Why There May Be No General, Rational and Descriptively Adequate Theory of Decision Under Risk

Graham Loomes, University of East Anglia, UK.

E-mail g.loomes@uea.ac.uk

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Abstract

This paper develops a parsimonious descriptive model which can account for a great many of the known systematic departures from the standard theory of individual decision under risk that have been discovered in controlled experiments. The paper argues that if this model is correct, there will be no theory of decision making which can accommodate the observed behaviour while also being both general and rational in the sense in which decision theorists normally use those terms. Some novel implications of the model are identified and compared with those of leading 'alternative' theories.

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1. Introduction

This paper develops a parsimonious descriptive model of risky choice which can organise much of the most influential experimental evidence of systematic departures from conventional decision theory. If this model is a reasonably good account of the judgments that generate the observed patterns in many individual decision experiments, it raises fundamental questions about the generality of theories and the role of experiments in testing existing theories and stimulating new ones.

Focusing on the very simplest experimental tasks – pairwise choices between lotteries involving no more than three outcomes and/or valuations of such lotteries – it will be shown that individuals who behave according to the model will violate every one of the key axioms/postulates of rational choice, excepting only transparent dominance. In other words, if the model of the data generating process is correct, there can be no descriptively adequate general theory of risky choice which is rational in the sense normally used by decision theorists.

The other side of this coin is that experimental tests of competing theories that revolve around pairwise choices and/or valuations may not be very informative about the relative merits of those theories in relation to more complex lotteries (i.e. more than three outcomes) and/or with respect to larger choice sets (i.e. more than two alternatives). A corollary of this is that it may be a mistake to suppose that a model which successfully explains particular phenomena in simple laboratory experiments can be extrapolated to more complex environments.

Clearly all this, if true, runs counter to basic presumptions made by many theorists and experimenters and threatens to undermine a substantial body of existing theoretical and empirical work (including, as a matter of fact, much of my own).

The rest of the paper is organised as follows. In the next section, the first part of the model is developed. This part focuses on the way that experimental participants process probabilities. To that end, it holds in abeyance the issue of just how they evaluate outcomes: it turns out that even on its own, the probability processing component of the model is sufficient to show that any theory which incorporates either the independence axiom / sure-thing principle or betweenness or transitivity is bound to fail at some point.

Having modelled the probability dimension, the paper applies essentially the same modelling strategy to outcomes. This allows various other well-attested phenomena to be accommodated; and additional novel implications are derived. The subsequent sections discuss various implications for current 'mainstream' theories and outlines some issues for experimental methodology and for policy prescription. However, before embarking on the next section, some remarks about two very deliberate 'omissions'.

First, there is no axiom system underpinning this model. Axioms are usually framed in terms of principles evaluated on their intrinsic merit and having general validity. However, a central proposition in this paper is that, with the exception of respect for transparent dominance – i.e. that more of a better thing is preferred to less so long as the more-less relationship is obvious – no general normative principle holds descriptively. This does not mean that the model is vacuous and that there is nothing it cannot accommodate, as will be shown in the sections below. But it is essentially a model of how people actually *do* behave when presented with the kinds of tasks frequently used in individual decision experiments, not a model of how they should behave in experiments and in decision making in non-experimental environments.

Second, although it is intended to be a model of how people actually behave, it *is* only a model, and therefore simplifies. In particular, although it is accepted that many participants' judgments are liable to be imprecise and somewhat variable, the exposition abstracts from that and presents the model *as if* those judgments are deterministic. I acknowledge that future attempts to apply statistical tests and/or estimate the parameters of the model will require some stochastic specification. I am also aware that different stochastic specifications may have very different implications for the hypotheses that are formulated and for the tests conducted (for a discussion of some of these issues, see Loomes, 2005).

However, by developing the model in deterministic form, the objective is to show that the results do not depend on some particular specification of random error. Such 'noise' in participants' responses is a fact of life, as are framing effects and failures of procedure invariance (Tversky and Kahneman, 1986), and such factors undoubtedly contribute to systematic violations of standard normative principles. But the claim being made in this paper is that such additional influences are not *necessary* in order to produce the many regularities organised by the model in this paper, and therefore the violations of the various normative principles cannot be dismissed as purely procedural, nor can they be mopped up by some catch-all 'error term'.

2. Modelling Probability Judgments

The starting point here is Rubinstein's (1988) intuition about the possible role of *similarity* in explaining the form of Allais paradox that has come to be referred to as the 'common ratio effect' – see Allais (1953) and Kahneman and Tversky (1979). Consider the following pair of choices between lotteries of the form (x, p; 0, 1-p) which offer payoff x with probability p and zero with probability 1-p:

Choice #1: L_1 = (30, 1) vs L_2 = (40, 0.80; 0, 0.20)

Choice #2: L_3 = (30, 0.25; 0, 0.75) vs L_4 = (40, 0.20; 0, 0.80)

The two lotteries in choice #2 can be regarded as scaled-down versions of their choice #1 counterparts, in that the probabilities of positive payoffs in choice #2 are reduced to a fraction – in this example, a quarter – of their choice #1 values, maintaining the ratio between them at 0.25 (hence the term 'common ratio'). According to the independence axiom of EU, there are only two permissible patterns of choice: if an individual prefers L_1 in choice #1, he should also prefer the scaled-down version in the form of L_3 in choice #2, and vice-versa; alternatively, if he prefers L_2 in the first choice, he should also prefer L_4 in the second. However, the evidence from numerous experiments involving variants on these parameters has shown a robust tendency for many respondents to violate independence by choosing the safer option L_1 in choice #1 but pick the riskier alternative L_4 in choice #2. The opposite combination of L_2 and L_3 is relatively rarely observed.

Similarity theory offers the following explanation for this pattern of behaviour. In choice #1, the lotteries differ substantially both on the payoff dimension and on the probability dimension; and although the expected value of L_2 – 32 – is somewhat higher than the certainty of 30 offered by L_1, the majority of respondents are liable to choose L_1, a result which Rubinstein ascribed to risk aversion. But in choice #2, the effect of scaling down the probabilities of the positive payoffs is to cause many respondents to consider those probabilities to be *similar* (or "approximately the same", or "inconsequential")[1], and therefore to give decisive weight to the *dis*similar payoff dimension, which favours L_4 over L_3.

[1] Rubinstein (1988, p.148) acknowledged and appeared to agree with a suggestion by Margalit that the phrase 'is approximately the same as' might be better than 'is similar to'; and Leland (1998) has been inclined to refer to the 'inconsequentiality' of the difference.

While this idea has some intuitive appeal and can be deployed to explain a number of other 'regularities' besides the common ratio effect (see, for example, Leland (1994), (1998)), it also has its limitations. In particular, a given pair of probabilities may be deemed to be inconsequential in one set of circumstances but that same pair may be decisive in another. To see this, consider the case where the choice is between L_4 = (40, 0.20; 0, 0.80) and L_{3*} = (40, 0.25; 0, 0.75). These two lotteries involve exactly the same probabilities that appear in choice #2 and that may be deemed 'inconsequential' in that context. But in the comparison between L_{3*} and L_4, the payoffs are the same, so that L_{3*} stochastically dominates L_4.

Evidence from experiments involving choices between lotteries where one transparently dominates the other suggests that in such cases dominance is very rarely violated, even when the differences are quite small – see, for example, Loomes and Sugden (1998). So in cases where there is no difference between payoffs, *any* perceptible difference between probabilities is liable to be decisive[2]. This poses the question of how the basic intuition behind similarity theory can be modelled in a way that preserves its distinctive features while at the same time making it possible for pairs of probabilities that appear inconsequential in some contexts to be considered decisive in others. That will be the main objective of the rest of this section.

So consider again choices such as #1 and #2, but this time set out as in Figure 1 below:

FIGURE 1 HERE

Let p be the probability of getting a payoff of 30 under the safer (S) lottery while q is the probability of 40 offered by the riskier (R) lottery. Under standard expected utility theory (EU), utility indices are assigned such that u(40) > u(30) > u(0). The advantage that R offers over S on the payoff dimension is the possibility of getting 40 rather than 30, a utility difference of u(40)-u(30); whereas the advantage of S over R on the payoff dimension is u(30)-u(0).

[2] The point is not restricted to cases involving dominance. Even if the payoffs were set so that one lottery did not dominate the other – for example, if L_{3*} offered 39.90 rather than 40 – we should still expect almost everyone to choose the latter, because it is easy to judge that the payoff difference is insignificant relative to the difference on the probability dimension, even though that probability difference is judged to be small.

Under EU, these contending advantages are weighted by their respective probabilities: q in the case of u(40)-u(30); and (p-q) in the case of u(30)-u(0). Thus, denoting strict preference by $\succ$ and indifference by ~, the EU representation of the choice problem can be expressed as:

$$S \underset{\prec}{\overset{\succ}{\sim}} R \Leftrightarrow (p\text{-}q).[u(30)\text{-}u(0)] \underset{<}{\overset{>}{=}} q.[u(40)\text{-}u(30)] \qquad (1)$$

which rearranges to:

$$S \underset{\prec}{\overset{\succ}{\sim}} R \Leftrightarrow (p\text{-}q)/q \underset{<}{\overset{>}{=}} [u(40)\text{-}u(30)]/[u(30)\text{-}u(0)] \qquad (2)$$

Call [u(40)-u(30)]/[u(30)-u(0)] 'the relative advantage of R over S on the payoff dimension' and correspondingly call $(p\text{-}q)/q$ 'the relative advantage of S over R on the probability dimension'. Under EU, preference is determined by whichever of these outweighs the other. Moreover, rescaling both p and q by the same factor leaves the inequality unaffected: whatever preference an EU maximiser has between S and R in choice #1, she should have exactly the same preference in choice #2 or in any other choice involving those three payoffs and the same ratio of p to q.

