Disinflation in a DSGE Perspective: Sacrifice Ratio or Welfare Gain Ratio? *

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Abstract

When used to examine disinflation monetary policies, the current workhorse dynamic stochastic general equilibrium model of business cycle fluctuations is able to quantitatively account for the main stylized facts in terms of recessionary effects and sacrifice ratio. We complement the transitional analysis of the short-run costs with a rigorous welfare evaluation and show that, despite the long-lasting economic downturn, disinflation entails non-zero overall welfare gains.

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1 Introduction

Disinflation is a long-standing issue in monetary economics. On the empirical side, there is ample evidence that disinflations generate short-run output losses. Indisputably, the key indicator to gauge the real costs of disinflation has been the sacrifice ratio, calculated as the ratio of the cumulative percentage output loss (i.e., the difference between actual and potential output) to the size of disinflation. Thus, the sacrifice ratio measures the real output cost per unit of permanent decrease in inflation. A host of empirical studies have estimated the costs of disinflation for various countries, using different econometric methods. In general, the findings vary greatly across countries, episodes or time periods and estimation methods. Gordon and King (1982) is an early assessment of the sacrifice ratio for the United States, based on the estimation of autoregressive Phillips curves (see, more recently, Andersen and Wascher, 1999). For euro-area countries, Cuñado and Gracia (2003) reports estimates of the sacrifice ratio ranging from 0.55 to 1.96. Ball (1994b) analyses specific disinflationary episodes in 19 moderate-inflation OECD countries between 1960 and 1991, and comes up with estimates of the sacrifice ratio ranging from 1.8 to 3.3 (see also Mankiw, 1999, and Zhang, 2005). Using the vector autoregression (VAR) methodology, Cecchetti and Rich (2001) find estimates of the sacrifice ratio between 1 and 10 for the United States, while Durand et al. (2007) studies twelve euro-area countries and reports substantially lower sacrifice ratios ranging from 0.23 to 0.75. In summary, among empirical studies there seems to be little disagreement on the following facts: (i) a disinflation generates a loss in output; (ii) the value of the sacrifice ratio varies across countries and time periods, but a plausible range is between 0.23 and 3.3.

On the theoretical side, however, there is a widespread view that the basic *linearized* New Keynesian DSGE model, as in Clarida, Gali and Gertler (1999), fails to replicate a costly disinflation. In a nutshell, because it is based on the Calvo (1983) price staggered mechanism, the basic New Keynesian DSGE model only delivers price stickiness but not inflation inertia. On the contrary, inflation is described as a forward-looking variable that can immediately adjust to a disinflation, without any output costs. Ball (1994a)

was among the first to point out this inconsistency of standard sticky price models, in which a disinflation could be followed by a boom rather than a slump (see also Burstein, 2006). In fact, in a subsequent paper, Ball (1995) introduces imperfect credibility as a necessary device to explain the observed output costs of a disinflationary policy. More recently, Erceg and Levin (2003) and Goodfriend and King (2005) introduce imperfect credibility in a standard New Keynesian model to explain the famous Volcker disinflation (see also Nicolae and Nolan, 2006). Mankiw (2001) also forcefully expresses the view that standard sticky price models cannot deliver inflation persistence and thus justify the costs of disinflation. Indeed, this drawback was one of the main reason that led Mankiw and Reis (2002) to propose a different model of price stickiness based on sticky information. The literature can then rationalize output costs of a disinflation by appealing to some form of imperfect credibility/information/rationality. It is however less conclusive on the size of the recession following a disinflation episodes.

The aim of this work is to give a quantitative assessment of the ability of the New Keynesian framework to match the stylized fact after a disinflation. In order to do that we need an operational model of business cycle fluctuations. In their seminal work, Christiano et al. (2005) (CEE, henceforth) show that a medium-scale New Keynesian model, enlarged to accommodate various nominal and real frictions, matched the business cycle fluctuations reasonably well. This model (or some slightly modified versions of it) has been widely and successfully employed both in empirical work (e.g., Smets and Wouters, 2003, Altig et al., 2004,) and in normative analysis (e.g., Schmitt-Grohé and Uribe, 2005).

Surprisingly, however, up to now no one has assessed the ability of the CEE model to quantitatively account for the costs of disinflation, and to address the issue of disinflation from a welfare perspective. This is what we do in this paper. We address two questions:

1. How successful is the current operational New Keynesian DSGE model of the business cycle at quantitatively replicating the empirical costs of disinflation and sacrifice ratio, without resorting to some form of imperfect credibility, imperfect information or irrationality in expectations?

2. How costly is a credible disinflation in terms of welfare?

Moreover, in oder to tiyng our hand as much as possible in aswering these questions, we deliberately restrain ourselves from changing any of the features of our reference model and the structural parameters values, as estimated or calibrated by CEE.[1]

The answer to the first question is: quite successful. The simulation of the model indicates that a credible disinflation leads to a prolonged decline in output, and that the value of the sacrifice ratio is in line with the available empirical evidence.

With regards to the second question, we work out a rigorous welfare evaluation of the costs of a disinflation, constructing a welfare-based sacrifice ratio. Interestingly, despite the prolonged slump in output, we show that *a disinflation implies welfare gains.* The size of these gains is very small: equal to a permanent increase in initial steady state consumption of 0.06-0.07% each period per each point of diminished inflation. More precisely, small long-run gains outweigh even smaller short-run costs. Surprisingly enough, the short run costs of a disinflation are negligible, despite the transitional economic downturn.

Finally, we want to raise a methodological consideration. In contrast with the standard practice in the literature of approximating the model structural equations, here we simulate numerically the original non-linear model. In our view this is crucial, because taking linear or log-linear approximations may rule out some important transmission mechanisms. Yun (2005), for instance, emphasizes the role of relative price dispersion, often neglected in linear models, in driving his results for optimal monetary policy. Also, money is non-superneutral in the CEE model. Ascari and Merkl (2007) shows that the use of log-linear approximations to study a disinflation may yield misleading results, since a disinflation implies a movement from one steady state to another one.

[1] A companion paper thoroughly analyzes how the different features of the CEE model, the parameter values and the monetary policy rule affect the costs of disinflation. For obvious length constraints, this kind of analysis is outside the scope of this paper.

2 An operational model of the business cycle

To study the effects of disinflationary monetary policy we rely on the operational medium-scale New Keynesian DSGE model developed in CEE and then used, among others, in Smets and Wouters (2003) and Schmitt-Grohé and Uribe (2005, 2007). In this section we discuss some key features of the model. A brief description of the structural equations and parameters calibration are given in the Appendix.

The model features both real and nominal frictions, which are deemed to be crucial to replicate the dynamic properties of the business cycle (see CEE for the United States and Smets and Wouters, 2003, for the euro area). Real frictions include: monopolistic competition in goods and labor markets, internal habit in consumption, variable capital utilization and adjustment costs in investment decisions. As for nominal frictions: prices and wages are sticky à la Calvo, with an indexation clause. In particular, in each period only a fraction of prices and wages are set optimally; prices and wages that cannot be reoptimized are automatically adjusted to keep up with the inflation rate recorded in previous period. Finally, money balances enter the model in two ways: households derive direct utility from holding real money balances (assumption of money-in-the-utility function) and entrepreneurs must hold nominal money balances to pay wages before production (assumption of cash-in-advance).

We depart from our reference models with regards to monetary policy. We assume the central bank sets the short-term nominal interest rate, i_t, according to the non-linear rule defined by

$$\frac{1+i_t}{1+i^*} = \left(\frac{1+\pi_t}{1+\pi^*}\right)^{\phi}, \text{ with } \phi > 1 \tag{1}$$

where π_t, π^* and i^* represent the inflation rate, the inflation target and the nominal interest rate target, respectively. Notice, from the standard consumption Euler equation, it must hold that $1 + i^* = (1 + \pi^*)/\beta$, where β is the representative household's subjective discount factor.

