# Innovation and Growth in Imperfect Financial Markets

Maurizio Iacopetta, SKEMA and OFCE (Sciences Po)
Raoul Minetti, Michigan State
Pietro F. Peretto, Duke

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# What we do

Schumpeter (1911, Theory of Economic Development) emphasized that innovation requires credit. A massive literature studies the role of financial markets in the growth process. However, not much has been done to understand role of finance in Schumpeterian theory of innovation-driven growth.

We elaborate on Schumpeter's vision as follows:

1. Two dimensions of innovations:
    - creation of new products/firms;
    - in-house incremental innovation by existing firms.
2. Moral Hazard: a source of financial frictions that can bite both
    - when new firms enter the market
    - and when incumbent firms innovate in-house.
3. How do frictions affect growth and market structure?
    - We study both direct (household) and bank financing.
    - Important insight: interdependence of decisions on two margins of innovation.

# Why is this important?

Crisis highlighted critical role of financial markets in economic activity, led to calls for far-reaching reforms. But,

*do we know what we are doing?*

Example — Financial market reforms may generate undesirable long-run effects: financial stability induced by reforms may come at the cost of more rigidity or unforseen frictions in financing firms, especially innovative ones (Economist, 2012).

**Our question**: What role do imperfect financial markets play in growth process led by firms' innovation?

A necessary first step toward understanding the more general issues of reform and regulation.

# Model structure

Final producers: Homogeneous good that is consumed, used to produce intermediate goods, or invested in R&D. (One-sector structure.) This good is the *numeraire*, so $P_Y \equiv 1$.

Intermediate producers: Develop new goods and set up operations to serve market (variety innovation or entry) and, when already in operation, invest in R&D internal to firm (quality innovation).

Households: Consume and save.

# Households

To focus analysis on financing of innovation, simplest possible characterization of consumption/saving:

- representative household chooses $C(t)$ to maximize

$$U(0) = \int_0^\infty e^{-\rho t} L(t) \log\left(\frac{C(t)}{L(t)}\right) dt, \quad L(t) = e^{\lambda t}, \ \rho > \lambda > 0$$

  subject to

$$\dot{A} = rA + wL - C;$$

- resulting consumption/saving plan is

$$r = \rho + \frac{\dot{C}}{C} - \lambda.$$

# Final producers: technology

$$Y = \int_0^N X_i^{\theta} \left( Z_i^{\alpha} Z^{1-\alpha} \frac{L}{N^{1-\sigma}} \right)^{1-\theta}, \quad Z \equiv \int_0^N (Z_j / N)\, dj$$

where:

- $0 < \theta, \alpha, \sigma < 1$;
- $Z_i^{\alpha} Z^{1-\alpha}$ **vertical** technology index, with $\alpha$ measure of private returns to quality and $1 - \alpha$ measure of social returns to quality;
- $N$ **horizontal** technology index, with $\sigma$ measure of social returns to variety.

# Final producers: behavior

Demand for good $i$:

$$X_i = N^{\sigma-1} \left( \frac{\theta}{P_i} \right)^{\frac{1}{1-\theta}} Z_i^{\alpha} Z^{1-\alpha} L.$$

Factor payments:

$$N \cdot PX = \int_0^N P_i X_i di = \theta Y;$$

$$wL = w \int_0^N L_i di = (1-\theta)\, Y.$$

# Intermediate producers: technologies

Production:

$$Cost_i = 1 \cdot \left( X_i + \phi Z_i^{\alpha} Z^{1-\alpha} \right),$$

where both the variable and the fixed cost are in units of final output.

Corporate innovation:

$$\dot{Z}_i = I_i,$$

where $I_i$ is in units of final output.

Entrepreneurial innovation:

$$\dot{N} = \frac{N}{\beta Y} \cdot E,$$

where $E$ is in units of final output.

