Unconventional government debt purchases as a supplement to conventional monetary policy*

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Abstract

In response to the Great Financial Crisis, the Federal Reserve, the Bank of England and many other central banks have adopted unconventional monetary policy instruments. We investigate if one of these, purchases of long-term government debt, could be a valuable addition to conventional short-term interest rate policy even if the main policy rate is not constrained by the zero lower bound. To do so, we add a stylised financial sector and central bank asset purchases to an otherwise standard New Keynesian DSGE model. Asset quantities matter for interest rates through a preferred habitat channel. If conventional and unconventional monetary policy instruments are coordinated appropriately then the central bank is better able to stabilise both output and inflation.

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1 Introduction

The Great Financial Crisis has seen the emergence of monetary policy instruments that are often described as "unconventional". The events of 2007-2008 forced monetary policy authorities to adopt new tools, even though they had little previous experience with them and there was considerable uncertainty about their likely impact. The general belief was that unconventional policy was an emergency response that would be phased out once the crisis was over. However, if it is designed carefully it may help a central bank reach its objectives even in non-crisis times.

This paper investigates whether the unconventional policy of central banks purchasing long-term government debt could be useful even after the Great Financial Crisis has passed. We obtain our results in a New Keynesian DSGE model with a stylised financial sector and a Taylor-type policy rule for central bank asset purchases. Asset quantities matter because of the behaviour of banks and the incompleteness of financial markets. Households can only transfer income between periods with the help of banks, who invest the deposits they receive into government bonds. The bank allocates deposits into government bonds of different maturities according to a perception that savers are heterogeneous with respect to their preferred investment horizons. Central bank purchases of long-term government bonds reduce the supply of long-term debt available to the private sector, which increases the marginal willingness of banks to pay for it. This reduces yields on long-term debt, discourages saving, and hence increases output and inflation. When the policy parameters are chosen optimally, a combination of conventional and unconventional policies leads to significantly lower losses compared to when the central bank uses only conventional policies.

Central bank purchases of government bonds in our model have an effect through a "preferred habitat" channel of the type identified by Modigliano and Sutch (1966), and later developed by Vayanos and Vila (2009). The idea is that investors see government bonds of different maturities as imperfect substitutes and so are willing to pay a premium on bonds of their preferred maturity. The quantities of assets available then matter for prices and returns; if the central bank purchases government debt of a particular maturity then the supply of that asset to the private sector is reduced, its price rises and its return falls. The preferred habitat channel operates in our model as banks hold government debt of different maturities in response to a perception that savers have heterogeneous investment horizons. The closest models to ours are Andrés et al. (2004) and Chen et al. (2012), although they consider different mechanisms and have less emphasis on policy implications.

The main focus of the current unconventional monetary policy literature is on credit policy, i.e. central bank purchases of private financial assets.[1] An important exception is Eggertsson and Woodford (2003), who examine central bank purchases of government debt. They argue that unconventional monetary policy works by acting as a signal for the future path of short-term

[1] See Cúrdia and Woodford (2011), Del Negro et al. (2011), Gertler and Karadi (2011), Gertler and Kiyotaki (2010), Gertler et al. (2012) and Kiyotaki and Moore (2012).

interest rates, so will be especially useful when the main policy rate is constrained by the zero lower bound. In their model, though, the risk-premium component of long-term interest rates is unaffected by any reallocation of assets between the central bank and the private sector. This is because risks are ultimately born by the private sector, even if government debt is purchased by the central bank. If the central bank makes losses on government debt then government revenue falls and taxes on the private sector have to rise to satisfy the government budget constraint.[2] This view is not supported by the empirical evidence in Bernanke et al. (2004), D'Amico et al. (2012), D'Amico and King (2013), Gagnon et al. (2011), Krishnamurthy and Vissing-Jorgensen (2011) and Neely (2010). Instead, these studies find strong evidence that central bank purchases of government debt have small but significant effects via the term premium component of long-term interest rates. The evidence supporting a preferred habitat mechanism comes from the "scarcity channel" in D'Amico et al. (2012) and the "safety channel" in Krishnamurthy and Vissing-Jorgensen (2011).

2 Model

The model economy consists of households, monopolistically competitive firms, banks, a treasury and a central bank. There is price stickiness, wages are assumed to be fully flexible, and firms use labour as the only input in the production of consumption goods.[3]

2.1 Households

The preferences of the representative household are given by

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left(\chi_t^C \frac{C_t^{1-\delta}}{1-\delta} - \chi_t^L \frac{L_t^{1+\psi}}{1+\psi} \right)$$

where $C_t \equiv \left[\int_0^1 C_t(i)^{\frac{\theta_t - 1}{\theta_t}} di \right]^{\frac{\theta_t}{\theta_t - 1}}$ is a CES consumption index composed to minimise cost and L_t is the time devoted to market employment. χ_t^C and χ_t^L are exogenous preference shock processes that evolve according to

$$\ln(\chi_t^C) = \rho_C \ln(\chi_{t-1}^C) + \varepsilon_t^C \quad \text{with } \varepsilon_t^C \sim N(0, \sigma_C^2) \tag{1}$$

$$\ln(\chi_t^L) = \rho_L \ln(\chi_{t-1}^L) + \varepsilon_t^L \quad \text{with } \varepsilon_t^L \sim N(0, \sigma_L^2) \tag{2}$$

The household maximises expected utility subject to the budget constraint:

$$P_t C_t + T_t + P_t^S S_{t,t+1} = S_{t-1,t} + W_t L_t + (1 - t_\pi)(P_t Y_t - W_t L_t) \tag{3}$$

[2] This intuition is given on page 5 of Cúrdia and Woodford (2010).

[3] The model incorporates elements from Benigno and Woodford (2005), Gali (2008) and Woodford (2003).

$P_t \equiv \left[\int_0^1 P_t(i)^{1-\theta_t} di\right]^{\frac{1}{1-\theta_t}}$ is the price of the composite consumption good, T_t is a lump-sum tax paid to the government and $S_{t,t+1}$ is the quantity of a savings device purchased from perfectly competitive banks at unit price $P_t^S < 1$ in period t. The household can secure a payment of $S_{t,t+1}$ in period $t+1$ by saving $P_t^S S_{t,t+1}$ in period t. W_t is the nominal market wage. Households own the firms producing consumption goods, so they receive dividend income $(P_t Y_t - W_t L_t)$ which is subject to tax at the rate t_π. All prices are measured in units of the numeraire good "money", which is not modelled.

The first-order conditions of the household's optimisation problem are

$$1 = \beta E_t \left[\frac{\chi_{t+1}^C}{\chi_t^C} \left(\frac{C_{t+1}}{C_t} \right)^{-\delta} \frac{1}{\Pi_{t+1}} \right] \frac{1}{P_t^S} \tag{4}$$

$$\frac{W_t}{P_t} = \frac{\chi_t^L}{\chi_t^C} L_t^\psi C_t^\delta \tag{5}$$

with $\Pi_{t+1} \equiv \frac{P_{t+1}}{P_t}$. Equation (4) is the intertemporal Euler equation that characterises the optimal consumption-savings decision. Equation (5) is the intratemporal condition describing optimality between labour supply and consumption.

2.2 Firms

There is a continuum of monopolistically competitive firms indexed by $i \in [0,1]$. Firm i's production function is $Y_t(i) = A_t L_t(i)^{\frac{1}{\phi}}$, where $L_t(i)$ is labour employed and A_t is an exogenous technology shock process that evolves according to

$$\ln(A_t) = \rho_A \ln(A_{t-1}) + \varepsilon_t^A \quad \text{with } |\rho_A| < 1 \text{ and } \varepsilon_t^A \sim N(0, \sigma_A^2) \tag{6}$$

As in Calvo (1983), only a fraction $1-\alpha$ of firms can adjust the price of their respective good in any given period. Let $P_t^*(i)$ be the price chosen by a firm that is able to reset its price in period t. The evolution of the aggregate price level is then described by

$$P_t = \left[(1-\alpha) P_t^*(i)^{1-\theta_t} + \alpha P_{t-1}^{1-\theta_t} \right]^{\frac{1}{1-\theta_t}}$$

θ_t is the time-varying elasticity of substitution between consumption goods. All firms that can change their price in period t choose $P_t(i)$ to maximise the expected discounted stream of their future profits

$$E_t \sum_{T=t}^{\infty} \alpha^{T-t} M_{t,T} \left[P_t(i) Y_T(i) - W_T L_T(i) \right]$$

subject to the relevant demand constraints. $M_{t,T} \equiv \beta^{T-t} \frac{\chi_T^C}{\chi_t^C} \frac{C_T^{-\delta}}{C_t^{-\delta}} \frac{P_t}{P_T}$ is the firm's stochastic discount factor, derived from the consumption Euler equation (4).

The first-order condition for price setting and the assumed price adjustment process imply that equilibrium inflation is given by[4]

$$\frac{1-\alpha\Pi_t^{\theta_t-1}}{1-\alpha} = \left(\frac{F_t}{K_t}\right)^{\frac{\theta_t-1}{\theta_t(\phi-1)+1}} \tag{7}$$

where

$$\begin{aligned} F_t &= \chi_t^C C_t^{-\delta} Y_t + \alpha\beta E_t \Pi_{t+1}^{\theta_t-1} F_{t+1} && (8) \\ K_t &= \frac{\theta_t \phi}{\theta_t - 1} \chi_t^L L_t^{\psi} \left(\frac{Y_t}{A_t}\right)^{\phi} \mu_t + \alpha\beta E_t \Pi_{t+1}^{\theta_t \phi} K_{t+1} && (9) \end{aligned}$$

F_t and K_t are auxiliary variables and

$$\ln\left(\frac{\theta_t}{\theta}\right) = \rho_\theta \ln\left(\frac{\theta_{t-1}}{\theta}\right) + \varepsilon_t^\theta \quad \text{with } \varepsilon_t^\theta \sim N(0, \sigma_\theta^2) \tag{10}$$

is the law of motion for the time-varying elasticity of substitution between consumption goods.

