Learning under Ambiguity when Information Acquisition is Costly: an Experiment

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Introduction

- Learning under ambiguity
- Learning is a costly process
- Information in continuous time rather than discrete

Related literature

Theory

- Marinacci (2002)
- Epstein (2007, 2010)
- Zimper and Ludwig (2009)
- **Epstein and Ji (forthcoming), Operations Research**

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Applications in finance

- Chen and Epstein (2002)
- Epstein and Schneider (2008)
- Miao (2009)
- Choi (2016)

Related literature

Experiments

- Trautmann and Zeckhauser (2013): subjects neglect learning opportunities.
- Ert and Trautmann (2014): sampling experience reverses preferences for ambiguity.
- Nicholls et al (2015): learning does not decrease violations of the sure-thing principle.
- Baillon et al (2017): more information leads to expected utility.
- Abdellaoui et al (2016):continuum of ambiguity degrees between the known and the unknown urn.

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How does exogenous learning affect ambiguity attitudes?
How do ambiguity attitudes affect the choice of how much to learn?

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Modified Ellsberg 2-urn thought experiment

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- Signals are provided via a diffusion process (Brownian motion) with drift equal to the bias towards blue
- There is a cost $c > 0$ per-unit-time of sampling (monetary or cognitive)

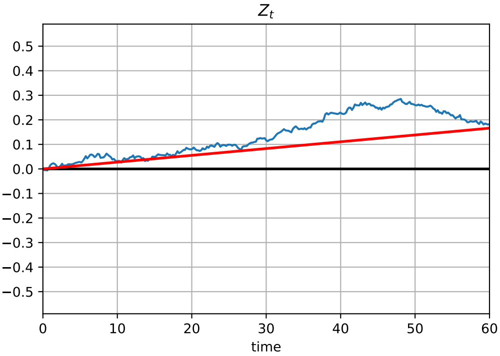

Figure 1: Diffusion process $Z_t = \theta t + \sigma B_t$

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Epstein and Ji (forthcoming)

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- ϵ measures the ambiguity attitude (perception), $\epsilon \in [0, 1]$

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- $\tau^* > 0$ which is given by $\tau^* = \min\{t \geq 0 : |Z_t| \geq \bar{z}\}$ with $\bar{z} = \frac{\sigma^2}{2\theta}[log(\frac{1+\epsilon}{1-\epsilon}) + log(\frac{\bar{r}}{1-\bar{r}})]$

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- $\bar{r}$ is the unique solution of $l(r) + l(\frac{1+\epsilon}{2}) = \frac{4\theta^3}{c\sigma^2}$

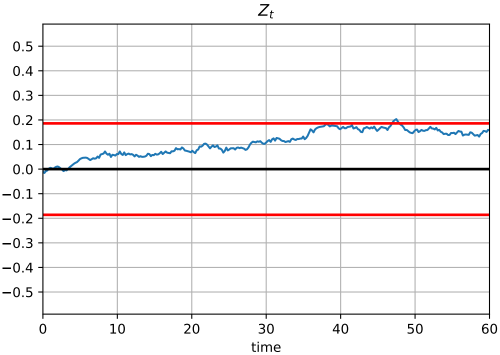

Figure 2: Stopping thresholds for: $\theta = 0.166$ (67 blue; 33 red), $\sigma = 0.1, \epsilon = 0.5, c = 0.01, \bar{z} = 0.186$

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Epstein and Ji (forthcoming)-Corollary B1

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Sampling time increases when:

- cost c falls
- σ and θ both increase in such away that $\frac{\theta}{\sigma^2}$ is constant

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- 4 treatments
- 1 minute of sampling
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- Payment on one random chosen round
- Choose an urn and a colour (also sample if they wish)
- Main prediction: $LH > HH > LL > HL$

This is a practice problem.

Bag A contains 50 Red and 50 Blue tokens.

