

Regulation of trades based on differences in beliefs*

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Abstract

Some trades based on differences in beliefs might cause more harm than good. Should they be restricted? If yes, how? We propose three limits on regulation aimed at protecting beneficial trades: *Unanimity* – the regulator should not object to trades with identical beliefs; *Autarky* – if the regulator does not object to two unrelated trades, both with identical beliefs, then it should not object to the mere juxtaposition of the trades; and, *Independence of Irrelevant Agents* – the regulator should consider solely the agents involved in the trade. We show that there is a unique policy within these three limits: Laissez-faire. Therefore regulation aimed at banning trades solely based on differences in beliefs is bound to ban beneficial trades too.

Keywords: heterogeneous beliefs; Pareto efficiency; regulation; speculative trading.

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1 Introduction

The subprime crisis has caused massive destruction of welfare reinforcing the suspicion that some trading activities on financial markets are potentially harmful. It was followed by a surge of scholarly interest in trying to identify and curb potentially harmful trades. Especially one category of trades has attracted lots of attention, namely trades based on differences in beliefs about the future.

Trades with heterogeneous beliefs are ex ante welfare improving for the agents involved. However different agents use different beliefs to evaluate trades. Hence the concern that agents make mistakes such that welfare gains turn out to be welfare losses. That is why some scholars consider Pareto optimality to be an inappropriate welfare criterion in case traders have heterogeneous beliefs. Several refinements of the Pareto domination have subsequently been proposed. These refinements aim at singling out, among all trades, those most likely to harm, and form the foundation for some tightening of the regulation of financial markets.

However, well-functioning financial markets create welfare by allowing agents to trade risks. Hence they must be protected from excessive regulation. For this end we propose three limits on regulation of trades with heterogeneous beliefs. First, the regulator should not object to trades with identical beliefs. Second, if the regulator does not object to two unrelated trades, both with identical beliefs, then it should not object to the mere juxtaposition of the trades. Third, the regulator should consider solely the agents involved in the trade. We show that there is a unique policy within these three limits: Laissez-faire.

Illustration

To illustrate the problem and describe more precisely our contribution, let us introduce a series of examples.

Example 1: (Gilboa et al., 2014) There are two risk neutral agents, Ann and Bill, both endowed with \$1. There are two states of the world: In the first state $s = 1$ the oil price is above \$100 one year from now and in the second state $s = 2$ it is below. Ann believes the probability of state $s = 1$ is $2/3$. Bill believes the probability of state $s = 1$ is $1/3$. Therefore their expected utility functions are

$$U_A(x_1, x_2) = \frac{2}{3}x_1 + \frac{1}{3}x_2 \quad \text{and} \quad U_B(x_1, x_2) = \frac{1}{3}x_1 + \frac{2}{3}x_2.$$

They want to trade one for one such that they both receive \$1 in the state they deem more probable and deliver \$1 in the state they deem less probable. This trade increases their expected utilities from 1 to $4/3$.

In Example 1 the sole reason for Ann and Bill to trade is the difference in their beliefs. They have identical endowments and attitudes toward risk. Moreover they are risk-neutral and their endowments are state-independent, so they have no insurance needs. Ann and Bill are both taking advantage of the difference in their beliefs to speculate against each other, betting on the other having mistaken beliefs. It is a pure bet. The trade could be considered to cause more harm than good.

Some other trades would typically *not* be considered harmful, e.g, when for each agent resources flow from the state deemed more probable to the state deemed less probable. An example follows.

Example 2: There are the same two states as in Example 1. There are two agents with a constant relative risk aversion of one. Cai is endowed with \$2 in state $s = 1$ and \$0.1 in state $s = 2$, and believes the probability of state $s = 1$ is $2/3$. Djal is endowed with \$0.1 in state $s = 1$ and \$2 in state $s = 2$, and believes the probability of state $s = 1$ is $1/3$. Their expected utility functions are

$$U_C(x_1, x_2) = \frac{2}{3} \ln(x_1) + \frac{1}{3} \ln(x_2) \quad \text{and} \quad U_D(x_1, x_2) = \frac{1}{3} \ln(x_1) + \frac{2}{3} \ln(x_2).$$

They want to trade one for one such that Djal receives \$1 in state $s = 1$ and Cai receives \$1 in state $s = 2$. This trade increases their expected utilities.