Two distinct features of (1) are worth stressing. First, our postulated nominal interest rate targeting rule does not respond to the output gap. The reason for this choice is the following. We think that a credible cold-turkey disinflation and countercyclical

monetary policy behavior cannot coexist: after a permanent reduction in the inflation target, any attempt to soften the output decline at the expense of higher inflation may call the monetary authority's credibility to curb inflation into questions. Second, our postulated nominal interest rate rule lacks an inertial term. Again, we think that the central bank's attitude should be history independent. Especially at the time the disinflation is implemented, the short-term nominal interest rate should be adjusted freely in the light of the new, lower inflation target. In sum, we envisage the disinflation period as a temporary pure inflation targeting regime, where the dominant concern for monetary policy is to bring down inflation.

Before analyzing the costs of disinflation, it is important to highlight two points. The first has to do with the deterministic steady state relationship between output and inflation. Although the degree of indexation in prices and wages is calibrated equal to one, money is non-superneutral. This is due to the cash-in-advance constraint on intermediate firms to pay wage bill. As illustrated in CEE, in this case the real marginal cost schedule depends on the nominal interest rate. Although this hypothesis is important to match the empirical impulse response functions and the overall short-run dynamics, it also affects the deterministic steady state. Even with full price and wage indexation, positive trend inflation yields real output cost. The higher the level of trend inflation, the higher the labor costs for firms, and, *ceteris paribus*, the lower the wage paid to workers. In response, households reduce their labor supply and employment falls. Firms in turn decrease their capital stock, because labor and capital are complements in the production function. Eventually, the level of output decreases. The long-run Phillips Curve is not vertical.[2] Given CEE calibration, these effects are rather minor: a permanent 1% reduction in inflation implies roughly a 0.1% increase in steady state output.[3]

[2]From an empirical point of view, it has been difficult to tackle this issue within the VAR literature as the Blanchard and Quah (1989) restriction, i.e. no long-run effects of aggregate demand shock on output, is typically used as an identifying restriction (see, for example, Cecchetti and Rich, 2001). However, when this restriction is not imposed, it does not follow automatically that output goes back exactly to its initial level (see Collard et al. 2006, Fève et al., 2007).

[3]It is important to stress that the assumption of full indexation of prices and wages rules out potential

The second point we want to draw attention to is methodological and concerns the solution of the model. We have just seen that in the CEE model money is non-superneutral. This means that changes in trend inflation affect the steady state level of output. In our view, then, whenever a policy experiment leads to a transition between two steady states, one should refrain from using standard solution methods based on local approximation. In these instances, it would be preferable and definitely more accurate to use non-linear solutions. And this is what we do in this paper. We simulate the perfect foresight transition path by numerically solving the non-linear model in DYNARE.[4]

3 The short-run effects of disinflation

In this section we study the short-run effects of disinflation in the non-linear operational New Keynesian DSGE model. As a preliminary step, we define the notion of disinflation in the context of our theoretical model. Before the disinflation, the economy is in a steady state characterized by a positive trend inflation π, which is pinned down by the inflation target π^*_{old}, i.e., $\pi = \pi^*_{\text{old}}$. At a certain point, say $t = 0$, the central bank unexpectedly, instantaneously and credibly reduces the inflation target from π^*_{old} to π^*_{new}, implementing what is commonly known as a cold-turkey disinflation. Agents acknowledge that the reduction of inflation target is permanent and do not expect any other policy surprise. Effectively, our disinflation experiment entails a transition between two steady states in a perfect foresight, non-linear model.

As regards the new inflation target we consider three cases: $\pi^*_{\text{new}} = \{0\%, 1\%, 2\%\}$. Disinflations aimed at achieving an inflation target of 1-2% are interesting for at least two reasons. Such targets come near to the actual inflation objectives at work in many

real effects arising from nominal rigidities. It is well-known that a positive steady state inflation rate increases steady state price and wage dispersion in the absence of full indexation, yielding an inefficiency loss on aggregate production (e.g., Ascari, 2004, Schmitt-Grohé and Uribe, 2005, Yun, 2005). In other words, with partial wage and/or price indexation the real effects of long-run inflation, and thus also the effects on welfare, would be much larger.

[4]For further details on DYNARE see the webpage: http://www.cepremap.cnrs.fr/dynare/.

central banks, e.g., the Reserve Bank of New Zealand, the Bank of Canada, the Bank of England and the European Central Bank.[5] Furthermore, an inflation target of 2% is not far from the recent estimates of US Federal Reserve's implicit inflation target.[6] Instead, the reason for studying cold-turkey disinflations aimed at achieving full price stability, i.e., $\pi^*_{\text{new}} = 0$, is more theory-based, as the recent literature on optimal monetary policy has thoroughly explained the reasons why full price stability is socially desirable (see, e.g., Woodford, 2003). Finally, we present results both for $\phi = 1.5$ and $\phi = 3$.

Figures 1 illustrates the dynamic adjustment of output, inflation, and nominal and real interest rates after cold-turkey disinflation aimed at achieving $\pi^*_{\text{new}} = 2\%$, when $\phi = 1.5$. Each panel reports transition paths starting off from different initial values of trend inflation, namely $\pi^*_{\text{old}} = \{3\%, 4\%, 5\%\}$.[7] In the non-linear CEE operational model, cold-turkey disinflations come with a sizable recession; the rate of inflation is highly persistent and gradually decreases towards the new target. Nominal and real interest rates increase on impact and then slowly revert to steady state.

When the central bank permanently reduces the inflation target, only a fraction of intermediate firms set optimal prices, because of the Calvo staggered adjustment mechanism.[8] Discounting the forthcoming decline of output, necessary to bring down inflation, optimizing firms lower their prices. The remaining firms, which instead are not allowed to optimize, simply index their unchanged prices to the previous period's

[5] Both in New Zealand and Canada the numerical inflation target extends from 1 to 3%. In the United Kingdom the explicit inflation objective is currently 2.5%, while in the euro area the European Central Bank has an inflation objective below, but close to, 2%.

[6] Leigh (2008) finds that in the period 1990-2004 the Federal Reserve's implicit inflation target varied in the range of 1-3%.

[7] Note that we focus on disinflation of relative small size. There are two main reasons. First, since the model features time-dependent pricing, the analysis is limited to just a moderate change in trend inflation. It would not be realistic to apply such a model to sizeable disinflations, in which case the average frequency of price changes can not be assumed constant. Second, we want to assess the quantitative prediction of this model to a disinflation relevant for monetary policy in the recent period. Just before the current economic crisis, the monetary policy problem in the EU was to realign inflation to the target, after the surge in oil, energy and food prices.

[8] Clearly, the same reasoning also applies to wage setters' behavior. Here, however, we primarily comment on intermediate firms' behavior and inflation dynamics.