2.3 Banks

The role of the representative bank is to determine the maturity composition of the aggregate savings device offered to households. Perfect competition in the banking sector ensures that all bank revenues are returned to households, but it is the bank that decides how deposits are invested into short-term and long-term government bonds. We assume when doing so that banks perceive households as heterogeneous with regards to their desired investment horizons, an assumption consistent with the preferred habitat literature where investors value characteristics of assets other than just their payoff. In the context of our model, this means that investors are perceived as having a preference for assets with maturities that match their preferred investment horizon. Assets of other maturities are viewed as only imperfect substitutes. As a result, the price of an asset with a given maturity is influenced by supply and demand effects local to that maturity, see Vayanos and Vila (2009), and the central bank can use purchases of government bonds to influence prices and yields at different maturities.[5]

Formally, in every period t the representative bank collects nominal deposits $P_t^S S_{t,t+1}$ from households, which it uses to purchase $B_{t,t+1}$ units of a short-term government bond and $Q_{t,t+\tau}$ units of a long-term government bond. The flow budget constraint of the representative bank is

$$P_t^S S_{t,t+1} = P_t^B B_{t,t+1} + P_t^Q Q_{t,t+\tau} \tag{11}$$

A unit of the short-term bond can be purchased at price P_t^B in t and has a payoff of one in the

[4]See Section A.1 in the appendix for the derivation.

[5]The assumption that households are *perceived* as having heterogeneous preferences over different investment horizons is made so the model remains within a standard representative agent framework. If households differ with respect to their *actual* preferences then a full heterogeneous agent model would be needed.

next period $t+1$. The long-term bond is traded at the price P_t^Q in t; a unit of this bond yields a payoff of $\frac{1}{\tau}$ in every period between $t+1$ and $t+\tau$.

The bank constructs the savings device S offered to households on the basis of a function V that reflects its perception of the different investment horizons preferred by heterogenous households. The allocation problem of the representative bank is

$$\max_{B_{t,t+1},Q_{t,t+\tau}} V\left(\frac{B_{t,t+1}}{P_t},\frac{Q_{t,t+\tau}}{P_t}\right)$$

subject to the flow constraint (11).[6] The asset demand schedules that solve the allocation problem depend on the functional form chosen for V. We prefer not to impose restrictive properties on the demand curves, so borrow from the empirical literature on demand estimation and adopt the flexible functional form of the Generalised Translog (GTL) model introduced by Pollak and Wales (1980). Rather than specifying V directly, the GTL model specifies the indirect utility function $V^* \equiv V(\frac{B^*}{P},\frac{Q^*}{P})$:

$$\begin{aligned}\log(V_t^*) &= a_0 + \sum_k a_1^k \log\left(\frac{P_t^k}{P_t^S s_t - P_t^B g^B - P_t^Q g^Q}\right) + \\ &\quad \frac{1}{2}\sum_k\sum_l a_2^{kl}\log\left(\frac{P_t^k}{P_t^S s_t - P_t^B g^B - P_t^Q g^Q}\right)\log\left(\frac{P_t^l}{P_t^S s_t - P_t^B g^B - P_t^Q g^Q}\right)\end{aligned}$$

with $k,l \in \{B,Q\}$, $a_2^{kl}=a_2^{lk}$ and $s_t \equiv \frac{S_{t,t+1}}{P_t}$. The asset shares $as^B \equiv \frac{P^B B}{P^S S}$ and $as^Q \equiv \frac{P^Q Q}{P^S S} = 1 - as^B$ can be derived using the logarithmic form of Roy's identity, see Barnett and Serletis (2008) and Section A.2 in the appendix for the details.

$$as_t^k = \frac{P_t^k g^k}{P_t^S s_t} + \left(1 - \frac{P_t^B g^B + P_t^Q g^Q}{P_t^S s_t}\right)\frac{a_1^k + \sum_l a_2^{kl}\log\left(\frac{P_t^l}{P_t^S s_t - P_t^B g^B - P_t^Q g^Q}\right)}{\sum_l a_1^l + \sum_k\sum_l a_2^{kl}\log\left(\frac{P_t^l}{P_t^S s_t - P_t^B g^B - P_t^Q g^Q}\right)}$$

To ensure that the above asset shares are the result of the bank's optimisation problem, the parameter space has to be restricted such that four "integrability conditions" hold. Under these conditions, quasi-homotheticity, $a_1 \equiv a_1^B$ and $a_2 \equiv a_2^{BB}$, the asset demands are:

$$\frac{B_{t,t+1}}{P_t} = g^B + \frac{P_t^S s_t - P_t^B g^B - P_t^Q g^Q}{P_t^B}\left[a_1 + a_2 \log\left(\frac{P_t^B}{P_t^Q}\right)\right] \tag{12}$$

$$\frac{Q_{t,t+\tau}}{P_t} = g^Q + \frac{P_t^S s_t - P_t^B g^B - P_t^Q g^Q}{P_t^Q}\left[1 - a_1 - a_2 \log\left(\frac{P_t^B}{P_t^Q}\right)\right] \tag{13}$$

[6]The optimisation problem is equivalent to that of a bank in a model where V aggregates the heterogeneous preferences of households over bonds of different maturities.

The income of the representative bank from their holdings of long-term bonds in period t is

$$\frac{1}{\tau}\sum_{j=1}^{\tau} Q_{t-j,t+\tau-j}$$

Perfect competition in the banking sector requires that banks return all their revenues from government bonds to depositors, so the return to a household depositing $P^S_{t-1}S_{t-1,t}$ is

$$S_{t-1,t} = B_{t-1,t} + \frac{1}{\tau}\sum_{j=1}^{\tau} Q_{t-j,t+\tau-j} \tag{14}$$

The prices P^B_t and P^Q_t of the short-term and long-term government bonds are known with certainty in period t but not in advance. The gross nominal interest rate paid on the one-period bond is

$$1 + i_t = \frac{1}{P^B_t} \tag{15}$$

For comparison purposes, it is possible to calculate an interest rate associated with the τ-period bond traded in t. It is implicitly defined by

$$\begin{aligned} P^Q_t &= \frac{\frac{1}{\tau}}{1+i^Q_t} + \frac{\frac{1}{\tau}}{\left(1+i^Q_t\right)^2} + \frac{\frac{1}{\tau}}{\left(1+i^Q_t\right)^3} + \ldots + \frac{\frac{1}{\tau}}{\left(1+i^Q_t\right)^\tau} \\ &= \frac{1}{\tau}\frac{1}{1+i^Q_t}\frac{1-\left(\frac{1}{1+i^Q_t}\right)^\tau}{1-\frac{1}{1+i^Q_t}} \end{aligned} \tag{16}$$

where i^Q_t is the per-period interest rate that equates the unit price P^Q_t with the value of the discounted payoff stream from a unit investment in the long-term bond.

An important feature of the bank behaviour is that the closed-form asset demand schedules are simple and intuitive, in contrast with previous contributions to the literature on unconventional monetary policy. Both demand curves are downward sloping for small enough values of a_2. Low asset quantities are associated with high asset prices, which allows unconventional monetary policy instruments to work through a scarcity channel. For example, central bank purchases of long-term bonds reduce the quantity of those bonds available to the banks and thus increase their price.

2.4 Government

The government's economic policy is implemented by the actions of a treasury and a central bank. We model the operations of the treasury in a simplified way to maintain the focus of our paper on monetary policy.[7]

The treasury issues both short and long-term government debt. Short-term bonds are issued in a quantity consistent with the central bank's setting of the short-term nominal interest rate. The quantity of long-term bonds issued in period t is $\bar{Q}_{t,t+\tau}$, determined by a simple rule

$$\frac{\bar{Q}_{t,t+\tau}}{P_t} = fY \tag{17}$$

to maintain constant real issuance of long-term bonds each period. $f > 0$ and Y is steady-state output. Long-term bonds can only be purchased from the treasury in the period they are issued and then have to be held until maturity, i.e. we follow Sargent (1987, pp. 102-105) and Andrés et al. (2004) in assuming that there is no secondary market for long-term government debt.[8]

The treasury uses lump-sum taxes to finance government spending. The government consumption good G_t has the same CES aggregator as the composite household consumption good and is given exogenously by

$$\ln\left(\frac{G_t}{G}\right) = \rho_G \ln\left(\frac{G_{t-1}}{G}\right) + \varepsilon_t^G \quad \text{with } \varepsilon_t^G \sim N(0, \sigma_G^2) \tag{18}$$

with G being the steady state value of government spending on consumption goods. Lump-sum taxes satisfy $T_t = P_t G_t$.