In Example 2 trading occurs between agents with different beliefs, but this is not crucial for justifying the trade. Indeed even if the two agents switched beliefs the trade would still increase their expected utilities. Consequently the agents do not seem to speculate against each other. Both agents are risk averse and the trade seems to be triggered by insurance needs stemming from large endowment shocks. This trade would typically be considered beneficial.

From a regulatory perspective it would be useful to separate potentially harmful, highly speculative trades, triggered by differences in beliefs (as in Example 1) from more beneficial trades, triggered by insurance needs (as in Example 2). To do this in a systematic way we adopt an axiomatic approach and propose three limits within which trades should not be banned in order to prevent excessive regulation.

Three limits on regulation

Let us proceed with more illustrating examples. Since the examples use the same four agents doing the same trades ending with the same final consumptions as in Examples 1 and 2, we summarize their characteristics and trades in Table 1.

Example 3: There are two agents: Ann from Example 1 and Cai from Example 2. They

	Endowment	Belief	Elementary utility	Net-trade	Consumption
Ann	(1,1)	$\left(\frac{2}{3}, \frac{1}{3}\right)$	$u(x) = x$	(1,-1)	(2,0)
Bill	(1,1)	$\left(\frac{1}{3}, \frac{2}{3}\right)$	$u(x) = x$	(-1,1)	(0,2)
Cai	(2,0,1)	$\left(\frac{2}{3}, \frac{1}{3}\right)$	$u(x) = \ln(x)$	(-1,1)	(1,1,1)
Djal	(0,1,2)	$\left(\frac{1}{3}, \frac{2}{3}\right)$	$u(x) = \ln(x)$	(1,-1)	(1,1,1)

Table 1: Characteristics and trades.

both believe that the probability of state $s = 1$ is $2/3$. They want to trade one for one such that Ann receives \$1 in state $s = 1$ and Cai receives \$1 in state $s = 2$.

In Example 3 trading is based on insurance needs caused by differences in endowments and attitudes toward risk, but absolutely not on differences in beliefs: Ann and Cai have identical beliefs. Unless the regulator has additional information, the standard notion of Pareto domination should not be questioned. Hence we propose a first limit on regulation: If all agents involved in a trade have identical beliefs, then the trade should be authorized. In other words the regulator should respect *Unanimity* (U).

The argument would be exactly the same in the following mirror image of Example 3.

Example 4: There are two agents: Bill from Example 1 and Djal from Example 2. They both believe that the probability of state $s = 1$ is $1/3$. They want to trade one for one such that Djal receives \$1 in state $s = 1$ and Bill receives \$1 in state $s = 2$.

To introduce our second limit on regulation, consider the juxtaposition of the two trades in Examples 3 and 4.

Example 5: There are four agents: Ann, Cai, Bill and Djal. Ann and Cai trade as described in Example 3 and Bill and Djal trade as described in Example 4.

There are agents with different beliefs in Example 5. Ann and Cai on the one side believe the probability of state $s = 1$ is $2/3$, and Bill and Djal on the other side believe the probability is $1/3$. However there is strictly no trade between agents with different beliefs, neither directly nor indirectly. Consequently trading is not in any sense based on differences in beliefs. The juxtaposition of the two trades does not introduce any speculation: Nobody is betting on others having mistaken beliefs. Therefore we propose a second limit on regu-

lation: If two trades in two autarkic economies, both involving agents with identical beliefs, are authorized, then the juxtaposed trade in the juxtaposed economy should be authorized. In other words, the regulator should respect *Autarky* (A).

Finally we propose a third limit on regulation, namely that the authorization of a trade should not depend on characteristics of agents not involved in the trade. In other words, the regulator should respect *Independence of Irrelevant Agents* (IIA). In Theorem 1 we show that Pareto domination is the unique regulatory principle satisfying U, A and IIA.

An interpretation of our result is that any regulation targeting potentially harmful trades based on differences in beliefs is bound to miss its target. Either it bans some trades with identical beliefs (in case it violates U), or it bans some trades because there are heterogeneous beliefs even though there is no trade involving agents with heterogeneous beliefs (in case it violates A) or it bans randomly (in case it violates IIA). Indeed regulation not respecting IIA would be random in the sense that authorization depends on characteristics of agents not involved in the trade. Regulation not respecting A would be invasive in the sense that unrelated trades would be used to ban each other. We claim that such a regulation would go beyond the agenda. Regulation not respecting U would be arbitrary unless the regulator has information about the probability distribution over the future states of the world. This is not the case studied in the present paper.