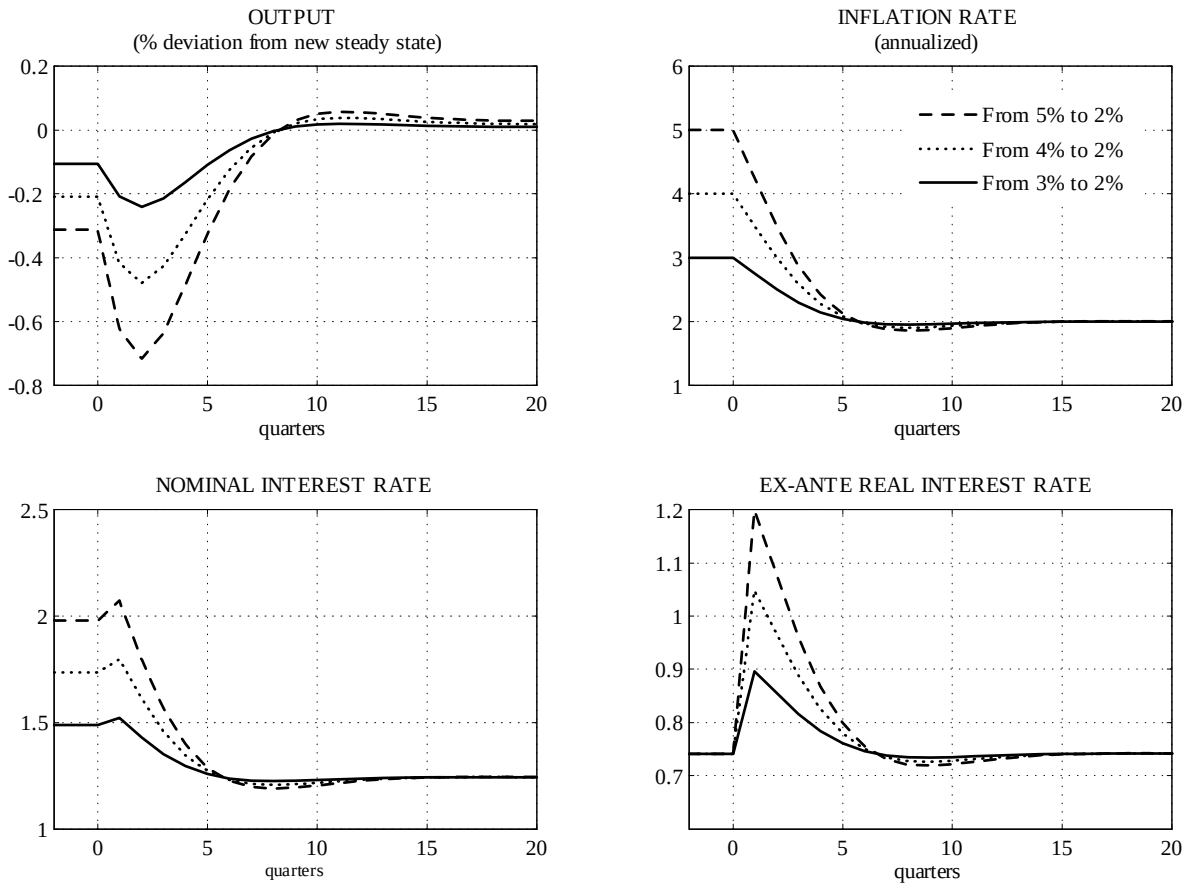

Figure 1: Cold-turkey disinflations aimed at achieving $\pi^*_{\text{new}} = 2\%$ with $\phi = 1.5$.

inflation rate. As a matter of fact, they increase their prices by $1 + \pi^*_{\text{old}}$. As shown in Figure 1, of these two conflicting pricing decisions the latter prevails. The aggregate price index continues to rise but at a slower rate. Thus, inflation rate decelerates.

As inflation does not immediately adjust to the new target, the central bank responds to the positive inflation gap $(\pi_1 - \pi^*_{\text{new}})$ with a monetary policy contraction. The central bank temporarily increases the policy rate, even though disinflation implies a lower steady state nominal interest rate. The follow-on rise of real interest rate reduce the aggregate demand: households postpone consumption and decrease investment spending. Furthermore, higher nominal interest rate increases intermediate firms' costs via the cash-in-advance constraint. Real wage drops, households supply less labor and

intermediate firms reduce the rate of capital utilization. Taken as a whole, the level of output falls. In successive periods, the inflation rate continues to adjust towards the new lower target, while the central bank starts cutting the nominal interest rate. Nonetheless, the real interest rate remains above steady state for several quarters. The economy enters a recession and the level of output hits bottom in the second quarter. Ultimately, the economy is successfully disinflated in about 15 quarters.

Figure 1 further shows that neither the qualitative dynamic adjustment nor the length of the recession and the time needed for inflation to reach the new steady state are affected by the initial level of trend inflation.[9] What the level of π^*_{old} does affect, however, is the amplitude of output fluctuation during the transition. As shown in the first column of Table 1, the percentage output drop (in deviation from the new steady state level) at the trough substantially worsens as π^*_{old} increases. At the trough, output drops by 0.24% for a disinflation from 3 to 2%, whereas it drops by 0.71% for a disinflation from 5 to 2%. Intuitively, higher values of π^*_{old} make optimizing firms cut prices more sharply, generating a larger drop in inflation and a greater rise in the real interest rate. It is interesting to note that regardless of the new inflation target, either π^*_{new} =1% or π^*_{new} =0, the percentage output drops at the trough are of the same magnitude for a given disinflation size, i.e., $\pi^*_{\text{old}} - \pi^*_{\text{new}}$.

Figure 2 illustrates the dynamic adjustment of output, inflation, and nominal and real interest rates after cold-turkey disinflations aimed at achieving $\pi^*_{\text{new}} = 2\%$, when $\phi = 3$. The effects of having a more hawkish central bank are intuitive. In general, the monetary policy is more restrictive (see the large increase in the nominal interest rate) and the output downturn more severe (see Table 1). Nevertheless, adjusting firms seems to behave much the same as in previous case (when $\phi = 1.5$). As a matter of fact, the adjustment path of inflation is surprisingly similar to the top-right panel in Figure 1. There is just a small difference in terms of adjustment speed: with $\phi = 3$, the cold-turkey disinflation is accomplished in about 12 quarters.

We have seen that cold-turkey disinflations produce significant recessions, but how

[9] We chose not to plot the dynamic adjustments for cold-turkey disinflations aimed at $\pi^*_{\text{new}} = 1\%$ and $\pi^*_{\text{new}} = 0$ as the transitions are qualitatively very similar to those in Figure 1.

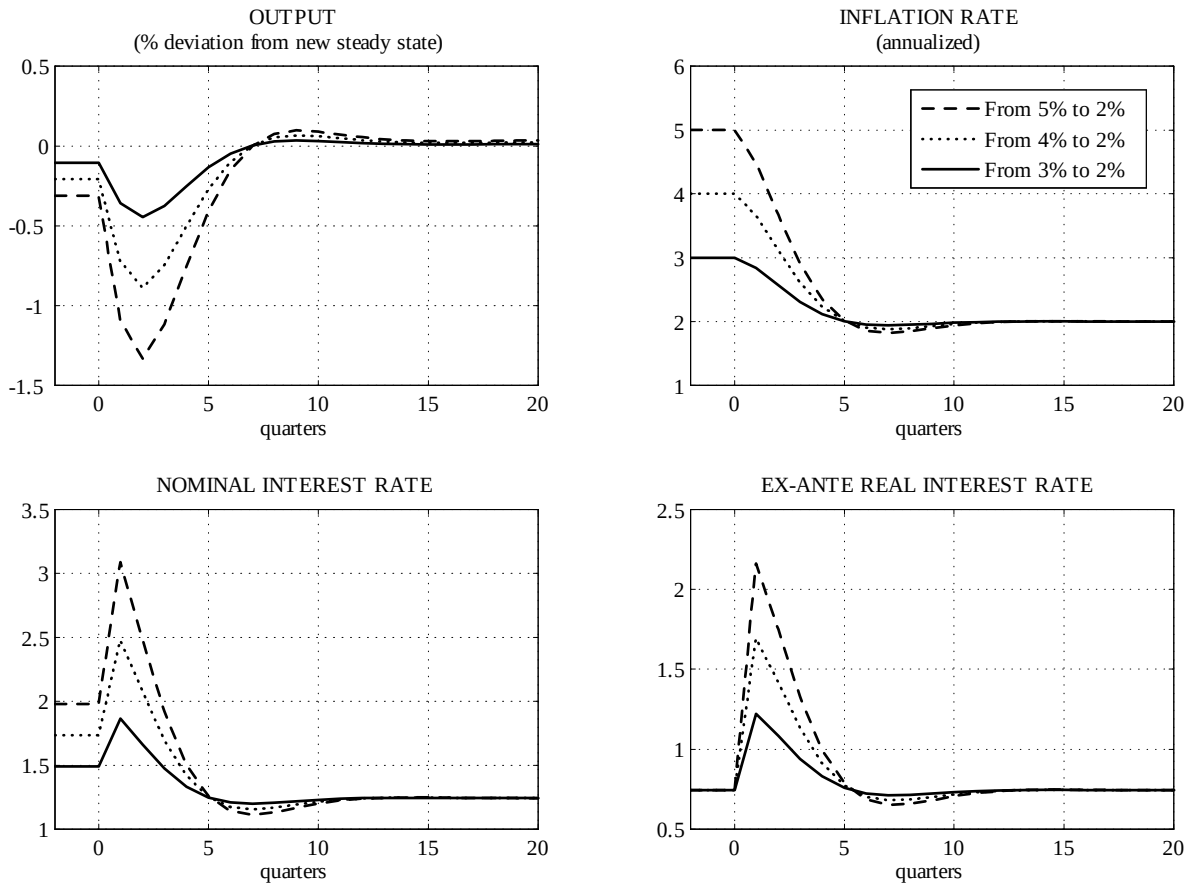

Figure 2: Cold-turkey disinflations aimed at achieving $\pi^*_{\text{new}} = 2\%$ with $\phi = 3$.

