# A Tax Reform Analysis of the Laffer Argument

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This version (first draft): 10 November 2006

## Abstract

This paper shows that tax reform techniques are well-suited to an examination of the Laffer argument, i.e., the possibility that an increase in a tax rate may reduce tax revenues (and vice versa). Our methodology allows us to examine the Laffer argument directly, without deriving the Laffer curve, which in turn allows us to conduct the analysis in a very general setting. Despite the high level of generality, we are able to reach some clear conclusions that provide formal support for the established intuitions that the Laffer effect requires: (i) a 'high' labour-income tax rate, and (ii) a 'large' labour supply response to wage changes. The notions of 'high' and 'large' are made precise in our framework. The analysis also provides indirect support for the intuition that it is never optimal for a government to operate on the downward-sloping segment of the Laffer curve. Finally, we show that our methods provide a theoretical framework for empirical investigation.

JEL classification codes: H2, H6.

Keywords: Laffer argument, tax reform.

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# 1. Introduction

The possibility that an increase in a tax rate may actually decrease total tax receipts (and vice versa) has featured prominently in many tax policy debates ever since Arthur Laffer famously drew his curve on a napkin in a Washington restaurant in the mid 1970s.[1] The Laffer argument is well known and quite simple: a tax rate of zero will (obviously) yield zero tax receipts, while a tax rate of 100 percent will also yield zero tax receipts as the private sector will not generate a tax base for no return. Thus an inverted U shaped curve is obtained in tax rate/tax revenue space, with tax revenues first increasing in the tax rate, reaching a peak, and then decreasing to reach zero at a tax rate of 100 percent. The increasing portion of the curve is known as the 'normal' segment, while the decreasing portion is known as the 'prohibitive' segment. Tax policy debates revolve around whether the economy is currently thought to be situated on the normal or prohibitive segment.

While the Laffer argument has been debated mostly by politicians, journalists, and analysts in the public domain, it has also been subjected to some rigorous analyses. Fullerton [1982] estimates the Laffer curve for the US economy using a computable general equilibrium model. Focusing on labour, he shows that the economy must be characterised by a very high labour tax rate, a very high labour supply elasticity, or some combination of the two, for the economy to be situated on the prohibitive segment of the Laffer curve. Indeed, it can be shown on the back-of-an-envelope (it might be more apt to say back-of-a-napkin!) that the Laffer effect requires unusually high labour tax rates and/or labour supply elasticities; see Varian [1993] for the simple calculations. Hansson and Stuart [2003]

[1] So the story goes, see Fullerton [1982].

provide recent estimates of the peaks of Laffer curves for a sample of OECD countries, which are again dependent upon labour tax rates and assumptions regarding labour supply elasticities.[2]

In order to estimate empirical Laffer curves, some restrictive assumptions regarding preferences, technologies, and the nature of the economy must, of course, be made. In a theoretical study, Malcomson [1986] uses a simple general equilibrium model with three goods (one consumption good, one public good, and labour) and identical consumers. Assuming well-behaved functional forms (but not necessarily the specific forms used in empirical studies), Malcomson [1986] shows that the Laffer curve may not be continuous and may not reach an interior maximum. The curve can be increasing in the labour tax rate until reaching a discontinuity at 100 percent, where tax receipts fall to zero. In a follow-up paper, Gahvari [1989] shows that the discontinuity identified by Malcomson [1986] disappears if government expenditures take the form of cash transfers to consumers, rather than being used to provide a public good. Such transfers are, however, rarely observed in practice.

Guesnerie and Jerison [1991] generalise the theoretical analysis further still by using a low-dimensional version (one consumption good, one public good, and labour) of the classic Diamond and Mirrlees [1971] general equilibrium tax model. The model has heterogeneous consumers, but the public good is assumed to be separable from the

[2] Some other recent studies have, in effect, focused on the Laffer argument indirectly by estimating 'taxable income elasticities', i.e., how reported levels of taxable income respond to tax rate changes. These estimates not only capture, for example, the labour supply response, but also capture possible increased tax evasion and avoidance in response to tax rate hikes. See Gruber and Saez [2002], Carroll and Hrung [2005], and Kopczuk [2005]. The estimated taxable income elasticity and assumed tax rates can then be used to examine the effect of tax rate changes on tax revenues.

consumption good and labour in each consumer's utility function. Guesnerie and Jerison [1991] show that the Laffer curve may have multiple local maxima, and in some cases it may never slope downwards.[3] They also note that it is not clear if their results can be extended to a model with many commodities.

In this paper, we show that an unrestricted version of the Diamond-Mirrlees model can be used — while maintaining the ability to obtain clear insights — by undertaking a tax reform style analysis of the Laffer argument. Tax reform analysis takes the existing tax system and its (possible) imperfections as its starting point, and examines the conditions under which there exist *small* changes in taxes that are equilibrium preserving and Pareto improving.[4] We take a similar approach to the examination of the Laffer argument. Starting in an arbitrary tax equilibrium, we characterise the conditions under which a small increase in the tax rate on labour income necessarily results in lower tax revenues. Thus, we can directly examine the Laffer argument without deriving the Laffer curve, which requires consideration of large changes in taxes. Intuitively, if the conditions for the Laffer effect to occur are satisfied, it could be said that the economy is situated on the prohibitive segment of the Laffer curve.

