Separation incentives and minimum wages in a job-posting search framework

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Preliminary Draft

October 9, 2013

Abstract

We present a job posting model of a labour market where jobs can potentially differ by characteristics other than wages and workers differ in their marginal willingness to pay for such characteristics. Assuming firms are not able to post-type dependent contracts, posting more than one contract may be desirable if these separate each type of worker. Understanding the interplay between these separation incentives and the standard search incentives is a key contribution of the paper. The paper then examines the implications for policies such as a minimum wage or ones which set minimum standards on these non-wage job characteristics. We show that policies that set standards on wages and the other job characteristics can increase the utility of the worst-off workers and may reduce inefficient forms of unemployment. Policies that only intervene in one aspect on the other hand may increase these forms of unemployment.

*The authors are grateful, without implicating, Hans-Martin Krozlig and Alan Carruth for comments and suggestions

1 Introduction

This paper uses the canonical job-posting search framework (Burdett and Mortensen, 1998, henceforth BM) to examine a labour market where jobs differ in ways apart from earnings (for example pace of production, stress levels, hours of work). Workers differ by their preferences over these non-wage characteristics. This implies that in addition to facing the standard trade-off created by matching frictions between profit-flow per worker and both recruitment and retention, firms will have incentives to separate heterogeneous workers by offering a variety of contracts. The interplay between separation incentives and the standard search incentives is the focus of the paper. The efficiency implications of this depend on whether separation constraints bind at the bottom of the income distribution. If they do this leads to inefficient contracts for the lowest paid workers, and potentially inefficiently high unemployment rates among those least willing to participate in the labour market. We also show that the efficiency case for interventions which set minimum standards on these non-wage job characteristics is strengthened in the presence of worker heterogeneity relative to policies that only intervene in one dimension.

The analysis of non-wage characteristics becomes non-trivial when their provision creates a trade-off between worker utility and firm profits. Such characteristics might include pace of production, hours of work, stress or a non-wage benefit such as child care or tied accommodation. In job-posting search models,[1] as Hwang, Mortensen and Reed (1998), Mortensen (2003), Manning (2003), Masui (2006) all show, firms will post bundles of job characteristics and wages if this trade off is non-linear. So, for example, if the

[1] In which each firm posts a single (usually unvarying) job specification that constitutes a take-it-or-leave it offer to any worker the firm is matched with. The benchmark BM model has been developed and extended in various ways, leading to a very well established class of models that has been used, amongst other purposes, to characterise both theoretically and empirically the nature of wage dispersion and job turnover (e.g. Van Den Berg and Ridder, 1998, Bontemps, Robin and and Van Den Berg 1999, and using productivity data, Postel-Vinay and Robin, 2006), explain wage-tenure profiles (Stevens, 2004, and Burdett and Coles, 2003), analyse the effects of minimum wage policies, describe the profile of wealth inequality (Lise, 2013) and recently to analyse business cycle dynamics (Coles and Mortensen, 2009 and Postel-Vinay and Moscarini, 2013). Of these only Postel-Vinay and Robin (2006) modifies the job-posting assumption by allowing a role for renegotiation between worker and firm, leading to an approach the is in some sense intermediate between a bargaining model and the monopsonistic competition framework.

relevant job characteristics are income and hours worked, firms will post a specific hours-income bundle rather than simply posting an hourly wage. These models generate equilibrium dispersion in worker utility (rather than simply in wages) and every job lies on a Pareto efficient 'contract curve'. Along this curve nicer jobs (in terms of non-wage characteristics) typically also have higher wages. The issue of heterogeneity in tastes has, however, yet to be addressed in this aspect of the job search literature.[2] The combination of a multi-dimensional contract space and worker heterogeneity allows the possibility of separating contracts: if workers differ in their preferences over job characteristics, firms will have incentives to post more than one type of job. The implication of these incentives for 'multiple posting' for the joint distribution of wages and job characteristics and the level of employment has not as yet been described in the job search model literature. We focus particularly on the qualitative implications for the worst-off workers.

The hypothesis of the paper is that upon making contact with the firm, the worker has a free choice of which job to apply for among those posted. That is, for reasons such as asymmetric information, worker morale, anti-discrimination legislation etc., the firm is not able to 'force' prospective recruits to apply for one type of job by withholding the others in a way that is contingent on her personal characteristics or situation (and so has to offer these various jobs on consistent terms across workers). Posting multiple jobs will only be of value to the firm if they separate the various types of worker along the classic lines of Rothschild and Stiglitz (1976). Among firms in which separation constraints bind, job bundles may not lie on the efficient contract curves that maximise profit-flow for a given utility. Other firms may only offer jobs that only one type of worker will accept, creating inefficient excess unemployment rates among those types of worker most averse to participating in the labor market.[3]

[2]In contrast to the the old compensating differentials literature, where all job contracts will lie on the zero profit curve and worker heterogeneity causes nicer jobs to have lower wages.

[3]In a modelling sense, these assumptions effectively imply using BM for the underlying search framework, rather than that of Postel-Vinay and Robin (2006, 2004, henceforth PVR). In the PVR, firms negotiate with workers contingent on their productivity and employment history about which they have complete information. In this context, worker preferences, assuming they are not directly observable, could be inferred from employment history. The role of new entrants in the labor market might make such a model quite complex since their initial choices would influence future offers and separation incentives would still apply. We might also think of preferences as a reduced form contingent on

Much of the paper discusses the consequences for minimum wage policies. We begin with a very simple and preliminary point (for which we were unable to find a reference in the literature) that though the BM model is often described as not producing a minimum wage spike, it produces one very straightforwardly in the presence of a relevant non-wage characteristic.[4] The more general point is that, with homogeneous workers, the welfare effects of minimum wage policies are 'second order' (this is described more precisely below) unless the policymaker undertakes the difficult task of observing and regulating the relevant non-wage characteristics; if it does not, the rise in wages is largely offset by an fall in the other aspects of job quality.

Worker heterogeneity will affect the efficiency properties of such minimum wage policies in two ways that can be summarised as follows:

i) Whereas a standard minimum wage is relatively 'innocuous' with homogeneity, with heterogeneity it can create inefficiency by increasing excess unemployment rates among those types of worker most averse to participating in the labor market.

ii) Since combined bounds on wages and non-wage characteristics eliminate this excess unemployment (even if it exists prior to the introduction of any minimum wage policy), heterogeneity strengthens the relative case for such regulation where it is practical. A more complex 'iso-profit' restriction on wages and non-wage characteristics would constitute a Pareto-improvement on these simple bounds.

Section 2 of the paper outlines the simple extension of the BM framework to include non-wage job characteristics. This is useful for development of the main body of the paper in sections 3 and 4, but is not new (see Hwang,

the worker's unmodeled domestic situation, noting that this is not an outside option as it applies to the worker both in and out of work. The consequent possibility that preferences are dynamic, which could be incorporated here without much consequence, would also lead to similar complications in the PVR framework. Both frameworks are often described as belonging to the same strand of the search literature (see e.g. Postel-Vinay and Moscarini (2013)); for a comparison see e.g. Burdett and Coles (2003, 2010), Mortensen (2003), Stevens (2004) and Postel-Vinay and Robin (2006, 2004)).

[4] Wage-tenure contract posting models, introduced by Stevens (2004) partly as an explanation of rising wage-tenure profiles, produce a minimum wage spike with risk neutrality, though not with risk aversion (Burdett and Coles, 2003). With non-wage characteristics, workers should also experience better non-wage characteristics with tenure in contract-posting models and a minimum wage spike could arise regardless of risk aversion.

Mortensen and Reed, 1998, Manning, 2003, and Mortensen, 2007) except perhaps for the very simple application to the minimum wage. Section 3 introduces worker heterogeneity and section 4 contains an analysis of the implications of minimum wage policies.

2 Non-wage characteristics with homogenous workers

We lay out briefly the basic analysis of non-wage characteristics with homogeneous workers in the BM framework following Mortensen (2007). As standard, we assume a zero real interest rate, a continuum of identical firms and workers of measure m and 1 respectively, and an exogenous job destruction rate δ. Suppose the bundle of non-wage characteristics is captured by a variable "stress" $s \in [0, 1]$ and $r \equiv 1 - s$ is "rest."

Each firm chooses a job offer (r, y) before any economic activity takes place, where y is the income of the worker. A job (r, y) provides instantaneous utility flow $\nu(r, y)$ to the worker and profit-flow per worker $p(r) - y$ to the firm where productivity $p(r)$ and $\nu(r, y)$ are strictly monotonically decreasing and increasing in r respectively. Dispersion in wages in the standard BM framework is replaced by dispersion in offer utility ν; suppose this has distribution function $F(\nu)$ across firms. Workers receive job offers from firms, drawn at random, which they can choose whether to accept. We treat the Poisson offer arrival rate λ as the same for unemployed and employed workers. In this case a worker accepts an offer iff it provides an improvement in instantaneous utility flow (appendix G describes the alternative). The utility flow of unemployment is $\underline{\nu}$.

Firms face a trade-off between maximising profits per worker recruited by making unattractive offers and maximising recruitment by making attractive ones. Therefore these offers must lie on an efficient contract curve that maximises firm profits subject to providing the worker with a certain utility. If p and ν are twice-differentiable and concave, the efficient contract curve must satisfy:

$$\frac{\nu_r(r, y)}{\nu_y(r, y)} = -p'(r). \tag{1}$$

Let $(r(\nu), y(\nu))$ be the offer on the contract curve (1) that provides utility

flow ν. If $J(\nu)$ is the present discounted value to a firm of a filled job that provides ν, then

$$J(\nu) = \frac{p(r(\nu)) - y(\nu)}{\delta + \lambda[1 - F(\nu)]}.$$

If u and $E(\nu)$ denote respectively the unemployment rate and the fraction of workers that are employed with a firm that makes an offer with utility flow ν or less, then for $\nu \geq \underline{\nu}$:

$$\dot{E}(\nu) = \lambda F(\nu)u - (\delta + \lambda[1 - F(\nu)])E(\nu), \tag{2}$$

$$\dot{u} = \delta(1 - u) - \lambda u. \tag{3}$$

The fraction of workers who will accept an offer $\nu \geq \underline{\nu}$ is $u + E(\nu)$, so a firm's expected profit $\pi(\nu)$ for each worker it contacts is $\pi(\nu) = (u + E(\nu))F(\nu)$; since all firms are identical, profit maximisation implies that $\pi(\nu)$ must be the same for all firms. In steady state $\dot{u} = \dot{E}(\nu) = 0$, and noting that the firm that makes the lowest offer must offer the reservation utility (since this firm by definition loses every employee that receives an offer from another firm, there is no recruitment advantage in sacrificing its flow of profits by offering anything higher), we have for $\underline{\nu} \leq \nu \leq \bar{\nu}$:

$$\frac{p(r(\nu)) - y(\nu)}{(\delta + \lambda[1 - F(\nu)])^2} = \frac{p(r(\underline{\nu})) - y(\underline{\nu})}{(\delta + \lambda)^2} \equiv \frac{\pi^{eq}}{\delta} \tag{4}$$

where $(\delta + \lambda)^2(p(r(\bar{\nu})) - y(\bar{\nu})) = \delta^2(p(r(\underline{\nu})) - y(\underline{\nu}))$.

Taking λ as exogenous, (1) and (4) describe the steady state together with the steady state versions of (2) and (3). If there is free entry and c is the exogenous cost of contacting a worker, λ falls as more firms enter the market so entry occurs until $\pi^{eq} = c$, unless $p(r(\underline{\nu})) - y(\underline{\nu}) < \delta c$ in which case no entry will occur. Note that, provided rest and income are normal goods, the contract curve (1) is upward-sloping in rest and income, so the worker who earns more also has a less stressful job. As described in Mortensen (2007), this 'some people have all the luck' result, which is in contrast to a pure compensating differential story, may not hold if there is a significant extent of heterogeneity of productivity among firms.

The Minimum Wage. The idea that the presence of non-wage characteristics means that minimum wage policies may not effectively raise the welfare of workers *who remain employed*, and might even have negative welfare effects, goes back at least to Wessels (1980). In the current search framework,

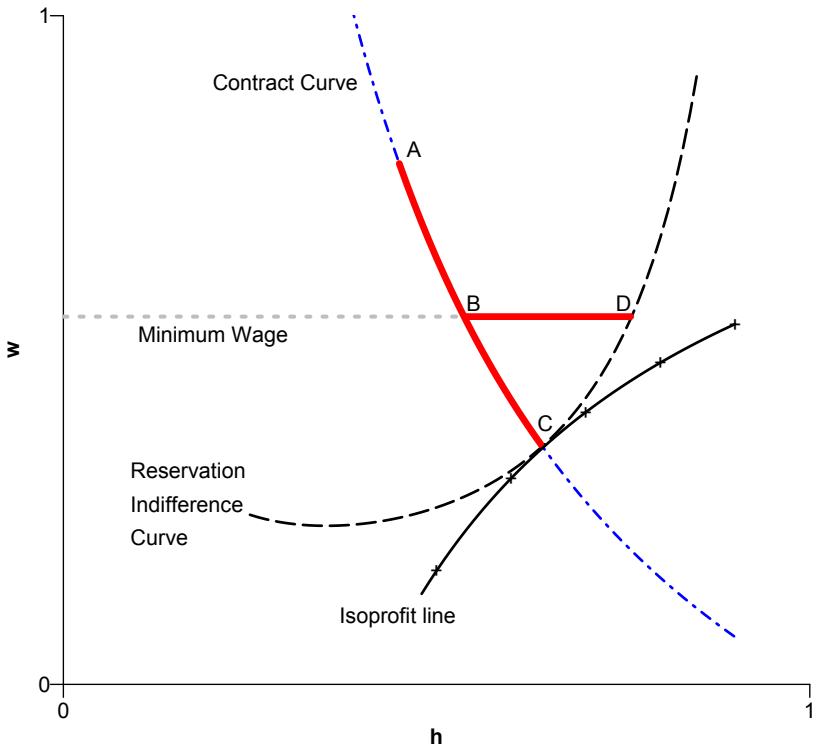

Figure 1: A minimum wage with non-wage characteristics

the effects of a minimum wage are easily calculated by seeing that the lowest offer will move from the point C to D as shown in figure 1 and the offer curve will adjust from lying on segments such as ABC to lying on ones such as ABD. This gives a new offer curve $(r(\nu), y(\nu))$ and we use equation (4) to calculate the distribution as before. Here we have used stress to represent hours worked $h = s$, so the minimum wage is a lower bound on $w = y/h$ rather than y, but this makes no difference to the argument. A minimum wage spike arises because dispersion in wages is replaced at the minimum wage by increased dispersion in the non-wage good; thus any minimum wage fails to increase the utility of the worst-off worker.

If we take a minimum wage that just bites and consider the effects of increasing it by a quantity δx, it is straightforward to see from equation (4) and figure 1 that the effects on the utility distribution of job offers and thus of employed workers are $o(\delta x^2)$. In this sense, the effects of minimum wage policies are 'second order.' This follows immediately from the fact that the iso-profit curve is tangent to the reservation indifference curve at the point C in figure 1, implying that effect on firm profits in equation (4) is second order. Thus the presence of a relevant non-wage characteristics makes the efficiency impacts of a minimum wage 'small.'[5] It is also straightforward to see that the impact of a minimum wage accompanied by a cap on working hours would be first order.

3 Heterogeneous Preferences

We now introduce heterogeneity in worker preferences, starting with the simplest possible framework. There are two types of worker, indexed by $i = 1, 2$, that differ only in their preferences $\nu_i(r, y)$ while maintaining the above assumptions of a zero real interest rate, exogenous offer arrival rates λ, and job destruction rate δ.[6]

We characterise two market equilibrium outcomes. In the first, separation constraints do not bind across the entire distribution and the equilibrium is essentially one of segmented markets. In the second, constraints bind at the bottom of the income distribution leading to inefficient outcomes but above a certain threshold constraints cease to bind and outcomes are again efficient. Whether this happens depends on where the reservation indifference curves of the two types of agent intersect in relation to the efficient contract curves of the two types of agent. Because of this, it greatly simplifies the exposition

[5]This remains the case if we allow job arrival rates to differ among employed and unemployed workers, where then the minimum wage creates a (second order) shift in reservation utility. The efficiency effects of the minimum wage that arise ordinarily in the BM framework if we introduce free entry or worker heterogeneity in reservation utility will similarly be of second order when non-wage characteristics are introduced as described above; an efficiency wage therefore becomes 'innocuous' unless it produces a large minimum wage spike.

[6]The outcome incorporating firm heterogeneity in productivity is described in appendix F. However, this does not greatly affect the consequences of heterogeneity in worker preferences that we are interested in here, principally because the most interesting effects of the latter occur at the bottom of the earnings distribution.

to start with the assumption that the offer arrival rates agents face do not vary according to whether they are employed or unemployed, since under this assumption the reservation indifference curves are exogenous. The qualitative implications of relaxing this assumption are discussed below where relevant and in appendix G.

