On the segregative properties of endogenous jurisdiction formation with a central government

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Abstract

This paper examines the segregative properties of endogenous processes of jurisdiction formation *à la* Tiebout in the presence of a central government who redistributes income across jurisdictions by maximizing a welfarist objective. Choice of of location by households, of local public good provision by jurisdiction, and of redistribution by the central government are assumed to be made simultaneously, taking the choices of others as given. Two welfarist objectives for the central government are considered in turn: Leximin and Utilitarianism. If the central government pursues a Leximin objective, it is easily shown that the only stable jurisdiction structure that can emerge is essentially the trivial one in which all households live in the same jurisdiction. A richer class of stable jurisdiction structures are compatible with a central utilitarian government. Yet, it happens that, if individual preferences are additively separable, the class of preferences that garantee the segregation of any stable jurisdiction structure remains unchanged by the presence of a central government.

1 Introduction

There is a wide presumption that decentralized processes of jurisdiction formation *à la* Tiebout (1956) lead individuals to self-sort into homogenous communities. In a recent paper, Gravel and Thoron (2007) investigate the validity of this intuition in the classical model of jurisdiction formation developed by Westhoff (1977) (see also Greenberg and Weber (1986) and Demange (1994) among many other contributions). In this model, households, who have the same preference for a local public good and a private good

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and who differ in wealth, must simultaneously choose their place of residence in a finite set of locations. Households who choose the same location form a jurisdiction and produce a local public good by applying a democratically chosen tax rate to all residents' wealth. Any simultaneous choice of residence by households thus generates a partition of these households into various jurisdictions that is referred to as a *jurisdiction structure*. The analysis of Gravel and Thoron (2007) concerns *stable* jurisdiction structures, which satisfy the additional property of being *robust to individual deviations*. The question raised by Gravel and Thoron (2007) is whether stable jurisdiction structures lead households to self sort, or segregate, themselves according to their wealth. The notion of segregation used is that known under the heading of *consecutiveness* in the coalition formation literature (see e.g. Greenberg and Weber (1986)). A jurisdiction structure is segregated in this sense if, for any two jurisdictions with different *per capita* wealth, the richest individual in the poorer jurisdiction is (weakly) poorer than the poorest individual in the richer jurisdiction. This form of segregation is clearly extreme and is interesting precisely for the reason that it represents a pure and "idealized" notion of segregation. Gravel and Thoron (2007) identify a condition on households preferences for the private and the local public good that is necessary and sufficient for the segregation of any stable jurisdiction structure. The condition requires households to consider public good to be either *always* a gross complement, or *always* a gross substitute to the private good. While stringent, and violated by several preferences, including additively separable ones, the condition is not implausible. For this reason, the analysis of Gravel and Thoron (2007) seems to provide some theoretical ground on the widespread belief that decentralized processes of jurisdiction formation are driven by segregative forces.

In this paper, we investigate the extent to which the segregative feature of endogenous jurisdiction formation is affected by the introduction of a central government. Introducing a central government in models of endogenous jurisdiction formation strikes us as an important step toward improving the realism of these models. In most countries, one finds indeed a mix of several levels of governments: central and local ones. It is therefore interesting to examine the consequence of central government's intervention on the segregative properties of the endogenous formation of local jurisdictions by freely mobile households.

Doing this requires one to specify:

1) the instruments available to the central government

2) the objective of the central government and

3) the nature of the interaction between the central government, the households and the local governments.

As for the first point, we assume that the central government redistributes local tax revenues across jurisdictions by maximizing some objective function. Although stylized, our modeling of the redistribution performed

by the central government does not provide a bad approximation of so-called horizontal equalization payments of the sort existing in Scandinavian countries(see e.g. Gravel and Poitevin (2006) for an examination of the normative properties of these equalization payments).

Concerning the objective of the central government, we assume it to be *welfarist*. Hence the central government is depicted as redistributing tax revenues across jurisdictions in such a way as to maximize an increasing function of the households' utilities (assumed to be interpersonally comparable and cardinally measurable, using Sen (1977a)'s terminology). Thanks to the classical work of Deschamps and Gevers (1978), there happens to be two "natural" types of such welfarist central governments: A Utilitarian one, which maximizes the sum of households utilities and a Leximin one, which maximizes the utility of the worse off households and, in case of indifference of the worse off household, switch to the second worse off and so on. We perform the analysis with the two objectives even though, as it turns out, the analysis is (much) more interesting when done with a utilitarian one.

As for the interaction between households, central and local governments, we model it as taking place simultaneously, as done, without central government, in the conventional Westhoff (1977)'s setting. Specifically, we define a stable jurisdiction structure with a central government to be an assignment, to every location, of a local tax rate, a central government net transfer, and a set of households such that:

1) the local tax rate is, in every jurisdiction, the favorite one of some member of the jurisdiction (say the median), given central government transfer and local tax base

2) the transfers given to jurisdictions maximize the welfarist objective of the central government, given the partition of households into jurisdictions and local tax rates and

3) each household weakly prefers its jurisdiction to any other, given the central government transfers, local tax rates and local public good provision.

The question addressed in this paper is whether the condition on households preference identified in Gravel and Thoron (2007) remains necessary and sufficient for ensuring the segregation of any stable jurisdiction structures in presence of a central welfarist government. If the condition remains necessary but ceases to be sufficient, we would interpret this as an indication that the presence of a central government mitigates the segregative tendencies of endogenous jurisdiction formation. Conversely, if the condition is not any more necessary but remains sufficient, we would interpret this as an evidence that central government intervention increases the self-sorting forces underlying jurisdiction formation (as segregation of any stable jurisdiction structure would then be observed on a wider class of individual preferences). If the same condition remains necessary and sufficient, we would conclude that central government intervention has no impact on the segregative properties of jurisdiction formation. A last, and interpretatively difficult, logical

possibility is for the condition to become neither necessary and sufficient for guaranteeing the segregation of a stable jurisdiction structure. As it happens however, this eventuality will not arise.

We examine this question by considering in turn a Leximin and a Utilitarian central government, making, in the later case, the extra assumption that the utility function used by the social planner is additively separable between the public and the private good. There is not much analysis to be performed in the case of a Leximin government. For we show easily in that case that, with one uninteresting exception, the only stable jurisdiction structure that can exist in the presence of a central government is the trivial one in which all households live in the same (grand) jurisdictions. As this grand jurisdiction is (trivially) segregated, segregation becomes (trivially) an inherent property of stability. Results are more interesting with a Utilitarian central government because the optimal redistribution by such a government is compatible with other jurisdiction structures than the trivial one. Yet we show in this paper that, if the central Utilitarian government uses additively separable utility functions, *the same condition* on household preference than that identified in Gravel and Thoron (2007) is necessary and sufficient for the segregation of any stable jurisdiction structure in the presence of central government. Hence it appears that central government intervention, at least when performed in the way modelled in this paper, has no impact on the segregative properties of endogenous processes of jurisdiction formation.

The rest of the paper is organized as follows. The next section introduces the basic notation and the model considered and shows how little interesting is the case where the Leximin government is considered. Section 3 states and proves the main result for the utilitarian government while section 4 concludes.

2 Endogenous formation of jurisdictions with a central government

The model we consider is similar to that of Gravel and Thoron (2007) and Westhoff (1977) with the exception that, contrary to these authors, we assume a *finite number*, rather than a continuum, of households. As will be clear - at least we hope - this change in the formal setting does not affect substantially the analysis done in Gravel and Thoron (2007) but facilitates the writing of the central government's objective (the Leximin objective is not easy to define if there is a continuum of individuals). On the other hand, the continuum setting is probably more natural for modeling processes of jurisdiction formation in which households are "small" relative to jurisdictions in the sense of assuming that their individual decision has no effect on the jurisdiction aggregate wealth and/or tax rate.

We assume that there are n households, indexed by h, taken from some

finite set N. Household h has a strictly positive wealth ω_h and we denote by $\omega = (\omega_1, ..., \omega_n)$ the distribution of wealth in the population written in such a way that $\omega_h \leq \omega_{h+1}$ for all $h = 1, .., n-1$. Households have all the same preference for a public good (Z) and a private good (x) that are represented by a twice differentiable, strictly increasing and strictly concave[1] utility function $U : \mathbb{R}^2_+ \to \mathbb{R}$. On occasion, we will also assume that households preferences are *additively separable* so that they can be represented by a utility function $U : \mathbb{R}^2_+ \to \mathbb{R}$ that can be written, for every $(\overline{Z}, \overline{x}) \in \mathbb{R}^2_+$, as:

$$U(\overline{Z}, \overline{x}) = f(\overline{Z}) + h(\overline{x}) \tag{1}$$

for some twice differentiable increasing and concave real valued functions f and g having both $\mathbb{R}_+$ as domain. Given any bundle of public and private good $(\overline{Z}, \overline{x}) \in \mathbb{R}^2_+$, we define $MRS(\overline{Z}, \overline{x})$, the *marginal rate of substitution of public good to private good* evaluated at $(\overline{Z}, \overline{x})$, by:

$$MRS(\overline{Z}, \overline{x}) = \frac{\partial U(\overline{Z}, \overline{x})/\partial Z}{\partial U(\overline{Z}, \overline{x})/\partial x} \tag{2}$$

As in Gravel and Thoron (2007), we find useful to express the condition on households preferences that is necessary and sufficient for the segregation of a stable jurisdiction in terms of the properties of the *Marshallian demands* associated with these preferences. For this purpose, we denote by $Z^M(p_Z, p_x, R)$ and $x^M(p_Z, p_x, R)$ the households' Marshallian demands for the public and the private good (respectively) when the prices for these two goods are p_Z and p_x and the household's income is R. Marshallian demand functions are the (unique under our condition) solution of the program:

$$\max_{Z,x} U(Z, x) \text{ subject to } p_Z Z + p_x x \leq R$$

Given the assumptions on U, Marshallian demands are differentiable functions of their arguments (except, possibly, at the boundary of $\mathbb{R}^2_+$). We emphasize that we view Marshallian demands as dual representations of preferences rather than description of behavior (households do not purchase local public good on competitive markets)/

We further assume that Marshallian demand for public good satisfies the following additional *regularity* condition (see Gravel and Thoron (2007) for a discussion of the meaning of this condition).