A way to circumvent that beliefs in unrelated trades are used to block trades is to limit the regulator to consider trades one-by-one. Unfortunately such a regulatory principle is subject to framing effects.

Framing effect

To illustrate the framing effect, consider a slight modification of Example 5.

Example 5’: There are four agents: Ann, Bill, Cai and Djal. Ann and Bill trade as described in Example 1 and Cai and Djal trade as described in Example 2.

Examples 5 and 5’ involve two different packages of trades, but in both cases the four agents start with the *same* initial allocation $((1, 1), (1, 1), (2, 0.1), (0.1, 2))$ and end up with the *same* final allocation $((2, 0), (0, 2), (1, 1.1), (1.1, 1))$. As we have argued in our discussions of Examples 1 and 2, in Example 5’ the trade between Ann and Bill would typically be banned, while the trade between Cai and Djal would be authorized. Similarly as we have argued in our discussions of Examples 3 and 4, in Example 5 the trades between Ann and Cai as well as Bill and Djal would be authorized. However we want to emphasize that there is no fundamental economic reason to differentiate between these two packages. They start from exactly the same initial allocation, end up at exactly the same final allocation and involve

exactly the same beliefs. The sole difference is in the accounting of the clearing house. The accounting is virtual and arbitrary, consequently we think that it should not matter. Hence a consistent regulation should not differentiate between these two lists of trades. Otherwise, it would be subject to a framing effect, a severe inconsistency in our eyes, pushing agents to waste time looking for trading partners who enable them to circumvent the regulatory principle.

In order to avoid framing effects in regulation of trades one-by-one we propose a fourth limit on regulation. It requires that two different patterns of trade starting from the same initial allocation and ending at the same final allocation should be treated identically. In other words, the regulator should respect *No-Framing* (NF). In Corollary 2 we consider finite lists of trades and show that the Pareto principle is the unique principle satisfying U, A and IIA for one trade and NF for finite lists of trades.

Related literature

Stiglitz (1989) discusses taxes on transactions in financial markets to curb trading based on differences in beliefs. In a series of papers Posner & Weyl (2013, 2013a, 2013b, 2014) advocate that benefit-cost analysis should be used to evaluate the consequences on welfare of new financial products. Though vast amounts of data documenting past trading might be available, it remains a challenge to compute welfare in case of heterogeneous beliefs.

Another strand of the literature has circumvented the challenge by proposing refinements of Pareto domination. Mongin (1997) suggests that the unanimity underpinning Pareto domination can be ‘spurious’, in the sense that it is based on differences in beliefs *and tastes* that offset each other. Hence the requirement that unanimity be founded on the same reasoning. Along this line, Gilboa et al. (2014) suggest that a trade should improve the welfare of every agent not only for her own belief but also for some hypothetical common belief. In the same vein, Gayer et al. (2014) strengthen the refinement in Gilboa et al.: A trade should improve the welfare of every agent for everybody’s beliefs. Following another route, Brunnermeier et al. (2014) consider a utilitarian welfare function with fixed weights on agents’ utilities and suggest that a trade should improve aggregate welfare for everybody’s beliefs.

Blume et al. (2014) use different welfare functions to evaluate different financial market designs. It is found that combinations of restrictions on the set of traded assets, borrowing limits and transaction taxes can offer substantial welfare gains relative to complete financial markets. Blume et al. (2014) and Posner & Weyl (2013) are both suggesting that financial innovation can destroy welfare.

Duffie (2014) criticizes this literature. First, there is no compelling philosophical justification for banning speculative trade between consenting agents. Second, there is no normative foundation for treating beliefs and tastes different: both are parts of personal

preferences. Third, beliefs and tastes cannot be separated: heterogeneous beliefs with state-independent tastes and identical beliefs with state-dependent tastes are indistinguishable from an observational point of view.

In the present paper we work under the assumption that beliefs and tastes can be separated and explore the possibility of having a regulatory principle banning potentially harmful trades. In Section 2 we introduce the setup and the axioms. In Section 3 we present the results and discuss how to extend them to ambiguous beliefs.

2 The model

To give the widest breadth to our contribution, we frame it in a full-fledged Arrow-Debreu economy.

The setup

There is a finite number of states $s \in \{1, \dots, S\}$. Let $\Delta = \{\pi \in \mathbb{R}_+^S \mid \sum_s \pi^s = 1\}$ be the set of probability distributions on the set of states. In every state s there is a finite number ℓ of goods.