Let $\gamma \in (0,1)$ be the fraction of type 1 workers. Type 1 workers are less averse to stress, so for all $r, y \geq 0$,[7]

$$\frac{\nu_{1,r}(r,y)}{\nu_{1,y}(r,y)} < \frac{\nu_{2,r}(r,y)}{\nu_{2,y}(r,y)}. \tag{5}$$

Each firm now has the option to post a pair of offers rather than a single one, and posting a pair of offers incurs no extra cost to the firm.[8] An offer is 'relevant' iff it recruits a strictly positive measure of workers in equilibrium, and we use the word 'offer' to mean 'relevant offer' throughout the paper. Since upon making contact with the firm workers freely choose which if any offer to accept, a firm can only post a pair of relevant offers if those offers separate the two types of worker. We describe a firm as a 'separating firm' if it posts a pair of (relevant) offers or as a 'single-offer' firm if it posts only one (relevant) offer. For separating firms, we also use the phrase 'type i offer' throughout the paper to refer to the offer that recruits (and only recruits) type i workers in equilibrium given the free choice of the worker (noting that this labelling does not refer to a choice on the part of the firm, but to the type of worker that would choose that offer).

The following three properties characterise the steady equilibria that we analyse. Up-to satisfying them, the equilibria we describe below will be unique.

Property 1. *There exist two offer curves* $({r_i}^o(\nu_i),\ {y_i}^o(\nu_i))$ *(formally of Hausdorff dimension* 1*) such that all offers accepted by type* $i \in \{1,2\}$ *agents lie on offer curve* i*, where* ν_i *is the utility flow of an offer on offer curve* i *to an agent of type* i*, and where both the functions* ${r_i}^o(\nu_i)$ *and* ${y_i}^o(\nu_i)$ *are piecewise continuously differentiable in* ν_i*.*

[7] For mathematical convenience, we will also the make the assumption that, when faced with two differing offers of equal utility, a type 1 worker will choose the offer with higher income whereas a type 2 worker chooses the offer with more rest.

[8] So once contact is made the worker is able to see the firm's entire menu of offers without additional costs that depend on the size of the menu. The fact that outcomes exist in which some firms only recruit one type of worker without these additional costs helps makes the efficiency implications clearer.

Property 2. *Let ranking i denote the ranking of firms can by the utility type i workers obtain from their preferred offer from each firm. As we go from the bottom of either ranking to the top, each of the constraints* (8) *and* (9) *only switch from binding to non-binding and vice-versa a finite number of times.*

Property 3. *Rankings* 1 *and* 2 *are the same. This ranking will also then be ranking of firm size in equilibrium.*

Property 1 excludes, for example, offers being continuously distributed over an area. Property 2 is a convenient technicality that is arguably unlikely to exclude any economically meaningful equilibrium. In fact we conjecture that no equilibria exist that do not satisfy these properties, but we do not include a formal proof of this here. Property 3 identifies the ranking of offers in which firms are most likely to be able to separate workers. Since separation constraints do not bind always for every firm, however, for each equilibrium that satisfies property 3 there will be continua of equilibria that does not satisfy property 3 but that are equivalent in terms of the allocation of resources.

For each type of worker there is an efficient contract curve analogous to (1) which maximises worker utility for a given profit-flow:

$$\frac{\nu_{i,r}(r,y)}{\nu_{i,y}(r,y)} = -p'(r). \tag{6}$$

Hence the contract curve for type 1 workers lies above the contract curve for type 2 workers (where words such as 'above' refer throughout to a diagram in (r, y) space with r on the horizontal axis and y on the vertical). The offer curves $(r_i^o(\nu_i), y_i^0(\nu_i))$ derived in equilibrium may or may not lie on these contract curves.

Consider the basic optimization problem for a separating firm i. Let (r_i, y_i) refer to the offer accepted by type i workers, and let $F_i(\nu_i)$ be the distribution of offers made on offer curve i, where ν_i is the utility flow that accrues to a type i agent from a given offer on offer curve i. Following the analysis of section 2, it will then wish to choose the two offers to maximise

$$\pi_1(r_1, y_1) + \pi_2(r_2, y_2) \text{ where } \pi_i(r_i, y_i) = \gamma_i \frac{(p(r_i) - r_i - y_i)}{(\delta + \lambda[1 - F_i(\nu_i(r_i, y_i))])^2} \tag{7}$$

subject to

$$\nu_1(r_1, y_1) \geq \nu_1(r_2, y_2) \tag{8}$$

$$\nu_2(r_2, y_2) \geq \nu_2(r_1, y_1) \tag{9}$$

where $\gamma = \gamma_1$ is the fraction of type 1 workers and $\gamma_2 = 1 - \gamma$. We can then note the following remarks:

Remark 1. If neither of the two constraints (8) and (9) bind for a particular firm, then each of its offers will lie on the respective contract curve.

Remark 2. Of the two constraints (8) and (9), clearly at most one can bind for each firm. Hence at least one of any firm's offers must lie on the efficient contract curves: if constraint (8)[9] does not bind, then (r_1, y_1) $[(r_2, y_2)]$ must lie on the efficient contract curve for type 1 [type 2] workers.

Remark 3. If (8) binds for a firm in equilibrium, the firm, holding its type 2 offer, can increase its type 1 offer (in terms of type 1 utility) and still satisfy both constraints. Since the firm's choice of type 1 offer must maximise its profit given its type 2 offer, in equilibrium this cannot increase its profits. This implies that wherever constraint (8) [(9)] binds, we must have $\pi_1'(\nu_1) \leq 0$ $[\pi_2'(\nu_2) \leq 0]$ along offer curve 1[2]. Where constraint (8) [(9)] strictly binds, we must have $\pi_1'(\nu_1) < 0$ $[\pi_2'(\nu_2) < 0]$ along offer curve 1[2].

Remark 4. By a similar argument to remark 3, if neither of the two constraints (8) and (9) bind, we must have $\pi_1'(\nu_1) = \pi_2'(\nu_2) = 0$ along the two offer curves.

In addition to strict concavity and twice-differentiability, we make the following assumption on worker utility. Suppose the provision of the the non-wage characteristic is bounded and its effect on productivity is both strictly monotonic and bounded, so we can then normalise the measure of stress s such that $s \in [0, 1]$ and that productivity is linear in stress. Given this normalisation, we assume that the preferences of workers $\nu_i(r, y)$ are homothetic in $r \equiv 1 - s$ and y.

Given the assumption of homotheticity, we can then assume wlog that $\nu_i(r, y)$ are both homogeneous of degree 1 and that $p = k + s = 1 + k - r$. Since it does not affect any of the results in the paper, for economy of exposition we will also take $k = 0$ – so that zero hours worked implies zero productivity. This does not affect any of the results in the paper. Hence

$$p(r) = 1 - r. \tag{10}$$

The assumption of homotheticity is one possible sufficient condition to produce a 'threshold' in the offer distribution. Building up the equilibrium distribution of offers from the smallest firm, once we reach a firm for which neither constraints (8) or (9) bind, homothetic utility provides a sufficient condition to show that the constraints do not bind for all larger firms.[9]

Thus the existence of binding separation constraints will depend on the smallest separating firm. This in turn depends on the reservation indifference curves of the two types of worker. Suppose all type i of workers have a reservation bundle that provides them utility flow $\underline{\nu}_i$ and the point where the two reservation indifference curves intersect (r_z, y_z) is such that Φ_i, the set of offers (r, y) that are acceptable to an agent of type i and that generate a positive profit flow for the firm (i.e. such that $\nu_i(r, y) \geq \nu_i(r_z, y_z)$ and $r + y \leq 1$), are both non-empty and of strictly positive measure for $i = 1, 2$. The smallest separating firm will not have binding separation constraints iff (r_z, y_z) lies between the two contract curves, i.e. iff

$$\frac{\nu_{1,r}(r_z, y_z)}{\nu_{1,y}(r_z, y_z)} \leq 1 \leq \frac{\nu_{2,r}(r_z, y_z)}{\nu_{2,y}(r_z, y_z)}. \tag{11}$$

3.1 Segmented markets

Suppose (11) holds as shown in figure 2. Let $(\underline{r}_1, \underline{y}_1)$ and $(\underline{r}_2, \underline{y}_2)$ be the points where the type 1 and 2 reservation indifference curves intersect the type 1 and 2 efficient contract curves (6) respectively. For an equilibrium that includes both types of worker to exist, both of these bundles must profitable so we assume that $\underline{r}_i + \underline{y}_i < 1$. The significance of (11) is that it implies the pair of offers $(\underline{r}_1, \underline{y}_1)$ and $(\underline{r}_2, \underline{y}_2)$ must separate the two types of worker. This leads to a simple equilibrium distribution of offers in which the offer curves coincide with the efficient contract curves (6) and in fact the markets behave essentially as if they were independent.

Let $(r_i(\nu), y_i(\nu))$ denote the point on the type i efficient contract curve that gives the type i worker utility ν. We can then define two distributions $F_i(\nu_i)$ on the supports $[\underline{\nu}_i, \bar{\nu}_i]$ with $i = 1, 2$ by the equation,

[9] The homotheticity assumption is not required in order to produce a numerical solution (see e.g. appendix F). It is also not required for theoretically characterising the equilibrium when minimum wages are combined with restrictions on non-wage characteristics.

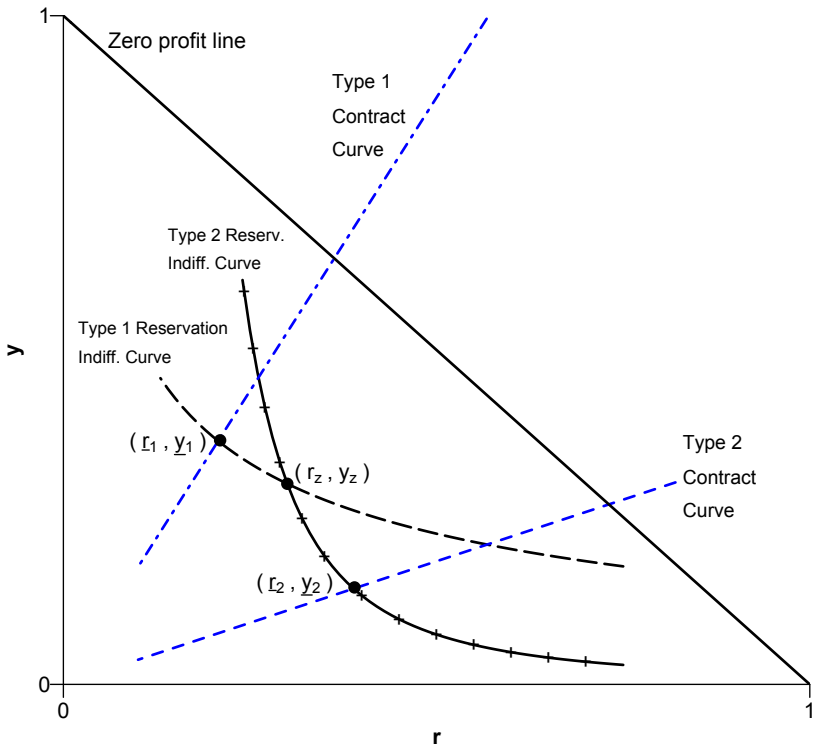

Figure 2: Independent Markets

$$\frac{(1 - r_i(\nu) - y_i(\nu))}{(\delta + \lambda[1 - F_i(\nu)])^2} = \frac{1 - \underline{r}_i - \underline{y}_i}{(\delta + \lambda)^2}, \tag{12}$$

where $\underline{\nu}_i = \nu_i(\underline{r}_i, \underline{y}_i)$ and $(\delta + \lambda)^2(1 - r_i(\bar{\nu}_i) - y_i(\bar{\nu}_i)) = \delta^2(1 - \underline{r}_i - \underline{y}_i)$. Note that F_i are continuous, strictly monotonic and one-to-one mappings between $[\underline{\nu}_i, \bar{\nu}_i]$ and $[0, 1]$ so $F_j^{-1}F_i$ is a well defined mapping from $[\underline{\nu}_i, \bar{\nu}_i]$ to $[\underline{\nu}_j, \bar{\nu}_j]$. Let us define a distribution of offers as follows.

Distribution 1. *Suppose $F_i(.)$ as given in (12) and all firms make a pair of offers $(r_i(\nu_i), y_i(\nu_i))$ for $i = 1, 2$ where $(r_i(\nu_i), y_i(\nu_i))$ lies on the type i*

efficient contract curve, so that each firm that makes an offer $(r_1(\nu_1), y_1(\nu_1))$ *on contract curve* 1 *makes an offer* $(r_2(\nu_2), y_2(\nu_2))$ *on contract curve* 2 *where* $\nu_2 = F_2^{-1}F_1(\nu_1)$. *Suppose also that* ν_1 *is distributed among firms with distribution function* $F_1(\nu_1)$ *on the support* $[\underline{\nu}_1, \bar{\nu}_1]$, *so* ν_2 *has distribution* $F_2(\nu_2)$ *on the support* $[\underline{\nu}_2, \bar{\nu}_2]$.

Proposition 1. *Suppose the offer distribution is that described by distribution 1. Then the pair of offers made by each firm separates the two types of worker.*

Proof. See appendix. □

It follows from proposition 1 and the analysis of section 2, that all firms will make equal profits, with profits per worker from each type of worker also equal across firms. It is also clear that no firm has any incentive to deviate. Take a firm moving its type i offer. Any movement can be decomposed into a movement along a type i indifference curve and along the type i contract curve. The former will diminish its profits (by the definition of the contract curve) while the latter will maintain them, subject to maintaining a separating pair of offers. Losing this separation must also diminish the firm's profits, since it will mean recruiting one type of worker off its efficient contract curve. Thus:

Corollary. *Together with a steady state unemployment rate of* $u = \frac{\delta}{\delta+\lambda}$ *for each type of worker, the offer distribution 1 constitutes a steady state equilibrium in which the distribution of offers is independent of the proportions of the two types of worker that exist in the labour force.*

Arguments for showing the uniqueness of this equilibrium, subject to satisfying properties 1 to 3 above, are left for section 3.2. In figure 2, the type i offer curve will then start at $(\underline{r}_i, \underline{y}_i)$ and move up contract curve i.

Suppose preferences are not correlated with outside options: that is both types of agent receive the same bundle in unemployment. With current assumptions, this must then be (r_z, y_z) and the segmented equilibrium must therefore appear quite unrealistic. All type 2 workers and potentially a strictly positive measure of type 1 workers will experience a *decrease* in rest upon entering unemployment and a strictly positive measure of type 2 workers will also experience an increase in income. However, we cannot rule out the segmented equilibrium described by distribution 1 theoretically for at least two possible reasons.

Firstly, the position of the reservation indifference curves may not be solely determined by the unemployment bundle, in which case the intersection of the reservation indifference curves (r_z, y_z) may not be the unemployment bundle, even if the latter does not differ among types. Suppose we adapt the model to be consistent with the empirical finding that job arrival rates are found to be greater for unemployed than for employed workers (Van den Berg and Ridder, 1998, do not find evidence for such a difference, but other studies often estimate a difference of up-to an order of magnitude). The most common approach is to assume an exogenous difference in offer arrival rates. As shown in BM, this causes reservation wages (or utilities here) to become endogenous, which here would imply that the intersection of the reservation indifference curves (r_z, y_z) will be displaced from the workers' actual unemployment bundle. Mortensen (2003), however, argues that making search intensity endogenous among workers is a theoretically more attractive (though more complex) approach to this issue. Since employees with good jobs have less incentive to search, this gives the result that the employed on average will have lower offer arrival rates than the unemployed. With this modelling approach, reservation utilities remain exogenous and so the argument against the segmented equilibrium remains if both types of agent have a common unemployment bundle. This is discussed further in appendix G.

Secondly, it might be natural to assume preferences over jobs are correlated with unemployment bundles. Imagine for instance that both types of worker share the same underlying preferences but type 2 agents have access to an additional source of income that is independent of employment status. Using y to denote earnings rather than total household income, type 2 agents have steeper indifference curves in (r, y) space (so equation 5 still holds). If the additional source of income precludes type 2 workers from receiving unemployment benefit when unemployed due to means-testing, type 2 agents will have a relatively inferior unemployment bundle to type 1 agents in (r, y) space. Alternatively, type 2 agents might be engaged in home production, which might make unemployment relatively more attractive for these agents if home-production is a closer substitute to income than rest.

Finally, note that in this equilibrium, a minimum wage will only initially affect type 2 workers and so long as it does so the effects will be essentially identical to those described in section 2.

3.2 Dependent markets

When (11) holds, the smallest firm can choose make an offer to each type of worker on the point of her reservation indifference curve that maximises profit flow to the firm, and this pair of offers will separate the two types of worker. In the resulting equilibrium, there is therefore no incentive for any firm to be a single-offer firm. We now suppose instead

$$\frac{\nu_{2,r}(r_z, y_z)}{\nu_{2,y}(r_z, y_z)} \leq 1 \tag{13}$$

as depicted in figure 3.