Yet the evidence from very many experiments using similar parameters to those shown above is that when p and q are scaled down, S is less frequently chosen: it is as if S's relative advantage on the probability dimension is progressively weakened relative to R's (unchanging) advantage on the payoff dimension.

The similarity intuition is that as p and q are scaled down, the *difference* between them becomes smaller, and this acts on perceptions so that although the *objective* ratio $(p\text{-}q)/q$ remains the same, the relative advantage is *perceived*, or judged, to be smaller. To model this intuition more formally, we require some functional form such that for the kinds of parameters involved in choices #1 and #2 the judgmental weight assigned to $(p\text{-}q)/q$ falls as the probabilities are scaled down.

However, before proposing such a functional form, let us establish a slightly more general analytical setting. The bulk of the experimental data used to test theories of risk are derived from decisions that can be represented in terms of pairs of alternative lotteries, each involving no more than three payoffs. This provides a basic framework, depicted in Figure 2 below, where the payoffs are $x_3 > x_2 > x_1$ and the probabilities of each payoff under the safer lottery I and under the riskier lottery J are, respectively, p_3, p_2, p_1 and q_3, q_2, q_1.

FIGURE 2 HERE

Denote the advantage that J has over I on the payoff dimension – that is, the subjective value of getting x_3 rather than x_2 – by z_J; and denote the probability of that advantage, namely (q_3–p_3), by a_J. At the same time, the payoff advantage of I over J is the difference between the subjective values of x_2 and x_1, denoted by z_I; and the corresponding probability advantage of I over J is (q_1-p_1), denoted by a_I.

However, if *actual* choices are based on how these relative advantages are judged or perceived, then what counts is how the *perceived* relative advantage of I over J on the probability dimension is weighed against the *perceived* relative advantage of J over I on the payoff dimension. It is *this* balance of judgments/perceptions which is the pivotal idea underpinning the descriptive model proposed in this paper and which is therefore called the *perceived relative advantage model* (PRAM)[3].

It is proposed that the *perceived relative advantage of I over J on the probability dimension* is some function of a_I and a_J, $\phi(a_I, a_J)$, while the *perceived relative advantage of J over I on the payoff dimension* is some function of z_J and z_I, $\xi(z_J, z_I)$. Thus expression (2) can be rewritten as:

[3] This may be the point to mention other models which have used related concepts, and to try to indicate in broad terms the difference between this model and those others. For example, Shafir et al. (1993) proposed an advantage model which accommodated a range of departures from EU. However, that model was concerned exclusively with choices between binary lotteries and money or probability equivalences for such lotteries. Being limited to binary lotteries necessarily restricted the scope of that model: by its nature, it cannot generate predictions about regularities such as the common consequence effect, mixed fanning, violations of betweenness or betweenness cycles, all of which are entailed by the present model, as will be shown below. In addition, Shafir et al. used different parameters for gains and losses, and invoked a particular principle to allow each of those parameters to vary further according to the nature of the task, whereas the present model applies the same person-specific parameters across the board. And although other models – by Mellers and Biagini (1994) and by Gonzalez-Vallejo (2002) for example – also deployed notions of similarity and imprecise judgments to fit various patterns in the data, they too were limited in scope and/or required parameters to vary from one class of task to another. A more detailed analysis of how these models relate to the one being proposed in this paper can be obtained from the author on request.

$$I \begin{array}{c}\succ\\ \sim\\ \prec\end{array} J \;\Leftrightarrow\; \phi(a_I, a_J) \begin{array}{c}>\\ =\\ <\end{array} \xi(z_J, z_I) \qquad (3)$$

Initially, consider the class of cases typical of the great majority of common ratio experiments, where $a_I < a_J$. (We shall in due course look at the implications when that inequality is reversed.) The similarity intuition is that $\phi(a_I, a_J)$ falls as the probabilities are scaled down. The scaling factor can be expressed as (a_I+a_J). When $(a_I+a_J) = 1$, as in choice #1, the probabilities are as scaled up as they can be ($p_2 = 1$). In that case, let us suppose that the perceived ratio coincides with the objective ratio, so that $\phi(a_I, a_J) = a_I/a_J$. However, when $(a_I+a_J) = 0.25$, as in choice #2, the probabilities q_3 and p_2 have been scaled down to one quarter of their choice #1 values. We therefore require a functional form such that, for cases where $a_I < a_J$, $\phi(a_I, a_J)$ falls as (a_I+a_J) falls, although never below 0. One simple way of modelling this is as follows:

$$\phi(a_I, a_J) = (a_I/a_J)^{(a_I+a_J)^{\alpha}} \quad \text{where } \alpha \leq 0 \qquad (4)$$

When $\alpha = 0$, the term $(a_I+a_J)^{\alpha} = 1$, so there is no systematic divergence between perceived and objective ratios: this is what is assumed in any theory where the decision maker always perceives ratios as they are and is not influenced by differences. So α may be thought of as a person-specific behavioural characteristic: someone for whom α is equal to 0 is someone who takes probabilities and their ratios just as they are. However, someone for whom α is less than 0 is inclined to have their judgment of ratios influenced by differences[4], so that for cases where $a_I < a_J$, $\phi(a_I, a_J) < a_I/a_J$ for all $(a_I+a_J) < 1$. Moreover, as α falls, the two diverge more and more. Thus a range of values of α across some population reflects interpersonal differences in the extent to which $\phi(a_I, a_J)$ diverges from a_I/a_J.

However, expression (4) may only be one element of the similarity judgment. To see why, and how it might be modified, consider some other choices within the Figure 2 framework. Figure 3 reproduces the $\{L_3, L_4\}$pair together with two other variants which involve the same values of a_I and a_J, but where the 0.75 probability of

[4] For other evidence of a widespread tendency for judgments about ratios and differences to become conflated see, for example, Birnbaum and Sutton (1992) and Baron (2001).

0 common to L_3 and L_4 is replaced by a 0.75 probability of 30 to give L_5 and L_6, and by a 0.75 probability of 40 to give L_7 and L_8.

FIGURE 3 HERE

Many authors have found it helpful to represent such choices visually by using a Marschak-Machina (M-M) triangle, as shown in Figure 4. The vertical edge of the triangle shows the probability of x_3 and the horizontal edge shows the probability of x_1. Any residual probability is the probability of x_2. The lotteries in the three choices in Figure 3 are depicted in Figure 4, together with L_1 (which is the same as L_5 – i.e. the certainty of 30) and L_2 from Figure 1, plus lottery L_9 = (40, 0.6; 30, 0.25; 0, 0.15), which is also included in Figure 4 for later use.

It is an implication of EU that an individual's preferences can be represented by linear and parallel indifference curves, where the gradient of those lines is a reflection of the individual's risk aversion – the more risk averse the individual, the steeper the gradient. The straight lines connecting L_1 to L_2, L_3 to L_4, L_5 to L_6 and L_7 to L_8 are all parallel, so that an EU maximiser who prefers the safer L_1 to the riskier L_2 also prefers L_3 to L_4, L_5 to L_6 and L_7 to L_8, and a less risk averse EU maximiser who prefers L_2 to L_1 also prefers L_4, L_6 and L_8 in their respective pairs.

FIGURE 4 HERE

The three pairs of lotteries from Figure 3 are connected by lines with the same gradient and the same length. So it might be tempting to think that they are equally similar. But inspection of Figure 3 suggests otherwise. Intuitively, L_3 and L_4 appear most similar, because they entail very similar probabilities of the same zero payoff: in fact, the ratio of those probabilities is 0.75/0.8. By contrast, L_5 and L_6 are arguably the least similar of the three, with L_5 offering a single payoff with certainty while L_6 offers a mix of all three payoffs, and with the ratio of the probabilities of the common payoff being 0.75/1. L_7 and L_8 may be seen as lying somewhere between the other two pairs: like $\{L_3, L_4\}$, each lottery involves two payoffs, as contrasted with the one-versus-three pattern in the $\{L_5, L_6\}$ pairing; but the ratio of the probabilities of the common payoff – 0.75/0.95 – is rather lower than that of $\{L_3, L_4\}$.

A possible objection to this is that many experiments have not displayed alternatives in the format used in Figure 3, but have simply presented them in something like the following form:

L_3: (30, 0.25; 0, 0.75)
L_4: (40, 0.20; 0, 0.80)

L_5: (30, 1)
L_6: (40, 0.20; 30, 0.75; 0, 0.05)

L_7: (40, 0.75; 30, 0.25)
L_8: (40, 0.95; 0, 0.05)

However, it could be argued that, if anything, this format goes even further towards reinforcing the idea that $\{L_3, L_4\}$ is the most similar pair, $\{L_5, L_6\}$ the least similar, with $\{L_7, L_8\}$ lying somewhere between[5].

There may be different ways of modelling this distinction, but one simple way involves generating a factor f_i for each payoff which indexes the degree of commonality between lotteries and using these to scaling down the term in (4). Defining $f_1 = [1-(p_1/q_1)]$, $f_2 = [1-(q_2/p_2)]$ and $f_3 = [1-(p_3/q_3)]$ and raising their product to the power β, where $\beta \geq 0$, allows this element to be incorporated as follows:

$$\phi(a_I, a_J) = (f_1 f_2 f_3)^\beta \left[(a_I/a_J)^{(a_I + a_J)^\alpha} \right] \tag{5}$$

If $\beta = 0$, the ratios of the probabilities of common consequences make no difference (as in EU). As β increases, $(f_1 f_2 f_3)^\beta$ falls, thereby reducing $\phi(a_I, a_J)$: which is

5 This is broadly in line – although for somewhat different reasons – with what was proposed by Buschena and Zilberman (1999). They suggested that when all pairs of lotteries are transformations of some base pair such as $\{L_1, L_2\}$, then the distances between alternatives in the M-M triangle would be primary indicators of similarity. However, they modified this suggestion with the conjecture that if one alternative but not the other involved certainty or quasi-certainty, this would cause the pair to be perceived as less similar, while if both alternatives had the same support, they would be regarded as more similar. That would give the same ordering, with $\{L_3, L_4\}$ as most similar of the three pairs and $\{L_5, L_6\}$ as least similar; and appears to be in line with the data on similarity judgments elicited from participants by Buschena and Zilberman.

to say, as an individual gives more and more weight to the degree of overlap between the common consequences, he judges the two alternatives to be more and more similar on the probability dimension. So β, like α, can be regarded as a person-specific characteristic, with some distribution of β's across the population.