# Intermediate producers: equilibrium concept (i)

**Value maximization**: firm $i$ chooses paths $P_i(t)$ and $I_i(t)$ to maximize

$$V_i(0) = \int_0^\infty e^{-\int_0^t r(s)ds} \Pi_i(t)\, dt,$$

where

$$\Pi_i = X_i(P_i - 1) - \phi Z_i^\alpha Z^{1-\alpha} - I_i,$$

subject to the incremental innovation technology and the demand curve coming from the final sector.

**Free entry**: the anticipated maximized value of the firm must equal the entry cost:

$$V_i^{\max}(0) = \frac{\beta Y(0)}{N(0)}.$$

# Intermediate producers: equilibrium concept (ii)

Looking at incentives on intensive and extensive margins:

- existing firm,

$$r = \frac{\partial \Pi_i / \partial Z_i}{q_i} + \frac{\dot{q}_i}{q_i} \quad \text{and} \quad q_i = 1;$$

- new firm,

$$r = \frac{\Pi_i}{V_i} + \frac{\dot{V}_i}{V_i} \quad \text{and} \quad V_i = \frac{\beta Y}{N}.$$

In **symmetric equilibrium**:

$$r = \alpha \left[ \frac{X \left( 1/\theta - 1 \right)}{Z} - \phi \right] \equiv r_Z;$$

$$r = \frac{\left[ X \left( 1/\theta - 1 \right) - \phi Z - I \right] N}{\beta Y} + \frac{\dot{Y}}{Y} - \frac{\dot{N}}{N} \equiv r_N.$$

# General equilibrium: output and role of firm size (i)

Symmetry plus factor markets equilibrium yields

$$Y = \theta^{\frac{2\theta}{1-\theta}} \cdot N^{\sigma} Z \cdot L.$$

Returns to innovation are functions of

$$x \equiv \frac{X\left(P-1\right)}{Z} = \frac{\text{gross cash flow}}{\text{quality}} = \textbf{``firm size''},$$

where in equilibrium

$$x = \theta\left(1-\theta\right) \cdot \frac{Y}{NZ} = \theta\left(1-\theta\right)\theta^{\frac{2\theta}{1-\theta}} \cdot \frac{L}{N^{1-\sigma}}.$$

# General equilibrium: output and role of firm size (ii)

Subtracting from final output cost of intermediate production yields

$$GDP \equiv G = \underbrace{(1-\theta)\left[\theta\left(1-\frac{\phi}{x}\right)+1\right]}_{\substack{\text{unit cost of intermediate firm}\\ \text{falls as scale of operation rises}}} \cdot Y.$$

Taking logs and time derivatives,

$$\frac{\dot{G}}{G} = \frac{\dot{Y}}{Y} + \xi(x)\frac{\dot{x}}{x}, \quad \xi(x) \equiv \frac{\theta\phi}{(1+\theta)x - \theta\phi}.$$

# General equilibrium: returns and saving behavior

Returns to innovation now read:

$$r = \alpha \left( x - \phi \right);$$

$$r = \frac{x - \phi}{\pi x} + z \left( 1 - \frac{1}{\pi x} \right) + \frac{\dot{x}}{x},$$

where $z \equiv \dot{Z}/Z = I/Z$ and

$$\pi \equiv \frac{\beta}{\theta \left( 1 - \theta \right)} = \frac{\beta Y/N}{X \left( P - 1 \right)} = \frac{\text{entry cost}}{\text{gross cash flow}}.$$

Assets market equilibrium requires

$$A = NV = \beta Y.$$

Then, household budget yields

$$0 = \rho - \lambda + \left( \frac{\dot{C}}{C} - \frac{\dot{Y}}{Y} \right) + \frac{1 - \theta - \left( C/Y \right)}{\beta} \Rightarrow \frac{C}{Y} = \left( \rho - \lambda \right) \beta + 1 - \theta.$$

# General equilibrium: dynamics

There exist threshold $x_N$ of firm size that triggers variety innovation and threshold $x_Z$ of firm size that triggers quality innovation.