It is useful to begin with a simple proposition that characterises the possibility of single-offer firms. Let (r_A, y_A) be the point where the type 2 reservation indifference curve intersects the zero-profit line $(r + y = 1)$. Recalling that Φ_i are the sets of profit-making offers (r, y) that are acceptable to an agent of type i (so that $r + y \leq 1$ and $\nu_i(r, y) \geq \nu_i(r_z, y_z)$), let Φ_A be the set of offers (r, y) such that $r + y \leq 1$ and $\nu_1(r, y) > \nu_1(r_A, y_A)$. Then:

Proposition 2. *Consider an offer made by a single-offer firm. This offer must then lie on the segment of the efficient contract curve 1 that lies in* $\Phi_1 \backslash (\Phi_A \bigcup \Phi_2)$*, and as a result is only accepted by type 1 workers.*

Proof. See appendix. □

Since proposition 2 implies that single-offer firms only make type 1 offers, the presence of single-offer firms will increase unemployment rates among type 2 workers; this is the 'type-dependent' unemployment we describe in section 3.3. For simplicity, in this section, we assume $r_z + y_z < 1$, in which case $\Phi_1 \backslash \Phi_A$ is an empty set and by proposition 2 all firms must make a pair of separating offers. We now identify the pair of offers made by the smallest firm. Using figure 3, we can make the following two observations:

i) The smallest firm's type 2 offer must lie on the type 2 reservation indifference curve since otherwise the firm can straightforwardly increase its profits, without affecting separation, by moving its type 2 offer along a type 1 indifference curve to the type 2 reservation indifference curve (since the type 2 offer must lie beneath the type 1 contract curve).

ii) Given its type 2 offer, the smallest firm's profit maximising type 1 offer lies on the type 1 indifference curve that goes through it, and by definition must be where this indifference curve intersects the type 1 efficient

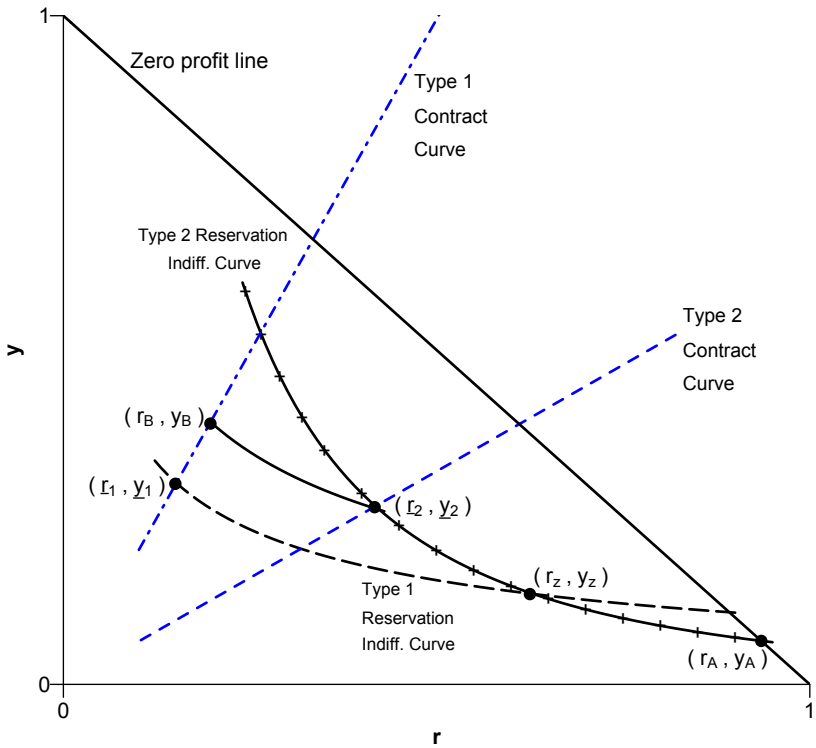

Figure 3: Dependent Markets

contract curve. A lower indifference curve leads to loss of separation while a higher one reduces profits.

Given these two observations, we can see that the lowest pair of separating offers must take the form of a type 1 offer $(r_1(\nu), y_1(\nu))$ and type 2 offer $(\hat{r}(\nu), \hat{y}(\nu))$, where $(\hat{r}(\nu), \hat{y}(\nu))$ is the point on the type 2 reservation indifference curve that gives a type 1 agent a utility flow ν. The lowest offer acceptable to a type 2 agent on the type 2 contract curve is $(\underline{r}_2, \underline{y}_2)$. We can see that if the lowest pair of offers did lie on the two contract curves, it would then have to be $\{(r_B, y_B), (\underline{r}_2, \underline{y}_2)\}$ where (r_B, y_B) is the point where the type 1 indifference curve through $(\underline{r}_2, \underline{y}_2)$; given $(\underline{r}_2, \underline{y}_2)$, (r_B, y_B)

is the lowest offer that can be made on the type 1 contract curve that separates the two types of worker. Writing $\nu_1^B \equiv \nu_1(r_B, y_B)(= \nu_1(\underline{r}_2, \underline{y}_2))$, in the above notation, this pair of offers can be written $\{(r_B, y_B), (\underline{r}_2, \underline{y}_2)\} = \{(r_1(\nu^B), y_1(\nu^B)), \hat{r}(\nu^B), \hat{y}(\nu^B)\}$.

The lowest pair of offers then takes the form $(\hat{r}(\nu_*), \hat{y}(\nu_*))$, $(r_1(\nu_*), y_1(\nu_*))$ where $\nu_* = \arg\max[a(\nu)]$ with

$$a(\nu) \equiv (1-\gamma)(1-\hat{r}(\nu)-\hat{y}(\nu)) + \gamma(1-r_1(\nu)-y_1(\nu)). \tag{14}$$

It is easy to see that ν_* must lie in the support $[\underline{\nu}_1, \nu_1^B]$: for $\nu \geq \nu^B$ both terms of $a(\nu)$ in (14) is decreasing in ν . For $\nu \in [\underline{\nu}_1, \nu_1^B]$, the first term of (14) is increasing in ν and the second term decreasing. If the lowest pair of offers lay on the two contract curves, this would correspond to $\nu_* = \nu^B$. We can see however that this cannot be the case: the iso-profit line is (by definition) tangent to the type 2 reservation indifference curve at (r_B, y_B). Thus decreasing ν by ε from ν_1^B decreases type 2 profits by $o(\varepsilon^2)$, but increases type 1 profits by order $o(\varepsilon)$. Hence:

Lemma 1. *$a(\nu)$ cannot be maximised on the support $[\underline{\nu}_1, \nu_1^B]$ at $\nu = \nu_1^B$.*

Proof. Compare $a(\nu_1^B)$ and $a(\nu_1^B - \varepsilon)$, considering the two terms of (14) separately. $(r_1(\nu_1^B - \varepsilon), y_1(\nu_1^B - \varepsilon))$ lies south-west of $(r_1(\nu_1^B), y_1(\nu_1^B)) = (r_B, y_B)$ on the type 1 efficient contract curve in figure 3. Since $(r_1(\nu_1^B - \varepsilon), y_1(\nu_1^B - \varepsilon))$ offers the worker ε lower utility it must offer the firm a higher profit flow, and $1 - r_1(\nu_1^B - \varepsilon) - y_1(\nu_1^B - \varepsilon)$ must exceed $1 - r_1(\nu_1^B) - y_1(\nu_1^B)$ by order ε. $(\hat{r}(\nu_1^B - \varepsilon), \hat{y}(\nu_1^B - \varepsilon))$ lies south-east of $(\hat{r}(\nu_1^B), \hat{y}(\nu_1^B)) = (\underline{r}_2, \underline{y}_2)$ on type 2's reservation indifference curve; the distance must be of order ε. Since $(\hat{r}(\nu_1^B - \varepsilon), \hat{y}(\nu_1^B - \varepsilon))$ lies of the type 2 efficient contract curve and offers the type 2 agent the same utility we must have $1 - \hat{r}(\nu_1^B - \varepsilon) - \hat{y}(\nu_1^B - \varepsilon) < 1 - \hat{r}(\nu_1^B) - \hat{y}(\nu_1^B)$. However, the type 2 reservation indifference curve is tangent to an iso-profit curve at $(\underline{r}_2, \underline{y}_2)$, so the difference between $1 - \hat{r}(\nu_1^B - \varepsilon) - \hat{y}(\nu_1^A - \varepsilon)$ and $1 - \hat{r}(\nu_1^B) - \hat{y}(\nu_1^B))$ must be of order ε^2. Hence there must some $\varepsilon > 0$ such that $a(\nu) > a(\nu_1^B)$ for all $\nu \in [\nu_1^B - \varepsilon, \nu_1^B]$. □

Since $\nu_* < \nu_1^B$, the lowest income type 2 workers will receive an inefficient contract off the type 2 efficient contract curve. The equilibrium solution then proceeds by deriving a set of differential equations for that describe the distribution of offers made by firms immediately larger than this firm under the assumption that (8) binds for these firms. Since, as noted in remarks 1

and 2, (8) binding implies that (9) does not bind, the type 1 offers of these firms must lie on the type 1 contract curve. If (8) ceases to bind, we also know that the type 2 offer will lie on the type 2 contract curve. Thus, as we verify below, the distribution of offers made by firms for which (8) binds is described by the solution to these differential equations up to the point where the type 2 offer curve intersects the type 2 efficient contract curve. For the remaining firms neither of the two constraints (8) or (9) will bind, resulting in a very similar distribution of offers to that of section 3.1.

Let us index each firm by $\tilde{\nu}_1$, the utility its type 1 offer gives type 1 workers. Since this offer is on the contract curve it must be $(r_1(\tilde{\nu}_1), y_1(\tilde{\nu}_1))$; let us denote its type 2 offer $(\bar{r}_2(\tilde{\nu}_1), \bar{y}_2(\tilde{\nu}_1))$. Suppose $F_i(\nu_i)$ are distribution functions of the type i offers where ν_i is the utility flow obtained by the type i worker, and $f_i(\nu_i)$ are the corresponding density functions. Combining the first order conditions from (7) with respect to r_1 and y_1 when (8) binds gives the type 1 contract curve; for $\bar{r}_2$ and $\bar{y}_2$ it gives:

$$f_2\left(\nu_2(\bar{r}_2, \bar{y}_2)\right) = \frac{\delta + \lambda(1 - F_2(\nu_2))}{2\lambda(1 - \bar{r}_2 - \bar{y}_2)} \frac{\nu_{1,y} - \nu_{1,r}}{\nu_{2,r}\nu_{1,y} - \nu_{2,y}\nu_{1,r}}\Bigg|_{\bar{r}_2, \bar{y}_2} \tag{15}$$

Since constraint (8) binds, i.e. $\nu_1(\bar{r}_2(\tilde{\nu}_1), \bar{y}_2(\tilde{\nu}_1)) = \tilde{\nu}_1$, we can also write:

$$\nu_{1,r}(\bar{r}_2, \bar{y}_2)\bar{r}_2'(\ \tilde{\nu}_1) + \nu_{1,y}(\bar{r}_2, \bar{y}_2)\bar{y}_2'(\tilde{\nu}_1) = 1. \tag{16}$$

From assumption 2, we also have $F_2\left(\nu_2(\bar{r}_2(\ \tilde{\nu}_1), \bar{y}_2(\tilde{\nu}_1))\right) = F_1\left(\tilde{\nu}_1\right)$ which gives

$$f_2\left(\nu_2(\bar{r}_2, \bar{y}_2)\right)\left(\nu_{2,r}(\bar{r}_2, \bar{y}_2)\bar{r}_2'(\tilde{\nu}_1) + \nu_{2,y}(\bar{r}_2, \bar{y}_2)\ \bar{y}_2{}'(\tilde{\nu}_1)\right) = f_1\left(\ \tilde{\nu}_1\right). \tag{17}$$

Finally, the each firm's profit is proportional, up to a multiplicative constant, to

$$\gamma \frac{(1 - r_1(\tilde{\nu}_1) - y_1(\tilde{\nu}_1))}{\left(\delta + \lambda\left[1 - F_1(\tilde{\nu}_1)\right]\right)^2} + (1 - \gamma)\frac{(1 - \bar{r}_2(\tilde{\nu}_1) - \bar{y}_2(\tilde{\nu}_1))}{\left(\delta + \lambda\left[1 - F_1(\tilde{\nu}_1)\right]\right)^2}$$

so, again differentiating with respect to $\tilde{\nu}_1$ gives us the following equi-profit condition:

$$\begin{aligned}\left(\delta + \lambda\left[1 - F_1(\tilde{\nu}_1)\right]\right)&\left\{-\gamma\left[\ r_1{}'(\tilde{\nu}_1) + y_1{}'(\tilde{\nu}_1)\right] - (1 - \gamma)\left(\bar{r}_2'(\tilde{\nu}_1) + \bar{y}_2'(\tilde{\nu}_1)\right)\right\} \\ &+2\lambda f_1(\tilde{\nu}_1)\left\{\gamma\left[1 - r_1 - y_1\right] + (1 - \gamma)\left[1 - \bar{r}_2 - \bar{y}_2\right]\right\} = 0.\end{aligned} \tag{18}$$

The type 1 offer curve $(r_1(\tilde{\nu}_1), y_1(\tilde{\nu}_1))$ is already known since it is the type 1 contract curve. Given initial values for $r_i(.)$, $y_i(.)$ and $F_i(.)$ equations (15),

(16), (17) and (18) can be solved uniquely to gives us the four unknowns $r_2'(.)$, $y_2'(.)$ and $f_i(.)$ (see appendix C). These differential equations in turn allow us to solve numerically for the path of the the type 2 offer curve $(\bar{r}_2(\tilde{\nu}_1), \bar{y}_2(\tilde{\nu}_1))$ and the two distribution functions $F_1(.)$ and $F_2(.)$.

Any solution to these differential equations assumes that constraint (8) binds and so we need to check that the solution is consistent with this; that is the solution should satisfy remark 3 in the discussion of section 3. Hence we require (i) $\pi_1(\tilde{\nu}_1) \leq 0$ globally across the type 1 offer curve, and (ii) $\pi_1(\tilde{\nu}_1) < 0$ in the part of distribution described the solution [of equations (15) to (18)]. By the equi-profit condition, $\pi_1 + \pi_2$ is constant across firms, so equivalently

$$\pi_2'(\tilde{\nu}_1) \geq 0. \tag{19}$$

must hold globally across the type 2 offer curve and strictly on the portion which is described by equations (15) to (18), where

$$\pi_2(\tilde{\nu}_1) = (1-\gamma)\frac{(1-\bar{r}_2(\tilde{\nu}_1) - \bar{y}_2(\tilde{\nu}_1))}{(\delta + \lambda\left[1 - F_1(\tilde{\nu}_1)\right])^2}.$$

We can then show however:

Proposition 3. *Any solution to the differential equations* (15) - (18) *will satisfy condition* (19) *iff the solution for the type 2 offer curve* $(\bar{r}_2, \bar{y}_2)$ *lies on or below the type 2 contract curve.* (19) *holds strictly iff* $(\bar{r}_2, \bar{y}_2)$ *lies below the type 2 contract curve.*

Proof. See appendix C. □

Hence we first solve for the pair of offers made by the smallest firm, $(\hat{r}(\nu_*), \hat{y}(\nu_*))$ and $(r_1(\nu_*), y_1(\nu_*))$ as above. Then the solution to (15) - (18) (using this pair of offers and $F_1(\nu_*) = F_2(\underline{\nu}_2) = 0$ as the initial conditions) describes the distribution of offers of the immediately larger firms, for all of which (8) also binds, up to the point where the type 2 offer under the solution first intersects the type 2 contract curve. Let $\tilde{\nu}_1 = \tilde{\nu}_1^\dagger$ and $\Delta = F_1(\tilde{\nu}_1^\dagger)$ at this point, so firm $\tilde{\nu}_1^\dagger$ makes a pair of offers $(r_i^\dagger, y_i^\dagger)$ with one on each contract curve, and write $\tilde{\nu}_2^\dagger = \nu_2(r_2^\dagger, y_2^\dagger)$. We can then show:

Proposition 4. *There exists no* $\varepsilon > 0$ *such that constraint* (8) *binds for every firm in the interval* $[\tilde{\nu}_1^\dagger, \tilde{\nu}_1^\dagger + \varepsilon)$ *and that all of these firms make equal profits. Therefore in equilibrium, due to property* 2*, there must exist an* $\varepsilon > 0$ *such that constraint* (8) *does not bind for any firm in the interval* $(\tilde{\nu}_1^\dagger, \tilde{\nu}_1^\dagger + \varepsilon]$.

