

AMBIGUITY ATTITUDES IN A LARGE REPRESENTATIVE SAMPLE*

Stephen G. Dimmock, Roy Kouwenberg, and Peter P. Wakker

December, 2012

ABSTRACT. We introduce a tractable method for measuring ambiguity attitudes, which requires only three observations and five minutes per subject, and we apply this method in a large representative sample. In addition to ambiguity aversion, we confirm a-insensitivity, a new ambiguity component recently found in laboratory studies. A-insensitivity means that people insufficiently discriminate between different levels of likelihood, often treating all likelihoods as fifty-fifty, which results in the overweighting of extreme events. Our ambiguity measurements can predict real economic decisions of the subjects; specifically a-insensitivity has a negative relation with stock market participation and private business ownership. Surprisingly, ambiguity aversion is not significantly related to stock market participation, except under high ambiguity perception. Reference dependence can explain our findings, and provides a promising direction for future research on ambiguity.

JEL Classification: D81, G11, D14, C83

Keywords: Ambiguity aversion, Uncertainty, Portfolio choice, Knightian uncertainty, Non-expected utility, Reference dependence, Stock market participation, Limited participation

Word count: 11.149

Corresponding author: Peter P. Wakker, Erasmus School of Economics, Erasmus University Rotterdam, P.O. Box 1738, Rotterdam, 3000 DR, The Netherlands, +31-(0)10 – 408.12.65 (O), +31-(0)10 – 408.12.78/12.64 (S), +31-(0)10 – 408.91.62 (F), Wakker@ese.eur.nl, <http://people.few.eur.nl/wakker/>

*: Kalok Chan, Patricio Dalton, Michael Haliassos, David Hirshleifer, Michael Keefe, Maik Dierkes, Elise Payzan-LeNestour, and Marno Verbeek provided helpful comments. CentERdata provided data and implemented the real incentives. Arthur van Soest and Marije Oudejans assisted with our survey. Netspar provided financial support. A preliminary version of this paper (June 2011) is available on the Netspar website at <http://arno.uvt.nl/show.cgi?fid=115422>.

1 Introduction

Despite the economic relevance of ambiguity (unknown probabilities), few empirical papers have studied the ambiguity attitudes of the general population, and there is little direct evidence on the relation between ambiguity attitudes and actual economic decisions. This paper develops a simple and tractable method for measuring ambiguity attitudes, requiring only three indifferences per subject. We apply our method to a large representative sample from the general population, paying €7,650 in real incentives to the subjects.

Besides ambiguity aversion, we find another relevant component of ambiguity attitudes: a-insensitivity (ambiguity-generated likelihood insensitivity). A-insensitivity implies that people do not sufficiently discriminate between different levels of ambiguity, transforming subjective likelihoods towards fifty-fifty. This leads to ambiguity seeking for low likelihoods, whereas it enhances ambiguity aversion for high likelihoods. As a result, people overweight extreme events. For example, an entrepreneur who is a-insensitive may be willing to accept the high uncertainty of starting a new business, because he overweights the unlikely event of success. Previous studies have demonstrated a-insensitivity among students in laboratory experiments. Our study is the first to demonstrate this component of ambiguity attitudes among the general population.

We then test whether the measured ambiguity attitudes can explain real-world economic decisions. Specifically, we test whether ambiguity attitudes can help to explain the non-participation puzzle: Many households do not participate in the stock market, which cannot be explained by standard portfolio choice models (Heaton and Lucas 1997; Merton 1969; Samuelson 1969). We find that a-insensitivity contributes to the explanation. Surprisingly, ambiguity aversion, if measured the traditional way,

does not contribute significantly for the full population. Ambiguity aversion has a significant negative relation with stock market participation only for those subjects who perceive stock returns to be very ambiguous. These results can be explained by reference dependent generalizations of existing theories of ambiguity. All results are robust to controlling for education, financial assets, income, age, family structure, risk aversion, trust, and financial literacy.

The outline of this paper is as follows. Section 2 discusses the prior literature. Section 3 shows that, contrary to common thinking, subjective probabilities can accommodate the Ellsberg paradox. Section 4 presents an efficient tool to analyze ambiguity, matching probabilities, and discusses their main characteristics. Two convenient indexes of ambiguity attitudes can be derived from matching probabilities (§5). The novelty of §§4-5 concerns the simplicity of measurement. Observing three matching probabilities, obtained in five minutes per subject, suffices to give the two indexes. Section 6 provides a theoretical foundation for our measurement method. Sections 7-8 present our experiment, and our results concerning the ambiguity attitudes of the general public. Section 9 presents our results concerning the relations between our indexes and actual financial decisions. Our simple measurements can predict the decisions that our subjects made, but some of our findings deviate from the predictions of the current literature. We then suggest an explanation and directions for future research (reference dependence of ambiguity attitudes). Section 10 concludes, followed by appendices.

2 Prior literature

Gilboa (1987), Gilboa and Schmeidler (1989), and Schmeidler (1989) introduced the first tractable quantitative theories of ambiguity. Their theories, and many follow-up theories, were normatively motivated (reviewed by Etner, Jeleva, and Tallon 2012). Tversky and Kahneman (1992) introduced a descriptive theory of ambiguity, adding the empirical realism of prospect theory to the axiomatically justified models of Gilboa and Schmeidler. These models, while useful points of departure, are too general for empirically fitting data.

Abdellaoui et al. (2011) introduced a specified version of prospect theory, the source method, which is tractable enough for fitting data. In a lab experiment, Abdellaoui et al. measured the entire risk and ambiguity attitudes of a sample of students, taking about a half hour per subject. We propose a method of measuring ambiguity attitudes that is considerably simpler than Abdellaoui et al.'s method, making it possible to measure the ambiguity attitude of nonacademic subjects within five minutes. Our method is based on matching probabilities, and measures only three indifferences per subject. These simple measurements can, nevertheless, predict stock market participation and private business ownership.

3 Subjective probabilities for the Ellsberg paradox after all

In our experiment, we used the classical (two-color) Ellsberg (1961) urn paradox to measure ambiguity attitudes. Ellsberg's paradox considers a known urn K and an ambiguous urn A. Urn K contains 50 yellow and 50 purple balls.¹ Urn A contains

¹ We did not use the classical Ellsberg colors red and black, as the color blind often cannot distinguish red from other colors.

100 yellow or purple balls in unknown proportion. The subject is asked to choose an urn, then a ball will be drawn randomly from that urn, and a prize may be awarded depending on the color of the ball drawn. Y_K denotes the event of a yellow ball drawn from urn K. P_K denotes the event of a purple ball drawn from urn K. Y_A and P_A are similarly defined. *Prospects* are event-contingent payments, also called acts or gambles in the literature. For example, $15_{Y_K}0$ denotes the prospect yielding €15 if event Y_K occurs, and €0 otherwise. The prevailing preferences are

$$15_{Y_K}0 > 15_{Y_A}0 \quad (3.1)$$

and

$$15_{P_K}0 > 15_{P_A}0. \quad (3.2)$$

That is, people prefer to gamble on the known urn rather than on the unknown urn (ambiguity aversion), regardless of the color. These preferences violate classical decision models using subjective probabilities, such as expected utility and subjective-probability-based generalizations, and imply the following contradiction. (Here $P(\cdot)$ indicates probability and the symbols P_K and P_A refer to purple.)

$$P(Y_K) > P(Y_A) \quad (3.3)$$

$$\frac{P(P_K)}{1} + \frac{P(P_A)}{1} > 1 : \text{Contradiction.} \quad (3.4)$$

Based on this reasoning, the common conclusion has been that subjective probabilities cannot accommodate Ellsberg's paradox. Chew and Sagi (2006, 2008) showed that, surprisingly, it is still possible to use subjective probabilities for Ellsberg's paradox. We can assign a probability of 0.5 not only to Y_K and P_K but also

to Y_A and P_A , but then assume more pessimistic weighting for A than for K—Eqs. 3.3 and 3.4 implicitly assumed identical weightings. The probabilities 0.5 of Y_A and P_A are subjective. The decision maker would adopt them as objective probabilities if he were ambiguity neutral, and so we call these subjective probabilities *ambiguity-neutral probabilities*. Smith (1969, p. 325) pleaded for using such probabilities.

Under ambiguity aversion, the ambiguity-neutral probabilities 0.5 of events Y_A and P_A are underweighted relative to the objective probabilities 0.5 of events Y_K and P_K . Thus the above preferences in the Ellsberg paradox can still be accommodated: for a chance to win €15, people prefer an objective probability of 0.5 to the same subjective (ambiguity-neutral) probability. Eqs. 3.3 and 3.4 hold with equality, but the preferences in Eqs. 3.1 and 3.2 still hold. Chew and Sagi's (2006, 2008) idea entails a remarkable revival of subjective probabilities for the Ellsberg paradox.²

Chew and Sagi (2006, 2008) gave preference conditions that justify the use of subjective ambiguity-neutral probabilities. These preference conditions were empirically verified by Abdellaoui et al. (2011), whose source method was based on Chew and Sagi's idea. In our Ellsberg experiment the conditions of Chew and Sagi follow almost trivially from symmetry in the colors. Hence we can also use the source method.

4 Matching probabilities to easily measure ambiguity attitudes

The main tool for our simple measurements of ambiguity attitudes is *matching probabilities*, which we explain using the ambiguous events in our experiment as an example. To measure the matching probability of event Y_A , we kept the ambiguous

² Multistage accommodations of Ellsberg's paradox also use probabilities, but in unconventional manners by violating the multiplication rule of conditioning (Ergin and Gul 2009; Halevy 2007; Klibanoff, Marinacci, and Mukerji 2005; Nau 2006; Yates and Zukowski 1976).

urn fixed, but changed the number of yellow balls in the known urn K. That is, urn K contained X yellow balls and $(100-X)$ purple balls. We found the number of X yellow balls in urn K that made the subject indifferent between gambling on Y_A and on Y_K (i.e., we found the number of yellow balls in urn K such that $15_{Y_K}0 \sim 15_{Y_A}0$). Details are in Appendix A. Then, for the ambiguous urn, the matching probability of event Y_A is $X/100$. We also call $X/100$ the matching probability of the ambiguity-neutral probability 0.5 of Y_A , and write

$$m(0.5) = X/100. \quad (4.1)$$

Under ambiguity aversion, $m(0.5) \leq 0.5$. The function, m, can depend on the source of ambiguity.