Condition 1: If there exists a public good price $\overline{p}_Z$, an income level R and a non-degenerate interval I of strictly positive real numbers such that, $Z^M(\overline{p}_Z, p_x, R) = Z^M(\overline{p}_Z, p'_x, R)$ for all prices p'_x and p_x in I, then, for all

[1] Our terminology for concavity and quasi-concavity of a function $f : A \to \mathbb{R}$ ($A \subseteq \mathbb{R}^k$) is as follows: f is strictly concave if, for every $\alpha \in]0, 1[$ and for every distinct a, $b \in A$, $f(\alpha a + (1-\alpha)b) > \alpha f(a) + (1-\alpha) f(b)$ and f is quasi-concave if, for every a, b, $x \in A$, and every $\alpha \in [0, 1]$, $f(a) \geq f(x)$ and $f(b) \geq f(x)$ imply $f(\alpha a + (1-\alpha)b) \geq f(x)$.

$(p_Z, p_x, R) \in \mathbb{R}^3_+$, we must have $Z^M(p_Z, p_x, R) = h(p_Z, R)$ for some function $h : \mathbb{R}^2_{++} \to \mathbb{R}_+$.

The problem considered in this paper is that of identifying the properties of the various *jurisdiction structures* that can emerge when households freely choose their location and share, when they choose the same location, the benefit of a local public good produced by local tax revenues and a central government grant. This intervention of the central government is the main distinctive ingredient of our model as compared to what is done in the literature (e.g. Gravel and Thoron (2007), Westhoff (1977), Greenberg (1983), Greenberg and Weber (1986) and Demange (1994)).

From a formal point of view, we view a jurisdiction structure with a central government as a triplet $S = (\{N_j\}_{j=1}^l, g, t)$ made of:

1) a collection $\{N_j\}_{j=1}^l$ of l non-empty sets of households, to be referred to as jurisdictions, satisfying $N_j \cap N_{j'} = \varnothing$ for all j and j' (with $j \neq j'$) in $\{1, ..., l\}$ and $\bigcup_{j=1} N_j = N$,

2) a vector $g = (g_1, ..., g_l\} \in \mathbb{R}^l$ of central government grants satisfying $\sum_{j=1}^{l} g_j = 0$ and $g_j \geq -\overline{\omega}_j$ for every $j \in \{1, ..., l\}$ where $\overline{\omega}_j = \sum_{i \in N_j} \omega_i$,

3) a vector $t = (t_1, ..., t_l) \in \mathbb{R}^l$ of local tax rates satisfying, for every $j = 1, ..., l$, $t_j \in [\frac{-g_j}{\overline{\omega}_j}, 1]$

In words, a jurisdiction structure with a central government is a partition of the set of households into l jurisdictions (condition 1), each of which receiving a (possibly negative) central government grant and being characterized by a (possibly negative) local tax rate. Condition 2) requires the central government to balance its budget so that the grants given to some jurisdictions can only come from taxes raised on others. It also limits the fiscal power of the central government to raise taxes in a given jurisdiction to the extent of this jurisdiction's tax base. Condition 3) requires jurisdiction's local tax rates to be less than one and greater than (the negative of) the ratio of the central government grant over the jurisdiction tax base. Since in every jurisdiction j, household h in N_j has access to $t_j\overline{\omega}_j + g_j$ units of public good and has $(1 - t_j)\omega_h$ units of wealth available for private consumption, the condition that $t_j \in [\frac{-g_j}{\overline{\omega}_j}, 1]$ guarantees that both $t_j\overline{\omega}_j + g_j \geq 0$ and $(1 - t_j)\omega_h \geq 0$ are satisfied. We notice that negative local tax rates are not unlikely in a world with a central government. A household living in a jurisdiction that receives a large grant may prefer local tax rate to be negative, and, therefore, use part of the central government grant in private spending.

Given any jurisdiction structure, we denote by n_j the number of households in N_j (with of course $\sum_{j=1}^{l} n_j = n$), and we write the set N of households as $N = \{1, ..., n_1, n_1 + 1, ..., n_1 + n_2, n_1 + n_2 + 1, ..., n - n_l + 1, ..., n\}$

in such a way that, for all $m = 0, ..., l-1$, one has $\omega_h \leq \omega_{h+1}$ for $h \in \{\sum_{i=0}^{m} n_i + 1, ..., \sum_{i=0}^{m+1} n_i\}$, using the convention that $n_0 = 0$.

We notice that the central government is not allowed to give (take) money directly to (from) households. If positive, the grant received from a central government can only be used to produce public good and, if negative, can only be taken out of the jurisdiction local tax revenue. While this modeling of the central government is certainly not representative of all kinds of central government intervention in multi-jurisdiction settings, it does not provide a bad description of the working of the horizontal equalization mechanisms such as those observed in Scandinavian countries.

Denote by $\Phi(\tau, \varpi, \omega_i, \gamma) = U(\tau\varpi + \gamma, (1-\tau)\omega_i)$ the utility received by a household with wealth ω_i in a jurisdiction with a local tax rate of τ, an aggregate wealth of ϖ and a central government grant of γ (assuming again that $\tau\varpi + \gamma \geq 0$ and $(1-\tau)\omega_i \geq 0$). The function $\Phi : [0,1] \times \mathbb{R}_+^2 \times \mathbb{R} \to \mathbb{R}$ so defined has several properties that we record in the following lemma (whose straightforward proof is omitted).

Lemma 1 *If U satisfies the conditions discussed above, Φ is a twice differentiable function of its four arguments, is strictly increasing and concave with respect to ω_i, ϖ and γ (taking τ as given) and is strictly concave and single peaked*[2] *with respect to τ (taking ω_i, ϖ and γ as given).*

The important property of Φ is its strict single peakedness. It implies that a household with wealth ω_i has a unique favorite tax rate $\tau^*(\varpi, \omega_i, \gamma)$ in any jurisdiction with wealth ϖ and central government grant γ to which it may belong. This unique favorite tax rate is the solution of the program:

$$\max_{\tau \in [\frac{-\gamma}{\varpi}, 1]} \Phi(\tau, \overline{\omega}, \omega_i, \gamma) \tag{3}$$

and is, for this reason, a continuous function of all its three arguments.

We are interested in the properties of the likely outcome of a free choice of a location by households in the presence of a central government. This likely outcome must be such that:

1) each household finds its location optimal, given central government redistribution and local choices of tax rate, assuming that it can move freely between locations and that it has no effect on jurisdictions' choices of tax rates and aggregate wealth,

2) the central government finds optimal its vector of equalization grants, given the partition of individuals into jurisdictions and jurisdictions' choices of tax rates

[2] A function $f : A \to \mathbb{R}$ ($A \subset \mathbb{R}$) is strictly single peaked if, for all a, b and $c \in A$ such that $a < b < c$, $f(c) > f(b) \Rightarrow f(b) > f(a)$ and $f(a) > f(b) \Rightarrow f(b) > f(c)$.

3) each jurisdiction finds its choice of tax rate and public good provision optimal, given its population, tax base and central government grant.

Concerning the last point, we adopt the view that each jurisdiction's choice of tax rate is minimally democratic in the sense that it is contained *between* the *smallest* and the *largest* favorite tax rates of the jurisdiction members. The rule for selecting this tax rate is inconsequential for the results. In many models of endogenous jurisdiction formation with public good provision where voting is assumed, the jurisdiction tax rate would the one that occupies the *median position* in the jurisdiction's distribution of favorite taxes. While the analysis of this paper applies to this particular rule of selection of local tax rates, they are valid for other rules as well. Given a jurisdiction structure $(\{N_j\}_{j=1}^{l}, g, t)$, define, for every jurisdiction $j \in \{1, ..., l\}$ the smallest and largest favorite tax rates t_{j*} and t_j^* respectively by:

$$\begin{aligned} t_{j*} &= \min_{i \in N_j} \arg \max_{\tau \in [\frac{-g}{\overline{\omega}}, 1]} \Phi(\tau, \overline{\omega}_j, \omega_i, g_j) \text{ and} \\ t_j^* &= \max_{i \in N_j} \arg \max_{\tau \in [\frac{-g}{\overline{\omega}}, 1]} \Phi(\tau, \overline{\omega}_j, \omega_i, g_j) \end{aligned}$$

We shall therefore assume that, in any jurisdiction j, the local tax rate t_j satisfies $t_j \in [t_{j*}, t_j^*]$.