The central bank sets the short-term nominal interest rate according to a Taylor rule:

$$\frac{1+i_t}{1+i} = \left(\frac{\Pi_t}{\Pi}\right)^{\gamma_\Pi} \left(\frac{Y_t}{Y}\right)^{\gamma_Y} \nu_t \tag{19}$$

Variables without time subscript denote steady-state values and ν_t is an interest rate shock term that evolves according to

$$\ln(\nu_t) = \rho_\nu \ln(\nu_{t-1}) + \varepsilon_t^\nu \quad \text{with } \varepsilon_t^\nu \sim N(0, \sigma_\nu^2) \tag{20}$$

Asset purchases are carried out according to a Taylor-type rule:

$$\frac{\bar{Q}_{t,t+\tau} - Q_{t,t+\tau}^{CB}}{\bar{Q}_{t,t+\tau}} = \left(\frac{\Pi_t}{\Pi}\right)^{\gamma_\Pi^{QE}} \left(\frac{Y_t}{Y}\right)^{\gamma_Y^{QE}} \xi_t \tag{21}$$

where $Q_{t,t+\tau}^{CB}$ is the quantity of long-term bonds purchased by the central bank in period t. The

[7] Only the net supply of long-term bonds matters in the model, so it would be possible to abstract from the supply of bonds by the treasury and work instead with an equation for the net supply of long-term bonds. The distinction is retained to avoid confusion between questions of issuance by the treasury and asset purchases by the central bank.

[8] If there is a secondary market in long-term government debt then long-term and short-term bonds become perfect substitutes and it is difficult to maintain the assumption of a preferred habitat across the two different types of bond.

logarithm of the shock ξ_t follows an AR(1) process.

$$\ln(\xi_t) = \rho_\xi \ln(\xi_{t-1}) + \varepsilon_t^\xi \quad \text{with } \varepsilon_t^\xi \sim N(0, \sigma_\xi^2) \tag{22}$$

The central bank is a net buyer of long-term bonds of the type issued in t if $Q_{t,t+\tau}^{CB} > 0$ and is a net supplier of those to the market if $Q_{t,t+\tau}^{CB} < 0$.[9] For example, if inflation and output are below their respective steady-state values then with $\gamma_\Pi^{QE} > 0$, $\gamma_Y^{QE} > 0$ and $\xi_t = \xi = 1$ the central bank stimulates the economy by purchasing long-term bonds. Conversely, if inflation and output lie above their steady-state values then asset sales are used to depress prices and productive activity.

The treasury and central bank are subject to a consolidated budget constraint that equates government revenue to government expenditures:

$$P_t^B B_{t,t+1} + P_t^Q \bar{Q}_{t,t+\tau} + T_t + t_\pi(P_t Y_t - W_t L_t) + \pi_t^{CB} = P_t G_t + B_{t-1,t} + \frac{1}{\tau}\sum_{j=1}^{\tau} \bar{Q}_{t-j,t+\tau-j} \tag{23}$$

π_t^{CB} are central bank profits (or losses) from asset purchases. The combination of perfect competition in the banking sector and goods markets together with lump-sum funding of current government expenditure means that the model's consolidated government budget constraint is always satisfied. Section A.4 of the appendix gives full details.

2.5 Market Clearing

The model is completed by conditions for clearing in bond, goods and labour markets.

Demand and supply in the market for long-term bonds are equated for

$$\bar{Q}_{t,t+\tau} = Q_{t,t+\tau} + Q_{t,t+\tau}^{CB} \tag{24}$$

The market for short-term bonds clears by assumption. In goods markets the aggregate resource constraint is

$$Y_t = C_t + G_t \tag{25}$$

Labour market clearing requires that hours supplied by the representative household L_t are equal to aggregate hours demanded by the firms, $\int_0^1 L_t(i)di$, implying that aggregate production is

$$Y_t = A_t \left(\frac{L_t}{D_t}\right)^{\frac{1}{\phi}} \tag{26}$$

where $D_t \equiv \int_0^1 \left[\frac{P_t(i)}{P_t}\right]^{-\theta_t \phi} di$ is a measure of price dispersion. The dynamics of the price disper-

[9] Since long-term bonds are only traded in the period in which they are issued, unconventional policy in our model should not be viewed as changing the existing stock of long-term bonds held by the central bank. It should instead be seen as affecting the supply of an asset available for purchase at a particular point in time.

sion term are given by[10]

$$D_t = (1-\alpha)\left(\frac{1-\alpha\Pi_t^{\theta_t-1}}{1-\alpha}\right)^{\frac{\theta_t\phi}{\theta_t-1}} + \alpha\Pi_t^{\theta_t\phi}D_{t-1} \tag{27}$$

Price dispersion is a source of inefficiency in the model. It acts in addition to the mark-up distortions created by the market power of firms.

2.6 Model Summary and Calibration

Equations (1)-(27) are sufficient to describe the behaviour of the endogenous variables in rational expectations equilibrium. Real variables are stationary, but for $\Pi > 1$ there is a positive trend in all nominal variables. Section A.6 of the appendix shows how we take a first order numerical approximation of a trend-stationary version of the model and simulate it around the steady state described in Section A.7 of the appendix. In what follows we define $s_t \equiv \frac{S_{t,t+1}}{P_t}$, $b_t \equiv \frac{B_{t,t+1}}{P_t}$ and $q_t \equiv \frac{Q_{t,t+\tau}}{P_t}$.

Table 1 gives an overview of the model calibration. The calibration of the parameters in the household and firm problems are standard and in line with Gali (2008) and Smets and Wouters (2003, 2007). The households discount factor is 0.99. The inverse of the intertemporal substitution elasticities in consumption and labour supply are 2 and 0.5 respectively. The intratemporal elasticity of substitution between consumption goods equals 6, which implies a steady-state mark-up of 20 per cent. The production function exhibits decreasing returns to scale. Price rigidity is calibrated to a relatively high value because wages are fully flexible in the model. Steady-state inflation corresponds to an annual inflation rate of close to 2 per cent. τ is 20 so the long-term bond has a maturity of 5 years. The government spending share of GDP is set to 40 per cent. This is at the upper end for the US and at the lower end for most European countries. Half of firm profits are paid to the households, the other half to the government, reflecting corporate taxes as well as public ownership of goods producing firms. The calibration of the asset supply and demand equations generates a realistic steady state in bond markets and a response to central bank asset purchases that is in line with empirical estimates.

We target steady-state asset prices P^B and P^Q that translate into annualised interest rates on 1-period and 20-period bonds of 4.5 and 5.5 per cent respectively, in line with the average interest paid on 3-month and 5-year US Treasury Bills between the start of the Greenspan era in 1987 and the end of 2007. The only parameterisation of the issuance equation for long-term bonds that is consistent with these steady-state bond prices and parameters already calibrated is f equal to 0.66. This parameterisation means that long-term government debt accounts for an acceptable 36 per cent share of the government's total steady-state outstanding debt obligations.

A realistic response to central bank asset purchases is achieved through the calibration of the parameters a_1, a_2, g^B and g^Q in the asset demand equations. We set $a_2 = 0$ without loss of

[10] See Section A.3 in the appendix for derivations.

Parameter	Value	Description
β	0.99	Households discount factor
δ	2	Inverse elasticity of intertemporal substitution in consumption
ψ	0.5	Inverse Frisch elasticity of labour supply
θ	6	Steady-state value of intratemporal elasticity of substitution
ϕ	1.1	Inverse of returns to scale in production
α	0.85	Degree of price rigidity
Π	1.005	Steady-state inflation
τ	20	Horizon of long-term bond
$\bar{g}$	0.4	Steady-state ratio of government spending to GDP
$t_{\mathcal{P}}$	0.5	Share of firm profits received by the government
f	0.66	Parameter in long-term bond supply rule
a_1	0.95	Asset demand
a_2	0	Asset demand
g^B	10.21	Asset demand (subsistence level of B)
g^Q	0.59	Asset demand (subsistence level of Q)
ρ_ν	0.1	Persistence of shock to Taylor rule
ρ_ξ	0.1	Persistence of shock to asset purchase rule
ρ_C	0.1	Persistence of consumption preference shock
ρ_L	0.7	Persistence of labour supply preference shock
ρ_G	0.1	Persistence of government spending
ρ_A	0.7	Persistence of technology shock
ρ_θ	0.95	Persistence of shock to elasticity of substitution
σ_ν	0.0025	Standard deviation of shock to Taylor rule
σ_ξ	0.0025	Standard deviation of shock to asset purchase rule
σ_C	0.0025	Standard deviation of consumption preference shock
σ_L	0.0025	Standard deviation of labour supply preference shock
σ_G	0.005	Standard deviation of government spending shock
σ_A	0.01	Standard deviation of technology shock
σ_θ	0.06	Standard deviation of shock to elasticity of substitution

Table 1: Calibration

generality because the logarithmic term in the asset demand equations is redundant when the model is solved to a first-order approximation. With this, the asset demand equation for the long-term bond can be written as

$$q_t = g^Q + \frac{1-a_1}{a_1}\frac{P_t^B}{P_t^Q}(b_t - g^B)$$

and g^B and g^Q can be interpreted as the parts of demand that do not react to relative prices, sometimes referred to as the "subsistence levels" of demand in consumer theory. a_1 determines the relative marginal gain in the objective of the bank from holding the short-term rather than the long-term bond, similar to the parameter in first-order homogeneous Cobb-Douglas preferences. The values of a_1, g^B, g^Q are chosen such that the yield on the long-term bond falls by 100 basis point (bp) if the central bank makes asset purchases that reduce the present discounted value of long-term payment obligations to the private sector by 1.5 per cent. In quantitative terms, this translates to the yield on long-term debt falling by 7 bps after central bank asset purchases that remove 100 trillion of long-term payment obligations to the private sector.[11] This is consistent with the empirical estimates of a 3-15 bps fall in long-term yields per 100 billion of asset purchases reported in various contributions, summarised in Table 1 of Chen et al. (2012).

The calibration of the shock processes emphasises the significance of shocks to government spending and the elasticity of substitution. Technology, monetary policy and preference shocks make a smaller contribution to overall volatility.