We can similarly measure $m(p)$ for ambiguous Ellsberg-urn probabilities other than $p=0.5$. In our experiment, we also considered a known urn containing 10 colors with 10 balls of each color versus an ambiguous urn containing 100 balls of 10 colors in unknown proportions. We use these urns to measure the matching probabilities $m(0.1)$ and $m(0.9)$. To measure $m(0.1)$ we consider gambles in which the subject wins a prize if the randomly selected ball is of one particular color. For example, $m(0.1)=0.14$ means that the subject is indifferent between gambling on one color with 14 of the 100 balls in the known urn versus gambling on one color from the ambiguous 10-color urn. This matching probability would imply ambiguity seeking, with the ambiguity-neutral probability 0.1 preferred to the objective probability 0.1. To measure $m(0.9)$ we consider gambles in which the subject wins a prize provided the randomly selected ball is *not* of one particular color. For example, $m(0.9)=0.8$

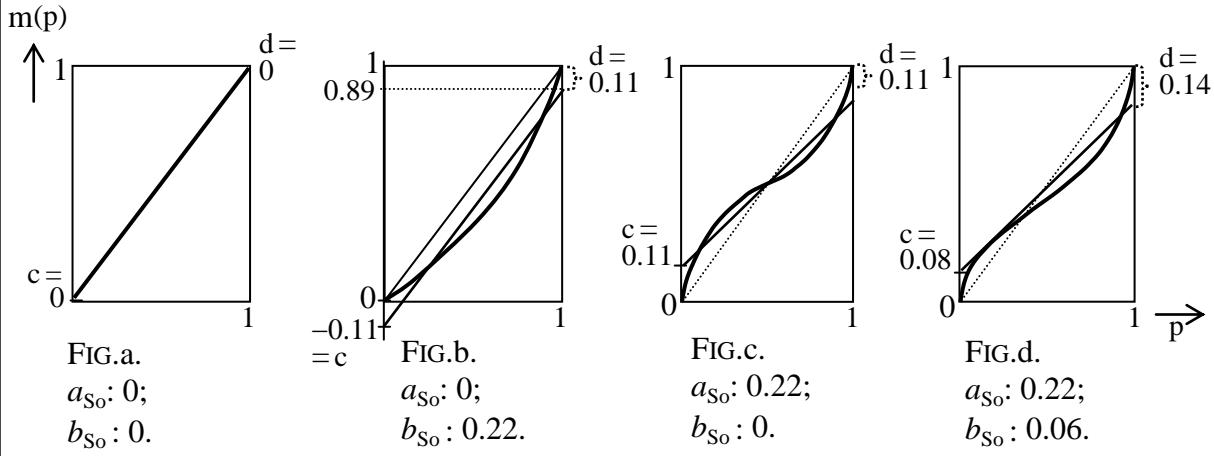
means that the subject is indifferent between gambling on 80 of the 100 balls in the known urn versus gambling on nine colors from the ambiguous 10-color urn.³

Although most current theoretical papers on ambiguity assume universal ambiguity aversion, as early as 1962 Ellsberg predicted prevailing ambiguity seeking for unlikely events, such as one color drawn from the ambiguous 10-color urn (see Ellsberg 2001 p. 203). Many empirical studies have confirmed Ellsberg's prediction (reviewed by Wakker 2010 §10.4.2), although only in laboratory studies, and it had not been tested outside the laboratory.

Figure 4.1 displays some possible shapes of matching functions $m(p)$, with the ambiguity-neutral p on the x-axis and $m(p)$ on the y-axis. The indexes, symbols, texts, and lines in the figures will be explained later and can be ignored for now. At present, we consider only the bold curves, designating matching probabilities. Fig. 4.1a displays ambiguity neutrality, where objective and ambiguity-neutral probabilities are weighted the same. That is, matching probabilities equal the ambiguity-neutral probabilities. Fig. 4.1b displays universal ambiguity aversion, with all ambiguity-neutral probabilities (for gains) matched by smaller objective probabilities. Fig. 4.1c displays a-insensitivity, with all matching probabilities moved towards fifty-fifty. Fig. 4.1d displays the prevailing empirical pattern, combining ambiguity aversion and a-insensitivity.

³ More precisely, then 20 balls in the known urn have the one losing color, and the other 80 balls have the nine winning colors. In our experiment we avoid the words “lose” or “losing”.

FIGURE 4.1. Quantitative indexes of ambiguity aversion (b_{So}) and a-insensitivity (a_{So})



5 Indexes of ambiguity attitudes derived from matching probabilities

Jaffray (1989 Eq. 10) and Kahn and Sarin (1988) used the following indexes of ambiguity attitudes (AA indexes), which we term *event-specific* indexes. Each AA_p shows the level of “local” ambiguity aversion for an Ellsberg-urn event with ambiguity-neutral probability p .

$$AA_{0.1} = 0.1 - m(0.1); \quad (5.1)$$

$$AA_{0.5} = 0.5 - m(0.5); \quad (5.2)$$

$$AA_{0.9} = 0.9 - m(0.9). \quad (5.3)$$

Ambiguity aversion implies positive values of these indexes, with matching probabilities below ambiguity-neutral probabilities. A-insensitivity corresponds with a positive value of $AA_{0.9}$ and a negative value of $AA_{0.1}$.

We also use the following global indexes of Abdellaoui et al. (2011), adapted to our matching probabilities (see Figure 4.1). First, we find the best-fitting line

between $m(p)$ and p (in terms of quadratic distance) on the open interval $(0,1)$. Say this line is

$$p \mapsto c + sp$$

with c the intercept and s the slope. We define:

$$a_{S_0} = 1 - s \text{ is the index of } a(\text{ambiguity-generated likelihood})\text{-insensitivity}, \quad (5.4)$$

and

$$b_{S_0} = 1 - s - 2c \text{ is the index of } \text{ambiguity aversion}. \quad (5.5)$$

Let $d = 1 - c - s$ be the distance of the regression line from 1 at $p=1$. Then index b_{S_0} can be written as: $b_{S_0} = d - c$.

One purpose of our study is to show that even when using only a very limited number of observations per individual (three matching probabilities), which are easy to obtain, the indexes still provide predictions about real-life decisions under ambiguity. Their ease of use makes these indexes attractive for applications.

Figure 4.1 illustrates the meaning of the indexes. Index b_{S_0} is an anti-index of the average height of the curve and, thus, it is a global index of ambiguity aversion. Index a_{S_0} is an anti-index of the steepness of the curve in the interior of its domain. It reflects lack of discrimination of intermediate levels of likelihood, transforming them towards 50%; i.e., it is a cognitive component (Wakker 2004). Baillon, Cabantous, and Wakker (2012) used similar indexes for matching probabilities of midpoints of probability intervals, rather than of ambiguity-neutral (subjective) probabilities as in Abdellaoui et al. (2011) and in our study.

6 Decision-theoretic foundation for matching probabilities

This section is not essential for readers focused on empirical implications and willing to accept our ambiguity indexes at face value. The section is essential, however, for a decision-theoretic justification of our method.

A *prospect* $\alpha_E\beta$ yields *outcome* α if *event* E occurs and outcome β otherwise.

Outcomes designate money and are nonnegative. E is an uncertain event, such as Y_A , and the decision maker is uncertain about whether the outcome of prospect $\alpha_E\beta$ will be α or β . Under prospect theory, and most ambiguity theories used today, for $\alpha \geq \beta$, the prospect is evaluated by

$$W(E)U(\alpha) + (1-W(E))U(\beta). \quad (6.1)$$

U denotes *utility* ($U(0)=0$), and W denotes a nonadditive *weighting function* ($W(\emptyset)=0$, $W(S)=1$ for the universal event S, and $A \supset B \Rightarrow W(A) \geq W(B)$). Using the tractable specification provided by Abdellaoui et al.'s (2011) source method, for $\alpha \geq \beta$, the prospect is evaluated by

$$w_{S_0}(P(E))U(\alpha) + (1-w_{S_0}(P(E)))U(\beta). \quad (6.2)$$

P denotes a subjective probability measure, justified by Chew and Sagi's (2006) conditions. Because these probabilities would be treated as objective probabilities by an ambiguity-neutral decision maker, we call them *ambiguity-neutral*. Here w_{S_0} , the *source function*, weights the ambiguity-neutral-probabilities, and is strictly increasing between the fixed points 0 and 1. Low values of w_{S_0} assign low weights to the best outcome, designating pessimism. The weighting of ambiguity-neutral probabilities through w_{S_0} can be different for different kinds of events. Thus the subscript So

indicates that w depends on the *source* of uncertainty, and can be different for the known versus the ambiguous urn. In general, other sources of uncertainty can be considered and can have different source functions, such as events regarding stock market returns.

We follow the convention of dropping the subscript S_0 if the source concerns known objective probabilities. We use the term risk for this case. Hence w denotes the probability weighting function for risk. For risk, we also write $\alpha_p\beta$ instead $\alpha_E\beta$, with p the *objective* probability of event E . It is convenient to define a function

$m_{S_0}(p) = w^{-1}w_{S_0}$, so that we can write

$$w_{S_0}(p) = w(m_{S_0}(p)). \quad (6.3)$$

m_{S_0} captures the difference between unknown and known probabilities. That is, m_{S_0} captures the ambiguity attitude. Hence it is called the *ambiguity function*.