As for the objective of the central government, we represent it by means of a social ordering[3] R^S, depending upon the jurisdiction structure S, and defined on the set $\mathbb{G}(S)$ of all vectors $G \in \mathbb{R}^l$ such that $g_j + t_j\overline{\omega}_j \geq 0$. For any g and $g' \in \mathbb{G}(S)$, we interpret the statement $g\ R^S\ g'$ as meaning "g is socially weakly better than g'". We further assume that R^S is welfarist (see e.g. Sen (1977b) or, for a recent account, Blackorby, Bossert, and Donaldson (2005)) in the sense that it compares alternative vectors of grants on the sole basis of the *distribution of utilities* achieved by households in the various jurisdictions of S as a result of these grants. Formally, R^S is welfarist if there exists an ordering R^U on the set $\mathbb{R}^n$ of utility vectors such that:

$$g\ R^S g' \Longleftrightarrow$$
$$\Phi(t_1, \overline{\omega}_1, \omega_1, g_1), ..., \Phi(t_l, \overline{\omega}_l, \omega_n, g_l)\ R^U\ \Phi(t_1, \overline{\omega}_1, \omega_1, g_1'), ..., \Phi(t_l, \overline{\omega}_l, \omega_n, g_l')$$

As is well-known, welfarism requires utilities to be interpersonally comparable and, sometimes, cardinally measurable as well. Following Deschamps and Gevers (1978), we adopt the view that utilities are cardinally measurable and interpersonally comparable. We also assume that the ordering R^U of utility vectors on which the central government ordering R^S is based is *Pareto inclusive* (increasing households utility levels *ceteris paribus* improves matters), *anonymous* (individual names don't matter) and satisfies

[3] An ordering is a reflexive, complete and transitive binary relation.

the so-called principle of "*independence with respect to unconcerned*" households (see e.g. Blackorby, Bossert, and Donaldson (2005), (p. 115)) according to which the ranking of two states should be independent from the utilities of the households who are indifferent between the states. We also assume that the central government has a minimal aversion to utility inequality as expressed in Hammond (1976)'s *minimal equity principle*.

With this specification of the central government's objective, we define formally our notion of stability as follows.

Definition 1 *A jurisdiction structure* $S = (\{N_j\}_{j=1}^{l}, g, t)$ *with a welfarist central government endowed with a social objective* R^S *is stable if*
1) for all $j \in \{1, ..., l\}$ *and all* $i \in N_j$, $\Phi(t_j, \overline{\omega}_j, \omega_i, g_j) \geq \Phi(t_{j'}, \overline{\omega}_{j'}, \omega_i, g_{j'})$ *for all* $j' \in \{1, ..., l\}$
2) for all $j \in \{1, ..., l\}$, $t_j \in [t_{j*}, t_j^*]$
3) $g \; R_S \; g'$ *for all* $g' \in \mathbb{G}(S)$.

This definition of stability rides on the assumption that households as well as local and central governments take their decision simultaneously, considering as given the behavior of others. While this way of proceeding generalizes naturally the (atemporal) Westhoff framework, it may be viewed as limiting the power of the central government to shape the process of jurisdiction formation. An alternative would have been to assume a *two-stage* setting in which the central government, like a Stackelberg leader, would play before households and local governments and would choose its redistributive grants by anticipating the impact of its choice on the stable jurisdiction structure that would emerge in a second stage. While this alternative strategy would have given more power to the central government to influence jurisdiction structures, it would have raised delicate modeling issues. First, choosing a set of equalization grants before knowing the jurisdiction structure that will prevail raises the problem of the financial viability of the equalizations grants. What if the central government choose in a first stage a grant for a jurisdiction which, after while, will be empty ? Second, and more importantly, there are many partitions of individuals into jurisdictions that can give rise to a stable jurisdiction structure if equalization payments scheme are appropriately chosen. Of course, everything else being the same, the central government, at least if it uses a Pareto inclusive social welfare function, would have a tendency to favour the "trivial" structure in which all households are in the same jurisdiction (and in which case the central government plays no role). The reason for this tendency is that, the local public good being non-rival, producing any quantity of it in a larger jurisdiction is always preferable from a social welfare view point because its cost can be shared by a larger number of tax payers. This of course does not mean that a central government who could play "before" local governments and households would always choose to grouping all households into a same

unique (grand) jurisdiction. As shown in Gravel and Poitevin (2005), a central government who can influence the jurisdiction structure may prefer separating households in several distinct jurisdictions rather than grouping them into a central one in order to fit better their different willingness to pay for the public good. Yet, the problem faced by the central government in that case is rather complex and we have chosen herein to ignore this possibility for the central government to play "before" the other agents

Turning now to the central government, we know from Deschamps and Gevers (1978) that there are *only two* welfarist social orderings that satisfy the properties mentioned above (Pareto, anonymity, independence with respect to unconcerned individuals, and Hammond's equity) and that use a cardinally meaningful and interpersonally comparable information on households' utilities. These are the *leximin* and the *utilitarian* ordering. The leximin ordering compares social states by lexicographically comparing the households' *positions* in the distribution of utilities associated to the states, with utilities ordered from the smallest to the largest. Formally, for every utility vector $u \in \mathbb{R}^n$, let $u_{(.)}$ denote the (ordered) permutation of u such that $u_{(i)} \leq u_{(i+1)}$ for $i = 1, ..., n-1$. The leximin ordering R^U_{LMIN} of the set $\mathbb{R}^n$ of all possible utility vectors is defined as follows:

$$\begin{aligned} u \; P^U_{LMIN} \; u' \quad &\Leftrightarrow \quad \exists \; i \in N \text{ such that } u_{(h)} = u'_{(h)} \text{ for all } h < i \text{ and } u_{(i)} > u'_{(i)} \\ u \; I^U_{LMIN} \; u' \quad &\Leftrightarrow \quad u_{(h)} = u'_{(h)} \text{ for all } h \in \{1, ..., n\} \text{ and} \\ u \; R^U_{LMIN} \; u' \quad &\Leftrightarrow \quad u \; P^U_{LMIN} \; u' \text{ or } u \; I^U_{LMIN} \; u' \end{aligned}$$

As it turns out, there is not much analysis to be performed with a Leximin central government. For, as established in the following proposition, and with one uninteresting exception, the only stable jurisdiction structure that can exist with a Leximin central government is the trivial jurisdiction structure in which all households are in the same jurisdiction.

Proposition 1 *A jurisdiction structure $S = (\{N_j\}_{j=1}^l, g, t)$ with a Leximin central government is stable iff either $l = 1$ or $\omega_1 = \omega_{n_1+1} = \omega_{n_1+n_2+1} = ... = \omega_{n-n_l+1}$.*

Proof. *By contraposition, assume that $S = (\{N_j\}_{j=1}^l, g, t)$ is a jurisdiction structure with a welfarist central government that is stable and such that $l \neq 1$ and that there exist j and $j' \in \{1, ..., l\}$ for which $\omega_{n_1+...+n_{j'-1}+1} \neq \omega_{n_1+...+n_{j-1}+1}$. We wish to show that the welfarist central government can not be leximin. Without loss of generality, assume that jurisdiction j contains one of the worse off household in the whole population so that:*

$$\Phi(t_j, \overline{\omega}_j, \omega_{n_1+...+n_{j-1}+1}, g_j) \leq \Phi(t_k, \overline{\omega}_k, \omega_i, g_k) \tag{4}$$

for all $k \in \{1, ..., l\}$ and $i \in N_k$. By stability one has:

$$\Phi(t_j, \overline{\omega}_j, \omega_{n_1+...+n_{j-1}+1}, g_j) \geq \Phi(t_{j'}, \overline{\omega}_{j'}, \omega_{n_1+...+n_{j-1}+1}, g_{j'}) \tag{5}$$

and

$$\Phi(t_{j'}, \overline{\omega}_{j'}, \omega_{n_1+...+n_{j'-1}+1}, g_{j'}) \geq \Phi(t_j, \overline{\omega}_j, \omega_{n_1+...+n_{j'-1}+1}, g_j) \qquad (6)$$

Either (i) $\omega_{n_1+...+n_{j'-1}+1} < \omega_{n_1+...+n_{j-1}+1}$ *or (ii)* $\omega_{n_1+...+n_{j'-1}+1} > \omega_{n_1+...+n_{j-1}+1}$. *If (i) holds, then using (5) and the fact that* Φ *is increasing with respect to private wealth, one gets:*

$$\Phi(t_j, \overline{\omega}_j, \omega_{n_1+...+n_{j-1}+1}, g_j) > \Phi(t_{j'}, \overline{\omega}_{j'}, \omega_{n_1+...+n_{j'-1}+1}, g_{j'})$$

in contradiction with (4). If (ii) holds, one gets, using (6) and the monotonicity of Φ *with respect to private wealth, that:*

$$\Phi(t_{j'}, \overline{\omega}_{j'}, \omega_{n_1+...+n_{j'-1}+1}, g_{j'}) > \Phi(t_j, \overline{\omega}_j, \omega_{n_1+...+n_{j-1}+1}, g_j)$$

so that household $n_1 + ... + n_{j-1} + 1$ *is strictly worse off than household* $n_1 + ... + n_{j'-1} + 1$ *who is the poorest (and therefore the worse off by monotonicity of* Φ*) in jurisdiction* j'. *Now, let* $W = \{k \in \{1, ..., l\} : \Phi(t_k, \overline{\omega}_k, \omega_{n_1+....+n_{k-1}+1}, g_k) = \Phi(t_j, \overline{\omega}_1, \omega_{n_1+...+n_{j-1}+1}, g_j)\}$ *and let, for all* $k \in W$, $\min(k)$ *be defined by:*

$$\min(k) = \{i \in \{\sum_{h=1}^{k-1} n_h + 1, ..., \sum_{h=1}^{k} n_h\} : \omega_i = \omega_{n_1+....+n_{k-1}+1}\}.$$