3 Conventional and Unconventional Monetary Policy

The possibility of purchasing long-term government debt presents the central bank with an additional tool to help stabilise the economy. To understand the implications of this it is useful to begin with a discussion of how conventional and unconventional monetary policy actions are transmitted in the model.[12]

3.1 Transmission Mechanisms

The response of the economy to an expansionary shock in the Taylor rule for the short-term nominal interest (19) rate is depicted in Figure 1; the response to an expansionary shock in the rule for long-term bond purchases is shown in Figure 2. Both figures are obtained under the calibration $\gamma_\Pi = 1.01$, $\gamma_Y = 0.3$ and $\gamma_\Pi^{QE} = 0$, $\gamma_Y^{QE} = 60$ for conventional and unconventional monetary policies. A value of 60 for γ_Y^{QE} implies in steady state that a one per cent decrease in output leads to the central bank buying approximately 45 per cent of the new long-term bonds issued in that period. This amounts to less than five per cent of all outstanding long-term debt.

The shock to the Taylor rule immediately decreases the short-term nominal interest rate and equivalently increases the price of the short-term bond. Demand for the short-term bond therefore falls. The subsequent rise in P^B also increases the price of the composite savings device available to households, leading to an decrease in savings $P^S s$ and a rise in both output and consumption. The rise in output is accompanied by inflation above its steady-state level. The shock has a less persistent impact on output than inflation because inflation induces relative price distortions that mitigate the rise in output.

The effects on the market for long-term bonds work through two channels. A decrease in savings shifts down the demand curve for the long-term bond, which puts downward pressure

[11] We arrive at our estimate of 7 bps as follows. Asset purchases that reduce the present discounted value of coupon payments on long-term debt payable to households in the model by 0.013 correspond to a 10 bps decrease in the annualised long-term yield. GDP in the US in 2007 was approximately 14 trillion. In our model, steady-state GDP is 1.3 so if 0.013/1.3 corresponds to 10 bps then 100/14000 corresponds to 7 bps.

[12] A full set of impulse response functions is available from the authors on request.

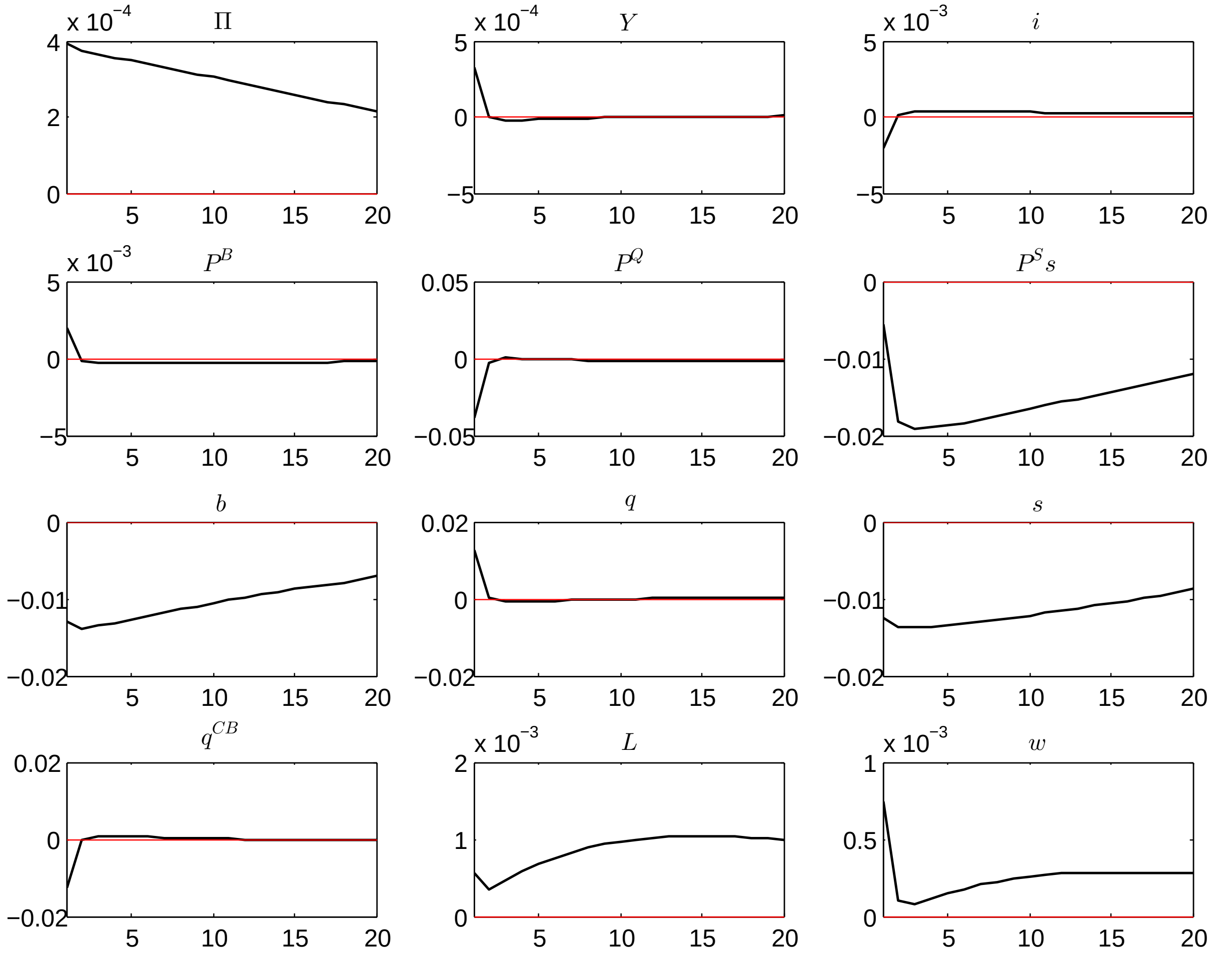

Figure 1: Response to an expansionary short-term nominal interest rate shock

on its price P^Q. In addition, output and inflation rise above their respective steady-state levels so the central bank uses unconventional monetary policy to sell long-term government bonds. This increases the quantity of long-term bonds available to the private sector, further decreasing their price.

Prices quickly return to their steady-state values once the shock to the short-term nominal interest rate has passed. The temporary decline in the yield of the short-term bond has a small negative effect on the wealth of households, forcing them to maintain lower levels of savings, consumption and leisure for a number of periods following the shock. A more elaborate description of public finances would eliminate this effect, since taxes should fall endogenously to reflect the lower interest payments on short-run debt. However, the wealth effect on consumption is very small so our assumption that taxes are exogenous does not significantly weaken our optimal policy exercises.

Figure 2 shows the response of the economy to an expansionary shock in the central bank's

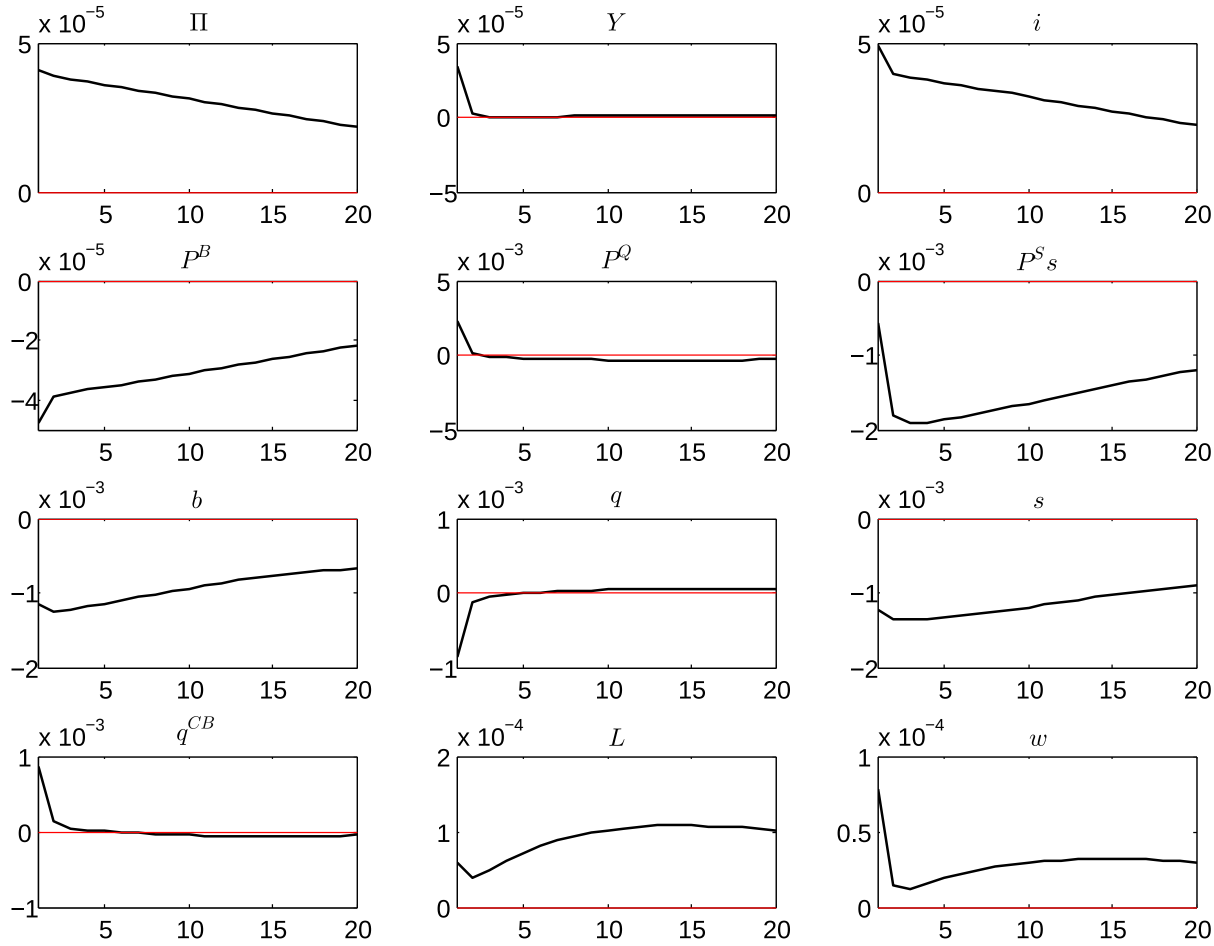

Figure 2: Response to an expansionary shock to purchases of the long-term bond

long-term bond purchase rule (21). The unanticipated rise in central bank purchases of long-term bonds q^{CB} reduces the supply of this asset to the private sector and pushes up its price. The increase in P^Q similarly increases the price of the composite savings device P^S, causing households to save less, consume more and supply more labour to firms. Output then increases and prices grow at a faster rate. The rise in the price P^Q of the long-term bond is associated with a decline in the long-term interest rate, as can be seen from equation (16) defining i^Q. This corresponds to a "flattening of the yield curve" effect generally attributed to purchases of long-term government securities.