Corollary. *In an equilibrium subject to properties* 1 *and* 2*, there exists no firm* $\tilde{\nu}_1$ *such that* $\tilde{\nu}_1 > \tilde{\nu}_1^\dagger$ *for which* (8) *binds.*

Proof. See appendix D. □

Hence, firms for which $\tilde{\nu}_1 \geq \tilde{\nu}_1^\dagger$ must then be described by a similar procedure to the segmented equilibrium of section 3.1.[10] We define $F_i(\nu)$ over the interval $[\tilde{\nu}_i^\dagger, \bar{\nu}_i]$ by

$$\frac{1 - r_i(\nu) - y_i(\nu)}{(\delta + \lambda[1 - F_i(\nu)])^2} = \frac{1 - r_i^\dagger - y_i^\dagger}{(\delta + \lambda(1 - \Delta))^2}, \tag{20}$$

where

$$\frac{1 - r_i(\bar{\nu}_i) - y_1(\bar{\nu}_i)}{\delta^2} = \frac{1 - r_i^\dagger - y_i^\dagger}{(\delta + \lambda(1 - \Delta))^2}.$$

The distribution of offers is such that firms in the interval $[\tilde{\nu}_1^\dagger, \bar{\nu}_1]$ make the pair of offers $(r_1(\tilde{\nu}_1), y_1(\tilde{\nu}_1))$ and $(r_2(\nu_2), y_2(\nu_2))$ with $\nu_2 = F_2 F_1^{-1}(\tilde{\nu}_1)$ and $\tilde{\nu}_1$ distributed on the interval $[\tilde{\nu}_1^\dagger, \bar{\nu}_1]$ with distribution function F_1. The pair offers made by each firm will separate the two types of worker by identical arguments to those used in the proof of proposition 1, and (20) and (18) ensure that the equi-profit condition holds across the entire distribution (the solution to (15) to (18) for $\tilde{\nu}_1 \in [\nu_*, \tilde{\nu}_1^\dagger]$ and by (20) for $\tilde{\nu}_1 \in [\tilde{\nu}_1^\dagger, \bar{\nu}_1]$). The first order conditions (15) and (18) and the arguments used in section 3.1 respectively ensure that no firm has any incentive to deviate. Hence together with equilibrium unemployment rate $u = \delta/(\delta + \lambda)$, this distribution of offers constitutes a steady state equilibrium.

Figure 4 shows an example of the solution with Cobb-Douglas utility and weights on income of 0.65 and 0.4 respectively for type 1 and 2 workers respectively, with $1 - r_z = y_z = 0.1$. The flow parameters δ and λ only affect the shape of the offer curves by influencing their starting points. The curved segment of the type 2 offer curve from (r_z, y_z) to the type 2 efficient contract curve represents the firms described by (15) - (18). As shown in the figure, it

[10] Similarly, we can prove the analogue of propostion 4 for constraint (9). The combination of these propositions implies that niether constraint binds for firms with $\tilde{\nu}_1 \geq \tilde{\nu}_1^\dagger$, and thus the distribution of offers for these firms must take the form implied by (20) subject to property 3. Since, for $\tilde{\nu}_1 < \tilde{\nu}_1^\dagger$, equations (15) to (18) have a unique solution for $r_2'(.)$, $y_2'(.)$ and $f_i(.)$ this gives the uniqueness of the equilibrium subject to properties 1 to 3. The combination of these propositions can also be slightly adapted to prove the uniqueness of the segmented equilibrium in section 3.1.

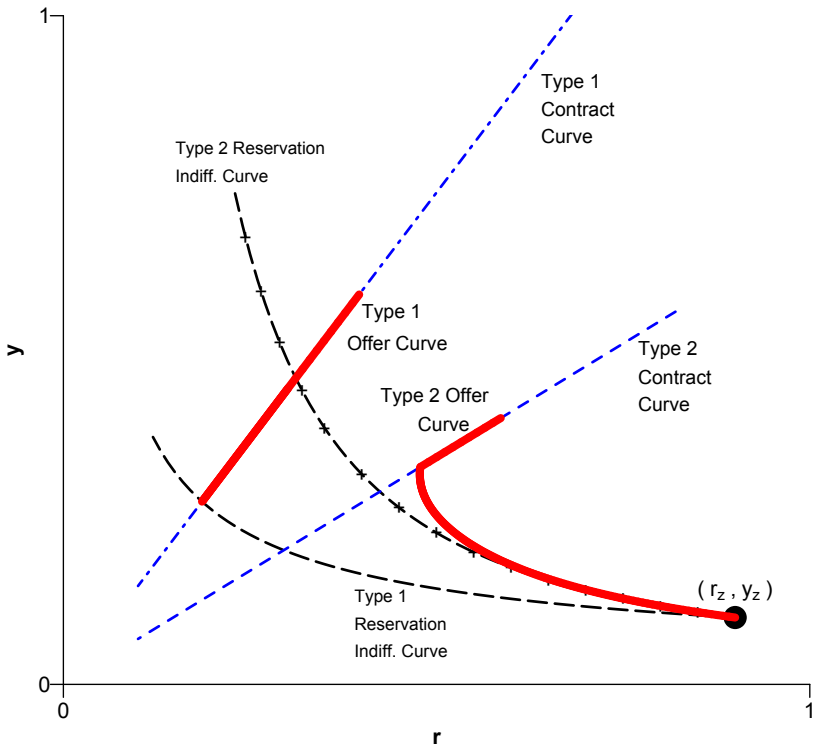

Figure 4: Equilibrium with dependent markets

is the lowest income workers who have inefficient contracts; given firm profit flow from their own job, they would prefer a job with more stress and more income.

Because the fraction of type 2 workers is relatively small (0.2) the pair of offers made by the lowest firm maximises the profit extracted from type 1 workers while offering the type 2 worker a wage equal to his or her marginal product; when the fraction is larger (0.5) as shown in figure 5, the distribution of offers is shifted upwards since the firm is less willing to sacrifice profits from type 2 workers in order to maximise those from type 1. Because of this, the entire offer distribution for type 1 workers, in terms of utility, is

shifted up: the worst-off type 2 worker is still indifferent between work and unemployment but if the fraction of type 2 workers is large enough, the worst-off type 1 worker will strictly prefer work to unemployment. As a result of this type of interaction, a minimum wage policy designed to help the lowest income (type 2) workers, may in fact only produce non-trivial welfare gains for type 1 workers. The worst-off type 1 workers, however, are indifferent between the type 1 and type 2 jobs in *their own firm*, whereas type 2 workers always strictly prefer their jobs to type 1 jobs in the same firm.

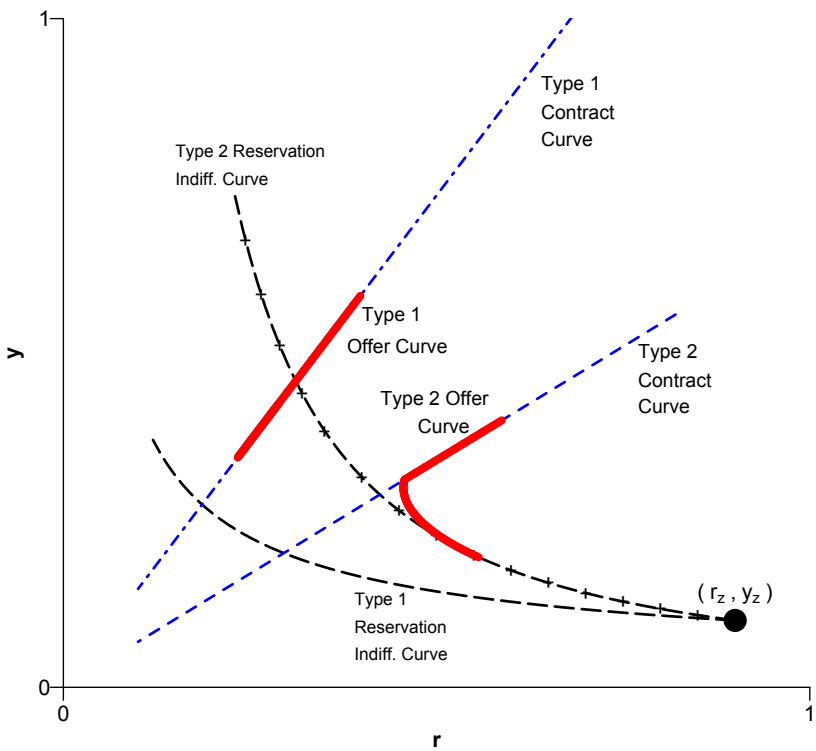

Figure 5: Equilibrium with dependent markets and an increased fraction of type 2 workers

3.3 Type-dependent Unemployment

In section 3.2 we ruled out the existence of single offer firms to simplify the exposition by assuming $r_z + y_z < 1$, i.e. that the point of intersection of the two reservation indifference curves lies below the zero-profit line. This rules out single-offer firms by proposition 2 since $r_z + y_z < 1$ implies that $\Phi_1 \backslash (\Phi_2 \cup \Phi_A)$ is an empty set. It is easy to show that the case $r_z + y_z = 1$ cannot be non-trivially different, so we now consider the case $r_z + y_z > 1$.

Let $\Phi_A^c \equiv \text{cl}(\Phi_A)$, the closure of Φ_A, in which case $\Phi_2 \subset \Phi_A^c$. The converse to proposition 2 follows straightforwardly: if a firm makes an offer in $\Phi_1 \backslash \Phi_A^c$, it must be a single-offer firm.[11] If $r_z + y_z > 1$, then $\Phi_1 \backslash \Phi_A^c$ is a non-empty set. No firm that makes an offer in $\Phi_1 \backslash \Phi_A^c$ can recruit type 2 workers. However, since making an offer in $\Phi_1 \backslash \Phi_A$ allows a greater profit flow from type 1 workers, such firms may exist in equilibrium, forgoing the recruitment of type 2 workers in order to maximise profits from type 1 workers.

Whether this happens again just depends again on how the smallest firm, that recruits only the unemployed and retains only those workers who have not received another offer, maximises profit flow. If it only makes one relevant offer it is easy to see that this using proposition 2 that this offer must be $(\underline{r}_1, \underline{y}_1)$. We then compare the profit flow derived from this offer to that by the pair of separating offers that maximises profit flow $(r_1(\nu_*), y_1(\nu_*))$ and $(\hat{r}(\nu_*), \hat{y}(\nu_*))$ where $\nu_* = \arg\max a(\nu)$ where $a(v)$ is defined as in equation (14). Hence if

$$a(\nu_*) \equiv (1-\gamma)(1-\hat{r}(\nu_*)-\hat{y}(\nu_*))+\gamma(1-r_1(\nu_*)-y_1(\nu_*)) \geq \gamma(1-\underline{r}_1-\underline{y}_1) \quad (21)$$

holds then all firms will make a pair of separating offers with the equilibrium derived with an identical procedure to that of section 3.2.

If (21) does not hold, then a strictly positive fraction of firms, denoted χ, will make an offer on the portion of the type 1 contract curve in $\Phi_1 \backslash \Phi_A$. All of these single-offer firms obtain a higher profit-flow from each type 1 worker than all separating firms which is their incentive for forgoing the recruitment of type 2 workers. Since all separating firms will recruit type 1 workers

[11] Suppose a separating firm makes a pair offers (r_i, y_i) for $i = 1, 2$ such that $(r_1, y_1) \in \Phi_1 \backslash \Phi_A^c$. Since $\Phi_2 \subset \Phi_A^c$, (r_1, y_1) can only recruit type 1 workers, and so, if the firm is separating, (r_2, y_2) must only recruit type 2 workers, in which case we must have $\nu_1(r_1, y_1) \geq \nu_1(r_2, y_2)$ and $(r_2, y_2) \in \Phi_2$. We can clearly see (diagrammatically) that if $(r_2, y_2) \in \Phi_2$, this must imply $\nu_1(r_2, y_2) \geq \nu_1(r_A, y_A)$. But by definition $\nu_1(r_1, y_1) < \nu_1(r_A, y_A)$ which is a contradiction.

from these single offer firms, their presence will increase the recruitment and therefore the profits of the smallest separating firm. In equilibrium χ will be such this firm makes the same profit as the firm making the single offer $(\underline{r}_1, \underline{y}_1)$, so

$$\frac{(1-\gamma)(1-\hat{r}(\nu_*)-\hat{y}(\nu_*))+\gamma(1-r_1(\nu_*)-y_1(\nu_*))}{(\delta+\lambda[1-\chi])^2} = \frac{\gamma(1-\underline{r}_1-\underline{y}_1)}{(\delta+\lambda)^2}. \quad (22)$$

where $\nu_* = \arg\max a(\nu)$ as in (21).

To see that (22) holds, imagine that that every unemployed type 2 worker that makes contact with a single-offer firm gets a 'ghost' job with the firm that generates zero profits for the firm but provides type 2 workers with a utility flow equal to their reservation bundle. Since this fiction does not affect the profits of any firm or the utility of any worker, the left-hand side of (22) represents the profits made by the smallest separating firm. Using this fiction allows us to keep the expression for total profits (7), and therefore the differential equations (15) to (18), unchanged, so long as we write $F_2(\underline{\nu}_2) = \chi$ as one of the initial conditions in the solution. This breaks down if the unemployed and employed face different offer arrival rates, in which case (22) is replaced by a slightly more complex set of equations (see appendix G).

The offer distribution of the fraction χ of single-offer firms must extend up the type 1 contract curve from $(\underline{r}_1, \underline{y}_1)$, the distribution $F_1(.)$ being given a straightforward application of the equi-profit condition following the analysis of section 2. Let ν_χ be defined by

$$\frac{1-r_1(\nu_\chi)-y_1(\nu_\chi)}{(\delta+\lambda[1-\chi])^2} = \frac{1-\underline{r}_1-\underline{y}_1}{(\delta+\lambda)^2}. \quad (23)$$

By the equi-profit condition, the largest single-offer firm must make a distinctly greater profit-flow from type 1 workers than the smallest separating firm, since both recruit the same measure of type 1 workers and the latter makes additional profits from recruiting type 2 workers. We can see this from (22) and (23) which imply $\nu_\chi < \nu_*$; $F_1(\nu) = \chi$ on the interval $\nu \in [\nu_\chi, \nu_*]$. On the interval $\nu \in [\underline{\nu}_1, \nu_\chi]$, we have,

$$\frac{1-r_1(\nu)-y_1(\nu)}{(\delta+\lambda[1-F_1(\nu)])^2} = \frac{1-\underline{r}_1-\underline{y}_1}{(\delta+\lambda)^2} \quad (24)$$

This gives $F_1(\nu)$ for $\nu \in [\underline{\nu}_1, \nu_*]$. The equilibrium is then built up from the smallest separating firm as described in section 3.2, using the pair of offers $(r_1(\nu_*), y_1(\nu_*))$ and $(\hat{r}(\nu_*), \hat{y}(\nu_*))$, and $F_1(\nu_*) = F_2(\underline{\nu}_2) = \chi$, as the initial conditions for solving (15) to (18), where, by definition $\nu_* = \nu_1(r_1(\nu_*), y_1(\nu_*))$ and $\underline{\nu}_2 = \nu_2(\hat{r}(\nu_*), \hat{y}(\nu_*))$. The unemployment rate for type 2 workers is now raised from $\frac{\delta}{\delta+\lambda}$ to $\frac{\delta}{\delta+\lambda(1-\chi)}$ and we label the difference between these two quantities as 'type-dependent unemployment.'

Type-dependent unemployment occurs despite the fact the firms could offer profitable vacancies that would recruit workers and so alleviate it, without incurring any additional vacancy costs. Thus, unlike the standard unemployment associated with search frictions, any strictly positive level of type-dependent is unambiguously inefficient. As described below, even if (21) holds, type-dependent unemployment can be caused by a minimum wage policy.

4 Minimum wage policies with heterogeneous workers

We now compare three different types of minimum wage policy; one a standard minimum wage while the other two involve a minimum wage combined with restrictions on non-wage characteristics. We first make a brief note on the underlying assumptions. Under the simplest assumptions, ignoring non-wage characteristics and worker heterogeneity, and with a fixed number of firms, a standard minimum wage will leave employment unaffected in the BM framework. Adding free entry leads to a negative relationship between minimum wage rises and employment, whereas adding heterogeneity in reservation wages leads to a positive one (see again Manning () and also Flinn () for an extended analysis in an alternate search framework). For reasons of tractability, however, we have left this last possibility (that is introducing heterogeneity in reservation bundles among each worker type) as an extension for future research. Hence the *absolute* effects of a standard minimum wage we find here on employment (which are negative, with or without free entry) would likely become ambiguous were we to include this feature. However, the *relative* qualitative differences in the effects of the three different types of policy are likely to be much more robust to alterations in the underlying assumptions of the search framework.

4.1 A Standard Minimum Wage Policy

If s is a bundle of non-wage characteristics that does not include hours worked, a standard minimum wage policy will take the form of a lower bound on income y denoted y^{min}. In that there are a variety of cases, the effects are relatively complex and summarised at towards end of this subsection. Note that in none of these cases, however, does the minimum wage succeed in raising the infimum of the support of the type 2 offer utility distribution.

Let us initially assume that $r_z + y_z < 1$ in order to best clarify the effects on type-dependent unemployment which does not exist under this assumption in the absence of market intervention, so that all firms are separating. Since all firms make two offers, the lowest paid workers will be of type 2 and a minimum wage must first bite on type 2 offers.

In figure 6, the points N_1 and N_2 denote the type 1 and type 2 offers made by the smallest firm without a minimum wage. Recalling the notation above, the point $(\underline{r}_i, \underline{y}_i)$ denotes the intersection of the type i contract curve and the type i reservation indifference curve, $(r_i(\nu), y_i(\nu))$ the point on the type i contract curve that provides the type i agent utility flow ν and $(\hat{r}(\nu), \hat{y}(\nu))$ the point on the type 2 reservation indifference curve that gives a type 1 agent utility flow ν. Then, as above, $\mathrm{N}_1 = (r_1(\nu_*), y_1(\nu_*))$ and $\mathrm{N}_2 = (\hat{r}(\nu_*), \hat{y}(\nu_*))$ where $\nu^* = \arg\max[a(\nu)]$ with

$$a(\nu) \equiv (1-\gamma)(1 - \hat{r}(\nu) - \hat{y}(\nu)) + \gamma(1 - r_1(\nu) - y_1(\nu)) \tag{25}$$

and $\nu_* \in [\underline{\nu}_1, \nu_1^B]$ where $\nu_1^B = \nu_1(r_B, y_B) = \nu_1(\underline{r}_2, \underline{y}_2)]$.

