Through Scarcity to Prosperity and Beyond: A Theory of the Transition to Sustainable Growth

Pietro F. Peretto

Duke University

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Resource economics emphasizes the role of **exhaustible** resources in generating diminishing returns to other inputs that worsen over time as resources run out. Two classic questions:

- Is level of consumption per capita sustainable? (Solow 1974)
- Is growth of consumption per capita sustainable? (Stiglitz 1974)


- escape from Malthusian trap of low consumption per capita?
- transition to sustained growth of consumption per capita?
Are these two ideas connected?

Resource economics studies the future behavior of economy under increasing scarcity. As we run out of natural inputs, and diminishing returns to man-made inputs reduce output per capita, how can we sustain our current — and growing — standards of living?

Growth economics studies the past transition to sustained growth experienced by advanced economies. It emphasizes population-technology interactions but ignores exhaustible resources, perhaps because modeling escape from the Malthusian regime requires that sustained growth be feasible in the first place.

So, the two ideas are two sides of the same coin and provide the foundation of a general theory of the interactions of population, resources and technology.
This paper integrates fertility choice and exhaustible resource dynamics in model of endogenous innovation to develop a theory of the transition from resource-based to knowledge-based growth.

- Initial phase where agents build up the economy by exploiting exhaustible natural resources to support population growth.
- Intermediate phase where agents turn on Schumpeterian engine of innovation-led growth in response to market expansion.
- Terminal phase where growth becomes driven by knowledge accumulation and no longer requires growth of physical inputs.

Last part is crucial: not only economics growth no longer requires growth of physical inputs, but also knowledge accumulation compensates for the exhaustion of the natural resource. The paper thus proposes a theory of de-coupling.
Final producers: Homogeneous good that is consumed, used to produce intermediate goods, or invested in R&D. (One-sector structure.) This good is the *numeraire*, so $P_Y \equiv 1$.

Intermediate producers: Develop new goods and set up operations to serve market (variety innovation or entry) and, when already in operation, invest in R&D internal to firm (quality innovation).

Households: Consume, save and set optimal path of population growth and resource use. For simplicity, they play the role of the extraction sector. (Alternative assumptions are feasible, of course.)
Representative final producer (i)

Technology:

\[ Y = N^{(\sigma-1)(1-\theta)} \int_0^N X_i^\theta (Z_i^\alpha Z^{1-\alpha} L^\gamma R^{1-\gamma})^{1-\theta} di, \quad Z \equiv \int_0^N \frac{1}{N} Z_j dj. \]

where:

- \( 0 < \theta, \gamma < 1 \) standard parameters that map into factor shares;
- \( Z_i^\alpha Z^{1-\alpha} \) **vertical** technology index, with \( \alpha \in [0, 1) \) measure of private returns to quality and \( 1 - \alpha \) measure of social returns to quality;
- \( N \) is **horizontal** technology index, with \( \sigma \in [0, 1) \) measure of social returns to variety (love-of-variety effect in production).
Demand for product $i$:

$$X_i = N^{\sigma - 1} \left( \frac{\theta}{P_i} \right)^{\frac{1}{1-\theta}} Z_i^\alpha Z^{1-\alpha} L^\gamma R^{1-\gamma}. $$

Factor payments:

$$N \cdot PX = \int_0^N P_i X_i di = \theta Y;$$

$$wL = \gamma (1 - \theta) Y;$$

$$pR = (1 - \gamma) (1 - \theta) Y.$$
Intermediate producers

Technologies:

\[ \text{Cost}_i = 1 \cdot \left( X_i + \phi Z_i^{\alpha} Z^{1-\alpha} \right); \]
\[ \dot{Z}_i = I_i. \]

Firm’s objective:

\[ V_i (0) = \int_0^\infty e^{-\int_0^t r(s)ds} \left[ X_i (t) (P_i (t) - 1) - \phi Z_i^{\alpha} (t) Z^{1-\alpha} (t) - I_i (t) \right] dt. \]