W *is clearly non-empty since* $j \in W$. *Moreover, if* $k \in W$ *and* $k \neq j$, *then* $\omega_{n_1+....+n_{k-1}+1} = \omega_{n_1+...+n_{j-1}+1}$. *Indeed assume by contradiction that* $\omega_{n_1+....+n_{k-1}+1} \neq \omega_{n_1+...+n_{j-1}+1}$. *Either (i)* $\omega_{n_1+...+n_{k-1}+1} < \omega_{n_1+...+n_j+1}$ *or (ii)* $\omega_{n_1+...+n_{k-1}+1} > \omega_{n_1+...+n_{j-1}+1}$.*As above, case (i) is incompatible, given (5), with (4) while case (ii) is incompatible, given (6), with the definition of* W *that* $\Phi(t_k, \overline{\omega}_k, \omega_{n_1+....+n_{k-1}+1}, g_k) = \Phi(t_j, \overline{\omega}_1, \omega_{n_1+...+n_{j-1}+1}, g_j)$ *for all* $k \in W$. *Let* $A = \bigcup_{k \in W} \min(k)$. *Hence* A *is the set of all households who are the worse off in the whole population in the jurisdiction structure* $S = (\{N_j\}_{j=1}^{l}, g, t)$. *By the reasoning just made, all households in* A *have the same wealth. Moreover* A *is a finite set. Now consider taking away from jurisdiction* j' *some amount of grant* Δ *and dividing it up equally among all jurisdictions in* k *so as to keep constant the central government budget constraint. For a suitably small* Δ, *this change in the central government transfer policy increase the well-being of all households in* A. *This shows that the original equalization grant vector* g *was not maximizing a Leximin ordering.* ■

Hence, except the unlikely jurisdiction structure in which all individuals who are the poorest of their jurisdictions have the same wealth, the only stable jurisdiction structure that can exist with a leximin central government

is the trivial one in which all individuals are pooled in the same jurisdiction. The intuition behind this result is quite simple. A leximin government wants to transfer money to the jurisdiction that contains the worse off households (who are clearly the poorest in their jurisdiction). By stability, these worse off households prefer staying in their jurisdictions than moving elsewhere while the poorest households in other jurisdictions also prefer staying where they are than moving to the jurisdiction containing the worse off households. Except if the wealth of these poorest households in all jurisdictions is the same, these two conditions for stability imply that the worse off household is strictly worse off than at least one household who is the poorest in its jurisdiction. But if this is the case, then the transfers provided by the central government are not optimal from a leximin point of view.

Hence, if one wants to go beyond this nihilistic message that no analysis of stable jurisdiction structures can be performed in presence of a central government, we must abandon the assumption that it could be Leximin and focus on the other possibility that it is Utilitarian.

For a Utilitarian government, there are several stable jurisdiction structures that differ from the (trivial) grand jurisdiction one, as illustrated in the following simple example.

Example 1 *There are seven households with utility function* $U(Z, x) = \ln(1 + Z) + x$*. Three households have a wealth of 2 and four households have a wealth of* $3/2$*. Consider the jurisdiction structure with two jurisdictions in which there is no redistribution by the central government (*$g = (0, 0)$*) and where the 2 households with wealth 2 are put in a jurisdiction while the four households with wealth* $3/2$ *are put in another. The tax rates* t_1 *and* t_2 *that will prevail in the two jurisdictions will be the favorite ones of the identical households who live there. Solving program (3), these optimal tax rates are easily found to be* $t_1 = 1/3$ *and* $t_2 = 1/2$*. We notice that with these tax rates, any household in jurisdiction 1 enjoys a utility level of* $\ln 3 + 4/3 \approx 2.4319$*, which is larger than the utility of* $\ln 4 + 1 \approx 2.386$ *the household would enjoy if it were to move to jurisdiction 2 and get the tax and public good package available there. Analogously, a household in jurisdiction 2 enjoys a utility level of* $\ln 4 + 3/4 \approx 2.1363$ *by staying where it is while the move to jurisdiction 1 would provide this household with a (lower) utility of* $\ln 3 + 1 \approx 2.0986$*. Hence households have no incentive to move from their jurisdiction. To see that a Utilitarian central government find optimal to perform no redistribution between jurisdictions, we must show that*

$$0 \in \arg\max_{\gamma} 3[\ln(3 + \gamma) + \frac{4}{3}] + 4[\ln(4 - \gamma) + \frac{3}{4}]$$

so that the following (first order) condition holds:

$$\frac{3}{3} = \frac{4}{4}$$

which is indeed the case. Hence we have a non trivial stable structure with a central government.

We want to find a condition on the household's utility function used by the utilitarian social planner that is necessary and sufficient for guaranteeing that any stable jurisdiction structure with a utilitarian central government will be wealth-segregated. This requires one to define what is meant by a wealth-segregated jurisdiction structure. We take the definition to be that used in Gravel and Thoron (2007) (see also Westhoff (1977) and Greenberg and Weber (1986)).

Definition 2 *A jurisdiction structure with a central government* $S = (\{N_j\}_{j=1}^{l}, g, t)$ *is wealth-segregated if, for every jurisdictions* j, $j' \in \{1, ..., l\}$, *and every households* h, i *and* $k \in N$, $(h,\ k \in \{\sum_{e=1}^{j-1} n_e + 1, ..., \sum_{e=1}^{j} n_e\},\ \omega_h < \omega_i < \omega_k$ *and* $i \in \{\sum_{e=1}^{j-1} n_e + 1, ..., \sum_{e=1}^{j} n_e\}) \Rightarrow t_j = t_{j'} = \frac{g_{j'} - g_j}{\overline{\omega}_j - \overline{\omega}_{j'}}$.

In words, a jurisdiction structure is wealth-stratified if, whenever a jurisdiction contains two households h and k with different levels of wealth, it also contains all households whose wealth levels are strictly between that of h and k or, if it does not contain those households, it is because they belong to some jurisdiction j' that offers the same tax rate and the same amount of public good than j (a logical, if not likely, possibility).

3 RESULTS

As in Gravel and Thoron (2007), the monotonicity of τ^* with respect to household's wealth (given jurisdiction's wealth and central government grant) will be a key element for guaranteeing the wealth segregation of stable jurisdiction structures. This property of monotonicity of the household's most preferred tax rate can be expressed conveniently in terms of standard consumer theory. In order to do this, we establish the following lemma.

Lemma 2 *Let* $(\overline{\omega}, \omega_i, \gamma) \in \mathbb{R}^2_{++} \times \mathbb{R}$. *Then for all* U *satisfying the above properties,* $\frac{1}{\overline{\omega}}(Z^M(\frac{1}{\overline{\omega}}, \frac{1}{\omega_i}, 1 + \frac{\gamma}{\overline{\omega}}) - \gamma)$ *is the solution of (3).*

Proof. *Since* $\frac{x^M(\frac{1}{\overline{\omega}}, \frac{1}{\omega_i}, 1+\frac{\gamma}{\overline{\omega}})}{\omega_i} \geq 0$ *and* $\frac{x^M(\frac{1}{\overline{\omega}}, \frac{1}{\omega_i}, 1+\frac{\gamma}{\overline{\omega}})}{\omega_i} \geq 0$, *the fact that the bundle* $(Z^M(\frac{1}{\overline{\omega}}, \frac{1}{\omega_i}, 1 + \frac{\gamma}{\overline{\omega}}),\ x^M(\frac{1}{\overline{\omega}}, \frac{1}{\omega_i}, 1 + \frac{\gamma}{\overline{\omega}}))$ *satisfies the consumer's budget constraint* $\frac{1}{\overline{\omega}} Z^M(\frac{1}{\overline{\omega}}, \frac{1}{\omega_i}, 1 + \frac{\gamma}{\overline{\omega}}) + \frac{x^M(\frac{1}{\overline{\omega}}, \frac{1}{\omega_i}, 1+\frac{\gamma}{\overline{\omega}})}{\omega_i} = 1 + \frac{\gamma}{\overline{\omega}} \Longleftrightarrow \frac{1}{\overline{\omega}}(Z^M(\frac{1}{\overline{\omega}}, \frac{1}{\omega_i}, 1 + \frac{\gamma}{\overline{\omega}}) - \gamma) + \frac{x^M(\frac{1}{\overline{\omega}}, \frac{1}{\omega_i}, 1+\frac{\gamma}{\overline{\omega}})}{\omega_i} = 1$ *implies obviously that* $\frac{1}{\overline{\omega}}(Z^M(\frac{1}{\overline{\omega}}, \frac{1}{\omega_i}, 1 + \frac{\gamma}{\overline{\omega}}) - \gamma) \in$

$[-\frac{\gamma}{\overline{\omega}}, 1]$. *Suppose by contradiction that* $\frac{1}{\overline{\omega}}(Z^M(\frac{1}{\overline{\omega}}, \frac{1}{\omega_i}, 1+\frac{\gamma}{\overline{\omega}}) - \gamma)$ *does not solve (3). That is, suppose that there exists* $\widehat{\tau} \in [0,1]$ *such that:*

$$U(\widehat{\tau}\overline{\omega} + \gamma, (1-\widehat{\tau})\omega_i) >$$

$$\begin{aligned} & U(\frac{1}{\overline{\omega}}(Z^M(\frac{1}{\overline{\omega}}, \frac{1}{\omega_i}, 1+\frac{\gamma}{\overline{\omega}}) - \gamma)\overline{\omega} + \gamma, (1 - \frac{1}{\overline{\omega}}(Z^M(\frac{1}{\overline{\omega}}, \frac{1}{\omega_i}, 1+\frac{\gamma}{\overline{\omega}}) - \gamma))\omega_i)) \\ = \; & U(Z^M(\frac{1}{\overline{\omega}}, \frac{1}{\omega_i}, 1+\frac{\gamma}{\overline{\omega}}), x^M(\frac{1}{\overline{\omega}}, \frac{1}{\omega_i}, 1+\frac{\gamma}{\overline{\omega}})). \end{aligned}$$