It may appear difficult to measure the function m_{S_0} . Seemingly we would have to measure utility U , risky weighting w , and uncertainty weighting w_{S_0} , as done by Abdellaoui et al. (2011), to distill m_{S_0} . Fortunately, the following result provides a convenient shortcut (Wakker 2010 Example 11.2.2).

THEOREM 6.1. The matching probability is the ambiguity function. \square

That is, the ambiguous prospect $\alpha_E\beta$ is equivalent to the risky prospect $\alpha_{m_{S_0}(E)}\beta$, yielding α with objective probability $m_{S_0}(E)$ and yielding β otherwise. (See Appendix B for the proof.) The economic intuition for this result is straightforward. Since our method involves a comparison between risky and ambiguous choices, with the same outcome in both cases, risk aversion and probability weighting for risk affect

both choices in the same manner, and are thus differenced out from the comparison. Thus we can immediately measure the ambiguity function from the matching probabilities described in §5, with no need to measure utilities or probability weighting. This observation is crucial for justifying the simple indexes introduced in the preceding section.

7 Experimental design

7.1 Sample and incentives

The data source for this study is a cross-section taken from the Longitudinal Internet Study for the Social Sciences (LISS) panel, a representative household survey conducted by CentERdata at Tilburg University in the Netherlands. Web Appendix A.1 gives further details. Half of the sample had one of their choices randomly selected and played for a possible real reward of €15, whereas the other half played for hypothetical rewards only. In total, 510 subjects won a reward, and we paid €7,650 in real incentives. To ensure that the rewards were credible, our instructions explained that LISS was responsible for determining and administering the rewards using funds provided by the researchers. LISS is credible to the subjects because it has previously paid them real incentives, and has reputational concerns due to its ongoing relationship with the subjects.

In the remainder of this paper, we focus on the group that received real incentives, as prior studies have shown that preferences measured using real

incentives are more reliable (Smith 1976).⁴ We further limit our sample to the subjects with complete data on financial assets. Thus, the results reported in the main text concern 675 subjects.

7.2 Stimuli in the core LISS panel and control variables added for our module

The LISS panel contains information about demographic characteristics, income, and asset ownership, summarized in Table 8.1. We also measured stock market participation, risk aversion, trust (Guiso, Sapienza, and Zingales 2008), and financial literacy (van Rooij, Lusardi, and Alessie 2011). Web Appendix A provides details regarding these questions.

7.3 Stimuli to measure ambiguity attitudes

The stimuli were mostly explained in §2, with further details provided in Appendix A. Because the measurement of ambiguity attitudes is central to our study, we first tested our method in a pilot experiment with students. The results were satisfactory (Web Appendix B).

7.4 Stimuli to check consistency

To test the consistency of the subjects' choices, our program generated two check questions following the three measurements of matching probabilities. The check questions were generated by taking each subject's elicited matching probability from

⁴ Consistent with the greater reliability of preferences elicited with real incentives, virtually all our stock market participation regressions are not significant under hypothetical choice. We report the hypothetical choice results in the Web Appendix.

the two-color question (ambiguity-neutral probability of winning of 0.5) and increasing (decreasing) this value by 20%. Inconsistency results if a subject preferred the ambiguous prospect in the first check question, and the unambiguous prospect in the second check question.

Table 8.1
Summary statistics

Income and total financial assets are reported at the household level. All other variables are reported at the individual level. The first three variables are dummy variables: Stock Market Participant indicates ownership of publicly traded stocks or equity mutual funds; Private Business Owner indicates ownership of equity of a private firm. Total Financial Assets is the sum of: bank accounts, investments, insurance, loans made to others, and other financial assets. Income is gross family income in euros per month. Risk Aversion is the CRRA coefficient derived from certainty equivalents. Trust refers to responses to a question that asks if others can be trusted (0-10 scale); high values indicate greater trust. Financial Literacy is a factor extracted from three questions measuring financial knowledge, following van Rooij, Lusardi, and Alessie (2011); high values indicate greater knowledge. Don't Know Returns is a dummy variable for individuals who answer "Don't Know" to a question about historical asset returns. See Web Appendix A for detailed definitions.

Variable	All	Non-Participants	Stock Market Participants
Stock Market Participant	20.4%	0.0	100.0
Private Business Owner	6.8%	4.8	14.5
Total Financial Assets	49,757	33,337	113,650
Income	3,821	3,584	4,747
Age	50.2	49.4	53.7
Female	52.1%	56.6	34.8
Household Size	2.5	2.5	2.4
Live with Partner	75.6%	71.5	76.8
<i>Education:</i>			
Low	9.4%	10.2	6.0
Intermediate/Low	28.6%	31.3	18.1
Intermediate/High	10.2%	10.2	10.1
Vocational 1	19.1%	19.4	18.1
Vocational 2	23.1%	21.6	28.9
University	9.6%	7.3	18.8
Risk Aversion	0.14	0.13	0.19
Trust	6.02	5.9	6.6
Financial Literacy	0.13	-0.01	0.72
Don't Know Returns	24.3%	28.1	9.4

8 Ambiguity attitudes and demographic variables

8.1 Demographic variables

Table 8.1 summarizes the characteristics of our sample. As stock market participation is our key dependent variable in Section 9, the table also separately summarizes the characteristics of non-participants and of the 20.4% of the subjects who do participate. Stock market participants are, on average, wealthier, better educated, and more likely to live with a partner.

8.2 Ambiguity aversion and a-insensitivity

Table 8.2
Ambiguity attitudes revealed by first round choices

The table shows the frequency distribution of subjects with ambiguity averse, ambiguity seeking, and ambiguity neutral attitudes at ambiguity-neutral probabilities of 0.10, 0.50, and 0.90. See Appendix A for a detailed description of our stimuli. For example, we offer subjects the choice between a known urn (K) with 50 yellow balls and 50 purple balls, and an ambiguous urn (A) with yellow and purple balls in unknown proportions. A preference for urn K (A) reveals ambiguity aversion (ambiguity seeking) at $p = 0.50$. Indifference implies ambiguity neutrality.

Ambiguity-neutral prob. p	0.10	0.50	0.90
Ambiguity Averse	33.8%	68.4	53.5
Ambiguity Seeking	48.9%	21.9	34.8
Ambiguity Neutral	17.3%	9.6	11.7

Table 8.2 shows that ambiguity aversion is prevalent for the two ambiguous events with moderate and high likelihood. For example, 68.4% of the subjects had $m(0.5) < 0.5$, consistent with common findings. For the unlikely event, however, ambiguity seeking is the modal response: 48.9% had $m(0.1) > 0.1$, whereas only 33.8% had $m(0.1) < 0.1$. The latter result is consistent with experimental evidence for a-insensitivity from laboratory studies, advanced in many psychological studies (Einhorn and Hogarth 1985; Tversky and Fox 1995), and recently confirmed in

economic experiments with revealed preferences and real incentives (reviewed by Wakker 2010 §10.4.2). It is, however, inconsistent with the universal ambiguity aversion commonly assumed in theoretical papers.

Table 8.3 summarizes the matching probabilities and our measures of ambiguity attitudes. The average matching probability $m(0.1)=0.22$ implies ambiguity seeking for low likelihoods. The average matching probabilities $m(0.5)=0.40$ and $m(0.9)=0.70$ imply ambiguity aversion. All three results are consistent with a-insensitivity. Further, Index $b=0.13$ shows ambiguity aversion and Index $a=0.40$ shows a-insensitivity.

Table 8.3
Statistics of the ambiguity attitude indexes

Rows 1-3 show the matching probabilities for the three ambiguity questions ($m(0.1)$, $m(0.5)$, $m(0.9)$). Rows 4-6 show the three indexes of ambiguity attitudes based on the differences between the objective and matching probabilities: $AA_{0.1}$ (Eq. 5.1); $AA_{0.5}$ (Eq. 5.2); $AA_{0.9}$ (Eq. 5.3). The last two rows show the overall indexes of ambiguity attitudes: Index b : Eq. 5.5 (ambiguity aversion); Index a : Eq. 5.4 (a-insensitivity).

Variable	Mean	Median	Std. Dev.	Min.	Max.
Matching Probability $m(0.1)$	0.22	0.10	0.25	0.02	0.98
Matching Probability $m(0.5)$	0.40	0.40	0.24	0.02	0.98
Matching Probability $m(0.9)$	0.70	0.89	0.32	0.02	0.98
$AA_{0.1}$	-0.12	0.0	0.25	-0.88	0.08
$AA_{0.5}$	0.10	0.11	0.24	-0.48	0.48
$AA_{0.9}$	0.21	0.01	0.32	-0.08	0.88
Index b (Ambiguity Aversion)	0.13	0.09	0.41	-0.97	0.97
Index a (A-Insensitivity)	0.41	0.29	0.44	-0.22	2.22

8.3 Inconsistencies

For the first check question (matching probability $m(0.5)$ increased by 20%), 11.9% of the subjects chose the ambiguous urn, implying inconsistency. For the second check question (matching probability $m(0.5)$ decreased by 20%), 35.0% chose the

unambiguous urn, implying inconsistency. Such inconsistencies are commonly found even in laboratory studies with students (Harless and Camerer 1994 p.1263).

Approximately 10% of the subjects choose “Indifferent” rather than directly contradicting their earlier choice.⁵ However, overall, responses to the check questions are significantly related to earlier responses ($p < 0.001$; Chi-square).

8.4 Relations between ambiguity attitudes and other variables

Table 8.4
Correlations between ambiguity attitude indexes, risk aversion, trust, and financial literacy

Variables 1-5 are defined in Table 8.3: Index *b*: Eq. 5.5 (ambiguity aversion); Index *a*: Eq. 5.4 (a-insensitivity); AA_{0.1}: Eq. 5.1; AA_{0.5}: Eq. 5.2; AA_{0.9}: Eq. 5.3. Variables 6-8 are defined in Table 8.1: Risk Aversion (CRRA coefficient), Trust, and Financial Literacy. Correlations that are *not* significant at the 0.10 level are italicized.