A consumer i is described by her consumption set $X_i \subset \mathbb{R}^\ell$, elementary utility function $u_i : X_i \rightarrow \mathbb{R}$ and belief $\pi_i \in \Delta$. If consumer i plans to consume the bundle $x_i = (x_i^1, \dots, x_i^S) \in X_i^S$, then her expected utility is $\sum_s \pi_i^s u_i(x_i^s)$. A firm j is described by its production set $Y_j \subset (\mathbb{R}_+^\ell)^S$ and $y_j = (y_j^1, \dots, y_j^S) \in Y_j$ is a production plan.

An *economy* E consists of a finite set of consumers $\mathcal{M} = \{1, \dots, m\}$, a finite set of firms $\mathcal{N} = \{1, \dots, n\}$ and total resources $r \in (\mathbb{R}^\ell)^S$ such that there are trading possibilities: $(\sum_i X_i) \cap (\{r\} + \sum_j Y_j) \neq \emptyset$. A *common-belief economy* is an economy E with $\pi_i = \pi_h$ for every $i, h \in \mathcal{M}$. For a pair of economies E and E' , the economy (E, E') is simply the juxtaposition of E and E' .

Consider an economy E . An *allocation* is an $m+n$ -tuple consisting of consumption plans and production plans (x, y) where $x = (x_i)_{i \in \mathcal{M}}$ and $y = (y_j)_{j \in \mathcal{N}}$. A *feasible allocation* is an allocation (x, y) for which $\sum_i x_i = r + \sum_j y_j$. For a feasible allocation (x, y) a *trade* is a $m+n$ -tuple (μ, ν) , where $\mu = (\mu_i)_{i \in \mathcal{M}}$ and $\nu = (\nu_j)_{j \in \mathcal{N}}$, such that $(x, y) + (\mu, \nu)$ is a feasible allocation. An *improving trade* is a trade (μ, ν) where for every i with $\mu_i \neq 0$,

$$\sum_s \pi_i^s u_i(x_i^s + \mu_i^s) > \sum_s \pi_i^s u_i(x_i^s).$$

An *improvement* is a feasible allocation and an improving trade $((x, y), (\mu, \nu))$.

A consumer i is affected by a trade (μ, ν) provided her bundle is changed $\mu_i \neq 0$. Let $\mathcal{M}(\mu) \subset \mathcal{M}$ be the set of consumers affected by a trade (μ, ν)

$$\mathcal{M}(\boldsymbol{\mu}) = \{i \in \mathcal{M} \mid \mu_i \neq 0\}.$$

Consider an improvement $((x, y), (\boldsymbol{\mu}, \boldsymbol{\nu}))$ for economy E and an improvement $((x', y'), (\boldsymbol{\mu}', \boldsymbol{\nu}'))$ for economy E' . Then the juxtaposition of the improvements $((x, x', y, y'), (\boldsymbol{\mu}, \boldsymbol{\mu}', \boldsymbol{\nu}, \boldsymbol{\nu}'))$ is an improvement for the juxtaposition of the economies (E, E') .

Improvements are submitted to a regulator which either authorizes or bans them. The regulator knows the characteristics of the agents but has no other information. In particular the regulator does not have any information about the likelihood of the different states. The regulator is described by a function that maps economies and improvements $(E, (x, y), (\boldsymbol{\mu}, \boldsymbol{\nu}))$ into recommendations $\{0, 1\}$ where $\Gamma(E, (x, y), (\boldsymbol{\mu}, \boldsymbol{\nu})) = 1$ corresponds to authorizing the improvement and $\Gamma(E, (x, y), (\boldsymbol{\mu}, \boldsymbol{\nu})) = 0$ corresponds to banning the improvement.

The upcoming questions are: What are desirable properties for authorization functions? How do authorization functions having these desirable properties perform?

Three desirable limits on regulation

Recall that speculative trades are characterized by two properties: agents having different beliefs; and, agents trading in order to take advantage of other agents having other beliefs.

A first question naturally arises: What should the policy recommend for common-belief economies and improvements? Well, since all agents have identical beliefs, trades are, by definition, not speculative. Consequently all such trades should be authorized. We term this property *Unanimity*.

