Labor Contracts, Equal Treatment and Wage-Unemployment Dynamics

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This version:March 2009

Abstract

This paper analyses a model in which firms cannot pay discriminate based on year of entry to a firm, and develops an equilibrium model of wage dynamics and unemployment. The model is developed under the assumption of worker mobility, so that workers can costlessly quit jobs at any time. Firms on the other hand are committed to contracts. Thus the model is related to Beaudry and DiNardo (1991). We solve for the dynamics of wages and unemployment, and show that real wages do not necessarily clear the labor market. Using sectoral productivity data from the post-war US economy, we assess the ability of the model to match the actual unemployment series. We also show that equal treatment follows in our model from the assumption of at-will employment contracting.

JEL Codes: E32, J41

Keywords: labor contracts, business cycle, unemployment, equal treatment, cohort effects, cross-contract restrictions

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1 Introduction

This paper develops a model in which it is assumed that firms cannot pay discriminate based on year of entry to a firm—there are no "cohort effects" within a firm—and develops an equilibrium model of wage dynamics and unemployment. The model is developed under the assumption of worker risk aversion, and also mobility, so that workers can costlessly quit jobs at any time. Firms on the other hand are risk neutral and are committed to contracts. Firms have to trade-off the desire to insure their risk-averse workers against the need to respond to market conditions to not only prevent their workers from quitting but also, because of equal treatment of workers, to take advantage of states of the world where labor is cheap. We solve for the dynamics of wages and unemployment when the only exogenous variable is productivity shocks.

We show that real wages exhibit a degree of downward rigidity due to the desire to insure incumbent workers. In periods in which workers' outside options are low, absent equal treatment, firms would offer a lower wage (and hence cheaper) contract to new entrants while holding constant incumbents' wages (for insurance reasons). With the equal treatment assumption, however, new entrants must be offered the same as incumbents from the point they join the firm. Firms will limit how far they allow wages to fall in order to avoid too much variability in the contract that incumbents signed up to; consequently the value of the contract may not fall sufficiently to match the outside option. That is, we find that there is a maximum rate of decline in the wage, *no matter how low the outside option falls.* The reverse is not true however; in periods when the outside option is high the continuation contract rises to match the outside option. Because of costless mobility of workers, firms have no choice but to do this.[1]

The linking of the pay of new hires to that of incumbents means that wage rigidity

[1] In the model we assume that the driving process in the economy is a productivity shock common to all firms. Since the outside option will be endogenized, the reader may wonder whether nonetheless equal treatment may dampen upward movements in wages: if *all* firms respond to positive shocks by not raising wages very much then the outside option will also not rise very much. However, this would create excess demand for labor and a firm would try to raise its a wage a fraction above other wages. We show that equilibrium can only occur when the contract specifies market clearing wages whenever the outside option constraint binds.

also has implications for employment levels.[2] Obviously wage rigidity for incumbents need not imply deviations from efficient outcomes so long as hiring is at the efficient level (in our base model workers only separate for exogenous reasons). We show however that (under certain conditions) firms hire up to the point where the real wage equals the marginal product of labor; to the extent then that wages do not correspond to market-clearing levels hiring will be inefficient; as argued above, we show that this occurs only in the direction of wages being too high leading to inefficiently low employment and an excess supply of labor.[3,4]

A striking feature of our solution is that the equilibrium paths for wages and employment depend only on the *realized sequence* of productivity shocks, and in a particularly simple fashion. Amongst other things it implies that we do not need to specify a probability distribution for the shocks in order to simulate the model. This allows us to take a very lightly parameterized model to the data, and we argue that even our very rudimentary model, when fed sectoral productivity shocks from the post-war U.S. economy, does reasonably well in accounting for unemployment fluctuations. The main contribution of the paper is thus to analyze the dynamic implications of equal treatment constraints, and to derive empirical implications of such constraints.

Our approach in the paper is to examine the implications for wage and employment dynamics of *imposing* equal treatment. We argue that there is a good deal of anecdotal evidence supporting the assumption, and that more formal testing has not rejected it. However, we will also argue that equal treatment can arise endogenously in an extension of our model. If contracting is at will, so that firms can costlessly terminate an individual worker's contract, then committing to equal treatment—if this is possible—will be an optimal strategy for a firm. The idea is that if it does not do so, then workers accepting a contract which offers insurance will anticipate being undercut in bad states of the world and substituted by cheaper workers. To be able to credibly offer insurance, then, which

[3] In Gertler and Trigari (2006) in the context of a matching model with staggered multiperiod wage bargaining, new hires are also brought in at the same wage as incumbents. This implies that the wage stickiness that results in the model is transmitted to the extensive margin via hiring decisions. This leads to amplification of the volatilities of vacancies and unemployment to empirically plausible levels.

[4] In the absence of cross-contract restrictions, long-term contracts *per se* do not appear to be able to generate plausible levels of unemployment volatility (Rudanko 2009).

reduces the ex ante wage bill, firms must commit not to undercut.[5]

The paper builds on Holmstrom (1983) and Beaudry and DiNardo (1991). The latter develop a model of labor contracting where a risk-neutral firm offers insurance to risk-averse employees but in which there is no worker commitment (perfect mobility). Wages follow a ratchet-like process, rising when productivity is higher than previously, but staying constant otherwise. The downward wage rigidity in their perfect mobility model provides insurance to the worker but does not directly affect employment decisions, unlike here.

An outline of the paper is as follows. We start by outlining the basic assumptions in Section 2. A simple two-period example is presented in Section 2.1. The general model is described and solved in Section 2.2. In 3.1 we allow for separation rates to vary with experience; in 3.2 we show that equal treatment arises in equilibrium if labor contracts are "at will," and we also discuss evidence and related literature; in 3.3 we argue that the results are robust to relaxing the worker mobility assumption. We then look at generalizing the model to include layoffs and firm-specific shocks in 3.4 and 3.5. In Section 4 we simulate the model using sectoral TFP data from the postwar US economy to generate predictions of unemployment movements. Finally Section 5 contains concluding comments.

2 The model

We start by outlining the basic ingredients of the model, then we present an example before we fill out further details and solve the general model.

Time runs from $t = 1, 2, 3 \ldots T$, where $T \geq 2$ is finite, and there is a single nonstorable consumption good each period. There are a large number of workers, assumed to be identical (we abstract from any tenure or experience effects on productivity), except for their birth/death dates. Workers are risk averse with per period twice differentiable utility

[5] See Menzio and Moen (2008), who in independent work also find that a lack of commitment not to replace leads to amplified employment fluctuations relative to a model in which the feature is absent. Their model and main focus differ in many respects from ours: the labor market is one of directed search, firms live for two periods only, and firm entry plays an important role in employment fluctuations. They model a more nuanced relationship between the wages of incumbents and new hires than our competitive framework permits. By contrast our model allows us to focus on richer wage and employment dynamics. (See also Menzio (2005) for a somewhat related mechanism.)

function $u(w)$, $u' > 0, u'' < 0$, where $w \geq 0$ is income, which must be consumed within the period—it is assumed that they can neither save nor borrow. There is no disutility of work, but hours are fixed, so that workers are either employed or unemployed. Assume that if workers are not employed in a period, they receive some low consumption level $\underline{c} \geq 0$. There is a large (but fixed) number of identical risk-neutral firms. Each firm has a diminishing returns technology with output at t given by $F(N, s_t)$, $\partial F/\partial N > 0, \partial^2 F/\partial N^2 < 0$, where N is labor input and s_t is the date t productivity shock (the sole source of fluctuations). It is assumed that a firm must always retain some (minimum measure of) workers each period.[6] Workers and firms discount the future with respective factors $\beta_w, \beta_f \in (0, 1]$.[7,8]

For an employed worker, there is a "staying" probability of $\delta \in (0, 1)$, each period, so workers exogenously separate from their firm with probability $1 - \delta$; moreover with probability μ separated workers must seek work at a different firm and with probability $1 - \mu$ they die (exit the market); the same death probability of $(1 - \delta)(1 - \mu)$ applies to the unemployed. Separation occurs at the end of a period so that separated workers who do not exit but find a job in the following period do not suffer any unemployment. The number dying is replaced by the same number of new entrants. We assume there are a large number of workers relative to the number of firms, but for notational simplicity we normalize the ratio of workers to firms to be one each period.[9] We assume that the "spot wage"/full employment ($N = 1$) solution is always greater than the unemployment consumption level, i.e., that $\partial F/\partial N(1, s_t) > \underline{c}$ all t.

The state of nature s_t, which is observed at the start of period t before any decisions are made, follows a Markov process, with initial fixed value s_1, finite state space S, and transition probabilities $\pi_{sr} \geq 0$.[10] Let $h_t \equiv (s_1, s_2, \ldots, s_t)$ be the history at t. The labor

[6] This can be motivated by an assumption that some knowledge must be passed on from one generation of workers to the next.

[7] Allowing for asymmetric discounting will be useful when we come to the empirical exercise.

[8] The model can be closed in a variety of ways without affecting the labor market equilibrium, for example by assuming that firms are owned by a class of agents who do not provide labor services, and who consume profits each period, with $\underline{c}$ being home production.

[9] Thus we take the fraction of a firm's workforce leaving to be exactly $(1 - \delta)$. If N was finite, then the fraction leaving a firm would be random, and it can be shown that the contract could be improved by conditioning on this. (An alternative assumption to N large would be to simply rule out contracts that condition on this fraction on the grounds that verification may be impossible.)

[10] We use a Markov process to fix ideas, although the arguments go through for more general stochastic processes.

market offers a worker currently looking for work (at the start of t) an expected utility (discounted to t) which is denoted by $\chi_t(h_t)$. We shall refer to this as *the outside option.*

We assume symmetry at the start of t between the situation of a worker who is currently employed and one who is not attached to a firm, by assuming that a worker who quits their current employer at t joins the non-assigned at the start of t and so also gets $\chi_t(h_t)$. In the base model, workers can costlessly quit. Thus a firm must offer at least $\chi_t(h_t)$ to prevent its workers from quitting. We distinguish this from the utility a firm must offer to hire, say $\underline{\chi}_t(h_t)$. The labor market is modelled as being competitive: if it clears then both χ_t and $\underline{\chi}_t$ equal the common utility offered by firms. It may be, however, that there is excess supply of labor. In this case, it is assumed that workers who are not assigned to a firm at the start of t (the surviving unemployed from $t-1$, plus the surviving separated from the end of $t-1$, together with new entrants and quitters at the start of t) will accept a contract if it offers at least the utility from remaining unemployed, so $\underline{\chi}_t$ is this value. Each non-assigned worker gets a job with the same probability, given by the number of (acceptable) new jobs divided by the number of non-assigned workers, and in this case χ_t would equal utility averaged across getting a job and remaining unemployed using this probability, and is thus no smaller than the latter, and hence no smaller than $\underline{\chi}_t$.[11] Since firms must retain some workers, they must offer at least χ_t and this will always guarantee they can also hire as many workers as they like. Thus the relevant participation constraint with costless worker mobility is that a firm must always offer at least χ_t.[12] (Until we consider worker commitment in Section 3.3, we do not need $\underline{\chi}_t$.)