With $\alpha < 0$, $\beta > 0$, but without assigning specific values to those parameters, it is possible to rank pairwise choices between lotteries from Figure 4 in order from the highest to the lowest $\phi(a_I, a_J)$. Table 1 shows these. Of course, there is some danger here of suggesting a rather spurious precision when figures are given to four decimal places, and it should be borne in mind that what both α and β signify are people's somewhat imprecise and impressionistic judgments or perceptions about the relative advantages on the probability as compared with the payoff dimensions. Nevertheless, in order to provide potentially refutable predictions, it is sufficient that the model generates an ordering over the various $\phi(a_I, a_J)$, with actual values unspecified but designated as 'levels', from Level 1 down to Level 6.

TABLE 1 HERE

It is not necessary at this stage to know exactly how the perceived relative advantage on the payoff dimension, $\xi(z_J, z_I)$, is evaluated: in all cases the set of payoffs is the same, so that however the evaluation is done, the value of $\xi(z_J, z_I)$ will be the same for a particular individual facing any of the choices: the question is simply how that value compares with the different $\phi(a_I, a_J)$. Assuming no errors, the various implications are set out in Table 2. This table shows how a series of known regularities may all be consistent with behaving according to PRAM.

TABLE 2 HERE

For behaviour to appear to be completely consistent with EU, it is necessary either that $\xi(z_J, z_I) >$ Level 1, so that the perceived advantage of J relative to I on the payoff dimension is such that the riskier lottery would be chosen in every case listed, or else that $\xi(z_J, z_I) <$ Level 6, in which case the safer lottery would always be chosen.

Any value of $\xi(z_J, z_I)$ between Level 1 and Level 6 will produce the common ratio effect. The size of this range relative to other ranges which produce different regularities may possibly help to explain why this is the violation of EU often most easily produced.

Because much of the early body of evidence focused on the common ratio effect, a number of the earlier non-EU models – Chew and MacCrimmon (1979) and Machina (1982) for example – characterised behaviour as if the individual's indifference curves were 'fanning out' from some point to the south-west of the right angle of the triangle and became flatter (less risk-averse) in the direction of the lower right-hand corner of the triangle. If this pattern were to operate consistently across the whole space of the triangle, it would entail the steepest curves being towards the top corner. However, as Camerer (1995) noted, later experiments often found some degree of fanning *in* towards that top corner (i.e. less risk aversion compared with $\{L_1, L_2\}$), which is consistent with values of $\xi(z_J, z_I)$ between Level 1 and Level 5). In response to this kind of evidence, some non-EU models – for example, Gul (1991) – were developed which have this 'mixed fanning' property.

L_6 is a probability mixture, or linear combination, of L_1 and L_2. According to EU, any such mixture of two 'more extreme' lotteries should not be preferred to both of them. So if L_1 is preferred to L_2, it should also be preferred to any mixture of itself and L_2, such as L_6. Equally, if L_2 is preferred to L_1, L_2 should also be preferred to any mixture of itself and L_1 – again, including L_6. Either way, choosing L_6 in preference to *both* L_1 and L_2 constitutes a violation of 'betweenness'. Yet such patterns have frequently been reported[6], and as Tables 1 and 2 show, they are consistent with the present model if $\xi(z_J, z_I)$ takes any value between Level 3 and Level 4.

Moreover, the model also entails the possibility of a *different* violation of betweenness: if $\xi(z_J, z_I)$ takes a value between Level 2 and Level 6, the mixture lottery L_9 will be *less* preferred than both L_1 and L_2. This region of the triangle has been less thoroughly explored experimentally, but one study that looked at mixtures like L_6 and L_9 along the L_1-L_2 chord – Bernasconi (1994) – found precisely the pattern entailed by the PRAM analysis.

The model also accommodates the other form of Allais paradox that has come to be known as the common consequence effect. This manifests itself as a switch from the safer alternative in choice #3.2 to the riskier alternative in choice #3.1, where

[6] Camerer (1995) concluded that the balance of evidence from a number of studies that examined the issue was consistent with patterns of indifference curves in the M-M triangle which are not linear, as EU entails, but which exhibit some degree of convexity close to the bottom edge of the triangle.

some probability of the intermediate payoff (in this case, a 0.75 chance of 30) is substituted in both lotteries by the same probability of a zero payoff. Under EU, this should make no difference to the balance of expected utilities; but if $\xi(z_J, z_I)$ lies between Level 4 and Level 6, that switch will occur.

The common consequence effect as identified here, and the 'usual' violation of betweenness (i.e. the one where L_6 is preferred to both L_1 and L_2), cannot both be exhibited by the same individual at the same time, since the former requires L_1 to be preferred to L_6 while the latter requires the opposite preference. However, if there is some variability between individuals in terms of their α's, β's and values of $\xi(z_J, z_I)$, we might see different members of the same *sample* exhibiting both regularities to some degree. Meanwhile, Table 2 shows that the same *individuals* can exhibit many other combinations of regularities – for example, the two 'opposite' violations of betweenness, or 'mixed fanning' alongside the common consequence effect.

Thus far, it might seem that the implications of the model developed in this paper are not very different from what might be implied by certain variants of rank dependent expected utility (RDEU) theories[7]. Some of those models can account for many of the regularities discussed above although, as Bernasconi (1994, p.69) noted, it is difficult for any particular variant to accommodate all of them via the same nonlinear transformation of probabilities into decision weights.

However, the modelling strategy in the present paper is *fundamentally* different from the strategy adopted by those theories in certain important respects, and leads to radical differences in the implications for regularities other than those discussed so far.

What is common to the most influential RDEU theories is that there may be rank-dependent transformations which allow violations of independence and/or betweenness while respecting both transitivity and stochastic dominance. To achieve that, each lottery L is evaluated separately and assigned some value index V(L), which is computed as the weighted sum of the values v(.) assigned to each payoff x_i offered by that lottery, with each $v(x_i)$ multiplied by its respective decision weight, π_i. In variants such as those listed in footnote 7, each π_i is determined by transforming the probability of each payoff according to that payoff's rank *within* the lottery. The

[7] An early form of this type of model was Quiggin's (1982) 'anticipated utility' theory. Subsequently, Starmer and Sugden (1989) proposed a form which incorporated a reference point and allowed for losses being treated differently from gains. Essentially the same basic idea is at the heart of Tversky and Kahneman's (1992) 'cumulative prospect theory'.

preference between any two alternatives is then assumed to depend on their respective V(.) indices.

But clearly, if each lottery is evaluated separately and independently of the other, there is no direct comparison between q_3 and p_3, nor between p_1 and q_1, so that the notion of the perceived relative advantage on the probability dimension, which is pivotal to PRAM, has no status in RDEU models.

If this distinction matters, we should expect to see some significant difference between the implications of PRAM and RDEU. And Table 2 shows just such a difference: if $\xi(z_J, z_I)$ takes a value between Level 1 and Level 3, it entails the non-transitive cycle $L_1 \succ L_2$, $L_2 \succ L_6$, $L_6 \succ L_1$; and if that value is in the range between Level 1 and Level 2, it also entails the cycle $L_1 \succ L_2$, $L_2 \succ L_9$, $L_9 \succ L_1$.

Neither of these implications can be accommodated by any RDEU model because, as noted above, RDEU models entail transitivity. So the possibility of systematic patterns of cyclical choice is a key feature of this model which distinguishes it from the RDEU family and any other model which entails transitivity. Moreover, as discussed shortly, the implication of systematic intransitivity is by no means limited to 'betweenness cycles' of the kind considered above.

The experimental evidence relating to betweenness cycles is limited: few studies seem to have set out to look for such cycles. One exception is reported by Buschena and Zilberman (1999), who examined choices between mixtures on two chords within the M-M triangle: they reported a significant asymmetric pattern of cycles along one chord, although no significant non-transitive asymmetries were observed along the other chord. More recently, Bateman et al. (2005) reported more such asymmetries: again, these were statistically significant in one area of the triangle; and in the expected direction, although not significantly so, in another area[8].

It might be objected that if the only thing that distinguishes PRAM from established RDEU models is the prediction of betweenness cycles for which there is only limited evidence, it would appear to be a rather small improvement. But that objection is unjustified, for four reasons.

First, that implication is not the only one which distinguishes PRAM from RDEU models, as will be shown in due course.

[8] Given the relative narrowness of the range within which $\xi(.,.)$ must lie in order to produce such cycles – see Tables 1 and 2 – their frequency in some areas but sparsity in other areas within the same triangle may not be surprising.

Second, it quite often happens that there is sparse pre-existing evidence of certain implications of new models: part of the process of theoretical development is to indicate the possible existence of patterns of data that have not been much reported before because existing theories gave no particular reason to look for them. For example, prior to the publication of regret theory (Bell, 1982; Fishburn, 1982; Loomes and Sugden, 1982) there was relatively little evidence about the effect on choices of different ways of juxtaposing payoffs, precisely because most 'prospect-based' theories placed no importance on juxtaposition issues and therefore few experiments were designed to test for them; but the lack of such evidence *just because it had never been looked for* should not have been (and was not) counted as an argument against regret theory. Indeed, it could be regarded as a positive quality that a new theory accounts for at least as much of the existing data as any extant model but also has some novel and untested implications.

