - c_a \geq R_{L\sigma}, R_{LL\sigma} \\ f_L \geq R_{LN}, R_{LLN} \end{gathered} \tag{14}$$

The contract problem ($\mathcal{P}_{pre-signal}$ below) sets optimal values of the probability of acquiring the signal α after the contract has been agreed but before the choice of signal strategy and realisation of revenues, the monitoring strategy m conditional on the possible signal values ($\sigma \in \{G, B\}$, if acquired, or N if not acquired),[4] and the repayments $R_{sr\sigma}, R_{srN}, R_{r\sigma}, R_{rN}$ due at the end, conditional on the reported state $r \in \{H, L\}$, the result of any audit and the signal values:

$$\begin{gathered} \max\ E\Pi_A \\ \text{s.t. } E\Pi_P \geq 0 \end{gathered}$$

and subject to the incentive constraints (12) and (13) corresponding to each possible state of the signal -either G or B if acquired or N if not acquired, and to the limited liability conditions (14).

[4] Notice that this is a slight abuse of notation in the interest of simplicity, since previously $\sigma \in \{G, B\}$.

3 Signal acquired before the contract offer

If the agent collects the signal before the contract offer, then the principal knows at the time of the contract offer which type of agent she is facing (high signal, low signal or no signal agent). The contract that is written if a signal σ is collected at time 1 will use probabilities $\pi_{H|\sigma} = \pi_{H\sigma}/(\pi_{H\sigma} + \pi_{L\sigma}), \pi_{L|\sigma} = \pi_{L\sigma}/(\pi_{H\sigma} + \pi_{L\sigma})$.

In principle the agent could collect the signal for sure or randomly before the contract is signed, of course he could also choose not to collect the signal.

If he decides to get the signal for sure before the contract is signed, then, since the signal received by the agent is publicly observable, the contract problem at date 2 will depend on the signal acquired $\sigma = G, B$.

The contract problem conditional on having received signal σ, $\mathcal{P}^{\sigma}_{post-signal}$, is then to choose $R_{sr\sigma}, R_{r\sigma}, m_\sigma$ to

$$\begin{aligned} \max\ & U_\sigma \\ \text{s.t. } & PC_\sigma \geq I \\ & TT_\sigma \end{aligned}$$

and to the relevant limited liability conditions, with U_σ, PC_σ and TT_σ as defined in (8), (11) and (12).

Notice that we don't have the no signal state here. This state occurs when the firm has decided not to acquire the signal at $t = 1$ and to wait until after the contract offer to decide whether to acquire or not. Automatically then we shift into the right hand branch of Fig 1.

4 Generic Properties of the Optimal Contract

Whatever the information gathering strategy, optimally each of the contracts $\mathcal{P}_{post-signal}, \mathcal{P}_{pre-signal}$ will display some common features:

(i) the participation constraint must bind since otherwise it would be possible to reduce $R_{H\sigma}, R_{HN}$ without violating any of the constraints and make the agent better off;

(ii) the truthtelling constraints must all bind:

after allowing for the participation constraint, the monitoring cost is a deadweight loss which ultimately subtracts from the expected gain to the firm. So whatever the detailed structure of repayments, it is optimal to minimise the probability of monitoring. From this it follows both that there must be maximum punishment and that any relevant truthtelling constraints must bind. The typical truthtelling constraint has the form

$$m_\sigma(R_{LH\sigma} - R_{L\sigma}) \geq R_{H\sigma} - R_{L\sigma}$$

so if any truthtelling constraint is slack m_σ can be reduced. If there is not maximum punishment, $R_{LH\sigma}$ can be increased and m_σ reduced in order to raise the firms expected payoff. Thus $R_{LH\sigma} = f_H - c_a$, $R_{LHN} = f_H$. Since there is truthtelling, the punishment repayments are never actually paid and so do not enter either the objective function or the participation constraint. However they serve to police the agent and setting them at their maximal levels allow costly monitoring to be minimised. If $0 < \alpha < 1$ then each state σ, N sometimes occurs and the truthtelling constraint for every state is then relevant. If one of the truthtelling constraints were slack, the above argument applies. If $\alpha = \{0, 1\}$ then at least one state σ, N never occurs and the associated truthtelling constraint is irrelevant. Then without loss of generality it can be taken to bind. It follows that we can express the monitoring probabilities in terms of repayments as $m_\sigma = (R_{H\sigma} - R_{L\sigma}) / (f_H - c_a - R_{L\sigma})$, $m_N = (R_{HN} - R_{LN}) / (f_H - R_{LN})$.

(iii) in nonaudited low states the repayments $R_{L\sigma}, R_{LN}$ are set to give zero rent to the agent: $R_{L\sigma} = f_L - c_a, R_{LN} = f_L$. This gives the agent the minimal incentive to cheat: if he cheats there is a chance he can profit by $f_H - R_{L\sigma}$ when $m < 1$ if not actually audited. In fact he earns the minimal rent possible from cheating when $R_{L\sigma}$ is set at its highest level possible.

(iv) in audited low states there is zero rent to the agent: $R_{LL\sigma} = f_L - c_a, R_{LLN} = f_L$. This happens because in all circumstances the ex ante marginal value of funds is higher to the firm than to the investor, that is an extra dollar of investment ex ante has more than an expected dollars payoff to the firm.

Demonstration of these properties is in the appendix. Generally they match the properties of contracts with costly monitoring but no signalling possibilities (an example for maximum deterrence is Baron and Besanko (1984), for binding low state feasibility in a commitment setting, Jost (1996)).

Using these properties

$$\begin{aligned} m_G &= \frac{R_{HG} - f_L + c_a}{f_H - f_L} \\ m_B &= \frac{R_{HB} - f_L + c_a}{f_H - f_L} \\ m_N &= \frac{R_{HN} - f_L}{f_H - f_L} \end{aligned}$$

4.1 The contract problems conditional on timing of signal acquisition

Having established the properties of the contract which are common to both scenarios, we use these to write the two contract problems faced by the agent in terms of just high state repayments.

If no signal is acquired before the contract offer, the contract problem $\mathcal{P}_{pre-signal}$ becomes $\mathcal{P}'_{pre-signal}$, i.e. one of choosing R_{HG}, R_{HB}, R_{HN} to

$$\max_{R_{HG}, R_{HB}, R_{HN}} \alpha \left[\pi_H \left(f_H - c_a\right) - \pi_{HG} R_{HG} - \pi_{HB} R_{HB}\right] + (1-\alpha)\, \pi_H \left(f_H - R_{HN}\right)$$

$$\begin{aligned} \text{s.t. } \alpha &\left\{ \left(\pi_{HG} R_{HG} + \pi_{HB} R_{HB} + \frac{\pi_{LG}}{f_H - f_L} \left[\left(f_H - c_a - R_{HG}\right)\left(f_L - c_a\right) + \left(R_{HG} - f_L + c_a\right)\left(f_L - c_a - c_m\right)\right] \right.\right. \\ &\left. + \frac{\pi_{LB}}{f_H - f_L} \left[\left(f_H - c_a - R_{HB}\right)\left(f_L - c_a\right) + \left(R_{HB} - f_L + c_a\right)\left(f_L - c_a - c_m\right)\right] \right\} \\ &+ (1-\alpha) \left\{ \pi_H R_{HN} + \frac{\pi_L}{f_H - f_L} \left[\left(f_H - R_{HN}\right) f_L + \left(R_{HN} - f_L\right)\left(f_L - c_m\right)\right] \right\} = I \\ &f_H - c_a \geq R_{H\sigma} \\ &f_H \geq R_{HN} \\ &0 \leq \alpha \leq 1 \end{aligned}$$

If the signal is acquired before the agent offers the contract then, when the signal has the value $\sigma = G, B$, using the generic properties above, the contract problem $\mathcal{P}^{\sigma}_{post-signal}$ becomes $\mathcal{P}'^{\sigma}_{post-signal}$:

$$\max_{R_{H\sigma}} \pi_{H|\sigma} \left(f_H - R_{H\sigma}\right) - \left(1 - \pi_{L|\sigma}\right) c_a \tag{15}$$

$$\text{s.t. } \pi_{H|\sigma} R_{H\sigma} + \pi_{L|\sigma} \left[f_L - c_a - \frac{R_{H\sigma} - f_L + c_a}{f_H - f_L} c_m\right] = I \tag{16}$$

$$f_H - c_a \geq R_{H\sigma} \geq f_L - c_a \tag{17}$$

5 Is it ever optimal to first get the signal?

The next expression compares the expected utility the firm gets in the left hand branch with the expected utility it gets in the right hand branch:

$$\begin{aligned} &\max_{R_{HG}, R_{HB}, R_{HN}} \left\{ \alpha \left[\pi_G U_G + \pi_B U_B\right] + (1-\alpha)\, U_N | \alpha \left[\pi_G PC_G + \pi_B PC_B\right] + (1-\alpha) PC_N = I \right\} \\ &\geq \alpha [\pi_G \max_{R_{HG}} \{U_G | PC_G = I\} + \pi_B \max_{R_{HB}} \{U_B | PC_B = I\}] + (1-\alpha) \max_{R_{HN}} \{U_N | PC_N = I\} \end{aligned}$$

Essentially the contract problem in the right hand branch is less constrained than that in the left hand branch. The values of R_{HG}, R_{HB}, R_{HN} which solve the second problem with contracts after the signal satisfy the ex ante participation constraint (9) in the first problem. Hence they are feasible choices in the contract written prior to the signal. So the best choices in the contract written before the signal must be at least as good as in the contracts written after the signal. Hence the agent can never strictly prefer to get the signal before offering the contract to the principal. In this sense the power of deciding when to acquire the signal (if at all) gives the agent a strategic advantage.

There will be a strict advantage to the agent in the right hand branch if the optimal repayments in the right hand branch are actually infeasible in the left hand branch. This occurs iff there is cross subsidisation of the principal between different signal states e.g. following a high signal the principal receives some rent in the right hand branch which subsidises a loss following a low signal. If this type of cross subsidisation occurs, then getting the signal after the contract must dominate getting it before the contract.

6 Optimal signal strategy after the contract

Working through the remaining first order conditions and variables, given that no information has been gathered before the contract offer, we deduce that there are three possible forms of information gathering with associated high state repayments and monitoring:

i. acquire the signal for sure;

ii. never acquire the signal;

iii. play a random strategy for acquiring the signal.

Of course the choice between these depends on the signal acquisition cost, the informativeness of the signal and also on the relative costs of signal acquisition and monitoring. These are expressed through various critical expressions which determine which signal acquisition strategy is optimal.

A first critical factor is the net cost of signal acquisition after the reductions in expected cost from monitoring that are possible following acquisition of the signal as measured by the sign of

$$NC_a = \pi_H c_a - \rho c_m$$

When the probability matrix is diagonal (i.e. the signal completely reveals the state), ρ reduces to $\pi_{HG}\pi_{LB} = \pi_H \pi_L$ and then $NC_a = \pi_H (c_a - \pi_L c_m)$, where the term in parentheses is the difference between the cost of signalling and the expected cost of monitoring, given that there is truth-telling. When the probability matrix is not purely diagonal, the relative cost of signal acquisition and monitoring are moderated by the imperfection with which the signal reveals the state. If the signal is completely uninformative, then $\rho = 0$ and the regret of having paid to acquire a signal is $\pi_H c_a$ (the signal cost for states in which you don't wish to monitor anyway).