We summarize the timing as follows. At date 1 each firm offers a *single* state-contingent wage contract $(w_t(h_t))_{t=1}^T$ *to which it is committed.* The fact that it cannot offer subsequent hires a different contract captures the equal treatment restriction. Workers accept contracts and period 1 production takes place. At the end of period 1, a firm loses a fraction $(1-\delta)$ of its workforce due to exogenous separation. At the start of each subse-

[11] The separated at the end of $t-1$ who manage to find work at t do not suffer a period of unemployment.

[12] Many of the fine details of how excess supply is modeled do not affect the solution for the paths of wages and employment in the symmetric equilibrium. For example, employment probabilities could differ among the different categories of the non-assigned, or the minimum utility that a non-assigned worker will accept could be higher than the unemployment utility; what is critical is that χ_t is no more than the utility offered by an employment contract.

quent period $t \geq 2$, firms and workers observe s_t. Attached workers may quit costlessly at this point and join the pool of those previously separated, the surviving unemployed from $t-1$ and new entrants (and receive χ_t). However, provided the continuation utility offered by the contract at least matches χ_t, the firm is able to retain its workers and hire in as many new workers as it requires from the pool of those previously separated. Production takes place and wages $w_t(h_t)$ are paid, and the period ends.

2.1 A two-period example

Before we proceed further, we shall deal with an example. This illustrates the main ideas while avoiding a number of complications. There are two periods ($t = 1, 2$). The state of nature s_t is represented by productivity a_t : a_1 is fixed and a_2 equals either $\underline{a}_2$ or $\overline{a}_2$ with equal probability, $\underline{a}_2 < a_1 < \overline{a}_2$. We assume no discounting, and $\mu = 1$ (so all separated workers remain in the market and there is no death).

The representative firm offers a wage contract in period 1, which we denote as $(w_1, \underline{w}_2, \overline{w}_2)$ using obvious notation, and has an employment plan $\left(N_1, \underline{N}_2, \overline{N}_2\right)$. Notice that the equal treatment restriction is captured by the fact that the firm is constrained to pay new hires, in period 2, $\underline{w}_2$ or $\overline{w}_2$ depending on the state, the same wage as the incumbents receive.

The problem faced by the firm[13] is to choose a contract and an employment plan to:

$$\max E \sum_{t=1}^{2} \left(F(N_t, a_t) - N_t w_t\right) \qquad \text{(Problem A1)}$$

subject to the *participation constraints*

$$u(w_1) + E\left(\delta u(w_2) + (1-\delta)\chi_2\right) \geq \chi_1 \tag{1}$$

at date 1, and

$$u(w_2) \geq \chi_2 \tag{2}$$

in both states in period 2. The value of the outside option in period 1, χ_1, is simply the equilibrium utility a worker can get from any other firm. If there is full employment in

[13] We ignore the possibility of layoffs, assuming that the loss of workers due to exogenous separation is larger than any desired reduction in the labor force.

period 2 then χ_2 is equal to the utility from the equilibrium wage because a worker who quits is guaranteed a job elsewhere. If however there is unemployment in period 2, χ_2 is less than the utility from the equilibrium wage because a worker who quits will get a new job with probability less than one.[14] If the period 2 participation constraint is binding in any state then this pins down the wage. Of more interest is what happens when the constraint does not bind. From the first-order conditions, $N_1 = \lambda u'(w_1)$ where λ is the multiplier on (1), and $\underline{N}_2 = \lambda\delta u'(\underline{w}_2)$ if (2) is not binding in the bad state. Thus

$$\frac{u'(\underline{w}_2)}{u'(w_1)} = \frac{\underline{N}_2}{\delta N_1}. \tag{3}$$

Equation (3) implies there is an optimal rate of fall of wages in this state even though a lower wage at $t = 2$ is feasible and would allow the firm to bring in new hires at a cheaper rate.

This is critical for our approach: wages may not fall fast enough to clear the labor market. Suppose that $F(N, a_t) = a_t LogN$, and $a_1 = 1$, a_2 is either 1.1 or 0.9, (i.e., productivity growth is either +10% or −10%) and $u(w) = -w^{-1}$. Let $\underline{c} = 0.8$ and $\delta = 0.87$. The marginal productivity of labor at full employment where $N = 1$ is a_t and this implies that the "spot market" solution (i.e., what would prevail in the absence of contracts) would have $w_t = a_t$.

The solution values here are $w_1 = 1$ (the spot solution), $\overline{w}_2 = 1.1$ (the spot solution) and $\underline{w}_2 = 0.966$ (> 0.9, the spot solution). N is determined according to the labor demand schedule (note that N only enters the maximand, not the constraints), so a contract wage above a_t implies the labor market does not clear. Here the wage in the bad state at $t = 2$ does not fall sufficiently to clear the labor market, and $\underline{N}_2 = 0.93$. The ability of workers to quit implies that in the good state the firm cannot stabilize the wage for wage smoothing purposes but must match the higher value of productivity, and there is full employment. In the low productivity state by contrast there is an unemployment rate of 7%, and the participation constraint is slack (a worker who quits will at best get the same wage at

[14] χ_1, the period 1 outside option, is equilibrium worker utility discounted to period 1, namely $u(w_1^*) + E(\delta u(w_2^*) + (1-\delta)\chi_2)$, where the * superscripts denote equilibrium values. If there is no unemployment at $t = 2$, then $\chi_2 = u(w_2^*)$; if there is (i.e., $N_2^* < 1$) $\chi_2 = ((N_2^* - \delta)/(1-\delta))\, u(w_2^*) + ((1 - N_2^*)/(1-\delta))\, u(\underline{c})$. Thus $\chi_2 < u(w_2^*)$ as $u(\underline{c}) < u(w_2^*)$ (see below).

another firm, but she may end up unemployed). Any attempt to cut $\underline{w}_2$ further will lead to an increase in overall wage costs because the need to compensate period 1 hires for the extra wage variability more than offsets the fact that period 2 hires could be brought in at a lower wage.

The example illustrates a number of features that arise in the general solution.

- Provided the firm is always hiring, employment is determined by wage = marginal productivity of labor. Intuitively, an extra worker employed at t can be 'neutralized' at $t+1$ by reducing hiring; thus profitability of an extra hire at t depends only on marginal productivity relative to the current wage as would hold in a static model.

- When there is full employment, the equilibrium wage equals marginal productivity at $N = 1$ *and the participation constraint binds;* otherwise the wage exceeds this spot solution *and the participation constraint is slack.*

- To solve the problem one can compute the wage change that follows from (3), i.e., ignoring the period 2 participation constraint in any state—this is a type of Euler equation which solves the problem of trading off the firm's desire to smooth the wages of its period 1 incumbents (which reduces the discounted wages it has to offer them) against the wish to take advantage of lower wages paid to new hires in period 2, and note that the period 1 wage is the spot solution.[15] If the implied wage from (3) in period 2 lies above the spot solution wage, then the period 2 participation constraint does not bind and there is unemployment; otherwise there is full employment and the wage is at the spot solution. We show below that the same argument can be used to compute the wage at any date $t+1$ given the wage at t.

- The latter implies that there is a degree of downward, but not upward, wage rigidity.

- Computing the wage in the second period by comparing the wage which satisfies the Euler equation with the spot wage also implies that the wage in each state can be computed independently of other states or their probabilities. This also

[15] N can be solved out in (3) by noting that wage equals marginal product of labor.

generalizes and implies—notably—that the model can be simulated by inputting a *realized* sequence of productivity shocks; that is, the solution does not require the *distribution* of shocks to be known.

How would the solution differ if the equal treatment constraint was *not* imposed? The optimal contract would then be of the type analyzed by Beaudry and DiNardo (1991): wages would be fully downward rigid, rising to the spot level in the good state; in the bad state new hires would be brought in at a lower wage (in fact at $\underline{a}_2$). There would be no unemployment.

2.2 The General Model

2.2.1 The Firm's problem

We work with a representative firm. At the start of date 1, after s_1 is observed, firms commit to contracts $(w_t(h_t))_{t=1}^T$, $w_t(h_t) \geq 0$, which we assume are not binding on workers. *We assume equal treatment:* a worker joining subsequently, at τ after history h_τ, is offered a continuation of this same contract: $(w_t(h_t))_{t=\tau}^T$. Each firm also has an employment plan $(N_t(h_t))_{t=1}^T$, where $N_t(h_t) \geq 0$. Let $V_t(h_t)$ denote the continuation utility from t onwards from the contract, defined recursively by:

$$V_t(h_t) = u(w_t(h_t)) + \beta_w E\left[\delta V_{t+1}(h_{t+1}) + (1-\delta)\mu\chi_{t+1} \mid h_t\right], \tag{4}$$

with $V_{T+1} = 0$, where E denotes expectation, and the term involving χ_{t+1} reflects the utility after exogenous separation but no death.

Our aim is to construct an equilibrium in which layoffs do not occur, as it substantially simplifies the analytics of the solution. In fact, we will state the optimization problem *imposing* no layoffs, to avoid complicating the statement of the problem, and later derive parameter restrictions for which this remains a solution even if layoffs are permitted.[16]. (In

[16] Thus given that the rate of separation is exogenous, movements in unemployment in our base model occur through changes in hiring. This is arguably consistent with the evidence reviewed in Hall (2005b) who argues that the separation rate is roughly constant (see also Pissarides (1986), Shimer (2005)), and that although job losses rise during recessions, the increase is usually very small in relation to the normal levels of separations. However, these conclusions have been disputed, e.g., Davis (2005), Fujita and Ramey (2007), Elsby, Michaels, and Solon (2009). Our base model does not account for the fact that layoffs

Section 3.4 below we discuss the implications of the case where layoffs are permitted and do occur in equilibrium. We argue in an example that the wage/unemployment dynamics are comparable to the no layoffs case.)

The problem faced by the firm, which takes $(\chi_t)_{t=1}^T$ as parametric, is:

$$\max_{(w_t(h_t))_{t=1}^T,(N_t(h_t))_{t=1}^T} E\left[\sum_{t=1}^T (\beta_f)^{t-1}\left(F(N_t(h_t),s_t)-N_t(h_t)w_t(h_t)\right)\right] \qquad \text{(Problem A)}$$

subject to

$$V_t\left(h_t\right) \geq \chi_t\left(h_t\right) \tag{5}$$

for all positive probability $h_t, T \geq t \geq 1$, and

$$N_t(h_{t-1},s) \geq \delta N_{t-1}(h_{t-1}) \tag{6}$$

for all positive probability h_{t-1}, all $s \in S$ with $\pi_{s_{t-1}s} > 0$, $T \geq t \geq 2$. (5) is the *participation constraint* that says that at any point the contract must offer at least what a worker can get by quitting, while (6) imposes that the firm may not layoff workers.[17]

2.2.2 Equilibrium

We shall use a $*$ superscript to denote equilibrium values. To close the model we impose an equation specifying the equilibrium determination, given $(w_t^*(h_t))_{t=1}^T, (N_t^*(h_t))_{t=1}^T$, of the outside option:

$$\chi_t = \frac{N_t^* - \delta N_{t-1}^*}{1-\delta N_{t-1}^*} V_t^* + \frac{1-N_t^*}{1-\delta N_{t-1}^*} U_t, \tag{7}$$

$N_0^* = 0$, where V_t^* is the equilibrium contract offer at t (the denominator is the number of workers not retained after $t-1$) and U_t is the discounted utility of a worker who fails to find work at t which is given by

$$U_t(h_t) = u(\underline{c}) + \beta_w\left(1-(1-\delta)(1-\mu)\right) E\left[\chi_{t+1} \mid h_t\right], \tag{8}$$

play a major role in downturns (e.g., Davis (2005)) as we treat separations as exogenous. The analysis of Section 3.4 below suggests that introducing layoffs would imply that their share of total separations is countercyclical, and the separation rate itself would become countercyclical.