Third, the fact that an existing model and a new model are both able to provide accounts for certain regularities should not automatically be taken to mean that the new model has nothing extra to offer in those respects. Except for those who take a purely instrumentalist view of such things, it also matters whether the new model provides an account which is more faithful to the actual data-generating process than the existing model. A new model may be able to provide a more coherent account of a whole set of regularities which existing models can only account for on a piecemeal basis (e.g. by changing the values of structural parameters from one regularity to another).

Fourth, it is not just a headcount of the number of regularities that matters, but how fundamental any particular one of them is in terms of the structures of different theories.

To illustrate these points, consider the following general experimental design. Select some set of points in the triangle diagram, such that $a_I < a_J$: those in Figure 4, perhaps, possibly supplemented by others on a different line with the same gradient – (30, 0.5; 0, 0.5) and (40, 0.4; 0, 0.6), say, plus the mixture (40, 0.2; 30, 0.25; 0, 0.55). Then take some x_2 and x_1 and hold these constant (so that z_I is fixed) while varying x_3 from a value small enough that the safer alternatives are chosen in every case up to a value big enough to cause the riskier alternatives to be chosen in every pair. Raising x_3 by appropriate increments, and thereby progressively increasing $\xi(z_J, z_I)$, should move each individual up Table 1 through the various levels.

Time limits and limits to participants' attention spans will place practical restrictions on the numbers of questions that any one person can be asked in any single experimental session, and therefore on the fineness of the incremental changes in x_3. So it would be unrealistic to expect in practice that the selected values of x_3 will result in every individual experiencing at least one $\xi(z_J, z_I)$ falling between each pair of adjacent levels in Table 1. Moreover, the finer the increments and the more questions being asked, the more noisy the data are likely to be. Nevertheless, if PRAM is a good model of the way many participants behave, we should expect the movements from safer to riskier alternatives to occur broadly in line with the ordering in Table 1 and we should be able to check the implications set out in Table 2[9].

In particular, we should expect to observe a significant number of individuals exhibiting betweenness cycles in the predicted direction (with those in the opposite direction being few enough to be attributed to noise). Were we to find this pattern, it would represent a very significant challenge not only to EU and RDEU models but to any theory which entails the existence of a well-behaved set of indifference curves in the M-M triangle, since the betweenness cycles predicted by PRAM are simply incompatible with the existence of any such well-behaved indifference map[10].

However, we can go further, in two directions. First, the above analysis was confined to the sorts of prospects which have featured most frequently in experiments: that is, ones where $a_I < a_J$. But if PRAM is to provide an account of behaviour involving lotteries with no more than three payoffs, its implications should be examined for cases where $a_I > a_J$. Moreover, since RDEU models claim to be even

[9] Note that this set of implications holds for cases such as the one depicted in the Figures so far where a_I/a_J is less than 1. In due course it will become apparent that other, somewhat different, sets of implications can be derived for other cases.

[10] The above experimental design has not (yet) been implemented. However, re-analysis of an earlier data set, though not originally designed for the purpose, turns out to yield some additional evidence that supports this distinctive implication of PRAM. Loomes and Sugden (1998) asked 92 respondents to make a number of pairwise choices in random order (with each choice being presented twice on separated occasions) in the course of which they faced six 'betweenness triples' where $a_I < a_J$ – that is, {18, 19, 20}, {21, 22, 23}, {26, 27, 28}, {29,30,31}, {34, 35, 36} and {37, 38, 39} in triangles III-VI. Individuals can be classified according to whether they a) exhibited no betweenness cycles, b) exhibited one or more cycles only in the direction consistent with PRAM, c) exhibited one or more cycles only in the opposite direction to that implied by PRAM, or d) exhibited cycles in both directions. 35 respondents never exhibited a cycle, and 11 recorded at least one cycle in both directions. However, of those who only cycled in one direction or the other, 38 cycled in the PRAM direction, as opposed to just 8 who cycled only in the opposite direction. If both propensities to cycle were equally likely to occur by chance, the probability of the ratio 38:8 is less than 0.000005; and even if all 11 'mixed cyclers' were counted against the PRAM implication, the probability of the ratio 38:19 would still be less than 0.01.

more general, their performance in this less familiar territory also requires examination. Second, the implications of PRAM are not confined to the M-M triangle: although they may only apply to pairwise comparisons involving no more than three payoffs at any one time, the model also applies to sequences of such choices, opening up the possibility of other forms of intransitivity. We shall come to those shortly, after considering some of the implications of PRAM when $a_I > a_J$.

Suppose that instead of offering a 0.8 chance of 40 in the most scaled-up choice, the riskier lottery offers a 0.4 chance of, say, 80. Without knowing more about the way an individual processes payoffs and derives her $\xi(z_J, z_I)$, we cannot say whether this 'new' lottery is more or less likely to be chosen over L_1 than was the original L_2, but that is not important for present purposes.

Label the 'new' lottery L_{12}, and re-label the original L_1 as L_{11}. On the basis of scaling down, mixing and substituting common consequences, we can generate L_{13} to L_{19} as listed in Table 3, which are the counterparts to the lotteries L_3 to L_9 in Tables 1 and 2.

TABLE 3 HERE

As in Table 1, it is possible to construct expressions for $\phi(a_I, a_J)$ for each pair. However, without knowing the values of α and β, it is no longer possible to provide a complete ordering as was done in Table 1. To illustrate,

$$\phi(a_{11}, a_{12}) = 1^{\beta}\left[1.5^{(1)^{\alpha}}\right] \quad = \ 1.5$$

while

$$\phi(a_{13}, a_{14}) = 0.1667^{\beta}\left[1.5^{(0.25)^{\alpha}}\right]$$

Whether $\phi(a_{13}, a_{14})$ is greater than or less than 1.5 depends on the values of α and β: the more positive β is, the smaller will be the first component of the expression, while the more negative α is, the larger will be the component in square brackets. However, what this allows is the possibility that scaling down the probabilities when $a_I > a_J$ *may* result in the *opposite* of the usual common ratio pattern: that is, instead of observations of L_1 & L_4 outnumbering L_2 & L_3, PRAM allows the possibility that if the alpha component outweighs the beta component, scaling down may increase $\phi(a_I, a_J)$ so that there may be some tendency for those choosing the riskier lottery in the scaled-up pair switching to the safer option in the scaled-down choice. And while this cannot be an unamiguous prediction in the absence of information about the values of α and β, it suggests a possibility which is contrary to the implications of RDEU models under standard assumptions about the probability-weighting function.

Moreover, it turns out that there is evidence which is consistent with PRAM in this respect and contrary to the RDEU implication: for example, Battalio et al. (1990) constructed a set of problems (Set 2 in their Table 7) which involved $(x_3–x_2) = \$14$ and $(x_2–x_1) = \$6$ with $a_I/a_J = 2.33$. Scaling down by a fifth resulted in 16 departures from EU (out of a sample of 33), with 10 of those switching from R in the scaled-up pair to S in the scaled-down pair, while only 6 exhibited the 'usual' common ratio pattern. Admittedly, this is only one study – most studies have not explored these sorts of parameters – but in conjunction with PRAM, it suggests there may be scope for closer examination of these sorts of parameters.

Although it is not possible to generate a complete ordering over the $\phi(a_I, a_J)$ applying to L_{11}-L_{19}, it is still possible to identify some partial orderings, as in Table 4.

TABLE 4 HERE

Some implications emerge from this. For example, even though the common ratio effect might be reversed if the influence of α is strong enough, α does not enter into the common consequence effect; and since the beta component causes $\phi(a_{11}, a_{16})$ to be greater than $\phi(a_{13}, a_{14})$, the usual common consequence may be expected to continue. But what PRAM suggests is that some – possibly many – of the implications of RDEU models for cases where $a_I < a_J$ may break down, or even be reversed, in cases where $a_I > a_J$ and where a_I/a_J becomes sufficiently large and/or (a_I+a_J) becomes sufficiently small. Besides the study by Battalio et al. cited above, some other studies have reported data seemingly at odds with, or at least unsupportive of, the usual RDEU configuration as applied to the M-M triangle. For example, Prelec (1990, p.255) also found variable degrees of fanning, and violations of betweenness in both directions, and concluded that "the relationship between the local attitude to risk and gamble value is, in general, nonmonotonic".

However, the implications of PRAM are not limited to lotteries within particular triangles. In particular, the model entails the possibility of other systematic intransitivity. Consider a triple of lotteries consisting of L_3 and L_4 as above, plus another lottery L_{10} = (55, 0.15; 0, 0.85), and consider the three pairwise choices L_3 vs L_4, L_4 vs L_{10} and L_3 vs L_{10}. The three pairwise comparisons are as shown in Table 5:

TABLE 5 HERE

Suppose, purely for illustrative purposes, that we take the perceived relative advantage of J over I on the payoff dimension simply to be the ratio of the subjective value differences, with those subjective values being drawn from a conventional v(.) function[11] with v(0) = 0: that is, $\xi(z_J/z_I) = [v(x_3)-v(x_2)]/v(x_2)$, which can be rewritten as $[v(x_3)/v(x_2)] - 1$.