The decline in savings shifts the demand curves for both types of bonds inwards, leading to a fall in the quantity of the short-term bond held by the private sector. The fall in holdings is though limited because the central bank subsequently follows its Taylor rule and increases the short-term nominal interest rate, thereby reducing the price of the short-term bond and restoring some of its demand. Savings $P^S s$ only gradually return to their steady-state level after the unexpected purchases of long-term bonds, where again we abstract from the small effect that changes in wealth would imply for taxes. Labour supply similarly remains elevated for a number of periods, which helps bring savings back to their steady-state level and ameliorates the negative output effects that inflation causes through price dispersion.

The impulse response functions suggest that both conventional and unconventional monetary policies may be effective at stabilising the economy. In what follows, the focus is on the benefits of allowing the central bank to purchase long-run government bonds as an unconventional policy that acts as a supplement to conventional short-term interest rate policy.

3.2 Optimised Policy Rules

Let the central bank loss function be

$$\Omega = \omega_\Pi \mathrm{Var}\left(\Pi_t - \Pi\right) + \omega_Y \mathrm{Var}\left(Y_t - Y\right) + \omega_i \mathrm{Var}\left(i_t - i\right) + \omega_Q \mathrm{Var}\left(i_t^Q - i^Q\right)$$

where ω_Π, ω_Y, ω_i and ω_Q are positive weights. Losses are therefore a weighted sum of the variances of inflation, output and interest rates. This specification allows for the volatility of both short-term and long-term interest rates to enter the central bank's considerations, although in the baseline case we set ω_i and ω_Q to zero.

The central bank sets the Taylor rule parameters γ_Π and γ_Y together with the asset purchase rule parameters γ_Π^{QE} and γ_Y^{QE}, subject to the constraints imposed by the model and assumed feasibility constraints $\gamma_\Pi \in [1, 6]$, $\gamma_Y \in [0, 6]$ and $\gamma_\Pi^{QE}, \gamma_Y^{QE} \in [0, 75]$. We allow for a high upper bound on the set of feasible Taylor rule parameters so that the benefits of unconventional monetary model do not arise solely when the feasibility constraints for γ_Π and γ_Y are binding. A lower upper bound on the set of Taylor rule parameters would likely make unconventional policy even more effective, as the central bank would then find it more difficult to stabilise the economy through conventional policy.

Table 2 shows optimised policy rules for different weights on inflation and output stabilisation in the central bank's loss function, with ω_i and ω_Q set to zero so the central bank is not concerned about stabilising interest rates. For each set of weights, the first row presents results when the central bank uses only conventional monetary policy. In this baseline case, coefficients on the asset purchase rule are set to zero and only the coefficients on the interest rate rule are optimised. For the next case, the central bank fixes the coefficients of the interest rate rule at their baseline values, but optimises over the coefficients in the asset purchase rule. This experiment shows the improvements attainable if the central bank starts purchasing long-term government bonds without internalising the potential effect this has on optimal interest rate policy. In the final case, the central bank acknowledges the complementarity of conventional and unconventional policies by jointly optimising over the parameters in the interest rate and asset purchase rules.

The benefits of unconventional monetary policy are small if the central bank's interest rate rule is not adjusted to account for purchases of long-term government bonds. However, when both policies are jointly optimised then the ability of the central bank to stabilise inflation

Weights (in %) $(\omega_\Pi, \omega_Y, \omega_i, \omega_Q)$	Interest Rate Rule (γ_Π, γ_Y)	Asset Purchase Rule $(\gamma_\Pi^{QE}, \gamma_Y^{QE})$	Var (Π_t) $\times 10^{-5}$	Var (Y_t) $\times 10^{-5}$	Var (i_t) $\times 10^{-4}$	Var (i_t^Q) $\times 10^{-4}$	Loss	$\Delta\pi^*$ equ.
70, 30, 0, 0	1.48, 2.22	0(f), 0(f)	9.33	1.43	1.59	0.85	6.96	
	1.48(f), 2.22(f)	0.05, 1.87	9.36	1.16	1.62	0.60	6.90	
	1.66, 0.00	0.00, 18.59	7.71	1.35	2.16	1.16	5.80	1.64%
80, 20, 0, 0	1.49, 2.16	0(f), 0(f)	9.30	1.53	1.58	0.88	7.75	
	1.49(f), 2.16(f)	0.04, 1.78	9.29	1.23	1.61	0.62	7.68	
	1.67, 0.00	0.00, 18.22	7.68	1.42	2.18	1.14	6.43	1.63%
90, 10, 0, 0	1.49, 2.11	0(f), 0(f)	9.28	1.62	1.58	0.91	8.52	
	1.49(f), 2.11(f)	0.04, 1.70	9.24	1.30	1.60	0.64	8.44	
	1.67, 0.00	0.00, 17.92	7.67	1.48	2.20	1.13	7.05	1.63%

(f): fixed, i.e. not chosen optimally

Table 2: Optimised policy rules without interest rate stabilisation

and output is greatly improved. The use of unconventional monetary policy reduces losses by more than 17 per cent compared to the baseline case, provided conventional monetary policy is adjusted accordingly. The optimal policy mix is dichotomised, assigning conventional short-term interest rate policy to react to inflation and unconventional asset purchasing policy to react to output. An important prerequisite for this result is that conventional and unconventional policies differ in their impact on inflation and output, so interventions on one margin are not exactly offset at another margin.

The final column of Table 2 provides an interpretation of welfare losses in terms of an equivalent change in annualised steady-state inflation, using a metric due to by Jensen (2002). For example, comparing the losses in the third row to those in the first row shows that not allowing the central bank to make asset purchases causes welfare losses equivalent to a rise in steady-state inflation of 1.64 percentage points. The derivation of the equivalent steady-state inflation measure is explained in Section A.5 of the appendix.

Weights (in %) $(\omega_\Pi, \omega_Y, \omega_i, \omega_Q)$	Interest Rate Rule (γ_Π, γ_Y)	Asset Purchase Rule $(\gamma_\Pi^{QE}, \gamma_Y^{QE})$	Var (Π_t) $\times 10^{-5}$	Var (Y_t) $\times 10^{-5}$	Var (i_t) $\times 10^{-4}$	Var (i_t^Q) $\times 10^{4}$	Loss	$\Delta\pi^*$ equ.
70, 20, 10, 0	1.55, 2.26	0(f), 0(f)	9.31	1.58	1.55	0.91	8.38	
	1.55(f), 2.26(f)	1.78, 4.33	9.50	1.60	1.18	1.07	8.15	
	1.00, 0.00	4.34, 16.14	8.45	1.31	0.89	1.53	7.07	1.74%
70, 10, 10, 10	1.51, 2.27	0(f), 0(f)	9.34	1.43	1.58	0.86	9.12	
	1.51(f), 2.27(f)	0.69, 4.07	9.46	1.10	1.48	0.61	8.82	
	1.36, 0.40	1.10, 11.66	8.24	1.50	1.46	0.54	7.91	1.67%

(f): fixed, i.e. not chosen optimally

Table 3: Optimised policy rules with interest rate stabilisation

Table 3 suggests that unconventional monetary policy is more effective when the central bank is also concerned about the volatility of interest rates. If only variability in the short-term interest rate is costly then the unconventional policy of purchasing long-term government bonds almost completely replaces conventional short-term interest rate policy. The optimal Taylor rule parameters are at the respective lower bounds of the feasibility sets. Large gains arise because long-term interest rate volatility from asset purchases is costless and substituting conventional policy with asset purchases avoids the losses that arise from movements in the short-term interest rate. The final part of the table shows that the gains from the unconventional tool are still significant when there is a positive weight on the variance of the long-term interest rate in the loss function. These gains are slightly smaller than in the previous case, but still larger than in Table 2 when the central bank is not concerned with interest rate volatility.

3.3 Loss Decomposition

To show the effect of the different policy regimes on the losses due to each type of shock, we focus on the case where the weights in the loss function are given by $(\omega_\Pi, \omega_Y, \omega_i, \omega_Q) = (80, 20, 0, 0)$. In Table 4 the variances of all but one shock are set to zero. The shock of interest is indicated in the first column and has its variance calibrated as in Table 1. Policy parameters, when not set to zero, take the value that is optimal when all shocks are operating jointly.