A second critical factor is the expected profit that remains in the project following a good signal

$$FEAS_G = \pi_{HG} f_H + (1 - \pi_{HG}) f_L - I - \pi_{LG} c_m - c_a$$

We can write the overall feasibility assumption (2)

$$\begin{aligned} 0 &< \pi_H f_H + \pi_L f_L - I - \pi_L c_m - c_a && (18) \\ &= FEAS_G + \pi_{HB}(f_H - f_L) - \pi_{LB} c_m && (19) \\ &= FEAS_G + FEAS_B && (20) \end{aligned}$$

which divides the ex ante profits of the project into those occurring after respectively a good and a bad signal. The sign of $FEAS_G$ is important in determining whether repayments can be set in some state-signal combinations to eliminate the incentive to cheat and hence the need for monitoring those state-signal combinations. If $FEAS_G > 0$ then there is a relatively high weight on high state revenues either because of the spread between f_H and f_L or because π_{HG} is high. The higher the spread of π_{HG} and π_{LG}, the more precise is the signal.

In the appendix we show when various forms of signal and monitoring strategy are optimal and derive the corresponding optimal signal acquisition and audit probabilities and high state repayments. Much depends on the critical factors above.

If $NC_a > 0$, it is optimal never to gather information ($\alpha = 0$). We have then the usual commitment contract (Border and Sobel (1987), Khalil and Parigi (1997)) with probabilities π_H, π_L. Truth-telling has to be policed by random monitoring and some rent is left to the firm in the high state, but zero rent in the low state whether audited or not. The firm can be induced never to collect the signal by setting the gain from signal acquisition $R_{HG} - R_{HB}$ equal to zero. Then the marginal benefit to the firm of giving even a tiny chance to signal acquisition is negative.

If $NC_a < 0$, it is optimal either to randomly gather information or to always gather information, depending on whether $c_a \geq 0$ and on the sign of $FEAS_G$.

If $c_a > 0$ and $FEAS_G \neq 0$, optimally $0 < \alpha < 1$. It is never optimal to leave the firm with any rent in the high state following a good signal realisation but it is always optimal to leave the largest rent possible to the firm in the high state following a bad signal realisation. Thus $R_{HG} = f_H - c_a$ and $R_{HB} = f_L - c_a$, which implies that $m_G = 1$ and $m_B = 0$. The intuition is as follows. Since $\rho > 0$, after a G signal it is more likely that state H is going to occur. Similarly, after a B signal it is more likely that $s = L$. Thus, in order to minimise monitoring costs while inducing participation, it is best to monitor after a G signal, where it is less likely that state L is going to occur ($\pi_{H|G} > \pi_{L|G}$), and never after a B signal. The powerful precision of the signal allows accurate targeting of monitoring.

The decision to monitor no signal states, when the signal is randomly acquired, depends instead on the sign of

$$FEAS_G = \pi_{HG} f_H + (1 - \pi_{HG}) f_L - I - \pi_{LG} c_m - c_a$$

the expected social gain of the project net of signal acquisition and monitoring costs at the interim stage after a G signal. If, as the result of mixed play of signal acquisition, the signal is not acquired when $FEAS_G > 0$, the expected social gain after a G signal is sufficiently high to make it unnecessary to monitor. Thus, the firm receives a rent ($R_{HN} = f_L$) and $m_N = 0$. If $FEAS_G < 0$, the expected social gain after a G signal is too low. Thus it is necessary to monitor also when no signal is acquired. The firm receives no rent ($R_{HN} = f_H$) and $m_N = 1$.

If, when $NC_a < 0$, either $FEAS_G = 0$ and/or $c_a = 0$, it is optimal to have full information gathering. If $FEAS_G = 0$ and $c_a > 0$, the firm only receives rent in the high state following a bad signal and is then never monitored. In this case the firm must have the incentive to always pay for the signal and this is assured by setting R_{HN} sufficiently high to punish the firm if it does not acquire the signal. But if $c_a = 0$ and $FEAS_G > 0$ the firm is always monitored following a bad signal but there is random monitoring following a good signal and then on average the firm receives a rent ($R_{HG} < f_H - c_a$). Finally if $c_a = 0$ and $FEAS_G < 0$ the firm receives some rent following a bad signal but zero rent following a good signal.

Finally there is a borderline case in which $NC_a = 0$. Then it turns out that an identical maximum payoff can be attained either from not collecting the signal at all but randomly monitoring high states, or from acquiring the signal randomly, and when it is acquired, setting $m_G = 1, m_B = 0$ and $0 < m_N < 1$. This is true whatever the sign of $FEAS_G$.

The formal detail of these general points is summarised in the following propositions.

Proposition 1 *When $NC_a > 0$, it is optimal to gather no information ($\alpha = 0$). The optimal contract has:*

$$R_{HN} = \frac{(I - \pi_L f_L)(f_H - f_L) - f_L \pi_L c_m}{\pi_H (f_H - f_L) - \pi_L c_m}$$

the value of the objective function is

$$U_{\alpha=0} = \frac{\pi_H (f_H - f_L)(\pi_H f_H + \pi_L f_L - I - \pi_L c_m)}{\pi_H (f_H - f_L) - \pi_L c_m} \tag{21}$$

$$m_N = \frac{R_{HN} - f_L}{f_H - f_L} = \frac{I - f_L}{\pi_H (f_H - f_L) - \pi_L c_m} < 1$$

This requires values of R_{HG}, R_{HB} such that

$$FOC_\alpha = -\frac{\pi_H (f_H - f_L) c_a}{\pi_H (f_H - f_L) - \pi_L c_m} + \frac{c_m \rho (R_{HG} - R_{HB})}{\pi_H (f_H - f_L) - \pi_L c_m} \leq 0$$

However by setting $R_{HG} = R_{HB}$ this is always possible.

This is the standard commitment result.

Proposition 2 *When $NC_a < 0$, it is optimal either to randomly gather information or to always gather information, depending on whether $c_a \geq 0$ and on the sign of $FEAS_G$.*

1. *If $c_a > 0$, $R_{HG} = f_H - c_a$ and $m_G = 1$; $R_{HB} = f_L - c_a$ and $m_B = 0$. However R_{HN}, m_N and α vary with the sign of $FEAS_G$. In particular:*

   (a) *If $FEAS_G > 0$, it is optimal to randomly gather information ($0 < \alpha < 1$ and increasing in c_m); $R_{HN} = f_L$ and $m_N = 0$. This gives:*

$$\alpha = \frac{I - f_L}{\pi_{HG}(f_H - f_L) - \pi_{LG}c_m - c_a} \tag{22}$$

$$U_{MSa} = \frac{(f_H - f_L)\{\pi_{HB}(I - f_L) + \pi_H(\pi_{HG}f_H + (1 - \pi_{HG})f_L - I - \pi_{LG}c_m - c_a)\}}{\pi_{HG}(f_H - f_L) - \pi_{LG}c_m - c_a} \tag{23}$$

   (b) *If $FEAS_G = 0$, it is optimal to always gather information ($\alpha = 1$); $R_{HN} = f_H$, $R_{HB} = f_L - c_a$, $R_{HG} = f_H - c_a$ with $m_N = 1$, $m_B = 0$, and $m_G = 1$. The value of the objective function is*

$$U^b_{\alpha=1} = \pi_{HB}(f_H - f_L) \tag{24}$$

   (c) *If $FEAS_G < 0$, it is optimal to randomly gather information ($0 < \alpha < 1$ and decreasing in c_m); $R_{HN} = f_H$ and $m_N = 1$. This gives*

$$\alpha = \frac{Ef - I - \pi_L c_m}{\pi_{HB}(f_H - f_L) - \pi_{LB}c_m + c_a} \tag{25}$$

$$U_{MSb} = \frac{(Ef - I - \pi_L c_m)\pi_{HB}(f_H - f_L)}{\pi_{HB}(f_H - f_L) - \pi_{LB}c_m + c_a} = \alpha\pi_{HB}(f_H - f_L) \tag{26}$$

2. *If $c_a = 0$, it is optimal to always gather information. However, the repayment structure and the signal dependent monitoring probabilities depend on the sign of $FEAS_G$.*

   (a) *when $FEAS_G > 0$, $R_{HB} = f_L$ and $m_B = 0$;*

$$R_{HG} = \frac{I(f_H - f_L) - ((1 - \pi_{HG})(f_H - f_L) + \pi_{LG}c_m)f_L}{\pi_{HG}(f_H - f_L) - \pi_{LG}c_m} < f_H$$

$$m_G = \frac{R_{HG} - f_L}{f_H - f_L} = \frac{I - f_L}{\pi_{HG}(f_H - f_L) - \pi_{LG}c_m} < 1$$

   The value of the objective function is

$$U^a_{\alpha=1} = \frac{(f_H - f_L)\{\pi_{HG}(\pi_H f_H + \pi_L f_L - I) - \pi_H\pi_{LG}c_m\}}{\pi_{HG}(f_H - f_L) - \pi_{LG}c_m} \tag{27}$$

   This requires

$$FOC_\alpha = \frac{\rho(R_{HN} - f_L)c_m}{\pi_{HG}(f_H - f_L) - \pi_{LG}c_m} > 0$$

   which always holds-remember $c_a = 0$ here.

   (b) *when $FEAS_G = 0$, we have case b analysed in part 1 of Proposition 2;*

   (c) *when $FEAS_G < 0$, $R_{HG} = f_H$, and $m_G = 1, 0 < m_B < 1$;*

$$R_{HB} = -\frac{(\pi_{HG}f_H + \pi_L f_L - I - \pi_{LG}c_m)(f_H - f_L)}{\pi_{HB}(f_H - f_L) - \pi_{LB}c_m} - \frac{\pi_{LB}c_m f_L}{\pi_{HB}(f_H - f_L) - \pi_{LB}c_m}$$

$$m_B = -\frac{\pi_{HG}f_H + (1 - \pi_{HG})f_L - I - \pi_{LG}c_m}{\pi_{HB}(f_H - f_L) - \pi_{LB}c_m} > 0$$

The value of the objective function is

$$U^c_{\alpha=1} = \frac{\pi_{HB}(f_H - f_L)(\pi_H f_H + \pi_L f_L - I - \pi_L c_m)}{\pi_{HB}(f_H - f_L) - \pi_{LB} c_m} \tag{28}$$

This requires

$$foc\alpha = \frac{(\pi_{LB}\pi_{HG} - \pi_{LG}\pi_{HB})(f_H - R_{HN})c_m}{\pi_{HB}(f_H - f_L) - \pi_{LB}c_m} > 0$$

This condition is automatically satisfied by $NC_a < 0$.

The above proposition covers the case in which $NC_a < 0$. When $NC_a = 0$, we have the following proposition.

Proposition 3 *If $NC_a = 0$ with any sign of $FEAS_G$, any combination of $R_{HN} \in [f_H, f_L]$ and $\alpha \in [0,1]$ which satisfy the participation constraint*

$$\alpha(\pi_{HG} f_H + (1 - \pi_{HG}) f_L - c_a - c_m \pi_{LG}) + (1 - \alpha)\left(\pi_H R_{HN} + \pi_L (f_L - \frac{R_{HN} - f_L}{f_H - f_L} c_m)\right) = I \tag{29}$$

is possible. The value of the objective function is thus

$$U_{MSc} = \frac{(f_H - f_L)(\pi_H f_H + \pi_L f_L - I - c_m \pi_L)\pi_H}{\pi_H(f_H - f_L) - \pi_L c_m} \tag{30}$$

which is identical to that with no signal acquisition,$\alpha = 0$.