large are these short-run output costs? To answer this question we borrow directly from the empirical literature on disinflation and define a model-consistent sacrifice ratio. In particular,

$$\text{SR} = -\frac{1}{\pi^*_{\text{old}} - \pi^*_{\text{new}}} \sum_{t=0}^{T} \left(\frac{Y_t - Y_{\text{new}}}{Y_{\text{new}}} \right), \tag{2}$$

where Y_{new} represents the steady state level of output at π^*_{new}. Thus, our measure indicates the cumulative percentage output loss the economy has to incur in order to achieve a 1% permanent reduction of steady state inflation. Two features of (2) are worth noticing. First, we define the sacrifice ratio by calculating the output loss in deviation from the new steady state. Second, we sum up the percentage output losses over the first T periods. In particular, the value of T is chosen to reflect the number of

periods inflation takes to converge to the new inflation target.[10]

Table 1 reports values of the model-consistent sacrifice ratios calculated both for $\phi = 1.5$ (and $T = 15$) and $\phi = 3$ (and $T = 12$). We note first of all that the theoretical sacrifice ratios are positive and in line with the existing empirical estimates (see the Introduction). In particular, the sacrifice ratio turns out to be approximately 1.05 when $\phi = 1.5$; whereas it takes up a slightly larger value, 1.62, when the central bank is relatively more concerned with inflation stabilization around the target, i.e., when $\phi = 3$. In fact, we have seen that in this case the ensuing recession is more severe. However, the size of disinflation does not seem to affect the sacrifice ratio. Varying the size of disinflation leads to a roughly proportional rescaling of output transition paths and this leaves the sacrifice ratio practically unchanged.

In summary, in the medium-scale operational New Keynesian DSGE model a cold-turkey permanent reduction in trend inflation entails sizable short-run output costs. To bring down trend inflation, say, from 4 to 2%, by means of a credible cold-turkey disinflation the economy would have to incur a cumulative output loss of either 2.1 or 3.2%, depending on the type of interest rate rule. The inflation adjustment would then be completed in about 4 or 3 years.

4 A welfare-based measure of the cost of disinflation

As already noted in Gordon and King (1982), the output loss from disinflation does not in itself have policy implications. A careful assessment must be made of the welfare cost of lost output *and* the welfare benefits of lower inflation. On the latter point, the recent monetary policy literature has largely emphasized the reasons why achieving full price stability is desirable (see Woodford, 2003 and the references therein). One notable advantage of working with microfounded structural models is that they provide a natural welfare metric, namely the representative household's value function. Hence, we can calculate a welfare-based indicator of the costs of disinflations, instead of just

[10] In particular we truncate the horizon at a point where the distance between actual inflation and the new inflation target is (in absolute value) less than 10^{-3}.

		$\phi = 1.5$		$\phi = 3$	
π^*_{old}	π^*_{new}	Output at trough	SR (T=15)	Output at trough	SR (T=12)
3%	2%	-0.24	1.04	-0.44	1.59
4%	2%	-0.47	1.03	-0.88	1.60
5%	2%	-0.71	1.02	-1.32	1.61
2%	1%	-0.24	1.05	-0.45	1.61
3%	1%	-0.48	1.04	-0.90	1.62
4%	1%	-0.72	1.03	-1.34	1.63
1%	0	-0.24	1.06	-0.45	1.63
2%	0	-0.49	1.05	-0.91	1.64
3%	0	-0.73	1.05	-1.36	1.65

Table 1: Short-run costs of disinflation. Output at the trough is expressed as a percentage deviation from the new steady state level.

focussing on an empirical one such as the sacrifice ratio.

Mimicking the construction of the sacrifice ratio, a measure of the welfare loss caused by disinflation can be calculated as the difference between the value function at time zero, V_0 (when the disinflation is actually implemented), and the value function at the initial steady state inflation, V_{old} (as if the disinflation was not implemented). More formally, our welfare-based sacrifice ratio can be defined as

$$\text{WSR} = -\left(\frac{V_0 - V_{\text{old}}}{\pi^*_{\text{old}} - \pi^*_{\text{new}}}\right). \tag{3}$$

Notice that V_0 represents the discounted sum of future stream of instantaneous utility as such it measures both the transition dynamics and the long-run effects of the disinflation. Paralleling the standard sacrifice ratio definition, WSR$>$ 0, if $V_0 - V_{\text{old}} < 0$. That is, the welfare-based sacrifice ratio is positive if the disinflation reduces welfare .

The consumption equivalent measure

A policy maker is interested in the welfare cost of implementing a disinflationary policy, but given that the utility function is not cardinal, a measure based on the value function is not very revealing. The difference $(\mathrm{V}_0 - \mathrm{V}_{\mathrm{old}})$ needs to be converted into consumption equivalent units. The consumption equivalent measure defines the constant fraction of consumption that households have to give up in each period in the starting steady state, to reach the value function that would obtain if the disinflation is implemented. Thus, it measures how much households have to suffer in terms of consumption loss, in order to reduce the inflation rate permanently by a certain amount.

The derivation of the welfare-based measure in terms of consumption equivalent units is straightforward. The initial value function, in case the central bank does not disinflate the economy and keeps inflation target permanently at π^*_{old}, is given by

$$\mathrm{V}_{\mathrm{old}} = \frac{1}{1-\beta}\left[\ln(1-b)c_{\mathrm{old}} - \frac{\phi_0}{2}h^2_{\mathrm{old}} + \frac{\left(m^h_{\mathrm{old}}\right)^{1-\sigma_m}}{1-\sigma_m}\right], \tag{4}$$

where c_{old}, h_{old} and m^h_{old} denote respectively consumption, hours worked and real money balances held by households in the initial steady state; ϕ_0 and σ_m are structural parameters.[11] Given the value of V_0, available from the numerical solution of the model, we then have to find the constant fraction of steady state consumption, i.e., λ, that solves the following equation

$$\mathrm{V}_0 = \frac{1}{1-\beta}\left[\ln(1-b)(1-\lambda)c_{\mathrm{old}} - \frac{\phi_0}{2}h^2_{\mathrm{old}} + \frac{\left(m^h_{\mathrm{old}}\right)^{1-\sigma_m}}{1-\sigma_m}\right]. \tag{5}$$

Thus, the consumption equivalent measure is given by

$$\lambda = 1 - \exp\left[(1-\beta)(\mathrm{V}_0 - \mathrm{V}_{\mathrm{old}})\right]. \tag{6}$$

Finally, our proposed welfare-based sacrifice ratio is obtained as[12]

$$\mathrm{SR}^{\mathbb{W}} = \frac{\lambda}{\pi^*_{\mathrm{old}} - \pi^*_{\mathrm{new}}}. \tag{7}$$

[11] See the Appendix for further details.

[12] Note that there is no minus in front of this ratio, to maintain a positive sign for a loss. Indeed, if V_0-$\mathrm{V}_{\mathrm{old}} < 0$, that is, if disinflation brings about a welfare loss, then $\lambda > 0$, and vice versa.