We study economy in the region $x > \max\{x_Z, x_N\} > \phi$ and approximate $\sigma/x \to 0$ so that firm size evolves according to **linear** differential equation:

$$\dot{x} = \nu \cdot (x^* - x),$$

where:

$$\nu \equiv \frac{(1-\sigma)\phi}{\pi x^*}; \quad x^* = \frac{(1-\alpha)\phi - \left(\rho + \frac{\sigma\lambda}{1-\sigma}\right)}{1-\alpha-\pi\left(\rho + \frac{\sigma\lambda}{1-\sigma}\right)}.$$

**Closed-form solution**:

$$x(t) = x_0 e^{-\nu t} + x^* \left(1 - e^{-\nu t}\right),$$

where $x_0$ is initial condition.

# The steady state: innovation rates and economic growth

Variety growth:

$$n^* \equiv \left(\frac{\dot{N}}{N}\right)^* = \frac{\lambda}{1-\sigma} > 0.$$

Quality growth:

$$z^* \equiv \left(\frac{\dot{Z}}{Z}\right)^* = \left[\frac{\alpha\left(\phi\pi - 1\right)}{1 - \alpha - \pi\left(\rho + \frac{\sigma\lambda}{1-\sigma}\right)} - 1\right]\left(\rho + \frac{\sigma\lambda}{1-\sigma}\right) > 0.$$

Final output and GDP per capita growth:

$$\left(\frac{\dot{Y}}{Y}\right)^* - \lambda = \left(\frac{\dot{G}}{G}\right)^* - \lambda = \frac{\sigma\lambda}{1-\sigma} + z^*.$$

# The steady state: comparative statics (i)

From saving behavior of household:

$$r^* = \rho + \frac{\sigma\lambda}{1-\sigma} + z^*.$$

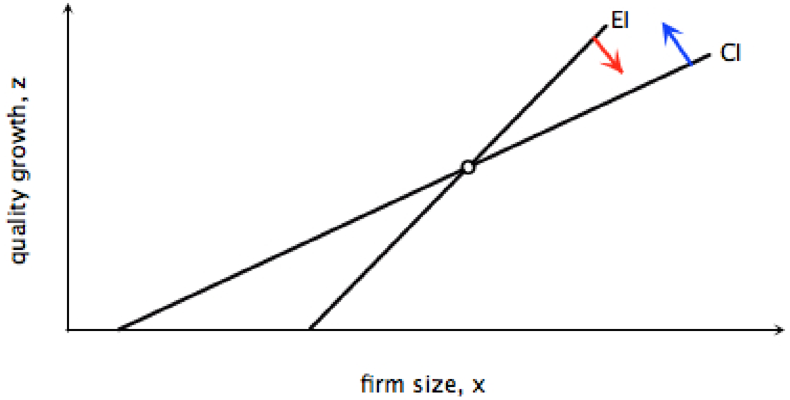
Manipulate returns to innovation to get:

$$z^* = \alpha \cdot (x^* - \phi) - \left(\rho + \frac{\sigma\lambda}{1-\sigma}\right); \qquad \text{(CI)}$$

$$z^* = x^* \left[1 - \pi \cdot \left(\rho + \frac{\sigma\lambda}{1-\sigma}\right)\right] - \phi. \qquad \text{(EI)}$$

Key concept: **no-arbitrage**.

# The steady state: comparative statics (ii)




When either $\alpha\uparrow$ or $\pi\uparrow$ we have $z^*\uparrow$ and $x^*\uparrow$.