Since the bundle $(\widehat{\tau}\overline{\omega} + g, (1-\widehat{\tau})\omega_i)$ *satisfies the (budget) constraint* $\frac{\widehat{\tau}\overline{\omega}+\tau}{\overline{\omega}} + \frac{(1-\widehat{\tau})\omega_i}{\omega_i} = 1 + \frac{\gamma}{\overline{\omega}}$, *this inequality is incompatible with the very definition of* $Z^M(\frac{1}{\overline{\omega}}, \frac{1}{\omega_i}, 1+\frac{\gamma}{\overline{\omega}})$ *and* $x^M(\frac{1}{\overline{\omega}}, \frac{1}{\omega_i}, 1+\frac{\gamma}{\overline{\omega}})$. ■

Lemma 2 states that, in a jurisdiction with aggregate wealth $\overline{\omega}$ and central government transfer γ, the favorite tax rate of a household with wealth ω_i can be viewed as the *expenditure* that the household would like to devote to the public good in excess of the central government grant if the prices of public and the private goods were $\frac{1}{\overline{\omega}}$ and $\frac{1}{\omega_i}$, and if this household had an income of $1 + \frac{\gamma}{\overline{\omega}}$. Interpreted in this fashion, the monotonicity of τ^* with respect to ω_i is *equivalent* to the monotonicity of the Marshallian demand for public good with respect to the price of the private good. In the language of standard consumer theory, this is equivalent to requiring the public good to be, at any price of public good, either always a gross *complement* to (if Z^M is monotonically *decreasing* with respect to p_x) or always a gross *substitute* for (if Z^M is monotonically *increasing* with respect to p_x) the private good.

As discussed in Gravel and Thoron (2007), without further assumptions on the household's preference, it is possible for the public good to be always a gross substitute of the private good at some price of the public good while being always a gross complement to the private good at some other price of the public good. Yet this possibility is ruled out if condition 1 above is imposed on the utility function.

Lemma 3 *For every* $U \in \mathbb{U}$, *the function* τ^* *that solves (3) is monotonic with respect to* ω_i *for any given jurisdiction level* $\overline{\omega}$ *and per capita wealth if and only if the public good is always either a gross complement to, or a gross substitute for, the private good.*

Let us refer to this property according to which the substitutability/ complementarity relationship between the public and private good is independent from all possible prices as to the **Gross Substitutability/ Complementarity (GSC) condition**. Although not unreasonable, the GSC condition is nonetheless a significant restriction that, as discussed in Gravel and Thoron (2007), can be violated even by additively separable preferences.

An information used in Gravel and Thoron (2007) to show that this condition is necessary and sufficient for guaranteeing the segregation of any stable jurisdiction structure is the structure of households' indifference curves in the tax-jurisdiction's wealth space. While this information is also useful in the present context, we need to account for the fact that the relevant space is now tree, rather than two, dimensional and must include central government grant as well as local tax rate and jurisdiction's aggregate wealth. The indifference surface of a household with wealth ω_i passing through some point $(\overline{\tau}, \overline{\omega}, \overline{\gamma})$ such that $\Phi(\overline{t}, \overline{\omega}, \omega_i, \overline{\gamma}) = \overline{\Phi}$ is the graph of the implicit function $f^{\overline{\Phi}} : [\frac{-\overline{\gamma}}{\overline{\omega}}, 1] \times \mathbb{R} \times \mathbb{R}_+ \longrightarrow \mathbb{R}_+$ defined by $\Phi(\tau, f^{\overline{\Phi}}(\tau, \gamma; \omega_i), \gamma, \omega_i) \equiv \overline{\Phi}$. The assumption imposed on U guarantees that the function $f^{\overline{\Phi}}$ exists and is derivable everywhere. Its partial derivative $f_\tau^{\overline{\Phi}}(\overline{\tau}, \omega_i, \overline{\gamma})$ with respect to τ evaluated at $(\overline{\tau}, \omega_i, \overline{\gamma})$ is given by:

$$f_\tau^{\overline{\Phi}}(\overline{\tau}, \omega_i, \overline{\gamma}) = \frac{1}{\overline{\tau}}[\frac{\omega_i}{MRS(\overline{\tau\omega} + \overline{\gamma}, (1 - \overline{\tau})\omega_i)} - \overline{\omega}] \tag{7}$$

where $\overline{\omega} = (\overline{\tau}, \omega_i, \overline{\gamma})$. The figure 1 below illustrate the shape of these indifference curves in the $(\tau, \overline{\omega})$ plane for a given value of $\overline{\gamma}$. Specifically, indifference curves of a household with private wealth ω_i are U-shaped and reach a minimum at this household's most preferred tax rate for the corresponding jurisdiction wealth level. It can be seen indeed that, at the minimum of an indifference curve, the term within the bracket of (7) is zero thanks to the first order conditions of (3)). Despite what figure1 suggests, indifference curves need *not* be globally convex. The only property that indifference curves possess is that of being "single caved" (monotonically decreasing at the left of the minimum and monotonically increasing at the right).

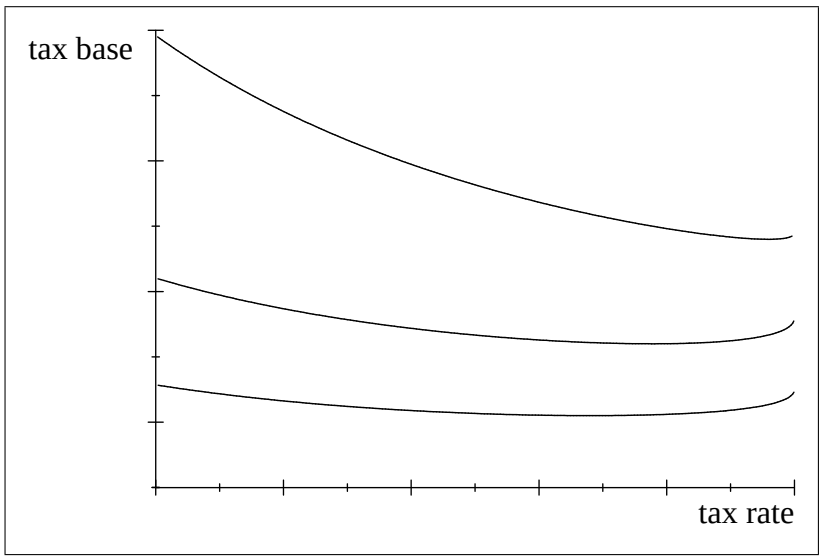

Figure 1

Analogously, one can fix the tax rate at $\overline{\tau}$ and examine the property of the derivative of $f^{\overline{\Phi}}$ with respect to the central government's grant. This partial derivative $f_{\gamma}^{\overline{\Phi}}(\overline{\tau}, \omega_i, \overline{\gamma})$ with respect to γ evaluated at $(\overline{\tau}, \overline{\gamma}; \omega_i)$ is given by:

$$f_{\gamma}^{\overline{\Phi}}(\tau, \overline{\gamma}; \omega_i) = \frac{-1}{\overline{\tau}} \tag{8}$$

Hence, when looked in the $(\gamma, \overline{\omega})$ space, indifference surfaces are straight line with negative slope (if at least $\overline{\tau}$ is positive). There is therefore a constant marginal trade off between tax base and central government grant as envisaged by a mobile household. This is of course not surprising since both central government grant and local tax base are perfectly substitutable ways of getting public expenditure in a given jurisdiction. The rate at which the household is willing to sacrifice local tax base in order to get more central government transfer depends obviously upon the local tax rate that converts tax base into public spending. The following picture shows a typical indifference surface in the tax rate, tax base and central government space in which location choice by households is made.

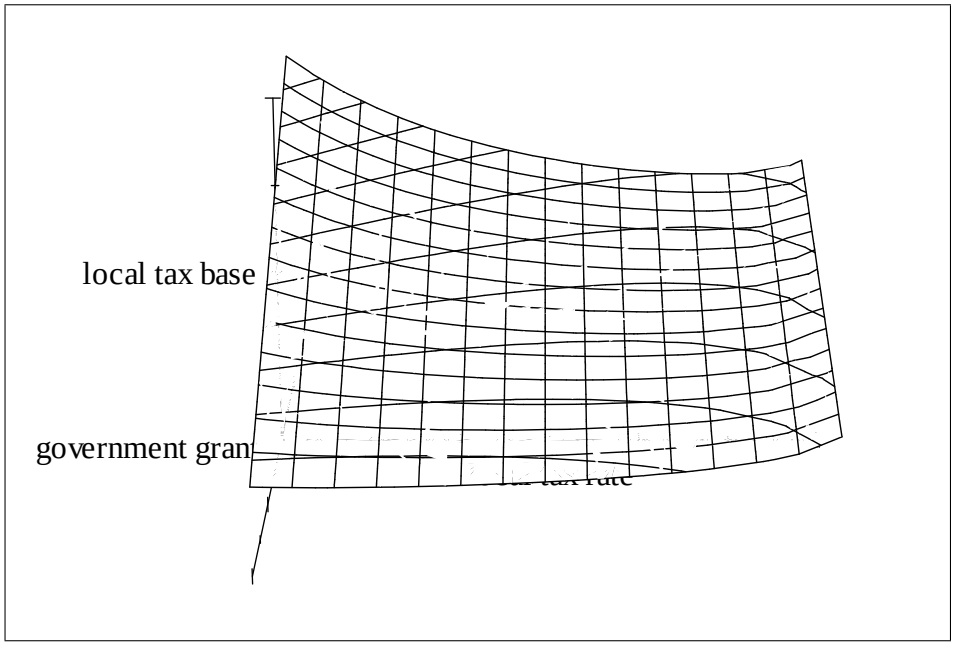

Figure 2

We first establish, in the following lemma, that the ordering of the *slopes* of these indifference curves at every point in the tax- jurisdictions wealth space (for a given level of central government grant) coincides with the ordering of the households' wealth if preferences for the public and the private good satisfy the GSC condition.