An individual will be indifferent between I and J when $\xi(z_J, z_I) = \phi(a_I, a_J)$. Applying this (after minor rearrangement) gives:

$$L_3 \sim L_4 \iff \frac{v(40)}{v(30)} = 1 + \phi(a_3, a_4) \qquad (6)$$

$$L_4 \sim L_{10} \iff \frac{v(55)}{v(40)} = 1 + \phi(a_4, a_{10}) \qquad (7)$$

$$L_3 \sim L_{10} \iff \frac{v(55)}{v(30)} = 1 + \phi(a_3, a_{10}) \qquad (8)$$

With a v(.) function with the standard properties it must be the case that

[11] The 'most' conventional such function would be the vN-M utility function; however, a wider class of value functions, including the forms of v(.) function used in RDEU models, will also suffice.

$$\frac{v(55)}{v(30)} = \frac{v(55)}{v(40)} \times \frac{v(40)}{v(30)} \qquad (9)$$

so that transitivity would require $1 + \phi(a_3, a_{10}) = [1 + \phi(a_3, a_4)] \times [1 + \phi(a_4, a_{10})]$. However, inspection of Table 5 shows that this will be true only if α and β are both equal to zero. If either $\alpha < 0$ or $\beta > 0$, $1 + \phi(a_3, a_{10}) > [1 + \phi(a_3, a_4)] \times [1 + \phi(a_4, a_{10})]$ so that $L_3 \succ L_{10}$. In other words, PRAM here allows the possibility of the choice cycle $L_{10} \succ L_4$, $L_4 \succ L_3$, $L_3 \succ L_{10}$, but not its opposite. This cycle will be denoted by RRS to signify that in each of the two more similar pairs the riskier (R) alternative is chosen, while in the least similar pair the safer (S) option is chosen.

Such cycles are, of course, consistent with similarity theory. However, there appear to have been few studies reporting them. One notable exception is a paper by Tversky (1969), which both Rubinstein (1988) and Leland (2002) acknowledge as influential, where *some* evidence was reported; but the data relating to lotteries were generated by just eight respondents, who had themselves been selected from an initial sample of eighteen on the basis of a 'screening' procedure which established a predisposition to violate transitivity in that particular direction. Recently, however, Bateman et al. (2005) reported that such cycles had been observed in two separate experiments, each involving around 150 participants. The main purpose of those experiments had been to explore the common ratio effect, and the data concerning cycles were to some extent a by-product. Nevertheless, in all four cases they observed, RRS cycles outnumbered SSR cycles to a highly significant extent.

On the other hand, that asymmetry is in quite the *opposite direction* to the patterns reported in a number of papers testing the implications of regret theory and/or examining the causes of the preference reversal phenomenon. The preference reversal phenomenon – see Lichtenstein and Slovic (1971), and Seidl (2002) for a review – occurs when individuals place a higher certainty equivalent value on one item in a pair, but pick the other item in a straight pairwise choice.

In the context of lotteries, this phenomenon has typically involved a lottery offering a fairly high chance of a moderate prize (a 'P-bet'), and an alternative offering a considerably lower chance of a rather bigger prize (a '\$-bet'). If the certainty equivalents of the two bets are denoted by CE(P) and CE(\$), the preference reversal phenomenon takes the form that many individuals state CE(\$) > CE(P) while exhibiting P $\succ$ \$ in a straight choice between the two. The opposite reversal – CE(P) > CE(\$) with \$ $\succ$ P – is relatively rarely observed.

For some sure sum of money M such that CE(\$) > M > CE(P), the common form of preference reversal translates into the choice cycle \$ $\succ$ M, M $\succ$ P, P $\succ$ \$. A number of studies have tested for the existence of such cycles, and their predominance over cycles in the opposite direction has been reported in several papers – see, for example, Tversky, Slovic and Kahneman (1990) and Loomes, Starmer and Sugden (1991). However, note that such a cycle involves choosing the safer alternative in each of the two more similar pairs {M, P} and {P, \$} while choosing the riskier alternative from the least similar pair {M, \$} – that is, it involves the cycle SSR, which is in exactly the opposite direction to the one which is consistent with Tversky (1969) and with the model in this paper, as developed so far. Does this constitute a refutation of the model? Or can such seemingly contradictory results from different studies be reconciled? That is the issue addressed in the next section.

3. Extending the Model to the Payoff Dimension

Up to this point, the main focus has been upon the degree of similarity between *probabilities* and little has been said about payoffs and the specification of $\xi(z_J, z_I)$. However, if interactions between ratios and differences are a general perceptual phenomenon, there is no reason to suppose that they operate on the probability dimension but not on the payoff dimension. In this section, the model will be extended accordingly.

In decision theory, it is usual to suppose that what individuals are concerned with on the payoff dimension is not the payoffs themselves but the utility of those payoffs: indeed, when the payoffs take some non-monetary form such as 'a weekend in Paris' or 'the amputation of a leg', there is no obvious alternative except to use some such index. To avoid any possible confusion with the notation from EU or RDEU, the primitives on the payoff dimension in this and subsequent sections will either be the payoffs themselves or some function of them expressed in terms of what might be thought of as 'basic' or 'choiceless' subjective value indices[12]. It will be

[12] The notion here is much the same as that proposed in Loomes and Sugden (1982): something that may be thought of as utility in the Bernouillian tradition – that is, the anticipated utility of any given

shown that such a function, denoted by c(x), may be smooth and everywhere weakly concave and yet be compatible with patterns that are sometimes taken to signify that v(.) is convex in some regions and/or kinked at certain points. In PRAM the working assumption is that c(.) is everywhere a weakly concave function of x (or of W + x, where W represents status quo wealth).

Let $c(x_i)$ be denoted by c_i for all i. Thus the payoff advantage of J over I, z_J, is given by $(c_3–c_2)$, and the payoff advantage of I over J is $z_I = (c_2–c_1)$. These terms are thus analogous to the probability differences a_I and a_J. On that basis, the most direct analogue to expression (4) would then be:

$$\xi(z_J, z_I) = (z_J/z_I)^\gamma \qquad (10)$$

where $\gamma \geq 1$ and plays a role which corresponds to $(a_I+a_J)^\alpha$ on the probability dimension[13].

To illustrate how this operates, apply expression (10) to pairwise choices between the three lotteries L_3, L_4 and L_{10}. Letting c(x) = x, we get $\xi(z_4, z_3) = (10/30)^\gamma$, $\xi(z_{10}, z_4) = (15/40)^\gamma$, and $\xi(z_{10}, z_3) = (25/30)^\gamma$. If preferences were linear in probabilities – so that $1 + \phi(a_{10}, a_3) = [1 + \phi(a_{10}, a_4)] \times [1 + \phi(a_4, a_3)]$ – transitivity would require $1 + \xi(z_{10}, z_3) = [1 + \xi(z_{10}, z_4)] \times [1 + \xi(z_4, z_3)]$. But this will only be the case when $\gamma = 1$; when the perceived relative advantage on the payoff dimension is specified as above, $1 + \xi(z_{10}, z_3) > [1 + \xi(z_{10}, z_4)] \times [1 + \xi(z_4, z_3)]$ for all $\gamma > 1$. Thus if preferences *were* linear in probabilities, this would allow the cycle $L_3 \succ L_4$, $L_4 \succ L_{10}$, $L_{10} \succ L_3$, – that is, an SSR cycle[14].

Of course, the point of PRAM applied to the probability dimension is to model preferences as *not* being linear in probabilities. But when probabilities are scaled *up* to the point where $(a_I+a_J) = 1$, the perceived ratio and the objective ratio are assumed to

payoff or consequence (which might, in many applications, be non-monetary) as it will be experienced in consumption, irrespective of how it was acquired.

[13] It is possible to configure γ more elaborately, with a 'scale' component analogous to (a_I+a_J) and a separate 'perception' element corresponding to α. However, none of the results in this paper depend on such a separation.

[14] Notice the resemblance between the inequality $1 + \xi(z_{10}, z_3) > [1 + \xi(z_{10}, z_4)] \times [1 + \xi(z_4, z_3)]$ and the condition that characterises regret theory – especially as specified in Loomes and Sugden (1987). In regret theory, the *net advantage* of one payoff over another is represented by the $\psi(. , .)$ function, which is assumed to be strictly convex, so that for all $x_3 > x_2 > x_1$, $\psi(x_3, x_1) > \psi(x_3, x_2) + \psi(x_2, x_1)$. Regret theory assumes preferences to be linear in probabilities, so for binary lotteries such as L_3, L_4 and L_{10} convexity of the $\psi(. , .)$ function allows SSR but not RRS cycles.

coincide. So for two of the three choices in the cycles that are the choice analogue of preference reversals – that is, for {M, \$} and for {M, P} – $\phi(a_I, a_J) = a_I/a_J$, so that the only 'distortion' occurs in the perception of probabilities in the {P, \$} choice: but since the P-bet often involves a probability of winning quite close to 1, the extent of the divergence from linearity may be relatively small.

Thus the inequality $1 + \xi(z_{10}, z_3) > [1 + \xi(z_{10}, z_4)] \times [1 + \xi(z_4, z_3)]$ on the payoff dimension may outweigh the inequality in the opposite direction on the probability dimension when probabilities are scaled up and most dissimilar, producing significantly more SSR cycles than RRS cycles; but when those probabilities are scaled down, the inequality $1 + \phi(a_3, a_{10}) > [1 + \phi(a_3, a_4)] \times [1 + \phi(a_4, a_{10})]$ on the probability dimension may weigh more heavily, overturning the effect of the payoff inequality and producing significantly more RRS than SSR cycles.

That rather striking implication of the model turns out to have some empirical support. Following the first two experiments reported in Bateman et al. (2005), a third experiment was conducted in which every pairwise combination of four scaled-up lotteries, together with every pairwise combination of four scaled-down lotteries, were presented in conjunction with two different sets of payoffs. All these choices were put to the same sample in the same sessions under the same experimental conditions. The results are reported in Day and Loomes (2005). As PRAM suggests, there was a tendency for SSR cycles to outnumber RRS cycles when the lotteries were scaled up, while the opposite asymmetry was observed among the scaled-down lotteries.