A minimum wage will not bite if it lies below N_2 in figure 6. Suppose it does bite, so $y^{min} > \hat{y}(\nu_*)$. We also assume initially that $y_{min} \leq \underline{y}_1$. Let (r_2^{min}, y^{min}) be the point where the line $y = y^{min}$ intersects the type 2 reservation indifference curve, and $\nu_1^{min} = \nu_1(r_2^{min}, y^{min})$, the utility obtained by a type 1 agent at that point. Thus $(r_2^{min}, y^{min}) = (\hat{r}(\nu_1^{min}), \hat{y}(\nu_1^{min}))$. Let Φ_1^{min} be the set of profit-making offers satisfying the minimum wage that give a type 1 agent a utility flow equal to or higher than this, so

$$\Phi_1^{min} = \{(r, y) \text{ s.t. } \nu_1(r, y) \geq \nu_1^{min},\ y \geq y^{min},\ r + y \leq 1\}.$$

We can see in figure 6 that upon the introduction of the minimum wage, an offer in $\Phi_1 \backslash \Phi_1^{min}$ can only recruit type 1 workers and no firm that makes such an offer can also make another offer that recruits type 2 workers. Hence any firm that posts an offer in $\Phi_1 \backslash \Phi_1^{min}$ must be a single-offer firm that only

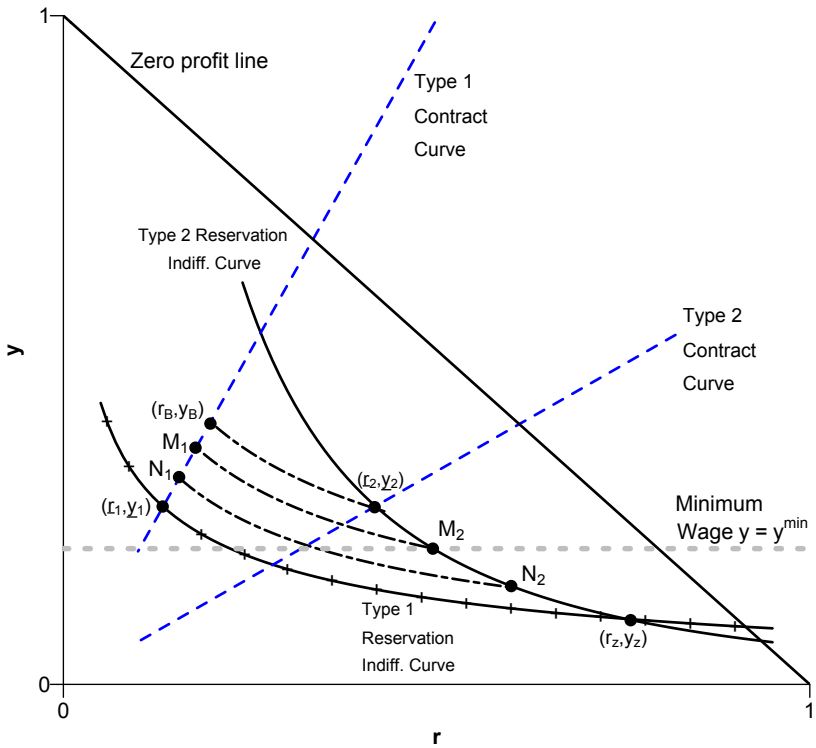

Figure 6: A Standard Minimum Wage. $\mathrm{N}_1 = (r_1(\nu_*), y_1(\nu_*))$, $\mathrm{N}_2 = (\hat{r}(\nu_*), \hat{y}(\nu_*))$, $\mathrm{M}_1 = (r_1(\nu_1^{min}), y_1(\nu_1^{min}))$ and $\mathrm{M}_2 = (\hat{r}(\nu_1^{min}), \hat{y}(\nu_1^{min}))$.

recruits type 1 workers. The lowest pair of separating offers a firm can make is $(r_1(\nu_*^{min}), y_1(\nu_*^{min}))$ and $(\hat{r}(\nu_*^{min}), \hat{y}(\nu_*^{min}))$ where

$$\nu_*^{min} = \arg\max a(\nu) \text{ s.t. } \nu_*^{min} \in [\nu_1^{min}, \nu_1^B].$$

If $a(\nu)$ is concave – which here is a conservative assumption in terms of the effects of the minimum wage[12] – then $\nu_*^{min} = \nu_1^{min}$, and the lowest pair of

[12] A sufficient condition for this is that $\nu_{1,y}\frac{\nu_{2,r}}{\nu_{2,y}} - \nu_{1,r}$, which due to homotheticity can be written as a function of $\frac{y}{r}$, is also increasing in $\frac{y}{r}$. This is satisfied if say the two types of agents have CES utility with a common elasticity of substitution.

separating offers then becomes M_1 and M_2 in figure 6.

Notice that the shift from N_2 to M_2 does not represent an increase in utility for the lowest paid type 2 employee but that the shift from N_1 to M_1 represents a discrete rise in utility for the lowest paid type 1 employee that belongs to a separating firm. If all firms remain separating this creates a discrete first-order improvement in the utility flow distribution of type 1 workers. Thus, paradoxically, even though the minimum wage bites only by constraining type 2 offers, it is the lowest paid type 1 workers who may see the biggest welfare gains. We can see that the profits of the smallest separating firm will fall, so if there is a free entry assumption, firm entry will decrease. If the standard BM result applies that firm entry is inefficiently high due to search costs, then a minimum wage can raise the welfare of type 1 workers will improving efficiency.

However this will not be so if the minimum wage creates type-dependent unemployment. Because offers in $\Phi_1 \backslash \Phi_1^{min}$ can allow a higher profit flow from type 1 workers, this creates an incentive for firms to become single-offer if this additional profit flow compensates forgoing type 2 recruitment. The smallest firm will recruit only unemployed workers, and if it is a single-offer firm it maximises its profit by posting $(\underline{r}_1, \underline{y}_1)$. It follows that type-dependent unemployment will exist iff:

$$(1-\gamma)(1-\hat{r}(\nu_*^{min})-\hat{y}(\nu_*^{min}))+\gamma(1-r_1(\nu_*^{min})-y_1(\nu_*^{min})) < \gamma(1-\underline{r}_1-\underline{y}_1). \quad (26)$$

If (26) holds, the mass of single-offer firms χ^{min} is given by:

$$\begin{aligned}(1-\gamma)(1-\hat{r}(\nu_*^{min})-\hat{y}(\nu_*^{min}))+\gamma(1-r_1(\nu_*^{min})-y_1(\nu_*^{min}))\\ = \frac{(\delta+\lambda[1-\chi^{min}])^2}{(\delta+\lambda)^2}\gamma(1-\underline{r}_1-\underline{y}_1).\end{aligned} \quad (27)$$

The left-hand sides of (26) and (27) must be decreasing in y^{min} (by definition) so, as might be expected, once a minimum wage causes type-dependent unemployment further rises in the minimum wage increase it. Rises in the minimum wage cause some separating firms to become single-offer. All of these firms will therefore cease to make a type 2 offer and will make a lower type 1 offer than they did prior to the rise: on figure 6, roughly speaking, it will shift a mass of type 1 offers in the vicinity of M_1 towards the vicinity of $(\underline{r}_1, \underline{y}_1)$. All firms that were single offer prior to the rise remain unaffected, and since the profit-flow of the smallest firm is unaffected so is the

profit-flow of all firms (by the equi-profit condition) and there is no effect on firm entry.[13] Thus the worst-off type 1 and 2 employees remain on their respective 2 reservation indifference curves (at $(\underline{r}_1, \underline{y}_1)$ and $(\hat{r}(\nu_1^{min}), \hat{y}(\nu_1^{min}))$ respectively). Unlike a rise in unemployment caused by reducing firm entry, type-dependent unemployment is associated with no reduction in search costs and so is unambiguously inefficient. The offer distribution of the fraction χ^{min} of single-offer firms is calculated straightforwardly as before following the analysis of section 2, replacing χ by χ^{min} in equations (23) and (24). The offer distribution for the remaining fraction $1 - \chi_{min}$ of separating firms is described in proposition 5.

Proposition 5. *The type 1 offer curve extends up the type 1 contract curve from* $(r_1(\nu_*^{min}), y_1(\nu_*^{min}))$. *The type* 2 *offer curve is described as follows.*

(i) If $y^{min} < \underline{y}_2$, *the offer curves are solved for as in section 3.2, using the pair of offers* $(r_1(\nu_*^{min}), y_1(\nu_*^{min}))$ *and* $(\hat{r}(\nu_*^{min}), \hat{y}(\nu_*^{min}))$, *and* $F_1(\nu_*^{min}) = F_2(\underline{\nu}_2) = F_2(\nu_2(\hat{r}(\nu_*^{min}, \hat{y}(\nu_*^{min}))) = \chi^{min}$ *as initial conditions for the differential equations* (15) *to* (18) *following section 3.3. There is no minimum wage spike.*

(ii) If $y^{min} > \underline{y}_2$, *the type 2 offer curve follows the minimum wage line* $y = y^{min}$ *to the type 2 contract curve, the offers on the line creating a minimum wage spike, and then extends up the type contract curve from there. Neither of the constraints* (8) *and* (9) *bind in equilibrium, except for the smallest separating firm for which* (8) *binds.*

Proof. See appendix E. □

We have assumed above that $y^{min} \leq \underline{y}_1$ which rules out any type 1 workers being employed at the minimum wage. If $y^{min} > \underline{y}_1$, then the minimum wage may bite directly on type 1 offers. No separating firm, however, can employ type 1 workers at the minimum wage,[14] so this will happen iff there is type-dependent unemployment. We can see that the distribution of type 1 offers

[13] We have not addressed the firms that stay separating. Because of the numerical solution procedure in section 3.2, we do not offer an analytical description of the effects minimum wage rises on the distribution of offers of these firms. However, since overall profit flow does not change, increased utility flows for type 1 employees, say, is likely to be offset by decreased utility flows for type 2. This must be the case for the largest separating firms that make both offers on the respective contract curves.

[14] It is straightforward to see that the type 2 offer of a separating firm must provide strictly lower income than its type 1 offer.

among single-offer firms, as in section 2, will now extend rightwards from the the type 1 reservation indifference curve along the minimum wage line, and then up the type 1 contract curve from the point of its intersection with the minimum wage line. Rises in the minimum wage will now lower the profit-flow made by the smallest single-offer firm but since the iso-profit curve at $\underline{y}_1$ is tangent to the reservation indifference curve, the latter effect is second order as previously described. Since rises in the minimum wage still cause first-order reductions in the profit-flow of the smallest separating firm, rises in the minimum wage must continue to increase type-dependent unemployment at least in the vicinity of $\underline{y}_1$. Thus for minimum wages in this vicinity and below, the effects described so far may then be briefly summarised as follows:

1. A standard minimum wage policy does not raise the lower the infimum of the support of the type 2 offer utility distribution, though it raises the income of the lowest paid type 2 workers.

2. Paradoxically, if a rise in the minimum wage does not cause type-dependent unemployment, it increases the welfare of type 1 workers, causing a first order shift in the support of their utility distribution. This happens despite the fact that type 1 workers have higher incomes prior to the market intervention, and occurs due to the separation constraints. Under free entry in this case, there will be first-order deterrent effects on firm entry.

3. Rises in the minimum wage will increase type-dependent unemployment if it already exists and can cause it if it does not. Once there is type-dependent unemployment, the worst off employees of both types will be at their reservation utility, and rises in the minimum wage negatively affect the welfare of type 1 workers among firms where the minimum wage bites. The mass of firms in the market will also be no longer affected by rises in the minimum wage under a free entry assumption.

There are two further possibilities we briefly mention. Suppose we apply the model to part- and full-time work, so stress is interpreted as working hours. In this case the minimum wage represents a lower bound on y/s.[15] If

[15] We previously 'normalised' our measure of stress so that it had a linear relationship with productivity. In the application to working hours, writing the minimum wage in

the minimum wage bites on type 2 offers (that is prior to introduction there are type 2 offers that do not satisfy the minimum wage) then the qualitative implications are as described above. However, a minimum wage could then bite directly only on type 1 offers, even in the absence of type-dependent unemployment.

A minimum wage could also only bite directly only on type 1 offers even if we do not interpret s as working hours. Suppose type-dependent unemployment already exists prior to introduction of the minimum wage (excluded from the above by the assumption by the assumption $r_z + y_z < 1$). A minimum wage will then only bite directly on type 1 offers iff $\underline{y}_1$ is lower than the income of the smallest type 2 offer prior to introduction (i.e. if $\underline{y}_1 < \hat{y}(\nu_*)$); this might occur when type 2 agents play a lesser role in the market, for example, if Φ_2 is 'small' relative to Φ_1. In both these instances, straightforward arguments show that the effects of the minimum wage parallel that of section 2 and 3.1; it produces second order effects of the offer distributions.[16]

4.2 Restrictions on Non-Wage Characteristics

In all the cases described above, a standard minimum wage policy fails to raise the worst-off type 2 employee's utility from his or her reservation utility, and may create excess and inefficient unemployment among these types of worker. While there are first-order positive effects on welfare arise for the worst-off type 1 employees in the absence of type-dependent unemployment these are reversed for once type-dependent unemployment exists. Therefore, in order to raise the welfare of the worst-off workers of both types the policy-maker needs to introduce regulations on the provision of non-wage characteristics, such as a cap on working hours for example. Clearly this may be a significant policy challenge but the theoretical consequences are at least worth examining; these issues are discussed further in the concluding comments.

this way, this becomes a technological assumption rather a normalisation: that each hour worked for the individual yields the same product. On the other hand, in such an application, it is perhaps a useful starting point to have a neutral assumption about productivity and hours worked.

[16]The sign of the effect on type-dependent unemployment (if it exists) is now negative, but also second order. With free-entry this reduction in type-dependent unemployment would be obtained through an increase in type 1 unemployment rates leaves type 2 unemployment rates unchanged.

In terms of the theory, we will also see that restricting non-wage characteristics provides the efficiency benefit of eliminating type-dependent unemployment. The theory here is also extremely simple and allows us to dispense with the assumption of homothetic utility and straightforwardly include any number of types; the application to working hours also goes through without any significant alteration.

4.2.1 Combined bounds on wages and non-wage characteristics

Suppose now the policymaker introduces a policy which consists of both a lower bound y^{min} on workers' incomes and a restriction on non-wage characteristics which takes the form of a lower bound r^{min} on rest. Trivially, if (r^{min}, y^{min}) lies strictly in the interior of Φ_1 and Φ_2, this form of policy must raise the utility of the worst-off employees above their reservation utility. We focus on this case, since it is the most interesting from a welfare point of view. We show that among this class of policies, those policies that do not produce a spike in the distributions of *both* wages and stress will be Pareto inefficient.

Since $(r^{min}, y^{min}) \in \Phi_1 \cap \Phi_2$, every offer that complies with the policy must be capable of recruiting both types of agent. Thus even if a firm only makes one offer, it will recruit both types of workers and hence there can be no type-dependent unemployment. The smallest firm – that only recruits the unemployed – must only make one offer, (r^{min}, y^{min}) (any another offer is better for the worker in terms of utility flow and worse for the firm in terms of profit-flow; there is no recruitment advantage in making a less profitable offer for the smallest firm, so it will choose (r^{min}, y^{min})). This firm therefore obtains equal profit flows per worker from each type of worker.