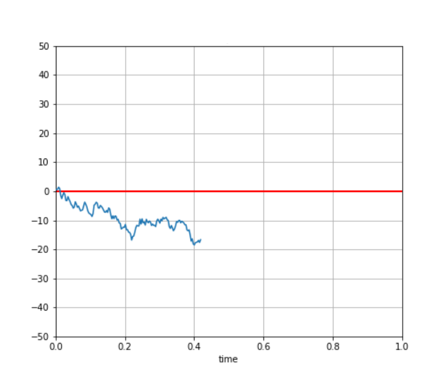
Bag C contains 50+p Blue tokens and 50-p Red tokens.

p is equal to 25 or -25 tokens.

You can buy information on p at a cost of 1.0 points per second.

You sampled for 25.2 seconds

at a total cost of 25.2 ECUs.



Red from Bag A

Blue from Bag A

Red from Bag C

Stop

Blue from Bag C

Results

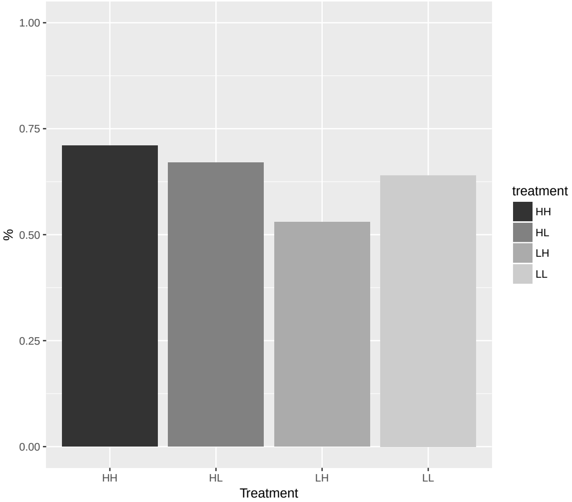


Figure 3: Percentage of trials where subjects sampled

Results

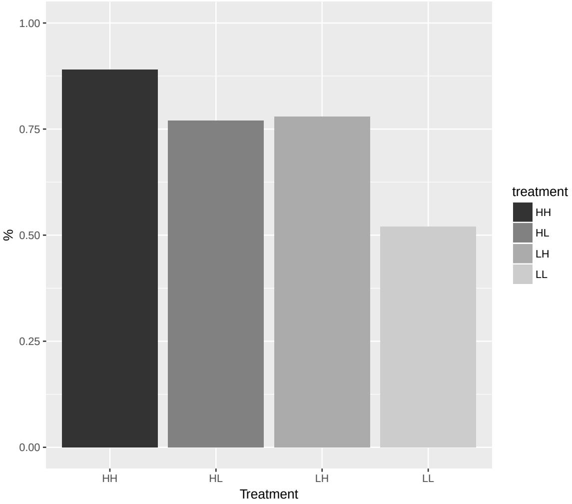


Figure 4: Percentage of trials where subjects did not sample and chose the risky urn.

Results

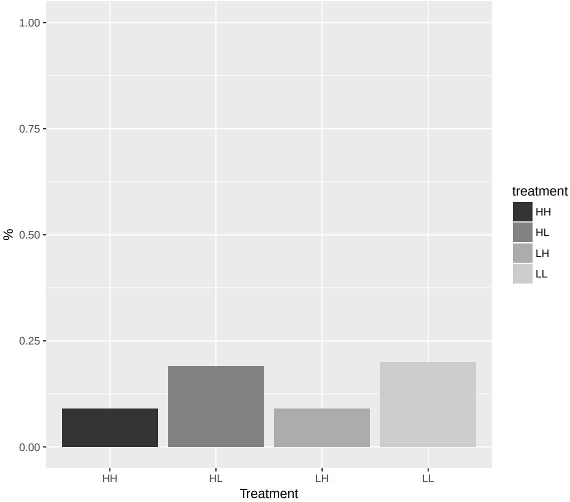


Figure 5: Percentage of trials where subjects sampled and chose the risky urn.