Lemma 4 *Assume that households preferences are represented by a utility function satisfying the properties mentioned above. Then, if Z^M is everywhere a gross substitute (resp. complement) to the private good, we have, at any $(\overline{\tau}, \overline{\omega}, \overline{\gamma}) \in]0,1[\times\mathbb{R}_{++}$, $f_{\tau}^{\overline{\Phi}_j}(\overline{\tau}, \overline{\gamma}; \omega_h) \leq$ (resp. $\geq$) $f_{\tau}^{\overline{\Phi}_k}(\overline{\tau}, \overline{\gamma}, \omega_k)$ for every h, k such that $\omega_h < \omega_k$ where, for every i, $\overline{\Phi}_i = \Phi(\overline{\tau}, \overline{\omega}, \overline{\gamma}; \omega_i)$.*

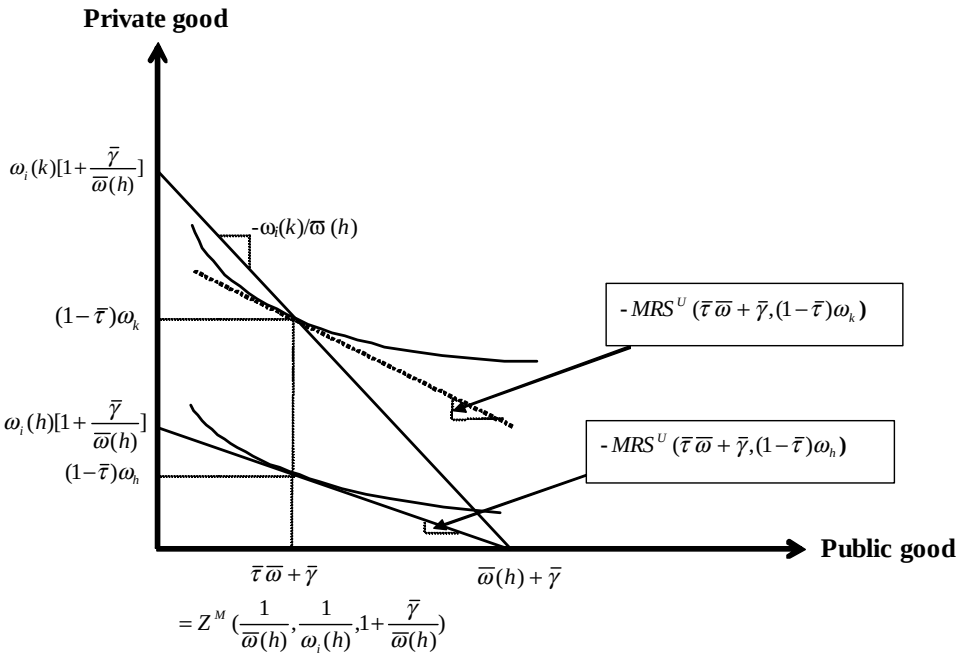

Figure 3.

Proof. *We provide the argument for the case of gross substitutability (the complementarity case being symmetric). Assume that Z^M is everywhere increasing with respect to p_x and let $(\overline{\tau}, \overline{\omega}, \overline{\gamma}) \in]0,1[\times\mathbb{R}_+ \times \mathbb{R}$ be a triple of tax rate, jurisdiction wealth and central government grant and let h and k be two households such that $\omega_h < \omega_k$. Refer to figure 3 and and define $\overline{\omega}(h)$ and $\omega_i(h)$ to be the numbers that generate public and private good prices $1/\overline{\omega}(h)$ and $1/\omega_i(h)$ which would lead a consumer with an income of $1 + \frac{\overline{\gamma}}{\overline{\omega}(h)}$ to choose the bundle $(\overline{\tau\omega}, (1-\overline{\tau})\omega_h)$ of public and private good. Hence $\overline{\omega}(h)$ and $\omega_i(h)$ satisfy the standard tangency and budget equality conditions:*

$$MRS^U(\overline{\tau\omega} + \overline{\gamma}, (1-\overline{\tau})\omega_j) = \frac{\omega_i(h)}{\overline{\omega}(h)}$$

and

$$\frac{\overline{\tau\omega} + \overline{\gamma}}{\overline{\omega}(h)} + \frac{(1-\overline{\tau})\omega_h}{\omega_i(h)} = 1 + \frac{\overline{\gamma}}{\overline{\omega}(h)}$$

Combining these two equations yields:

$$\frac{(1-\overline{\tau})}{\overline{\omega}(h)-\overline{\tau\omega}} = \frac{MRS^U(\overline{\tau\omega}+\overline{\gamma},(1-\overline{\tau})\omega_h)}{\omega_h} \tag{9}$$

Define now $\omega_i(k)$ to be a level of household wealth which would generate private good price $1/\omega_i(k))$ that is just sufficient to enable a consumer the same income of $1+\frac{\overline{\gamma}}{\overline{\omega}(h)}$ and facing public good price $1/\overline{\omega}(h)$ to afford the bundle $(\overline{\tau\omega}+\overline{\gamma},(1-\overline{\tau})\omega_k)$. This $\omega_i(k)$ (which is clearly larger than $\omega_i(h)$ if $\omega_k > \omega_h$) is defined by the budget constraint equality:

$$\begin{aligned} \frac{\overline{\tau\omega}+\overline{\gamma}}{\overline{\omega}(h)} + \frac{(1-\overline{\tau})\omega_k}{\omega_i(k)} &= 1+\frac{\overline{\gamma}}{\overline{\omega}(h)} \\ &\Longleftrightarrow \\ \omega_i(k) &= \frac{\overline{\omega}(h)(1-\overline{\tau})\omega_k}{\overline{\omega}(h)-\overline{\tau\omega}} \end{aligned} \tag{10}$$

Now, since Z^M is increasing with respect to p_x, we must have $Z^M(\frac{1}{\overline{\omega}(h)},\frac{1}{\omega_i(h))},1+\frac{\overline{\gamma}}{\overline{\omega}(h)}) \geq Z^M(\frac{1}{\overline{\omega}(h)},\frac{1}{\omega_i(k)},1+\frac{\overline{\gamma}}{\overline{\omega}(h)})$ and, therefore (see figure 3), the slope of the indifference curve passing through $(\overline{\tau\omega}+\overline{\gamma},(1-\overline{\tau})\omega_k)$ must be, in absolute value, less than the price ratio $\omega_i(k)/\overline{\omega}(h)$. Formally, this amounts to saying that:

$$MRS^U(\overline{\tau\omega}+\overline{\gamma},(1-\overline{\gamma})\omega_k) \leq \frac{\omega_i(k)}{\overline{\omega}(h)}$$

or, using (10):

$$\frac{MRS^U(\overline{\tau\omega}+\overline{\gamma},(1-\overline{\tau})\omega_k)}{\omega_k} \leq \frac{(1-\overline{\tau})}{\overline{\omega}(h)-\overline{t\omega}} \tag{11}$$

Combining inequality (11) and equality (9), we get:

$$\begin{aligned} \frac{MRS^U(\overline{\tau\omega}+\overline{\gamma},(1-\overline{\tau})\omega_k)}{\omega_k} &\leq \frac{MRS^U(\overline{\tau\omega}+\overline{\gamma},(1-\overline{\tau})\omega_h)}{\omega_h} \\ &\Longleftrightarrow \\ \frac{\omega_k}{MRS^U(\overline{\tau\omega}+\overline{\gamma},(1-\overline{\tau})\omega_k)} &\geq \frac{\omega_j}{MRS^U(\overline{\tau\omega}+\overline{\gamma},(1-\overline{\tau})\omega_h)} \end{aligned}$$

which, using the definition of $f_\tau^{\overline{\Phi}_j}$ provided by (7), establishes the result. ■

This lemma thus implies that the projection of the indifference surfaces of households with different wealth in the tax rate and jurisdiction wealth space are *single crossing* in that space for any exogenous central government grants. As they are ordered in the same direction for any level of central government grant, this means that the indifference surface themselves must have a tangent plane that is ordered as per the wealth.

We now establish that if the utility functions aggregated by the Utilitarian central government are additively separable, the GSC condition is *necessary* for the wealth segregation of any stable jurisdiction structure. That is to say, for any violation of the GSC condition, it is possible to construct a stable jurisdiction structure with a utilitarian central government who sum additively separable utility functions that is not segregated as per definition 2.