The first column of Table 2 reports the values of $\mathrm{SR}^{\mathbb{W}}$. The main result can be stated as:

Result 1. Our proposed welfare-based sacrifice ratio calculated in a medium-scale New Keynesian DSGE model for different disinflation experiments takes on negative values. This means that disinflation is welfare improving.[13]

Therefore, when discussing the effects of disinflation policies it would be more appropriate to use the notion of *welfare gain ratio* rather than *sacrifice ratio*, as in the empirical literature. We think this is a novel and interesting result. The empirical literature on disinflation focuses only on the short-run costs in terms of output (or unemployment), but neglects any long-run gain. We show, on the contrary, that in a medium-scale DSGE monetary model of the business cycle a disinflationary policy is welfare improving.

Moreover, note that the welfare gain from disinflating: (i) decreases with the size of the disinflation; and (ii) decreases with the starting level of inflation, for a given size of disinflation.

A second notable result from Table 2 is:

Result 2. The size of $\mathrm{SR}^{\mathbb{W}}$, however, is small: the welfare gain is equivalent to an extra 0.06% of consumption each period.

Actually, the results are possibly even more striking, if we disentangle the short-run welfare costs of a disinflation during the transition dynamics and the long-run welfare gains stemming from greater price stability. In fact, in the standard medium-scale DSGE macro model, even though a disinflation entails a deep and prolonged recession, whose implied sacrifice ratio is in line with the empirical evidence, the short-run welfare costs of such a painful adjustment path are plainly insignificant.

[13] This result does not depend on the inclusion of real money balances in the utility function. We also calculated a similar measure without taking into account the gain in utility coming from the increase in real money balances in the new steady state. The measure would then be about 2/3 of the values reported in Table 2.

π^*_{old}	π^*_{new}	$\text{SR}^{\mathbb{W}}$ Total Welfare costs ($\times 10^{-2}$)		$\text{SR}^{\mathbb{W}}_{\infty}$ Long-run Welfare costs ($\times 10^{-2}$)	$\text{SR}^{\mathbb{W}}-\text{SR}^{\mathbb{W}}_{\infty}$ Short-run Welfare costs ($\times 10^{-2}$)	
		$\phi = 1.5$	$\phi = 3$		$\phi = 1.5$	$\phi = 3$
3%	2%	-6.46	-6.38	-7.23	0.77	0.85
4%	2%	-6.39	-6.32	-7.18	0.79	0.86
5%	2%	-6.35	-6.27	-7.13	0.79	0.86
2%	1%	-6.55	-6.48	-7.34	0.78	0.86
3%	1%	-6.49	-6.41	-7.29	0.80	0.87
4%	1%	-6.44	-6.36	-7.24	0.80	0.87
1%	0	-6.67	-6.59	-7.46	0.80	0.87
2%	0	-6.59	-6.52	-7.40	0.81	0.89
3%	0	-6.54	-6.46	-7.35	0.81	0.89

Table 2: Welfare-based sacrifice ratios.

To show this, we follow the same line of reasoning above and define:

(i) the long-run costs in terms of consumption equivalent units:

$$\lambda_{\infty} = 1 - \exp\left[(1-\beta)\left(\text{V}_{\text{new}} - \text{V}_{\text{old}}\right)\right] \tag{8}$$

where V_{new} and V_{old} denote the value function in the new and old inflation steady states. The above indicator can be expressed per unit of diminished inflation to yield a *long-run welfare-based sacrifice ratio:*[14]

$$\text{SR}^{\mathbb{W}}_{\infty} = \frac{\lambda_{\infty}}{\pi^*_{\text{old}} - \pi^*_{\text{new}}}; \tag{9}$$

[14]Note that we use a consistent definition as above for $\text{SR}^{\mathbb{W}}_{\infty}$. In fact, if V_{new}-$\text{V}_{\text{old}} < 0$ (that is, if disinflation brings about a welfare loss) then $\lambda > 0$, and vice versa.

(*ii*) the *short-run welfare-based sacrifice ratio* is then given by

$$\mathrm{SR}^{\mathbb{W}} - \mathrm{SR}^{\mathbb{W}}_{\infty} = \frac{\exp\left[(1-\beta)\left(\mathrm{V}_{\mathrm{new}} - \mathrm{V}_{\mathrm{old}}\right)\right] - \exp\left[(1-\beta)\left(\mathrm{V}_{0} - \mathrm{V}_{\mathrm{old}}\right)\right]}{\pi^{*}_{\mathrm{old}} - \pi^{*}_{\mathrm{new}}}. \tag{10}$$

Table 2 reports the long-run welfare gains and the short-run welfare costs in consumption equivalent units for various disinflation experiments. The order of magnitude of the short-run welfare costs is roughly 0.008-0.009% of initial consumption. Therefore, the long-run gains quantitatively dominate, though they too are very small (roughly 0.07%). The main message from Table 2 is that a disinflation is going to be welfare improving of the order of an increase of initial consumption of 0.06-0.07% each period per point of diminished inflation. That is, the welfare effects of a disinflation are scarcely significant, despite high short-run costs in terms of output losses.

This stands in sharp contrast with the consensus view of the effects of a credible disinflation. What is the intuition for these results? To illustrate this point, let us consider the case with $\phi = 3$. Figure 3 displays the paths of consumption and employment, expressed in deviation from the new steady state, together with value of the utility function. The disinflation induces a prolonged recession that causes both consumption and employment to be below their new (and higher) steady state values for some periods. Consumption and employment, however, have opposite effects on the utility function of the representative agent. It follows that the net effects of the recession on the utility of the representative agent is ambiguous. The decrease in consumption dominates in the impact period, dragging the utility function down, from the second period on the effects of the dynamics of employment take over, and the utility function is above its new higher long-run value. Moreover, it will stay there for all the remaining periods of the recession. This is because the drop in employment is bigger in percentage terms, and slightly more sluggish. It follows that the positive impact of employment is quite effective in counterbalancing the negative effect of lower consumption. Overall, the transition entails a short-run cost, as shown above, but of a negligible order of magnitude. Finally, the value of the utility function without counting the real money balances term is also depicted in Figure 3, to clarify that the role of the real money balances term in the utility function in the above results is nil.