Notice that differentiating (29) the sign of $\frac{d\alpha}{dR_{HN}}$ is the opposite to the sign of $FEAS_G$:

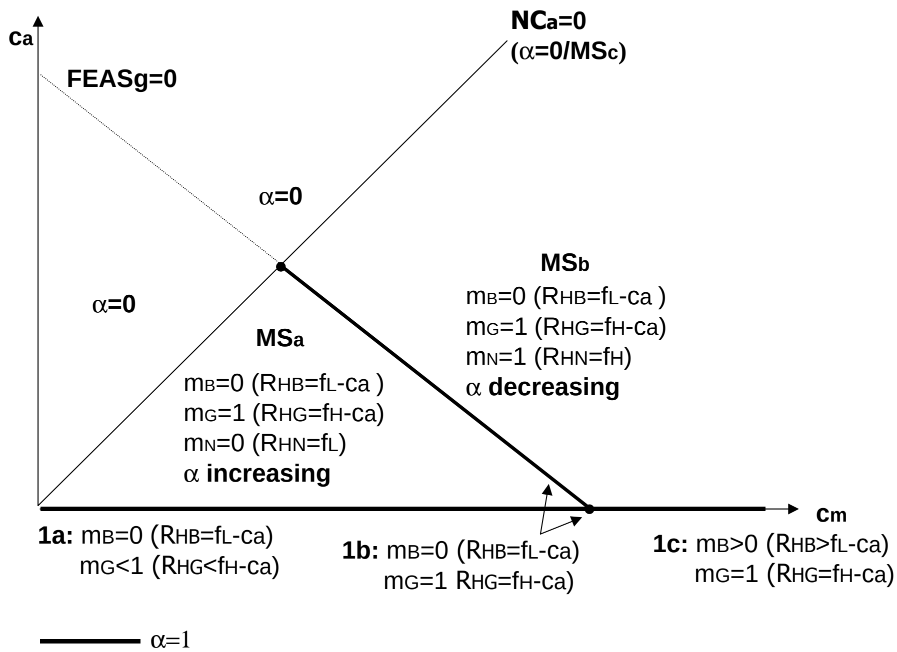
$$\frac{d\alpha}{dR_{HN}} = \frac{-\left[(f_H - f_L)\pi_H - \pi_L c_m\right](f_H - f_L)\left[\pi_{HG} f_H + (1 - \pi_{HG}) f_L - c_m \pi_{LG} - c_a\right]}{\left\{(c_a - \pi_{HG}(f_H - R_{HN}) + \pi_{HB}(R_{HN} - f_L))(f_H - f_L) + \left[\pi_{LG}(f_H - R_{HN}) - \pi_{LB}(R_{HN} - f_L)\right] c_m\right\}^2};$$

This means that when $FEAS_G < 0$, increasing the probability of getting the signal is associated with a higher return R_{HN}, and thus a higher probability of monitoring m_N.

One interesting example is when the signal is totally informative: if $\sigma = G$, then state H is going to occur and although $m_G = 1$, there is no need to monitor an L report, because such a report is never made. If $\sigma = B$, then state L is going to occur, which is not monitored either because optimally $m_B = 0$ results optimally. What about monitoring no signal states? Our results say that $m_N = 1$ when $FEAS_G < 0$ and zero otherwise. With a perfectly informative signal, $FEAS_G = \pi_{HG} f_H + (1 - \pi_{HG}) f_L - I - \pi_{LG} c_m - c_a = \pi_H f_H + \pi_L f_L - I - c_a \gg 0$, which implies that only one of the two mixed strategy cases arises. Thus ex-ante information gathering totally substitutes for ex post monitoring, even when randomly gathered. For this result, need perfect correlation between signal and state. If no perfect correlation, then there is some monitoring in G signal states which turn out to be low.

Propositions 1 to 3 are summarised in the following table and diagram:

$FEAS_G$	exist	NC_a	Optimum
> 0	$\alpha = 0, ms_a, ms_c, \alpha = 1a$	> 0	$\alpha = 0$
		$= 0$	$\alpha = 0/ms_c$
		< 0	ms_a ($1a$ if $c_a = 0$)
$= 0$	$\alpha = 0, ms_c, \alpha = 1b$	> 0	$\alpha = 0$
		$= 0$	$\alpha = 0/ms_c/1b$
		< 0	$1b$
< 0	$\alpha = 0, ms_b, ms_c, \alpha = 1c$	> 0	$\alpha = 0$
		$= 0$	$\alpha = 0/ms_c$
		< 0	ms_b ($1c$ if $c_a = 0$)



The diagram shows how the optimal signal acquisition and monitoring strategy varies with the costs of the signal and of monitoring for a fixed joint probability distribution of revenue and the signal. For joint costs above $FEAS_G = 0$ randomised signal acquisition post-contract is never optimal and depending on the relative costs of signalling and monitoring, either the signal is always or never acquired. For relatively low joint costs (when $FEAS_G > 0$) it is never optimal to get the signal for sure but either to acquire it randomly or never.

6.1 Signal strategy as a function of acquisition and monitoring costs

Although the above diagram shows how the optimal monitoring and signal acquisition strategy vary qualitatively with $NC_a, FEAS_G$, it is of interest to see how α varies with c_a and $c_{m.}$

6.1.1 Signal strategy as a function of monitoring costs

When $NC_a < 0$, the appendix shows that

$$\frac{\partial \alpha}{\partial c_m} > 0;\ \frac{\partial^2 \alpha}{\partial c_m^2} > 0$$

if $FEAS_G \geq 0$ (i.e. when U_{MSa} is optimal), while

$$\frac{\partial \alpha}{\partial c_m} < 0;\ \frac{\partial^2 \alpha}{\partial c_m^2} < 0$$

if $FEAS_G > 0$ (i.e. when U_{MSb} is optimal).

Moreover, for given $c_a > 0$, $\alpha(c_m)$ reaches a maximum at $\alpha = 1$ when $FEAS_G = 0$.

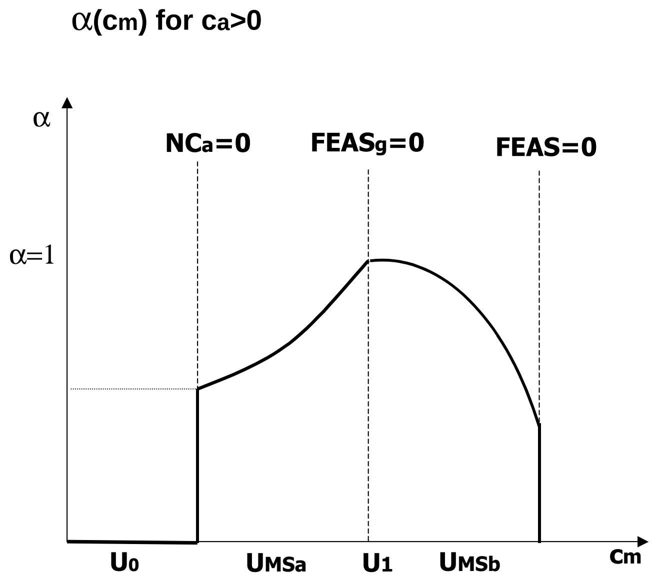
Thus, for given values of the other parameters, so long as c_a is not too high, then as c_m increases from zero, initially $NC_a > 0$ and optimally $\alpha = 0$. Then, after a critical value of c_m, NC_a switches sign from positive to negative, $FEAS_G$ is positive and U_{MSa} becomes the optimal strategy. Further increases in c_m lead to increases in α until $\alpha = 1$, when $FEAS_G = 0$. At this value of c_m, $U_{MSa} = U_{MSb} = U_{MSc}$ and the optimal strategy switches into U_{MSb} with decreasing values of α as c_m continues to rise.

For the intermediate range of monitoring cost, U_{MSa} is the optimal strategy, reports without the signal are never monitored because ex ante there is sufficient revenue in the good signal states ($FEAS_G > 0$) to meet the participation constraint. As the monitoring cost rises, the optimal probability of getting the signal also rises. This increases the amount of monitoring of good signal cases and also raises monitoring cost. But since

$FEAS_G > 0$, for each good signal case that is monitored the investor gets a surplus that covers the extra monitoring cost.

For high values of the monitoring cost, $FEAS_G < 0$ and optimally $m_N = 1$. Here there is insufficient revenue from monitoring good signal types to cover the investment cost and so all no signal types have to be monitored.

As c_m increases in this range, the amount of monitoring and the monitoring cost increase, both because α falls and because the per-unit monitoring cost is increased. Overall this is optimal because the reduction in α reduces the signal acquisition cost.



6.1.2 Signal strategy as a function of acquisition costs

We can draw a similar diagram showing how α varies with c_a.

When $NC_a < 0$, the appendix shows that

$$\frac{\partial \alpha}{\partial c_a} > 0; \ \frac{\partial^2 \alpha}{\partial c_a^2} > 0$$

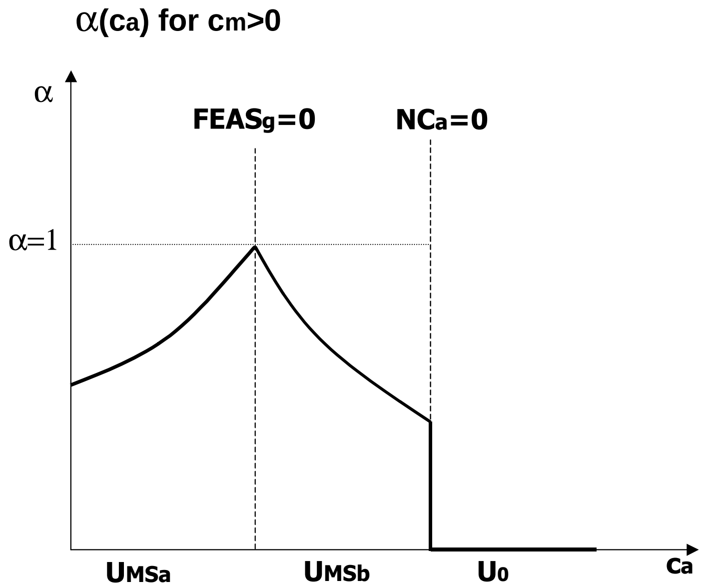
if $FEAS_G \geq 0$ (i.e. when U_{MSa} is optimal), while

$$\frac{\partial \alpha}{\partial c_a} < 0; \ \frac{\partial^2 \alpha}{\partial c_a^2} > 0$$

if $FEAS_G > 0$ (i.e. when U_{MSb} is optimal).

Moreover, for given $c_m > 0$, $\alpha(c_a)$ reaches a maximum at $\alpha = 1$ when $FEAS_G = 0$.

Thus, for given values of the other parameters, so long as c_m is not too high, then as c_a increases from zero, initially α increases in c_a (for $c_a = 0$, $\alpha = 1$. Then for $c_a > 0$, α drops to some positive value and then starts increasing). U_{MSa} is the optimal strategy with $FEAS_G > 0$ and $NC_a < 0$. Then, after a critical value of c_a, $FEAS_G$ switches sign from positive to negative, α reaches its maximum $(= 1)$ and U_{MSb} becomes the optimal strategy. Further increases in c_a lead to decreases in α until $\alpha = 0$, when $NC_a = 0$. At this value of c_a, $U_{MSc} = U_{\alpha=0}$. For higher values of c_a, $\alpha = 0$ becomes the optimal strategy.



In the appendix we also plot the optimal payoff for the agent as a function of c_a and c_m.

6.2 Cross-subsidisation

The ex ante participation constraint can be written as

$$\alpha\left\{\pi_G PC_G + \pi_B PC_B\right\} + (1-\alpha) PC_N = I$$

When $NC_a < 0$ and $FEAS_G \geq 0$ if it is optimal to randomly gather information as in MSa, the participation constraints following different signals are:

$$PC_G = \pi_{H|G} f_H + \pi_{L|G} f_L - \pi_{L|G} c_m - c_a \gtreqless I$$
$$PC_B = f_L - c_a < I$$
$$PC_N = f_L < I$$

Knowing the sign of PC_B and PC_N, we deduce that $PC_G > 0$.