Variable	(1) <i>b</i>	(2) <i>a</i>	(3) AA _{0.1}	(4) AA _{0.5}	(5) AA _{0.9}
(1) Index <i>b</i> (Amb. Aversion)	1				
(2) Index <i>a</i> (A-Insensitivity)	0.22	1			
(3) AA _{0.1}	0.71	-0.46	1		
(4) AA _{0.5}	0.77	0.05	0.45	1	
(5) AA _{0.9}	0.79	0.73	0.26	0.40	1
(6) Risk Aversion	-0.15	-0.16	-0.03	-0.09	-0.20
(7) Trust	<i>-0.04</i>	<i>-0.06</i>	<i>0.01</i>	<i>-0.04</i>	<i>-0.05</i>
(8) Financial Literacy	<i>-0.01</i>	<i>-0.13</i>	0.08	<i>-0.03</i>	<i>-0.08</i>

Table 8.4 shows that the two ambiguity attitude indexes are positively correlated, which is not surprising if both indexes are related to irrationality (in the sense of deviating from expected utility; Smith 1969, p. 325). Their correlation is not very strong, though, confirming that they capture different components of ambiguity attitudes. The ambiguity attitude indexes are not strongly correlated with trust or financial literacy.

⁵ Unfortunately, there was a coding error in the implementation of the survey. For the first and second check questions 28.2% and 36.2%, respectively, of the subjects were presented a choice that was too similar to their initial choice. The inconsistencies are significantly higher for the subjects that received these erroneous check questions.

Table 8.5
Regressions for demographics predictors of ambiguity attitudes

The dependent variables are defined in Table 8.3: index *b*: Eq. 5.5 (ambiguity aversion); Index *a*: Eq. 5.4 (a-insensitivity); AA_{0.1}: Eq. 5.1; AA_{0.5}: Eq. 5.2; AA_{0.9}: Eq. 5.3. The independent variables are defined in Table 8.1: Risk Aversion (CRRA coefficient), Trust, Financial Literacy, Don't Know Returns (proxy for perceived ambiguity). The education controls are five dummy variables for highest level of education achieved (base category is primary school). The regressions include constants but these are not displayed for brevity's sake. The t-statistics are calculated using standard errors clustered by household.

	Index <i>b</i>	Index <i>a</i>	AA _{0.1}	AA _{0.5}	AA _{0.9}
Risk Aversion	-0.161 *** [3.35]	-0.130 *** [3.18]	-0.058 [1.35]	-0.100 ** [2.13]	-0.189 *** [4.14]
Trust	-0.025 [0.58]	0.012 [0.28]	-0.021 [0.49]	-0.039 [0.91]	-0.004 [0.09]
Financial Literacy	0.015 [0.31]	-0.120 ** [2.42]	0.099 * [1.94]	0.014 [0.28]	-0.056 [1.20]
Don't Know Returns	0.120 [1.20]	-0.181 ** [1.98]	0.260 *** [2.77]	0.032 [0.32]	0.004 [0.04]
Total Fin. Assets	0.062 [0.64]	-0.134 [1.58]	0.125 [1.33]	0.104 [1.04]	-0.052 [0.56]
Total Fin. Assets Squ.	-0.079 [0.97]	0.038 [0.52]	-0.071 [0.92]	-0.119 [1.42]	-0.010 [0.13]
Income	-0.064 [0.44]	-0.047 [0.32]	-0.008 [0.06]	-0.081 [0.57]	-0.057 [0.37]
Income Squared	0.092 [0.68]	0.057 [0.44]	0.031 [0.30]	0.093 [0.75]	0.085 [0.58]
Age	-0.014 [0.08]	0.350 * [1.87]	-0.293 * [1.69]	0.049 [0.24]	0.163 [0.89]
Age Squared	-0.020 [0.10]	-0.286 [1.48]	0.212 [1.13]	-0.065 [0.30]	-0.154 [0.81]
Female	0.007 [0.09]	-0.037 [0.47]	-0.010 [0.13]	0.094 [1.17]	-0.048 [0.58]
Household Size	0.044 [1.14]	-0.010 [0.30]	0.034 [0.94]	0.057 [1.37]	0.017 [0.46]
Live with Partner	-0.059 [0.49]	0.087 [0.73]	-0.074 [0.63]	-0.126 [1.05]	0.035 [0.28]
Educ. (joint p-value)	0.021 **	0.138	0.002 **	0.353	0.061 *
Adjusted - R ²	0.023	0.057	0.033	0.0003	0.041
# Observations	675	675	675	675	675

To test whether the ambiguity attitude indexes capture information that is distinct from previously studied demographic characteristics, Table 8.5 regresses the ambiguity attitude indexes on the full set of control variables for stock market participation used in Section 9. Here and throughout, the symbols *, **, and *** indicate significance at the 0.10, 0.05, and 0.01 levels, respectively. To be consistent with subsequent regressions, we standardize all non-binary variables. The adjusted- R^2 values show that the control variables jointly explain only between 0.03% and 5.7% of the variation in ambiguity attitudes. The indexes have little relation with financial assets, income, age, or education. Higher a-insensitivity is associated with lower financial literacy, but the overall explanatory power is low (adj. $R^2 = 5.7\%$). The indexes a and b are negatively related to risk aversion. The ambiguity variables are clearly not a proxy for low education or financial illiteracy, but contain independent empirical information. von Gaudecker et al. (2011, p. 684) similarly found that individual choices contain much more information than what is captured by sociodemographic variables.

9 Ambiguity attitudes and household portfolio choice

To investigate the validity of our measures of ambiguity attitudes and their economic importance for other sources of uncertainty, this section investigates whether they can help to explain financial decisions.

9.1 Background on the non-participation puzzle

The non-participation puzzle refers to the empirical fact that many households do not participate in the stock market, while standard portfolio choice models predict that they should, provided that the stock market has a positive risk premium. As shown by Heaton and Lucas (1997), Merton (1969), and Samuelson (1969), non-participation is difficult to explain with standard preferences given the historical returns on equities. Although economic frictions, such as participation costs, may explain some non-

participation, frictions cannot explain a large fraction of non-participation (Andersen and Nielsen 2011), especially among wealth households.

Several theoretical papers have shown that ambiguity aversion can plausibly explain the participation puzzle (Bossaerts et al. 2010; Cao, Wang, and Zhang 2005; Easley and O’Hara 2009). Despite these theoretical models, there have only been a few non-experimental, empirical studies of ambiguity aversion and stock market participation (Kezdi and Willis 2009; Guiso, Sapienza, and Zingales 2008). These used answers to probabilistic questions as proxies for ambiguity aversion. Our study elicits ambiguity attitudes directly, using a measurement method that is descriptively and psychologically justified, and is based on a behaviorally founded decision model.

9.2 Predictions based on the present literature

Based on the present theoretical literature, it is natural to expect a negative relation between ambiguity aversion (our index b_{S0}) and stock market participation (Bossaerts et al. 2010; Dow and Werlang 1992). This relation will be stronger when agents perceive a higher level of ambiguity (Cao, Wang, and Zhang 2005; Easley and O’Hara 2009).

Based on the present literature, it is less clear what the effect of a-insensitivity (index a_{S0}) on stock market participation will be. A-insensitivity implies an extremity orientation, where the best and the worst outcomes are overweighted. The effect thus depends on the relative importance of overweighting very good outcomes versus overweighting very bad outcomes. It also depends on the perceived ambiguity associated with very good and very bad outcomes.

An additional assumption underlying the above predictions is that ambiguity attitudes for some sources of uncertainty (e.g., Ellsberg urns), while not identical, still

are related to those of other sources (e.g., future stock returns). Finding such relations is important for the general study of ambiguity, and our paper can be interpreted as a confirmation of such relations. As we will see, ambiguity attitudes concerning the traditional Ellsberg urns, using only three measurements per individual, can predict actual decisions where the ambiguity concerns the stock market and private business ownership.

9.3 Ambiguity attitudes and stock market participation: results, with some deviations from predictions

Table 9.1 shows the results of logit regressions in which the dependent variable indicates stock market participation. In all specifications, we control for financial assets, income, age, their squared values, and education, gender, household size, and family composition (following Alessie, Hochguertel, and van Soest 2002). These variables control for many things, including participation costs: wealth enables subjects to pay the fixed costs of entering the stock market and education helps subjects to learn about investing.

To simplify interpretations, we standardize all non-binary variables and report marginal effects; i.e., the reported marginal effects show the (absolute) change in the probability of stock market participation given a one standard deviation change in the independent variable (or a change from zero to one for dummy variables). Because the 675 subjects belong to 609 distinct households, in all regressions we cluster the standard errors by household to avoid overstating significance due to within-household correlations.

Table 9.1
Ambiguity attitudes and stock market participation

This table shows logit regressions with stock market participation as the dependent variable. The ambiguity attitude variables are defined in Table 8.3: Index *a*: Eq. 5.4 (a-insensitivity); index *b*: Eq. 5.5 (ambiguity aversion); AA_{0.1}: Eq. 5.1; AA_{0.5}: Eq. 5.2; AA_{0.9}: Eq. 5.3. The other independent variables are defined in Table 8.1. The regressions include constants but these are not displayed for brevity's sake. The t-statistics are calculated using standard errors clustered by household.