In short, if perceived relative advantage as modelled in this paper operates on *both* dimensions – and there seems no reason why, if it operates at all, it should operate only on one dimension and not the other – then it sets up interesting tensions and offers an account of all of the regularities considered above, and more besides, as discussed in the next section.

4. Some Further Implications

4.1 The Shape of c(.) and the Magnitude of β

Consider the choice between the certainty of x_2 and a lottery offering x_3 and x_1, both with probability 0.5. Here $\phi(a_I, a_J) = 1$, irrespective of the individual's values of α and β. Thus an individual will be indifferent between I and J when $\xi(z_J, z_I) = 1$.

Examining such cases and observing the relationship between $(x_3–x_2)$ and $(x_2–x_1)$ would give insights into the extent to which it can be assumed that $c(x_i) = x_i$ for all i, or whether concavity of c(.) needs to be assumed.

People's aversion to actuarially fair 50-50 gambles in the domain of gains – and even more so, when x_2 is the *status quo* while x_3 is a gain relative to current wealth and x_1 is a loss of the same magnitude relative to current wealth – has been taken as a sign of risk aversion, compounded perhaps by loss aversion. However, the PRAM analysis does not operate in terms of either risk aversion or loss aversion as individual characteristics. Under PRAM, the individual characteristics are represented by α, β and γ operating on the probabilities and choiceless subjective value indices given by c(x).

On this basis, suppose that some set of payoffs $x_3 > x_2 > x_1 \geq 0$ have been identified such that the individual is indifferent between the certainty of x_2 and a 50-50 lottery paying x_3 or else x_1. Holding those payoffs constant but scaling down the probabilities of x_3 and x_2 will result in a change in $\phi(a_I, a_J)$ attributable solely to the 'beta component' of that expression, since the 'alpha component' is held equal to 1 under these conditions; and this allows an exploration of the sign and magnitude of β.

4.2 Mixed 'Risk Attitudes'

For another perspective on the way that the PRAM framework can accommodate what appears to be a manifestation of within-person mixed attitudes to risk, consider the implied pattern of certainty equivalents for binary lotteries with the same means but different degrees of skewness. To illustrate, consider lotteries of the form $(x_3, q_3; 0, 1\text{-}q_3)$ with the means held constant at some x_2, so that $x_2 = x_3 \times q_3$. Since we are dealing with certainty equivalents, $(a_I+a_J) = 1$, and $\phi(a_I, a_J)$ is therefore simply $(1–q_3)/q_3$. To keep things as simple as possible, let $c(x_i) = x_i$ for all i. Then the ratio $z_J/z_I = (1–q_3)/q_3 = \phi(a_I, a_J)$, so that $\xi(z_J, z_I) = [\phi(a_I, a_J)]^\gamma$. If $\gamma > 1$, then whenever $\phi(a_I, a_J)$ is less than 1, $\xi(z_J, z_I)$ is even smaller, so that the certainty is preferred to the lottery, implying that the certainty equivalent of the lottery must be less than its expected value – an observation that is conventionally taken to signify risk aversion; but whenever $\phi(a_I, a_J) > 1$, $\xi(z_J, z_I)$ is bigger, so that the lottery is preferred, and the certainty equivalent of the lottery will be greater than its expected value – conventionally interpreted as risk seeking.

Notice that these seemingly mixed risk attitudes are obtained even when c(.) is assumed to be linear. In other words, the patterns are not – as they would conventionally be interpreted – due to curvature in the utility/value function, but rather are the result of the way that relative advantages on the payoff dimension are perceived: when z_J is small relative to z_I, the effect of $\gamma > 1$ is to reduce the weight on the riskier lottery, whereas when z_J is large relative to z_I, perceptual influences enhance the appeal of the riskier alternative. If c(.) were concave, the result would be modified somewhat: when $x_2/x_3 = 0.5$, $(c_3-c_2)/c_2 < 0.5$, so the certainty would be strictly preferred for $q_3 = 0.5$ – and for some range of values below 0.5, depending on the curvature of c(.) and the value of γ. Nevertheless, it could easily happen that below some value of q_3 there is a range of probabilities for which the certainty equivalent of the lottery would be greater than its expected value.

4.3 The Reflection Effect and 'Loss Aversion'

Since Kahneman and Tversky (1979), a number of studies have shown that when the sign in front of all payoffs is changed from positive to negative, the preference between two alternatives is liable to be reversed. This has been interpreted as evidence that the value function is concave for gains relative to the status quo but convex for losses over some range in the vicinity of the status quo reference point (usually also supposing the gradient of the function to be steeper for losses than for gains of the same magnitude, and with a kink at the status quo).

The PRAM framework also implies the reflection effect, but without any convexity of the c(.) function in the domain of losses, nor any kink at x = 0 or anywhere else. Indeed, as shown below, it produces the effect even when c(x) = x.

Figure 5 reproduces the two choices from Figure 1, but with all positive payoffs replaced by losses of the corresponding magnitudes. The requirement for L_1 to be preferred to L_2 as depicted in Figure 1 is that $\phi(a_I, a_J) > \xi(z_J, z_I)$. Since $\phi(a_I, a_J) = 0.25$ in this case, $L_1 \succ L_2$ requires $(10/30)^\gamma < 0.25$, which will hold iff $\gamma > 1.26186$.

FIGURE 5 HERE

In the choice between L_{-1} and L_{-2}, the effect of reversing the signs on the payoffs is to invert both a_I/a_J and z_J/z_I, so that preference depends on the relative magnitudes of $\phi(0.8, 0.2)$ and $\xi(30, 10)$. If $\gamma > 1.26186$, as was required for $L_1 \succ L_2$, then $(30/10)^{\gamma} > 4$, so that $L_{-2} \succ L_{-1}$. Thus PRAM implies the reflection effect[15].

It is also easy to see that if the probabilities of losses are scaled down so as to produce L_{-3} and L_{-4}, $\xi(z_J, z_I)$ still has whatever value it had for the $\{L_{-1}, L_{-2}\}$ choice, but $\phi(a_I, a_J)$ will be increased as the ratio of 4 from a_I/a_J is raised to the power $(0.25)^{\alpha}$. If this raises $\phi(a_I, a_J)$ to the extent that $\phi(a_I, a_J) > \xi(z_J, z_I)$, the safer alternative L_{-3} will be chosen, implying a switch from choosing R in the scaled-up loss pair to choosing S in the scaled-down pair – a reflection of the standard common ratio effect observed in the domain of gains and another regularity reported in the literature (again, see Kahneman and Tversky, 1979, or Battalio et al., 1990, among others).

What if c(.) were everywhere concave? For lotteries such as L_2, where $q_3 > 0.5$, the effect would be to reinforce the appearance of risk aversion in the domain of gains. For lotteries involving a 50-50 chance of a gain and a loss, individuals would also behave in a risk averse manner, which in (Cumulative) Prospect Theory terms would be represented by the value function being steeper for losses than for gains of the same absolute magnitude. For choices between prospects such as L_{-1} and L_{-2}, the outcome would depend on the interaction between the curvature of c(.) and the value of γ. For modest losses of the sort examined in incentive-compatible experiments, it could well be the case that the impact of γ relative to any curvature of c(.) could produce risk seeking behaviour. Thus it is at least arguable that behaviour which has been interpreted in terms of a value function that is concave for gains but convex for losses and steeper for losses than for gains of the same magnitude, might actually be generated by the processes modelled by PRAM with c(.) everywhere concave.

4.4 Preference Reversals: Money Equivalents and Probability Equivalents

Earlier it was shown how the PRAM analysis allowed the choice cycle analogue of the preference reversal phenomenon, namely $\$ \succ M$, $M \succ P$, $P \succ \$$. It follows that the sure money equivalent of the \$-bet, $M_{\$}$, is strictly greater than the sure money equivalent of the P-bet, M_P. While this may not be the only influence on the certainty equivalent values that individuals state in experiments, it is nevertheless consistent with the preference reversal phenomenon, which could therefore be

[15] Although the point here is illustrated with a numerical example, it is easy to see that this is a general result, since what it hinges on is the inversion of a_I/a_J and z_J/z_I while the individual's α, β and γ remain the same.

expected to occur even if those other influences were not at work. And because it is entails the reflection effect, the PRAM analysis also accommodates the opposite asymmetry when losses are involved, as reported in Loomes & Taylor (1992).

Moreover, there is another form of reversal which the model can explain. In addition to a P-bet and a \$-bet, consider some lottery T which offers an even higher payoff than the \$-bet but with a lower probability. Since the probability of winning offered by the \$-bet is usually (a lot) less than 0.5, \$ and T may both be regarded as scaled-down. So if we can identify some {P, \$} pair such that an individual has a *slight* preference for the \$-bet, the balance of perceived relative advantages would allow an RRS cycle that would translate into T $\succ$ \$, \$ $\succ$ P, P $\succ$ T.

If the individual is then asked to give a *probability equivalent* by adjusting the probability of winning the payoff of T until she is indifferent between P and T, she will adjust it *upwards* from its initial value. Let this probability equivalent of P be denoted by PE(P). Asked to undertake the corresponding task to establish the probability equivalent of the \$-bet, the initial preference T $\succ$ \$ requires that she adjust the original probability of the T payoff *downwards*. Thus PE(\$) < PE(P) at the same time as \$ $\succ$ P. So the PRAM analysis is consistent not only with the predominance of the classic money preference reversal, but also with the opposite asymmetry when value is elicited in probability equivalence form.