Unanimity (U) *Consider a common-belief economy and an improvement $(E, (x, y), (\boldsymbol{\mu}, \boldsymbol{\nu}))$. Then*

$$\Gamma(E, (x, y), (\boldsymbol{\mu}, \boldsymbol{\nu})) = 1.$$

It seems that there is a consensus that the policy should authorize all improvements in case of identical beliefs. It can be seen as a classical unanimity condition: if all consumers have a common belief and strictly prefer $(x, y) + (\boldsymbol{\mu}, \boldsymbol{\nu})$ to (x, y) , then the improvement should be authorized.

Violating the U axiom would make sense in case the regulator had additional knowledge. However the starting point of the paper is that the regulator does not have any information about the likelihood of the different states.

A second question naturally arises: What should the regulator do when there is actually no trade between agents with different beliefs (and thus no one tries to take advantage of

other agents having other beliefs)? Suppose the regulator authorized two independent improvements in two autarkic common-beliefs economies. Should these trades be banned if now submitted to the regulator as one trade - i.e. the mere juxtaposition of the two original ones? Well, since juxtaposing the trades does not introduce any speculation or betting, we argue that the regulator should not change its mind. After all, agents solely trade with other agents having the same belief as themselves, hence the trade is not speculative, and therefore should be authorized. We term this property *Autarky*.

Autarky (A) Consider a pair of common-belief economies and improvements $(E, (x, y), (\mu, \nu))$ and $(E', (x', y'), (\mu', \nu'))$. If

$$\Gamma(E, (x, y), (\mu, \nu)) = \Gamma(E', (x', y'), (\mu', \nu')) = 1,$$

then

$$\Gamma((E, E'), (x, x', y, y'), (\mu, \mu', \nu, \nu')) = 1.$$

The A axiom eliminates some obvious forms of excessive inertia. Consider again the introductory examples, and imagine N replicas of the pair Ann and Cai (Example 3) and one pair of Bill and Djal (Example 4). For a large N , the economy is almost showing homogeneous beliefs. For a rule violating A, the presence of an outlier (here Bill and Djal), not involved in any trade with the rest of the economy, would block the entire economy.

In addition to U and A, we find it desirable that policy recommendations depend on the consumers affected by the improvement and nothing else. In particular, policy recommendations should neither depend on production, nor on consumers unaffected by the improvement. We term this property *Independence of Irrelevant Agents*.

Independence of Irrelevant Agents (IIA) Consider a pair of economies and improvements $(E, (x, y), (\mu, \nu))$ and $(E', (x', y'), (\mu', \nu'))$ with $\mathcal{M}(\mu) = \mathcal{M}'(\mu')$. If for every $i \in \mathcal{M}(\mu)$,

$$(X_i, u_i, \pi_i) = (X'_i, u'_i, \pi'_i) \text{ and } (x_i, \mu_i) = (x'_i, \mu'_i),$$

then

$$\Gamma(E, (x, y), (\mu, \nu)) = \Gamma(E', (x', y'), (\mu', \nu')).$$

If a policy violates the IIA axiom, then it can be subject to manipulation by misrepresenting the agents not affected by the considered improvement. In particular, consider a consumer who is not trading. She does not reveal any information about her characteristics; hence her characteristics are easy to misrepresent by the regulator or other agents with stakes in the improvement.

3 Consistent regulation

In the present section we first show that our three axioms characterize Pareto domination. Afterwards we consider an alternative to the A axiom and extensions to finite lists of trades and ambiguous beliefs.

Laissez-faire

Our three axioms characterize Pareto domination.

Theorem 1 *An authorization function Γ satisfies U, A and IIA if and only if for all economies and improvements $(E, (x, y), (\mu, \nu))$,*

$$\Gamma(E, (x, y), (\mu, \nu)) = 1.$$

Proof: We leave it to the reader to check that the authorization function with $\Gamma(E, (x, y), (\mu, \nu)) = 1$ for all $(E, (x, y), (\mu, \nu))$ satisfies U, A and IIA. Consequently we shall focus on the converse claim.