Despite the model's high level of generality, we are able to obtain some clear conclusions, in particular:

[3] Guesnerie and Jerison [1991] also address the normative question of how relevant the Laffer argument is for social choice amongst tax equilibria.

[4] Tax reform therefore differs from optimal tax analysis, which pays no attention to the existing tax system and implicitly assumes that the government is free to implement large changes in taxes to obtain an optimum. The tax reform approach was pioneered by Guesnerie [1977], and developed further by Diewert [1978] and Weymark [1979]. More recently, tax reform techniques have been used by Murty and Russell [2005] to analyse externalities, and by Krause [2006] to analyse the incidence of capital taxation.

- If the status quo tax system is Pareto efficient, an increase in the labour-income tax rate cannot result in the Laffer effect;
- The tax rate on labour income must be 'high' in an economy that is subject to the Laffer effect;
- Labour supply must be 'very sensitive' to changes in wages in an economy that is subject to the Laffer effect.

These conclusions are consistent with current thinking about the Laffer argument; thus we establish that such thinking passes a rigorous general equilibrium test. Moreover, we are able to:

- Give precise meaning to the notions that the tax rate on labour income must be 'high' and that labour supply must be 'very sensitive' to changes in wages;
- Characterise precisely what an economy must 'look like' if the Laffer effect is to occur, where the characterisation exercise provides a theoretical framework for an empirical investigation into the possibility of the Laffer effect in real economies.

A recent literature, e.g., Agell and Persson [2001] and Novales and Ruiz [2002], has used endogenous growth models to examine if the Laffer effect is more likely to occur in the longer run. Our model is static, but the analysis is in the same spirit, in the sense that we are interested in how the Laffer argument is affected by changes in models and modelling techniques. Section 2 describes the model, while Section 3 discusses the tax reform methodology we employ and presents the results. Section 4 contains some concluding comments, including a discussion of some weaknesses with our approach. Proofs and many of the mathematical details are relegated to two appendices.

## 2. The Model

The economy has $k$ consumers, indexed by $i$ = 1, ..., $k$. Consumer $i$ chooses his (net of endowment) consumption vector $x_i \in \mathbb{R}^n$, and his labour supply $l_i \in [0, 1]$,[5] to solve the following programme:

$$V_i(q, \omega, g) = \max_{x_i, l_i} \{ U_i(x_i, l_i, g) \mid qx_i \leq \omega l_i \} \tag{2.1}$$

where $V_i(\cdot)$ is the indirect utility function, $U_i(\cdot)$ is the direct utility function with $\nabla_{x_i} U_i(\cdot) \gg 0^{(n)}$, $\nabla_{l_i} U_i(\cdot) < 0$, and $\nabla_g U_i(\cdot) > 0$ where $g$ is a public good provided by the government.[6] The consumer price vector corresponding to the commodities is $q = p + t$, where $p$ is the producer price vector corresponding to the commodities and $t$ is a vector of commodity taxes. The consumer wage is $\omega = w - \tau$, where $w$ is the producer price of labour and $\tau$ is the per-unit tax on labour income. The consumers have no profit income, as we make the standard assumption that the government taxes away all pure profit.[7] Under standard assumptions regarding preferences (namely, local non-satiation and strict convexity), the solution to (2.1) yields each consumer's (net) commodity demand and labour supply functions:

$$x_i(q, \omega, g) \quad \text{and} \quad l_i(q, \omega, g) \tag{2.2}$$

[5] We assume that each consumer is endowed with one unit of time. Time not used to supply labour is consumed as leisure.

[6] Vector notation: $z \geq \bar{z} \Leftrightarrow z_j \geq \bar{z}_j \ \forall j$, $z > \bar{z} \Leftrightarrow z_j \geq \bar{z}_j \ \forall j \wedge z \neq \bar{z}$, and $z \gg \bar{z} \Leftrightarrow z_j > \bar{z}_j \ \forall j$.

[7] Guesnerie and Jerison [1991] also make this assumption. Alternatively, one could assume that the production side of the economy is characterized by constant returns to scale, which implies zero profits in equilibrium.

The production of the $n$ private commodities is undertaken by a single, aggregate, profit-maximising firm, as there are no aggregation problems on the supply side in the absence of production externalities (as is assumed). Thus, there is no loss in generality by assuming a single firm. The firm's closed and strictly convex technology set is $Y \subset \mathbb{R}^{n+1}$. The firm's profit maximisation problem can be stated as:

$$\pi(p, w) = \max_{x,l} \{ px - wl \mid \langle x, l \rangle \in Y \} \tag{2.3}$$

The firm's profit function is $\pi(p, w)$. Application of Hotelling's Theorem to the profit function yields the firm's output-supply and input-demand functions:

$$\nabla_p \pi(\cdot) = x(p, w) \qquad \text{and} \qquad \nabla_w \pi(\cdot) = -l(p, w) \tag{2.4}$$

where $x$ is the (net) supply vector of private commodities, and $l$ is the firm's demand for labour.

The government uses commodities and labour to produce the public good according to the following technology:

$$g \leq f(x_g, l_g) \tag{2.5}$$

where $f(\cdot)$ is strictly concave and increasing in all its arguments, and $x_g$ and $l_g$ denote the employment of commodities and labour to produce the public good.