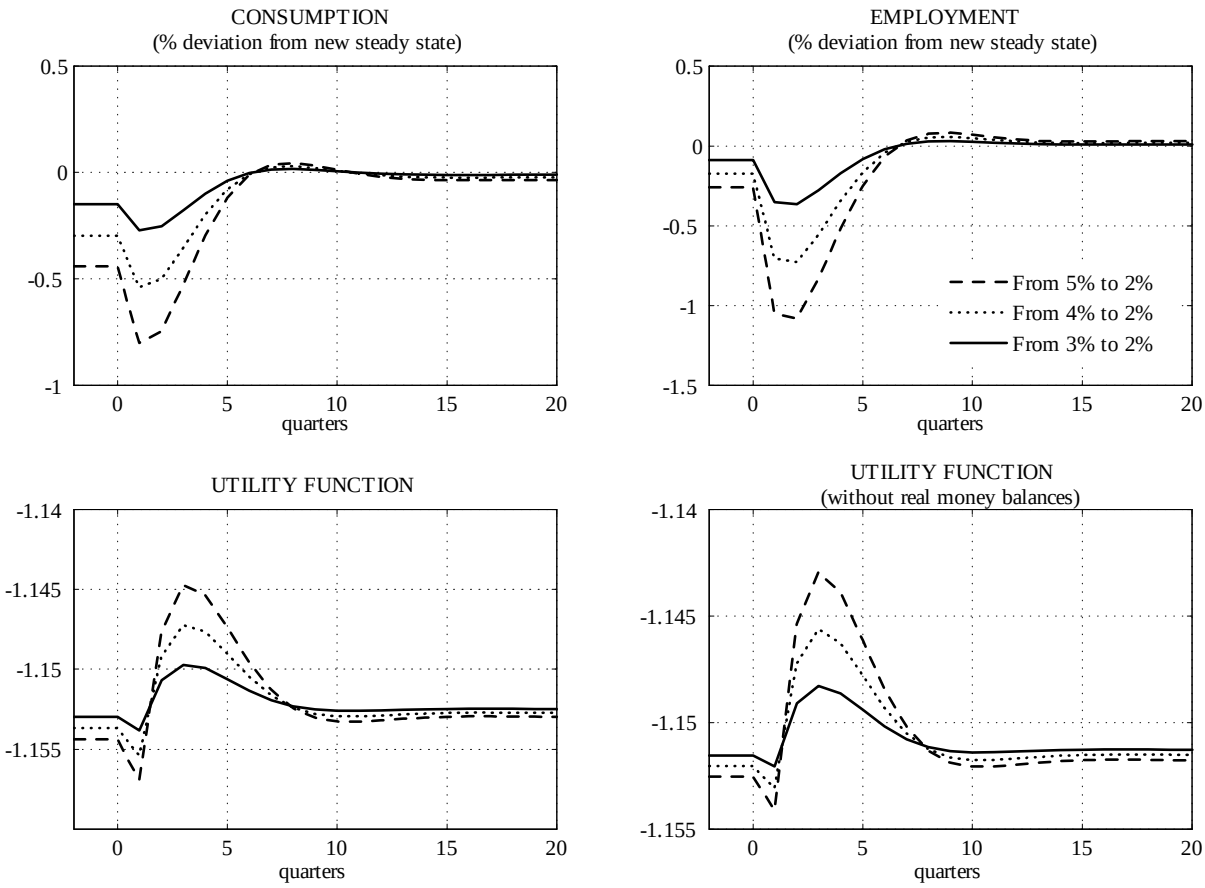

Figure 3: Cold-turkey disinflations aimed at achieving $\pi^*_{\text{new}} = 2\%$ with $\phi = 3$.

This result obviously hinges on the representative agent assumption, that is, on complete markets and risk-sharing. The welfare analysis based on a representative agent framework cannot take into account, for example, the fact that some people may suffer a very big drop in utility during recessions because they lose their jobs and do not have access to financial markets. This heterogeneity and composition effect is missing by construction. However, we believe our results have two notable interpretations. First, taken at face value, our findings simply show that disinflations, in particular, and recessions, in general, could be less of a problem than is normally thought, if the economy could provide an efficient risk-sharing mechanism among agents (by means of capital markets or some public welfare system). In this sense, this is once again the Lucas' negligible

costs of business cycle result. Second, if one is skeptical about the actual relevance of the welfare outcomes, then, at the very least, our results cast serious doubts on using these DSGE models for welfare evaluation without "inspecting the mechanism". In particular, the whole literature on optimal policy problems or on the ranking of different monetary policy rules is bound to be based on mechanism similar to the ours.

5 Conclusions

Disinflation is an important topic in monetary economics and the subject of a vast literature. However, there is a widespread consensus that the New Keynesian models cannot explain the cost of disinflation observed in the data, for which they need to resort to lack of credibility or information.

The logic of the policy experiments laid out in this paper is clear. We investigate whether the workhorse DSGE model of the US business cycle, namely the CEE model, can quantitatively account for the sacrifice ratio and the overall adjustment dynamics after a disinflation. We think such an ability is an essential requisite of an operational monetary model.

Our results show that a perfectly credible cold-turkey disinflation entails a sizable and long-lasting recession in the CEE model. In addition, the values of the sacrifice ratio are in line with those estimated in the empirical literature.

Moreover, we conduct a rigorous welfare evaluation of the costs of disinflation, proposing a welfare-based sacrifice ratio. Surprisingly enough, despite a deep and prolonged recession the short-run costs of a disinflation are negligible in terms of consumption equivalent units. A disinflation would actually imply miniscule welfare gains, since in the CEE model money is not superneutral (despite full indexation) and there are very small long-run welfare gains that overcome the short-run costs.

The finding that the CEE model can replicate the main facts after a disinflation is at odds with the consensus in the literature and may be good news for the New Keynesian models. But this does not mean that some of the model's features or mechanisms should not be improved to tackle the disinflation question. In fact, we think that testing the

CEE model with respect to disinflationary policies has proved useful to shed light on important aspects for current and future research.

First, it will be important to understand how each of the different features of the CEE model quantitatively affects our results. A thorough investigation of this issue is outside the scope of this paper for obvious length constraint, but it is developed in a companion paper, which focuses on the role of the monetary policy and of price indexation. Regarding monetary policy, the companion paper investigates different interest rate rules (with responses to output and to lagged interest rate), money supply rules, the role of anticipation and of gradualism, and higher sizes of disinflation. Moreover, the companion paper shows how the role of price indexation would depend on the way monetary policy is implemented.

Second, the role of price indexation should be investigated further. Indexation is indeed a reduced form assumption that can act as a substitute for many other more structural phenomena. There is a macroeconomic reduced form equivalence of different microeconomic models, so that a similar effect can actually come from irrational price setters (rule of thumbers), inattentive price setters or lack of credibility, and hence sluggish expectation adjustment.

Third, a Calvo time-dependent price setting model would need indexation in order not to have unpalatable long-run implications of a permanent change in inflation due to the large effects of price dispersion in that model. Moreover, although we look only at moderate rates of inflation, for which the Calvo parameter defining the frequency of price adjustment can be considered constant, ideally one would like to work with a model where the changes in the average inflation level induce firms to revise their behavior[15]. In other words, a time-dependent model is particularly exposed to the Lucas critique when used to analyze changes in the average inflation rate. Last but not least, Klenow and Kryvtsov (2008) has recently shown that the many price adjustments occur on the intensive margin rather than on the extensive margin. Embedding what Klenow and Kryvtsov (2008) calls a second-generation model of state-dependent pricing

[15] For disinflation dynamics in small models with endogenous pricing, see Almeida and Bonomo (2002), Bonomo and Carvalho (2004), Burstein (2006). .

in the CEE framework would solve all these problems at once: no need for indexation to remedy the unpalatable long-run effects, shelter from the Lucas critique, and the intensive margin. Moreover, as we know from Burnstein (2006) this could generate interesting non-linearities regarding the effects of large vs. small disinflations.

Finally, our welfare results are rather surprising. The abandonment of the risk sharing assumption, together with a proper account of heterogeneity among agents regarding the impact of a recession on their welfare, may overturn our results.

Fortunately, current research and the recent contributions to the New Keynesian literature are taking up all these challenges.

References

Almeida, H. and M. Bonomo (2002). Optimal state-dependent rules, credibility and inflation inertia. *Journal of Monetary Economics 49*, 1317–1336.

Altig, D., L. J. Christiano, M. Eichenbaum, and J. Linde (2004). Firm-specific capital, nominal rigidities, and the business cycle. Technical Report wp 04-16, Federal Reserve Bank of Clevelend.

Andersen, P. S. and W. L. Wascher (1999). Sacrifice ratios and the conduct of monetary policy in conditions of low inflation. BIS wp No. 82.

Ascari, G. (2004). Staggered prices and trend inflation: Some nuisances. *Review of Economic Dynamics 7*, 642–667.

Ascari, G. and C. Merkl (2007). Real wage rigidities and the cost of disinflations. IZA dp No. 3049, forthcoming in *The Journal of Money, Credit and Banking.*

Ball, L. (1994a). Credible disinflation with staggered price-setting. *American Economic Review 84*, 282–289.

Ball, L. (1994b). What determines the sacrifice ratio? In N. G. Mankiw (Ed.), *Monetary Policy*, pp. 155–182. Chicago: University of Chicago Press.

Ball, L. (1995). Disinflation with imperfect credibility. *Journal of Monetary Economics 35*, 5–23.

Blanchard, O. J. and D. Quah (1989). The dynamic effects of aggregate demand and supply disturbances. *American Economic Review 79*, 655–673.

Bonomo, M. and C. Carvalho (2004). Endogenous time-dependent rules and inflation inertia. *Journal of Money, Credit and Banking 36*(6), 1015–1041.

Burstein, A. T. (2006). Inflation and output dynamics with state-dependent pricing decisions. *Journal of Monetary Economics 53*, 1235–1257.

