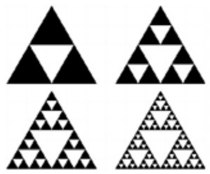
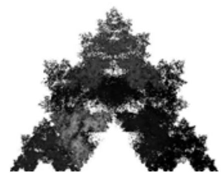
University of York

Department of Economics



Fractal analysis and inverse problems: models, methods and applications



Teacher: Dr. Davide La Torre, Ph.D., Associate Professor of Mathematics, Department of Economics, Business and Statistics, University of Milan.

Contact information:
Mobile: 0039 333 8860227
E-mail: davide.latorre@unimi.it
Home page: www.economia.unimi.it/latorre

Course objectives and description:
After recalling some mathematical backgrounds, the lectures will focus on the notions of fractals, iterated function systems and inverse problems; very recent applications of these complex nonlinear structures to stochastic growth models, financial forecasting and statistical estimation will be shown and analyzed in details.
Mandelbrot (1975) introduced the term 'fractal' to characterize spatial or temporal phenomena that are continuous but not differentiable. Every attempt to split a fractal into smaller pieces results in the resolution of more structure. Fractal properties include scale independence, self-similarity, complexity, and infinite length or detail.
The landmark papers by Hutchinson and Barnsley and Demko showed how systems of contractive maps with associated probabilities (called "iterated function systems" by the latter), acting in a parallel manner, either deterministically or probabilistically, could be used to construct fractal sets and measures.
There is an ongoing research programme on the construction of appropriate IFS-type operators, or generalized fractal transforms (GFT), over various spaces, i.e., function spaces and distributions, vector-valued measures, integral transforms and wavelet transforms (see www.uwaterloo.ca). The action of a GFT on an element u of the complete metric space (X,d) under consideration can be summarized as follows:

- it produces a set of N spatially-contracted copies of u,
- it then modifies the values of these copies by means of a suitable range-mapping
- it recombines these copies using an appropriate operator to produce the element v in X, v = Tu.

In each of the above-mentioned cases, the fractal transform T is guaranteed to be contractive when the parameters defining it satisfy appropriate conditions specific to the metric space of concern. In this situation, Banach's fixed point theorem guarantees the existence of a unique fixed point u = Tu. The inverse problem of fractal-based approximation is as follows: given an element y, can we find a fractal transform T with fixed point u so that d(y,u) is sufficiently small. However, the search for such transforms is enormously complicated. Thanks to a simple consequence of Banach's fixed point theorem known as the Collage Theorem, most practical methods of solving the inverse problem seek to find an operator T for which the collage distance d(u,Tu) is as small as possible.
In more general situations, fractal based methods attempt to discover and exploit inter-scale relations for prediction and control of the phenomena. In many cases they have been used for solving inverse problems arising in models described by systems of differential equations (with deterministic or stochastic coefficients). An inverse problem is the task that often occurs in many branches where the values of some model parameters must be obtained from the observed data.
A general inverse problem can be formulated as follows: given a family of problems J_λ depending upon a parameter $\lambda \in P \subseteq \Re^n$ and a target object u, find values of the parameter $\bar{\lambda}$ such that the corresponding solutions of J_λ are ``close'' to u. Inverse problems can be formulated for many mathematical problems and studied by different techniques; in this research project we are interested into fractal-based techniques for the solution of inverse problems arising from differential equations with initial and boundary conditions. Most natural phenomena or the experiments that explore them are subject to small variations in the environment within which

they take place. As a result, data gathered from many runs of the same experiment may well show differences that are most suitably accounted for by a model that incorporates some randomness.

Course prerequisites: Calculus.

Lectures: 5 lectures, two hours each.

Course description:

1. Mathematical background, Hausdorff measure and dimension, alternative definitions of dimension, fractals.
2. Fractals defined by transformation, self-similar and self-affine sets, iterated function systems, iterated function systems with probabilities, applications to stochastic growth models.
3. Dynamical systems, random dynamical systems, fractal estimation, fractal simulation, Brownian motion.
4. Inverse problems for fractals and related topics. Formulation, solution.
5. Differential equations and random differential equations. Inverse problems using fractal-based methods.

Main references:

1. K.Falconer, Fractal geometry: mathematical foundations and applications, J.Wiley & Sons, 1990.
2. O.Grabbe, Chaos & Fractals in Financial Markets.
3. B. Forte and E.R. Vrscay, 1. Theory of generalized fractal transforms, and 2. Inverse problem methods for generalized fractal transforms, in Fractal Image Encoding and Analysis, Ed. Y. Fisher (Springer Verlag, Heidelberg, 1998).
4. H. Kunze and E.R. Vrscay, Solving inverse problems for ODEs using the Picard contraction mapping, Inverse Problems, 15, 7450-770, 1999.
5. H.Kunze, D. La Torre, E.R.Vrscay, Contractive multifunctions, fixed point inclusions and iterated multifunction systems, J.Math.Anal.Appl., 303, 2007.
6. H. Kunze, D. La Torre and E.R. Vrscay, Random fixed point equations and inverse problems by collage method, J. Math. Anal. Appl., 334, 2007.
7. H. Kunze, D. La Torre and E.R. Vrscay, Solving inverse problems for DEs by collage method and applications to variational optimization, Proceedings of International Symposium on Inverse Problems, Design And Optimization (IPDO-2007) Miami Beach, Florida, U.S.A., April 16-18, 2007.
8. H. Kunze, D. La Torre and E.R. Vrscay, Solving inverse problems for random equations and applications, Proceedings of International Symposium on Inverse Problems, Design And Optimization (IPDO-2007) Miami Beach, Florida, U.S.A., April 16-18, 2007.
9. S.M. Iacus and D. La Torre, A comparative simulation study on the IFS distribution function estimator, Nonlinear Analysis: Real World Applications, 6, 774-785 (2005).
10. D.La Torre, F. Mendivil and E.R. Vrscay, Iterated function systems on multifunctions, Math Everywhere - Deterministic and Stochastic Modelling in Biomedicine, Economics and Industry, Springer, 2006, 125-138.
11. E.Peter, Fractal market analysis. Applying chaos theory to economics and investments, J.Wiley & Sons, 1994.