Equilibrium is obtained if and only if:

$$\sum_i x_i(q, \omega, g) + x_g - x(p, w) \leq 0^{(n)} \tag{2.6}$$

$$l(p, w) + l_g - \sum_i l_i(q, \omega, g) \leq 0 \tag{2.7}$$

$$g - f(x_g, l_g) \leq 0 \tag{2.8}$$

Equations (2.6) and (2.7) are market clearing conditions for the $n$ private commodities and labour. Equation (2.8) requires that the provision of the public good be technically feasible. It is shown in Appendix 1 that if all the equations in (2.6) and (2.7) are satisfied as equalities, the government's budget is exactly balanced. But if some of these equations are satisfied as inequalities, the government's budget will be in surplus. An equilibrium is said to be *tight* when (2.6) – (2.8) are satisfied as equalities, and *non-tight* when some of these equations are satisfied as inequalities.

# 3. A Characterisation of the Laffer Effect

Consider an arbitrarily given tight equilibrium of our economy, where the corresponding tax system may or may not be optimal in any sense of the word. We are interested in whether a small (modelled as differential) increase in the tax rate on labour income *necessarily* moves the economy to a neighbouring equilibrium which has a lower level of tax revenues (holding all other taxes constant). To this end, we define a *policy reform* as a vector $\mathrm{d}P := \langle \mathrm{d}p, \mathrm{d}t, \mathrm{d}w, \mathrm{d}\tau, \mathrm{d}g, \mathrm{d}x_g, \mathrm{d}l_g \rangle$, where the government has direct control over the taxes $t$ and $\tau$, as well as over the level ($g$) and method of production ($x_g$ and $l_g$) of the public good. Changes in these instruments may induce changes in producer prices, $p$ and $w$, in order to maintain equilibrium.[8] Specifically, a policy reform is equilibrium preserving if and only if:

[8] More formally, $\mathrm{d}P$ gives the government $3n + 4$ instruments, but it must satisfy the $n + 2$ equilibrium equations in (2.6) – (2.8). This suggests by the Implicit Function Theorem $2n + 2$ degrees of freedom in picking equilibria in the neighbourhood of the status quo equilibrium. There are in fact only $2n$ degrees of freedom, however, as two degrees of freedom are lost due to the consumers' demand and supply functions being homogenous of degree zero in consumer prices, and the firm's supply and demand functions being homogenous of degree zero in producer prices. See chapter 2 in Guesnerie [1995] for a detailed discussion of these issues.

$$-\nabla Z \mathrm{d}P \geq 0^{(n+2)} \tag{3.1}$$

where $\nabla Z$ is the $(n + 2) \times (3n + 4)$ Jacobian matrix (with respect to d$P$) associated with equations (2.6) – (2.8) and is defined explicitly as:

$$\nabla Z := \begin{pmatrix} \sum_i \nabla_q x_i - \nabla_p x & \sum_i \nabla_q x_i & \sum_i \nabla_\omega x_i - \nabla_w x & -\sum_i \nabla_\omega x_i & \sum_i \nabla_g x_i & \mathrm{I}^{(n\times n)} & 0^{(n)} \\ \nabla_p l - \sum_i \nabla_q l_i & -\sum_i \nabla_q l_i & \nabla_w l - \sum_i \nabla_\omega l_i & \sum_i \nabla_\omega l_i & -\sum_i \nabla_g l_i & 0^{(n)} & 1 \\ 0^{(n)} & 0^{(n)} & 0 & 0 & 1 & -\nabla_{x_g} f & -\nabla_{l_g} f \end{pmatrix}$$

The set of policy reforms that include no changes in the commodity taxes are those that satisfy:

$$\begin{pmatrix} 0^{(n\times n)} & \mathrm{I}^{(n\times n)} & 0^{(n)} & 0^{(n)} & 0^{(n)} & 0^{(n\times n)} & 0^{(n)} \end{pmatrix} \mathrm{d}P = 0^{(n)} \tag{3.2}$$

The average (and marginal) tax rate on labour income can be written as:

$$\frac{w\sum_i l_i(q,\omega,g) - \omega\sum_i l_i(q,\omega,g)}{w\sum_i l_i(q,\omega,g)} = \frac{w-\omega}{w} = \frac{\tau}{w}$$

Thus, the set of policy reforms that include an increase in the tax rate on labour income are those that satisfy:

$$\begin{pmatrix} 0^{(n)} & 0^{(n)} & -1 & 1 & 0 & 0^{(n)} & 0 \end{pmatrix} \mathrm{d}P > 0 \tag{3.3}$$

The set of policy reforms that do not include a decrease in the level of the public good are those that satisfy:

$$\begin{pmatrix} 0^{(n)} & 0^{(n)} & 0 & 0 & 1 & 0^{(n)} & 0 \end{pmatrix} \mathrm{d}P \geq 0 \tag{3.4}$$

Suppose there does not exist a policy reform that satisfies (3.1) – (3.4). Then all policy reforms that satisfy (3.1) – (3.3) must violate (3.4). That is, all policy reforms that are equilibrium preserving, that involve no changes in the commodity taxes, and that involve an

increase in the tax rate on labour income, must also involve a lower level of the public good. Given that the government is free to change the combination of inputs $x_g$ and $l_g$ in order to minimise the cost of producing the public good, the lower level of the public good must necessarily be the result of lower tax revenues accruing to the government. Thus, an economy in which there does not exist a policy reform that satisfies (3.1) – (3.4) can be interpreted as an economy in which an increase in the tax rate on labour income necessarily results in lower tax revenues, i.e., the Laffer effect. Such an economy could be interpreted as being situated on the prohibitive segment of the Laffer curve, although, strictly speaking, this is not correct since we have not derived a Laffer curve.