In symmetric equilibrium:

\[ \max_{P_i, I_i} V_i \Rightarrow r = \alpha \left( \frac{X}{Z} \left( \frac{1}{\theta} - 1 \right) - \phi \right) \equiv r_Z; \]
\[ V_{i \text{max}} = \beta \frac{Y}{N} \Rightarrow r = \frac{X \left( \frac{1}{\theta} - 1 \right) - \phi Z - I}{\beta \frac{Y}{N}} + \frac{\dot{Y}}{Y} - \frac{\dot{N}}{N} \equiv r_N. \]
Representative household

Chooses \( C(t), L(t), R(t) \) and \( b(t) \) to maximize

\[
U_0 = \int_0^\infty e^{-\rho t} \left[ \log \left( \frac{C(t)}{M(t)} M^\eta + 1(t) \right) + f(b(t)) \right] dt, \quad \rho, \eta > 0,
\]

subject to:

\[
\dot{A} = rA + wL + pR - C - \Psi bM, \quad M \geq L \geq 0, \quad \psi > 0;
\]

\[
\dot{M} = M (b - \delta), \quad M_0 > 0, \quad \delta > 0;
\]

\[
S_0 \geq \int_0^\infty R(t) \, dt, \quad R \geq 0, \quad S_0 > 0, \quad \dot{S} = -R.
\]

What’s new here?

- \( M \) evolves according to fertility choice \( b \); preference for \( b \) increasing and (weakly) concave, i.e, \( f' > 0, f'' \leq 0 \).
- \( S \) is stock of exhaustible resource that evolves according to extraction choice \( R \) (for simplicity, extraction cost is zero).
Household behavior (i)

First-order conditions

- for control variables $C$, $b$, $R$:

$$1 = \lambda_A C; \quad f'(b) + \lambda_M M = \lambda_A \Psi M; \quad \lambda_A \rho = \lambda_S,$$

where the $\lambda$s denote the shadow values of $A$, $M$ and $S$;

- for state variables $A$, $M$, $S$:

$$r + \frac{\dot{\lambda}_A}{\lambda_A} = \rho;$$

$$\frac{\eta}{M} + \lambda_A (w - \Psi b) + \left( \frac{\dot{\lambda}_M}{\lambda_M} + \frac{\dot{M}}{M} \right) = \rho;$$

$$\frac{\dot{\lambda}_S}{\lambda_S} = \rho.$$

Plus usual transversality conditions.
Let \( c \equiv C/Y \) and \( h \equiv \lambda MM \).

Conditions for \( C \) and \( A \) yield Euler equation

\[
r = \rho + \frac{\dot{C}}{C} = \rho + \frac{\dot{c}}{c} + \frac{\dot{Y}}{Y}.
\]

To simplify, cost of reproduction in units of the final good (taken as given since depends on aggregate variables that household does not control):

\[
\Psi = \psi \frac{Y}{M}.
\]
Household behavior (iii)

Conditions for $C$, $b$, $A$ and $M$ yield fertility rule

$$f'(b) + h = \frac{\psi}{c}$$

and asset-pricing equation

$$\dot{h} = \rho h - \eta - \frac{1}{c} \left( \frac{wM}{Y} - \psi b \right).$$

Conditions for $C$, $R$, $S$ and the Euler equation yield Hotelling rule

$$\frac{p}{C} = \lambda S \Rightarrow \frac{\dot{p}}{p} = \rho + \frac{\dot{C}}{C} = r.$$
Equilibrium extraction (i)

Flow of resource supplied by household equals final sector demand

\[ pR = (1 - \gamma)(1 - \theta)Y. \]

Log-differentiating and using Hotelling rule

\[
\frac{\dot{R}}{R} = \frac{\dot{Y}}{Y} - \frac{\dot{p}}{p} = \frac{\dot{Y}}{Y} - r = - \left( \frac{\dot{c}}{c} + \rho \right).
\]