Suppose Γ satisfies U, A and IIA and consider $(E, (x, y), (\mu, \nu))$. Define $|\mathcal{M}(\mu)|$ economies $(E_i)_i$ with one consumer i , one firm $j = i$ with production set Z_j and total resources x_i for every i . Assume $0, \mu_i \in Z_j$ so a possible production set for firm $j = i$ is

$$Z_j = \{z_j \in (\mathbb{R}_+^\ell)^S \mid \exists \lambda \in [0, 1] : z_j = \lambda \mu_i\}.$$

According to U, $\Gamma(E_i, (x_i, 0), (\mu_i, \mu_i)) = 1$ for every i . Then, according to A,

$$\Gamma((E_i)_i, (x_i, 0)_i, (\mu_i, \mu_i)_i) = 1.$$

Then, according to IIA, $\Gamma(E, (x, y), (\mu, \nu)) = 1$. □

Theorem 1 remains valid in case of pure exchange economies rather than productive economies. In the proof the economy E_i should be an economy with one consumer i with endowment e_i where $e_i^{sk} = \max\{x_i^{sk}, x_i^{sk} + \mu_i^{sk}\}$ for every s and k . Then both x_i and $x_i + \mu_i$ are possible consumption bundles for consumer i in economy E_i .

Our theorem is disturbing for anyone sympathetic to the general agenda of protecting consumers against the consequences of trade based on distorted beliefs. A way to escape the implication of our theorem would be to regulate trades one-by-one. But as illustrated in the introduction, such a principle would be subject to framing effects unless it respects our Pattern Independence axiom introduced below.

Discussion of related literature

Gilboa et al. (2014) propose a *common belief criterion* to separate (dubious) improvements triggered by differences in beliefs, e.g., trade 1, from the (unquestionable, if ‘*de gustibus non est disputandum*’) improvements triggered by differences in insurance needs as in trades 2, 3 and 4. They require that the improvement remains a Pareto improvement for some hypothetical common belief. The resulting refinement of Pareto domination is denoted *No-Betting-Pareto (NBP) dominance*.

Regulation based on NBP dominance would satisfy U and IIA, but violate A. Indeed, applied to the introductory series of examples, an NBP-compliant regulator would authorize trades 2, 3 and 4, but ban trades 1 and 5. The same would hold for regulation based on the criterion proposed in Gayer et al. (2014). The reason why a NBP-compliant regulator would ban trade 5 is that there is no common belief under which both Ann and Bill would be better off. The problem is neither that they trade with each other nor that they bet on their trading partner having mistaken beliefs as in trade 1 – they both trade with someone having the same belief as they themselves have. The problem is that their independent and unrelated actions cannot be rationalized by a common belief. And this lack of rationalization could lead to reservations about whether the improvement following trade 5 is actually an improvement.

Our sense is that the reservations builds on considerations that are very pervasive. Opening the way to coercion based on the fact that people live with the same state space would drive public action to very intrusive shores. That is why we propose to restrict individual freedom solely in situations where agents with different beliefs actually trade. Therefore we are looking for a regulatory principle that treats the case of Ann and Bill differently depending on whether trade 1 or trade 5 are evaluated. Unfortunately, as Theorem 1 shows, no regulatory principle does that consistently.

There are principles which satisfy A and IIA, but violate U. One example is the case of a reverse use of the common belief criterion according to which an improvement is banned if there exists a common belief for which the status quo Pareto dominates the trade. Another example is the case of a regulatory principle based on Brunnermeier et al. (2014).

Brunnermeier et al. (2014) propose a *belief-neutral* welfare criterion, which takes a large set of beliefs as being ‘reasonable’, and requires the improvement to dominate according to all common beliefs in this set. If the regulator chooses a reasonable set, and sticks to it regardless of the involved agents’ beliefs, then it respects A and IIA, but violates U. If the regulator chooses, as a reasonable set of beliefs, some subset of the convex hull of the involved agents’ beliefs, then it respects U and IIA, but violates A. In the latter case, we have nothing to add to the previous discussion. In the former case, the regulator is not just a referee being watchful that agents do not speculate against each other, but an actor imposing its views on the likelihood of the states of the world.

Lastly, and for the sake of completeness, there are regulatory principles satisfying U and A, but violating IIA: Consider for example a principle that bans all trades involving agents with heterogeneous beliefs in pure exchange economies and authorizes all other trades in all other economies.

Autarky versus (weak) no framing

The A axiom considers two independent improvements in two different economies. Alternatively two independent improvements affecting disjoint sets of consumers in the same economy could be considered. To illustrate this alternative, let us modify slightly the introductory examples.

Consider an economy consisting of the four agents Ann, Bill, Cai and Djal.

Example 3’: Ann and Cai are trading as in Example 3 while Bill and Djal do nothing.

Example 4’: Bill and Djal are trading as in Example 4 while Ann and Cai do nothing.