[17] More precisely, (6) implies layoffs are not *needed*. However the definition of $V_t(h_t)$ in (4) implies that a worker remains with the firm unless exogenously separated, so together these two assumptions rule out layoffs.

i.e., the utility from the reservation wage plus future utility assuming survival (with probability $(1-(1-\delta)(1-\mu)))$ from not having a job at the beginning of $t+1$. In interpreting (7), note that there are two cases: market clearing and excess labor supply. If the labor market at time t clears, $N_t^*(h_t)=1$, then from (7) $\chi_t(h_t)$ must match the utility offered by other firms. In symmetric equilibrium, each other firm is offering an identical contract, and so it is the utility associated with this, $V_t^*(h_t)$, which must be matched. If, on the other hand, there is excess supply of labor,[18] $N_t^*(h_t)<1$, the outside opportunity will depend on the probability of getting a job, and recall we assumed that all unattached workers have equal probability of getting work.[19]

Given the endpoint condition $\chi_{T+1}=0$, (7), (4) and (8) uniquely determine U_t, V_t^* and χ_t.

We can summarize:

Definition 1 $\left((w_t^*(h_t))_{t=1}^T, (N_t^*(h_t))_{t=1}^T\right)$, *where* $0 \leq N_t^*(h_t) \leq 1$ *all* h_t, *constitutes* a symmetric no layoff equilibrium *(SNLE) if it solves Problem A where* $(\chi_t)_{t=1}^T$ *is determined recursively from (4), (7) and (8).*

Lemma 1 below gives *necessary* conditions for wage changes in an optimal contract.

Lemma 1 *In an optimal contract with perfect mobility (i.e., a solution to Problem A),*

$$\frac{u'(w_{t+1}(h_t,s))}{u'(w_t(h_t))} \leq \frac{\beta_f N_{t+1}(h_t,s)}{\delta\beta_w N_t(h_t)} \tag{9}$$

must hold; it can only hold strictly (<) if the participation constraint binds at (h_t, s).

Lemma 1 implies that there will be a *lower bound on the fall of real wages*; even if the labor market is slack at $t+1$, the firm will not want to cut the wage too far because of the desire to insure incumbents. Once this point is reached, the wage will not fall faster

[18] Intuitively, as noted earlier, the case of excess demand for labor cannot arise in equilibrium, as an infinitesimally small increase in the wage would cure the individual firm's supply problem. In contrast, because of equal treatment the case of excess supply can arise since workers cannot undercut.

[19] When there is unemployment ($N_t^*<1$), we assumed that firms can hire at U_t, the utility of the unemployed. It follows from (7) that $U_t<\chi_t$, which justifies the earlier discussion that matching the outside option χ_t is the appropriate constraint.

no matter how low the supply price of outside workers (so new hires will strictly want to work for the firm in this case).

We can prove Lemma 1 with the aid of a simple variational argument.[20] The general strategy is to start with an optimal wage contract and then consider the change in profits arising from shuffling wages over any two adjacent periods. Then by optimality of the original contract we know that the resulting changes in profits must be nonpositive. This nonpositivity condition will in turn give us a bound on the rate of change of marginal utilities and hence wages between two periods. We end up with a condition that limits the rate of fall of wages in a recession. It will transpire that this plus a "wage-equals-marginal-product" result proved below will completely specify the dynamics of equilibrium wages for any given sequence of productivity shocks.

Proof of Lemma 1. Starting from the optimal contract, consider reshuffling wages between t and $t+1$ in state s, to backload them. Increase the wage at $t+1$ after state s by a small amount Δ, and cut the wage at t by x so as to leave the worker indifferent; do not change the contract otherwise. This implies that

$$\pi_{s_t s}\delta\beta_w u'\left(w_{t+1}(h_t,s)\right)\Delta - u'\left(w_t(h_t)\right)x \simeq 0. \tag{10}$$

This backloading satisfies all participation constraints: worker utility rises at $t+1$, and so from this point on constraints are satisfied; similarly, participation constraints are also satisfied both after h_t and earlier because utility is held constant over the two periods.

The change in profits (viewed from h_t) arising from the backloading is

$$\Delta P = -\pi_{s_t s}\beta_f N_{t+1}\Delta + N_t x$$

(where $N_{t+1} \equiv N_{t+1}\left(h_t, s\right)$ etc.). Using (10) to eliminate x gives the change in profits as

$$\Delta P \simeq -\pi_{s_t s}\beta_f N_{t+1}\Delta + \frac{\pi_{s_t s}\delta\beta_w u'\left(w_{t+1}(h_t,s)\right)N_t\Delta}{u'\left(w_t(h_t)\right)}. \tag{11}$$

[20] We can also use standard recursive arguments to derive (9) and solve the model. However Lemma 1 does not require any of the technical restrictions we later introduce. In fact it applies quite generally, relying only on risk sharing subject to a sequence of participation constraints, and so might serve as a general test of equal treatment models. It would still apply if employment determination is modelled differently, for example due to a non-competitive product market, or subject to adjustment costs. It holds even if there are layoffs provided layoff probabilities can be committed to (see Footnote 43 below).

The change in profits cannot be positive by optimality of the original contract, i.e., $\Delta P \leq 0$, so using (11) and rearranging we establish (9) (i.e., by considering Δ sufficiently small the approximation in (11) can be made arbitrarily precise).

To prove the second part of the lemma, consider frontloading wages, i.e., repeat the above arguments but for a decrease in the wage at $t+1$ of Δ, offset by an increase of x at t. Note that in this case the $t+1$ participation constraint *will* be violated if it is binding initially. By an analogous argument to the above, frontloading is profitable if

$$\frac{u'\left(w_{t+1}(h_t, s)\right)}{u'\left(w_t(h_t)\right)} < \frac{\beta_f N_{t+1}(h_t, s)}{N_t(h_t)\delta\beta_w},$$

i.e., if (9) holds with strict inequality. In this case the $t+1$ participation constraint must bind, as otherwise frontloading would be undertaken and profits would increase. ■

The critical idea is that backloading the contract cannot violate participation constraints; hence the contract will always be backloaded at least until the optimal dynamic first-order condition is attained, or beyond if the participation constraint at $t+1$ binds. So in an optimal contract there is an unconstrained wage Euler equation, or "target marginal utility growth rate":

$$\frac{u'\left(w_{t+1}(h_t, s)\right)}{u'\left(w_t(h_t)\right)} = \frac{\beta_f N^s_{t+1}}{N_t\delta\beta_w} \tag{12}$$

which will be maintained, unless a binding participation constraint at $t+1$ forces it to be lower. Equivalently, this puts a lower bound on how fast real wages can decline, but a tight labor market at $t+1$ can imply that wage growth is not against this bound. Note that this lemma applies whether or not the firm is hiring at t or $t+1$.

To understand the intuition for (12), suppose $\beta_f = \beta_w$, so marginal utility grows at rate $N^s_{t+1}/N_t\delta$. In the absence of equal treatment, this rate would be unity—incumbents' wages are stabilized (as in Beaudry and DiNardo) if the future participation constraint is not binding. Under equal treatment, if $N^s_{t+1}/N_t\delta > 1$, however, the firm is taking on additional workers at $t+1$ who will receive the same wage as the incumbents; hence the future wage is taken into account with a larger weight by the firm than by the incumbent worker, and this imparts a downward bias to the future incumbent wage in comparison with the discrimination case.

One implication of this is that the smaller is δ (i.e., the higher the rate of turnover), the smaller is the degree of downward rigidity. Intuitively, as turnover approaches 100%, there is very little to be gained by smoothing incumbents' wages, so the firm will cut wages whenever it can. Du-Caju, Fuss, and Wintr (2007) is the one paper we are aware of which has looked at wage rigidity and turnover, and indeed finds that firms with higher rates of turnover do appear to have wages that are less rigid downwards.[21]

We now establish that in equilibrium wages are always equal to the marginal product of labor. Together with the conditions in Lemma 1 this completely characterizes the dynamics of equilibrium wages for any given productivity sequence.

To proceed, we shall provisionally assume that firms always hire (at all h_t) in equilibrium. That is to say, we proceed on the supposition that the constraint (6) in problem A never binds in the solution. We characterize the solution if this is the case, and later find conditions on a specific parametrization for which the solution satisfies this property. Finally we verify that this is also a solution to the problem even if layoffs are permitted (i.e., without (6)). Under this assumption, employment is determined by a standard marginal productivity equation:

Lemma 2 *If in an SNLE hiring takes place at every h_t, then $N_t^*(h_t)$ satisfies*

$$\partial F(N_t^*(h_t), s_t)/\partial N = w_t^*(h_t). \tag{13}$$

Proof. Suppose that $\partial F(N_t^*(h_t), s_t)/\partial N > w_t^*(h_t)$. It is feasible to increase current hiring holding the wage contract constant, and consider this as the only change to the firm's plan: An increase in current hiring by $\Delta > 0$, for Δ small enough, and holding the wage constant at $w_t^*(h_t)$, would lead to an increase in current profits. At the same time, holding employment at $t+1$ constant at $N_{t+1}^*(h_{t+1})$ in all states (so hiring falls by $\delta\Delta$), is feasible for Δ small enough given hiring is positive at $t+1$. Thus there is an increase in profits at t, and no change at other dates, contradicting profit maximization. A symmetric

[21] They argue that this is consistent with a turnover-cost efficiency wage model: firms which suffer high costs from a given level of turnover are more likely to invest in long-term contracting mechanisms, including more rigid wages, in order to minimize turnover. Thus lower turnover and more rigidity will go together in the data.

argument, using the fact that current hiring is positive so can be *reduced* by Δ, and that $t+1$ employment can be increased by $\delta\Delta$, rules out $\partial F(N_t^*(h_t), s_t)/\partial N < w_t^*(h_t)$. ■

Now, suppose that at some t, the participation constraint binds. Then there must be full employment and the wage is determined by marginal productivity at full employment:

Lemma 3 *Consider an SNLE in which hiring always occurs; then the participation constraint binds at h_t if and only if $N_t^*(h_t) = 1$; moreover if the constraint binds then $w_t^*(h_t) = \partial F(1, s_t)/\partial N$.*

Proof. (i) Suppose that $N_t^*(h_t) < 1$. Under the hiring hypothesis, we know from Lemma 2 that $\partial F(N_t^*(h_t), s_t)/\partial N = w_t^*(h_t) > \underline{c}$ by the assumption on $\underline{c}$ and diminishing marginal productivity (i.e., $w_t^*(h_t) \leq \underline{c}$ would imply $N_t^*(h_t) > 1$). So a worker who fails to get a job is strictly worse off, and thus $V_t^*(h_t) > \chi(s_t)$ (cf. (7) and (8)) and so the participation constraint does not bind. On the other hand, if $N_t^*(h_t) = 1$ then from (7), $V_t^*(h_t) = \chi(s_t)$ and the constraint binds. The equilibrium wage follows directly from Lemma 2. ■

We define $\underline{w}_s^* = \partial F(1, s)/\partial N$, which in view of the above lemma is the equilibrium wage when the participation constraint binds in state s. Then we can summarize:

Corollary 1 *In an SNLE with hiring, if at $t+1$ the participation constraint is not binding, wages are updated according to (12); if it is binding, then $w_{t+1}^* = \underline{w}_{s_{t+1}}^*$.*

2.3 Using Specific Functional Forms

To proceed to an explicit solution, we put more structure on the problem.[22] This will allow us to assert that the wage updating rule is of the following simple form: given w_t^* compute w_{t+1} under the hypothesis that the participation constraint at $t+1$ is not binding; if $w_{t+1} > \underline{w}_{s_{t+1}}^*$ then the hypothesis is confirmed and w_{t+1} is the equilibrium wage; otherwise the constraint *is* binding and the equilibrium wage will be at $\underline{w}_{s_{t+1}}^*$. The

[22] We also need the problem faced by the firm to be concave; concave production and utility functions are not sufficient to guarantee this.

structure will also allow us to demonstrate sufficient conditions for the symmetric hiring equilibrium to exist.