By Motzkin's Theorem,[9] there does not exist a policy reform d$P$ that satisfies (3.1) – (3.4) if and only if there exist real numbers $\mu \geq 0^{(n+2)}$, $\theta \in \mathbb{R}^n$, $\alpha > 0$, and $\delta \geq 0$ such that:

$$\theta\begin{pmatrix} 0^{(n\times n)} & \mathrm{I}^{(n\times n)} & 0^{(n)} & 0^{(n)} & 0^{(n)} & 0^{(n\times n)} & 0^{(n)} \end{pmatrix} +$$

$$\alpha\begin{pmatrix} 0^{(n)} & 0^{(n)} & -1 & 1 & 0 & 0^{(n)} & 0 \end{pmatrix} + \delta\begin{pmatrix} 0^{(n)} & 0^{(n)} & 0 & 0 & 1 & 0^{(n)} & 0 \end{pmatrix} = \mu\nabla Z \qquad (3.5)$$

The system of equations in (3.5) characterises what the economy must 'look like' for an increase in the tax rate on labour income to necessarily result in lower tax revenues. Expanding (3.5) yields the following lemma:

**LEMMA 1**

*Consider any tight equilibrium of our economy. An increase in the tax rate on labour income necessarily results in lower tax revenues if and only if there exist real numbers* $\mu \geq 0^{(n+2)}$, $\theta \in \mathbb{R}^n$, $\alpha > 0$, *and* $\delta \geq 0$ *such that*:

[9] See Appendix 1 for a statement of this theorem.

$$0^{(n)} = \langle \mu_1, ..., \mu_n \rangle \left[ \sum_i \nabla_q x_i(\cdot) - \nabla_p x(\cdot) \right] + \mu_{n+1} \left[ \nabla_p l(\cdot) - \sum_i \nabla_q l_i(\cdot) \right] \tag{3.6}$$

$$\theta = \langle \mu_1, ..., \mu_n \rangle \sum_i \nabla_q x_i(\cdot) - \mu_{n+1} \sum_i \nabla_q l_i(\cdot) \tag{3.7}$$

$$-\alpha = \langle \mu_1, ..., \mu_n \rangle \left[ \sum_i \nabla_\omega x_i(\cdot) - \nabla_w x(\cdot) \right] + \mu_{n+1} \left[ \nabla_w l(\cdot) - \sum_i \nabla_\omega l_i(\cdot) \right] \tag{3.8}$$

$$\alpha = -\langle \mu_1, ..., \mu_n \rangle \sum_i \nabla_\omega x_i(\cdot) + \mu_{n+1} \sum_i \nabla_\omega l_i(\cdot) \tag{3.9}$$

$$\delta = \langle \mu_1, ..., \mu_n \rangle \sum_i \nabla_g x_i(\cdot) - \mu_{n+1} \sum_i \nabla_g l_i(\cdot) + \mu_{n+2} \tag{3.10}$$

$$0^{(n)} = \langle \mu_1, ..., \mu_n \rangle - \mu_{n+2} \nabla_{x_g} f(\cdot) \tag{3.11}$$

$$0 = \mu_{n+1} - \mu_{n+2} \nabla_{l_g} f(\cdot) \tag{3.12}$$

*where all derivatives are evaluated in the status quo equilibrium.*

At this point it is worth mentioning that Lemma 1 provides a theoretical foundation for an empirical investigation into the possibility of the Laffer effect in real economies. The information requirements of (3.6) – (3.9) are estimates of aggregate demand and supply price derivatives (or elasticities) for the commodities and labour. In principle, these can be estimated using market data and econometric techniques. Equation (3.10) requires estimates of how commodity demand and labour supply vary with the level of public goods. A standard separability assumption on preferences, however, could be made to ensure that $\nabla_g x_i(\cdot) = 0^{(n)}$ and $\nabla_g l_i(\cdot) = 0$, which would remove the need for such estimates (although at the cost of some loss in generality). Equations (3.11) and (3.12) require estimates of the marginal productivities of commodities and labour in producing public goods, which in principle can also be estimated. Once such data are obtained, the task then would be to

check (say with a computer) whether the Lemma 1 conditions can be satisfied. A simple numerical example of an economy that satisfies Lemma 1 is provided in Appendix 2.