In Examples 3’ and 4’, there are different beliefs in the economy, but all agents involved in the trade have common beliefs. Under IIA and U, there is no reason to restrict such trades. Suppose now that trade 5 is banned, then we face an awkward situation: Depending on how it is presented, either trade 5, or trades 3’ and 4’, the same trade would be either authorized, or banned. Such a situation would have the flavor of framing.

To eliminate this (weak) form of framing in regulation, we propose an alternative to A: If two trades *involving disjoint sets of agents* are both authorized, then the juxtaposed trade should be authorized too. In other words, the regulator should respect *Weak No Framing* (WNF). In Corollary 1 we show that A and WNF are equivalent under IIA.

Weak No Framing (WNF) Consider an economy E , an allocation (x, y) and three improving trades (μ, ν) , (μ', ν') and (μ'', ν'') with: $\mathcal{M}(\mu) \cap \mathcal{M}(\mu') = \emptyset$; $\pi_i = \pi_h$ for every $i, h \in \mathcal{M}(\mu)$ and $\pi'_i = \pi'_h$ for every $i, h \in \mathcal{M}(\mu')$; and, $\mu''_i = \mu_i$ for every $i \in \mathcal{M}(\mu)$, $\mu''_i = \mu'_i$ for every $i \in \mathcal{M}(\mu')$ and $\mu''_i = 0$ otherwise. If

$$\Gamma(E, (x, y), (\mu, \nu)) = \Gamma(E, (x, y), (\mu', \nu')) = 1,$$

then

$$\Gamma(E, (x, y), (\mu'', \nu'')) = 1.$$

The WNF axiom eliminates some forms of framing, but not all. Indeed framing effects arising from trades in one part of the economy affecting the evaluation of trades in another part of the economy are eliminated. But trades rather than patterns of trade are evaluated making Examples 5 and 5’ indistinguishable. Therefore framing effects caused by the pattern of

trade are not eliminated. Consequently, WNF can be seen as a weak version of no framing effects. However WNF is sufficient to pin down Pareto domination.

Corollary 1 *Suppose Γ satisfies IIA. Then Γ satisfies A if and only if it satisfies WNF.*

Proof: Suppose Γ satisfies A. Consider an economy E and three improvements $((x, y), (\mu, \nu))$, $((x, y), (\mu', \nu'))$ and $((x, y), (\mu'', \nu''))$ with: $\mathcal{M}(\mu) \cap \mathcal{M}(\mu') = \emptyset$; $\pi_i = \pi_h$ for every $i, h \in \mathcal{M}(\mu)$ and $\pi'_i = \pi'_h$ for every $i, h \in \mathcal{M}(\mu')$; and, $\mu''_i = \mu_i$ for every $i \in \mathcal{M}(\mu)$, $\mu''_i = \mu'_i$ for every $i \in \mathcal{M}(\mu')$ and $\mu''_i = 0$ otherwise. If

$$\Gamma(E, (x, y), (\mu, \nu)) = \Gamma(E, (x, y), (\mu', \nu')) = 1,$$

then according to A,

$$\Gamma((E, E), (x, x, y, y), (\mu, \mu', \nu, \nu')) = 1.$$

Therefore according to IIA, $\Gamma(E, (x, y), (\mu'', \nu'')) = 1$.

Suppose Γ satisfies WNF. Consider two economies and improvements $(E, (x, y), (\mu, \nu))$ and $(E', (x', y'), (\mu', \nu'))$. If

$$\Gamma(E, (x, y), (\mu, \nu)) = \Gamma(E', (x', y'), (\mu', \nu')) = 1,$$

then according to IIA,

$$\Gamma((E, E'), (x, x', y, y'), (\mu, 0, \nu, 0)) = \Gamma((E, E'), (x, x', y, y'), (0, \mu', 0, \nu')) = 1$$

Therefore according to WNF, $\Gamma((E, E'), (x, x', y, y'), (\mu, \mu', \nu, \nu')) = 1$. \square

No framing

In order to tackle framing effects arising from packaging the same trade in different ways, we need a richer setting where initial allocations and finite lists of trades (rather than one trade) are evaluated. In such a setting the regulator can distinguish between different packages of trades starting from the same initial allocation and ending at the same final allocation. We consider such a setting and show in Corollary 2 that the unique policy robust to framing is the standard Pareto policy.