# Welfare

Utility flow is

$$\log\left(\frac{C(t)}{L(t)}\right) = \underbrace{\left(\frac{\sigma\lambda}{1-\sigma}+z^*\right)t}_{\text{steady-state growth}} + \underbrace{\left(\frac{\sigma}{1-\sigma}+\frac{\alpha}{\nu}x_0\right)\left(1-\frac{x^*}{x_0}\right)\left(1-e^{-\nu t}\right)}_{\text{transitional dynamics}}.$$

Substituting in welfare functional and integrating yields:

$$U = \frac{1}{\rho-\lambda}\left[\left(\frac{\sigma\lambda}{1-\sigma}+z^*\right)\frac{1}{\rho-\lambda}+\frac{\frac{\sigma\nu}{1-\sigma}+\alpha x_0}{\rho-\lambda+\nu}\left(1-\frac{x^*}{x_0}\right)\right].$$

Note: there is a **growth/variety tradeoff** here, because

$$\log N^*(t) = \frac{1}{1-\sigma}\log\left(\theta(1-\theta)\,\theta^{\frac{2\theta}{1-\theta}}\frac{L(t)}{x^*}\right).$$

# Introducing financial frictions

Firms need external finance:

- creation of new varieties, i.e., foundation of new firms;
- incremental innovation within existing firms.

Two cases:

- borrow directly from households;
- borrow from banks. *(Not today since analysis not finished yet.)*

We consider **moral hazard**: agents (i.e., founders, entrepreneurs, managers, CEOs, whatever else we want to call them) can abscond loan of financier and use it to produce private benefit.

# The frictions: foundation

Extraction of entrepreneur's private benefit occurs according to

$$\gamma \cdot \underbrace{\frac{\beta Y(0)}{N(0)}}_{\text{size of loan}}, \quad \gamma \in [0, 1].$$

If entrepreneur "runs away" with loan, firm does not start and $V_i(0) = 0$. Then, value of entrepreneurial project is $V_i(0) - \frac{\beta Y(0)}{N(0)} = -\frac{\beta Y(0)}{N(0)}$.

Outside option of financier (household) is to consume funds.

# The frictions: management

Manager can commit to optimal price plan but not to optimal R&D plan.

Extraction of manager's private benefit occurs according to

$$\delta \cdot \underbrace{I_i(t)}_{\text{size of loan}}, \quad \delta \in [0,1].$$

If manager "runs away" with loan at time $t$, innovation does not occur and $q_i(t) = 0$. Then, value of R&D project is $I_i(t) \cdot [q_i(t) - 1] = -I_i(t)$.

Outside option of financier (household) is to consume funds.

Given capital-theoretic structure of incremental innovation, we can focus on marginal values.

# The incentives (i)

Key: **Interdependence** of incentive and participation constraints on extensive and intensive margins.

Need to solve simoultaneously moral hazard problems in foundation and management.

# The incentives (ii)

Incentive constraint of entrepreneur:

$$\underbrace{\gamma \cdot \frac{\beta Y(0)}{N(0)}}_{\substack{\text{private benefit} \\ \text{of entrepreneur}}} \leq \underbrace{(1 - a_i(0))}_{\substack{\text{equity share} \\ \text{of entrepreneur}}} \cdot \underbrace{V_i(0)}_{\substack{\text{present value} \\ \text{of firm at entry}}}.$$

Participation constraint of financier:

$$\underbrace{\frac{\beta Y(0)}{N(0)}}_{\substack{\text{outside option} \\ \text{of financier}}} \leq \underbrace{a_i(0)}_{\substack{\text{equity share} \\ \text{of financier}}} \cdot \underbrace{(1 - b_i(t))}_{\substack{\text{equity share} \\ \text{of manager}}} \cdot \underbrace{V_i(0)}_{\substack{\text{present value} \\ \text{of firm at entry}}}.$$

# The incentives (iii)

Incentive constraint of manager:

$$\underbrace{\delta \cdot 1}_{\substack{\text{private benefit} \\ \text{of manager}}} \leq \underbrace{a_i\left(0\right)}_{\substack{\text{equity share} \\ \text{of financier}}} \cdot \underbrace{\left(1 - b_i\left(t\right)\right)}_{\substack{\text{equity share} \\ \text{of manager}}} \cdot \underbrace{q_i(t)}_{\substack{\text{present value} \\ \text{of project}}}.$$