From Lemma 1 we obtain the following results (all proofs are in Appendix 1):

**THEOREM 1**

*Consider a tight equilibrium of our economy in which the tax system is Pareto efficient. In such an economy, an increase in the tax rate on labour income cannot result in the Laffer effect.*

Theorem 1 provides formal support for the intuition that it is not optimal for a government to operate on the prohibitive segment of the Laffer curve.[10] An optimal tax system (which we take to satisfy at least the condition of Pareto optimality) is necessarily characterised by the marginal benefit of a change in each tax being equated to its marginal cost. For example, a tax decrease which reduces the consumer price of some good will boost consumption of that good and welfare. This is the benefit. The cost is that the increase in demand must be met by an increase in supply by transferring resources from other sectors of the economy. Theorem 1 implies that the tax system of an economy subject to the Laffer effect cannot be characterised by the condition that marginal benefit equals marginal cost for each tax. In particular, the labour-income tax rate is 'too high' in the following sense:

**COROLLARY 1**

*In an economy subject to the Laffer effect, the labour-income tax rate is at its highest level possible that is consistent with the status quo levels of the commodity taxes and public good, and satisfaction of the equilibrium conditions (2.6) – (2.8).*

[10] Although in models of tax competition this intuition may not hold. See Hindriks [2001] and Keen and Kotsogiannis [2003].

In other words, the economy looks *as if* the government has attempted to maximise the labour-income tax rate, subject to the equilibrium constraints, $t = \hat{t}$, and $g = \hat{g}$, where $\hat{t}$ and $\hat{g}$ denote the status quo levels of the commodity taxes and public good. On the other hand, to achieve a Pareto-efficient equilibrium, the government attempts to minimise the labour-income tax rate, because consumer welfare is a decreasing function of the tax.

**THEOREM 2**

*Consider any tight equilibrium of our economy in which consumer demand is such that* $\nabla_{\omega} x_i(\cdot) \geq 0^{(n)}$ *for all i (i.e., demand for all commodities is non-decreasing in the consumer wage). If the income effect of an increase in the consumer wage on labour supply is not dominated by the substitution effect, then an increase in the tax rate on labour income cannot result in the Laffer effect.*

Theorem 2 is relatively easy to explain. Consider first the income effect of an increase in the tax rate on labour income. If all commodities and leisure are (in aggregate) normal goods, then there is a decrease in demand for the commodities and an increase in labour supply. All else equal, this results in excess supply of the commodities and labour, with a corresponding shift in the government's budget from balance to surplus.[11] Thus, there is no Laffer effect. Labour income is a function of the consumer wage, and in a general equilibrium framework an increase in the labour-income tax rate will induce wage changes. This results in income and substitution effects, so rather than assuming normality we make the assumption that $\nabla_{\omega} x_i(\cdot) \geq 0^{(n)}$. A similar argument applies to labour supply. The

[11] Recall that in a tight equilibrium the government's budget is balanced, and in a non-tight equilibrium the government's budget is in surplus. We always assume that the status quo equilibrium is tight so that the system (2.6) – (2.8) can be differentiated.

substitution effect of a decrease in the consumer wage is necessarily negative, but if leisure is a normal good the income effect is positive. If the income effect dominates the substitution effect, then a decrease in the consumer wage increases labour supply and, all else equal, creates excess supply in the labour market and a corresponding budget surplus.

Theorem 2 provides support for the established thinking that labour supply must be sensitive to changes in wages (i.e., the substitution effect must be large) in order for Laffer-like phenomena to occur. The labour supply effect must be large in the sense made precise by the following corollary:

**COROLLARY 2**

*In an economy subject to the Laffer effect, the labour supply response to a change in the consumer wage must be large in the sense that*:

$$\sum_i \nabla_\omega l_i(\cdot) > \frac{\nabla_{x_g} f(\cdot)}{\nabla_{l_g} f(\cdot)} \sum_i \nabla_\omega x_i(\cdot) \qquad \Leftrightarrow \qquad \nabla_{l_g} f(\cdot) \sum_i \nabla_\omega l_i(\cdot) > \nabla_{x_g} f(\cdot) \sum_i \nabla_\omega x_i(\cdot)$$

*where all derivatives are evaluated in the status quo equilibrium.*

The ratio $\nabla_{x_g} f(\cdot) \big/ \nabla_{l_g} f(\cdot)$ is a vector of technical rates of substitution of each commodity for labour in production of the public good. Corollary 2 suggests that the labour supply response to an increase in the consumer wage, valued at the marginal product of labour in public good production, must be greater than the commodity demand response to an increase in the consumer wage, valued at the marginal products of commodities in public good production. The marginal productivities in public good production are used to put the labour supply response and commodity demand response in comparable units. Put simply, for the Laffer effect to occur, an increase in the labour-income tax rate must reduce labour

supply by more than commodity demand, so that the total value of excess supply is non-positive. This rules out the possibility of a move from budget balance to budget surplus.

## 4. Concluding Comments

In this paper we have characterised the conditions under which an increase in the tax rate on labour income necessarily results in lower tax revenues, but we have not attempted to characterise when a *decrease* in the labour-income tax rate *increases* tax revenues. At first thought one might expect that the analysis could simply be reversed, in that it is characterised when a decrease in the labour-income tax rate requires an increase in the public good. But this cannot be interpreted as the generation of extra tax revenues. An increase in the public good could be obtained with the same, or even lower, level of tax revenues if the status quo method of producing the public good were inefficient. For this reason, our analysis only considers when an increase in the tax rate on labour income can necessarily result in lower tax revenues.

Nevertheless, we believe our model and methodology provide important insights into the Laffer argument. At its core, the Laffer argument asks the following question: Given the current state of the economy and its tax system, will a small increase (decrease) in tax rates yield lower (higher) tax revenues? The very nature of the tax reform approach makes it well-suited to answer this question. Moreover, we have examined the Laffer argument in a very general setting, which ensures that our results are valid for all well-behaved classes of preferences and technologies. This also makes our characterisation result (Lemma 1) directly applicable to empirical testing.