Let us also assume for now that the policymaker chooses (r_{min}, y_{min}) so it lies between the two contract curves, so that

$$\frac{\nu_{1,r}(r_{min}, y_{min})}{\nu_{1,y}(r_{min}, y_{min})} \leq 1 \leq \frac{\nu_{2,r}(r_{min}, y_{min})}{\nu_{2,y}(r_{min}, y_{min})}. \tag{28}$$

Clearly the smallest firm must make a single offer (r_{min}, y_{min}) that will recruit both types of worker. Suppose we conjecture an equilibrium where all the remaining firms are separating and neither of the constraints (8) and (9) strictly bind. If this conjecture holds these firms must not only make equal profits, but the ratio of the profit flow per worker extracted from its type 1 to type 2 offer must be constant across all these firms (as it was

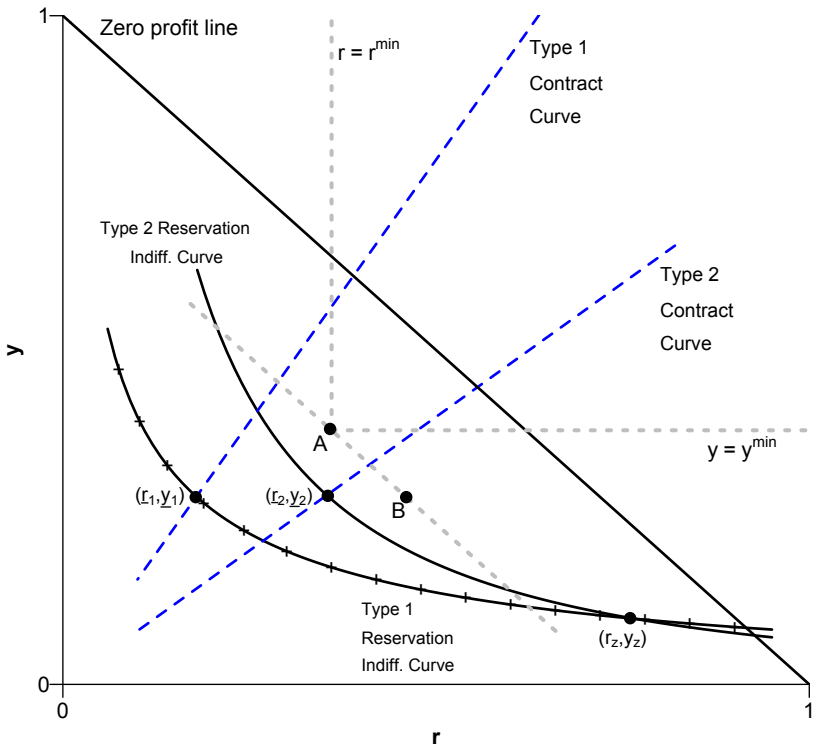

Figure 7: Bounds on non-wage characteristics

in section 3.1) since otherwise firms would be able to exploit the fact that neither constraints (8)) and (9) strictly bind to increase their profits. But this ratio is unity for the smallest firm since both types of worker accept the same offer. Since by standard arguments the distributions of offers must be continuous from the smallest firm, the ratio must also be arbitrarily close to unity for firms that are sufficiently small. Since the ratio is constant, it must therefore be unity for all firms in the equilibrium. In other words, the pair of separating offers made by all firms except the smallest will lie on the same iso-profit line. Then define $\underline{x} \equiv r_{min} + y_{min}$ and $r_i^o(x)$ and $y_i^o(x)$ by

$$\underset{r_i^o \geq r_{min},\, y_i^o \geq y_{min}}{\arg\max} \quad \nu_i(r_i^o, y_i^o) \; s.t. \; r_i^o + y_i^o = x \tag{29}$$

where x has a distribution $F(x)$ which is given by the equi-profit condition:

$$\frac{1-x}{(\delta+\lambda[1-F(x)])^2} = \frac{1-\underline{x}}{(\delta+\lambda)^2}. \tag{30}$$

Then equations (28) and (29) imply that the pair of offers $(r_i^o(x), y_i^o(x))$ must separate the two types of worker even when the constraints $r_i^o \geq r_{min}$ and $y_i^o \geq y_{min}$ strictly bind. We can easily check that this distribution of offers constitutes an equilibrium in which no firm has an incentive to deviate by similar arguments to those given at the end of section 3.1. An example of an equilibrium distribution is shown in figure ***. Due to (28), there is a strictly positive mass of type 1 workers for whom the constraint $r_i^o \geq r_{min}$ strictly binds, and similarly a strictly positive mass of type 2 workers on the minimum wage spike $y_i^o = y_{min}$, with the remaining offers on the respective contract curves.

Thus all firms expect the smallest one are separating. Since the smallest firm is of measure zero, and anyway recruits both types of worker, there can be no type-dependent unemployment. The worst-off workers of each type will receive the offer (r_{min}, y_{min}), and so by choosing this appropriately the policymaker can produce non-trivial gains in welfare for both types of workers. Hence the joint constraints on income and non-wage characteristics eliminates any type-dependent unemployment and allows the policymaker to raise welfare for all workers. However, the policy also forces a strictly positive mass of firms to make offers off their efficient contract curves. Finally, for completeness, we can also very simply show that the policymaker ought to choose (r_{min}, y_{min}) so it lies between the two contract curves as implied by equation (28):

Lemma 2. *Suppose the policymaker chooses (r_{min}, y_{min}) so that it does not lie on one of or between the two contract curves. The distribution of offers described by equations* (29) *and* (30) *describes an equilibrium distribution of offers in which a strictly positive measure of firms are single offer firms, but in which there is no type-dependent unemployment. However there exists a (r'_{min}, y'_{min}) that satisfies* (28) *that would result in a Pareto improvement over this equilibrium.*

Proof. The first part of the lemma follows immediately from the arguments above; note that the offer made by single-offer firms will be accepted by both types of worker. Suppose (r_{min}, y_{min}) lies beneath both contract curves. Let

(r'_{min}, y'_{min}) be the intersection of the iso-profit line through (r_{min}, y_{min}) so $r'_{min} + y'_{min} = r_{min} + y_{min}$, and the type 2 contract curve. It is straightforward to see that this produces an identical distribution of profits among firms (and an identical number of firms with free entry) and makes all type 1 and type 2 workers better off – the intuition for this is that all offers that changes are moved along an iso-profit line closer to the contract curve of each type agent, which must improve utility. If (r_{min}, y_{min}) lies above both contract curves, then let (r'_{min}, y'_{min}) be the intersection of the iso-profit line through (r_{min}, y_{min}) and the type 1 contract curve. An identical argument applies. □

4.2.2 'Iso-profit' minimum wage policies

If we allow a stress- or rest-contingent minimum wage policy, it is straightforward to produce a Pareto improvement on the outcome just described. This is done by introducing lower bound on income $y_{min}(r)$ which is a function of rest, such that $y_{min}(r)$ follows an iso-profit line; we label this as an 'iso-profit' minimum wage policy. This allows the policymaker to eliminate inefficient contracts and type-dependent unemployment while providing arbitrarily high (subject to non-negative profits) welfare gains for both types of worker. Suppose for a job offering rest r, we impose a minimum wage $y_{min}(r) = \underline{x} - r$ where $1-\underline{x}$ is then the maximum profit-flow per worker a firm can derive. By exactly the same reasoning as in section 4.2, the equilibrium will take a form where each firm makes a pair of offers $(r_i^o(x), y_i^o(x))$ for $i = 1, 2$ by

$$\arg\max_{r_i^o, y_i^o} \nu_i(r_i^o, y_i^o) \ s.t. \ r_i^o + y_i^o = x. \tag{31}$$

where again x has a distribution $F(x)$ given by the equi-profit condition:

$$\frac{1-x}{(\delta + \lambda[1 - F(x)])^2} = \frac{\underline{x}}{(\delta + \lambda)^2}. \tag{32}$$

Figure *** shows the nature of the equilibrium. Let us assume that $\underline{x}$ is such that the iso-profit line intersects the contract curve of each type of worker within its respective reservation indifference curve. It is straightforward to see that this policy eliminates type-dependent unemployment, by identical arguments used to those used in section 4.2.1. The smallest firm with the worst-off workers will recruit both types of worker, with the worst-off type i worker (at this firm) having a job $(r_i^o(\underline{x}), y_i^o(\underline{x}))$. Since equation (31)

implies that $(r_i^o(x), y_i^o(x))$ will lie on contract curve i, thus eliminating any inefficient contracts, it is also straightforward to see that this policy provides a Pareto improvement on that of section 4.2.1: for a given $\underline{x}$, the distribution of firm profits is identical with the two policies, whereas the workers that would be 'on the spikes' in section 4.2.1 receive strictly higher utility here.

5 Concluding comments and possible extensions

This paper introduces separation constraints into the classic wage-posting Burdett-Mortensen framework where workers are heterogenous in their preferences over a non-wage job characteristic. It shows that when these constraints do not bind for the smallest firms, the markets for each type of worker are independent; where they do, two sorts of inefficiencies arise. First it can create inefficiently excessive unemployment rates among the type of worker least willing to participate in the labour market. Second contracts at the bottom of the wage distribution for this group are inefficient. Standard minimum wage policies may exacerbate this excess unemployment, but policies that also regulate non-wage characteristics may eliminate it.

Clearly, implementing these sorts of policies would in practice be quite difficult. Ideally, the policymaker would be able to both observe and regulate the various non-wage characteristics which the firm can practically vary, and know the impact their provision had on profitability per worker. In that sense, the paper is supportive of industry-specific regulation such as the recently abolished agricultural wages board in the U.K. which took into account factors such as tied accommodation.

The analysis here ought to extend to the contract-posting models of Stevens (2004) and Burdett and Coles (2003) in a way that is at least conceptually straightforward: workers should progress 'up' their relevant offer curves with tenure: abstracting from the technical details of deriving these, they should not look too different, qualitatively, from those derived in sections 3.1 and 3.2. There are clearly a variety of possible theoretical extensions such as introducing more, and possibly a continuum of, types and unobserved heterogeneity in worker productivity. In addition, allowing heterogeneity in workers' reservation bundles would provide a richer analysis of the effects of minimum wage policies.

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Appendices

A Proof of Proposition 1

Proposition 1: Suppose all firms make a pair of offers $(r_i(\nu_i), y_i(\nu_i))$ on the type i efficient contract curve, with $i = 1, 2$, where ν_1 has distribution $F_1(\nu_1)$ on the support $[\underline{\nu}_1, \bar{\nu}_1]$ and $\nu_2 = F_2^{-1}F_1(\nu_1)$ where $F_i(.)$ are given in (12), so ν_2 has distribution $F_2(\nu_2)$ on the support $[\underline{\nu}_2, \bar{\nu}_2]$. Then the pair of offers made by each firm separates the two types of worker.

Proof. Let C_i denote the loci of contract curves for type i. Since $p'(r) = 1$, the slope of type i's indifference curve must equal unity on C_i. It is straightforward to show that C_i are upward-sloping, and the assumption of homothetic utility implies that C_1 and C_2 take the form of rays through the origin in (r, y) space. Consider any firm that posts a pair of offers $(r_1(\nu_1), y_1(\nu_1))$ and $(r_2(\nu_2), y_2(\nu_2))$, where ν_2 is defined $\nu_2 \equiv F_2^{-1}F_1(\nu_1)$ as in the proposition. We need to show that these offers separate the two types of worker. From equation 12 we know that

$$\frac{1 - r_1(\nu_1) - y_1(\nu_1)}{1 - r_2(\nu_2) - y_2(\nu_2)} = \frac{1 - r_1(\underline{\nu}_1) - y_1(\underline{\nu}_1)}{1 - r_2(\underline{\nu}_2) - y_2(\underline{\nu}_2)} \equiv K \tag{33}$$

for some constant K. Now consider a type 1 indifference curve that offers a type 1 agent utility ν_1. It intersects C_1 by definition at $(r_1(\nu_1), y_1(\nu_1))$ and we denote its intersection with C_2 by $(\hat{r}_2(\nu_1), \hat{y}_2(\nu_1))$. Since the contract curves maximise profit flow for a given level of utility, we must have for all $\nu_1 \in [\underline{\nu}_1, \bar{\nu}_1]$:

$$r_1(\nu_1) + y_1(\nu_1) < \hat{r}_2(\nu_1) + \hat{y}_2(\nu_1). \tag{34}$$

Let

$$\Gamma(\nu_1) \equiv \frac{1 - r_1(\nu_1) - y_1(\nu_1)}{1 - \hat{r}_2(\nu_1) - \hat{y}_2(\nu_1)}. \tag{35}$$

Given the position of (r_z, y_z) relative to the contract curves, we can see that $r_2(\underline{\nu}_2) + y_2(\underline{\nu}_2) \leq \hat{r}_2(\underline{\nu}_1) + \hat{y}_2(\underline{\nu}_1)$, so $\Gamma(\underline{\nu}_1) \geq K$ in (33). By definition, both $(r_2(\nu_2), y_2(\nu_2))$ and $(\hat{r}_2(\nu_1), \hat{y}_2(\nu_1))$ lie on C_2. If $\Gamma(\nu_1)$ is strictly increasing on $\nu_1 \in (\underline{\nu}_1, \bar{\nu}_1]$ then $\hat{r}_2(\nu_1) + \hat{y}_2(\nu_1) > r_2(\nu_2) + y_2(\nu_2)$ and since C_2 is upward-sloping, then $\hat{r}_2(\nu_1) > r_2(\nu_2)$ and $\hat{y}_2(\nu_1) > y_2(\nu_2)$ for all $\nu_1 \in (\underline{\nu}_1, \bar{\nu}_1]$. Hence type 1 workers must then prefer $(r_1(\nu_1), y_1(\nu_1))$ to $(r_2(\nu_2), y_2(\nu_2))$ on $\nu_1 \in [\underline{\nu}_1, \bar{\nu}_1]$ (and strictly on $\nu_1 \in (\underline{\nu}_1, \bar{\nu}_1]$) since they are by definition indifferent between $(r_1(\nu_1), y_1(\nu_1))$ and $(\hat{r}_2(\nu_1), \hat{y}_2(\nu_1))$.

To show that $\Gamma(\nu_1)$ is in fact strictly increasing on $\nu_1 \in (\underline{\nu}_1, \bar{\nu}_1]$ we use the assumption of homothetic utility. Since C_i are rays:

$$r_1 dy_1 = y_1 dr_1 \ ; \ \hat{r}_2 d\hat{y}_2 = \hat{y}_2 d\hat{r}_2. \tag{36}$$

By the definitions of $(r_1(\nu_1), y_1(\nu_1))$ and $(\hat{r}_2(\nu_1), \hat{y}_2(\nu_1))$, we can also write

$$\nu_{1,r}\big|_{\hat{r}_2,\hat{y}_2} d\hat{r}_2 + \nu_{1,y}\big|_{\hat{r}_2,\hat{y}_2} d\hat{y}_2 = \nu_{1,r}\big|_{r_1,y_1} dr_1 + \nu_{1,y}\big|_{r_1,y_1} dy_1 = d\nu_1 \tag{37}$$

Using (36), (37) and the degree 1 homogeneity of $\nu_1(.,.)$, we then have

$$\begin{aligned} dr_1 + dy_1 &= \frac{dr_1 + dy_1}{\nu_{1,r}\big|_{r_1,y_1} dr_1 + \nu_{1,y}\big|_{r_1,y_1} dy_1} d\nu_1 \\ &= \frac{r_1 + y_1}{\nu_{1,r}\big|_{r_1,y_1} r_1 + \nu_{1,y}\big|_{r_1,y_1} y_1} d\nu_1 = \frac{r_1 + y_1}{\nu_1} d\nu_1. \end{aligned} \tag{38}$$

Similarly,

$$\begin{aligned} d\hat{r}_2 + d\hat{y}_2 &= \frac{d\hat{r}_2 + d\hat{y}_2}{\nu_{1,r}\big|_{\hat{r}_2,\hat{y}_2} d\hat{r}_2 + \nu_{1,y}\big|_{\hat{r}_2,\hat{y}_2} d\hat{y}_2} d\nu_1 \\ &= \frac{\hat{r}_2 + \hat{y}_2}{\nu_{1,r}\big|_{\hat{r}_2,\hat{y}_2} \hat{r}_2 + \nu_{1,y}\big|_{\hat{r}_2,\hat{y}_2} \hat{y}_2} d\nu_1 = \frac{\hat{r}_2 + \hat{y}_2}{\nu_1} d\nu_1. \end{aligned} \tag{39}$$

Equations (34), (38) and (39) then combine imply $\Gamma(\nu_1)$ is in strictly increasing on $\nu_1 \in (\underline{\nu}_1, \bar{\nu}_1]$.

Considering a type 2 indifference curve would similarly show type 2 agents must then strictly prefer $(r_2(\nu_2), y_2(\nu_2))$ to $(r_1(\nu_1), y_1(\nu_1))$. Thus the offers are separating. □

B Proof of Proposition 2

Proposition 2: Consider an offer made by a single-offer firm. This offer must then lie on the segment of the efficient contract curve 1 that lies in $\Phi_1 \backslash \Phi_A$, and as a result is only accepted by type 1 workers. Conversely, if a firm makes an offer in $\Phi_1 \backslash \Phi_A^c$, it must be a single-offer firm.

Proof. Suppose the firm posts only one relevant offer (r, y) and suppose $(r, y) \in \Phi_A \backslash \Phi_2$. By definition, only type 1 workers accept (r, y), and also $\nu_1(r, y) > \nu_1(r_A, y_A)$ and $r_A + y_A = 1$ (zero profits) with $\nu_2(r_A, y_A) = \nu_2(r_z, y_z)$. Since Φ_2 is of strictly positive measure, by continuity, for some $\varepsilon > 0$, there must exist an offer $(\hat{r}, \hat{y})$ such that $r_A + y_A \leq 1 - \varepsilon$, $\nu_2(\hat{r}, \hat{y}) > \nu_2(r_z, y_z)$and $\nu_1(r, y) > \nu_1(\hat{r}, \hat{y})$. Hence making the pair of offers (r, y) and $(\hat{r}, \hat{y})$ must increase the firm's profits.

Suppose $(r, y) \in \Phi_2 \backslash \Phi_1$. This offer will by definition only recruit type 2 workers. We can also see diagrammatically, that since (r_z, y_z) lies below the type 2 contract curve, so must (r, y). But then the firm can clearly increase its profits by posting the offer on the type 2 contract curve which provides the type 2 worker with equal utility to (r, y).