Shock	Interest Rate Rule (γ_Π, γ_Y)	Asset Purchase Rule $(\gamma_\Pi^{QE}, \gamma_Y^{QE})$	Var (Π_t) $\times 10^{-5}$	Var (Y_t) $\times 10^{-5}$	Var (i_t) $\times 10^{-4}$	Var (i_t^Q) $\times 10^{-4}$	Loss
ν	1.49, 2.16	0, 0	0.511	0.057	0.065	0.050	0.420
	1.49, 2.16	0.04, 1.78	0.485	0.047	0.067	0.039	0.397
	1.67, 0.00	0.00, 18.22	0.433	0.065	0.145	0.047	0.359
ξ	1.49, 2.16	0, 0	0.005	0.000	0.001	0.005	0.004
	1.49, 2.16	0.04, 1.78	0.005	0.000	0.001	0.005	0.004
	1.67, 0.00	0.00, 18.22	0.005	0.000	0.001	0.003	0.004
ε^C	1.49, 2.16	0, 0	0.218	0.077	0.028	0.021	0.190
	1.49, 2.16	0.04, 1.78	0.243	0.071	0.032	0.018	0.209
	1.67, 0.00	0.00, 18.22	0.144	0.075	0.041	0.095	0.130
ε^L	1.49, 2.16	0, 0	0.015	0.002	0.002	0.002	0.012
	1.49, 2.16	0.04, 1.78	0.015	0.001	0.002	0.001	0.012
	1.67, 0.00	0.00, 18.22	0.014	0.002	0.004	0.001	0.012
ε^G	1.49, 2.16	0, 0	1.551	0.544	0.196	0.152	1.350
	1.49, 2.16	0.04, 1.78	1.729	0.502	0.226	0.128	1.484
	1.67, 0.00	0.00, 18.22	1.026	0.532	0.288	0.678	0.927
ε^A	1.49, 2.16	0, 0	0.663	0.068	0.070	0.072	0.544
	1.49, 2.16	0.04, 1.78	0.642	0.052	0.075	0.054	0.524
	1.67, 0.00	0.00, 18.22	0.615	0.077	0.173	0.023	0.508
θ	1.49, 2.16	0, 0	6.337	0.781	1.222	0.578	5.226
	1.49, 2.16	0.04, 1.78	6.170	0.561	1.209	0.374	5.048
	1.67, 0.00	0.00, 18.22	5.445	0.668	1.529	0.293	4.489

Table 4: Losses by type of shock

4 Conclusion

Our results call for central banks to consider purchasing long-term government bonds, even when the economy is not in financial crisis. Unconventional asset purchases of this type are a valuable addition to the tool kit of a central bank trying to stabilise inflation and output, whatever the state of the economy and not just when the short-term nominal interest rate is constrained by the zero lower bound. In order to reap the full benefit, though, it is important to coordinate unconventional and conventional monetary policy. In our model, this means that the short-term interest rate should respond to inflation while the central bank's purchases of long-term debt should respond to output. Unconventional monetary policy plays an even more important role if the central bank is additionally concerned about interest rate volatility. The 'Maturity Extension Program and Reinvestment Policy" announced by the Federal Reserve in September 2011 is an example of the type of unconventional monetary policy measure we advocate.[13] Under this programme, the Federal Reserve lengthens the average maturity of its government bond portfolio by selling short-term treasuries and re-investing the proceeds in long-term treasuries. Our results suggest that such operations may become increasingly important even when financial crisis has passed.

The benefit of unconventional monetary policy in our model is indicative of more general gains available from the coordination of monetary policy and the debt management of fiscal authorities. In our model, central bank purchases of long-term government bonds are equivalent to a reduced issuance of these bonds by the fiscal authority. Whilst our results abstract from active debt management by the treasury, Borio and Disyatat (2010) argues that fiscal authorities may be tempted to reduce debt financing costs by increasing the maturity of new debt at a time when long-term interest rates are relatively low. This is problematic, because it implies that any attempt by the central bank to use unconventional asset purchases to stimulate the economy may be unwound by the treasury issuing more long-term government debt.

[13] In reference to a comparable programme from the 1960s, this policy is sometimes referred to as "Operation Twist" or "Operation Twist Again."

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A Appendix

A.1 Firms

The profit maximisation problem of a firm i that can adjust its price in period t is

$$\max_{P_t(i)} E_t \sum_{T=t}^{\infty} \alpha^{T-t} M_{t,T} \left[P_t(i) Y_T(i) - W_T L_T(i) \right] \tag{28}$$

subject to

$$Y_T(i) = Y_T \left[\frac{P_t(i)}{P_T} \right]^{-\theta_t} \tag{29}$$

Equation (29) is the aggregate demand for firm i's product.[14] Firm i's production function implies that $L_T(i) = \left[\frac{Y_T(i)}{A_T} \right]^{\phi}$ which, together with (29), can be used to rewrite i's optimisation problem as

$$\max_{P_t(i)} E_t \sum_{T=t}^{\infty} \alpha^{T-t} M_{t,T} \left\{ P_t(i) Y_T \left[\frac{P_t(i)}{P_T} \right]^{-\theta_t} - W_T \left(\frac{Y_T}{A_T} \right)^{\phi} \left[\frac{P_t(i)}{P_T} \right]^{-\theta_t \phi} \right\} \tag{30}$$

The first-order condition is

$$E_t \sum_{T=t}^{\infty} \alpha^{T-t} M_{t,T} \left\{ Y_T \left[\frac{P_t^*(i)}{P_T} \right]^{-\theta_t} - \theta_t Y_T \left[\frac{P_t^*(i)}{P_T} \right]^{-\theta_t} + \theta_t \phi \frac{W_T}{P_T} \left(\frac{Y_T}{A_T} \right)^{\phi} \left[\frac{P_t^*(i)}{P_T} \right]^{-\theta_t \phi - 1} \right\} = 0 \tag{31}$$

By substituting in for the real wage from (5), and using the definition of the stochastic discount factor, the first-order condition can be expressed as

$$\begin{aligned} & E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \chi_T^C C_T^{-\delta} Y_T \left(\frac{P_T}{P_t} \right)^{\theta_t - 1} \\ & = \left[\frac{P_t}{P_t^*(i)} \right]^{\theta_t \phi + 1 - \theta_t} E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \frac{\theta_t \phi}{\theta_t - 1} \chi_T^L L_T^{\psi} \left(\frac{Y_T}{A_T} \right)^{\phi} \left(\frac{P_T}{P_t} \right)^{\theta_t \phi} \end{aligned} \tag{32}$$

Defining

$$F_t \equiv E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \chi_T^C C_T^{-\delta} Y_T \left(\frac{P_T}{P_t} \right)^{\theta_t - 1} \tag{33}$$

$$K_t \equiv E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \frac{\theta_t \phi}{\theta_t - 1} \chi_T^L L_T^{\psi} \left(\frac{Y_T}{A_T} \right)^{\phi} \left(\frac{P_T}{P_t} \right)^{\theta_t \phi} \tag{34}$$

[14] This demand schedule is standard in models with monopolistically competitive producers, see Woodford (2003) for a derivation.

followed by rearranging yields

$$\frac{P_t^*(i)}{P_t} = \left(\frac{K_t}{F_t}\right)^{\frac{1}{\theta_t(\phi-1)+1}} \tag{35}$$

Benigno and Woodford (2005) show in an analogous problem that (35) is both necessary and sufficient for an optimum. Note that F_t and K_t can be written recursively:

$$F_t = \chi_t^C C_t^{-\delta} Y_t + \alpha\beta E_t \Pi_{t+1}^{\theta_t - 1} F_{t+1} \tag{36}$$

$$K_t = \frac{\theta_t \phi}{\theta_t - 1} \chi_t^L L_t^{\psi} \left(\frac{Y_t}{A_t}\right)^{\phi} + \alpha\beta E_t \Pi_{t+1}^{\theta_t \phi} K_{t+1} \tag{37}$$

The above equations are those shown in the paper.

The price level in the economy is

$$P_t = \left[(1-\alpha) P_t^*(i)^{1-\theta_t} + \alpha P_{t-1}^{1-\theta_t}\right]^{\frac{1}{1-\theta_t}} \tag{38}$$

Rearranging yields

$$\frac{P_t^*(i)}{P_t} = \left(\frac{1-\alpha\Pi_t^{\theta_t-1}}{1-\alpha}\right)^{\frac{1}{1-\theta_t}} \tag{39}$$

Combining the above equation with (35) gives

$$\frac{1-\alpha\Pi_t^{\theta_t-1}}{1-\alpha} = \left(\frac{F_t}{K_t}\right)^{\frac{\theta_t-1}{\theta_t(\phi-1)+1}} \tag{40}$$

which is the same as equation (7) in Section 2.2.

A.2 Asset Demand Equations

In the GTL model, the asset shares $as^B \equiv \frac{P^B B}{P^S S}$ and $as^Q \equiv \frac{P^Q Q}{P^S S} = 1 - as^B$ are

$$as_t^k = \frac{P_t^k g^k}{P_t^S s_t} + \left(1 - \frac{P_t^B g^B + P_t^Q g^Q}{P_t^S s_t}\right) \frac{a_1^k + \sum_l a_2^{kl} \log\left(\frac{P_t^l}{P_t^S s_t - P_t^B g^B - P_t^Q g^Q}\right)}{\sum_l a_1^l + \sum_k \sum_l a_2^{kl} \log\left(\frac{P_t^l}{P_t^S s_t - P_t^B g^B - P_t^Q g^Q}\right)} \tag{41}$$

where $k, l \in \{B, Q\}$ and symmetry requires that $a_2^{kl} = a_2^{lk}$.