Calvo, G. A. (1983). Staggered prices in a utility-maximising framework. *Journal of Monetary Economics 12*, 383–398.

Cecchetti, S. G. and R. W. Rich (2001). Structural estimates of the U.S. sacrifice ratio. *Journal of Business and Economic Statistics 19*, 416–427.

Christiano, L. J., M. Eichenbaum, and C. L. Evans (2005). Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of Political Economy 113*(1), 1–45.

Clarida, R., J. Galí, and M. Gertler (1999). The science of monetary policy: A new keynesian perspective. *Journal of Economic Literature 37*, 1661–1707.

Collard, F., P. Fève, and J. Matheron (2006). What happens after a disinflation policy shock. mimeo.

Cuñado, J. and F. P. D. Gracia (2003). Sacrifice ratios: Some lessons from EMU countries, 1960-2001. *International Review of Applied Economics 17*, 327–337.

Durand, J.-J., M. Huchet-Bourdon, and J. Licheron (2007). Sacrifice ratio dispersion within the Euro Zone: What can be learned about implementing a single monetary policy? forthcoming in *International Review of Applied Economics*.

Erceg, C. J. and A. T. Levin (2003). Imperfect credibility and inflation persistence. *Journal of Monetary Economics 50*, 915–944.

Fève, P., J. Matheron, and J.-G. Sahuc (2007). Disinflation shocks in the eurozone: A DSGE perspective. mimeo.

Goodfriend, M. and R. G. King (2005). The incredible Volcker disinflation. *Journal of Monetary Economics 52*, 981–1015.

Gordon, R. J. and S. King (1982). The output cost of disinflation in traditional and vector autoregressive models. *Brookings Papers on Economic Activity 1*, 205–242.

Klenow, P. J. and O. Kryvtsov (2008). State-dependent or time-dependent pricing: Does it matter for recent U.S. inflation? *The Quarterly Journal of Economics 123*(3), 863–904.

Leigh, D. (2008). Estimating the Federal Reserve's implicit inflation target: A state space approach. *Journal of Economic Dynamics and Control 32*, 2013–2030.

Mankiw, G. N. (1999). *Macroeconomics*. New York: W.H. Freeman and Company.

Mankiw, G. N. (2001). The inexorable and mysterious tradeoff between inflation and unemployment. *The Economic Journal*, C45–C61.

Mankiw, N. G. and R. Reis (2002). Sticky information versus sticky prices: A proposal to replace the New Keynesian Phillips Curve. *Quarterly Journal of Economics 117*, 1295–1328.

Nicolae, A. and C. Nolan (2006). The impact of imperfect credibility in a transition to price stability. *Journal of Money, Credit, and Banking 38*(1), 47–66.

Schmitt-Grohé, S. and M. Uribe (2005). Optimal fiscal and monetary policy in a medium-scale macroeconomic model. In M. Gertler and K. Rogoff (Eds.), *NBER Macroeconomics Annual*, pp. 383–425. Cambridge, MA: MIT Press.

Schmitt-Grohé, S. and M. Uribe (2007). Optimal inflation stabilization in a medium-scale macroeconomic model. In K. Schmidt-Hebbel and R. Mishkin (Eds.), *Monetary Policy Under Inflation Targeting*, pp. 125–186. Santiago, Chile: Central Bank of Chile.

Smets, F. and R. Wouters (2003). An estimated dynamic stochastic general equilibrium model of the euro area. *Journal of the European Economic Association 1*, 1123–1175.

Woodford, M. (2003). *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton: Princeton University Press.

Yun, T. (2005). Optimal monetary policy with relative price distortions. *American Economic Review 95*, 89–108.

Zhang, L. H. (2005). Sacrifice ratios with long-lived effects. *International Finance 8*, 231–262.

A The Christiano, Eichenbaum and Evans (2005) Model

In this Appendix we describe the CEE model, closely following the outline in Schmitt-Grhoe and Uribe (2005).

Households

There is a continuum of infinitely-lived households whose expected intertemporal utility function is given by

$$U_0 = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u \left(c_t - bc_{t-1}; h_t^s; m_t^h \right) \right\}, \tag{11}$$

where E_0 defines the mathematical expectation operator conditional on the information set available at time 0, β is the subjective discount factor, function $u \left(c_t - bc_{t-1}; h_t^s; m_t^h \right)$ is well-behaved and increasing in consumption c_t and money holdings m_t^h, while decreasing in hours worked h_t^s. Preferences display habit in consumption levels, measured by the parameter b.

There is a continuum of final goods indexed by $i \in [0,1]$, which are aggregated in the usual CES consumption bundle c_t

$$c_t = \left[\int_0^1 c_{it}^{\frac{\eta-1}{\eta}} di \right]^{\frac{\eta}{\eta-1}}, \tag{12}$$

where the parameter η indicates the elasticity of substitution between different varieties of goods. The standard household problem defines the optimal demand of good i, given by $c_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\eta} c_t$, where P_t is the general price index given by $P_t = \left[\int_0^1 P_{it}^{1-\eta} di \right]^{\frac{1}{1-\eta}}$.

There is a continuum of labour services h_{jt}, $j \in [0,1]$, which are combined according to the following technology

$$h_t^d = \left[\int_0^1 h_{jt}^{\frac{\tilde{\eta}-1}{\tilde{\eta}}} dj \right]^{\frac{\tilde{\eta}}{\tilde{\eta}-1}}, \tag{13}$$

where $\tilde{\eta}$ is the elasticity of substitutions of labour types. The standard cost minimization problem for the firms yields the labour-specific demand function given by $h_{jt} = \left(\frac{W_{jt}}{W_t} \right)^{-\tilde{\eta}} h_t^d$, where W_{jt} is the wage paid to labor type j and W_t is a wage index defined as $W_t = \left[\int_0^1 W_{it}^{1-\tilde{\eta}} di \right]^{\frac{1}{1-\tilde{\eta}}}$. The total labor supply is found by integrating

labour-specific demand functions, to obtain h_t^s

$$h_t^s \equiv \int_0^1 h_{jt} dj = h_t^d \int_0^1 \left(\frac{w_{jt}}{w_t}\right)^{-\tilde{\eta}} dj. \tag{14}$$

Agents owns physical capital k_t that depreciates at rate δ. The capital accumulation equation is

$$k_{t+1} = (1-\delta) k_t + i_t \left[1 - S\left(\frac{i_t}{i_{t-1}}\right)\right], \tag{15}$$

where the function S introduces the adjustment cost on investment and satisfies the properties that $S(1) = S'(1) = 0$, $S''(1) > 0$. The model also features variable capacity utilization of physical capital, denoted by u_t,. The cost of capital then depends on the degree of utilization and it is given by $a(u_t)$. Agents rent capital to firms at a real interest rate r_t^k and decide also over the utilization rate. There are complete markets for state contingent assets, such that all agents choose the same level of consumption.