# Appendix 1

## I. The Government's Budget.

The government's budget surplus *BS* can be written as:

$$BS = \sum_i (q - p)x_i + \sum_i (w - \omega)l_i + \pi(p, w) - px_g - wl_g \tag{A.1}$$

where the first term represents receipts from commodity taxation, the second term is receipts from taxing labour, the third term is from the total taxation of pure profits, and the last two terms represent the government's expenditures on commodities and labour to produce the public good. Rewriting the profits term in (A.1) yields:

$$BS = \sum_i (q - p)x_i + \sum_i (w - \omega)l_i + px - wl - px_g - wl_g \tag{A.2}$$

Under the assumption of local non-satiation, the consumers will satisfy their budget constraints with equality, which implies that $qx_i = \omega l_i$ for all *i*. Thus (A.2) reduces to:

$$BS = -p\sum_i x_i + w\sum_i l_i + px - wl - px_g - wl_g \tag{A.3}$$

Rearranging (A.3) yields:

$$BS = p(x - x_g - \sum_i x_i) + w(\sum_i l_i - l_g - l) \tag{A.4}$$

Market clearing requires that the terms in parentheses in (A.4) be non-negative. Thus, the government's budget is exactly balanced in a tight equilibrium, and the government's budget is in surplus in a non-tight equilibrium.

## II. Motzkin's Theorem of the Alternative.

Let $A$, $C$, and $D$ be $n_1 \times m$, $n_2 \times m$, and $n_3 \times m$ matrices, respectively, where $A$ is non-vacuous (not all zeros). Then *either*

$$Az \gg 0^{(n_1)} \qquad Cz \geq 0^{(n_2)} \qquad Dz = 0^{(n_3)}$$

has a solution $z \in \mathbb{R}^m$, *or*

$$y_1 A + y_2 C + y_3 D = 0^{(m)}$$

has a solution $y_1 > 0^{(n_1)}$, $y_2 \geq 0^{(n_2)}$, and $y_3$ sign unrestricted, *but never both*. A proof of Motzkin's Theorem can be found in Mangasarian [1969].

## III. Proof of Theorem 1.

1. We first derive the equations that implicitly characterise a Pareto-efficient tax system. A policy reform increases the welfare of consumer $i$ if and only if $\mathrm{d}V_i(\cdot) = \nabla V_i \mathrm{d}P > 0$, where $\nabla V_i$ is the gradient of consumer $i$'s indirect utility function with respect to d$P$, i.e., $\nabla V_i := \langle \nabla_q V_i, \nabla_q V_i, \nabla_\omega V_i, -\nabla_\omega V_i, \nabla_g V_i, 0^{(n)}, 0 \rangle$. Starting in an initial tight equilibrium, if there does not exist a policy reform that is equilibrium preserving and Pareto improving, then the status quo equilibrium is Pareto efficient. Let $\nabla V$ be the $k \times (3n + 4)$ matrix formed by the vectors $\nabla V_i$. By Motzkin's Theorem, if there does not exist a policy reform d$P$ such that $-\nabla Z \mathrm{d}P \geq 0^{(n+2)}$ and $\nabla V \mathrm{d}P \gg 0^{(k)}$, then there exist two vectors of real numbers $\lambda > 0^{(k)}$ and $\bar{\mu} \geq 0^{(n+2)}$ such that:

$$\lambda \nabla V = \bar{\mu} \nabla Z \tag{A.5}$$

The system of equations (A.5) characterises the set of Pareto-efficient equilibria. Expanding (A.5) yields:

$$\sum_i \lambda_i \nabla_q V_i = \langle \bar{\mu}_1, ..., \bar{\mu}_n \rangle \left[ \sum_i \nabla_q x_i(\cdot) - \nabla_p x(\cdot) \right] + \bar{\mu}_{n+1} \left[ \nabla_p l(\cdot) - \sum_i \nabla_q l_i(\cdot) \right] \tag{A.6}$$

$$\sum_i \lambda_i \nabla_q V_i = \langle \bar{\mu}_1, ..., \bar{\mu}_n \rangle \sum_i \nabla_q x_i(\cdot) - \bar{\mu}_{n+1} \sum_i \nabla_q l_i(\cdot) \tag{A.7}$$

$$\sum_i \lambda_i \nabla_\omega V_i = \langle \bar{\mu}_1, ..., \bar{\mu}_n \rangle \left[ \sum_i \nabla_\omega x_i(\cdot) - \nabla_w x(\cdot) \right] + \bar{\mu}_{n+1} \left[ \nabla_w l(\cdot) - \sum_i \nabla_\omega l_i(\cdot) \right] \tag{A.8}$$

$$-\sum_i \lambda_i \nabla_\omega V_i = -\langle \bar{\mu}_1, ..., \bar{\mu}_n \rangle \sum_i \nabla_\omega x_i(\cdot) + \bar{\mu}_{n+1} \sum_i \nabla_\omega l_i(\cdot) \tag{A.9}$$

$$\sum_i \lambda_i \nabla_g V_i = \langle \bar{\mu}_1, ..., \bar{\mu}_n \rangle \sum_i \nabla_g x_i(\cdot) - \bar{\mu}_{n+1} \sum_i \nabla_g l_i(\cdot) + \bar{\mu}_{n+2} \tag{A.10}$$