Integrating and defining average growth rate of extraction flow between time 0 and time \( t \) as \( \varepsilon(t) \equiv \frac{1}{t} \int_0^t \left( \frac{\dot{c}(s)}{c(s)} + \rho \right) ds \) yields

\[ R(t) = R_0 e^{-\varepsilon(t)t}. \]
Equilibrium extraction (ii)

Substituting into

\[ S_0 = \int_0^\infty R(t) \, dt \]

yields

\[ R_0 = \left[ \int_0^\infty e^{-\varepsilon(t)t} \, dt \right]^{-1} \cdot S_0. \]

constant that depends on fundamentals

Therefore, the extraction rule at time \( t \) is

\[ R(t) = \frac{e^{-\varepsilon(t)t}}{\int_0^\infty e^{-\varepsilon(t)t} \, dt} \cdot S_0. \]
In symmetric equilibrium aggregate output is

$$Y = \kappa N^\sigma Z M^\gamma R^{1-\gamma}, \quad \kappa \equiv \theta^{\frac{2\theta}{1-\theta}}$$

where $N^\sigma Z$ is TFP. Output per capita is

$$y \equiv \frac{Y}{M} = \kappa N^\sigma Z \left(\frac{R}{M}\right)^{1-\gamma}.$$

Let $n \equiv \dot{N}/N$, $z \equiv \dot{Z}/Z$ and $g \equiv \dot{y}/y$. At time $t$, output per capita growth rate is:

$$g = \underbrace{\sigma n + z}_{\text{TFP growth}} - \underbrace{(1-\gamma) \left(m + \dot{c}/c + \rho\right)}_{\text{growth drag}}.$$
Transition dynamics: differential equation for firm size

Final producer pays total compensation \( N \cdot PX = \theta Y \) to intermediate producers who set \( P = 1/\theta \). Hence, \( NX = \theta^2 Y \). Let \( \mu \equiv P - 1 \) and \( x \equiv X/Z \). Reduced-form production function yields

\[
x = \theta^2 Y / NZ = \theta^2 \kappa M \gamma R^{1-\gamma} / N^{1-\sigma}.
\]

Returns to innovation become:

\[
r = \alpha (\mu x - \phi) ;
\]

\[
r = \frac{1}{\beta \theta^2} \left( \mu - \frac{\phi + z}{x} \right) + \frac{\dot{x}}{x} + z.
\]

Note: firm-level decisions depend on quality adjusted firm size \( x \), which follows differential equation

\[
\frac{\dot{x}}{x} = \frac{\dot{Y}}{Y} - n - z = \gamma m - (1 - \gamma) \left( \frac{\dot{c}}{c + \rho} \right) - \frac{(1 - \sigma) n}{\text{market growth}} - \frac{\gamma m - (1 - \gamma) \left( \frac{\dot{c}}{c + \rho} \right)}{\text{market fragmentation}}.
\]
Population growth net of resource exhaustion drives growth of market for intermediate goods.

Adjusting for market fragmentation due to product proliferation yields dynamics of firm size, the driver of agents’ investment decisions in quality and variety.

Whether these decisions support positive output per capita growth depends on whether growth of TFP is larger than growth drag.

This is the classic condition for sustainability derived by Stiglitz (1974, see also Brock and Taylor 2005), with the difference that in this model TFP growth is endogenous and not necessarily positive.
Lemma 1 There are two main regimes, one where entrants are inactive and one where they are active. In the latter the consumption ratio $c$ and the fertility rate $b$ jump to their respective steady-state values $c^*$, $b^*$ and remain constant throughout the transition driven by the evolution of firm size $x$. Moreover, the resource input $R$ follows an exponential process with constant rate of exhaustion $\rho$, i.e., $R(t) = \rho S_0 e^{-\rho t}$. 
Lemma 2  Let $x_N$ denote the threshold of firm size that triggers variety innovation and $x_Z$ the threshold of firm size that triggers quality innovation. Assume $\frac{\rho \beta \theta^2 \phi}{\mu - \rho \beta \theta^2} < \rho + \gamma m^*$, where $m^*$ is the constant (endogenous) growth rate of population in an equilibrium with active entrants. Then $x_N < x_Z$.