	(1)	(2)	(3)	(4)
Index <i>b</i> (Amb. Aversion)	0.010 [0.60]	0.013 [0.75]		
Index <i>a</i> (A-Insensitivity)	-0.034** [2.26]	-0.029* [1.87]		
AA _{0.1}			0.042** [2.23]	0.038* [1.93]
AA _{0.5}			-0.016 [0.89]	-0.013 [0.68]
AA _{0.9}			-0.018 [1.12]	-0.012 [0.78]
Risk Aversion		-0.002 [0.12]		-0.002 [0.11]
Trust		0.027* [1.90]		0.027* [1.85]
Financial Literacy		0.081*** [3.79]		0.080*** [3.78]
Total Financial Assets	0.158*** [5.66]	0.142*** [5.02]	0.159*** [5.70]	0.143*** [5.03]
Total Fin. Assets Squ.	-0.092*** [3.16]	-0.082*** [2.85]	-0.093*** [3.22]	-0.084*** [2.90]
Income	0.098* [1.87]	0.073 [1.46]	0.097* [1.84]	0.073 [1.45]
Income Squared	-0.054 [1.14]	-0.039 [0.90]	-0.054 [1.12]	-0.039 [0.88]
Age	0.098 [1.22]	0.100 [1.21]	0.098 [1.23]	0.098 [1.19]
Age Squared	-0.068 [0.86]	-0.070 [0.86]	-0.067 [0.86]	-0.068 [0.84]
Female	-0.114*** [4.22]	-0.085*** [3.10]	-0.111*** [4.11]	-0.082*** [3.00]
Household Size	-0.013 [0.77]	-0.011 [0.69]	-0.012 [0.74]	-0.011 [0.68]
Live with Partner	-0.030 [0.66]	-0.028 [0.63]	-0.031 [0.69]	-0.029 [0.65]
Education (joint p-value)	0.279	0.57	0.291	0.616
Pseudo - R ²	0.186	0.216	0.188	0.218
# Observations	675	675	675	675

The results in column (1) of Table 9.1 show:

- i. The coefficient for ambiguity aversion is not significant;
- ii. The coefficient for a-insensitivity is significant and has a negative relation with stock market participation.

Finding (ii) is confirmed by the results in column (3), in which we consider the three event-specific ambiguity attitude variables: non-participants are more likely to overweight low ambiguity-neutral probabilities. This result supports the arguments of Barro (2006, 2009), Drechsler (2011), Liu, Pan, and Wang (2005), and Pan (2002), who argue that rare, low likelihood disasters have a significant effect on investment decisions.

The implied economic magnitudes of the effects of a-insensitivity are large. A change of one standard deviation in index a implies a 3.4 percentage point change in the probability of stock market participation. This is a change of 16.7% relative to the mean participation rate, and is equivalent to the economic effect of a €27,000 change in financial assets. The results in column (3) imply that a change of one standard deviation in AA_{0.1} is associated with a 4.2 percentage point change in the probability of stock market participation.

Additional tests, in Appendix C, rule out a number of alternative interpretations of our findings for ambiguity attitudes. These include risk aversion, trust, optimism, education, quantitative skill, and financial literacy.⁶ In unreported results, we also ran the regressions without control variables. In all cases, the significances of the ambiguity variables were maintained and were usually stronger.

⁶ As a further test of whether ambiguity attitudes simply measure a lack of financial sophistication, we estimate similar logit regressions, but with the dependent variable equal to one if the subject owns a checking account. The idea being that ownership of a checking account signals at least some, minimal level of financial sophistication, but should not be related to ambiguity attitudes. In this specification, the coefficients on the ambiguity attitude variables are small and insignificantly different from zero.

9.4 Further analyses to partially explain the deviating findings: perceived ambiguity

One potential explanation for the insignificance of ambiguity aversion is that some subjects do not, or to a lesser extent, perceive future stock returns as a source of uncertainty. In our survey, we asked subjects which asset class provides the best long-term average return: stocks, bonds, savings accounts, or "Don't know". Subjects who choose the answer "Don't know" are arguably less familiar with the stock market, which will enhance the perception of ambiguity. Hence we use answers "Don't Know" as a proxy for perceived ambiguity. Details are in Web Appendix A; see question 2 there.

Table 9.2 includes interaction terms between the ambiguity attitude indexes and the dummy for "don't know returns", our proxy for perceived ambiguity. Ambiguity aversion does have a negative effect on stock market participation for subjects who perceive stock returns as highly ambiguous.⁷ Subjects whose ambiguity aversion is one standard deviation above the mean *and* who do not know which asset class tends to give highest return over longer periods of time are 10.3 percentage points less likely to participate in the stock market. By contrast, ambiguity aversion has virtually no relation with stock market participation for subjects who chose stocks, bonds, or deposits as the asset that normally has the highest long-term return.

⁷Note that we control for the direct effect of "Don't Know" in the regressions, and so the interaction term does not simply measure a lack of financial knowledge. It is the combination of aversion to ambiguity and a lack of financial knowledge that is measured by the interaction term.

Table 9.2
Ambiguity attitudes, perceived ambiguity, and stock market participation

This table shows logit regressions with stock market participation as dependent variable. The regressions include interaction terms of the ambiguity attitude variables with “Don’t Know Returns” (see question 2 in Web Appendix A.4). All other variables are the same as in Table 9.1. The regressions include constants but these are not displayed for brevity’s sake. The t-statistics are calculated using standard errors clustered by household.

	(1)	(2)	(3)	(4)
Index <i>b</i> (Amb. Aversion)	0.026 [1.41]	0.027 [1.38]		
Index <i>b</i> * Don’t Know	-0.103*** [3.30]	-0.104*** [3.24]		
Index <i>a</i> (A-Insensitivity)	-0.037** [2.30]	-0.033** [2.02]		
Index <i>a</i> * Don’t Know	0.005 [0.13]	0.020 [0.42]		
AA _{0.1}			0.059*** [2.91]	0.055*** [2.62]
AA _{0.1} * Don’t Know			-0.095** [2.05]	-0.106** [2.00]
AA _{0.5}			-0.022 [1.11]	-0.019 [0.93]
AA _{0.5} * Don’t Know			0.045 [0.89]	0.047 [0.84]
AA _{0.9}			-0.008 [0.46]	-0.004 [0.25]
AA _{0.9} * Don’t Know			-0.092* [1.86]	-0.075 [1.51]
Don’t Know Returns	-0.112** [2.45]	-0.071 [1.54]	-0.119** [2.43]	-0.078 [1.60]
Risk Aversion		-0.0001 [0.01]		0.001 [0.09]
Trust		0.028** [1.97]		0.027* [1.90]
Financial Literacy		0.072*** [3.27]		0.070*** [3.24]
Controls and Constant	Yes	Yes	Yes	Yes
Pseudo - R ²	0.203	0.226	0.210	0.232
# Observations	675	675	675	675

The interaction between a-insensitivity (index a) and ambiguity perception is not significant. A-insensitivity affects the decision weights of extreme outcomes, whereas the question about asset returns is framed in terms of the general tendency of returns. Thus this question is more related to the center of the distribution (mean or median), and does not speak to the extremes.⁸

9.5 Reference dependence of ambiguity attitudes as an explanation of our findings and a direction for future research

Our empirical finding for ambiguity aversion differs from common views (see §5), which predict a negative relation with stock market participation. To explain our deviating finding, we first note that measurement error is unlikely to explain our results related to ambiguity aversion for several reasons. First, our estimates of ambiguity aversion are very similar to those found in laboratory studies. Second, we find significant results for a-insensitivity, which is measured using the same set of questions. Also, the standard errors on the coefficients for ambiguity aversion and a-insensitivity in Table 9.1 are nearly identical. Thus our results are not due to a lack of power. This is further supported by the significant interactions between ambiguity aversion and perceived ambiguity.

Our findings are best explained by reference dependence. Stock market participation typically involves both gains and losses and, hence, reference dependence is relevant. Several studies have found that reference dependence is

⁸ In results not reported in this paper (see Web Appendix E), we create interaction terms between the ambiguity attitude indexes and an indicator variable for subjects whose answer to the asset return question was incorrect (i.e., subjects who stated that bonds or savings accounts normally give higher returns than stocks over long time periods). An incorrect answer identifies subjects who lack financial knowledge but are nevertheless confident of their knowledge. These interaction terms are not significant, which supports our interpretation that it is the interaction of ambiguity attitudes with perceived ambiguity, rather than with correct knowledge, which is relevant.

empirically as important for ambiguity (reviewed by Wakker 2010 p. 354) as it is for risk (Barberis, Huang, and Santos 2001; Kahneman 2003; Köszegi and Rabin 2006).

Those empirical studies considered only lab data. Our field data confirm their findings.

Ambiguity aversion (or seeking) for losses has been found to deviate considerably from gains, usually even being the opposite). This reflection of ambiguity attitudes is similar to the reflection found for risk attitudes, where risk seeking rather than risk aversion prevails for losses. Reflection is one of the cornerstones of prospect theory (Kahneman and Tversky 1979). See the review in Wakker (2010 p.354). Here again, ambiguity amplifies the deviations from expected utility found under risk. Because of reflection, the common and almost universally adopted measurements of ambiguity aversion (gains in the Ellsberg paradox), also adopted in our study, will be only weakly related to investment decisions. Indeed, consistent with this argument we find a relation only for the subjects who perceived ambiguity most strongly.

In contrast, a-insensitivity is not different for losses than for gains even in theory, because reflecting the overweighting of extremes leads to the same overweighting of extremes again. Reflecting ambiguity aversion, as shown in Fig. 4.1b, leads to ambiguity seeking, but reflecting Fig. 4.1c (a-insensitivity) does not change it. This has been confirmed in two empirical studies (Abdellaoui, Vossmann, and Weber 2005; Baillon and Bleichrodt 2012). For a-insensitive individuals, loss aversion (the extra weighting of losses) then aggravates the overweighting of extreme losses relative to the overweighting of extreme gains. Thus a-insensitivity generates a negative relation with stock market participation, as in our findings.

Web appendix C presents simulations based on reference-dependent prospect theory for ambiguity that explain all our findings. Hence, we suggest as a promising direction for future research the study of ambiguity attitudes for losses, and their relations to actual decisions. As yet, little is known about ambiguity attitudes for losses.