Proposition 2 *Stable jurisdictions structures with a utilitarian government who use additively separable utility functions are segregated for any economy only if the preferences represented by the utility function satisfies the GSC condition.*

Proof. *Assume that the utility function used by the utilitarian planner is additively separable and that the GSC condition is violated. Hence, there exist individual wealth levels a, b and $c \in \mathbb{R}_+$ satisfying $a < b < c$ such that*

$$\tau^*(a, \varpi, \overline{\gamma}) = t^*(c, \varpi, \overline{\gamma}) > t^*(b, \varpi, \overline{\gamma}) \tag{12}$$

for some aggregate wealth level ϖ and central government grant $\overline{\gamma}$ where τ^* is the solution of program (3). Using lemma 2, we can write (12) as

$$\begin{aligned}
\frac{1}{\varpi}(Z^M(\frac{1}{\varpi}, \frac{1}{a}, 1 + \frac{\overline{\gamma}}{\varpi}) - \overline{\gamma}) &= \frac{1}{\varpi}(Z^M(\frac{1}{\varpi}, \frac{1}{c}, 1 + \frac{\overline{\gamma}}{\varpi}) - \overline{\gamma}) \\
&> \frac{1}{\varpi}(Z^M(\frac{1}{\varpi}, \frac{1}{b}, 1 + \frac{\overline{\gamma}}{\varpi}) - \overline{\gamma}) \\
&\Leftrightarrow \\
Z^M(\frac{1}{\varpi}, \frac{1}{a}, 1 + \frac{\overline{\gamma}}{\varpi}) &= Z^M(\frac{1}{\varpi}, \frac{1}{c}, 1 + \frac{\overline{\gamma}}{\varpi}) \\
&> \frac{1}{\varpi}(Z^M(\frac{1}{\varpi}, \frac{1}{b}, 1 + \frac{\overline{\gamma}}{\varpi}) \\
&\Leftrightarrow \textit{(homogenous of degree 0 of Marshallian demand)} \\
Z^M(\frac{1}{\theta}, \frac{1}{\alpha}, 1) &= Z^M(\frac{1}{\theta}, \frac{1}{\delta}, 1) > Z^M(\frac{1}{\theta}, \frac{1}{\beta}, 1)
\end{aligned}$$

after defining θ, α, β and δ by:

$$\begin{aligned}
\theta &= \overline{\omega} + \overline{\gamma} \\
\alpha &= \frac{a(\overline{\omega} + \overline{\gamma})}{\overline{\omega}} \\
\beta &= \frac{b(\overline{\omega} + \overline{\gamma})}{\overline{\omega}} \\
\delta &= \frac{c(\overline{\omega} + \overline{\gamma})}{\overline{\omega}}
\end{aligned}$$

Hence, the existence of three individual wealth levels a, b and $c \in \mathbb{R}_+$ satisfying $a < b < c$ and such that (12) holds entails the existence of three wealth levels α, β and δ satisfying $\alpha < \beta < \delta$ such that $Z^M(\frac{1}{\theta}, \frac{1}{\alpha}, 1) =$

$Z^M(\frac{1}{\theta}, \frac{1}{\delta}, 1) > Z^M(\frac{1}{\theta}, \frac{1}{\beta}, 1)$ *holds for some* $\theta > 0$. *Consider now an economy with* n_α *households with wealth* α, n_β *households with wealth* β *and* n_δ *households with wealth* δ. *Let us construct a stable jurisdiction structure with a utilitarian government that is not segregated. Specifically, we are going to put households with wealth* α *and* δ *in one jurisdiction - called it 1- and households with wealth* β *in another, 2 say. For this purpose we are going to choose number of households* n_α, n_β *and* n_δ *so that:*

$$n_\alpha \alpha + n_\delta \gamma = n_\beta \beta = \theta \tag{13}$$

and

$$\frac{n_\beta}{n_\alpha + n_\delta} = \frac{\partial f(Z^M(\frac{1}{\theta}, \frac{1}{\alpha}, 1)/\partial Z}{\partial f(Z^M(\frac{1}{\theta}, \frac{1}{\beta}, 1)/\partial Z} \tag{14}$$

Equation (13) guarantees that the two jurisdictions have the same aggregate wealth θ. *Using lemma 2 we know that households in jurisdiction 1 will all chose tax rate* $t_1 = \frac{Z^M(\frac{1}{\theta}, \frac{1}{\alpha}, 1)}{\theta} = \frac{Z^M(\frac{1}{\theta}, \frac{1}{\delta}, 1)}{\theta}$ *while households in jurisdiction 2 will choose a tax rate of* $t_2 = \frac{Z^M(\frac{1}{\theta}, \frac{1}{\beta}, 1)}{\theta} < t_1$. *Without any central government redistribution, such a jurisdiction structure would be stable. The purpose of equation (14) is to characterize the optimal choice by the utilitarian social planner of zero redistribution across jurisdictions under the assumption that household's utility function is additively separable and writes therefore as per (1). The only things that remains to be shown is that we can always find numbers* n_α, n_β *and* n_γ *such that (13) and (14) hold. Clearly, since* α, β, δ *and* θ *are given, we have that* $n_\beta = \theta/\beta$. *Hence, we only need to find numbers* n_α *and* n_β *such that:*

$$n_\alpha = \frac{\theta}{\alpha} - \frac{\delta}{\alpha} n_\gamma \tag{15}$$

and:

$$n_\alpha = \frac{\theta \partial f(Z^M(\frac{1}{\theta}, \frac{1}{\beta}, 1))/\partial Z}{\beta \partial f(Z^M(\frac{1}{\theta}, \frac{1}{\alpha}, 1))/\partial Z} - n_\gamma$$

From the first order condition that defines Marshallian demands, we can write the later equation (using additive separability) as: :

$$n_\alpha = \frac{\theta \partial h(x^M(\frac{1}{\theta}, \frac{1}{\beta}, 1))/\partial x}{\alpha \partial h(x^M(\frac{1}{\theta}, \frac{1}{\alpha}, 1))/\partial x} - n_\gamma \tag{16}$$

Since the preferences represented by the utility function are additively separable, no good is inferior and, as a result, the private good is not a Giffen good. Hence x^M *is decreasing with respect to its own price so that, since* $\frac{1}{\beta} < \frac{1}{\alpha}$, $x^M(\frac{1}{\theta}, \frac{1}{\beta}, 1) > x^M(\frac{1}{\theta}, \frac{1}{\alpha}, 1)$ *and, since* h *is concave,* $\partial h(x^M(\frac{1}{\theta}, \frac{1}{\beta}, 1)/\partial x < \partial h(x^M(\frac{1}{\theta}, \frac{1}{\alpha}, 1)/\partial x$. *Hence the intercept of the linear equation (16) is smaller*

than that of equation (15). Moreover, the abscissa at the origin $n_\delta^0(1)$ *of equation (15) is given by:*

$$n_\delta^0(1) = \frac{\theta}{\delta}$$

while the abscissa at the origin $n_\delta^0(2)$ *of equation (16) is of course:*

$$\begin{aligned}
n_\gamma^0(2) &= \frac{\theta\partial h(x^M(\frac{1}{\theta},\frac{1}{\beta},1)/\partial x}{\alpha\partial h(x^M(\frac{1}{\theta},\frac{1}{\alpha},1)/\partial x} \\
&= \frac{\theta\partial f(Z^M(\frac{1}{\theta},\frac{1}{\beta},1)/\partial Z}{\beta\partial f(Z^M(\frac{1}{\theta},\frac{1}{\alpha},1)/\partial Z} \quad \textit{(by separability and the definition of Marshallian demands)} \\
&= \frac{\theta\partial f(Z^M(\frac{1}{\theta},\frac{1}{\beta},1)/\partial Z}{\beta\partial f(Z^M(\frac{1}{\theta},\frac{1}{\delta},1)/\partial Z} \quad \textit{(since } Z^M(\frac{1}{\theta},\frac{1}{\delta},1) = (Z^M(\frac{1}{\theta},\frac{1}{\alpha},1)\textit{)} \\
&= \frac{\theta\partial h(x^M(\frac{1}{\theta},\frac{1}{\beta},1)/\partial x}{\gamma\partial h(x^M(\frac{1}{\theta},\frac{1}{\delta},1)/\partial x} \quad \textit{(by separability and the definition of Marshallian demands)}
\end{aligned}$$

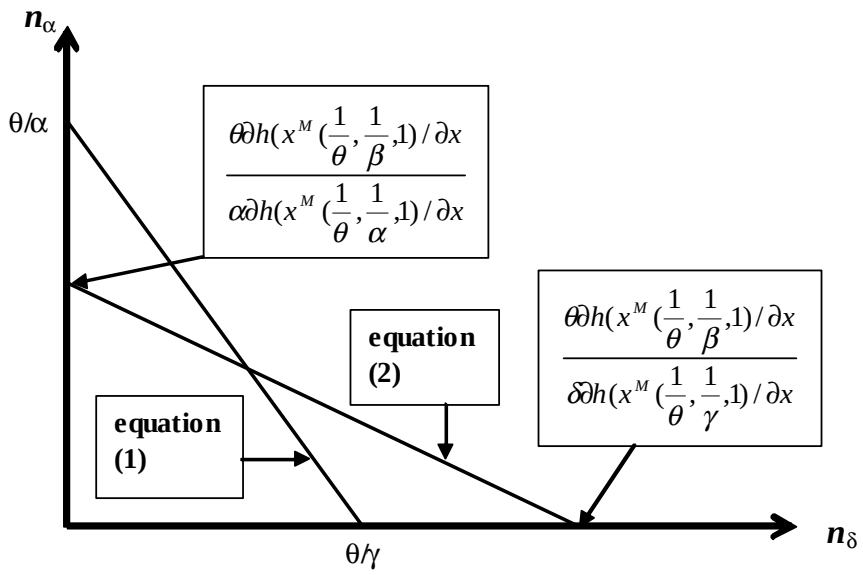

Figure 4.