Suppose instead $(r, y) \in \Phi_2 \cup \Phi_1$. Let $(\tilde{r}_i(\varepsilon), \tilde{y}_i(\varepsilon))$ be the point on contract curve i that gives the firm a profit $r - y + \varepsilon$. (r, y) clearly cannot lie on both contract curves; let us assume that it does not lie on contract curve 2. Since, by definition, points on contract curve i maximise profits subject to providing a given utility flow to a type i worker, we must have the two following strict inequalities $\nu_2(\tilde{r}(0), \tilde{y}(0)) > \nu_2(r, y)$ and $\nu_i(\tilde{r}_i(0), \tilde{y}_i(0)) > \nu_j(\tilde{r}_i(0), \tilde{y}_i(0))$ for $i \neq j$. Hence there must exist an $\varepsilon > 0$ such that $\nu_2(\tilde{r}_2(\varepsilon), \tilde{y}_2(\varepsilon)) > \nu_2(r, y)$ and the pair of offers $(\tilde{r}_1(0), \tilde{y}_1(0))$ and $(\tilde{r}_2(\varepsilon), \tilde{y}_2(\varepsilon))$, which then also must lie in $\Phi_1 \cup \Phi_2$, is separating. This pair of offers must increase the profits of the firm, since its recruitment and profit flow from type 2 workers is increased, while its recruitment and profit flow from type 1 workers is maintained or increased. A similar argument applies if we assume (r, y) does not lie on contract curve 1.

Hence if the firm posts only one relevant offer it must lie outside $\Phi_2 \cup \Phi_A$ and can only recruit type 1 workers. The offer must lie in Φ_1 of course since otherwise the firm will not recruit at all, and standard profit maximisation arguments imply that it must then lie on contract curve 1.

□

C Proof of Proposition 3

Proposition 3: Any solution to the differential equations (15) - (18) will satisfy condition (19) if and only if (or where and only where) the solution for the type 2 offer curve $(\bar{r}_2, \bar{y}_2)$ lies below or on the type 2 contract curve. If $(\bar{r}_2, \bar{y}_2)$ lies on the type 2 contract curve then (19) holds with equality.

Proof. The profit the firm makes from its type 2 offer is:

$$\pi_2(\tilde{\nu}_1) = (1-\gamma)\frac{1-\bar{r}_2(\tilde{\nu}_1)-\bar{y}_2(\tilde{\nu}_1)}{(\delta+\lambda\left[1-F_1(\tilde{\nu}_1)\right])^2}.$$

Taking the derivative with respect to $\tilde{\nu}_1$:

$$\pi_2'(\tilde{\nu}_1) = \frac{1-\gamma}{(\delta+\lambda\left[1-F_1(\tilde{\nu}_1)\right])^2}\left\{\frac{\left[1-\bar{r}_2(\tilde{\nu}_1)-\bar{y}_2(\tilde{\nu}_1)\right]2\lambda}{\delta+\lambda\left[1-F_1(\tilde{\nu}_1)\right]}f_1(\tilde{\nu}_1)-(\bar{r}_2'(\tilde{\nu}_1)+\bar{y}_2'(\tilde{\nu}_1))\right\}. \tag{40}$$

Remembering that $F_2\left(\nu_2(\bar{r}_2(\tilde{\nu}_1),\bar{y}_2(\tilde{\nu}_1))\right) = F_1(\tilde{\nu}_1)$, and substituting from (15), (17) and then (16),

$$\pi_2'(\tilde{\nu}_1) = \frac{1-\gamma}{(\delta+\lambda\left[1-F_1(\tilde{\nu}_1)\right])^2}\left\{\frac{\nu_{2,y}-\nu_{2,r}}{\nu_{2,r}\nu_{1,y}-\nu_{2,y}\nu_{1,r}}\right\} \tag{41}$$

Since at any point a type 2 indifference curve is steeper than a type 1, $\nu_{2,r}\nu_{1,y}-\nu_{2,y}\nu_{1,r}>0$, and so (41) is positive if and only if

$$\nu_{2,y}-\nu_{2,r}\geq 0. \tag{42}$$

But (42) is satisfied if and and only if $(\bar{r}_2, \bar{y}_2)$ lies on or below the type 2 contract curve, and hence proposition 3. The expression (41) is also useful in that it allows us to eliminate $\bar{r}_2'$ and $\bar{y}_2'$ from (18), giving

$$f_1 = \frac{(\delta+\lambda\left[1-F_1\right])}{2\lambda(1-r_1-y_1)}\left({y'}_1(\tilde{\nu}_1)+{r'}_1(\tilde{\nu}_1)-\frac{1-\gamma}{\gamma}\left(\left.\frac{\nu_{2,y}-\nu_{2,r}}{\nu_{2,r}\nu_{1,y}-\nu_{2,y}\nu_{1,r}}\right|_{\bar{r}_2,\bar{y}_2}\right)\right) \tag{43}$$

where we drop the $(\tilde{\nu}_1)$ argument from the functions r_1, y_1, F_1 and f_1. Combining 43 and 15 then gives us an expression for $\frac{f_1}{f_2}$ that we label $K(\tilde{\nu}_1)$ as follows

$$K = \frac{1-\bar{r}_2-\bar{y}_2}{1-r_1-y_1}\left(\left.\frac{\nu_{2,r}\nu_{1,y}-\nu_{2,y}\nu_{1,r}}{\nu_{1,y}-\nu_{1,r}}\right|_{\bar{r}_2,\bar{y}_2}({y'}_1(\tilde{\nu}_1)+{r'}_1(\tilde{\nu}_1))-\frac{1-\gamma}{\gamma}\left(\left.\frac{\nu_{2,y}-\nu_{2,r}}{\nu_{1,y}-\nu_{1,r}}\right|_{\bar{r}_2,\bar{y}_2}\right)\right) \tag{44}$$

dropping $(\tilde{\nu}_1)$ again as above. Then (16) and (17) straightforwardly give

$$\bar{y}_2'(\tilde{\nu}_1) = \left.\frac{\nu_{2,r} - K(\tilde{\nu}_1)\nu_{1,r}}{\nu_{1,y}\nu_{2,r} - \nu_{1,r}\nu_{2,y}}\right|_{\bar{r}_2,\bar{y}_2} \tag{45}$$

and

$$\bar{r}'_2(\tilde{\nu}_1) = \left.\frac{\nu_{2,y} - K(\tilde{\nu}_1)\nu_{1,y}}{\nu_{1,r}\nu_{2,y} - \nu_{1,y}\nu_{2,r}}\right|_{\bar{r}_2,\bar{y}_2}. \tag{46}$$

Equations (43), (45) and (46) can then be solved numerically to give the type 2 offer curve $(\bar{r}_2(\tilde{\nu}_1), \bar{y}_2(\tilde{\nu}_1))$ and the distribution function $F_1(.)$. The solution for $F_2(.)$ follows from this. Note that, like the efficient contract curves, the shape of the type 2 offer curve, as opposed to the distribution of offers along it, is independent of δ and λ. □

D Proof of Proposition

Proposition 4: There exists no $\varepsilon > 0$ such that constraint (8) binds for every firm in the interval $[\tilde{\nu}_1^\dagger, \tilde{\nu}_1^\dagger + \varepsilon)$ and that all of these firms make equal profits. Therefore in equilibrium, due to property 2, there must exist an $\varepsilon > 0$ such that constraint (8) does not bind for any firm in the interval $(\tilde{\nu}_1^\dagger, \tilde{\nu}_1^\dagger + \varepsilon]$.
Corollary: In an equilibrium subject to properties 1 and 2, there exists no firm $\tilde{\nu}_1$ such that $\tilde{\nu}_1 > \tilde{\nu}_1^\dagger$ for which (8) binds.

Proof. Suppose there does exist an $\varepsilon > 0$ such that constraint (8) binds for every firm in the interval $[\tilde{\nu}_1^\dagger, \tilde{\nu}_1^\dagger + \varepsilon)$. In this interval the distribution of offers must be described by the solution to equations (15) - (18). Since (8) binds, condition (19) must hold, and so the type 2 offers of all these firms must lie on or below the type 2 contract curve by proposition 3.

Each firm $\tilde{\nu}_1$ makes a pair of offers the lie on the same type 1 indifference curve that provides a type 1 utility flow $\tilde{\nu}_1$. The type 1 of will lie on the type contract curve and so is $(r_1(\tilde{\nu}_1), y_1(\tilde{\nu}_1))$; we denote the type 2 offer $(\bar{r}_2(\tilde{\nu}_1), \bar{y}_2(\tilde{\nu}_1))$ as in section 3.2. Consider the right differentials dr_1, dy_1, $d\bar{r}_2$ and $d\bar{y}_2$ in response to a differential change $d\tilde{\nu}_1 > 0$ at $\tilde{\nu}_1 = \tilde{\nu}_1^\dagger$ (where both offers lie on the respective contract curves). Since type 2 offers must lie on or beneath the type 2 contract curve which itself is a ray through the origin,

we must have $d\bar{y}_2/d\bar{r}_2 \leq \bar{y}_2/\bar{r}_2$. From the proof of propostion 1,

$$\begin{aligned} dr_1 + dy_1 &= \frac{dr_1 + dy_1}{\nu_{1,r}|_{r_1,y_1}\, dr_1 + \nu_{1,y}|_{r_1,y_1}\, dy_1} d\nu_1 \\ &= \frac{r_1 + y_1}{\nu_{1,r}|_{r_1,y_1}\, r_1 + \nu_{1,y}|_{r_1,y_1}\, y_1} d\nu_1 = \frac{r_1 + y_1}{\nu_1} d\nu_1 \end{aligned} \tag{47}$$

and

$$d\bar{r}_2 + d\bar{y}_2 = \frac{d\bar{r}_2 + d\bar{y}_2}{\nu_{1,r}|_{\bar{r}_2,\bar{y}_2}\, d\bar{r}_2 + \nu_{1,y}|_{\bar{r}_2,\bar{y}_2}\, d\bar{y}_2} d\nu_1.$$

Since $(\bar{r}_2, \bar{y}_2)$ lies below the type 1 contract curve, $\nu_{1,r}|_{\bar{r}_2,\bar{y}_2} < \nu_{1,y}|_{\bar{r}_2,\bar{y}_2}$. Hence

$$d\bar{r}_2 + d\bar{y}_2 > \frac{\bar{r}_2 + \bar{y}_2}{\nu_{1,r}|_{\bar{r}_2,\bar{y}_2}\, \bar{r}_2 + \nu_{1,y}|_{\bar{r}_2,\bar{y}_2}\, \bar{y}_2} d\nu_1 = \frac{\bar{r}_2 + \bar{y}_2}{\nu_1} d\nu_1. \tag{48}$$

But by arguments used in the proof of proposition 1, this implies $d\bar{r}_2 + d\bar{y}_2 > dr_1 + dy_1$, which in turn, by the equi-profit condition, implies that profits made from type 2 workers are falling along the type 2 offer curve, i.e. a violation of (19). Hence we have a contradiction.

The second part of the proposition then follows from property 2 in section 3: if it were not true we could construct an infinite sequence of firms where constraint (8) alternately binds and does not bind thus violating the property.

For the proof of the corollary, we again use a proof by contradiction. Suppose it is not true, and let $\tilde{\nu}_1 = \tilde{\nu}_1^b$ denote the smallest firm for which (8) binds such that $\tilde{\nu}_1 > \tilde{\nu}_1^\dagger$; from the second part of the proposition such a firm must exist. Thus for all firms $\tilde{\nu}_1 \in (\tilde{\nu}_1^\dagger, \tilde{\nu}_1^b)$ constraint (8) does not bind, and so their type 2 offers must lie on the type 2 contract curve. Therefore, by continuity, the type 2 offer of the firm $\tilde{\nu}_1^b$ must lie on the type 2 contract curve; both of its offers lie on the respective contract curves and by definition on the same type 1 indifference curve. The arguments used in proposition 1, however, then imply that this firm must make smaller profits from type 2 workers than all firms in the interval $[\tilde{\nu}_1^\dagger, \tilde{\nu}_1^b)$. But then firm $\tilde{\nu}_1^b$ can increase its profits by making a lower type 2 offer without affecting the separation of its offers, and so (8) will not bind. Hence we have a contradiction. □

E Proof of Proposition 5

Proposition 5: The type 1 offer curve extends up the type 1 contract curve from $(r_1(\nu_*^{min}), y_1(\nu_*^{min}))$. The type 2 offer curve is described as follows.

(i) If $y^{min} < \underline{y}_2$, the offer curves are solved for as in section 3.2, using the pair of offers $(r_1(\nu_*^{min}), y_1(\nu_*^{min}))$ and $(\hat{r}(\nu_*^{min}), \hat{y}(\nu_*^{min}))$, and $F_1(\nu_*^{min}) = F_2(\underline{\nu}_2) = F_2(\nu_2(\hat{r}(\nu_*^{min}, \hat{y}(\nu_*^{min}))) = \chi^{min}$ as initial conditions for the differential equations (15) to (18) following section 3.3. There is no minimum wage spike.

(ii) If $y^{min} > \underline{y}_2$, the type 2 offer curve follows the minimum wage line $y = y^{min}$ to the type 2 contract curve, the offers on the line creating a minimum wage spike, and then extends up the type contract curve from there. Neither of the constraints (8) and (9) bind in equilibrium, except for the smallest separating firm for which (8) binds.

Proof. Case (i). For case (i) note that from previous results, we simply need to show that $\bar{y}'_2(\tilde{\nu}_1) > 0$ everywhere on the part of the type 2 offer curve that is given by the solution of the differential equations (15) to (18). Since (r_2^{min}, y^{min}) lies beneath the type 2 contract curve, and constraint (8) binds at the initial condition, this implies that minimum wage constraint binds at no point in the solution except at the initial condition; thus the solution given by (15) to (18) remains valid till the type 2 offer curve reaches contract curve as before, and there is no minimum wage spike. We therefore show this using the solution to differential equations (15) to (18) described in proposition 3 and appendix C does indeed imply $\bar{y}'_2(\tilde{\nu}_1) > 0$. Let the profit the firm makes from its type 1 offer be:

$$\pi_1(\tilde{\nu}_1) = \gamma \frac{1 - r_1(\tilde{\nu}_1) - y_1(\tilde{\nu}_1)}{(\delta + \lambda [1 - F_1(\tilde{\nu}_1)])^2}.$$

Since $\pi_1 + \pi_2$ is constant from the equi-profit condition, and $\pi_2' > 0$ from proposition C,

$$\frac{\pi_2(\tilde{\nu}_1)}{\pi_1(\tilde{\nu}_1)} = C \frac{1 - \bar{r}_2(\tilde{\nu}_1) - \bar{y}_2(\tilde{\nu}_1)}{1 - r_1(\tilde{\nu}_1) - y_1(\tilde{\nu}_1)} \tag{49}$$

for some constant C, is increasing in $(\tilde{\nu}_1)$.

Suppose $y(\tilde{\nu}_1)$ is not strictly increasing in $\tilde{\nu}_1$. Then for some $\tilde{\nu}_1$ in the solution, at $\tilde{\nu}_1 = X$ say, a sufficiently small change $d\tilde{\nu}_1$ in $\tilde{\nu}_1$ produces a change dr_1, dy_1, $d\bar{r}_2$ and $d\bar{y}_2$ in r_1, y_1, $\bar{r}_2$ and $\bar{y}_2$ respectively with $d\bar{y}_2 \leq 0$. As shown in proposition 1,

$$dr_1 + dy_1 = \frac{r_1 + y_1}{\tilde{\nu}_1} d\tilde{\nu}_1. \tag{50}$$

But, since $\nu_{1,y}(\bar{r}_2, \bar{y}_2) > \nu_{1,r}(\bar{r}_2, \bar{y}_2)$ and $d\bar{y}_2 < 0$,

$$d\bar{r}_2 + d\bar{y}_2 > d\bar{r}_2 + \frac{\nu_{1,y}(\bar{r}_2, \bar{y}_2)}{\nu_{1,r}(\bar{r}_2, \bar{y}_2)} d\bar{y}_2 = \frac{d\tilde{\nu}_1}{\nu_{1,r}(\bar{r}_2, \bar{y}_2)}. \tag{51}$$

Using the degree 1 homogeneity of ν_1, we then have

$$d\bar{r}_2 + d\bar{y}_2 > \frac{\bar{r}_2 + \frac{\nu_{1,y}}{\nu_{1,r}}\Big|_{\bar{r}_2,\bar{y}_2} \bar{y}_2}{\tilde{\nu}_1} d\tilde{\nu}_1 > \frac{\bar{r}_2 + \bar{y}_2}{\tilde{\nu}_1} d\tilde{\nu}_1. \tag{52}$$

But since (r_1, y_1) lies on the type 1 contract curve (and therefore maximises profit flow subject to a given type 1 utility flow) and offers equal type 1 utility to $(\bar{r}_2, \bar{y}_2)$, we must have $\bar{r}_2 + \bar{y}_2 > r_1 + y_1$. Thus (50) and (52) imply that $\frac{\pi_2(\tilde{\nu}_1)}{\pi_1(\tilde{\nu}_1)}$ is strictly decreasing at $\tilde{\nu}_1 = X$, which is a contradiction.