Lemma 3  Expenditure behavior of household is

$$c + \psi b = \begin{cases} 
\theta^2 \left( \mu - \frac{\phi}{x} \right) + 1 - \theta & \phi < x \leq x_N \\
\rho \beta \theta^2 + 1 - \theta & x > x_N 
\end{cases}.$$
Dynamical system consists of fertility and expenditure rules derived from household’s first order conditions, three differential equations in $c$, $h$, $x$, transversality condition on $x$, and initial value

$$x_0 = \theta^2 \kappa M_0^\gamma R_0^{1-\gamma} / N_0^{1-\sigma}.$$ 

As in Dasgupta and Heal (1974), $R_0$ in numerator is a choice. So, given $M_0$ and $N_0$, $x_0$ is set so to select path $c(t)$ that satisfies $S_0 = \int_0^\infty R(t) \, dt$ (recall analysis of equilibrium extraction).

Nothing crucial hinges on presence or specific functional form of $f(b)$. I thus posit $f(b) = 0$ and reduce fertility rule to

$$h = \psi / c,$$

which allows me to eliminate $h$ and work in terms of $c$ and $x$ only.
Transition dynamics: the regime with no entry

After some algebra, the system reduces to:

\[
\frac{\dot{c}}{c} = \frac{\eta + 1}{\psi} c - \frac{\theta^2}{\psi} \left( \mu - \frac{\phi}{x} \right) - \frac{(1 - \gamma)(1 - \theta)}{\psi} - \rho;
\]

\[
\frac{\dot{x}}{x} = -\frac{1 + (1 - \gamma)\eta}{\psi} c + \frac{\theta^2}{\psi} \left( \mu - \frac{\phi}{x} \right) + (1 - \gamma + \gamma^2) \frac{1 - \theta}{\psi} - \gamma \delta.
\]

Key: feedback between consumption and firm size, that, because of no innovation, is driven solely by evolution of population and resource stock.
Transition dynamics: the regime with entry

In this case we have:

\[
\frac{\dot{c}}{c} = \frac{\eta + 1}{\psi} c - \frac{\rho \beta \theta^2 + (1 - \gamma)(1 - \theta)}{\psi} - \rho;
\]

\[
\frac{\dot{x}}{x} = \gamma m^* - (1 - \gamma) \rho - (1 - \sigma) n(x).
\]

Key:

- rate of entry \( n \) is a function of \( x \);
- unstable differential equations for \( c \) and \( h \) do not depend on \( x \) (because model has no scale effect) and, consequently, \( c \) and \( h \) jump to their steady-state values \( c^* \) and \( h^* \) and determine \( b = b^* \) and \( m^* = b^* - \delta \) at all times (Lemma 1).
The transition to sustained growth: phase diagram, unique equilibrium path
The transition to sustained growth: innovation rates and law of motion of firm size in entry region

When economy crosses threshold $x_N$, rates of innovation are:

$$n = \begin{cases} 
\frac{1}{\beta \theta^2} \left( \mu - \frac{\phi}{x} \right) - \rho & x \leq x_Z \\
\frac{(1-\alpha)(\mu x - \phi) + \gamma (m^* + \rho) - \sigma \rho}{\beta \theta^2 x - \sigma} - \rho & x > x_Z 
\end{cases}$$

$$z = \begin{cases} 
0 & x \leq x_Z \\
\frac{(\mu x - \phi) \left( \alpha - \frac{\sigma}{\beta \theta^2 x} \right) - \gamma (m^* + \rho) + \sigma \rho}{1 - \frac{\sigma}{\beta \theta^2 x}} & x > x_Z 
\end{cases}$$