Similarly when $NC_a < 0$ and $FEAS_G < 0$ it is optimal to randomly gather information as in MSb and the participation constraints are

$$PC_G = \pi_{H|G} f_H + \pi_{L|G} f_L - \pi_{L|G} c_m - c_a \gtreqless I$$
$$PC_B = f_L - c_a < I$$
$$PC_N = \pi_H f_H + \pi_L f_L - \pi_L c_m > I$$

$PC_G \gtreqless I$ can be written as $\pi_{HG} f_H + (1-\pi_{HG}) f_L - I - \pi_{LG} c_m - c_a \gtreqless \pi_B (f_L - c_a)$. Since in MSb $FEAS_G < 0$, and the $RHS > 0$ (for $f_L \geq c_a$), this implies that the inequality is negative, i.e. $PC_G < I$ and the principal gets a loss in this state-signal combination. Moreover $PC_G + PC_B - I = FEAS_G < 0$ so there is cross subsidisation from the signal received states to the no signal state.

Hence we know that there is cross subsidisation and the agent will always strictly prefer to offer the contract prior to revealing whether any signal has been acquired (and its value) to the principal. So the right hand branch of the game tree strictly dominates the left hand branch, i.e. the contract with ex-post information gathering strictly dominates the one with ex-ante information gathering. This means that before deciding which information acquisition strategy to adopt, full, random or no information gathering, the agent will always wait until after the contract offer. So the overall conclusion is that it may be optimal to get the signal either for sure or randomly but only after the contract has been agreed.

The intuition is as follows. There are incentive effects of the signal which work indirectly through the participation constraint of the principal and the truth-telling constraints on the agent. In particular, when the signal is collected after the contract offer, the principal's participation is to be satisfied ex ante. Thus, it is possible to minimise observation cost by monitoring only when it is less likely that the low state occurs, i.e. when $\sigma = G$, and never when the low state is more likely to occur, i.e. $\sigma = B$.

When the signal is collected before the contract offer, the participation constraints are to be satisfied for each signal state and thus need to monitor both after a good signal state $\sigma = G$ and after a bad signal state $\sigma = B$. The saving in monitoring cost after a good signal is more than offset by the higher monitoring cost that have to be incurred after a bad signal.

7 Conclusions

We have considered a contract problem in which a firm proposes a project to the investor and who subsequently has risky revenues from which to repay the investor. The investor can pay a cost to observe the private revenues of the firm. The firm can choose the contract to offer (repayments in various states, the audit strategy of the lender) and can also choose at any time to get a costly, and (imperfectly) informative signal about future revenues, whose realisation is freely available to the investor. The question is should the firm get the signal and if so when? What impact does the signal have on the audit strategy and the structure of repayments in the contract?

There are three factors involved in the signalling. Firstly since the acquisition of the signal reduces the uncertainty about future revenues, it allows the contract to be written with more precise separation of the possible types of firm. Secondly since both the signal and ex post monitoring of low state revenue reports are costly, it allows for a trade off in control devices between paying to reduce uncertainty by getting a costly signal and paying for monitoring. Thirdly there are incentive effects of the signal which work indirectly through the participation constraint of the investor and the truth telling constraint on the firm. For example if the signal is obtained, when it is good, the investors participation constraint is relaxed which allows a lower use of costly monitoring of low state reports.

The literature does not allow for either the endogenous timing of acquisition of the signal, or for imperfect correlation between the signal and the resulting revenues. This yields elements of corner solution type results e.g. if there is a bad signal prior to the contract being agreed and the investor proposes the contract, it is then optimal to write such an unattractive contract that it is refused.

We find that when the agent writes the contract and any signal (which in general is imperfectly correlated with the state of revenues) acquired is public information to the lender, there is no gain in the agent acquiring the signal prior to offering the contract to the principal. It may be optimal to always acquire the signal after the contract has been agreed with the investor, or to acquire the signal randomly at this time. But sometimes it is optimal for the firm to remain uninformed about his type after the contract has been offered, and to hide his type by never collecting the signal. Which of these is best depends on the relative costs of monitoring state reports and of acquiring the signal, on the correlation between the signal and revenues and on the profitability of the project at the interim stage in various signal situations.

Thus our results on optimal timing of signal acquisition suggest that models in which the agent must acquire the signal prior to the contract are only optimal if the institutional context enforces this. And in contrast to results in models in which signals perfectly reveal the state, one can do better by allowing the agent to randomise signal acquisition.

In our framework the signal is not purely strategic, it has strategic implications but also since it is informative to all parties, it is socially valuable. We have assumed that if acquired, the signal is publicly available. An alternative scenario is that in which if the firm acquires the signal it is private unless the firm chooses to reveal it. This raises a lot of new issues: for example can the firm cheat on signal revelation as well as on revenue reports? Moreover we have assumed commitment to auditing by investors which allows us to impose truthful revenue reports. What happens without commitment?

A No Signal acquired Prior to the Contract

A.1 Generic Properties of the Contract

(i) the participation constraint must bind since otherwise the high state repayments can be reduced

(ii) and (iii) In $\mathcal{P}_{pre-signal}$ the truthtelling constraints must bind and there must be maximum punishment $R_{HL\sigma} = f_H - c_a, R_{HLN} = f_H$.

The truthtelling constraint is

$$m_\sigma(R_{HL\sigma} - R_{L\sigma}) \geq R_{H\sigma} - R_{L\sigma} > 0 \qquad \sigma = G, B, N$$

The strict inequality holds since if for any σ, $R_{H\sigma} = f_L \geq R_{L\sigma}$ expected repayments are too low to satisfy the participation constraint since $I > f_L$.

It then follows that both $R_{HL\sigma} \geq R_{H\sigma}$ and also since the LHS>0, $m_\sigma > 0$. The binding participation constraint has the form

$$\begin{aligned} &\alpha \sum_\sigma \{\pi_{H\sigma} R_{H\sigma} + \pi_{L\sigma}\left(\left(1-m_\sigma\right) R_{L\sigma} + m_\sigma\left(R_{LL\sigma} - c_m\right)\right)\} \\ &+ \left(1-\alpha\right)\left[\pi_H R_{HN} + \pi_L\left(\left(1-m_N\right) R_{LN} - m_N\left(R_{LLN} - c_m\right)\right)\right] = I \end{aligned}$$

Hence solving for the expected payoffs in terms of the expected monitoring cost and investment size and putting the result in the objective function

$$\begin{aligned} U &= \alpha \sum_\sigma \{\pi_{H\sigma}\left(f_H - R_{H\sigma}\right) + \pi_{L\sigma}\left(f_L - \left(1-m_\sigma\right) R_{L\sigma} - m_\sigma R_{LL\sigma}\right) - c_a\} \\ &\quad + \left(1-\alpha\right)\{\pi_H\left(f_H - R_{HN}\right) + \pi_L\left(f_L - \left(1-m_N\right) R_{LN} - m_N R_{LLN}\right)\} \\ &= Ef - \alpha c_a - \alpha\pi_{L\sigma} m_\sigma c_m - (1-\alpha)\pi_L m_N c_m - I \end{aligned}$$

in which case whatever the values of the repayments, the agent is best off with the lowest probability of monitoring. This just reflects that part of the deadweight loss of the asymmetric information arises from the costly monitoring, the agent is better off the lower this deadweight loss. The optimum must always minimise the probability of monitoring.

If either truthtelling is slack or there is less than maximum punishment the level of monitoring for any signal strategy can be reduced by either increasing the punishment repayment or by imposing binding truthtelling.

(iv) in nonaudited low state repayments there is zero surplus to the firm:

after imposing binding truthtelling constraints and maximum punishment for cheating following monitoring, the contract problem becomes

$$\begin{aligned} &\max\ \alpha\Bigg\{\{\pi_{HG}\left(f_H - R_{HG}\right) + \pi_{LG}\left(f_L - \left(1 - \frac{R_{HG} - R_{LG}}{f_H - c_a - R_{LG}}\right) R_{LG} - \frac{R_{HG} - R_{LG}}{f_H - c_a - R_{LG}} R_{LLG}\right) \\ &+ \pi_{HB}\left(f_H - R_{HB}\right) + \pi_{LB}\left(f_L - \left(1 - \frac{R_{HB} - R_{LB}}{f_H - c_a - R_{LB}}\right) R_{LB} - \frac{R_{HB} - R_{LB}}{f_H - c_a - R_{LB}} R_{LLB}\right) - c_a\}\Bigg\} \\ &+ (1-\alpha)\Bigg\{\pi_H\left(f_H - R_{HN}\right) + \pi_L\left[f_L - \left(1 - \frac{R_{HN} - R_{LN}}{f_H - R_{LN}}\right) R_{LN} - \frac{R_{HN} - R_{LN}}{f_H - R_{LN}} R_{LLN}\right]\Bigg\} \\ \text{s.t.}\quad &\alpha\Bigg\{\pi_{HG} R_{HG} + \pi_{HB} R_{HB} + \pi_{LG}\left[\left(1 - \frac{R_{HG} - R_{LG}}{f_H - c_a - R_{LG}}\right) R_{LG} + \frac{R_{HG} - R_{LG}}{f_H - c_a - R_{LG}}\left(R_{LLG} - c_m\right)\right] + \\ &\pi_{LB}\left(1 - \frac{R_{HB} - R_{LB}}{f_H - c_a - R_{LB}}\right) R_{LB} + \frac{R_{HB} - R_{LB}}{f_H - c_a - R_{LB}}\left(R_{LLB} - c_m\right)\Bigg\} \\ &+ (1-\alpha)\Bigg\{\pi_H R_{HN} + \pi_L\left(1 - \frac{R_{HN} - R_{LN}}{f_H - R_{LN}}\right) R_{LN} + \frac{R_{HN} - R_{LN}}{f_H - R_{LN}}\left(R_{LLN} - c_m\right)\Bigg\} \geq I \end{aligned}$$

Then for example the FOC wrt R_{LG} is

$$\alpha\pi_{LG}\{(-(f_H - c_a - R_{LLG})\frac{(f_H - c_a - R_{HG})}{(f_H - c_a - R_{LG})^2} + \lambda\frac{(f_H - c_a - R_{LLG} + c_m)(f_H - c_a - R_{HG})}{(f_H - c_a - R_{LG})^2}\}$$

This is negative and hence $R_{LG} = f_L - c_a$. Similar arguments hold for R_{LB} and R_{LN}. Hence low state nonaudited repayments are set at the highest level possible to give zero rent to the firm.