From henceforth assume each firm has technology given by, at time t,

$$F(N, s_t) = M_t + a_t N^{1-\alpha}/(1-\alpha), \tag{14}$$

where $\alpha > 0, \alpha \neq 1$, $a_t > 0$, and $M_t \geq 0$. (M_t, a_t) will evolve according to a Markov process. Note that for $\alpha > 1$, F has an upper bound given by M_t, which given that we are modelling short-run production functions at the establishment or plant level, may be appropriate. We also assume henceforth that workers have per-period utility functions of the constant relative risk aversion family with coefficient $\gamma > 0$, $\gamma \neq 1$, described by $u(c) = c^{1-\gamma}/(1-\gamma)$.[23] Finally we assume that $\alpha\gamma > 1$.

The "target" rate of wage growth (i.e., if unconstrained at $t+1$) becomes, from (12),

$$\frac{w_{t+1}}{w_t} = \left(\frac{\lambda N_t}{N_{t+1}}\right)^{\frac{1}{\gamma}}, \tag{15}$$

where $\lambda \equiv \frac{\delta\beta_w}{\beta_f}$. Under the hiring assumption, we also have that the marginal product of labor equals $a_t N_t^{-\alpha}$, so that using (13),

$$N_t = a_t^{\frac{1}{\alpha}} w_t^{-\frac{1}{\alpha}}. \tag{16}$$

Combining (15) and (16) yields an equation for the evolution of wages if unconstrained at $t+1$:

$$\frac{w_{t+1}}{w_t} = \lambda^{\frac{\alpha}{\alpha\gamma-1}} \left(\frac{a_{t+1}}{a_t}\right)^{\frac{-1}{\alpha\gamma-1}} \equiv \xi\left(\frac{a_{t+1}}{a_t}\right) \tag{17}$$

where the function $\xi(.)$ simplifies notation. Moreover if firms are constrained at $t+1$, then as $N_{t+1} = 1$, $w_{t+1} = \underline{w}^*_{s_{t+1}} = a_{t+1}$ (from Lemma 3).

We can now state a more precise version of Corollary 1 and also provide a sufficient condition for this to be not only a no layoff equilibrium, but also an equilibrium of the model in which layoffs are permitted (see Appendix for proof).

Proposition 1 *(i) In an SNLE with positive hiring, wages will satisfy*

$$w^*_{t+1} = \max\left\{\xi\left(\frac{a_{t+1}}{a_t}\right) w^*_t, a_{t+1}\right\}, \tag{18}$$

[23] For $\alpha = 1$, we can specify $F(N, s_t) = M_t + \log(N)$, and for $\gamma = 1$, $u(c) = \log(c)$; all results go through.

where $w_1^* = a_1$ and employment is determined by the marginal productivity condition.

(ii) A sufficient condition for existence of this equilibrium is

$$\frac{a_{t+1}}{a_t} > \lambda^{\frac{1}{\gamma}} \delta^{\frac{\alpha\gamma-1}{\gamma}} = \delta^{\alpha} \left(\frac{\beta_w}{\beta_f}\right)^{\frac{1}{\gamma}} \tag{19}$$

for all (a_t, a_{t+1}) that occur with positive probability. (iii) Under (19) this is also an equilibrium where layoffs are allowed.[24]

For $\beta_w = \beta_f$, condition (19) requires that the maximum rate of *fall* of productivity should be smaller than the exogenous turnover rate raised to the power of α; the condition is equivalent to ensuring that labor demand does not fall beyond exogenous separations in the putative equilibrium, so hiring remains positive.[25] In this situation, layoffs are of no value to the firm and they can only reduce the value of a contract to a worker.

To illustrate the solution, we take the productivity series for one of the sectors that we use in the empirical exercise (see Section 4 for details); Figure 1 displays simulated wages using the following parameters: $\alpha = 0.9$, $\gamma = 6.6$, $\beta_w = \beta_f$ (so $\lambda = \delta$) and $\delta = 0.92$, 0.85 respectively in the left and right panels. Recall that the productivity level equals the spot wage. Whenever the wage lies above productivity, the labor market fails to clear (and the participation constraint does not bind); the larger the percentage gap, the larger is unemployment (as $N_t = (w_t/a_t)^{-\frac{1}{\alpha}}$). At the lower value for δ, wages fall faster when the participation constraint does not bind, and unemployment will be lower.

3 Extensions

3.1 Experience dependent δ

In this section we allow for δ to vary with the experience of an individual; this is relevant for calibrating the model. What we wish to model is the idea that more recent entrants

[24] Assuming that layoff probabilities are part of the contract to which the firm is committed (these could be cohort dependent). For simplicity we rule out layoff pay, so a worker laid off in period t simply receives χ_t.

[25] For a range of productivity falls above this maximum rate, outcomes differ from spot outcomes: starting from full employment in some state a_t, we need the wage to fall by less than the spot wage. Thus we need, using $w_t = a_t$, from (17), $w_{t+1} = \lambda^{\frac{\alpha}{\alpha\gamma-1}} \left(\frac{a_{t+1}}{a_t}\right)^{\frac{-1}{\alpha\gamma-1}} a_t > a_{t+1}$ which can be rewritten as $\frac{a_{t+1}}{a_t} < \lambda^{\frac{1}{\gamma}}$.

are more likely to separate than more experienced workers, so we assume that a worker with experience τ has a probability of surviving at the end of the period of $\underline{\delta}_\tau < 1$, where τ is the worker's age (experience). Thus at any date there will be heterogeneity across workers, although we do not allow firms to condition wages on the staying probability. For simplicity in this section, we assume that all separations involve exiting the labor market, $\mu = 0$, so all exogenous turnover involves entry into and exit from this labor market.

Workers with different experience will now evaluate a firm's contract and outside opportunities differently, so the previous expressions have to be modified slightly, and there will be a separate participation constraint for each worker vintage. The participation constraints become[26]:

$$V_t^\tau(h_t) \equiv u(w_t(h_t)) + E\left[\sum_{t'=t+1}^{T} \beta_w^{t'-t}\left(\prod_{\tilde{t}=1}^{t'-t} \underline{\delta}_{\tau+\tilde{t}-1}\right) u(w_{t'}(h_{t'})) \mid h_t\right] \tag{20}$$

$$\geq \chi_t^\tau, \tag{21}$$

for $t = 1, \ldots, T$, and $\tau \leq t$, where $V_t^\tau(h_t)$ is the continuation utility offered by the contract at time t to a worker with τ periods' experience (including the current one), and where the outside option χ_t^τ is as before a weighted average of $V_t^{\tau *}$ (the equilibrium value of V_t^τ; i.e., $w_t(h_t)$ replaced by $w_t^*(h_t)$ in (20)) and U_t^τ (defined by $U_t^\tau(h_t) = u(\underline{c}) + \beta_w \underline{\delta}_\tau E\left[\chi_{t+1}^{\tau+1} \mid h_t\right]$, etc.), with the weights depending on the employment probability, defined as before.[27]

The firm's problem now is as in Problem A but with (5) replaced by (21). We show that if the equilibrium is characterized for the problem where *all* workers separate at the same time dependent rate, with this rate given by the oldest workers' separation rate, then this also characterizes an equilibrium of the model with experience dependent rates.

Proposition 2 *Provided it entails positive hiring rates at all dates, the following constitutes a symmetric equilibrium for the model with experience dependent separation rates: wages satisfy*

$$w_{t+1}^* = \max\left\{\xi_{t+1}\left(\frac{a_{t+1}}{a_t}\right) w_t^*, a_{t+1}\right\}, w_1^* = a_1, \tag{22}$$

[26] It is assumed that the firm wishes to retain all its cohorts. This could be justified by sufficient turnover costs.

[27] I.e., assuming this is independent of age, although this can be relaxed without affecting the argument.

where $\xi_t(\cdot)$ is defined as in (17) but now with $\lambda \equiv \frac{\underline{\delta}_t \beta_w}{\beta_f}$, while employment is determined by the usual marginal productivity condition (13).