Case (ii). For case (ii) the shape of the type 2 offer curve follows immediately from continuity of the offer distribution and the observation that no profit maximising firm will make a type 2 offer that lies above both the minimum wage line and both the type 2 contract curve. In this case the type 2 offer could be replaced by an offer on the same type 2 indifference curve but lower income, increasing profit-flow without affecting recruitment or separation, since it must be less attractive to type 1 workers. Thus the type 2 offer curves has two adjoining segments, one on the horizontal minimum wage line and the other on the type 2 contract curve. Let us label these A and B respectively.

Let the type 2 offer curve be $(r_2^o(\nu_2), y_2^o(\nu_2))$, indexed by the type 2 utility. Let F_1 be defined on the support $[\nu_*^{min}, \bar{\nu}_1]$ by

$$\frac{1 - r_1(\nu) - y_1(\nu)}{(\delta + \lambda[1 - F_1(\nu)])^2} = \frac{1 - r_1(\nu_*^{min}) - y_1(\nu_*^{min})}{(\delta + \lambda)^2}, \tag{53}$$

where $(\delta + \lambda)^2(1 - r_1(\bar{\nu}_1) - y_1(\bar{\nu}_1)) = \delta^2(1 - r_1(\nu_{min}^*) - y_1(\nu_{min}^*))$, and F_2 on $[\nu_2(r_{2,min}, \underline{y}_{min}), \bar{\nu}_2]$ by

$$\frac{(1 - r_2^o(\nu) - y_2^o(\nu))}{(\delta + \lambda[1 - F_2(\nu)])^2} = \frac{1 - r_2^{min} - y^{min}}{(\delta + \lambda)^2}. \tag{54}$$

where $(\delta + \lambda)^2(1 - r_2^o(\bar{\nu}_2) - y_2^o(\bar{\nu}_2)) = \delta^2(1 - r_2^{min} - y^{min})$.

We can then show, following a very similar method to the proof of proposition 1, that the offers $(r_1(\nu_1), y_1(\nu_1))$ and $(r_2^o(\nu_2), y_2^o(\nu_2))$ where $\nu_2 =$

$F_2^{-1}F_1(\nu_1)$ separate the two types of worker and so constitute an equilibrium distribution of offers in which neither of the constraints (8) and (9) strictly bind except for the smallest firm. Consider the ratio of profit flows

$$\frac{1 - r_2^o - y_2^o}{1 - r_1 - y_1}$$

of two offers on each offer curve lying on the same type 1 indifference curve of utility ν_1. By very similar arguments to those used in case (i) and proposition 1 (for segments A and B of the type 2 offer curve respectively), this ratio is strictly decreasing in ν_1, implying a type 1 agent strictly prefers $(r_1(\nu_1), y_1(\nu_1))$ to $(r_2^o(\nu_2), y_2^o(\nu_2))$ for $\nu_1 > \nu_*^{min}$. Similarly the same ratio for another two offers lying on the same type 2 indifference curve of utility ν_2 can be shown to be strictly increasing in ν_2, so implying a type 2 agent the profit ratio strictly prefers $(r_2^o(\nu_2), y_2^o(\nu_2))$ to $(r_1(\nu_1), y_1(\nu_1))$ for all $\nu_2 \in [\nu_2(r_2^{min}, y^{min}), \bar{\nu}_2]$. □

F Heterogeneous Firm Productivity

We maintain the assumption productivity is linear in stress $s \equiv 1-r$, but now allow productivity p to be heterogeneous among firms so that p is distributed on the support $[1, \infty)$ with distribution $\Gamma(p)$. Suppose the productivity of a filled job (r, y) in a firm with productivity p produces a profit flow $p(1-r)-y$ for the firm (writing profit flow as $p-r-y$ this would lead to the marginally simpler case where average productivity of firms does not affect the marginal product of stress, and the shape of the contract curves remains unchanged from the analysis above).

Let us for simplicity start with the assumption that the intersection of the reservation indifference curves for the two types (r_z, y_z) is such that $r_z + y_z \leq 1$, so that all firms are separating in equilibrium. As standard in the BM framework, in equilibrium more productive firms will make more attractive offers and we index firms by their productivity p. Thus suppose a firm p makes a pair of offers $(r_i(p), y_i(p))$ $i = 1, 2$ where $(r_i(p), y_i(p))$ is accepted by the type i worker and the distributions $F_i(\nu_i(r_i(p), y_i(p)))$ are defined as above. Property 3 in section 3 above then implies:

$$F_i(\nu_i(r_i(p), y_i(p))) = \Gamma(p). \tag{55}$$

The firm then chooses the two offers to maximise $\sum_{i=1,2} \pi_i(r_i(p), y_i(p))$ where

$$\pi_i(r_i, y_i) = \gamma_i \frac{p(1 - r_i(p)) - y_i(p)}{(\delta + \lambda[1 - F_i(\nu_i(r_i(p), y_i(p)))])^2} \tag{56}$$

subject to

$$\nu_1(r_1(p), y_1(p)) \geq \nu_1(r_2(p), y_2(p)) \tag{57}$$

$$\nu_2(r_2(p), y_2(p)) \geq \nu_2(r_1(p), y_1(p)). \tag{58}$$

The pair of offers made by the firm with $p = 1$ is solved for as discussed in section 3. We then, as standard, obtain a set of differential equations the solution of which gives the offer curves for $p > 1$. The nature of the differential equations at a 'point' p in the solution path will depend on whether constraints (57) or (58) bind or not. For brevity will discuss only the two cases where neither constraint binds or constraint (57) binds. As in section 3, the latter will represent the case that holds for the firm $p = 1$ when (r_z, y_z) lies beneath the two contract curves that would be drawn if all firms had productivity 1.

Suppose neither constraint binds. Then both offers will lie on their respective contract curves

$$\frac{\nu_{i,r}(r_i(p), y_i(p))}{\nu_{i,y}(r_i(p), y_i(p))} = p. \tag{59}$$

We drop the arguments of ν_i or any of its partial derivatives below only if the arguments are $(r_i(p), y_i(p))$ (so $\nu_i \equiv \nu_i(r_i(p), y_i(p))$ etc.). Totally differentiating (55) and (59) wrt p gives

$$F_i'(\nu_i) \left\{\nu_{i,r} r_i'(p) + \nu_{i,y} y_i'(p)\right\} = \Gamma'(p) \tag{60}$$

and

$$\left\{\frac{\nu_{i,rr}}{\nu_{i,y}} - \frac{\nu_{i,r}\nu_{i,ry}}{\nu_{i,y}^2}\right\} r_i'(p) + \left\{\frac{\nu_{i,ry}}{\nu_{i,y}} - \frac{\nu_{i,r}\nu_{i,yy}}{\nu_{i,y}^2}\right\} y_i'(p) = 1. \tag{61}$$

Then, substituting (55) in the first-order condition from (56) wrt y_i (the first-order condition wrt r_i then becoming redundant due to the imposition of (59)), we have

$$\delta + \lambda[1 - \Gamma(p)] = 2\lambda \left\{p(1 - r_i(p)) - y_i(p)\right\} F'(v_i)\nu_{i,y}.$$

Thus (60) becomes

$$pr_i'(p) + y_i'(p) = \frac{2\lambda\Gamma'(p)\left\{p(1-r_i(p)) - y_i(p)\right\}}{\delta + \lambda[1-\Gamma(p)]}. \tag{62}$$

The equations (61) and (62) then give the required solution for $r_i'(p)$ and $y_i'(p)$. Suppose instead constraint (57) binds. Then $(r_1(p), y_1(p))$ lies on the type 1 contract curve, so $r_1'(p)$ and $y_1'(p)$ are given by (61) and (62) above. They will not hold, however, for $r_2'(p)$ and $y_2'(p)$, which are obtained as follows. We assume that constraint (57) continues to bind in the vicinity of p. We then have

$$\nu_{2,r} r_2'(p) + \nu_{2,y} y_2'(p) = \nu_{1,r} r_1'(p) + \nu_{1,y} y_1'(p). \tag{63}$$

The first order conditions from (56) wrt r_2 and y_2 when (57) binds combine to give

$$F'_2(\nu_2) = \frac{\delta + \lambda(1-\Gamma(p))}{2\lambda(p(1-r_2(p)) - y_2(p))} \frac{p\nu_{1,y} - \nu_{1,r}}{p\nu_{2,r}\nu_{1,y} - \nu_{2,y}\nu_{1,r}}\Bigg|_{r_2,y_2}. \tag{64}$$

We can then see that (60), (63) and (64) combine to give the solution for $r_2'(p)$ and $y_2'(p)$. This remains valid up to the point where $r_2(p)$ and $y_2(p)$ satisfy (59), at which point the unconstrained equations (61) and (62) give the correct solution for $r_2'(p)$ and $y_2'(p)$. Note that we do not characterise here where the separating constraints will bind as we do in the main text; checking for this is simply incorporated into the numerical algorithm.

If $r_z + y_z > 1$, then the possibility of type dependent unemployment arises according to the same conditions given in section 3. We then simply solve for the distribution of single-offer firms along the type 1 contract curve according to (60) and (62), with a simple calculation allowing us to check at any point in the solution whether the 'next firm' can increase its profits by making a pair of separating offers. The solution for the separating firms then proceeds as above.

G Differing offer arrival rates for employed and unemployed workers

Suppose the exogenous offer arrival rates for employed and unemployed workers are respectively λ and λ^u. Let $\underline{\nu}_i$ and ν_i^R denote respectively the type

i agent's utility flow obtained in unemployment and reservation utility flow respectively; if $\lambda^u = \lambda$ these will be the same, but we now assume $\lambda^u > \lambda$ in which case ν_i^R become endogenous. We write $k = \lambda/\delta$ and $k^u = \lambda^u/\delta$.

In the segmented market equilibrium of section 3.1, we can exploit the homothetic utility assumption to derive a closed form expression for the reservation utilities ν_i^R following the original Burdett and Mortensen (1998) paper very closely. Due to homotheticity, the contract curves are rays through the origin, with $(r_i(\nu), y_i(\nu))$ denoting the point on the type i contract curve that gives a type i agent utility flow ν. Taking $\nu_i(r, y)$ w.l.o.g. as homogenous of degree 1, for any given $\nu_i(r, y)$ there will exist constants h_i^r and h_i^y such that $r_i(\nu) = h_i^r \nu$ and $y_i(\nu) = h_i^y \nu$. Let us write $h_i = h_i^r + h_i^y$. The condition for the two reservation indifference curves to intersect between the two contract curves that implies the segmented market equilibrium is then:

$$\nu_2(h_1^r, h_1^y) < \frac{\nu_2^R}{\nu_1^R} < \frac{1}{\nu_1(h_2^r, h_2^y)}. \tag{65}$$

The equivalent of equation (12) becomes

$$\frac{1 - h_i \nu}{(\delta + \lambda[1 - F_i(\nu)])^2} = \frac{1 - h_i \nu_i^R}{(\delta + \lambda)^2}, \tag{66}$$

and so

$$F_i(\nu) = \frac{1 + k}{k} \left[1 - \left(\frac{1 - h_i \nu}{1 - h_i \nu_i^R} \right)^{\frac{1}{2}} \right]. \tag{67}$$

The fact that profit flow is linear in ν allows us to follow Burdett and Mortensen (1998) almost exactly in the following derivations:

$$\nu_i^R = \frac{(1 + k)^2 \underline{\nu}_i + (k^u - k)k/h_i}{(1 + k)^2 + (k^u - k)k} \tag{68}$$

and

$$\frac{\nu_2^R}{\nu_1^R} = \frac{(1 + k)^2 \underline{\nu}_2 + (k^u - k)k/h_2}{(1 + k)^2 \nu_1 + (k^u - k)k/h_1}. \tag{69}$$

For an example, let us take the Cobb-Douglas case, writing

$$\nu_i(r, y) = \frac{r^{1-\alpha_i} y^{\alpha_i}}{\alpha_i^{\alpha_i} (1 - \alpha_i)^{1-\alpha_i}}. \tag{70}$$

This representation gives the convenient expression for the contract curves $y_i(\nu) = \alpha_i \nu$ and $r_i(\nu) = (1-\alpha_i)\nu$ and so in the above notation $h_i = 1$. The utility flow level $\nu_i = 1$ then represents the best job a type i agent could get in the absence of frictions: the one on its contract curve that gives zero profit to the firm. Substituting (69) into (65), we get

$$\frac{\alpha_1^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}}{\alpha_2^{\alpha_1}(1-\alpha_2)^{1-\alpha_1}} > \frac{(1+k)^2\underline{\nu}_2 + (k^u - k)k}{(1+k)^2\underline{\nu}_1 + (k^u - k)k} > \frac{\alpha_1^{\alpha_2}(1-\alpha_1)^{1-\alpha_2}}{\alpha_2^{\alpha_2}(1-\alpha_2)^{1-\alpha_2}}. \tag{71}$$

Since the expression $\phi^{\alpha}(1-\phi)^{(1-\alpha)}$ is maximised as a function of ϕ at $\phi = \alpha$, the first and last terms of (71) are greater and less than 1 respectively. Hence we can see that (71) will be satisfied if k^u is sufficiently large.

Numerically, k^u can be pinned down by the unemployment rate, and for example Flinn (2005) gives a typical range of estimates for the ratio k/k^u to be between 0.1 and 0.3. With these values ***

For completeness, we sketch a solution procedure if there exogenous differences in arrival rates and a segmented equilibrium (i.e. condition (71)) does not hold. We must use a numerical solution as in section 3.2. We start with initial guesses for the reservation utilities ν_i^R. Given these guesses, if there is type-dependent unemployment the smallest single-offer firm will make an offer at $(r_1(\nu_1^R), y_1(\nu_1^R))$. Let

$$a(\nu, \chi) \equiv (1-\gamma)c(\chi)(1 - \hat{r}(\nu) - \hat{y}(\nu)) + \gamma(1 - r_1(\nu) - y_1(\nu)) \tag{72}$$

where

$$c(\chi) = \frac{[1+k^u]\,[1+k(1-\chi)]}{[1+k^u(1-\chi)]\,[1+k]}, \tag{73}$$

and define

$$\nu_*(\chi) \equiv \underset{\nu}{\operatorname{argmax}}\; a(\nu, \chi). \tag{74}$$

The condition for an absence of type-dependent unemployment is then

$$a(\nu_*(0), 0) \geq \gamma(1 - r_1(\nu_1^R) - y_1(\nu_1^R)) \tag{75}$$

in which case we set $\chi = 0$. Otherwise χ is given by the solution to

$$\left(\frac{1+k}{1+k(1-\chi)}\right)^2 a(\nu_*(\chi), \chi) = \gamma(1 - r_1(\nu_1^R) - y_1(\nu_1^R)) \tag{76}$$

and χ denotes the fraction of single-offer firms. In the presence of type-dependent unemployment, we will continue the convention of writing $F_2(\nu_2^R) = \chi$ as described in section 3.3. Unlike section 3.3, we have to modify the expression for total profits (7): up-to a multiplicative constant, firms now can be shown to maximise $\pi_1(r_1, y_1) + \pi_2(r_2, y_2)$ where

$$\begin{aligned}\pi_1(r_1, y_1) &= \gamma \frac{(p(r_1) - r_1 - y_1)}{(1 + k[1 - F_1(\nu_1(r_1, y_1))])^2} \\ \pi_2(r_2, y_2) &= (1-\gamma)c(\chi) \frac{(p(r_2) - r_2 - y_2)}{(1 + k[1 - F_2(\nu_2(r_2, y_2))])^2}. \end{aligned} \tag{77}$$

We can see that (77) is identical to (7) if either $k^u = k$ or $\chi = 0$ since both imply $c = 1$. Hence, the differential equations (15) to (18) in section 3.2 that we solve to give the distribution of offers for separating firms remain the same, except that γ is replaced by γ' where

$$\gamma' = \frac{\gamma}{(1 - c(\chi))\gamma + c}.$$

The equilibrium is then built up from the smallest separating firm as described in section 3.2, using the pair of offers $(r_1(\nu_*(\chi)), y_1(\nu_*(\chi)))$ and $(\hat{r}(\nu_*(\chi)), \hat{y}(\nu_*(\chi)))$, and $F_1(\nu_*(\chi)) = F_2(\nu_2^R) = \chi$ as the initial conditions for these differential equations, with the type 1 offers of single-offer firms distributed along the type 1 contract curve starting from $(r_1(\nu_1^R), y_1(\nu_1^R))$ as described in section 3.3. Having thus solved for $F_i(\nu_i)$, the initial guesses ν_i^R are correct if

$$\nu_i^R = \underline{\nu}_i + (k^u - k) \int_{\nu_i^R}^{\infty} \frac{1 - F_i(\nu)}{1 + k(1 - F_i(\nu))} d\nu. \tag{78}$$

The justification for (78) is identical to that of equation (5) on Burdett and Mortensen (1998). We therefore iterate numerically by adjusting ν_i^R suitably according to the discrepancies between the left- and right-hand sides of (78) until we obtain a solution within a required tolerance.