9.6 Ambiguity attitudes and private business ownership

Table 9.3
Ambiguity attitudes and private business ownership

This table shows logit regressions. The dependent variable indicates ownership of equity in a private business. The independent variables are the same as in Table 9.1. The regressions include constants but these are not displayed for brevity's sake. The t-statistics are calculated using standard errors clustered by household.

	<u>Private Business</u>	
	(3)	(4)
Index b (Amb. Aversion)	-0.007 [0.59]	
Index a (A-Insensitivity)	-0.025** [2.29]	
AA _{0.1}		0.019 [1.58]
AA _{0.5}		-0.009 [0.89]
AA _{0.9}		-0.024** [2.06]
Risk Aversion	-0.003 [0.29]	-0.002 [0.25]
Trust	0.0003 [0.03]	0.0004 [0.04]
Financial Literacy	0.027** [1.99]	0.026* [1.95]
Controls and Constant	Yes	Yes
Pseudo - R ²	0.193	0.194
# Observations	675	675

As an additional test of whether ambiguity attitudes elicited for one source of uncertainty (Ellsberg urns) can explain economic choices related to a different source

of uncertainty, we test the relation between the ambiguity attitudes and private business ownership. The distribution of the returns to private business ownership is highly ambiguous (Moskowitz and Vissing-Jorgensen 2002 p. 745). Column (1) and (2) of Table 9.3 show the results of logit regressions in which the dependent variable is 1 if the subject owns equity in a private business. We find a negative relation between a-insensitivity (index a) and private business ownership. The implied magnitude is highly economically significant. For example, in column (1) a one standard deviation increase in a-insensitivity implies a 2.5 percentage point decrease in the probability of owning private business equity, which is large given that only 6.8% of the subjects own private equity (a 36.8% change relative to the mean).

These findings are similar to those for stock market participation in Table 9.1, but one difference is that private business ownership is more negatively affected by the tendency to underweight highly likely ambiguous events rather than by the tendency to overweight unlikely ambiguous events (column (4) of Table 9.3). A possible explanation is that many of the private business owners in our sample work in professions such as dentistry, law, or accounting, in which the probability of success is relatively high. Individuals may also overestimate their likelihood of success in business.

10 Conclusion

This paper provides a very simple method for measuring ambiguity attitudes and deriving indexes, using only three matching probabilities that can be elicited from subjects in five minutes. We provide a decision-theoretic justification for the indexes, and apply our method to a large representative sample. Besides ambiguity aversion, a-insensitivity (failing to distinguish between different levels of uncertainty) is

prevalent. Thus earlier findings with students in laboratories are now confirmed in the general population. Our simple measures can predict actual stock market participation. This paper provides the first direct empirical evidence that ambiguity attitudes affect stock market participation for the general population.

The new tools of decision under ambiguity can help to explain investor behavior in financial markets, in particular the non-participation puzzle, beyond what previously identified variables can explain. A-insensitivity has a clear negative relation with participation. Ambiguity aversion has a negative relation with participation only for those subjects who perceive stock returns as highly ambiguous. Our results support the arguments of Easley and O'Hara (2009) and Guiso, Sapienza, and Zingales (2008) that regulations decreasing the uncertainty of stock market outcomes may increase stock market participation. Our results further suggest that incorporating reference dependence of ambiguity is a promising direction for future research.

Dimmock (dimmock@ntu.edu.sg) is at Nanyang Technological University. Kouwenberg (kouwenberg@ese.eur.n) is at Mahidol University and Erasmus University Rotterdam. Wakker (wakker@ese.eur.nl) is at Erasmus University Rotterdam.

Appendix A. Matching probabilities for ambiguity attitudes

FIGURE A.1

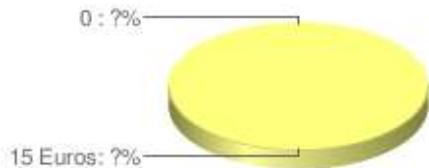
Question 1: Choosing Between Two Boxes with Yellow and Purple Balls

In this game you can choose between box A or box K, both containing 100 balls, which can be either yellow or purple. One ball will be drawn from the box you have chosen. You win 15 euros if a **Purple** ball is drawn.

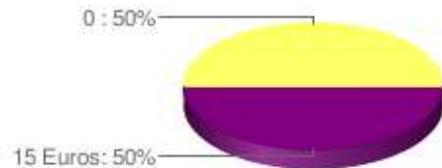
For Box K you can see the exact proportion of yellow balls and purple balls below. Box A also contains yellow and purple balls, but the proportions are not shown in advance. Hence, both boxes contain 100 balls with two different colors (yellow and purple). The composition of yellow and purple balls is known (K) for box K and unknown (A) for box A.

Please select the box of your choice: A or K. If you think both boxes are equally attractive, you can select Indifferent.

Choice A



Choice K



Which option do you prefer? (You win 15 euros if a **Purple** ball is drawn)

Choice U

Indifferent

Choice K

Note: if you prefer a different winning color then use the drop box.

Purple	▼	Select Color
--------	---	--------------

Following a practice question (whose results are not used in our analyses), we asked subjects three sets of questions that involved choices between an ambiguous and an unambiguous prospect.⁹ The first question provided a choice between an urn containing yellow and purple balls in unknown proportions, Choice A, and an urn

⁹ In our instructions we avoided terms such as lottery or gamble (often used in the literature) so as to avoid undesirable connotations. We used the terms choices and options for what we call prospects in this paper.

with known proportions of yellow and purple balls (initially equal proportions), Choice K. A subject could choose an urn (A or K) and a color to bet on (purple or yellow). Then a ball was drawn randomly from the urn¹⁰ chosen by the subject and a prize (€15) was gained if the ball drawn had the chosen color. For example, Figure A.1 depicts the initial choice in the first round.

The default winning color was purple,¹¹ but the subject could select a different winning color in the first round of the question. This extra option served to avoid suspicion (Brewer and Fellner 1965; Pulford 2009; Zeckhauser 1986, p. S445).

If the subject chose “Choice K”, then in the next round Choice K was made less attractive by bisecting between 0% and 50%, and the subject was presented Figure A.2.

If the subject selected Choice K, then Choice K was once again made less attractive by bisecting between 0% and 25%. If the subject selected the urn with unknown proportions of the two colors, Choice A, then Choice K was made more attractive, by bisecting between 25% and 50%. This process continued until the subject selected “Indifferent”, or the subject reached the maximum number of six iterations. If after six iterations the subject had not yet chosen “Indifferent”, we took the average of the lower and upper bounds. Our survey then took the subject to Question 2.

The second question was similar to the first, but now both urns contained 10 different colors of balls. The known urn, Choice K, initially had 10 balls of each of the 10 colors. The ambiguous urn, Choice U, also contained 100 balls, but with unknown proportions of the 10 colors. The subject could win a prize of €15 if the ball

¹⁰ In our survey we use the term “box” instead of “urn” because it is more natural.

¹¹ We use only colors that can be distinguished by most color blind.

drawn from the urn selected (Choice A or Choice K) had the chosen color. The natural ambiguity-neutral probability of winning here was 0.1, in view of symmetry of colors. The third ambiguous event also concerned similar urns with 10 different colors of balls, but now the prize was won under 9 of the 10 colors. This event has ambiguity-neutral probability of 0.9 and its matching probability is $m(0.9)$.

FIGURE A.2

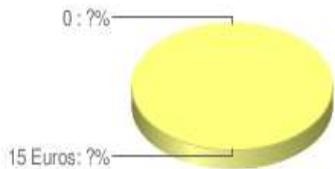
Question 1: Choosing Between Two Boxes with Yellow and Purple Balls

We play the same game again, but with a different proportion of yellow and purple balls in Box K (see below). Everything else is the same.

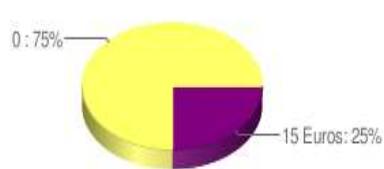
You can choose between box A or box K, both containing 100 balls, which can be either yellow or purple. One ball will be drawn from the box you have chosen. You win 15 euros if a **Purple** ball is drawn.

Please select the box of your choice: A or K. If you think both boxes are equally attractive, you can select Indifferent.

Choice A



Choice K



Which option do you prefer? (You win 15 euros if a **Purple** ball is drawn)

Choice U

Indifferent

Choice K

In sum, we considered three different ambiguous events, with ambiguity-neutral probabilities of winning of 0.5, 0.1, and 0.9, respectively. For each event, we derived matching probabilities from choices using bisection, varying the objective probability of winning, and stopping after the subject chooses “Indifferent” or after the 6th round.

Although bisection takes longer, it provides more reliable data than directly asking for an indifference value (Noussair, Robin, and Ruffieux 2004).

In the actual survey, the first, second, and third set of questions were always asked in the order just described, so as to begin with the simplest question. For expositional purposes, however, in this paper we order the variables by their ambiguity-neutral probability (first 0.1, second 0.5, and third, 0.9).

It is of course psychologically possible that subjects perceive gambling on one of two colors differently than gambling on five of ten colors. Because we feel that there was little reason to distinguish the two urns, and to keep our analysis tractable, we treat the two ambiguous urns as one source.

Subjects may answer strategically when a random incentive system is used with chained questions, as in our survey. Although it is possible that subjects did so in our bisections, it is unlikely that they were aware of it. This problem is more likely to arise with a sample of trained economic students than with a representative sample from the general population. If it had occurred, it would have increased ambiguity seeking. Subjects would have needed time to discover the chained nature of series, and an understanding of a maximum of six chained questions. Thus they would primarily become more ambiguity seeking for the last questions in our experiment, which always concerned the high likelihoods (0.9) of winning. However, here we found strong ambiguity aversion.