Proof. Consider a symmetric equilibrium of a model in which *all* agents have time (but *not* experience) dependent separation rates $1-\delta_t = 1-\underline{\delta}_t$ (so all agents separate at the same rate at t, irrespective of when they entered). For time dependent separation rates (22) and (13) characterize the equilibrium, directly from arguments already developed *mutatis mutandis,* where existence is handled as before. That is, for wages w_t^* given by (22), and employment determined by the marginal productivity condition, maximizing profits (as in Problem A) with constraints given by (21), for $t = 1, \ldots, T$, and $\tau = t$ (but not $\tau < t$) is solved by these same values. It must follow that $(w_t^*, N_t^*)_{t=1}^T$ remains a solution to the problem of maximizing the same objective function subject to the *additional* constraints (21), for $t = 1, \ldots, T$, *and* $\tau < t$ provided it satisfies these additional constraints. This follows immediately, as $U_t^\tau \leq V_t^{\tau *}$ (repeating earlier arguments), so that $\chi_t^\tau \leq V_t^{\tau *}$. ■

To understand the intuition for this result, consider $T = 3$, $\beta_w = \beta_f$ and suppose that shocks are deterministic and such that at $t = 2$ there will be full employment and at $t = 3$ there is unemployment (so in period 2 productivity is high and in period 3 it is low). Assume that workers born at $t = 1$ ("the old") have a higher survival rate, $\underline{\delta}_2$, at the end of period 2 than those born in period 2 ("the young"), $\underline{\delta}_2 > \underline{\delta}_1$. As in the proof, construct the equilibrium contract assuming that the young have the same survival rates as the old (i.e., at the end of period 2 they survive with probability $\underline{\delta}_2$). This will have $\acute{w}_1 = a_1$, $w_2 = a_2$ and $w_3 = (\underline{\delta}_2)^{\frac{\alpha}{\alpha\gamma-1}} \left(\frac{a_3}{a_2}\right)^{\frac{-1}{\alpha\gamma-1}}$. The claim is that this remains an equilibrium when the young have a different survival rate. Can there be any change in w_2 and w_3 that would lead to a profitable deviation? Note first that the participation constraints of *both* old and young hold with equality at $t = 2$ by symmetry of the equilibrium and the fact that there is full employment—the outside option is to get an identical contract from a different firm. Participation constraints are slack at $t = 3$ as there is unemployment, so we only have to worry about maintaining the constraints at $t = 2$. In the putative equilibrium, wages fall at the rate given by the old's Euler condition; satisfying the young's would require a larger fall. Suppose that wages are moved, as in the variational argument we used to establish

Lemma 1, slightly in the direction of satisfying the Euler condition for the young, that is, frontloaded from period 3 to 2, while holding the young's utility from $t = 2$ constant. This would increase profits. However the change makes the old worse off and would violate their participation constraint as they put more weight on the fall in w_3 than do the young. If instead the utility of the old is held constant, profits are maximized subject to this constraint, by definition of the Euler equation. So any frontloading will reduce profits if both constraints are to be satisfied, and there is no profitable deviation.[28]

Suppose that the staying probability reaches a maximum after $\bar{\tau}$ periods, so that a worker with $\tau \geq \bar{\tau} > 1$ years' experience remains with their firm with a probability of δ^O, with $\delta^O > \underline{\delta}_\tau$ all $\tau < \bar{\tau}$. The consequence of the equilibrium characterized by Theorem 2 is that once the first $\bar{\tau}$ periods have elapsed, so there are fully experienced workers in the workforce, the updating equation is that which pertains to δ^O. Since the turnover rate for experienced workers is substantially below that for new hires, this suggests that average turnover rates are not relevant for calibrating δ.[29]

3.2 On the Equal Treatment Constraint

So far we have simply imposed equal treatment as a constraint. In this section we analyze why equal treatment might arise, and briefly discuss existing theories and evidence.

In the absence of the equal treatment constraint, a firm will offer a lower cost contract to new hires in bad states of the world than the continuation of incumbents' contracts. However, suppose at the same time that the firm cannot commit to retaining incumbents, so that it can terminate contracts if it so wishes. Then in bad states it will have an

[28] Can this same argument be used to show that a wage change corresponding to the young's survival rate is also an equilibrium? The answer is no. It is true that the same logic as above applies to show that no change to w_2 and w_3 can improve profits starting from this alternative putative equilibrium. Nevertheless, consider backloading wages slightly so as to keep the utility of the young constant from $t = 2$. There is a second-order loss of profits, but the old benefit from a first-order utility increase at $t = 2$. This allows w_1 to be cut without violating the old's period 1 participation constraint, and overall profits increase, so a profitable deviation exists. In the equilibrium discussed in the text, a first-order utility increase for the young at $t = 2$ has no value as w_1 cannot be cut without violating the old's period 1 constraint.

[29] For example, Royalty (1998) shows from the NLSY that during the first year of tenure 55% of men will stay in the same job. By the time they reach 7 years of tenure (the nature of the data did not allow for longer tenure), this has risen to 90% even amongst the young workers covered in the survey (the figures for women were similar).

incentive to replace incumbents by cheaper new hires.[30] The ability to wage discriminate may in fact be detrimental to the firm since incumbents will anticipate that the firm will end their contract in bad states, replacing them by cheaper labor, and this inability to commit to retain the worker will ultimately increase wage costs as firms cannot credibly commit to insure a worker. Inability to commit to retain incumbents may follow from courts being unable to distinguish between a voluntary quit and one that is enforced by the employer, for example by making working conditions unpleasant, or by dismissing workers on the basis of minor contract violations. Alternatively it may be that the law stipulates that the employment relationship is "at will".[31]

In such an environment, a firm may thus be better off if it is able *to commit to equal treatment,* assuming a technology exists for doing this, and we argue this is true in our model. Thus we make two changes to our previous assumptions. First, although firms can still commit to wage contracts, we dispense with the equal treatment restriction *so these contracts may be cohort contingent*—with firms committing to a different wage contract only when each new cohort is employed. However firms have the option, at the outset, to commit to equal treatment, in which case they sacrifice the ability to offer cohort dependent contracts to subsequent hires. Secondly, *the firm* can terminate its relationship with a worker at any point without cost (previously only workers were allowed to terminate). Thus, although there is commitment to wages should the relationship continue, either party can walk away costlessly at any point.[32]

[30] There is some evidence for a concern about replacement existing among incumbent workers when faced with the possibility of two-tier wages (Bewley 1999, p. 146).

[31] The common law doctrine of at-will employment recognises "that where an employment was for an indefinite term, an employer may discharge an employee 'for good cause, for no cause, or even for cause morally wrong, without being thereby guilty of legal wrong'." (Wisconsin Supreme Court, 1983).

[32] A repeated-game reputation model is one way of justifying this environment. Suppose that there is no commitment; however workers can observe wages paid by firms, but *not* observe hiring and separations. Firms can offer cohort dependent *implicit* contracts: wage contracts which are enforced by bad continuation equilibria in which workers expect the deviant firm to pay low wages in the future. A firm may have a reputation for sticking to an equal treatment implicit wage contract; if it deviates either by cheating on its incumbents or by offering new hires a lower wage, this is observed by all workers, who punish it by believing it will henceforth only pay spot wages. Our arguments (which extend to an infinite horizon version straightforwardly) show that the equal treatment contract solution derived earlier is an equilibrium of this game which maximizes profits across all equilibria, provided the reputation forces are strong enough to sustain it (discount factors sufficiently close to 1). This follows as it is the best commitment policy subject to at-will employment. Again, if equal treatment is not the wage policy, then firms will have an incentive to replace more expensive cohorts and given that this is not observable, no punishment will ensue.

The following argument can be extended to the general case[33] but we consider for simplicity the example of Section 2.1. We claim that the solution presented there remains a solution under these new assumptions: firms would prefer to commit to equal treatment if they can do so. To see this, suppose that all other firms commit and follow the equilibrium as earlier. Consider a potential deviant firm, which chooses not to commit to equal treatment: at the start of period 2 it can hire at a different wage from that promised to incumbents. It will hire in the bad state at the workers' reservation utility χ_2, paying less than other firms. If the wage contract it offered to the period 1 hires pays $\underline{w}_2 > u^{-1}(\chi_2)$ the firm will replace these workers. (Likewise if $\underline{w}_2 < u^{-1}(\chi_2)$ they would quit.) Given that the period 1 hires will anticipate this, it then follows that the firm could offer them an equally attractive contract that gives them a wage in the bad state of $u^{-1}(\chi_2)$, and it will have no incentive to replace them. Thus this is a contract with equal treatment given that the firm will still hire in new workers at $u^{-1}(\chi_2)$ (and a similar argument can be used in the good state), which offers as much profit to the firm. However we know that there is a more profitable equal treatment contract, one which offers some insurance in the bad state, and so the firm is (strictly) better off committing ex ante to equal treatment.[34]

A related argument has been made in the insider-outsider context by, amongst others, Gottfries (1992).[35] In that paper, outsiders have reservation wages below any wage that insiders might receive even in "good" states of the world, and wages are kept constant in the face of rising demand to prevent too much surplus leaking to outsiders.[36] Likewise, Moore (2007) shows that if it is necessary to retain at least one worker to train the new employees, then there is a unique von Neumann-Morgenstern stable set consisting of configurations in which all workers receive the same wage.

[33] Details are available on request.

[34] A similar argument can be made even when the firm can commit in advance to future cohorts' contracts: it can do no worse than commit to future contracts that are the same as the first cohort's contract.

[35] Gottfries and Sjostrom (2000) allow the firm to fix a termination payment for workers, payable irrespective of who initiates termination, which in principle should allow outsiders to be brought in at lower pay without creating incentives for replacement of insiders; on the other hand it creates a moral hazard problem: insiders have an increased incentive to leave and turnover may increase. It is shown that the turnover effect may stop the firm from offering termination payments. A similar argument can be made here if we allow for on-the-job search (which does not, *per se,* affect our equilibrium) so that termination payments may induce incumbents who find a job elsewhere to leave, and if we introduce sufficiently convex turnover costs.

[36] See also Carruth and Oswald (1987).

Survey evidence appears to suggest that internal equity is important, although explanations centre more on maintaining workforce morale than the fear of replacement. Bewley (1999) suggests that violations of equal treatment are unusual, particularly in the primary sector, because morale and hence productivity would suffer. Similar findings exist for other countries: "Managers responded that hiring underbidders would violate their internal wage policy" (Agell and Lundborg (2003, p.7), based on a Swedish survey); in a British survey, Kaufman (1984) reported that almost all managers viewed bringing in similarly qualified workers at lower wage rates as "infeasible." Akerlof and Yellen (1990) argue that personnel management texts treat the need for equitable pay as virtually self-evident.

There is little direct empirical evidence on the issue of equal treatment. A notable exception is Baker, Gibbs, and Holmstrom (1994), who examined the pay of managerial employees in a single firm over time. They found that incumbents' pay tends to move together, but the pay of entrants is significantly more variable, which suggests that the pay of new hires may be more subject to outside conditions than that of incumbents.[37] A recent study of Swedish pay by Kwon and Milgrom (2005) finds that if cohort effects for labor market entry and occupation entry are included in addition to firm entry cohort effects, the former two are procyclical in line with expectation, while the latter actually appear *counter*cyclical. Thus a worker entering the labor market in a downturn will tend to do worse than those already active, but entering a firm in a downturn does not of itself lead to a lower wage than that received by incumbents; in fact the opposite appears to be the case (which Kwon and Myersson suspect is due to unobserved heterogeneity). In another recent paper, Haefke, Sonntag, and van Rens (2007) look at the wage cyclicality of new hires from entrants to the labor market and the unemployed, and find a significant degree of procyclicality relative to that of the continuously employed. This would certainly appear to go against our symmetric single sector model, although it should be noted that they are not comparing the wage cyclicality of new hires with that of those already in employment *within the same firm* (or sector).

[37] Wachter and Bender (2007) run a similar analysis on an entire German manufacturing sector for which they can track workers between firms. They find substantial variability in firm-entry cohort effects across time (in line with the findings of Baker, Gibbs, and Holmstrom (1994)), but also *across firms*. Market wide conditions do not appear to be responsible for this. This suggests that the findings of Baker, Gibbs, and Holmstrom (1994) may not reflect business cycle effects.