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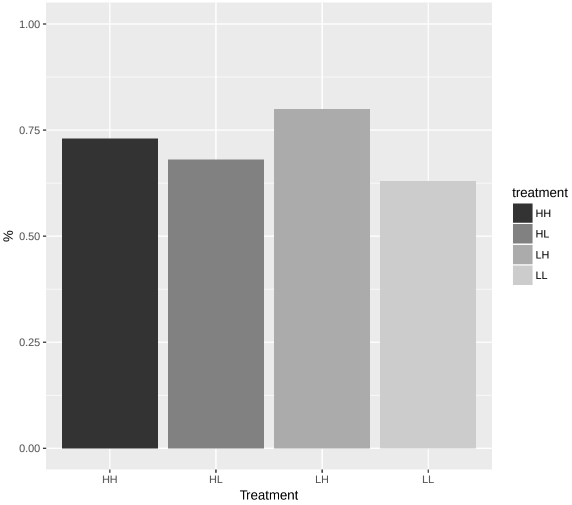

Figure 6: Percentage of trials where subjects sampled, chose the ambiguous urn and chose the correct colour.

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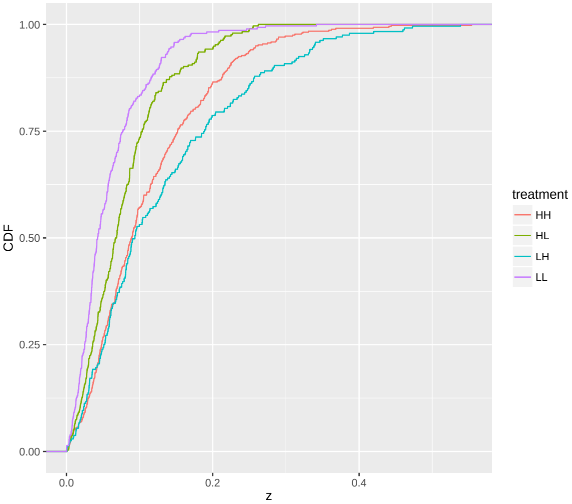

Figure 7: CDF of thresholds.

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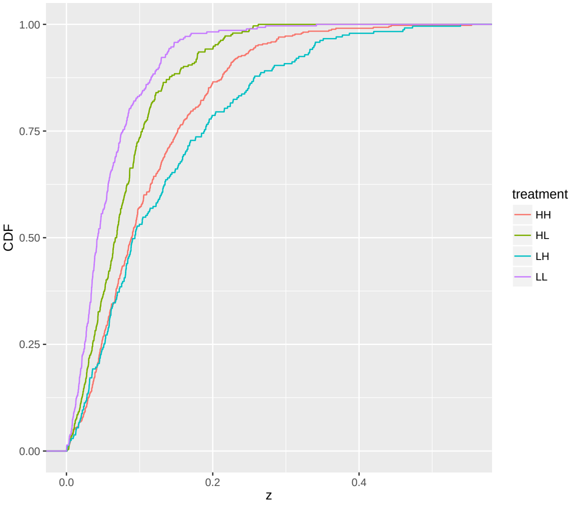

Figure 7: CDF of thresholds.

Pred: $LH > HH > LL > HL$ Vs. Obs: $LH > HH > HL > LL$

	Estimate	Std. Error	t value	Pr(>—t—)	
(Intercept)	0.106291	0.005324	19.96305	3.51E-77	***
dHL	-0.03183	0.005763	-5.52266	4.06E-08	***
dLH	0.01964	0.006147	3.195249	0.001432	**
dLL	-0.05205	0.005823	-8.93942	1.37E-18	***
round	0.000345	0.000371	0.929444	0.352839	

Table 1: Treatment effects. Dependent variable: absolute value of threshold Z_t. HH is the baseline.

Summary and extensions

- Predictions of the theory seem to be confirmed
- Demand for information
- Some subjects never sample
- Some subjects never sample but choose the ambiguous urn
- High variation in the sampling time
- Identify and estimate ϵ to classify subjects

Thank you!