(v) There is zero rent in audited low states $R_{LL\sigma} = f_L - c_a, R_{LLN} = f_L$:

setting $R_{LG} = R_{LB} = f_L - c_a$ and $R_{Ln} = f_L$, and forming a Lagrangian $\mathcal{L}$ with multiplier λ on the participation constraint (repayments are all strictly positive so only upper corners matter) and deriving the first order conditions:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial R_{LLG}} &= \alpha\pi_{LG}(\lambda-1)\frac{(R_{HG}-f_L+c_a)}{(f_H-f_L)} \geq 0, R_{LLg} \leq f_L - c_a \\
\frac{\partial \mathcal{L}}{\partial R_{LLB}} &= \alpha\pi_{LB}(\lambda-1)\frac{R_{HB}-f_L+c_a}{(f_H-f_L)} \geq 0, R_{LLb} \leq f_L - c_a \\
\frac{\partial \mathcal{L}}{\partial R_{LLN}} &= (1-\alpha)(\pi_{LG}+\pi_{LB})(\lambda-1)\frac{(R_{HN}-f_L)}{(f_H-f_L)} \geq 0, R_{LLn} \leq f_L \\
\frac{\partial \mathcal{L}}{\partial R_{HG}} &= \alpha(\lambda-1)(\pi_{HG} - \frac{\pi_{LG}(f_L-c_a-R_{LLg})}{(f_H-f_L)}) - \alpha\frac{\lambda\pi_{LG}c_m}{(f_H-f_L)} \geq 0, R_{HG} \leq f_H - c_a \\
\frac{\partial \mathcal{L}}{\partial R_{HB}} &= \alpha(\lambda-1)(\pi_{HB} - \frac{\pi_{LB}(f_L-c_a-R_{LLb})}{(f_H-f_L)}) - (1-\alpha)\frac{\lambda\pi_{LB}c_m}{(f_H-f_L)} \geq 0, R_{HB} \leq f_H - c_a \\
\frac{\partial \mathcal{L}}{\partial R_{HN}} &= (1-\alpha)(\lambda-1)(\pi_{HG}+\pi_{HB} - \frac{(\pi_{LG}+\pi_{LB})(f_L-R_{LLn})}{(f_H-f_L)}) - (1-\alpha)\frac{(1-\lambda)(\pi_{LG}+\pi_{LB})c_m}{(f_H-f_L)} \geq 0, R_{HN} \leq f_H \\
\frac{\partial \mathcal{L}}{\partial \alpha} &= -c_a + \frac{\pi_{LG}(R_{HN}-R_{HG}-c_a)c_m}{(f_H-f_L)} + \frac{\pi_{LB}(R_{HN}-R_{HB}-c_a)c_m}{(f_H-f_L)} \\
&\quad +(\lambda-1)(\pi_{HG}(R_{HG}-R_{HN}) + \pi_{LG}(\frac{(f_H-c_a-R_{HG})(f_L-c_a)}{(f_H-f_L)} + \frac{(R_{HG}-f_L+c_a)(R_{LLG}-c_m)}{(f_H-f_L)}) \\
&\quad +\pi_{HB}(R_{HB}-R_{HN}) + \pi_{LB}(\frac{(f_H-c_a-R_{HB})(f_L-c_a)}{(f_H-f_L)} + \frac{(R_{HB}-f_L+c_a)(R_{LLB}-c_m)}{(f_H-f_L)}) \\
&\quad -(\pi_{LG}+\pi_{LB})(\frac{(f_H-R_{HN})}{(f_H-f_L)}f_L + \frac{(R_{HN}-f_L)(R_{LLN}-c_m)}{(f_H-f_L)}))
\end{aligned}$$

we find that at the optimum $\lambda > 1$ - the ex ante marginal value of funds is higher to the firm than to the investor. If not, the first order conditions wrt $R_{H\sigma}, R_{HN}$ are all negative, which would mean that all repayments $R_{LL\sigma}, R_{H\sigma}$ are at most f_L. But this would yield insufficient revenue to the principal to satisfy the participation constraint. It then follows that the first order conditions with respect to the truthful low state monitored repayments are always positive in each state σ, N and hence optimally $R_{LL\sigma} = f_L - c_a, R_{LLN} = f_L$.

A.2 Optimal High State Repayments and Monitoring Conditional on Signal Strategy

Using $R_{L\sigma} = R_{LL\sigma} = f_L - c_a$, $R_{LN} = R_{LLN} = f_L$ reduces the objective function and the participation constraint to

$$\max\ \alpha\left\{\pi_{HG}\left(f_H - R_{HG} - c_a\right) + \pi_{HB}\left(f_H - R_{HB} - c_a\right)\right\} + (1-\alpha)\,\pi_H\left(f_H - R_{HN}\right)$$

$$\text{s.t. } \alpha\left\{\pi_{HG}R_{HG} + \pi_{HB}R_{HB} + \pi_L\left(f_L - c_a\right) - \left[\pi_{LG}\left(R_{HG} - f_L + c_a\right) + \pi_{LB}\left(R_{HB} - f_L + c_a\right)\right]\frac{c_m}{f_H - f_L}\right\} +$$
$$+ (1-\alpha)\left[\pi_H R_{HN} + \pi_L f_L - \pi_L\frac{R_{HN} - f_L}{f_H - f_L}c_m\right] \geq I$$

with the remaining FOC's as

$$\frac{\partial \mathcal{L}}{\partial R_{HG}} = \alpha(\lambda-1)\pi_{HG} - \alpha\frac{\lambda \pi_{LG} c_m}{f_H - f_L} \tag{31}$$

$$\frac{\partial \mathcal{L}}{\partial R_{HB}} = \alpha(\lambda-1)\pi_{HB} - \alpha\frac{\lambda \pi_{LB} c_m}{f_H - f_L} \tag{32}$$

$$\frac{\partial \mathcal{L}}{\partial R_{HN}} = (1-\alpha)(\lambda-1)\pi_{H} - (1-\alpha)\frac{\lambda \pi_{L} c_m}{f_H - f_L} \tag{33}$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial \alpha} &= -c_a - \lambda\frac{c_m}{f_H - f_L}\left[\pi_{LG}(R_{HG}-R_{HN}) + \pi_{LB}(R_{HB}-R_{HN}) + \pi_L c_a\right] \\ &\quad +(\lambda-1)\left(\pi_{HG}(R_{HG}-R_{HN}) + \pi_{HB}(R_{HB}-R_{HN}) - \pi_L c_a\right)\end{aligned}$$

There are two sets of factors which determine when different cases are optimal. Firstly there are conditions which determine the feasibility of a particular type of solution, for example revenues in different states, relative costs of signal acquisition and monitoring and the informativeness of the signal must lie in appropriate ranges to be able to incentivise the firm correctly. Secondly, where alternative forms of solution could be used, there is a question of which of them is optimal. The choice between them will depend on further aspects of the relative costs and revenues.

A.2.1 No information gathering (α=0)

In this case,

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial \alpha} &= -c_a + \lambda\frac{c_m}{f_H - f_L}\left[\pi_{LG}(R_{HN}-R_{HG}-c_a) + \pi_{LB}(R_{HN}-R_{HB}-c_a)\right] \\ &\quad +(\lambda-1)\left(\pi_{HG}(R_{HG}-R_{HN}) + \pi_{HB}(R_{HB}-R_{HN}) - \pi_L c_a\right) < 0\end{aligned}$$

Solving (33) for λ

$$\lambda = \frac{\pi_H\left(f_H - f_L\right)}{\pi_H\left(f_H - f_L\right) - \pi_L c_m}$$

and then deriving R_{HN} from the participation constraint

$$R_{HN} = \frac{\left(I - \pi_L f_L\right)\left(f_H - f_L\right) - f_L \pi_L c_m}{\pi_H\left(f_H - f_L\right) - \pi_L c_m}$$

Note that $f_L < R_{HN} < f_H$ since

$$\begin{aligned}f_L - R_{HN} &= \frac{(f_H - f_L)(f_L - I)}{\pi_H\left(f_H - f_L\right) - \pi_L c_m} < 0 \\ f_H - R_{HN} &= \frac{(f_H - f_L)(\pi_H f_H + \pi_L f_L - I - \pi_L c_m)}{\pi_H\left(f_H - f_L\right) - \pi_L c_m} > 0\end{aligned}$$

Given this, the FOC on α has the form

$$\frac{\partial \mathcal{L}}{\partial \alpha} = -\frac{\pi_H\left(f_H - f_L\right)c_a}{\pi_H\left(f_H - f_L\right) - \pi_L c_m} + \frac{c_m \rho(R_{HG} - R_{HB})}{\pi_H\left(f_H - f_L\right) - \pi_L c_m} \leq 0$$

where R_{HG}, R_{HB} are set to be any value such that $foc\alpha < 0$. In particular we can set $R_{HG} = R_{HB} \geq f_L$ in which case $foc\alpha < 0$.

The value of the objective function is

$$U_{\alpha=0} = \frac{\pi_H\left(f_H - f_L\right)\left[\pi_H f_H + \pi_L f_L - I - \pi_L c_m\right]}{\pi_H\left(f_H - f_L\right) - \pi_L c_m}$$

This establishes Proposition 1.

A.2.2 Full information gathering (α=1)

Here the critical repayments to be determined are R_{HG}, R_{HB}. The first order conditions (31) and (32) have the form

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial R_{HG}} &= (\lambda-1)\pi_{HG} - \frac{\lambda \pi_{LG} c_m}{(f_H - f_L)} \\
\frac{\partial \mathcal{L}}{\partial R_{HB}} &= (\lambda-1)\pi_{HB} - \frac{\lambda \pi_{LB} c_m}{(f_H - f_L)}
\end{aligned}
$$

Since $fochg > fochb$ when $\alpha > 0$ the solution must have one of three forms:
(1a) R_{HG} interior, $R_{HB} = f_L - c_a$;
(1b) $R_{HG} = f_H - c_a, R_{HB} = f_L - c_a$;
(1c) $R_{HG} = f_H - c_a, R_{HB}$ interior

1a

$$obj1a = \frac{(f_H - f_L)\left\{\pi_{HG}\left(\pi_H f_H + \pi_L f_L - I - \pi_{LG} c_m + c_a\right) - \pi_{HB}\pi_{LG} c_m\right\}}{\pi_{HG}\left(f_H - f_L\right) - \pi_{LG} c_m}$$

Need

$$foc\alpha = \frac{-1}{\pi_{HG}\left(f_H - f_L\right) - \pi_{LG} c_m}\left\{\pi_{HG} c_a \left(f_H - f_L\right) - \rho c_m \left(R_{HN} - f_L\right)\right\} > 0$$

We must choose $R_{HN} \leq f_H$ so that

$$R_{HN} \geq f_L + \frac{\pi_{HG} c_a}{\rho c_m}(f_H - f_L)$$

This is always possible if $\rho c_m \geq \pi_{HG} c_a$ (which is always true if NCa<0). In this case setting $R_{HB} = f_L - c_a$ and solving from the participation constraint for R_{HG}

$$R_{HG} = \frac{((1-\pi_{HG})(f_H - f_L) + \pi_{LG} c_m)(c_a - f_L) + I(f_H - f_L)}{\pi_{HG}(f_H - f_L) - \pi_{LG} c_m}$$

We require $f_H - c_a > R_{HG} > f_L - c_a$.

$$
\begin{aligned}
f_H - c_a - R_{HG} &= \frac{(f_H - f_L)(\pi_{HG} f_H + (1-\pi_{HG}) f_L - \pi_{LG} c_m - c_a - I)}{\pi_{HG}(f_H - f_L) - \pi_{LG} c_m} > 0 \\
f_L - c_a - R_{HG} &= \frac{(f_H - f_L)(f_L - I - c_a)}{\pi_{HG}(f_H - f_L) - \pi_{LG} c_m} < 0
\end{aligned}
$$

i.e. $FEAS_G > 0$.

1b

$$obj1b = \pi_{HB}(f_H - f_L)$$

It needs $PC = 0$, which reduces to $FEAS_G = 0$.

1c

$$obj1c = \frac{\pi_{HB}(f_H - f_L)\left(\pi_H f_H + \pi_L f_L - I - \pi_L c_m - c_a\right)}{\pi_{HB}\left(f_H - f_L\right) - \pi_{LB} c_m}$$

Need

$$foc\alpha = -\frac{\pi_{HB}(f_H - f_L) c_a}{\pi_{HB}\left(f_H - f_L\right) - \pi_{LB} c_m} + \frac{(\pi_{LB}\pi_{HG} - \pi_{LG}\pi_{HB})(f_H - R_{HN}) c_m}{\pi_{HB}\left(f_H - f_L\right) - \pi_{LB} c_m} > 0$$

$$
\begin{aligned}
R_{HB} &= \frac{\left(1-\pi_{HB}\right)\left(f_H - f_L\right) + \pi_{LB} c_m}{\pi_{HB}\left(f_H - f_L\right) - \pi_{LB} c_m} c_a - \frac{(\pi_{HG} f_H \left(f_H - f_L\right) + \pi_L f_L \left(f_H - f_L\right) - \pi_{LG} c_m \left(f_H - f_L\right) + \pi_{LB} c_m f_L -}{\pi_{HB}\left(f_H - f_L\right) - \pi_{LB} c_m} \\
&= -\frac{\left(\pi_{HG} f_H + \pi_L f_L - I - \pi_{LG} c_m - \left(1-\pi_{HB}\right) c_a\right)\left(f_H - f_L\right)}{\pi_{HB}\left(f_H - f_L\right) - \pi_{LB} c_m} - \frac{\pi_{LB} c_m \left(f_L - c_a\right)}{\pi_{HB}\left(f_H - f_L\right) - \pi_{LB} c_m}
\end{aligned}
$$

Check that $R_{HB} \geq f_L - c_a$

$$-\frac{(f_H - f_L)(\pi_{HG} f_H + (1-\pi_{LG}) f_L - I - \pi_{LG} c_m - c_a)}{\pi_{HB}(f_H - f_L) - \pi_{LB} c_m} \geq 0$$

Thus need $FEAS_G \leq 0$.

$$m_B = -\frac{\pi_{HG} f_H + (1-\pi_{HG}) f_L - I - \pi_{LG} c_m - c_a}{\pi_{HB}(f_H - f_L) - \pi_{LB} c_m} \geq 0$$

so long as $FEAS_G < 0$. For $m_B < 1$

$$-\frac{\pi_H f_H + (1-\pi_H) f_L - I - \pi_L c_m - c_a}{\pi_{HB}(f_H - f_L) - \pi_{LB} c_m} < 0$$

Notice that, given our feasibility assumption, R_{HB} strictly interior.