In our experiment we measure ambiguity attitudes using Ellsberg urns, which is a different source of uncertainty than the uncertainty about stocks. We use Ellsberg urns for several reasons. First, they are most commonly used in the literature, in a contrasting setup as in our experiment (Fox and Tversky 1995). Second, ambiguity-neutral probabilities immediately result from symmetry arguments for such urns,

without the need to measure ambiguity-neutral probabilities as Abdellaoui et al. (2011) had to do for natural events. It was important for our study that the measurement was as tractable as possible. Third, an additional reason for not using choices between bonds and stocks was to avoid the possibility of reverse causality, which could occur if subjects made choices in the survey that they feel are consistent with their actual portfolio choices. They might do so to avoid cognitive dissonance (Bertrand and Mullainathan 2001). Finally, it is also useful to investigate relations between ambiguity attitudes towards different sources.

Appendix B. Proof

PROOF OF THEOREM 6.1. Assume, for $\alpha > \beta$, that $\alpha_E\beta \sim \alpha_q\beta$, implying that q is the matching probability of event E and of ambiguity-neutral probability $P(E)$. This implies

$$w(m_{S_0}(P(E)))U(\alpha) = w(q)U(\alpha)$$

$$w(m_{S_0}(P(E))) = w(q)$$

$$m_{S_0}(P(E)) = q.$$

□

Appendix C. Alternative explanations

Tables 9.1 and 9.2 show a relation between the elicited ambiguity attitudes and stock market participation. A potential concern, however, is that the elicited ambiguity attitudes inadvertently measured some other concept.

C.1 Risk aversion

Although we measure ambiguity attitudes relative to risky attitudes, and the two concepts are conceptually distinct (Liu, Pan, and Wang 2005 p. 149-152), they may still be statistically related (Abdellaoui et al. 2011, Figures 12 and 13; Bossaerts et al. 2010; Lauriola, Levin, and Hart 2007; Qiu and Weitzel 2012; our Table 8.4). Including the responses for risk aversion, however, does not alter the significance of our ambiguity attitudes. Table 9.1 shows that individuals who overweight low ambiguity-neutral probabilities have lower participation rates. This result is directionally inconsistent with the possibility that our ambiguity indexes inadvertently measure risk aversion. The interaction terms in Table 9.2 show that, consistent with our predictions, ambiguity aversion has a negative relation with stock market participation for individuals who do not know the long-term returns of various asset classes. There is no clear reason to expect risk aversion to interact with the perceived ambiguity of stock returns (in unreported robustness tests we find insignificant results for an interaction between our risk aversion variable and the do-not-know variable). Hence this result provides indirect evidence that the ambiguity attitude indexes do not inadvertently measure risk aversion.

It may be surprising that risk aversion had little effect in our study. One reason is that we measure pure decision-theoretic risk aversion, whereas many studies used questions specifically about stock market risk, which may create a mechanical relation (Web Appendix A.5). A second reason is that we, following common conventions in the field, measured risk aversion using gains. However, stock returns involve losses where risk attitudes can be different and even opposite to those for gains, as predicted by prospect theory. Noussair, Trautmann, and van de Kuilen (2011) used a different

subsample of the LISS panel, and similarly found no relation between conventional measures of risk aversion and financial decisions.

C.2 Trust

Guiso, Sapienza, and Zingales (2008) show that trust is positively associated with stock market participation. They argue that trust and ambiguity attitudes are distinct concepts, but there is some conceptual similarity. In our results, Tables 9.1 and 9.2 show that controlling for trust does not affect our findings. Ambiguity aversion could arise if subjects assume that ambiguous situations are biased (distrust). However, this is unlikely to drive our results. First, subjects tend to overweight low likelihoods, whereas distrust would predict underweighting. Second, the correlations between trust and the ambiguity attitudes are very low. Finally, the elicitation procedure was designed to minimize any effect from distrust. We allowed subjects to choose their own winning color, and the instructions clearly stated that LISS would administer the rewards using funds provided by the researchers.

C.3 Optimism

Puri and Robinson (2007) show that optimistic subjects are more likely to participate in the stock market and have more favorable economic expectations. Although the LISS panel does not contain the questions used in their study, LISS does contain questions about economic expectations for a subset of our sample (347 subjects). None of the correlations between economic expectations and the ambiguity attitude indexes are significant. We also find that the ambiguity attitude indexes are not correlated with the responses to questions concerning depression, positive attitudes, or life satisfaction.

Table C.1
Ambiguity attitudes and stock market participation: subsamples

This table shows logit regressions with stock market participation as dependent variable. Columns (2), (4), and (6) include interaction terms with the “Don’t Know” dummy and the ambiguity indexes. The independent variables are the same as in Table 9.1. The subsamples are limited to Tertiary Education (only subjects who have completed some form of tertiary education), Questions Were Clear (stated that the ambiguity attitude questions were clear or very clear), and Check Questions Not Inconsistent (did not violate their earlier choices when responding to the check questions). The regressions include constants but these are not displayed for brevity’s sake. The t-statistics are calculated using standard errors clustered by household.

	<u>Tertiary Education</u>		<u>Questions Were Clear</u>		<u>Check Questions Not Inconsistent</u>	
	(1)	(2)	(3)	(4)	(5)	(6)
Index <i>b</i> (Amb. Aversion)	0.028 [1.08]	0.046 [1.58]	0.009 [0.44]	0.021 [0.94]	0.014 [0.72]	0.029 [1.28]
Index <i>b</i> * Don’t Know		-0.166 *** [2.77]		-0.086 ** [2.37]		-0.106 *** [3.00]
Index <i>a</i> (A-Insensitivity)	-0.038 [1.54]	-0.051 * [1.93]	-0.041 ** [2.26]	-0.046 ** [2.41]	-0.035 ** [2.02]	-0.040 ** [2.15]
Index <i>a</i> * Don’t Know		0.150 * [1.69]		0.034 [0.68]		0.036 [0.70]
Don’t Know Returns		-0.027 [0.37]		-0.059 [1.20]		-0.073 [1.45]
Risk Aversion	-0.017 [0.79]	-0.017 [0.79]	-0.001 [0.05]	-0.001 [0.05]	0.001 [0.04]	0.003 [0.19]
Trust	0.030 [1.31]	0.030 [1.34]	0.024 [1.56]	0.025 * [1.67]	0.029 * [1.89]	0.030 ** [2.01]
Financial Literacy	0.127 *** [3.62]	0.124 *** [3.63]	0.059 ** [2.43]	0.050 ** [2.02]	0.082 *** [3.68]	0.074 *** [3.16]
Controls and Constant	Yes	Yes	Yes	Yes	Yes	Yes
Pseudo - R ²	0.242	0.257	0.228	0.235	0.209	0.220
# Observations	350	350	545	545	539	593

C.4 Education, quantitative skill, and financial literacy

The results in Table 8.5 show that education has little explanatory power for the ambiguity indexes, suggesting that the indexes are not proxies for low quantitative skills. We found similar results in our pilot experiment on a sample of upper year university students who had already completed courses in statistics, calculus, finance, and economics. Financial Literacy, while related to stock market participation, does not eliminate the significance of ambiguity attitudes, further suggesting that quantitative skills do not explain ambiguity attitudes.

We further explore this possibility in Table C.1, by excluding subjects with low levels of education or who found the elicitation procedure confusing. In columns (1) and (2), the sample is restricted to subjects who have completed some form of tertiary education. In columns (3) and (4), the sample is restricted to subjects who, at the end of our survey module, stated that the ambiguity elicitation questions were either clear or very clear. In columns (5) and (6), the sample is restricted to subjects whose answers to the check questions did not violate their earlier choices. The results are similar to the full sample.

As a further test of whether the ambiguity attitude indexes measure a lack of sophistication or unfamiliarity with financial decision making, columns (1) and (2) of Table C.2 estimate a logit model in which the dependent variable is 1 if the subject has a bank account. Presumably, subjects without a bank account are relatively unsophisticated or lack financial expertise. The pseudo R-square of 0.14 indicates that the control variables can explain a sizeable proportion of the variation in bank account ownership. However, the coefficients on the ambiguity indexes are all insignificant.

Table C.2.
Ambiguity attitudes and bank account ownership

This table shows logit regressions. The dependent variable indicates ownership of a bank account. The independent variables are the same as in Table 8.1. The regressions include constants but these are not displayed for brevity's sake. The t-statistics are calculated using standard errors clustered by household.

	<u>Bank Account</u>	
	(1)	(2)
Index b (Amb. Aversion)	0.011 [1.03]	
Index a (A-Insensitivity)	-0.005 [0.46]	
AA _{0.1}		0.004 [0.35]
AA _{0.5}		0.012 [1.00]
AA _{0.9}		-0.003 [0.21]
Risk Aversion	0.007 [0.57]	0.007 [0.57]
Trust	-0.013 [1.15]	-0.012 [1.13]
Financial Literacy	0.014 [1.19]	0.014 [1.16]
Controls and Constant	Yes	Yes
Pseudo - R ²	0.145	0.146
# Observations	675	675

Web Appendix:

http://people.few.eur.nl/wakker/pdf/dkw_liss_amb_finance_webappendix.pdf

References

Abdellaoui, Mohammed, Aurélien Baillon, Laetitia Placido, & Peter P. Wakker (2011) “The Rich Domain of Uncertainty: Source Functions and Their Experimental Implementation,” *American Economic Review* 101, 695–723.