Law of motion of firm size is

$$\frac{\dot{x}}{x} = \begin{cases} 
\gamma (m^* + \rho) - \sigma \rho - (1 - \sigma) \frac{\mu x - \phi}{\beta \theta^2 x} & x_N < x \leq x_Z \\
\gamma (m^* + \rho) - \sigma \rho - (1 - \sigma) \frac{(1-\alpha)(\mu x - \phi) + \gamma (m^* + \rho) - \sigma \rho}{\beta \theta^2 x - \sigma} & x > x_Z 
\end{cases}$$
The transition to sustained growth: phase diagram, non-unique equilibrium path or failure to launch?
For $x_N < x < x_Z$ economy follows linear differential equation

$$\dot{x} = \bar{\nu} (\bar{x}^* - x).$$

Integrating between time $T_N$ and time $t$ yields

$$x(t) = x_N e^{\bar{\nu}(T_N - t)} + \bar{x}^* \left( 1 - e^{\bar{\nu}(T_N - t)} \right).$$

If $\bar{x}^* > x_Z$, then there exists value

$$T_Z = T_N + \frac{1}{\bar{\nu}} \log \left( \frac{\bar{x}^* - x_N}{\bar{x}^* - x_Z} \right)$$

such that $x(T_Z) = x_Z$, which is date when economy turns on quality growth.
The transition to sustained growth: analytical solution (ii)

Note: $T_Z$ is watershed moment when de-coupling becomes feasible.

For $t > T_Z$ economy follows *nonlinear* differential equation and converges to

$$x^* = \frac{1 - \frac{(1-\sigma)(1-\alpha)\phi}{\gamma(m^*+\rho)+\sigma\rho}}{\beta\theta^2 - \frac{(1-\sigma)(1-\alpha)\mu}{\gamma(m^*+\rho)+\sigma\rho}}.$$

Time path of income per capita growth is

$$g(t) = \begin{cases} 
\sigma n(t) - (1 - \gamma) (m^* + \rho) & 0 \leq t \leq T_Z \\
\sigma n(t) + z(t) - (1 - \gamma) (m^* + \rho) & t > T_Z 
\end{cases}.$$

With this expression in hand, we can investigate one of the central questions of the paper.
Is long-run growth sustainable? (i)

If economy fails to cross threshold $x_Z$, it converges to semi-endogenous growth rate

$$\bar{g}^* = \sigma \gamma \left( m^* + \rho \right) - \frac{\rho}{1 - \sigma} - (1 - \gamma) (m^* + \rho).$$

Sustainability condition is

$$\bar{g}^* > 0 \quad \text{iff} \quad \sigma > (1 - \gamma) \frac{m^* + \rho}{m^*}.$$

- Holds for small values of $m^* + \rho$.
- Important: given $\rho$, this condition holds for fast population growth.
- Implication: Given non-zero exhaustion rate, sustainable growth requires sufficiently high fertility.
If economy crosses threshold $x_Z$, it converges to (fully) endogenous growth rate

$$g^* = \alpha (\mu x^* - \phi) - m^* - \rho.$$ 

Sustainability condition is

$$g^* > 0 \text{ iff } \alpha \left( 1 - \frac{(1-\sigma)(1-\alpha)\phi}{\gamma(m^*+\rho)+\sigma \rho} \right) > m^* + \rho,$$

- Also holds for low values of $m^* + \rho$.
- However: given $\rho$, this condition holds for slow population growth.
- Implication: Given non-zero exhaustion rate, sustainable growth requires sufficiently low fertility.
Final thoughts

Model makes rather loose predictions about population growth rate. In fact, it only says that it is constant. Not quite satisfactory.

- Can we develop model with tighter predictions, e.g., long-run population level has to be constant in world of limited resources?
- Caveat: It should **not** be driven by Malthusian-like mechanism that ties population size to the resource stock.

Model is also very stark in its treatment of resource supply to preserve tractability.

- Can we do better?
- Likely. To gain analytical insight, however, need to be quite clever as things get easily quite complex since we are adding state variables and non-linearities.