A.2.3 Random Information Gathering

We can then establish a useful lemma which relates the marginal value of funds to the investor and the firm in different interim scenarios following acquisition of the signal:

Lemma 1 *If* $0 < \alpha < 1$ *then*
(1) $fochg > fochb$
(2)$fochg = 0 \Rightarrow fochn < 0, fochb < 0;$
(3) $fochn = 0 \Rightarrow fochg > 0, fochb < 0;$
(4) $fochb = 0 \Rightarrow fochg > 0, fochn > 0;$
(5) $fochg < 0 \Rightarrow fochn < 0.$

Proof. (1)

$$\begin{aligned} fochb/\alpha &= ((\lambda-1)\pi_{HB} - \frac{\lambda \pi_{LB} c_m}{(f_H - fL)} \\ &< (\lambda-1)\pi_{HG} - \frac{\lambda \pi_{LG} c_m}{(f_H - fL)} \\ &= fochg/\alpha \end{aligned}$$

so $fochg \leq 0 \Rightarrow fochb < 0; fochb \geq 0 \Rightarrow fochg > 0$

(2) If $fochg = 0$

$$\frac{\lambda - 1}{\lambda} = \frac{\pi_{LG} c_m}{\pi_{HG}(f_H - f_L)}$$

so

$$\begin{aligned} sign(fochn) &= sign(\frac{\pi_{LG} c_m}{\pi_{HG}(f_H - f_L)}(\pi_{HG} + \pi_{HB}) - \frac{(\pi_{LB} + \pi_{LG}) c_m}{f_H - f_L}) \\ &= sign(\pi_{LG}(\pi_{HG} + \pi_{HB}) - (\pi_{LB} + \pi_{LG})\pi_{HG}) \\ &= sign(\pi_{LG}\pi_{HB}) - \pi_{LB}\pi_{HG}) < 0 \end{aligned}$$

So $fochg = 0 \Rightarrow fochn < 0$. From (1) if $fochg = 0$ then $fochb < 0$.

(3) If $fochn = 0$

$$\frac{\lambda - 1}{\lambda} = \frac{(\pi_{LG} + \pi_{LB}) c_m}{(\pi_{HG} + \pi_{HB})(f_H - f_L)}$$

$$\begin{aligned} sign(fochg) &= sign(\frac{\lambda - 1}{\lambda}\pi_{HG} - \frac{\pi_{LG} c_m}{f_H - f_L}) \\ &= sign(\frac{(\pi_{LG} + \pi_{LB})}{(\pi_{HG} + \pi_{HB})}\pi_{HG} - \pi_{LG}) \\ &= sign((\pi_{LG} + \pi_{LB})\pi_{HG} - \pi_{LG}(\pi_{HG} + \pi_{HB})) \\ &= sign(\pi_{LB}\pi_{HG} - \pi_{LG}\pi_{HB}) > 0 \end{aligned}$$

and

$$\begin{aligned}
sign(fochb) &= sign(\frac{\lambda-1}{\lambda}\pi_{HB} - \frac{\pi_{LB}c_m}{f_H - f_L}) \\
&= sign(\frac{(\pi_{LG}+\pi_{LB})}{(\pi_{HG}+\pi_{HB})}\pi_{HB} - \pi_{LB}) \\
&= sign((\pi_{LG}+\pi_{LB})\pi_{HB} - \pi_{LB}(\pi_{HG}+\pi_{HB})) \\
&= sign(\pi_{LG}\pi_{HB} - \pi_{LB}\pi_{HG}) < 0
\end{aligned}$$

(4) $fochb = 0 \Rightarrow$ by (1) that $fochg > 0$. Then if $fochb = 0$

$$\frac{\lambda-1}{\lambda} = \frac{\pi_{LB}c_m}{\pi_{HB}(f_H - f_L)}$$

$$\begin{aligned}
sign(fochn) &= sign(\frac{\pi_{LB}c_m}{\pi_{HB}(f_H - f_L)}(\pi_{HG}+\pi_{HB}) - \frac{(\pi_{LB}+\pi_{LG})c_m}{f_H - f_L}) \\
&= sign(\pi_{LB}(\pi_{HG}+\pi_{HB}) - \pi_{HB}(\pi_{LB}+\pi_{LG})) \\
&= sign(\pi_{LB}\pi_{HG} - \pi_{HB}\pi_{LG}) > 0
\end{aligned}$$

(5) Similarly if $fochg < 0$ then $fochn < 0$ since

$$\begin{aligned}
fochg &< 0 \Rightarrow \frac{(\lambda-1)}{\lambda} < \frac{\pi_{LG}c_m}{\pi_{HG}(f_H - fL)} \\
\frac{fochn}{(1-\alpha)\pi_H\lambda} &= \frac{(\lambda-1)}{\lambda} - \frac{\pi_L c_m}{\pi_H(f_H - fL)} \lesseqgtr 0
\end{aligned}$$

Since

$$\frac{\pi_{LG}c_m}{\pi_{HG}(f_H - f_L)} < \frac{\pi_L c_m}{\pi_H(f_H - f_L)}, fochg < 0 \; \Rightarrow \frac{fochn}{(1-\alpha)\pi_H\lambda} < 0$$

and so $fochn < 0$. ■

>From the Lemma it follows that if $0 < \alpha < 1$ there are various possible cases for a potential optimum with random information gathering.

1. $fochg, fochn, fochb > 0$

   Then

   $$R_{HG} = f_H - c_a = R_{HB}, R_{HN} = f_H$$

   and the participation constraint is

   $$\pi_H f_H + \pi_L f_L - \pi_L c_m - \alpha c_a - I > 0$$

   so this cannot be an optimal outcome.

2. $fochg > fochn > 0 \geq fochb$

   There are two subcases:

   2.1 $fochg > fochn > 0 > fochb$. When $fochb < 0$ we have

   $$R_{HG} = f_H - c_a, R_{HB} = f_L - c_a, R_{HN} = f_H$$

   The first order condition on α becomes

   $$\pi_{LB}c_m - c_a - (\lambda - 1)(\pi_{HB}(f_H - f_L) - \pi_{LB}c_m + c_a) = 0$$

   which is possible if $PFEAS$=$\pi_{HG}(f_H - f_L) > c_A + \pi_L c_m$. The participation constraint has the form

   $$\alpha\{\pi_{HG}f_H + (1-\pi_{HG})f_L - \pi_{LG}c_m - c_a\} + (1-\alpha)[\pi_H f_H + \pi_L f_L - \pi_L c_m] = I$$

Solving for α

$$\alpha = \frac{\pi_H f_H + \pi_L f_L - I - \pi_L c_m}{\pi_{HB}\left(f_H - f_L\right) + c_a - \pi_{LB} c_m} < 1$$

$\alpha < 1$ needs $FEAS_G = \pi_{HG} f_H + (1 - \pi_{HG}) f_L - I - \pi_{LG} c_m - c_a < 0$.
The objective is

$$U_{MSb} = \frac{(\pi_H f_H + \pi_L f_L - I - \pi_L c_m)\pi_{HB}\left(f_H - f_L\right)}{\pi_{HB}\left(f_H - f_L\right) + c_a - \pi_{LB} c_m}$$

Case 3b of Prop 1

2.2 $fochg > fochn > 0 = fochb$
When $fochb = 0$ we can solve for λ from $fochb = 0$

$$\lambda = \frac{\pi_{HB}\left(f_H - f_L\right)}{\pi_{HB}\left(f_H - f_L\right) - \pi_{LB} c_m}$$

and then the first order condition wrt α becomes

$$foc\alpha = -\frac{\pi_{HB}\left(f_H - f_L\right) c_a}{\pi_{HB}\left(f_H - f_L\right) - \pi_{LB} c_m} < 0$$

so this case is impossible.

3. $fochg > 0 \geq fochn > fochb$

case 3 has two subcases:

3.1 $fochg > 0 > fochn > fochb$
In this case, $R_{HN} = f_L, R_{HB} = f_L - c_a, R_{HG} = f_H - c_a$. The participation constraint has the form

$$\alpha\left(\pi_{HG}\left(f_H - f_L\right) - c_a - \pi_{LG} c_m + f_L\right) + (1-\alpha) f_L = I$$

and solving this for α

$$\alpha = \frac{I - f_L}{\pi_{HG}\left(f_H - f_L\right) - c_a - \pi_{LG} c_m}$$

For $\alpha < 1$ need $FEAS_G = \pi_{HG} f_H + \left(1 - \pi_{HG}\right) f_L - I - \pi_{LG} c_m - c_a > 0$.
Putting α into the objective gives a payoff of

$$U_{msa} = \frac{\left(f_H - f_L\right)\left(\pi_{HG}(\pi_H f_H + \pi_L f_L - I - \pi_H(\pi_{LG} c_m + c_a)\right)}{\pi_{HG}\left(f_H - f_L\right) - \pi_{LG} c_m - c_a}$$

Case 3a Prop 1

3.2 $fochg > 0 = fochn > fochb : R_{HG} = f_H - c_a, R_{HB} = f_L - c_a, R_{HN}$ interior
When $fochn = 0$, solving for λ

$$\lambda = \frac{\pi_H\left(f_H - f_L\right)}{\pi_H\left(f_H - f_L\right) - \pi_L c_m}$$

and the first order condition on α becomes

$$\begin{aligned} foc\alpha &= -\frac{\pi_H\left(f_H - f_L\right) c_a}{\pi_H\left(f_H - f_L\right) - \pi_L c_m} + \frac{c_m(\pi_{LB}\pi_{HG} - \pi_{LG}\pi_{HB})\left(f_H - f_L\right)}{\pi_H\left(f_H - f_L\right) - \pi_L c_m} \\ &\Rightarrow \pi_H c_a = \rho c_m \end{aligned}$$

The objective is

$$(\pi_H R_{HN} - \pi_{HB} f_L - \pi_{HG} f_H)\alpha + \pi_H(f_H - R_{HN})$$

and the participation constraint is

$$\alpha(\pi_{HG} f_H + \left(1 - \pi_{HG}\right) f_L - \pi_{LG} c_m - c_a) + (1-\alpha)(\pi_H R_{HN} + \pi_L(f_L - \frac{(R_{HN} - f_L) c_m}{(f_H - f_L)})) - I = 0$$

Solving the participation constraint for α

$$\alpha = \frac{(I - \pi_H R_{HN} - \pi_L f_L)(f_H - f_L) + \pi_L (R_{HN} - f_L) c_m}{(\pi_{HG} f_H + \pi_{HB} f_L - \pi_H R_{HN} - \pi_{LG} c_m - c_a)(f_H - f_L) + \pi_L (R_{HN} - f_L) c_m}$$

and replacing in the objective

$$objmsc = \frac{\pi_H (f_H - R_{HN}) \{\pi_{HG} f_H + (1-\pi_{HG}) f_L - I - \pi_{LG} c_m - c_a\} - \pi_{HB} [(R_{HN} - f_L)(\pi_H (f_H - f_L) - \pi_L c_m) - (I - f_L)(f_H - f_L)]}{(\pi_{HG} f_H + \pi_{HB} f_L - \pi_H R_{HN} - \pi_{LG} c_m - c_a)(f_H - f_L) + \pi_L (R_{HN} - f_L) c_m} (f_H - f_L)$$

However this solution is only valid if $\pi_H c_a = \rho c_m$. Setting $c_a = \rho c_m / \pi_H$ in the objective the latter reduces to

$$U_{MSc} = \frac{\pi_H (\pi_H f_H + \pi_L f_L - I - \pi_L c_m)}{\pi_H (f_H - f_L) - \pi_L c_m} (f_H - f_L)$$

Case 3c of Proposition 1

4. $0 \geq fochg > fochn > fochb$

   This has two subcases:

   4.1 $0 > fochg > fochn > fochb$

   This case is impossible-it makes all repayments at most f_L and so violates the participation constraint.