Butler and Loomes (2005) reported an experiment which elicited both certainty equivalents and probability equivalents for a particular {P, \$} pair via an iterative choice procedure. They observed the standard asymmetry whereby reversals of the form P $\succ$ \$ but CE(\$) > CE(P) outnumbered the opposite money reversals to a highly significant extent; at the same time, they observed probability reversals of the form \$ $\succ$ P but PE(P) > PE(\$) significantly outnumbering those in the opposite direction. Since the elicitation of certainty equivalents and probability equivalents play a significant role in informing various important areas of public policy[16], the fact that both are prone to disparities between choice and valuation, and moreover prone to disparities in opposite directions, may be a cause of some practical concern.

However, there is an even more distinctive implication of PRAM, as follows. The \$-bet offers some payoff $x_\$$ with probability q, while the P-bet offers x_P with

[16] For example, certainty equivalents are elicited to estimate the value of reducing the risks of death and injury to guide public safety policy, while probability equivalents (often referred to as 'standard gambles') have been widely used to elicit values of health states for cost utility purposes in health care provision.

probability p. Typically, q is small and p is several times larger, so that (p-q) often exceeds q to a considerable extent. Supposing (p-q) > q and c(x) = x, and denoting the certainty equivalent of the \$-bet by $M_\$$, we can express the point at which an individual is indifferent between the \$-bet and the P-bet as:

$$\text{P} \sim \$ \;\Leftrightarrow\; [(p-q)/q]^{p^{\alpha}} = [(x_\$ - x_P)/x_P]^{\gamma} \qquad (13)$$

The certainty equivalent of the \$-bet, $M_\$$, can be expressed as:

$$\$ \sim M_\$ \;\Leftrightarrow\; [(1-q)/q] = [(x_\$ - M_\$)/M_\$]^{\gamma} \qquad (14)$$

It can be seen immediately that so long as (p-q)/q is greater than 1, there will exist a value of p^{α} such that $M_\$ = x_P$: that is to say, an individual could be indifferent between P and \$ and also state a certainty equivalent for the \$-bet which is equal to the positive payoff offered by the P-bet. Higher values of p^{α} are therefore compatible with P $\succ$ \$ and at the same time $M_\$ > x_P$. This latter is what Fishburn (1988) called a ‘strong reversal’: that is, a case where the P-bet is chosen even though the certainty equivalent of the \$-bet is strictly greater than the positive payoff offered by the P-bet.

Even models such as regret theory, which can accommodate preference reversals up to a point by relaxing transitivity, cannot accommodate strong reversals. Nor can strong PE reversals[17] be reconciled with any existing formal theories (at least, not in their deterministic forms). Yet strong reversals *are* a feature of the evidence: indeed, in the Butler and Loomes (2005) data, more than 40% of both CE and PE reversals were ‘strong’; and PRAM can accommodate them.

5. Concluding Remarks

The past thirty years has seen the development of an array of ‘alternative’ theories which try in different ways to account for the many well-established regularities observed in individual decision experiments: see Starmer (2000) for a

[17] Strong PE reversals occur when \$ $\succ$ P in the straight choice while the probability equivalent of the P-bet is strictly greater than the chance offered by the \$-bet of receiving its positive payoff. A parallel analysis to the one for certainty equivalents shows that strong PE Reversals are also allowed by PRAM.

review of “the hunt for a descriptive theory of choice under risk”; and Rieskamp et al. (in press) for a review from a more psychological perspective.

However, no single theory has so far been able to organise more than a subset of the evidence. This has been something of a puzzle, because all of the regularities in question are generated by the same kinds of people. In fact, in some experiments, the very same group of individuals exhibit many of them one after the other in the same session. So it would seem that there really ought to be a single model of individual decision making under risk that is able to account for *most if not all* of them.

It has been argued above that PRAM (or something very much like it) offers a solution to that puzzle by representing the way that many participants in experiments make pairwise choices and judge equivalences in cases where there are no more than three payoffs – this being the nature of the great majority of experimental designs. Using some simple propositions about perception and judgment, PRAM shows how a typical sample of participants will, between them, be liable to exhibit *all* of the following regularities: the common ratio effect; the common consequence effect; mixed fanning; violations of betweenness; betweenness cycles; ‘similarity’ cycles; ‘regret’ cycles; the reflection effect and mixed risk attitudes; and preference reversals with money and with probability equivalents[18].

However, while pairwise comparisons involving no more than three payoffs may be the staple diet of individual decision experiments, they are only a small subset of the kinds of risky decisions which are of interest to psychologists, economists and decision theorists. Yet if those same individuals were asked to make decisions involving more complex lotteries and/or more alternatives, we should expect them to be able to do so. So is there some more general form of PRAM that characterises behaviour in those richer tasks as well as in the simpler cases that have been the focus of the bulk of individual decision experiments to date?

It seems unlikely. The key to explaining the various choice and equivalence tasks discussed above is that many individuals make somewhat impressionistic assessments in cases where there are just two contending advantages on both the payoff and the probability dimensions. If there were larger numbers of different

[18] Actually, this is not an exhaustive list: in earlier drafts it was shown how the model entailed other regularities, including certain violations of the ‘reduction of compound lotteries’ axiom and ambiguity aversion; but this list suffices to demonstrate the scope of the model, and details of how the model accounts for other patterns are available from the author.

payoffs, individuals would no doubt find means of processing them, but there is more room for various different ways in which they might do so.

Moreover, even if each alternative only involved at most two outcomes, asking respondents to process three or more lotteries at a time might well modify pairwise judgments and/or introduce additional considerations. Indeed, in certain cases, it could hardly be otherwise. For example, consider cases where PRAM allows intransitive cycles over three lotteries. Suppose individuals who exhibit such cycles are asked to rank-order the three lotteries. Clearly, in order to do so, they must reverse at least one of the orderings expressed in their pairwise choices. However, it is not obvious that different individuals would use the same means of deciding what to do in such circumstances: a variety of other perceptual or judgmental influences might be brought into play.

So it is far from obvious that there is any straightforward way of generalising PRAM to include more complex lotteries and/or larger choice sets. For many decision theorists and for social scientists wanting to apply theory to cases involving such lotteries and/or choice sets, this may be an inconvenient and discomforting message. But this is not a criticism of PRAM: if people really do process simple experimental tasks differently from the way they process more complex decision problems, it will be necessary to develop experimental and other empirical methods suitable for those more complex environments in order to discover what kinds of models function best in *those* domains. It may then be possible to identify the salient features that distinguish one domain from another, and the modifications required when moving across domain boundaries.

Besides a desire for generality, theorists also like to have models with appealing normative properties. There is a long tradition in decision theory that involves identifying what 'rational' individuals 'ought' to do and supposing that such features are a *sine qua non* for any 'good' theory. Two things may be said about this.

First, on closer examination it turns out that some of these normative principles may not be quite so compelling after all. For example, if human preferences allow for interactions between different outcomes of the same lottery (through, say, the formation of expectations and the anticipation of, and reaction to, the possibilities of those expectations being exceeded or disappointed), the independence axiom may not hold – see, for example, Bell (1985), Loomes and Sugden (1986). Put another way, requiring that the independence axiom *should* hold may be tantamount to disallowing those features of human preferences. Likewise, if human preferences allow for interactions between the contrasting outcomes of different lotteries (through, say, the anticipation of regret if the chosen alternative yields a worse outcome than the foregone alternative) then imposing transitivity amounts to saying that such features of preferences are inadmissible.

Second, incorporating features into a theory on the grounds of their normative desirability is bound to cause problems when they come to be tested: if the data are actually generated by a PRAM-like process, trying to make the data fit such theories is rather like trying to shoehorn an Ugly Sister's foot into one of Cinderella's elegant glass slippers. And the price that has to be paid to try to accommodate the mismatch between theory and data is that supplementary assumptions or forms of special pleading may need to be invoked.

To illustrate this point, consider regret theory. Because it seemed compelling to have preferences linear in probabilities, there were regularities that regret theory could not easily accommodate. For example, if common ratio and common consequence problems were presented in a form that preserved the act-state juxtaposition and thereby invoked Savage's (1954) sure-thing principle, no systematic switching was predicted by regret theory. The 'solution' was to assume that the lotteries were statistically independent of each other, so that scaling down the probabilities allowed greater weight to be given to juxtapositions where regret favoured the riskier alternative. But the fact is that these effects do *not* in practice require statistical independence. By building in a normatively-driven requirement that preferences should be linear in probabilities, regret theory could not accommodate some of the effects that arise from the $\phi(., .)$ component in PRAM without such qualifying assumptions.

On the other hand, the reason why regret theory succeeded in accommodating certain violations of transitivity that many other models could not explain was that it tapped into the kind of behaviour entailed by the $\xi(., .)$ component of PRAM. In contrast, by designing RDEU so as to maintain transitivity, modellers imposed restrictions that are incompatible with the behaviour generated by $\phi(., .)$ and $\xi(., .)$. Violations of transitivity and the preference reversal phenomenon therefore require recourse to ‘outside’ considerations[19]. More generally, if PRAM is an essentially correct model of how perceptions operate on the probability dimension, neither RDEU nor any other theory which entails non-intersecting indifference curves everywhere in the M-M triangle can be descriptively adequate.

In the face of such a seemingly irreconcilable descriptive-normative conflict, what can or should be done? This paper cannot provide any simple or definitive answer to that question. That debate is still to come. What this paper has sought to show is that the question should not be postponed in the vain hope that someone may eventually devise a theory which simultaneously satisfies the requirements of generality, normative acceptability and descriptive adequacy: such a theory is a chimera, and we would be better advised to attend instead to the question of what to do in terms of prediction and prescription in the absence of any such theory.