$$0^{(n)} = \langle \bar{\mu}_1, ..., \bar{\mu}_n \rangle - \bar{\mu}_{n+2} \nabla_{x_g} f(\cdot) \tag{A.11}$$

$$0 = \bar{\mu}_{n+1} - \bar{\mu}_{n+2} \nabla_{l_g} f(\cdot) \tag{A.12}$$

where all derivatives are evaluated in the status quo equilibrium. Equations (A.6) – (A.12) can be interpreted as the first-order conditions for a Pareto optimum. Each number $\lambda_i$ can be interpreted as the welfare weight of consumer *i*, and the vector $\langle \bar{\mu}_1, ..., \bar{\mu}_{n+2} \rangle$ can be interpreted as the multipliers attached to the $n$ + 2 equilibrium constraints in (2.6) – (2.8). That is, if the government were to choose its policy instruments to maximise a social welfare function $W(V_1, V_2, ..., V_k)$ subject to (2.6) – (2.8), then $\lambda_i = \partial W(\cdot) / \partial V_i$ and $\langle \bar{\mu}_1, ..., \bar{\mu}_{n+2} \rangle$ is the vector of multipliers attached to the $n$ + 2 constraints in (2.6) – (2.8).

2. We show that there does not exist an equilibrium of our economy in which both (3.6) – (3.12) and (A.6) – (A.12) can be satisfied. Suppose $\mu_{n+2}$ = 0. Then from (3.11) and (3.12) we have $\langle \mu_1, ..., \mu_n \rangle = 0^{(n)}$ and $\mu_{n+1}$ = 0. Then from (3.9) we have $\alpha$ = 0 which is a contradiction. Hence $\mu_{n+2} > 0$. From (3.11) and (3.12) we have:

$$\nabla_{x_g} f(\cdot) = \frac{\langle \mu_1, ..., \mu_n \rangle}{\mu_{n+2}} \qquad \text{and} \qquad \nabla_{l_g} f(\cdot) = \frac{\mu_{n+1}}{\mu_{n+2}} \tag{A.13}$$

Suppose $\bar{\mu}_{n+2} = 0$. Then from (A.11) and (A.12) we have $\langle \bar{\mu}_1, ..., \bar{\mu}_n \rangle = 0^{(n)}$ and $\bar{\mu}_{n+1} = 0$. Then from (A.10) we have $\sum \lambda_i \nabla_g V_i = 0$ which is a contradiction. Hence $\bar{\mu}_{n+2} > 0$. From (A.11) and (A.12) we have:

$$\nabla_{x_g} f(\cdot) = \frac{\langle \bar{\mu}_1, ..., \bar{\mu}_n \rangle}{\bar{\mu}_{n+2}} \qquad \text{and} \qquad \nabla_{l_g} f(\cdot) = \frac{\bar{\mu}_{n+1}}{\bar{\mu}_{n+2}} \tag{A.14}$$

Let $\beta := \mu_{n+2} / \bar{\mu}_{n+2}$. From (A.13) and (A.14) we have $\langle \mu_1, ..., \mu_n \rangle = \beta \langle \bar{\mu}_1, ..., \bar{\mu}_n \rangle$ and $\mu_{n+1} = \beta \bar{\mu}_{n+1}$. Equation (3.9) can now be written as:

$$\frac{\alpha}{\beta} = -\langle \bar{\mu}_1, ..., \bar{\mu}_n \rangle \sum_i \nabla_\omega x_i(\cdot) + \bar{\mu}_{n+1} \sum_i \nabla_\omega l_i(\cdot) \tag{A.15}$$

The right-hand sides of (A.15) and (A.9) are identical, but the left-hand side of (A.15) is positive while the left-hand side of (A.9) is non-positive, yielding a contradiction. ■

**IV. Proof of Corollary 1.**

Consider the following hypothetical maximisation problem. Choose $p$, $t$, $w$, $\tau$, $g$, $x_g$, and $l_g$ to maximise $\alpha(\tau - w)$ subject to (i) $t = \hat{t}$, (ii) $g = \hat{g}$, and (iii) the equilibrium conditions (2.6) – (2.8), where $\alpha > 0$ and $\hat{t}$ and $\hat{g}$ denote the status quo levels of the commodity taxes and public good. Let $\theta = \langle \theta_1, ..., \theta_n \rangle$ denote the multipliers on constraint (i), let $\delta$ denote the multiplier on constraint (ii), and let $\mu = \langle \mu_1, ..., \mu_{n+2} \rangle$ denote the multipliers on constraint (iii). By forming the Lagrangian and deriving the first-order conditions, equations (3.6) – (3.12) are obtained. Thus, an economy that is subject to the

Laffer effect is an economy that 'looks like' the government has attempted to maximise the labour-income tax rate, subject to the equilibrium constraints, $t = \hat{t}$, and $g = \hat{g}$. ■