   4.2 $0 = fochg > fochn > fochb$

   So *Rhg* interior, $R_{HN} = f_L, Rhb = f_L - c_a$. Solving $fochg$ for λ

$$\lambda = \frac{\pi_{HG} (f_H - f_L)}{\pi_{HG} (f_H - f_L) - \pi_{LG} c_m}$$

   and using this in $foca$ gives

$$foc\alpha = -\frac{\pi_{HG} (f_H - f_L) c_a}{\pi_{HG} (f_H - f_L) - \pi_{LG} c_m} < 0$$

   so this case is also impossible.

B Rankings

To determine which signal acquisition strategy provides the optimal contract when no signal is collected prior to writing the contract we can perform pairwise comparisons of payoffs. Comparing the various expressions of the objective functions in pair, dividing all the rankings by $f_H - f_L$, we have:

$$\frac{U_{\alpha=0} - U_{msb}}{f_H - f_L} = \frac{(\pi_H f_H + \pi_L f_L - \pi_L c_m - I)(\pi_H c_a - \rho c_m)}{[\pi_H (f_H - f_L) - \pi_L cm] [\pi_{HB} (f_H - f_L) + ca - \pi_{LB} cm]}$$

$$\frac{U_{\alpha=0} - U_{msa}}{f_H - f_L} = \frac{I - f_L}{[\pi_H (f_H - f_L) - \pi_L c_m] [\pi_{HG} (f_H - f_L) - c_a - \pi_{LG} c_m]} [\pi_H c_a - \rho c_m]$$

$$\begin{aligned} \frac{U_{\alpha=0} - U_{\alpha=1a}}{f_H - f_L} &= U_{msc} - U_{\alpha=1c} = \frac{(\pi_{HG} c_a - \rho c_m)}{\pi_{HG} (f_H - f_L) - \pi_{LG} c_m} + \frac{[\pi_H f_H + \pi_L f_L - \pi_L c_m - I] \rho c_m}{[\pi_H (f_H - f_L) - \pi_L cm] [\pi_{HG} (f_H - f_L) - \pi_{LG} c_m]} \\ &= \frac{\pi_{HG} c_a [\pi_H (f_H - f_L) - \pi_L c_m] - (I - f_L) \rho c_m}{[\pi_H (f_H - f_L) - \pi_L c_m] [\pi_{HG} (f_H - f_L) - \pi_{LG} c_m]} \end{aligned}$$

$$\begin{aligned} \frac{U_{msb} - U_{\alpha=1c}}{f_H - f_L} &= -\frac{(\pi_{HG} f_H + (1 - \pi_{HG}) f_L - I - \pi_{LG} c_m - c_a)(\pi_{HG} c_a - \rho c_m)}{(\pi_{HB} (f_H - f_L) + c_a - \pi_{LB} c_m)(\pi_{HG} (f_H - f_L) - \pi_{LG} c_m)} \\ &= -\frac{(\pi_{HG} c_a - \rho c_m) FEAS_G}{(\pi_{HB} (f_H - f_L) + c_a - \pi_{LB} c_m)(\pi_{HG} (f_H - f_L) - \pi_{LG} c_m)} \end{aligned}$$

$$
\begin{aligned}
\frac{U_{msa}-U_{\alpha=1a}}{f_H-f_L} &= \frac{\pi_{HG}c_a\left(\pi_{HG}f_H+(1-\pi_{HG})f_L-I-\pi_{LG}c_m-c_a\right)}{\left(\pi_{HG}\left(f_H-f_L\right)-\pi_{LG}c_m-c_a\right)\left(\pi_{HG}\left(f_H-f_L\right)-\pi_{LG}c_m\right)} \\
& \frac{\pi_{HG}c_a}{\left(\pi_{HG}\left(f_H-f_L\right)-\pi_{LG}c_m-c_a\right)\left(\pi_{HG}\left(f_H-f_L\right)-\pi_{LG}c_m\right)}FEAS_G
\end{aligned}
$$

$$
\begin{aligned}
\frac{U_{msb}-U_{msa}}{f_H-f_L} &= -\frac{\left[\pi_{HG}f_H+(1-\pi_{HG})f_L-I-c_a-\pi_{LG}c_m\right]\left(\pi_H c_a-c_m\rho\right)}{\left[\pi_{HB}\left(f_H-f_L\right)+c_a-\pi_{LB}c_m\right]\left[\pi_{HG}\left(f_H-f_L\right)-\pi_{LG}c_m-c_a\right]} \\
&= -\frac{FEAS_G\left(\pi_H c_a-c_m\rho\right)}{\left[\pi_{HB}\left(f_H-f_L\right)+c_a-\pi_{LB}c_m\right]\left[\pi_{HG}\left(f_H-f_L\right)-\pi_{LG}c_m-c_a\right]}
\end{aligned}
$$

Given the sign of the expressions we already know, this amounts to find the sign of the following expressions

$$sign\left(U_{\alpha=0}-U_{msb}\right)=sign\left(\pi_H c_a-\rho c_m\right)$$

$$sign\left(U_{\alpha=0}-U_{msa}\right)=sign\frac{\left[\pi_H c_a-\rho c_m\right]}{\left[\pi_{HG}\left(f_H-f_L\right)-\pi_{LG}c_m-c_a\right]}$$

$$sign\left(U_{\alpha=0}-U_{\alpha=1a},U_{msc}-U_{\alpha=1c}\right)=sign\left(\pi_{HG}c_a-\frac{I-f_L}{\pi_H(f_H-f_L)-\pi_L c_m}\rho c_m\right)$$

$$sign\left(U_{msb}-U_{\alpha=1c}\right)=sign\left(-\left(\pi_{HG}c_a-\rho c_m\right)FEAS_G\right)$$

$$sign\left(U_{msa}-U_{\alpha=1a}\right)=sign\left(\frac{FEAS_G}{\pi_{HG}\left(f_H-f_L\right)-\pi_{LG}c_m-c_a}\right)$$

$$sign(U_{msb}-U_{msa})=sign\left(-\frac{FEAS_G\left(\pi_H c_a-c_m\rho\right)}{\pi_{HG}\left(f_H-f_L\right)-\pi_{LG}c_m-c_a}\right)$$

$$sign\left(U_{\alpha=0}-U_{\alpha=1c}\right)=sign\left(-\frac{\left(\pi_H f_H+\pi_L f_L-I-\pi_L c_m\right)\rho c_m}{\left(\pi_H\left(f_H-f_L\right)-\pi_L c_m\right)\left(\pi_{HB}\left(f_H-f_L\right)-\pi_{LB}c_m\right)}+\frac{\pi_{HB}c_a}{\pi_{HB}\left(f_H-f_L\right)-\pi_{LB}c_m}\right)$$

$$sign\left(U_{msb}-U_{\alpha=1c}\right)=-\frac{\pi_{HB}\left(f_H-f_L\right)c_a FEAS_G}{\left(\pi_{HB}\left(f_H-f_L\right)-\pi_{LB}c_m\right)\left(\pi_{HB}\left(f_H-f_L\right)-\pi_{LB}c_m+c_a\right)}>0$$

Remark 2 *Notice that* $\pi_H c_a-\rho c_m<0$ *and* $FEAS_G<0$ *imply that*

$$\pi_{HG}c_a<\frac{I-f_L}{\pi_H(f_H-f_L)-\pi_L c_m}\rho c_m$$

and thus that $\alpha=1\succ\alpha=0/ms_c$.

Proof. Setting $NC'_a=\pi_{HG}c_a-\rho c_m$ we know that

$$
\begin{aligned}
\pi_H c_a\left(FEAS_G\right)+\left(NC'_a\right)\left(I-f_L\right)+c_a\left(NC_a-I+f_L\right) &< 0 \\
\pi_H c_a\left(FEAS_G\right)+NC'_a\left(I-f_L\right)+c_a\left\{\pi_H c_a-\rho c_m+f_L-I\right\} &< 0
\end{aligned}
$$

Adding and subtracting $c_a\pi_{HG}\pi_{HB}\left(f_H-f_L\right)$ and expanding ρ, the term in curly brackets can be written as

$$c_a\left\{\pi_{HG}\left[\pi_{HB}\left(f_H-f_L\right)-\pi_{LB}c_m+c_a\right]-\pi_{HB}\left[FEAS_G\right]\right\}$$

$$
\begin{aligned}
\pi_H c_a\left(FEAS_G\right)+\left(NC'_a\right)\left(I-f_L\right)+c_a\left\{\pi_{HG}\left[\pi_{HB}\left(f_H-f_L\right)-\pi_{LB}c_m+c_a\right]-\pi_{HB}\left[FEAS_G\right]\right\} &< 0 \\
\pi_{HG}c_a\left(FEAS_G\right)+\left(NC'_a\right)\left(I-f_L\right)+c_a\left\{\pi_{HG}\left[\pi_{HB}\left(f_H-f_L\right)-\pi_{LB}c_m+c_a\right]\right\} &< 0
\end{aligned}
$$

Using $FEAS_G$ and NC'_a, the expression above reduces to

$$\pi_{HG}c_a\left(\pi_H(f_H-f_L)-\pi_L c_m\right)-\left(I-f_L\right)\rho c_m<0$$

which proves the result. ■

C Signal strategy as a function of acquisition and monitoring costs

C.1 How the signal acquisition strategy varies with c_m

When $NC_a < 0$ and $FEAS_G \geq 0$,

$$\begin{aligned}
\frac{\partial \alpha}{\partial c_m} &= \frac{\pi_{LG}\left(I-f_L\right)}{\left(\pi_{HG}\left(f_H-f_L\right)-\pi_{LG}c_m-c_a\right)^2} > 0 \\
\frac{\partial^2 \alpha}{\partial c_m^2} &= \frac{1}{\left(\pi_{HG}\left(f_H-f_L\right)-\pi_{LG}c_m-c_a\right)^2}\left(\pi_{LG}\frac{\partial \alpha}{\partial c_m}\left(\pi_{HG}\left(f_H-f_L\right)-\pi_{LG}c_m-c_a\right)+\pi_{LG}^2\alpha\right) \\
&= \frac{2\pi_{LG}^2\alpha}{\left(\pi_{HG}\left(f_H-f_L\right)-\pi_{LG}c_m-c_a\right)^2} > 0
\end{aligned}$$