Participation constraint of financier:

$$\underbrace{1}_{\substack{\text{outside option} \\ \text{of financier}}} \leq \underbrace{a_i\left(0\right)}_{\substack{\text{equity share} \\ \text{of financier}}} \cdot \underbrace{b_i\left(t\right)}_{\substack{\text{equity share} \\ \text{of financier}}} \cdot \underbrace{q_i(t)}_{\substack{\text{present value} \\ \text{of project}}}.$$

# The incentives (iv)

Bang-bang solution for manager/financier problem plus free entry for entrepreneur/financier problem imply that all constraints hold with equality.

From manager/financier constraints:

$$\delta = \frac{1 - b_i(t)}{b_i(t)} \Rightarrow b_i(t) = \frac{1}{1+\delta}.$$

From entrepreneur/financier constraints:

$$\gamma = \frac{(1 - b_i(t))\, a_i(0)}{1 - a_i(0)} \Rightarrow a_i(0) = \frac{1}{1 + \frac{\gamma\delta}{1+\delta}}.$$

# The incentives (v)

From first-order condition:

$$\underbrace{\frac{\delta}{\left(1-b_i\left(t\right)\right)a_i\left(0\right)}}_{\text{effective innovation cost}}\cdot 1 = q_i(t) \quad \Rightarrow \quad (1+\gamma+\gamma\delta)\cdot 1 = q_i(t).$$

From free-entry condition:

$$\underbrace{\frac{\gamma}{1-a_i\left(0\right)}\cdot\frac{\beta Y\left(0\right)}{N\left(0\right)}}_{\text{effective entry cost}} = V_i(0) \quad \Rightarrow \quad (1+\gamma+1/\delta)\cdot\frac{\beta Y\left(0\right)}{N\left(0\right)} = V_i(0).$$

# Returns to innovation and saving behavior with frictions

Returns to corporate and entrepreneurial innovation are:

$$r = \alpha_f \cdot (x - \phi), \quad \alpha_f \equiv \frac{\alpha}{1 + \gamma + \gamma\delta};$$

$$r = \frac{x - \phi}{\pi_f x} + z\left(1 - \frac{1}{\pi_f x}\right) + \frac{\dot{x}}{x}, \quad \pi_f \equiv \frac{\beta_f}{\theta(1-\theta)}, \quad \beta_f \equiv \beta(1 + \gamma + 1/\delta).$$

Wealth/output and consumption/output ratios are:

$$\frac{A}{Y} = \beta_f \quad \text{and} \quad \frac{A}{G} = \frac{A}{Y}\frac{Y}{G} = \frac{\beta_f}{(1-\theta)\left[\theta\left(1 - \frac{\phi}{x}\right) + 1\right]};$$

$$\frac{C}{Y} = (\rho - \lambda)\beta_f + 1 - \theta \quad \text{and} \quad \frac{C}{G} = \frac{C}{Y}\frac{Y}{G} = \frac{\frac{\rho-\lambda}{1-\theta}\beta_f + 1}{\theta\left(1 - \frac{\phi}{x}\right) + 1}.$$

# General equilibrium dynamics with frictions

Again, we study economy in the region $x > \max\{x_Z, x_N\} > \phi$ and approximate $\sigma/x \to 0$ so that firm size evolves according to **linear** differential equation:

$$\dot{x} = \nu_f \cdot \left(x_f^* - x\right),$$

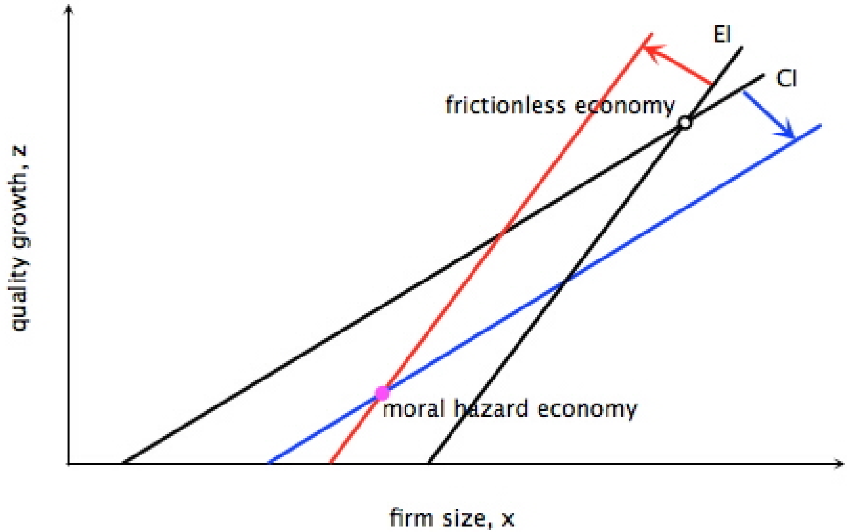
where:

$$\nu_f \equiv \frac{(1-\sigma)\,\phi}{\pi_f x_f^*}; \quad x_f^* = \frac{(1-\alpha_f)\,\phi - \left(\rho + \frac{\sigma\lambda}{1-\sigma}\right)}{1 - \alpha_f - \pi_f\left(\rho + \frac{\sigma\lambda}{1-\sigma}\right)}.$$

Using explicit solution for path $x(t)$, we obtain welfare:

$$U_f = \frac{1}{\rho-\lambda}\left[\left(\frac{\sigma\lambda}{1-\sigma} + z_f^*\right)\frac{1}{\rho-\lambda} + \frac{\frac{\sigma\nu_f}{1-\sigma} + \alpha_f x_0}{\rho-\lambda+\nu_f}\left(1 - \frac{x_f^*}{x_0}\right)\right].$$

# The "delta" friction: steady state

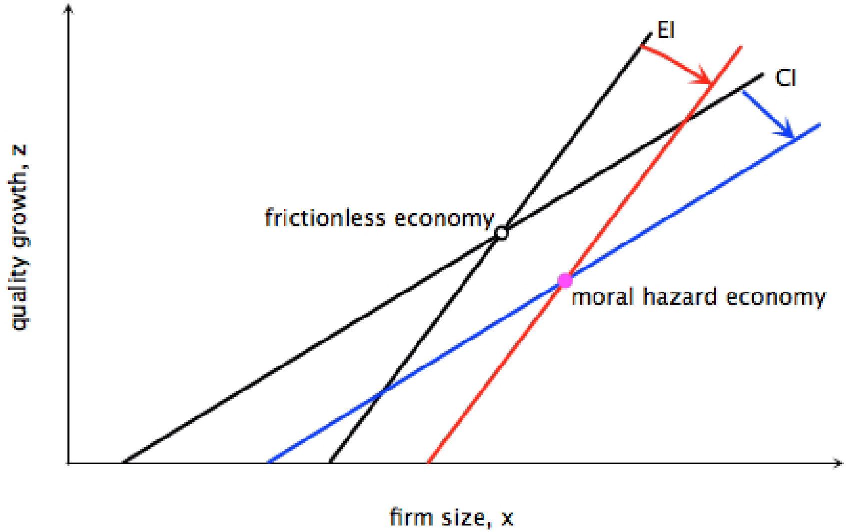


# The "delta" friction: dynamics

Because $\pi_f = (1 + \gamma + 1/\delta)\,\pi$ and $\alpha_f = \alpha / (1 + \gamma + \gamma\delta)$, the $\delta$ friction raises the return to entry and lowers the return to in-house innovation. The interaction of the two margins yields that:

- steady-state firm size falls (intuition: firms can service debt contracted at foundation with smaller gross profit flow because they do less in-house R&D);
- the eigenvalue of the dynamics is larger so that economy exhibits faster mean reversion (intuition: ?????);
- steady-state growth falls and the path of the mass of firms is higher (because of more entry in transition);
- overall, ambiguous change in welfare (but good reasons to think it is negative, see expression above).