Household first order conditions are hence given by

$$u_{c_t}\left(c_t - bc_{t-1}; h_t^s; m_t^h\right) + u_{c_t}\left(c_{t+1} - bc_t; h_{t+1}^s; m_{t+1}^h\right) = \lambda_t \tag{16}$$

$$u_{h_t}\left(c_t - bc_{t-1}; h_t^s; m_t^h\right) = -\lambda_t \frac{w_t}{\tilde{\mu}_t} \tag{17}$$

$$q_t = \beta \frac{\lambda_{t+1}}{\lambda_t}\left[q_{t+1}(1-\delta) + r_{t+1}^k u_{t+1} - a(u_{t+1})\right] \tag{18}$$

$$q_t \lambda_t \left[1 - S\left(\frac{i_t}{i_{t-1}}\right) - \left[S_i\left(\frac{i_t}{i_{t-1}}\right)\right] i_t\right] - \beta q_{t+1}\lambda_{t+1} S_i\left(\frac{i_{t+1}}{i_t}\right) i_{t+1} = \lambda_t \tag{19}$$

$$a_{u_t}(u_t) = r_t^k \tag{20}$$

$$u_{m_t^h}\left(c_t - bc_{t-1}; h_t^s; m_t^h\right) + \beta \frac{\lambda_{t+1}}{\pi_{t+1}} = \lambda_t. \tag{21}$$

Wages are sticky à la Calvo, and $1-\tilde{\alpha}$ is the probability of being able to reset wages in the next period. If wages can not be re-optimized, the CEE model assumes that wage are updated anyway according to past inflation, such that: $w_{j,t+1} = w_{j,t}\pi_t^{\tilde{\chi}}$ where $\tilde{\chi}$ is the degree of indexation to past inflation. Define $\tilde{w}_t$ as the optimal wage set every period t. The union chooses the optimal wage maximizing the utility function given by equation (12), subject to the demand for labour in the specific market $h_{jt} = \left(\frac{w_{jt}}{w_t}\right)^{-\tilde{\eta}} h_t^d$

and the probability of not being able to re-optimize in future periods. The resulting first order condition is

$$E_t \sum_{s=0}^{\infty} (\beta\tilde{\alpha})^s \lambda_{t+s} \left(\frac{\tilde{w}_t}{w_{t+s}}\right)^{-\tilde{\eta}} h_{t+s}^d \prod_{k=1}^{s} \left(\frac{\pi_{t+k}}{\pi_{t+k-1}^{\tilde{\chi}}}\right)^{\tilde{\eta}} \left[\frac{\tilde{\eta}-1}{\tilde{\eta}} \frac{\tilde{w}_t}{\prod_{k=1}^{s}\left(\frac{\pi_{t+k}}{\pi_{t+k-1}^{\tilde{\chi}}}\right)} - \frac{w_{t+s}}{\tilde{\mu}_{t+s}}\right] = 0. \tag{22}$$

All the reset optimal wages are identical in all labour markets.

Firms

Each good is produced by a firm that monopolistically supply its own variety using a production technology of the form

$$z_t F(k_{it}, h_{it}) - \psi,$$

where z_t is an aggregate technology factor common across firms, and ψ represents a fixed cost of production. The production function $F(k_{it}, h_{it})$ is well-behaved and it is the same across firms. Final goods can be used for consumption, investment, public expenditure and to pay cost of capital utilization. Each firm faces the following demand function

$$y_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\eta} y_t, \tag{23}$$

where

$$y_t = c_t + i_t + g_t + a(u_t) k_t. \tag{24}$$

Firms rent capital from households on a competitive market and must pay a fraction ν of wages at the beginning of the period in cash. Therefore, their money demand function is

$$m_{it}^f = \nu w_t h_{it} \tag{25}$$

The firms' problem is then to maximize the expected value of future profits, under their demand function (23) and the cash-in-advance constraint (25). The first order conditions with respect to capital and labour services are

$$mc_{it} z_t F_{k_{it}}(k_{it}, h_{it}) = r_t^k \tag{26}$$

$$mc_{it} z_t F_{h_{it}}(k_{it}, h_{it}) = w_t \left[1 + \nu \frac{R_t - 1}{R_t}\right]. \tag{27}$$

Since F is homogeneous of degree one, equation (26) and equation (27) imply that all firms have the same marginal costs and aggregation across firms is straightforward.

Prices are sticky à la Calvo. Every period each firm can choose a new price of its own good with a probability $1-\alpha$. As for wages, the prices that cannot be reset optimally are likewise automatically updated according to past inflation, such that: $P_{it} = P_{it-1}\pi_{t-1}^{\chi}$, where χ is the degree of price indexation. The first order condition for the optimal price is

$$E_t \sum_{s=0}^{\infty} r_{t,t+s} P_{t+s} \alpha^s \left(\frac{\tilde{P}_t}{P_t}\right)^{-\eta} y_{t+s} \prod_{k=1}^{s} \left(\frac{\pi_{t+k}}{\pi_{t+k-1}^{\chi}}\right)^{\eta} \left[\frac{\eta-1}{\eta}\frac{\tilde{P}_t}{P_t}\prod_{k=1}^{s}\left(\frac{\pi_{t+k-1}^{\chi}}{\pi_{t+k}}\right) - mc_{i,t+s}\right] = 0. \tag{28}$$

Again, all the reset optimal prices are identical for all goods.

The government

Government expenditure is financed through lump-sum taxes and seigniorage

$$g_t = \tau_t + m_t - \frac{m_{t-1}}{\pi_t}. \tag{29}$$

where m_t denotes real money balances and $\pi_t \equiv P_t/P_{t-1}$ is the (gross) inflation rate at time t. The government minimizes the costs of acquiring the composite good; hence given public expenditure, the government's absorption of a single type of good is $g_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\eta} g_t$.

To close the model we postulate that monetary policy uses the simple non-linear nominal interest rate rule as described in the paper.

Equilibrium

The model equilibrium conditions are

$$
\begin{array}{rl}
\text{Money market:} & m_t = m_t^h + m_t^f \\
\text{Labor market:} & h_t^s = \int_0^1 h_{it}^d di \\
\text{Capital market:} & \int_0^1 k_{it} di = u_t k_t \\
\text{Good } i \text{ market:} & z_t F\left(k_{it}, h_{it}\right) = \left(c_t + g_t + i + a\left(u_t\right) k_t\right) \left(\frac{P_{it}}{P_t}\right)^{-\eta} \\
\begin{array}{c}\text{Aggregate} \\ \text{Goods market}\end{array} : & z_t h_t^d F\left(\frac{u_t k_t}{h_t^d}, 1\right) = \left(c_t + g_t + i + a\left(u_t\right) k_t\right) \int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\eta} di
\end{array}
$$

where $s_t \equiv \int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\eta}$ is the price dispersion generated by price staggering, causing a wedge between aggregate supply and aggregate absorption. Similarly,θ wage staggering gives rise to wage dispersion, given by $\tilde{s}_t \equiv \int_0^1 \left(\frac{w_{jt}}{w_t}\right)^{-\tilde{\eta}} dj$, see (14).

Functional forms and calibration

As in Schmitt-Grohé and Uribe (2005), we assume the following functional forms:

$$
\begin{aligned}
u\left(c_t - bc_{t-1}; h_t^s; m_t^h\right) &= \ln(c_t - bc_{t-1}) - \frac{\phi_0}{2} h_t^2 + \phi_1 \frac{\left(m_t^h\right)^{1-\sigma_m}}{1-\sigma_m} \\
F\left(u_t k_t, h_t^d\right) &= \left(u_t k_t\right)^{\theta} \left(h_t^d\right)^{1-\theta} \\
S\left(\frac{i_t}{i_{t-1}}\right) &= \frac{\kappa}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2 \\
a\left(u_t\right) &= \gamma_1\left(u_t - 1\right) + \frac{\gamma_2}{2}\left(u_t - 1\right)^2 .
\end{aligned}
$$

The parameters values, taken from CEE, are listed in the Table 3.

Parameter	Value	Description
β	$1.03^{-0.25}$	Time discount rate
θ	0.36	Share of capital
ψ	0.5827	Fixed cost (guarantee zero profits in steady state)
δ	0.025	Depreciation of capital
ν	1	Fraction of wage bill subject to CIA constraint
η	6	Elasticity of substitution of different varieties of goods
$\tilde{\eta}$	21	Elasticity of substitution of labour services
α	0.6	Probability of not setting a new price each period
$\tilde{\alpha}$	0.64	Probability of not setting a new wage each period
b	0.65	Degree of habit persistence
ϕ_0	1.1196	Preference parameter
ϕ_1	0.5393	Preference parameter
σ_m	10.62	Intertemporal elasticity of money
κ	2.48	Investment adjustment cost parameter
χ	1	Price indexation
$\tilde{\chi}$	1	Wage indexation
γ_1	0.0324	Capital utilization cost function parameter
γ_2	0.000324	Capital utilization cost function parameter
z	1	Steady state value of technology shock

Table 3: Calibration of parameters in the Christiano, Eichenbaum and Evans (2005).