Now an *improvement* is a feasible allocation and a finite list of n trades $((x, y), (\mu_a, \nu_a)_a)$ such that every trade (μ_c, ν_c) is an improving trade of the allocation obtained by starting with the feasible allocation and adding the previous trades to it $(x, y) + \sum_{b < c} (\mu_b, \nu_b)$. A *one-trade improvement* is an improvement with one trade $((x, y), (\mu, \nu))$.

With the change from one-trade improvements to multi-trade improvements the authorization function changes to a function from economies and multi-trade improvements $(E, (x, y), (\mu_a, \nu_a)_a)$ to multi-authorizations of trades $\{0, 1\}^n$. Banning a trade can change

a subsequent trade from being improving to not being improving. However no assumption needs to be made about evaluation of non-improving trades.

Our three properties U, A and IIA carry over to the setting with finite lists of trades provided they are restricted to one-trade improvements. In addition to these three properties, we propose that regulation of a trade should depend on the intermediate allocations from which it starts and ends and nothing else. Hence, regulation of a trade should be independent of the patterns of trade preceding the allocations from which it starts and at which it ends. We term this property *Pattern Independence*.

Pattern Independence Consider an economy and an improvement $(E, (x, y), (\mu_a, \nu_a)_a)$. If for every $b < c$,

$$\Gamma_b(E, (x, y), (\mu_a, \nu_a)_a) = 1,$$

then

$$\Gamma_c(E, (x, y), (\mu_a, \nu_a)_a) = \Gamma(E, (x, y) + \sum_{b < c} (\mu_b, \nu_b), (\mu_c, \nu_c)).$$

The PI axiom does not eliminate framing effects, but it implies that regulation of lists of trades should be trade by trade. However, PI is sufficient to pin down Pareto domination in a setting with lists of trades.

Corollary 2 An authorization function Γ satisfies U, A and IIA for all one-trade improvements and PI for all improvements if and only if for all economies and improvements $(E, (x, y), (\mu_a, \nu_a)_a)$,

$$\Gamma(E, (x, y), (\mu_a, \nu_a)_a) = (1, \dots, 1).$$

Proof: We leave it to the reader to check that the authorization function with

$$\Gamma(E, (x, y), (\mu_a, \nu_a)_a) = (1, \dots, 1)$$

for all $(E, (x, y), (\mu_a, \nu_a)_a)$ satisfies U, A, IIA and PI as specified in Corollary 2. Consequently we shall focus on the converse claim.

Suppose Γ satisfies U, A, IIA and PI and consider $(E, (x, y), (\mu_a, \nu_a)_a)$. According to U, A and IIA as shown in Theorem 1, for every c , $\Gamma_c(E, (x, y) + \sum_{b < c} (\mu_b, \nu_b), (\mu_c, \nu_c)) = 1$. Then, according to PI, $\Gamma_c(E, (x, y), (\mu_a, \nu_a)_a) = 1$ for every c . \square

Corollary 2 is reminiscent of Proposition 3 in Gayer et al. (2014) which states that if the set of expected utility profiles is rectangular and convex, then the transitive closure of the preference relation defined by No-Betting-Pareto dominance is the standard Pareto domination. However it must be noted that typically economies do not satisfy rectangularity.

Ambiguous beliefs

Our analysis and results straightforwardly extend to the case of ambiguous beliefs where consumers have sets of beliefs $\Pi_i \subseteq \Delta$, not necessarily singletons. Preferences can be as in e.g. Bewley (2002) or Gilboa & Schmeidler (1989) according to which consumer i prefers bundle x_i to bundle x'_i provided respectively $\sum_s \pi_i^s u_i(x_i^s) \geq \sum_s \pi_i^s u_i(x'_i{}^s)$ for all $\pi_i \in \Pi_i$ or $\min_{\pi_i \in \Pi_i} \sum_s \pi_i^s u_i(x_i^s) \geq \min_{\pi_i \in \Pi_i} \sum_s \pi_i^s u_i(x'_i{}^s)$.

A *common-belief economy* in case of ambiguity is an economy E where consumers have identical sets of beliefs: $\Pi_i = \Pi_h$ for every $i, h \in \mathcal{M}$. Given this extended definition of common-belief economies, all our axioms readily extend. It could be argued that coincidence of sets of beliefs is less likely than coincidence of singleton beliefs (if, e.g., beliefs are taken from a finite set, as percentages); in which case our axioms become even weaker. Finally it can be checked immediately that our results remain valid.

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