Demand curves are quasi-homothetic if they have linear Engel curves, i.e. the demand for an asset increases linearly with income. We impose quasi-homotheticity to limit wealth effects, which implies that $a_2^{BB} + a_2^{QB} = 0$ and $a_2^{BQ} + a_2^{QQ} = 0$. Together with symmetry this implies that $a_2^{BB} = a_2^{QQ} = -a_2^{BQ} = -a_2^{QB}$. Making the common normalisation $a_1^B + a_1^Q = 1$ and defining

$a_1 \equiv a_1^B$ and $a_2 \equiv a_2^{BB}$, the asset shares are

$$
\begin{aligned}
as_t^B &= \frac{P_t^B g^B}{P_t^S s_t} + \left(1 - \frac{P_t^B g^B + P_t^Q g^Q}{P_t^S s_t}\right) \times \\
&\left[a_1 + a_2 \log\left(\frac{P_t^B}{P_t^S s_t - P_t^B g^B - P_t^Q g^Q}\right) - a_2 \log\left(\frac{P_t^Q}{P_t^S s_t - P_t^B g^B - P_t^Q g^Q}\right)\right] \quad (42)\\
as_t^Q &= \frac{P_t^Q g^Q}{P_t^S s_t} + \left(1 - \frac{P_t^B g^B + P_t^Q g^Q}{P_t^S s_t}\right) \times \\
&\left[1 - a_1 - a_2 \log\left(\frac{P_t^B}{P_t^S s_t - P_t^B g^B - P_t^Q g^Q}\right) + a_2 \log\left(\frac{P_t^Q}{P_t^S s_t - P_t^B g^B - P_t^Q g^Q}\right)\right] \quad (43)
\end{aligned}
$$

which after rearranging is

$$
\begin{aligned}
\frac{B_{t,t+1}}{P_t} &= g^B + \frac{P_t^S s_t - P_t^B g^B - P_t^Q g^Q}{P_t^B}\left[a_1 + a_2 \log\left(\frac{P_t^B}{P_t^Q}\right)\right] \quad (44)\\
\frac{Q_{t,t+\tau}}{P_t} &= g^Q + \frac{P_t^S s_t - P_t^B g^B - P_t^Q g^Q}{P_t^Q}\left[1 - a_1 - a_2 \log\left(\frac{P_t^B}{P_t^Q}\right)\right] \quad (45)
\end{aligned}
$$

Marshallian demands satisfy i) positivity, ii) adding up, iii) homogeneity of degree zero in prices and income and iv) symmetry and negative semi-definiteness of the matrix of substitution effects. Thus, the above demand system is only consistent with the firm's maximisation problem stated in section 2.3 if it satisfies these four "integrability conditions" (Barnett and Serletis, 2008). (44) and (45) satisfy ii) and iii), i) will be satisfied by an adequate calibration and iv) remains to be checked post simulation.

A.3 Price Dispersion

The labour market clearing condition is

$$L_t = \int_0^1 L_t(i)di \quad (46)$$

or, using the production function of firm i

$$L_t = \int_0^1 \left[\frac{Y_t(i)}{A_t}\right]^\phi di \quad (47)$$

Together with (29) this implies

$$L_t = \int_0^1 \left\{\frac{Y_t}{A_t}\left[\frac{P_t(i)}{P_t}\right]^{-\theta_t}\right\}^\phi di = \left(\frac{Y_t}{A_t}\right)^\phi \int_0^1 \left[\frac{P_t(i)}{P_t}\right]^{-\theta_t \phi} di \quad (48)$$

Defining $D_t \equiv \int_0^1 \left[\frac{P_t(i)}{P_t}\right]^{-\theta_t\phi} di$ and rearranging yields

$$Y_t = A_t \left(\frac{L_t}{D_t}\right)^{\frac{1}{\phi}} \tag{49}$$

The law of motion of price dispersion (27) is derived as follows. Since only a share $1-\alpha$ of firms adjusts their price in every period, the dispersion of prices in t can be written

$$\int_0^1 \left[\frac{P_t(i)}{P_t}\right]^{-\theta_t\phi} di = (1-\alpha)\left[\frac{P_t^*(i)}{P_t}\right]^{-\theta_t\phi} + \alpha \int_0^\alpha \left[\frac{P_{t-1}(i)}{P_t}\right]^{-\theta_t\phi} di \tag{50}$$

where without loss of generality it is assumed that the firms that are not able to reset their price in t are those indexed by $i \in [0,\alpha]$. After simple manipulations one obtains

$$\int_0^1 \left[\frac{P_t(i)}{P_t}\right]^{-\theta_t\phi} di = (1-\alpha)\left[\frac{P_t^*(i)}{P_t}\right]^{-\theta_t\phi} + \alpha\Pi_t^{\theta_t\phi} \int_0^\alpha \left[\frac{P_{t-1}(i)}{P_{t-1}}\right]^{-\theta_t\phi} di \tag{51}$$

Since the distribution of prices among the firms that are unable to change their price in $t-1$ is the same as the distribution of all prices in $t-1$, the above equation becomes

$$\int_0^1 \left[\frac{P_t(i)}{P_t}\right]^{-\theta_t\phi} di = (1-\alpha)\left[\frac{P_t^*(i)}{P_t}\right]^{-\theta_t\phi} + \alpha\Pi_t^{\theta_t\phi} \int_0^1 \left[\frac{P_{t-1}(i)}{P_{t-1}}\right]^{-\theta_t\phi} di \tag{52}$$

and adequate substitutions yield the final result

$$D_t = (1-\alpha)\left(\frac{1-\alpha\Pi_t^{\theta_t-1}}{1-\alpha}\right)^{\frac{\theta_t\phi}{\theta_t-1}} + \alpha\Pi_t^{\theta_t\phi} D_{t-1} \tag{53}$$

A.4 Government Budget Constraint

The budget constraint of the representative household is given by

$$P_tC_t + T_t + P_t^S S_{t,t+1} = S_{t-1,t} + W_tL_t + (1-t_\pi)(P_tY_t - W_tL_t) \tag{54}$$

Additionally, we have

$$\begin{aligned}
Y_t &= C_t + G_t && (55)\\
P_t^S S_{t,t+1} &= P_t^B B_{t,t+1} + P_t^Q Q_{t,t+\tau} && (56)\\
S_{t-1,t} &= B_{t-1,t} + \frac{1}{\tau}\sum_{j=1}^{\tau} Q_{t-j,t+\tau-j} && (57)\\
\bar{Q}_{t,t+\tau} &= Q_{t,t+\tau} + Q_{t,t+\tau}^{CB} && (58)
\end{aligned}$$

Central bank profits from asset purchases are given by

$$\pi_t^{CB} \equiv \frac{1}{\tau}\sum_{j=1}^{\tau} Q_{t-j,t+\tau-j}^{CB} - P_t^Q Q_{t,t+\tau}^{CB} \tag{59}$$

and firm profits are equal to $\int_0^1 [P_t(i)Y_t(i) - W_t L_t(i)] = P_t Y_t - W_t L_t$.

Using the relationships listed above, the household budget constraint can be re-written as follows.

$$\begin{aligned}
&P_t C_t + T_t + P_t^S S_{t,t+1} \\
&= S_{t-1,t} + W_t L_t + (1-t_\pi)(P_t Y_t - W_t L_t) && (60)\\
&P_t C_t + T_t + P_t^B B_{t,t+1} + P_t^Q Q_{t,t+\tau} + t_\pi (P_t Y_t - W_t L_t) \\
&= B_{t-1,t} + \frac{1}{\tau}\sum_{j=1}^{\tau} Q_{t-j,t+\tau-j} + P_t Y_t && (61)\\
&T_t + P_t^B B_{t,t+1} + P_t^Q (\bar{Q}_{t,t+\tau} - Q_{t,t+\tau}^{CB}) + t_\pi (P_t Y_t - W_t L_t) \\
&= B_{t-1,t} + \frac{1}{\tau}\sum_{j=1}^{\tau} (\bar{Q}_{t-j,t+\tau-j} - Q_{t-j,t+\tau-j}^{CB}) + P_t G_t && (62)\\
&T_t + P_t^B B_{t,t+1} + P_t^Q \bar{Q}_{t,t+\tau} + t_\pi (P_t Y_t - W_t L_t) + \pi_t^{CB} \\
&= B_{t-1,t} + \frac{1}{\tau}\sum_{j=1}^{\tau} \bar{Q}_{t-j,t+\tau-j} + P_t G_t && (63)
\end{aligned}$$

This is the government budget constraint, Equation (23).