On the other hand when $NC_a < 0$ and $FEAS_G < 0$

$$\begin{aligned}
\frac{\partial \alpha}{\partial c_m} &= -\frac{\pi_L-\alpha\pi_{LB}}{\left(\pi_{HB}\left(f_H-f_L\right)-\pi_{LB}c_m+c_a\right)^2} < 0 \\
\frac{\partial^2 \alpha}{\partial c_m^2} &= -\frac{2\pi_{LB}\left(\pi_L-\alpha\pi_{LB}\right)}{\left(\pi_{HB}\left(f_H-f_L\right)-\pi_{LB}c_m+c_a\right)^2} < 0
\end{aligned}$$

The $\alpha\left(c_m\right)$ function reaches its maximum at the value of c_m at which $FEAS_G = 0$ ($c_m = \left(\pi_{HG}f_H+(1-\pi_{HG})f_L-I-c_a\right)/\pi_{LG}$). At this value, the function has a kink: the slope changes from $\frac{\partial \alpha}{\partial c_m} = \frac{\pi_{LG}}{I-f_L}$ to $\frac{\partial \alpha}{\partial c_m} = \frac{\pi_{LG}^2}{\rho(f_H-f_L)-\pi_{LB}(I-f_L)-\pi_L c_a} < 0$.[5]

C.2 How the signal acquisition strategy varies with c_a

When $NC_a < 0$ and $FEAS_G \geq 0$,

$$\begin{aligned}
\frac{\partial \alpha}{\partial c_a} &= \frac{I-f_L}{\left(\pi_{HG}\left(f_H-f_L\right)-\pi_{LG}c_m-c_a\right)^2} > 0 \\
\frac{\partial^2 \alpha}{\partial c_a^2} &= \frac{2\left(I-f_L\right)}{\left(\pi_{HG}\left(f_H-f_L\right)-\pi_{LG}c_m-c_a\right)^3} > 0
\end{aligned}$$

If $NC_a < 0$ and $FEAS_G < 0$,

$$\begin{aligned}
\frac{\partial \alpha}{\partial c_a} &= -\frac{Ef-I-\pi_L c_m}{\left(\pi_{HB}\left(f_H-f_L\right)-\pi_{LB}c_m+c_a\right)^2} < 0 \\
\frac{\partial^2 \alpha}{\partial c_a^2} &= \frac{2\left(Ef-I-\pi_L c_m\right)}{\left(\pi_{HB}\left(f_H-f_L\right)-\pi_{LB}c_m+c_a\right)^3} > 0
\end{aligned}$$

C.3 How utility varies with acquisition and monitoring costs

We can plot U against c_a and c_m.

[5] For this to be negative, we need that $pfeas_B = \pi_{HB}\left(f_H-f_L\right)-\pi_{LB}c_m > 0$ at the value of c_m at which $FEAS_G = 0$. Because $FEAS_B + FEAS_G = FEAS > 0$, this is always true and the sign of the derivative is negative:

$$\begin{aligned}
FEAS_B + FEAS_G &= FEAS \\
\pi_{HB}\left(f_H-f_L\right)-\pi_{LB}c_m+\left(\pi_{HG}f_H+\left(1-\pi_{HG}\right)f_L-I-\pi_{LG}c_m-c_a\right) &= \pi_H f_H+\pi_L f_L-I-\pi_L c_m-c_a
\end{aligned}$$

Put it differently, the value of c_m that makes $FEAS_G = 0$ must be less than the value of c_m that makes $pfeas_B = 0$:

$$c_{m|pfeasB} = \frac{\pi_{HB}}{\pi_{LB}}\left(f_H-f_L\right) > \left(\pi_{HG}f_H+\left(1-\pi_{HG}\right)f_L-I-c_a\right)/\pi_{LG} = c_{m|FEASg}$$

$$\pi_{LG}\pi_{HB}\left(f_H-f_L\right) > \pi_{LB}\left(\pi_{HG}f_H+\left(1-\pi_{HG}\right)f_L-I-c_a\right).$$

For $c_a > 0$ and increasing c_m, first optimally $\alpha = 0$ and utility is given by $U_{\alpha=0}$ (21) ($NC_a \geq 0$ and $FEAS_G \geq 0$)

$$\begin{aligned}
\frac{\partial U_{\alpha=0}}{\partial c_m} &= -\frac{\pi_H \pi_L (f_H - f_L)(I - f_L)}{(\pi_H (f_H - f_L) - \pi_L c_m)^2} < 0 \\
\frac{\partial U^2_{\alpha=0}}{\partial c^2_m} &= -\frac{2\pi_H \pi_L^2 (f_H - f_L)(I - f_L)}{(\pi_H (f_H - f_L) - \pi_L c_m)^3} < 0
\end{aligned}$$

For higher c_m, then optimally $\alpha > 0$ and utility is given by U_{MSa} (23) ($NC_a < 0$ and $FEAS_G \geq 0$)

$$\begin{aligned}
\frac{\partial U_{MSa}}{\partial c_m} &= -\frac{\pi_{LG} \pi_{HG} (f_H - f_L)(I - f_L)}{(\pi_{HG} (f_H - f_L) - \pi_{LG} c_m - c_a)^2} < 0 \\
\frac{\partial U^2_{MSa}}{\partial c^2_m} &= -\frac{2\pi^2_{LG} \pi_{HG} (f_H - f_L)(I - f_L)}{(\pi_{HG} (f_H - f_L) - \pi_{LG} c_m - c_a)^2} < 0
\end{aligned}$$

Last, for sufficienty high c_m, optimally still $\alpha > 0$, but utility is given by U_{MSb} (26) ($NC_a < 0$ and $FEAS_G < 0$) and

$$\frac{\partial U_{MSb}}{\partial c_m} = \frac{\pi_{HB} (f_H - f_L) [-\pi_L (\pi_{HB} (f_H - f_L) + c_a) + \pi_{LB} (Ef - I)]}{(\pi_{HB} (f_H - f_L) - \pi_{LB} c_m + c_a)^2}$$

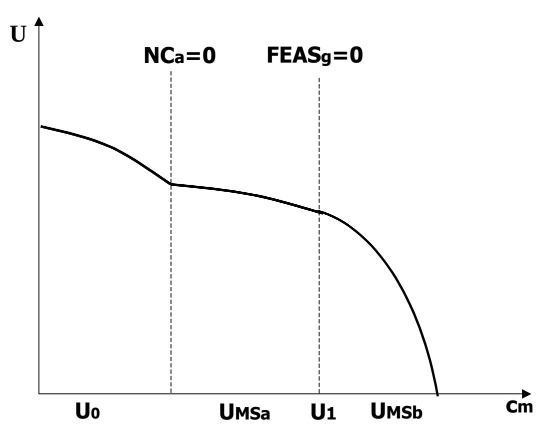
which can be written as

$$\frac{\pi_{HB} (f_H - f_L) \{\pi_{LB} FEAS_G - \pi_{LG} [\pi_{HB} (f_H - f_L) - \pi_{LB} c_m + c_a]\}}{(\pi_{HB} (f_H - f_L) - \pi_{LB} c_m + c_a)^2} < 0$$

which is certainly negative given that $FEAS_G < 0$. Last

$$\frac{\partial U^2_{MSb}}{\partial c^2_m} = -\frac{2\pi_{LB} \pi_{HB} (f_H - f_L) \{\pi_{LB} FEAS_G - \pi_{LG} [\pi_{HB} (f_H - f_L) - \pi_{LB} c_m + c_a]\}}{(\pi_{HB} (f_H - f_L) - \pi_{LB} c_m + c_a)^3} < 0$$

U(cm) for ca>0



Plotting U against c_a, for sufficiently high c_m $\left(c_m > \frac{\pi_H (\pi_{HG} f_H + (1-\pi_{HG}) f_L - I)}{\pi_H \pi_{LG} + \rho}\right)$, first optimally $\alpha > 0$ and utility is given by U_{MSa} (23) ($NC_a < 0$ and $FEAS_G \geq 0$)

$$\frac{\partial U_{MSa}}{\partial c_a} = -\frac{\pi_{HG} (I - f_L)(f_H - f_L)}{(\pi_{HG} (f_H - f_L) - \pi_{LG} c_m - c_a)^2} < 0$$

$$\frac{\partial U^2_{MSa}}{\partial c^2_a} = -\frac{2\pi_{HG} \pi_{LG} (I - f_L)(f_H - f_L)}{(\pi_{HG} (f_H - f_L) - \pi_{LG} c_m - c_a)^3} < 0$$

For higher c_a, optimally still $\alpha > 0$ but utility is given by U_{MSb} (26) ($NC_a < 0$ and $FEAS_G < 0$)

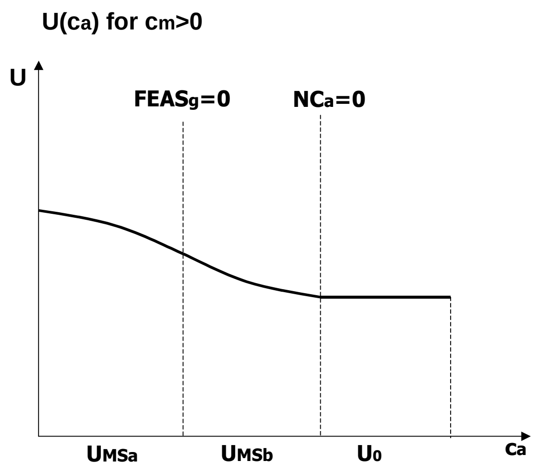
$$\frac{\partial U_{MSb}}{\partial c_a} = -\frac{(Ef - I - \pi_L c_m)\,\pi_{HB}\,(f_H - f_L)}{(\pi_{HB}\,(f_H - f_L) - \pi_{LB} c_m + c_a)^2} < 0$$

$$\frac{\partial U^2_{MSb}}{\partial c_a^2} = \frac{\pi_{LB}\,(Ef - I - \pi_L c_m)\,\pi_{HB}\,(f_H - f_L)}{(\pi_{HB}\,(f_H - f_L) - \pi_{LB} c_m + c_a)^3} > 0$$

Last, for sufficienty high c_a, optimally $\alpha = 0$ and utility is given by $U_{\alpha=0}$ (21) ($NC_a \geq 0$ and $FEAS_G < 0$) and

$$\frac{\partial U_{\alpha=0}}{\partial c_a} = 0$$

Notice that if $c_m \leq \frac{\pi_H(\pi_{HG} f_H + (1-\pi_{HG}) f_L - I)}{\pi_H \pi_{LG} + \rho}$, case MS_b does not arise.



References

[1] Baron, D. and R.B. Myerson (1982), "Regulating a Monopolist with Unknown Costs," *Econometrica*, **50** (4), 911-930.

[2] Bennardo, A. (2007), "Information Gathering, Disclosure and Contracting in Competitive Markets," mimeo.

[3] Crémer, J. and F. Khalil (1992), "Gathering information before signing a contract," *American Economic Review*, **82** (3), 566-578.

[4] Crémer, J. and F. Khalil (1994), "Gathering information before the contract is offered: The case with two states of nature," *European Economic Review*, **38**, 675-682.

[5] Crémer, J., F. Khalil and J.C, Rochet (1998a), "Contracts and Productive Information Gathering," *Games and Economic Behavior*, **25**, 174-193.

[6] Crémer, J., F. Khalil and J.C. Rochet (1998b), "Strategic Information Gathering before a Contract is Offered," *Journal of Economic Theory*, **81**, 163-200.

[7] Khalil, F. and B. Parigi (1998), "Loan Size as a Commitment Device," *International Economic Review*, **39**, 135-150.

[8] Khalil, F. and B Parigi (2001), "Screening, Monitoring and Consumer Credit," *mimeo.*

[9] Lewis, T.R. and D.E.M. Sappington (1997), "Information management in incentive problems," *Journal of Political Economy,* **105**, 796 821.

[10] Mookherjee, D. and I. Png (1989), "Optimal Auditing, Insurance and Redistribution," *Quarterly Journal of Economics*, **104**, 399-415.