- Abdellaoui, Mohammed, Frank Vossmann, & Martin Weber (2005) “Choice-Based Elicitation and Decomposition of Decision Weights for Gains and Losses under Uncertainty,” *Management Science* 51, 1384–1399.
- Alessie, Rob J. M., Stefan Hochguertel, & Arthur van Soest (2002) “Household Portfolios in the Netherlands.” In Luigi Guiso, Michael Haliassos, & Tullio Jappelli (eds.) *Household Portfolios*, The MIT Press, Cambridge, MA.
- Andersen, Steffen & Kasper Meisner Nielsen (2011) “Participation Constraints in the Stock Market: Evidence from Unexpected Inheritance due to Sudden Death,” *Review of Financial Studies* 24, 1667–1697.
- Baillon, Aurélien & Han Bleichrodt (2012) “Testing Ambiguity Models through the Measurement of Probabilities for Gains and Losses,” School of Economics, Erasmus University, Rotterdam, the Netherlands.
- Baillon, Aurélien, Laure Cabantous, & Peter P. Wakker (2012) “Aggregating Imprecise or Conflicting Beliefs: An Experimental Investigation Using Modern Ambiguity Theories,” *Journal of Risk and Uncertainty* 44, 115–147.
- Barberis, Nicholas, Ming Huang, & Tano Santos (2001) “Prospect Theory and Asset Prices,” *Quarterly Journal of Economics* 116, 1–53.
- Barro, Robert J. (2006) “Rare Disasters and Asset Markets in the Twentieth Century,” *Quarterly Journal of Economics* 121, 823-866.
- Barro, Robert J. (2009) “Rare Disasters, Asset Prices, and Welfare Costs,” *American Economic Review* 99, 243-264.
- Bertrand, Marianne & Sendhil Mullainathan (2001) “Do People Mean What They Say? Implications for Subjective Survey Data,” *American Economic Review, Papers and Proceedings* 91, 67–72.

- Bossaerts, Peter, Serena Guarnaschelli, Paolo Ghirardato, & William Zame (2010) “Ambiguity and Asset Prices: An Experimental Perspective,” *Review of Financial Studies* 23, 1325–1359.
- Brewer, K.R.W. & William Fellner (1965) “The Slanting of Subjective Probabilities—Agreement on Some Essentials,” *Quarterly Journal of Economics* 77, 657 –663.
- Cao, H. Henry, Tan Wang, & Harold H. Zhang (2005) “Model Uncertainty, Limited Market Participation, and Asset Prices,” *Review of Financial Studies* 18, 1219–1251.
- Chew, Soo Hong & Jacob Sagi (2006) “Event Exchangeability: Probabilistic Sophistication without Continuity or Monotonicity,” *Econometrica* 74, 771–786.
- Chew, Soo Hong & Jacob S. Sagi (2008) “Small Worlds: Modeling Attitudes toward Sources of Uncertainty,” *Journal of Economic Theory* 139, 1–24.
- Dow, James & Sérgio R.C. Werlang (1992) “Uncertainty Aversion, Risk Aversion and the Optimal Choice of Portfolio,” *Econometrica* 60, 197–204.
- Drechsler, Itamar (2011) “Uncertainty, Time-Varying Fear, and Asset Prices,” *Journal of Finance*, forthcoming.
- Easley, David & Maureen O’Hara (2009) “Ambiguity and Nonparticipation: The Role of Regulation,” *Review of Financial Studies* 22, 1817–1843.
- Einhorn, Hillel J. & Robin M. Hogarth (1985) “Ambiguity and Uncertainty in Probabilistic Inference,” *Psychological Review* 92, 433–461.
- Ellsberg, Daniel (1961) “Risk, Ambiguity and the Savage Axioms,” *Quarterly Journal of Economics* 75, 643–669.

- Ellsberg, Daniel (2001) “*Risk, Ambiguity and Decision.*” Garland Publishers, New York. Original Ph.D. dissertation: Ellsberg, Daniel (1962) “Risk, Ambiguity and Decision.” Harvard University, Cambridge, MA.
- Ergin, Haluk & Faruk Gul (2009) “A Theory of Subjective Compound Lotteries,” *Journal of Economic Theory* 144, 899–929.
- Etner, Johanna, Meglena Jeleva, & Jean-Marc Tallon (2012) “Decision Theory under Ambiguity”, *Journal of Economic Surveys* 26, 234–270.
- Fox, Craig R. & Amos Tversky (1995) “Ambiguity Aversion and Comparative Ignorance,” *Quarterly Journal of Economics* 110, 585–603.
- Gilboa, Itzhak (1987) “Expected Utility with Purely Subjective Non-Additive Probabilities,” *Journal of Mathematical Economics* 16, 65–88.
- Gilboa, Itzhak & David Schmeidler (1989) “Maxmin Expected Utility with a Non-Unique Prior,” *Journal of Mathematical Economics* 18, 141–153.
- Guiso, Luigi, Paola Sapienza, & Luigi Zingales (2008) “Trusting the Stock Market,” *Journal of Finance* 63, 2557–2600.
- Halevy, Yoram (2007) “Ellsberg Revisited: An Experimental Study,” *Econometrica* 75, 503–536.
- Harless, David W. & Colin F. Camerer (1994) “The Predictive Utility of Generalized Expected Utility Theories,” *Econometrica* 62, 1251–1289.
- Heaton, John & Deborah Lucas (1997) “Market Frictions, Savings Behavior, and Portfolio Choice,” *Macroeconomic Dynamics* 1, 76–101.
- Jaffray, Jean-Yves (1989) “Linear Utility Theory for Belief Functions,” *Operations Research Letters* 8, 107–112.
- Kahn, Barbara E. & Rakesh K. Sarin (1988) “Modeling Ambiguity in Decisions under Uncertainty,” *Journal of Consumer Research* 15, 265–272.

- Kahneman, Daniel (2003) “Maps of Bounded Rationality: Psychology for Behavioral Economics,” *American Economic Review* 93, 1449–1475.
- Kahneman, Daniel & Amos Tversky (1979) “Prospect Theory: An Analysis of Decision under Risk,” *Econometrica* 47, 263–291.
- Kezdi, Gabor & Robert J. Willis (2009) “Stock Market Expectations and Portfolio Choice of American Households,” Working Paper, University of Michigan.
- Klibanoff, Peter, Massimo Marinacci, & Sujoy Mukerji (2005) “A Smooth Model of Decision Making under Ambiguity,” *Econometrica* 73, 1849–1892.
- Köszegi, Botond & Matthew Rabin (2006) “A Model of Reference-Dependent Preferences,” *Quarterly Journal of Economics* 121, 1133–1165.
- Lauriola, Marco, Irwin P. Levin, & Stephanie S. Hart (2007) “Common and Distinct Factors in Decision Making under Ambiguity and Risk: A Psychometric Study of Individual Differences,” *Organizational Behavior and Human Decision Processes* 104, 130–149.
- Liu, Jun, Jun Pan, & Tan Wang (2005) “An Equilibrium Model of Rare-Event Premia and Its Implication for Option Smirks,” *Review of Financial Studies* 18, 131–164.
- Merton Robert C. (1969) “Lifetime Portfolio Selection Under Uncertainty: The Continuous-Time Case,” *Review of Economics and Statistics* 51, 247–257.
- Moskowitz, Tobias J. & Annette Vissing-Jorgensen (2002) “The Returns to Entrepreneurial Investment: A Private Equity Premium Puzzle?,” *American Economic Review* 92, 745–778.
- Nau, Robert F. (2006) “Uncertainty Aversion with Second-Order Utilities and Probabilities,” *Management Science* 52, 136–145.

- Noussair, Charles, Stephane Robin, & Bernard Ruffieux (2004) "Revealing Consumers' Willingness-to-Pay: A Comparison of the BDM Mechanism and the Vickrey Auction," *Journal of Economic Psychology* 25, 725–741.
- Noussair, Charles, Stefan T. Trautmann, & Gijs van de Kuilen (2011) "Higher Order Risk Attitudes, Demographics, and Financial Decisions," CentER Discussion Paper, 2011-055, 2011-055, Tilburg University, the Netherlands.
- Pan, Jun (2002) "The Jump-Risk Premia Implicit in Options: Evidence from an Integrated Time-Series Study," *Journal of Financial Economics* 63, 3–50.
- Pulford, Briony D. (2009) "Is Luck on My Side? Optimism, Pessimism, and Ambiguity Aversion," *Quarterly Journal of Experimental Psychology* 62, 1079–1087.
- Puri, Manju, & David T. Robinson (2007) "Optimism and Economic Choice," *Journal of Financial Economics* 86, 71–99.
- Qiu, Jianying & Utz Weitzel (2012), "Reference Dependent Ambiguity Aversion: Theory and Experiment," mimeo.
- Samuelson, Paul A. (1969) "Lifetime Portfolio Selection by Dynamic Stochastic Programming," *Review of Economics and Statistics* 51, 239–246.
- Schmeidler, David (1989) "Subjective Probability and Expected Utility without Additivity," *Econometrica* 57, 571–587.
- Smith, Vernon L. (1969) "Measuring Nonmonetary Utilities in Uncertain Choices: The Ellsberg Urn," *Quarterly Journal of Economics* 83, 324–329.
- Smith, Vernon L. (1976) "Experimental Economics: Induced Value Theory," *American Economic Review* 66, 274–279.
- Tversky, Amos & Craig R. Fox (1995), "Weighing Risk and Uncertainty," *Psychological Review* 102, 269-283.

- Tversky, Amos & Daniel Kahneman (1992) "Advances in Prospect Theory: Cumulative Representation of Uncertainty," *Journal of Risk and Uncertainty* 5, 297–323.
- van Rooij, Maarten, Annamaria Lusardi, & Rob Alessie (2011) "Financial Literacy and Stock Market Participation," *Journal of Financial Economics* 101, 449–472.
- von Gaudecker, Hans-Martin, Arthur van Soest, & Erik Wengström (2011) "Heterogeneity in Risky Choice Behavior in a Broad Population," *American Economic Review* 101, 664–694.
- Wakker, Peter P. (2004) "On the Composition of Risk Preference and Belief," *Psychological Review* 111, 236–241.
- Wakker, Peter P. (2010) "*Prospect Theory: for Risk and Ambiguity.*" Cambridge University Press, Cambridge, UK.
- Yates, J. Frank & Lisa G. Zukowski (1976) "Characterization of Ambiguity in Decision Making," *Behavioral Science* 21, 19–25.
- Zeckhauser, Richard J. (1986) "Comments: Behavioral versus Rational Economics: What You See Is What You Conquer," *Journal of Business* 59, S435–S449.