A.5 Loss Conversion

We define the loss from the "baseline" policy as

$$\Omega^b = \omega_\Pi \mathrm{Var}\,(\Pi_t^b - \Pi^b) + \omega_Y \mathrm{Var}\,(Y_t^b - Y^b) + \omega_i \mathrm{Var}\,(i_t^b - i^b) + \omega_Q \mathrm{Var}\,(i_t^{Q,b} - i^{Q,b}) \tag{64}$$

and similarly the loss from an alternative policy that we would like to compare with the baseline policy as

$$\begin{aligned}
\Omega^a &= \omega_\Pi \mathrm{Var}\,(\Pi_t^a - \Pi^a) + \omega_Y \mathrm{Var}\,(Y_t^a - Y^a) + \omega_i \mathrm{Var}\,(i_t^a - i^a) + \omega_Q \mathrm{Var}\,(i_t^{Q,a} - i^{Q,a}) && (65)\\
&= \omega_\Pi \mathrm{E}\,(\Pi_t^a - \Pi^a)^2 + \omega_Y \mathrm{Var}\,(Y_t^a - Y^a) + \omega_i \mathrm{Var}\,(i_t^a - i^a) + \omega_Q \mathrm{Var}\,(i_t^{Q,a} - i^{Q,a}) && (66)
\end{aligned}$$

The equivalent change in steady state inflation $\Delta\pi^*$ is then implicitly defined as

$$\omega_\Pi \mathrm{E}\,[\Pi_t^a + \Delta\pi^* - \Pi^a]^2 + \omega_Y \mathrm{Var}\,(Y_t^a - Y^a) + \omega_i \mathrm{Var}\,(i_t^a - i^a) + \omega_Q \mathrm{Var}\,(i_t^{Q,a} - i^{Q,a}) = \Omega^b \tag{67}$$

the constant additional amount of inflation that would raise the loss from the alternative policy to the level of the baseline policy. Solving for $\Delta\pi^*$ gives

$$\omega_\Pi \left[\mathrm{E}\left(\Pi_t^a - \Pi^a\right)^2 + (\Delta\pi^*)^2 \right] + \omega_Y \mathrm{Var}\left(Y_t^a - Y^a\right) + \omega_i \mathrm{Var}\left(i_t^a - i^a\right) + \omega_Q \mathrm{Var}\left(i_t^{Q,a} - i^{Q,a}\right)$$
$$= \Omega^b \tag{68}$$
$$\Omega^a + \omega_\Pi (\Delta\pi^*)^2 = \Omega^b \tag{69}$$
$$\Delta\pi^* = \sqrt{\frac{\Omega^b - \Omega^a}{\omega^\Pi}} \tag{70}$$

which when annualised is the measure used in the text.

A.6 Stationary Model

Definitions: $w_t \equiv \frac{W_t}{P_t}$, $s_t \equiv \frac{S_{t,t+1}}{P_t}$, $b_t \equiv \frac{B_{t,t+1}}{P_t}$, $q_t \equiv \frac{Q_{t,t+\tau}}{P_t}$, $\bar{q}_t \equiv \frac{\bar{Q}_{t,t+\tau}}{P_t}$, $q_t^{CB} \equiv \frac{Q_{t,t+\tau}^{CB}}{P_t}$

$$C_t + P_t^S s_t + G_t = \frac{s_{t-1}}{\Pi_t} + t_{\mathcal{P}} w_t L_t + (1 - t_{\mathcal{P}}) Y_t \tag{71}$$
$$1 = \beta E_t \left[\frac{\chi_{t+1}^C}{\chi_t^C} \left(\frac{C_{t+1}}{C_t} \right)^{-\delta} \frac{1}{\Pi_{t+1}} \right] \frac{1}{P_t^S} \tag{72}$$
$$w_t = \frac{\chi_t^L}{\chi_t^C} L_t^\psi C_t^\delta \tag{73}$$
$$\frac{1 - \alpha \Pi_t^{\theta_t - 1}}{1 - \alpha} = \left(\frac{F_t}{K_t} \right)^{\frac{\theta_t - 1}{\theta_t(\phi - 1) + 1}} \tag{74}$$
$$F_t = \chi_t^C C_t^{-\delta} Y_t + \alpha\beta E_t \Pi_{t+1}^{\theta_t - 1} F_{t+1} \tag{75}$$
$$K_t = \frac{\theta_t \phi}{\theta_t - 1} \chi_t^L L_t^\psi \left(\frac{Y_t}{A_t} \right)^\phi \mu_t + \alpha\beta E_t \Pi_{t+1}^{\theta_t \phi} K_{t+1} \tag{76}$$
$$s_t = b_t + \frac{1}{\tau} \left(q_t + \sum_{k=1}^{\tau - 1} \frac{q_{t-k}}{\prod_{j=0}^{k-1} \Pi_{t-j}} \right) \tag{77}$$
$$1 + i_t = \frac{1}{P_t^B} \tag{78}$$
$$P_t^Q = \frac{1}{\tau} \frac{1}{1 + i_t^Q} \frac{1 - \left(\frac{1}{1 + i_t^Q} \right)^\tau}{1 - \frac{1}{1 + i_t^Q}} \tag{79}$$
$$b_t = g^B + \frac{P_t^S s_t - P_t^B g^B - P_t^Q g^Q}{P_t^B} \left[a_1 + a_2 \log\left(\frac{P_t^B}{P_t^Q} \right) \right] \tag{80}$$
$$q_t = g^Q + \frac{P_t^S s_t - P_t^B g^B - P_t^Q g^Q}{P_t^Q} \left[1 - a_1 - a_2 \log\left(\frac{P_t^B}{P_t^Q} \right) \right] \tag{81}$$
$$\bar{q}_t = fY \tag{82}$$

$$\frac{1+i_t}{1+i} = \left(\frac{\Pi_t}{\Pi}\right)^{\gamma_\Pi} \left(\frac{Y_t}{Y}\right)^{\gamma_Y} \nu_t \tag{83}$$

$$\frac{\bar{q}_t - q_t^{CB}}{\bar{q}_t} = \left(\frac{\Pi_t}{\Pi}\right)^{\gamma_\Pi^{QE}} \left(\frac{Y_t}{Y}\right)^{\gamma_Y^{QE}} \xi_t \tag{84}$$

$$\bar{q}_t = q_t + q_t^{CB} \tag{85}$$

$$Y_t = C_t + G_t \tag{86}$$

$$Y_t = A_t \left(\frac{L_t}{D_t}\right)^{\frac{1}{\phi}} \tag{87}$$

$$D_t = (1-\alpha)\left(\frac{1-\alpha\Pi_t^{\theta_t-1}}{1-\alpha}\right)^{\frac{\theta_t\phi}{\theta_t-1}} + \alpha\Pi_t^{\theta_t\phi} D_{t-1} \tag{88}$$

$$\ln(\chi_t^C) = \rho_C \ln(\chi_{t-1}^C) + \varepsilon_t^C \tag{89}$$

$$\ln(\chi_t^L) = \rho_L \ln(\chi_{t-1}^L) + \varepsilon_t^L \tag{90}$$

$$\ln(A_t) = \rho_A \ln(A_{t-1}) + \varepsilon_t^A \tag{91}$$

$$\ln\left(\frac{\theta_t}{\theta}\right) = \rho_\theta \ln\left(\frac{\theta_{t-1}}{\theta}\right) + \varepsilon_t^\theta \tag{92}$$

$$\ln\left(\frac{G_t}{G}\right) = \rho_G \ln\left(\frac{G_{t-1}}{G}\right) + \varepsilon_t^G \tag{93}$$

$$\ln(\nu_t) = \rho_\nu \ln(\nu_{t-1}) + \varepsilon_t^\nu \tag{94}$$

$$\ln(\xi_t) = \rho_\xi \ln(\xi_{t-1}) + \varepsilon_t^\xi \tag{95}$$

A.7 Steady State

This section describes the steady state of the stationary model summarised in A.6.

$$A = \chi^C = \chi^L = \mu = \nu = \xi = 1 \tag{96}$$

$$\Pi = 1.005 \tag{97}$$

$$\theta = 6 \tag{98}$$

$$D = \frac{1-\alpha}{1-\alpha\Pi^{\theta\phi}} \left(\frac{1-\alpha\Pi^{\theta-1}}{1-\alpha}\right)^{\frac{\theta\phi}{\theta-1}} \tag{99}$$

$$Y = \left[\frac{1-\alpha\beta\Pi^{\theta-1}}{1-\alpha\beta\Pi^{\theta\phi}} \frac{\theta\phi}{\theta-1} D^\psi (1-\bar{g})^\delta \left(\frac{1-\alpha\Pi^{\theta-1}}{1-\alpha}\right)^{\frac{\theta(\phi-1)+1}{\theta-1}}\right]^{\frac{1}{1-\delta-\phi(\psi+1)}} \tag{100}$$

$$G = \bar{g}Y \tag{101}$$

$$C = Y - G \tag{102}$$

$$F = \frac{YC^{-\delta}}{1-\alpha\beta\Pi^{\theta-1}} \tag{103}$$

$$K = \frac{1}{1-\alpha\beta\Pi^{\theta\phi}} \frac{\theta\phi}{\theta-1} D^\psi Y^{\phi(\psi+1)} \tag{104}$$

$$L = DY^\phi \tag{105}$$

$$
\begin{aligned}
w &= L^{\psi} C^{\delta} && (106)\\
P^S &= \frac{\beta}{\Pi} && (107)\\
q^{CB} &= 0 && (108)\\
\bar{q} &= fY && (109)\\
q &= \bar{q} && (110)\\
s &= \frac{\Pi}{1-\beta}\left[C + G - t_{\mathcal{P}} wL - (1 - t_{\mathcal{P}})\, Y\right] && (111)\\
b &= s - \frac{q}{\tau}\left(1 - \frac{1}{\Pi^{\tau}}\right)\frac{\Pi}{\Pi - 1} && (112)
\end{aligned}
$$

The asset demand equations (113) and (114) jointly determine P^B and P^Q

$$
\begin{aligned}
b &= g^B + \frac{P^S s - P^B g^B - P^Q g^Q}{P^B}\left[a_1 + a_2 \log\left(\frac{P^B}{P^Q}\right)\right] && (113)\\
q &= g^Q + \frac{P^S s - P^B g^B - P^Q g^Q}{P^Q}\left[1 - a_1 - a_2 \log\left(\frac{P^B}{P^Q}\right)\right] && (114)
\end{aligned}
$$

Finally,

$$
i = \frac{1}{P^B} - 1 \tag{115}
$$

and i^Q is given implicitly by

$$
P^Q = \frac{1}{\tau}\frac{1}{1+i^Q}\frac{1 - \left(\frac{1}{1+i^Q}\right)^{\tau}}{1 - \frac{1}{1+i^Q}} \tag{116}
$$