There are, of course, ways in which firms may be able to side-step equal treatment norms or rules, for example by using bonuses, varying seniority pay relativities according to hiring conditions, etc. However, the approach of the paper will still go through, albeit in an attenuated fashion, to the extent that equal treatment cannot be completely side-stepped.[38]

3.3 Worker Commitment

We assumed that workers are not committed to contracts, and hence it is the ex post mobility of workers which drives the wage dynamics. Suppose we drop the assumption that workers can costlessly quit the firm, for example by assuming that there is a mobility cost suffered if a worker changes jobs. Because of equal treatment, very little changes. If there is a symmetric equilibrium with mobility costs in which firms hire every period, then it must be identical to a symmetric hiring equilibrium with ex post mobility since essentially the same participation constraint needs to be satisfied each period. To see this, recall from Section 2 that the hiring constraint is $V_t \geq \underline{\chi}_t$. But if the continuation contract offers enough to hire a new worker in full employment states, then it will also offer enough to prevent a worker from leaving in such states because $\underline{\chi}_t = \chi_t$. In unemployment states, where hiring a worker is cheaper than retaining an incumbent, then $\underline{\chi}_t < \chi_t$,under our assumptions. But in such states in a symmetric equilibrium the participation constraint does not bind so this does not affect the optimum.

However the converse may not be true: an equilibrium with fully mobile workers may not be one with mobility costs. It may pay firms to choose *not to hire* in some periods (to avoid increases in wages) and to let $V_t(h_t)$ fall below $\underline{\chi}_t$. In the mobility case a firm doing this would lose its incumbent workers too, something by assumption it wants to avoid. The two cases *will* coincide if however we additionally assumed that a firm must always hire some workers to replace separated workers; this could be justified if there are 'key' workers who cannot be replaced by reallocating incumbents and new workers must be hired and

[38] Suppose that new hires can be brought in at a wage that can be below that of incumbents by some maximum amount. A very similar argument to that given in Lemma 1 still applies to the incumbent wage, but the fact that the new hire wage can be lower than the incumbent wage in low productivity states will attenuate employment variability.

trained in these jobs. With this assumption the participation constraint $V_t \geq \underline{\chi}_t = \chi_t$ must be satisfied in each full employment state, which is sufficient to guarantee identical solutions.

3.4 Incorporating Layoffs

So far the model has been solved assuming that the environment is such that layoffs are never desirable. If shocks are sufficiently adverse to make layoffs optimal, the simple equilibrium characterization derived above no longer holds. We discuss how our results are modified in the context of the example of Section 2.1 by including a sufficiently adverse shock that the demand for labor falls at a faster rate than the exogenous separation rate. We show that the nature of the solution when we allow for layoffs does not change too much.

We suppose that the firm does not commit to its layoff strategy (see the discussion of the termination assumption in Section 3.2), and that workers are unable to observe the number of hires in their firm. (This latter assumption implies that the firm does not worry about externalities from period 1 hiring on the beliefs of workers about period 2 layoff probabilities.) We also rule out layoff pay for simplicity. Period 1 hires will factor in the optimal layoff behavior of the firm in period 2, given the wage contract (which, as before, is committed to). Suppose that a new state in period 2 is introduced, with productivity $a_2 = 0.7$. The period 1 participation constraint previously evaluated period 2 utility in each state as $(\delta u(w_2) + (1-\delta)\,\chi_2)$; in the layoff state this must now be replaced by $((0.7/\widehat{w}_2)\,/\,(1/w_1))\,u\,(\widehat{w}_2) + (1 - (0.7/\widehat{w}_2)\,/\,(1/w_1))\,\chi_2$ where the contract wage in the new state is denoted by $\widehat{w}_2$, and the term $(0.7/\widehat{w}_2)\,/\,(1/w_1)$—the ratio of employment in the two periods[39]—is the probability of a worker not being separated or laid off; moreover $\chi_2 = u\,(\underline{c})$ since there is no prospect of an unattached worker getting a job. If this new state replaces the previous bad state and occurs with probability 0.5 as before, the new equilibrium has $w_1 = 1$ (the spot solution), $\overline{w}_2 = 1.1$ (the spot solution as

[39] With unobservability and no layoff pay, employment satisfies equality between wage and marginal product in period 1 as there is no future cost to varying employment today, and the same is true in the layoffs state in period 2 due to no layoff pay.

before), $\widehat{w}_2 = 0.91$, and employment in the new state $\widehat{N} = 0.77$, so around 12% of the *non-exogenously-separated* workers are laid off in the worst state. The ratio of unemployment to the fall in productivity from period 1 in the very bad state is slightly higher than the original example, which suggests that the quantitative employment response to shocks may not be dissimilar in a model with layoffs.[40]

Some of the previous results are still valid (in general, not just in the example); the equality between wage and marginal product still holds as already noted;[41] similarly, whenever there is full employment (and hence when the participation constraint binds) the wage is equal to the spot wage. However, the Euler condition previously derived for future states $(t+1)$ in which the participation constraint fails to bind but layoffs do not occur (here, the original bad state), no longer holds if there are layoffs in other states at $t+1$. In the two-period example we find wages falling more rapidly. The reason is that frontloading wages (relative to the Euler equation solution) has a benefit: by making the current (at t) wage higher and so cutting current employment, the layoff probability in layoff states at $t+1$ is reduced, thus relaxing the current and earlier participation constraints.[42] In layoff states at $t+1$, the benefit from frontloading is even greater because a lower $t+1$ wage increases employment and thus further reduces the layoff probability. This implies that the evolution of wages depends not only on realized shocks as before, but also on the distribution of future shocks, making simulation computationally more difficult.[43]

[40] Extending this example to more periods gives similar results.

[41] If employment levels were observable to workers, then this would not be true. If a firm is going to layoff workers in some state in the following period, the cost to hiring an extra worker today exceeds the wage as there is an externality on the participation constraint. In the two-period example, hiring an extra worker at the start reduces the ex ante utility of the contract as it increases the likelihood of layoff (period 2 optimal employment levels remain unchanged). Using this alternative assumption in the example only marginally affects the equilibrium, however.

[42] We can see this in the example if we have the original bad state and the new one both occurring with probability one third. Then $\underline{w}_2 = 0.93$ (< 0.966, the previous solution). Layoffs as a fraction of total separations are positive in the worst state, implying that this fraction is countercyclical, as is the rate of separation.

[43] If the firm *can* commit to the layoff rate (or equivalently to its employment plan), things are reversed: we find that the Euler equation (12) always holds, but hiring may not be on the labor demand curve if there are either current layoffs or the possibility of layoffs next period. Intuitively, the ability to commit to employment implies that wage changes do not have indirect effects on participation constraints through changing layoff probabilities, and so the argument underlying Lemma 1 goes through. To be precise, when layoffs occur the Euler equation holds provided we use the actual retention rate—taking into account layoffs—instead of δ. This implies the contract would offer optimal wage smoothing to those who were retained (i.e., constant wages if discount rates are equal). However employment would no longer be on the labor demand curve, there now being a positive externality of higher employment on previous participation constraints.

3.5 Firm Specific Shocks

So far it has been assumed that firms are subject to a common shock. This allowed for a simple solution to be derived, but the base model can be generalized straightforwardly to the case of firm specific shocks. This seems particularly important given that idiosyncratic shocks at plant level appear to be large relative to even sectoral shocks (Davis, Haltiwanger, and Schuh 1996). In terms of how employment responds to negative shocks (critical in terms of the unemployment predictions), we show that in the two period model idiosyncratic shocks have only a second-order effect on the aggregate prediction.

Suppose that the specification is the same as in Section 2.3 except that firm j now receives productivity a_t^j at time t. For a firm that is always hiring, Lemmas 1 and 2 continue to hold (they depend only on problem A, i.e., the firm optimizing, taking χ_t as parametric). Thus (17) still determines the target wage, i.e.,

$$w_{t+1}^j / w_t^j = \xi \left(a_{t+1}^j / a_t^j \right), \tag{23}$$

whenever the participation constraint for a firm does not bind at $t+1$. We also have $N_t^j = \left(a_t^j\right)^{\frac{1}{\alpha}} \left(w_t^j\right)^{-\frac{1}{\alpha}}$ from (16). However we lose the simplicity of the symmetric case in that a simple comparison between the target and spot wages to determine whether the participation constraint binds, is no longer always possible; moreover, when the participation constraint binds for a firm, we can no longer assert that the wage must be equal to the spot wage.

We can illustrate what happens in the two period example considered in Section 2.1. First consider perturbing the example slightly by adding a small (multiplicative) equiprobable idiosyncratic shock to period 2 productivity, 0.99 or 1.01. Little changes, and in fact the earlier principles still hold good. As firms are all identical in period 1 the equilibrium wage remains as before at $w_1 = 1$ for all firms. In period 2, in the good aggregate state (where half of all firms receive a productivity of 0.99×1.1, the others of 1.01×1.1), the wage remains at 1.1. The reason is that the target wages for both types of firm will be below the spot wage and both constraints will bind. As this is the final period this implies both wages must be equal (since the constraint is the same for both) at the

spot wage (which happens to be as before). In the bad aggregate state, as the constraint will not bind for either, both types will set their wages equal to target levels (0.976 and 0.957, using (23) and $w_1^j = 1$) and unemployment remains approximately the same (see below).

Suppose that the size of the idiosyncratic shock is increased. Does this affect the relationship between bad aggregate shocks and unemployment? The answer is that in the bad aggregate state unemployment will be smaller, but it is a second-order effect. To see this, if the idiosyncratic shock is now $1 \pm \varepsilon$, then provided neither constraint binds, from (23) and (16), employment equals $(\underline{a}_2)^2 \left(1 + \varepsilon^2\right) / \delta$. Thus for idiosyncratic shocks of $\pm 10\%$, employment will rise by 1%, and unemployment would fall from 7% to 6%, relative to the original example.

However, for larger idiosyncratic shocks/different aggregate shocks a non-degenerate wage distribution can occur with full employment. Suppose for example that the bad aggregate shock is such that in the absence of idiosyncratic shocks, the participation constraint only just binds in equilibrium (so target and spot wages are equal). This requires $\underline{a}_2 = \delta^{0.5}$. Now with idiosyncratic shocks of $1 \pm \varepsilon$, the target wages lie either side of the spot wage, and there is excess demand for labor (of $(\underline{a}_2)^2 \varepsilon^2 / \delta$ from above). To restore equilibrium, the lower wage will rise until the excess demand is choked off, yielding full employment but asymmetric wages, with the higher wage firms being unconstrained.[44]

4 Unemployment Simulations

In this section we simulate the model using sectoral productivity series from the post-war US economy to see how the model performs. Obviously the model is very stripped down,

[44] With the possibility of asymmetric configurations, we have to refine our definition of the outside option. If we continue with the setting of the previous section in which workers commit, it is the ability to hire which is the relevant constraint. So when there is unemployment, it is natural to assume in this competitive environment that a firm only has to match the utility from being unemployed ($\underline{\chi}_2 = U_2$). If there is full employment but non-uniform wages, then the utility of the lowest wage at which firms are hiring would be the natural value for $\underline{\chi}_2$. This specification is being implicitly followed in the analysis of the example. The case of costless worker mobility would preclude asymmetric full employment configurations as all workers in lowest paying firms would quit in the expectation of getting a higher wage. In the case considered in the example, the lower wage would have to rise until the risk of unemployment was sufficient to deter this from happening.

and we consider this more of an indicative exercise than an attempt at a rigorous empirical analysis.