**V. Proof of Theorem 2.**

Let $I_i$ denote consumer *i*'s income. Use the Slutsky equation to obtain:

$$\nabla_{\omega} l_i(q, \omega, g) = \nabla_{\omega} h_i(q, \omega, g, u) + \nabla_{I_i} l_i(q, \omega, g) l_i \qquad \forall\, i$$

where $h_i(\cdot)$ is the Hicksian labour supply function (with *u* denoting the consumer's utility level). Standard results in consumer theory ensure that the substitution effect $\nabla_{\omega} h_i(\cdot)$ is non-negative, and if leisure is a normal good the income effect $\nabla_{I_i} l_i(\cdot) l_i$ is non-positive. So if the income effect is not dominated by the substitution effect, $\nabla_{\omega} l_i(\cdot) \leq 0$. If commodity demand is such that $\nabla_{\omega} x_i(\cdot) \geq 0^{(n)}$ for all *i*, and if $\nabla_{\omega} l_i(\cdot) \leq 0$ for all *i*, then the right-hand side of (3.9) is non-positive, contradicting the left-hand side which is positive. ■

**VI. Proof of Corollary 2.**

From equation (3.9) we obtain:

$$-\langle \mu_1, ..., \mu_n \rangle \sum_i \nabla_{\omega} x_i(\cdot) + \mu_{n+1} \sum_i \nabla_{\omega} l_i(\cdot) > 0 \tag{A.16}$$

In proving Theorem 1 we showed that $\mu_{n+2} > 0$. Thus:

$$\frac{-\langle \mu_1, ..., \mu_n \rangle}{\mu_{n+2}} \sum_i \nabla_{\omega} x_i(\cdot) + \frac{\mu_{n+1}}{\mu_{n+2}} \sum_i \nabla_{\omega} l_i(\cdot) > 0 \tag{A.17}$$

Using equations (3.11) and (3.12) we obtain:

$$-\nabla_{x_g} f(\cdot) \sum_i \nabla_{\omega} x_i(\cdot) + \nabla_{l_g} f(\cdot) \sum_i \nabla_{\omega} l_i(\cdot) > 0 \tag{A.18}$$

Rearranging (A.18) completes the proof. ■

## Appendix 2

We present an example of a simple economy in which an increase in the tax rate on labour income would result in the Laffer effect. The economy has two commodities ($n$ = 2) and $k$ consumers. The key features of the economy are summarised in the Jacobian matrix:

$$\nabla Z = \begin{pmatrix} -2 & 0 & -1 & 0 & 2 & -1 & 0 & 1 & 0 & 0 \\ 0 & -2 & 0 & -1 & 2 & -1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & -4 & 3 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 & -2 \end{pmatrix}$$

By Lemma 1, there does not exist a policy reform d$P$ such that:

$$-\nabla Z \mathrm{d}P \geq 0^{(4)}$$

$$\begin{pmatrix} 0^{(2\times 2)} & \mathrm{I}^{(2\times 2)} & 0^{(2)} & 0^{(2)} & 0^{(2)} & 0^{(2\times 2)} & 0^{(2)} \end{pmatrix} \mathrm{d}P = 0^{(2)}$$

$$\begin{pmatrix} 0^{(2)} & 0^{(2)} & -1 & 1 & 0 & 0^{(2)} & 0 \end{pmatrix} \mathrm{d}P > 0$$

$$\begin{pmatrix} 0^{(2)} & 0^{(2)} & 0 & 0 & 1 & 0^{(2)} & 0 \end{pmatrix} \mathrm{d}P \geq 0$$

if and only if there exist real numbers $\mu \geq 0^{(4)}$, $\theta \in \mathbb{R}^2$, $\alpha > 0$, and $\delta \geq 0$ such that:

$$\theta \begin{pmatrix} 0^{(2\times 2)} & \mathrm{I}^{(2\times 2)} & 0^{(2)} & 0^{(2)} & 0^{(2)} & 0^{(2\times 2)} & 0^{(2)} \end{pmatrix} +$$

$$\alpha \begin{pmatrix} 0^{(2)} & 0^{(2)} & -1 & 1 & 0 & 0^{(2)} & 0 \end{pmatrix} + \delta \begin{pmatrix} 0^{(2)} & 0^{(2)} & 0 & 0 & 1 & 0^{(2)} & 0 \end{pmatrix} = \mu \nabla Z$$

The reader can check that $\mu = \langle \tfrac{1}{4}, \tfrac{1}{4}, \tfrac{1}{2}, \tfrac{1}{4} \rangle$, $\theta = \langle -\tfrac{1}{4}, -\tfrac{1}{4} \rangle$, $\alpha = 1$, and $\delta = \tfrac{1}{4}$ satisfy the required conditions.

It should be noted that there is nothing particularly unusual about the economy described in $\nabla Z$. For example, the consumers' and producer's demand and supply derivatives satisfy the appropriate signs. For simplicity we have assumed that demand for

each commodity is dependent upon only its own price. (This would be true, for example, if the consumers' preferences were Cobb-Douglas.) We have also implicitly assumed that commodities and labour are separable from the public good in the consumers' utility functions. This ensures that $\nabla_g x_i(\cdot) = 0^{(2)}$ and $\nabla_g l_i(\cdot) = 0$ for all $i$. The only potentially unusual feature of the economy is that $\sum \nabla_\omega l_i(\cdot) > 0$ is relatively large, as reflected in the large (absolute) values of $-4$ and 3 in columns five and six of $\nabla Z$. As intuition and Corollary 2 suggest, a necessary condition for the Laffer effect is that labour supply be very sensitive to wage changes.

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