Now, again, since the private good is not inferior (and therefore not Giffen) thanks to additive separability, we have that $x^M(\frac{1}{\theta},\frac{1}{\beta},1) < x^M(\frac{1}{\theta},\frac{1}{\delta},1)$ *and, by concavity of* h*, that* $n_\delta^0(2) > n_\delta^0(1) > 0$*. Hence, the two straight lines represented by equations (15) and (16) are as in figure 4) and cross in the*

strictly positive orthant. There is therefore a set of positive numbers n_α and n_δ that satisfy equations (15) and (16) and this completes the proof. ■

In the next proposition, we establish, without additive separability, the converse proposition that requiring households preferences to satisfy the GSC condition is sufficient for guaranteeing that any stable jurisdiction structure with a Utilitarian central government is wealth segregated.

Proposition 3 *Assume that households preference satisfy the GSC condition. Then, any stable jurisdiction structure with a Utilitarian central government is wealth segregated.*

Proof. *We sketch the argument for the case where the public good is everywhere a gross complement to the private good. Assume therefore that Z^M is decreasing with respect to p_x and, by contradiction, let $S = (\{N_j\}_{j=1}^l, g, t)$ be a jurisdiction structure that is not wealth-stratified. Hence, there are jurisdictions j and $j' \in \{1, ..., l\}$ (with $j \neq j'$), and households h, i and $k \in N$ with $\omega_h < \omega_j < \omega_k$ for which one has h and $k \in N_j$, $i \in N_{j'}$ and either $t_j \neq t_{j'}$ or $t_j\overline{\omega}_j + g_j \neq t_{j'}\omega_j + g_{j'}$. It is clear that if only one of the two inequalities $t_j \neq t_{j'}$ and $t_j\overline{\omega}_j + g_j \neq t_{j'}\omega_j + g_{j'}$ holds, then the jurisdiction structure can not be stable because there would be unanimity of the members of one of the jurisdictions j and j' to go to the jurisdiction with the low tax rate (if $t_j \neq t_{j'}$ and $t_j\overline{\omega}_j + g_j = t_{j'}\omega_j + g_{j'}$) or to the jurisdiction with the largest public good provision (if $t_j = t_{j'}$ and $t_j\overline{\omega}_j + g_j \neq t_{j'}\omega_j + g_{j'}$). Hence we can assume that both $t_j \neq t_{j'}$ and $t_j\overline{\omega}_j + g_j \neq t_{j'}\omega_j + g$ hold. Define now $\overline{\omega}$ by:*

$$\begin{aligned} t_j\overline{\omega} + g_{j'} &= t_j\overline{\omega}_j + g_j \\ &\Leftrightarrow \\ \overline{\omega} &= \overline{\omega}_j + \frac{g_j - g_{j'}}{t_j} \end{aligned}$$

Clearly we have that:

$$U(t_j\overline{\omega}+g_{j'}, (1-t_j)\omega_m) = \Phi(t_j, \overline{\omega}, \omega_m, g_{j'}) = U(t_j\omega_j+g_j, (1-t_j)\omega_m) = \Phi(t_j, \overline{\omega}_j, \omega_m, g_j) \tag{17}$$

for every household m. For this non-stratified jurisdiction structure to be stable, we must have:

$$\begin{aligned} \Phi(t_j, \overline{\omega}_j, \omega_h, g_j) &\geq \Phi(t'_j, \overline{\omega}_{j'}, \omega_h, g_{j'}) \\ \Phi(t_j, \overline{\omega}_j, \omega_i, g_j) &\leq \Phi(t'_j, \overline{\omega}_{j'}, \omega_i, g_{j'}) \end{aligned}$$

and

$$\Phi(t_j, \overline{\omega}_j, \omega_k, g_j) \geq \Phi(t'_j, \overline{\omega}_{j'}, \omega_k, g_{j'})$$

or, using (17):

$$\Phi(t_j, \overline{\omega}, \omega_h, g_{j'}) \geq \Phi(t'_j, \overline{\omega}_{j'}, \omega_h, g_{j'}) \tag{18}$$

$$\Phi(t_j, \overline{\omega}, \omega_i, g_{j'}) \leq \Phi(t'_j, \overline{\omega}_{j'}, \omega_i, g_{j'}) \tag{19}$$

and

$$\Phi(t_j, \overline{\omega}_j, \omega_k, g_{j'}) \geq \Phi(t'_j, \overline{\omega}_{j'}, \omega_k, g_{j'}) \tag{20}$$

We have seen by lemma 4 that the slopes of indifference curves in the space of all combinations of local tax rate and jurisdiction aggregate wealth are ordered as per the individual wealth for any level of central government grant and, therefore, in particular for the level $g_{j'}$. Hence indifference curves of households h, i and k at the combination (t_j, ϖ) must be as they are depicted in figure 5. Clearly from this figure, unless the indifference curves of households k and i or h and i cross in the wrong order at some point (such as (τ, ϖ'')), the set of combinations of local tax rates and jurisdiction aggregate wealth that household i considers weakly worse (given a central government grant of $g_{j'}$) than

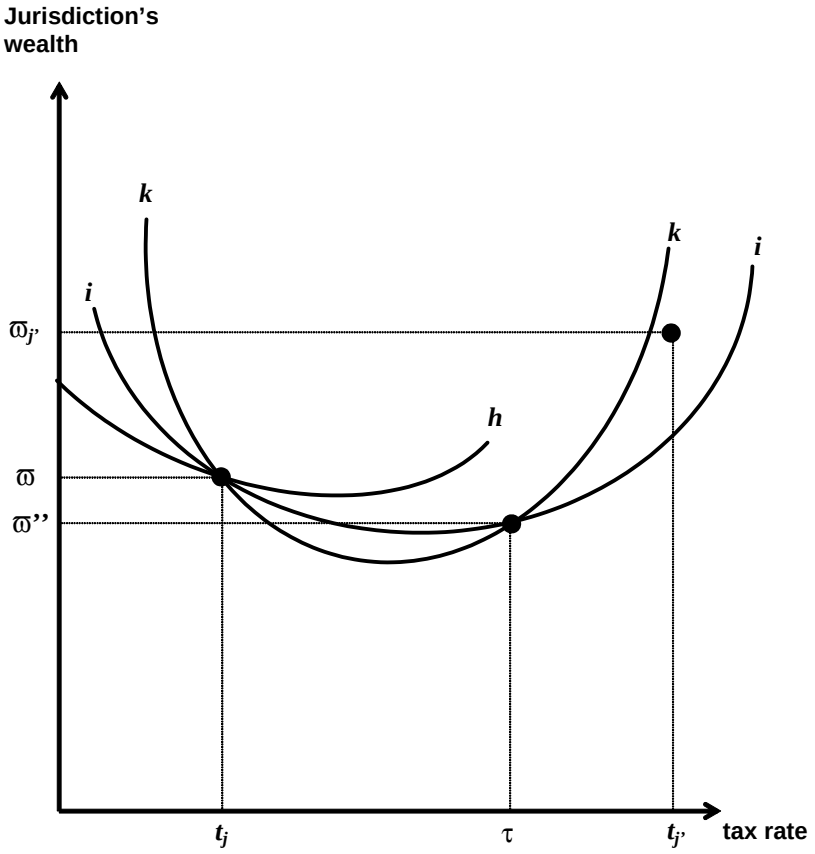

Figure 5

(t_j, ϖ) is contained in the set of such combinations that either household h or household k considers strictly worse than (t_j, ϖ). Hence, unless indifference curves of two households cross in the "wrong" order at a point such as

(τ, ϖ'') , inequalities (18)-(20) can not simultaneously hold for distinct combinations (t_j, ϖ) and $(t_{j'}, \varpi_{j'})$ of local tax rates and jurisdiction tax rates. Hence the jurisdiction structure can not be stable. ■

4 Conclusion

The main conclusion of this paper is that the welfarist intervention of a central government does not alter substantially the segregative properties of endogenous jurisdiction formation, at least when this jurisdiction formation is modelled within a framework *à la* Whestoff. Specifically, the condition that it is necessary and sufficient to impose on household preferences for guaranteeing the wealth segregation of any stable jurisdiction is not affected by the presence of a central government who redistribute wealth across jurisdictions. This of course does not mean that central government intervention does not affect jurisdiction formation. As the Leximin government case dramatically reveals, the redistributive behavior of the central government tends to reduce the number of stable jurisdiction structure. Yet, the remaining stable jurisdictions structures that remain will be segregated under exactly the same conditions on household's preferences than would be the case without central government.

While we believe that the message according to which central government intervention does not modify the segregative forces underlying Tiebout-like processes of jurisdiction formation is of some interest, it is worth recalling the limitations of the analysis on which it stands. For one thing, the result is obtained, at least for its necessity part, under the assumption that preferences are additively separable and that the utility function summed by the utilitarian central government is also additive. It would be nice to relax this assumption. Another limitation of the analysis lies perhaps in the simultaneous setting in which the decisions by households, central and local government are considered. As discussed in the paper, an alternative approach would be to assume a form of "leadership" of the central government in the process of jurisdiction formation. A third limitation of the analysis is the horizontal equalization performed by the central government that has been assumed. While this form of equalization is performed in several "real world" environments (Scandinavian countries for instance), it does not provide a proper account of the *vertical* equalization performed in many countries (including France and Canada) in which equalization grants given to jurisdictions are financed by taxation levied on households. Extending the analysis of this paper over these limitations, as well as many others, seems to us a worthy objective for future research.

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