In the one sector model studied above, unemployment falls to zero whenever recent productivity shocks are not too unfavorable. Using a multisector model in which each sector is subject to idiosyncratic productivity shocks we will obtain more realistic unemployment levels because it is less likely that all labor markets will simultaneously clear; moreover when the average productivity shock is positive, there will tend to be more sectors with low unemployment and consequently aggregate employment is likely to be lower. Naturally this exercise depends on how well correlated the sectoral shocks are.

We use U.S. manufacturing industry multifactor productivity processes for 17 sectors plus a residual non manufacturing sector, as provided by the Bureau for Labor Statistics (BLS), and then aggregate the model's predictions made for each of these sectors.[45] This fixes the variability of shocks and their correlation across sectors, and also allows us to generate a simulated unemployment series which can be directly compared to the data. We make the extreme assumption that each sector is otherwise independent, so that the sectoral labor markets are completely segmented.[46] As we shall see, even though the model is very lightly parametrized (two degrees of freedom for wages and three for unemployment), feeding it these sectoral shocks can lead to unemployment predictions that correspond reasonably well to the data.

As Proposition 1 makes clear, given knowledge of the model's parameters, given an initial time period where there was full employment and given a TFP series it is possible

[45] We use TFP rather than labor productivity series. See Chang and Hong (2006) for reasons why using the latter for this kind of exercise is problematic.

[46] We use this data as it is the only sectoral TFP series available for such a long time scale and collected on a consistent basis; TFP data for other broad sectors such as services are only available from the early 1970s onwards. It is also extreme to assume that these sectors map exactly into genuinely distinct and separate labor markets. Suppose however that this level of disaggregation is too fine and there is mobility of labor between sector A and sector B. Then this corresponds to the case analyzed in Section 3.5. The analysis there suggests that any resulting bias might not be large provided shocks across these sectors are not too asymmetric. Specifically, if the participation constraint is not binding in either sector, we saw that this leads only to a second-order error in the two-period model in terms of the predicted unemployment rate. The error would be larger if the two sectors have sufficiently different shocks that the constraint binds in one sector but not in the other. Then because the binding constraint censors the impact of the more positive shock on employment, our model would predict lower employment than the aggregate shock in the combined sector would warrant.

to generate the sectoral "real wage" series that would be predicted by our theory.[47] Recall that we are able to solve the model on this basis because of the convenient property that the solution depends only on actual realizations of the random processes, and not on their distributions. It is then possible to derive the corresponding implications for unemployment rates. An example of this exercise for one of the sectors was discussed in Section 2.3. The wage solution depends only on two composite parameters, $\alpha\gamma$ and $\lambda^{\frac{\alpha}{\alpha\gamma-1}}$. Thus varying α and γ but keeping their product constant does not affect the solution for wages provided λ is also varied to keep $\lambda^{\frac{\alpha}{\alpha\gamma-1}}$ constant. The unemployment series will vary with $-1/\alpha$ however. The latter measures the elasticity of labor demand and this determines the extent to which a wage greater than the spot wage ($w_t > a_t$) translates into the level of unemployment. Thus a lower value for α will magnify fluctuations in sectoral unemployment, if the wage solution is constant.

Treating each sector as a separate economy of fixed size, and given values for α, γ and λ, individual predicted wage series were generated for each of the 17 two digit manufacturing sectors for which TFP data are available from the BLS and for the residual sector (whose TFP is constructed as the weighted difference of total nonfarm business TFP and manufacturing TFP, all in logs).[48] An aggregate unemployment index was then constructed as the weighted average of the individual sector simulations with weights given by employment shares.[49,50]

We show a number of calibrations in Figure 2, with values for α lying between 0.7 and 1.4 (labor demand elasticities between -1.4 and -0.7) and γ between 3 and 10. We also report the correlation coefficient, ρ, between simulated and actual unemployment. Standard estimates of long-run elasticities of labor demand can be larger in absolute size

[47] To ensure that the no layoff condition (19) is always satisfied, we note that δ, β_w and β_f only affect the equilibrium wage and employment series through the composite parameter λ, but (19) can be made to hold for given λ by reducing δ and varying the ratio of β_w to β_f (i.e., for any parameterization there exists an equivalent parameterization in terms of the implied solution such that the condition holds).

[48] We assume full employment in each sector in 1949 in order to fix initial values, and discard the first 6 years.

[49] We added 4% to our simulated series to allow for a constant level of frictional unemployment.

[50] The (fixed) employment weights were taken from the middle year of the sample and the manufacturing sector as a whole was assumed to be 50% larger than the residual sector - roughly consistent with the average relative actual sizes over the period. Repeating the analysis with weights varying over time produces similar results; formally this would however require an extension of the model to allow for variable sectoral employment pools (although this extension is relatively straightforward).

than -2 while short-run elasticity estimates are frequently estimated to be as close to zero as -0.3 (Hamermesh 1993).[51] The values for γ are on the high side for risk-aversion, although it is intertemporal consumption smoothing that governs the evolution of wages, and here elasticities of substitution (the inverse of γ) are often estimated to be very low (e.g., Hall (1988)). The first two panels show that for a value of λ of 0.85, and $\alpha = 0.7$, wages fall sufficiently quickly in response to negative productivity shocks that there is little unemployment except when shocks are particularly adverse in the early 1980s, unless γ is set high. The next two panels show that a somewhat higher value of λ combined with a lower value for γ yield unemployment fluctuations that correspond better with reality. In particular the volatility of actual unemployment is reasonably well matched, as are the peaks and troughs of the actual series through to the late 1980s.[52] In the final two panels λ is set close to 1. The fifth shows the parameterization that in fact minimizes the average absolute value between the two series (here, frictional unemployment is set to 3.8%)[53]. In the final panel there is insufficient downward movement of wages towards the latter part of the period for simulated unemployment to return closer to observed levels. To summarize: to get reasonable simulations it is necessary to avoid wages falling too fast, which requires that γ and λ are not simultaneously too small.[54]

Recall that if $\beta_f = \beta_w$, then $\delta = \lambda$. A value of δ of 0.95, for example, then corresponds to a separation rate of 5%, far lower than that observed from annual data which is typically around 30% or more. However, for a number of reasons $1 - \delta$ cannot be read off from the average empirical labor turnover rate. Recall that in Section 3.1 we established an equilibrium such that the wage equation depends in the long run on the staying probability

[51] Estevão and Wilson (1998), analyzing BLS manufacturing data for a similar period that we study, found a short-run demand elasticity ranging between close to zero and -0.71 with aggregate data, and of between -0.5 and -0.89 at the 4-digit industry level for manufacturing

[52] The spikes in unemployment in the mid 1970s and early 1980s involved substantial layoffs (see, e.g., Davis (2005)), something we have abstracted from. However, the analysis of Section 3.4 suggests that an extension of the model to include layoffs may lead to broadly similar predictions, with the layoffs playing a larger role in downturns.

[53] With λ varying between 0.8 and 1.0, α between 0.6 and 3.0, and γ between 1.0 and 10.0.

[54] We also find that wage falls are of an empirically reasonable size. Across the 18 sectors, for $\alpha = 0.7, \gamma = 5$ and $\delta = 0.95$, the most negative wage changes lie between -2% and -3% in half of the sectors, with the range being between -0.7% and -5.1% (mean of –2.45%). Elsby (2009) charts the distribution of real wage changes in the PSID over a relatively low inflation period (so surprise inflation is less likely to lead to unanticipated real wage falls), 1983-1992; real wage falls rarely exceed about 6%, with a spike around 2-4%. Likewise Christophides and Stengos (2003) find from Canadian wage contract data in the unionized sector that most real wage reductions in the 1990s were of the order of 1-2%.

of the most experienced workers, which is far lower than average turnover, and a value for δ above 0.9 would be more reasonable (see Section 3.1). Secondly, for tractability reasons, we have constructed a model with no layoffs in equilibrium; layoffs however enter the empirical determination of turnover. Thus δ is a larger number than the average staying rate which includes layoffs. Suppose for example that a sector in our analysis is in fact made up of two independent subsectors, one of which receives a large negative shock and lays off workers and the other a smaller negative shock which does not entail layoffs, where both started from full employment. If the ratio of the employment response to the shock is similar in both sectors, as was true in the example with layoffs considered in Section 3.4, then although the aggregate employment response may be similar to that in the model without layoffs, the empirical turnover rate will be greater than $1 - \delta$. Thirdly, we abstracted from firm turnover; if this was included as an exogenous death rate, λ would measure the *ratio* of the worker survival rate to the firm survival rate.[55]

Finally, of course, it may be that wage rigidity of incumbents is due to factors other than risk-aversion. Even if this were true, the impact of equal-treatment may be analyzed in much the same way as we have done here (in particular, ongoing research suggests that the property that wages are downward but not upward rigid follows in a wider class of equal-treatment models with outside option constraints).

5 Closing Comments

This paper has analyzed a model in which firms cannot pay discriminate based on year of entry to a firm. The trading-off of wage insurance for incumbents against the desire to be flexible in the hiring wage paid to new hires leads to wages which do not always clear the labor market and have a degree of downward rigidity. On the other hand, the need to hire means that wages have to respond to sufficiently positive shocks, so that wages in the long-run respond to productivity movements. Even though fluctuations in the model are wholly driven by labor demand variations, we find that these two features imply that the model gives a reasonable account of unemployment fluctuations in recent US history.

[55] Although the model would also have to be modified to allow the introduction of new firms.

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5.1 Appendix: Proof of Proposition 1

(i) We have just shown that w^*_{t+1} must equal one of the arguments of the max operator, depending on whether or not the participation constraint binds at $t+1$. Suppose first that $\xi\left(\frac{a_{t+1}}{a_t}\right)w^*_t > a_{t+1}$, which given $\alpha\gamma > 1$, can be rewritten as $w^*_t > \left(a_t^{-1}a_{t+1}^{\alpha\gamma}\lambda^{-\alpha}\right)^{1/(\alpha\gamma-1)}$. Suppose that the participation constraint binds at $t+1$ (so $w^*_{t+1} = a_{t+1}$ and $N_{t+1} = 1$) contrary to assertion. Lemma 1 implies that $\frac{w^*_{t+1}}{w^*_t} \geq \left(\frac{\lambda N_t}{N_{t+1}}\right)^{\frac{1}{\gamma}}$ with equality unless the participation constraint binds at $t+1$. Thus $a_{t+1}/w^*_t \geq \left(\lambda a_t^{\frac{1}{\alpha}} w_t^{*-\frac{1}{\alpha}}/1\right)^{1/\gamma}$, or equivalently $w^*_t \leq \left(a_t^{-1}a_{t+1}^{\alpha\gamma}\lambda^{-\alpha}\right)^{1/(\alpha\gamma-1)}$. So we have a contradiction. Alternatively, suppose that $\xi\left(\frac{a_{t+1}}{a_t}\right)w^*_t < a_{t+1}$, and suppose that $w^*_{t+1} = \xi\left(\frac{a_{t+1}}{a_t}\right)w^*_t$. But this implies that labor demand exceeds unity, which is incompatible with equilibrium. Finally if $\xi\left(\frac{a_{t+1}}{a_t}\right)w^*_t = a_{t+1}$, then whether the participation constraint either binds or does not, w^*_{t+1} equals this common value. To show that $w^*_1 = a_1$, note that in an optimal contract the participation constraint binds at the initial date ($t = 1$): if it did not, the firm would increase profits by cutting $w_1(s_1)$ holding the remainder of the contract fixed, and would still satisfy all participation constraints. Thus by Lemma 3 $N^*_t(h_t) = 1$, so $w^*_1 = a_1$.