[19] For example, Schmidt, Starmer and Sugden (2005) have proposed a ‘third-generation’ form of prospect theory which aims to explain preference reversal. Their explanation involves allowing the reference point to be different for a selling task than for a direct choice. However, while this might account for reversals in cases where values are elicited in a *selling* framework, it does not account for the analogous SSR choice cycles discussed earlier, and still less for the phenomenon when values are elicited via a buying task, such as those reported in Lichtenstein and Slovic (1971) and in Loomes, Starmer and Sugden (2005).

Figure 1

<table>
<tr><td></td><td colspan="2">1</td></tr>
<tr><td>L_1</td><td colspan="2">30</td></tr>
<tr><td>L_2</td><td>40</td><td>0</td></tr>
<tr><td></td><td>0.8</td><td>0.2</td></tr>
</table>

<table>
<tr><td></td><td colspan="2">0.25</td><td>0.75</td></tr>
<tr><td>L_3</td><td colspan="2">30</td><td>0</td></tr>
<tr><td>L_4</td><td>40</td><td colspan="2">0</td></tr>
<tr><td></td><td>0.2</td><td colspan="2">0.8</td></tr>
</table>

Figure 2

<table>
<tr><td></td><td>p_3</td><td colspan="3">p_2</td><td>p_1</td></tr>
<tr><td>I</td><td>x_3</td><td colspan="3">x_2</td><td>x_1</td></tr>
<tr><td>J</td><td colspan="2">x_3</td><td>x_2</td><td colspan="2">x_1</td></tr>
<tr><td></td><td colspan="2">q_3</td><td>q_2</td><td colspan="2">q_1</td></tr>
</table>

Figure 3

Choice #3.1

<table>
<tr><td></td><td colspan="2">0.25</td><td>0.75</td></tr>
<tr><td>L_3</td><td colspan="2">30</td><td>0</td></tr>
<tr><td>L_4</td><td>40</td><td colspan="2">0</td></tr>
<tr><td></td><td>0.2</td><td colspan="2">0.8</td></tr>
</table>

Choice #3.2

<table>
<tr><td></td><td colspan="3">1</td></tr>
<tr><td>L_5</td><td colspan="3">30</td></tr>
<tr><td>L_6</td><td>40</td><td>30</td><td>0</td></tr>
<tr><td></td><td>0.2</td><td>0.75</td><td>0.05</td></tr>
</table>

Choice #3.3

<table>
<tr><td></td><td>0.75</td><td colspan="2">0.25</td></tr>
<tr><td>L_7</td><td>40</td><td colspan="2">30</td></tr>
<tr><td>L_8</td><td colspan="2">40</td><td>0</td></tr>
<tr><td></td><td colspan="2">0.95</td><td>0.05</td></tr>
</table>

Figure 4

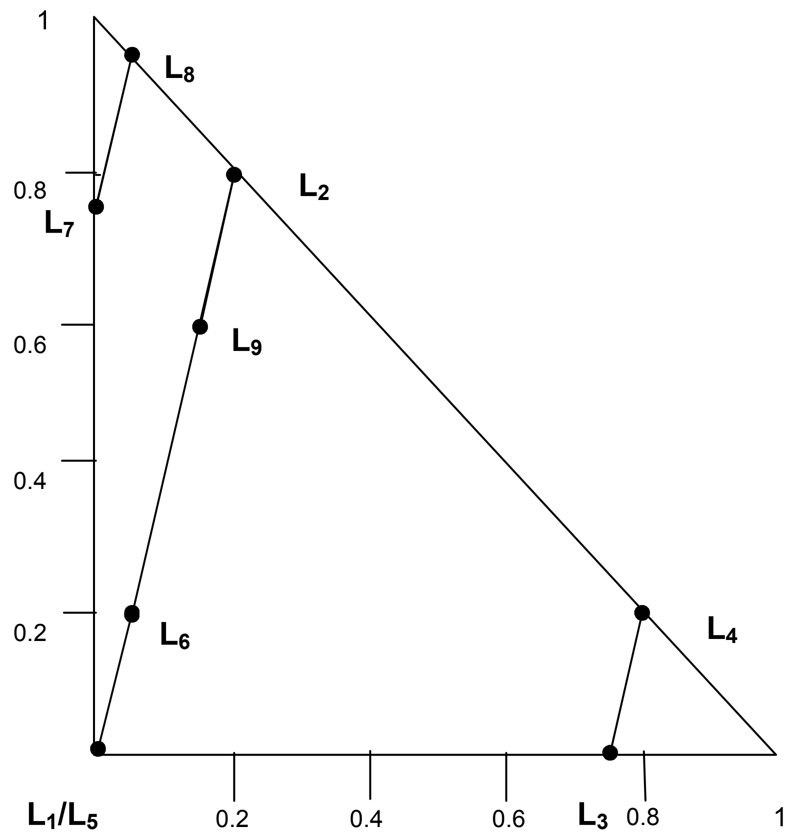

Figure 5

	1	
L_{-1}	–30	
L_{-2}	–40	0
	0.8	0.2

	0.25	0.75
L_{-3}	–30	0
L_{-4}	–40	0
	0.2	0.8

TABLE 1

Pairs Ranked by Perceived Relative Advantage on Probability Dimension

		Level
$\{L_1, L_2\}$	$\phi(a_1, a_2) = 1^{\beta}\left[0.25^{(1)^{\alpha}}\right]$	1
$\{L_1, L_9\}$	$\phi(a_1, a_9) = 0.75^{\beta}\left[0.25^{(0.75)^{\alpha}}\right]$	2
$\{L_6, L_2\}$	$\phi(a_6, a_2) = 0.5625^{\beta}\left[0.25^{(0.75)^{\alpha}}\right]$	3
$\{L_1, L_6\}$	$\phi(a_1, a_6) = 0.25^{\beta}\left[0.25^{(0.25)^{\alpha}}\right]$	4
$\{L_7, L_8\}$	$\phi(a_7, a_8) = 0.2105^{\beta}\left[0.25^{(0.25)^{\alpha}}\right]$	5
$\{L_9, L_2\}$	$\phi(a_9, a_2) = 0.0625^{\beta}\left[0.25^{(0.25)^{\alpha}}\right]$	6
$\{L_3, L_4\}$	$\phi(a_3, a_4) = 0.0625^{\beta}\left[0.25^{(0.25)^{\alpha}}\right]$	6

TABLE 2

Implications of Different Levels of $\xi(z_J/z_I)$

Position of $\xi(z_J, z_I)$	**Nature of Regularity**
Level 6 > $\xi(z_J, z_I)$	Consistent with EU: Safer lottery always chosen
Level 1 > $\xi(z_J, z_I)$ > Level 6	Common ratio effect: $L_1 \succ L_2$ but $L_3 \prec L_4$
Level 1 > $\xi(z_J, z_I)$ > Level 5	'Fanning in' in top corner: $L_1 \succ L_2$ but $L_7 \prec L_8$
Level 2 > $\xi(z_J, z_I)$ > Level 6	Betweenness violated: L_9 less preferred than both L_1 and L_2
Level 3 > $\xi(z_J, z_I)$ > Level 4	Betweenness violated: L_6 preferred to both L_1 and L_2
Level 4 > $\xi(z_J, z_I)$ > Level 6	Common consequence effect: $L_1 \succ L_6$ but $L_3 \prec L_4$
Level 1 > $\xi(z_J, z_I)$ > Level 3	Transitivity violated: $L_1 \succ L_2$; $L_2 \succ L_6$; but $L_1 \prec L_6$
Level 1 > $\xi(z_J, z_I)$ > Level 2	Transitivity violated: $L_1 \succ L_2$; $L_2 \succ L_9$; but $L_1 \prec L_9$
$\xi(z_J, z_I)$ > Level 1	Consistent with EU: Riskier lottery always chosen

TABLE 3

L_{11} (= L_{15}): (30, 1)

L_{12}: (80, 0.40; 0, 0.60)

L_{13}: (30, 0.25; 0, 0.75)

L_{14}: (80, 0.10; 0, 0.90)

L_{16}: (80, 0.10; 30, 0.75; 0, 0.15)

L_{17}: (80, 0.75; 30, 0.25)

L_{18}: (80, 0.85; 0, 0.15)

L_{19}: (80, 0.30; 30, 0.25; 0, 0.45)

TABLE 4

Pairs Ranked by Perceived Relative Advantage on Probability Dimension

{L11, L12} $\phi(a_{11}, a_{12}) = 1^{\beta}\left[1.5^{(1)^{\alpha}}\right]$

{L11, L19}: $\phi(a_{11}, a_{19}) = 0.75^{\beta}\left[1.5^{(0.75)^{\alpha}}\right]$

> {L16, L12}: $\phi(a_{16}, a_{12}) = 0.5625^{\beta}\left[1.5^{(0.75)^{\alpha}}\right]$

{L11, L16} $\phi(a_{11}, a_{16}) = 0.25^{\beta}\left[1.5^{(0.25)^{\alpha}}\right]$

> {L13, L14} $\phi(a_{13}, a_{14}) = 0.1667^{\beta}\left[1.5^{(0.25)^{\alpha}}\right]$

> {L17, L18} $\phi(a_{17}, a_{18}) = 0.1176^{\beta}\left[1.5^{(0.25)^{\alpha}}\right]$

> {L19, L12} $\phi(a_{19}, a_{12}) = 0.0625^{\beta}\left[1.5^{(0.25)^{\alpha}}\right]$

TABLE 5

$$\phi(a_3, a_4) = (0.05/0.80)^{\beta}\left[0.25^{(0.25)^{\alpha}}\right]$$

$$\phi(a_4, a_{10}) = (0.05/0.85)^{\beta}\left[0.33^{(0.20)^{\alpha}}\right]$$

$$\phi(a_3, a_{10}) = (0.10/0.85)^{\beta}\left[0.67^{(0.25)^{\alpha}}\right]$$

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