# The "gamma" friction: steady state



# The "gamma" friction: dynamics

Because $\pi_f = (1 + \gamma + 1/\delta)\, \pi$ and $\alpha_f = \alpha / (1 + \gamma + \gamma\delta)$, the $\gamma$ friction lowers both the return to entry and to in-house innovation. The interaction of the two margins yields that all effects are, in principle, ambiguous. Let's take figure at face value and say that:

- steady-state firm size rises;
- eigenvalue of the dynamics is larger so that economy exhibits faster mean reversion (intuition: ?????);
- steady-state growth falls but the path of the the mass of firms is higher;
- overall, they yield an ambiguous change in welfare.

# Welfare: a more refined exercise (i)

What is the welfare cost of financial frictions?

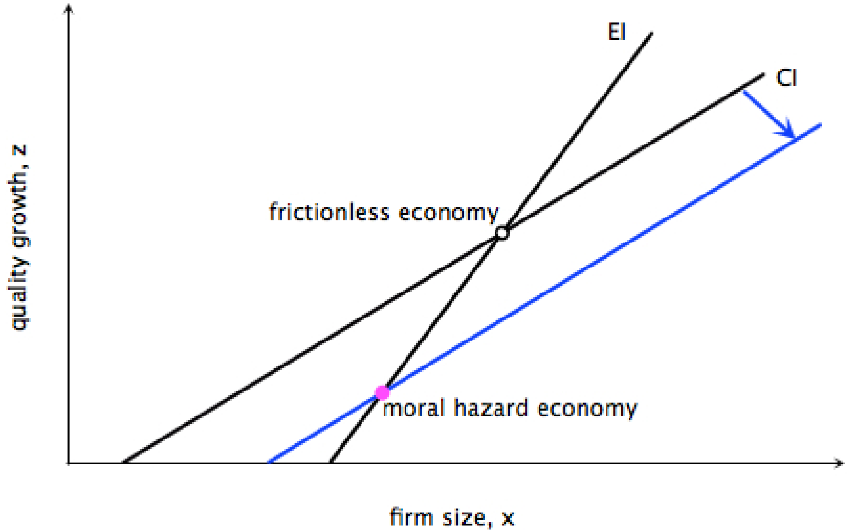
We can think about this as follows:

- the baseline path is the case with frictions;
- the welfare gain of eliminating frictions is the expression for the case without frictions calculated using $x_0 = x_f^*$, that is,

$$U - U_f^* = \frac{1}{\rho - \lambda}\left[\frac{z^* - z_f^*}{\rho - \lambda} + \frac{\frac{\sigma\nu}{1-\sigma} + \alpha x_f^*}{\rho - \lambda + \nu}\left(1 - \frac{x^*}{x_f^*}\right)\right].$$

- The graphical analysis suggests that $\gamma$ and $\delta$ push the EI line in opposite directions. Say that they roughly cancel out and think of dynamics when EI line does not move while CI line shifts up.

# Welfare: a more refined exercise (ii)



# Welfare: a more refined exercise (iii)

Hence, $z^* > z_f^*$ and $x^* > x_f^*$.

Welfare change from elimination of frictions:

$$U - U_f^* = \frac{1}{\rho - \lambda} \left[ \underbrace{\frac{z^* - z_f^*}{\rho - \lambda}}_{>0} + \frac{\frac{\sigma \nu}{1-\sigma} + \alpha x_f^*}{\rho - \lambda + \nu} \underbrace{\left( 1 - \frac{x^*}{x_f^*} \right)}_{<0} \right].$$

Conclusion: Elimination of frictions allows faster quality growth **and** larger mass of firms.