(ii) Using the putative solution, the condition for hiring to occur at $t+1$ is

$$N^*_{t+1} = a_{t+1}^{\frac{1}{\alpha}} w_{t+1}^{*-\frac{1}{\alpha}} > \delta N^*_t = \delta a_t^{\frac{1}{\alpha}} w_t^{*-\frac{1}{\alpha}}. \tag{24}$$

If firms are constrained at $t+1$ then $N = 1$ and hiring is positive; if they are not, then (17) holds, and after simplification the condition becomes (19), hence provided this is satisfied on all positive probability paths (24) is satisfied.

We next consider a relaxed version of the problem faced by a potential deviant firm (i.e., where $(\chi_t)_{t=1}^T$ is fixed at the putative equilibrium levels) and show that the firm cannot improve on the putative equilibrium contract and hence the latter is optimal also in Problem A. The relaxed problem is as before but the firm now has no employment constraints, so that it solves Problem A without the constraint (6) (that is, it can costlessly reduce its workforce at any time, and only has to respect the participation constraints, which do not take into account layoffs, this despite the fact that a worker in calculating his utility from the contract should take into account the layoff possibility). We call this Problem A^R.

Consider the static problem of maximizing profits given that workers receive utility u, so that $w = \left((1-\gamma)\,u\right)^{1/(1-\gamma)}$. Substituting from (16) for N (this must hold in the

static problem), yields profits of

$$\Pi\left(u, a_t\right) \equiv M_t + \frac{a_t^{\frac{1}{\alpha}} \alpha \left(\left(1-\gamma\right) u\right)^{-\frac{1-\alpha}{\alpha(1-\gamma)}}}{1-\alpha}. \tag{25}$$

As $\alpha\gamma > 1$, this is a strictly concave function of u. We can formulate Problem A^R faced by the firm as:

$$\max_{(u_t(h_t))_{t=1}^T} E\left[\sum_{t=1}^{T} (\beta_f)^{t-1} \Pi\left(u_t(h_t), a_t\right)\right] \tag{Problem A^R}$$

$$\text{subject to} \qquad \tilde{V}_t\left(h_t\right) \geq \chi\left(h_t\right) \tag{26}$$

for all positive probability $h_t, T \geq t \geq 1$, where

$$\begin{aligned} \tilde{V}_t\left(h_t\right) &= u_t(h_t) + \\ & E\left[\sum_{t'=t+1}^{T} (\beta_w)^{t'-t} \left[\delta^{t'-t} u_t(h_{t'}) + \delta^{t'-t-1}\left(1-\delta\right)\chi_{t'}\right] \mid h_t\right]. \end{aligned} \tag{27}$$

Thus the maximand is strictly concave and the constraints are linear. The Slater condition is satisfied by, for all h_t, $u_t(h_t) = u(w^*(h_t)+\varepsilon)$, for $\varepsilon > 0$. Moreover it is straightforward to show that the Kuhn-Tucker conditions are satisfied at the putative equilibrium contract, hence the necessary conditions given by (18) are *sufficient* for existence in the relaxed problem A^R. Thus provided (19) holds, our putative solution solves Problem A^R and satisfies positive hiring, and so this must also be solution to Problem A.

(iii) Consider the problem in which layoffs are permitted; call it Problem B (for brevity's sake we omit its statement). Consider now a feasible plan in Problem B which involves layoffs occurring. Suppose we implement the same wage (i.e., utility) plan in Problem A^R; (26) must hold given that any cohort facing a layoff probability in Problem B will get weakly less continuation utility than $\tilde{V}_t$, since employment always offers weakly more utility than layoff by the participation constraint. Since, given w_t, and hence u_t, per-period profits are maximized in Problem A^R, the solution to the latter must weakly dominate the solution to Problem B. Putting this together, if (19) holds, our solution is also a solution to Problem B.

static problem), yields profits of

$$\Pi\left(u, a_t\right) \equiv M_t + \frac{a_t^{\frac{1}{\alpha}} \alpha \left(\left(1-\gamma\right) u\right)^{-\frac{1-\alpha}{\alpha(1-\gamma)}}}{1-\alpha}. \tag{25}$$

As $\alpha\gamma > 1$, this is a strictly concave function of u. We can formulate Problem A^R faced by the firm as:

$$\max_{(u_t(h_t))_{t=1}^T} E\left[\sum_{t=1}^{T} (\beta_f)^{t-1} \Pi\left(u_t(h_t), a_t\right)\right] \tag{Problem A^R}$$

$$\text{subject to} \qquad \tilde{V}_t\left(h_t\right) \geq \chi\left(h_t\right) \tag{26}$$

for all positive probability $h_t, T \geq t \geq 1$, where

$$\begin{aligned} \tilde{V}_t\left(h_t\right) &= u_t(h_t) + \\ & E\left[\sum_{t'=t+1}^{T} (\beta_w)^{t'-t} \left[\delta^{t'-t} u_t(h_{t'}) + \delta^{t'-t-1}\left(1-\delta\right)\chi_{t'}\right] \mid h_t\right]. \end{aligned} \tag{27}$$

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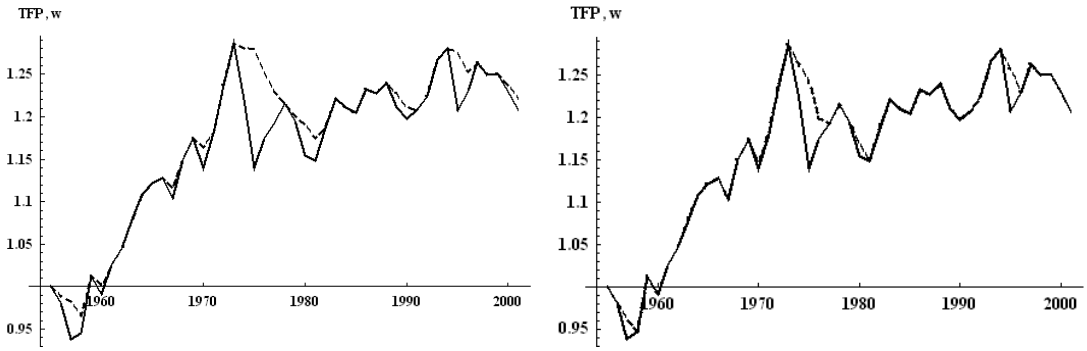
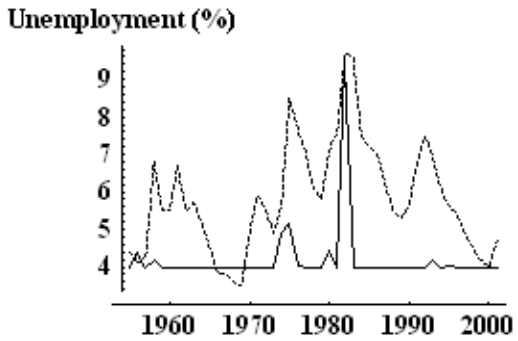
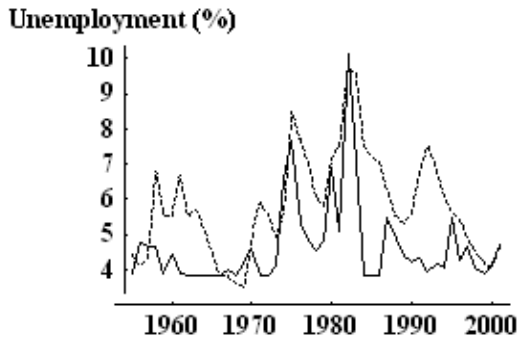


$\alpha = 0.9$, $\gamma = 6.6$, $\delta = 0.92$ $\alpha = 0.9$, $\gamma = 6.6$, $\delta = 0.85$

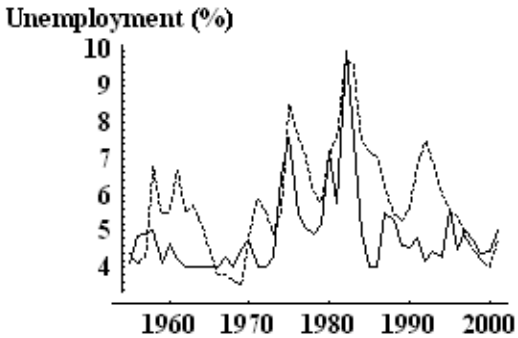
Figure 1: TFP (solid) and wage simulation (broken line)



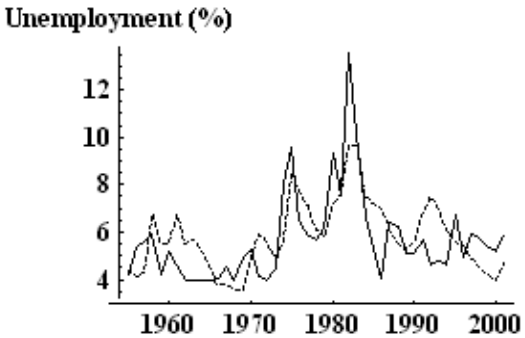
$\alpha = 0.7, \gamma = 3, \lambda = 0.85; \rho = 0.44$



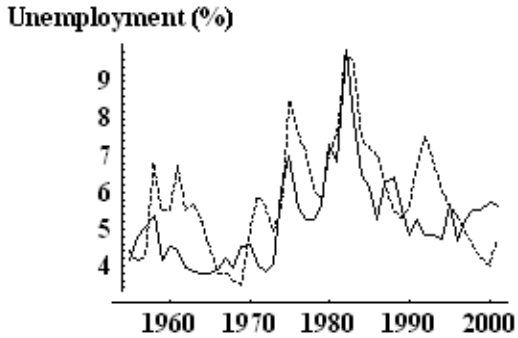
$\alpha = 0.7, \gamma = 10, \lambda = 0.85; \rho = 0.6$



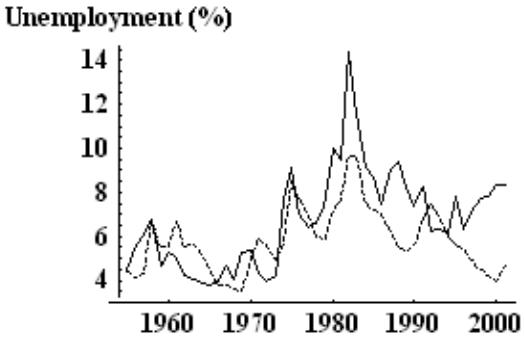
$\alpha = 0.9, \gamma = 6.6, \lambda = 0.92; \rho = 0.63$



$\alpha = 0.7, \gamma = 5, \lambda = 0.95; \rho = 0.66$



$\alpha = 1.4, \gamma = 3.4, \lambda = 0.984; \rho = 0.69$



$\alpha = 0.9, \gamma = 5, \lambda = 0.99; \rho = 0.64$

Figure 2. Unemployment simulations (solid line) and actual annual US rate (broken line), 1955-2001