1 Individual rights

1.1 Sen's impossibility of a Paretian liberal

Chapter 2 has taught us that a set of rather mild looking conditions imposed on an Arrovian social welfare function creates the dictatorship of one person. Whenever this individual prefers any x over any y from the set of alternatives X, society prefers x over y by necessity, and this holds for all pairs of alternatives from X and for all profiles in the domain of the social welfare function. Arrow called such an individual a dictator, and most people will agree that a person with such a wide–ranging power is unacceptable for a democratic society. The reader should remember that this dictatorial power refers to all–encompassing social alternatives. We do not want one person to decide whether or not our country goes to war, we do not want this person to decide all by himself whether the country pursues a policy that benefits a few and makes many citizens suffer nor do we want one single person to determine that our country execute a cultural revolution.

There are, however, good arguments for allowing citizens to exercise 'local decisiveness', i.e., be dictatorial with respect to narrowly defined spheres which are the citizens' private business. This idea can be found in J.S. Mill's 'On Liberty' where the author speaks about a circle around every human being upon which nobody should trespass. We do not want the government to decide which religion we practise nor do we want busybody to determine whether we read 'Playboy' or not. It is a distinct feature of a democratic society that its members are free to exercise certain rights over private spheres. In other words, local decisiveness not only makes a lot of sense but it is also a vital characteristic of a liberal society.

Sen (1970a, 1970b) was the first to model the exercise of individual rights within the social choice context and to derive what has become known as 'the impossibility of a Paretian liberal'. Let us assume that each individual $i \in N$ has a 'protected or recognized private sphere' D_i consisting of at least one pair of personal alternatives over which this individual is decisive both ways in the social choice process, i.e., $(x, y) \in D_i$ iff $(y, x) \in D_i$ with $x \neq y$. D_i is called symmetric in this case. And decisiveness means that whenever $(x, y) \in D_i$ and xP_iy, then xPy and whenever $(y, x) \in D_i$ and yP_ix, then yPx for society. Clearly, the alternatives in each pair should be distinct.

Note that the present notation with respect to decisiveness is slightly different from the one in chapter 2 but the meaning is exactly the same. A rights–system then is an assignment of ordered pairs of states to individuals, viz. an n–tuple $D = (D_1, D_2, \ldots, D_n) \in \Omega(n)$, where $\Omega(n)$ stands for the n–fold cartesian product of Ω, the set of all non–empty subsets of $X \times X$. A short example may help to illustrate. Let us assume that $X = \{a, b, c, d\}$ and that there are only two individuals. Then a rights–

system $D = (D_1, D_2)$ for this two–person society could be such that $D_1 = ((a, b), (b, a))$ and $D_2 = ((c, d), (d, c))$. So individual 1 would possibly be decisive both ways over the pair (a, b) and the same would hold for person 2 over the pair (c, d). In this case, there would, for example, be no individual rights over the pairs (b, c) and (a, d).

In his original presentation of the 'liberal paradox', Sen (1970a) used the concept of a social decision function. This is a collective choice rule or social aggregation mechanism, the range of which is restricted to social preference relations that generate a choice function. The latter concept was introduced in chapter 1. The reader will remember that a necessary and sufficient condition for a choice function to be defined over a finite set X is that the binary relation R be reflexive, complete and acyclical over X.

Sen required the collective choice rule to satisfy the following three properties:

Condition U (Unrestricted Domain). The domain of the collective choice rule includes every logically possible set of individual orderings on $X(\mathcal{E}' = \mathcal{E})$.

Condition P (Weak Pareto Principle). For any x, y in X, if every member of society strictly prefers x to y, then xPy.
Note that conditions U and P have the same definition as in chapter 2.

Condition L (Liberalism). For each individual i, there is at least one pair of personal alternatives $(x, y) \in X$ such that the individual is decisive both ways in the social choice process. Therefore, $(x, y) \in D_i$ and xP_iy imply xPy and $(y, x) \in D_i$ and yP_ix imply yPx for society.

The reader should note that Sen never claimed that condition L would adequately describe the multi–faceted character of the concept of liberalism. Sen (1970a) wrote that this condition 'represents a value involving individual liberty that many people would subscribe to' (p. 153).

In order to strengthen his impossibility result, Sen weakened the condition of liberalism further. He required that the decisiveness over at least one pair of alternatives be given not to all members of society but to at least two individuals. Sen called this condition 'minimal liberalism' (condition L^*).

The result then is

THEOREM 4.1 (Sen's Impossibility of a Paretian Liberal (1970)). There is no social decision function that satisfies conditions U, P and L^*.

Proof. The proof is very simple. We shall suppose that person i is decisive over (x, y) and person j is decisive over (z, w), i.e., $(x, y) \in D_i$ and $(z, w) \in D_j$. We shall assume that these two pairs have no element in common (the other cases are as easy to treat as this one). Let us suppose that xP_iy, zP_jw and for all $k \in \{1, 2\}$: wP_kx and yP_kz. From condition L^*, we obtain xPy and zPw. From condition P, we get wPx and yPz so that we arrive at xPy, yPz, zPw and wPx. This outcome clearly violates the property of acyclicity of the social preference relation so that no social decision function exists that fulfils conditions P and L^* under unrestricted domain.

Sen's own illustration of his impossibility theorem has won a certain fame over the decades. His example, in contrast to the case in the proof above, contains only three alternatives, with some overlap in the decisiveness structure. There are three alternatives revolving around reading a copy of Lady Chatterley's Lover by D.H. Lawrence. Alternative a says that Mr A, the prude, reads this copy. Alternative b prescribes that Mr B, the lascivious, reads the book, and alternative c specifies that neither A nor B studies the novel. Mr A prefers most that nobody reads the book, next that he reads the book and last that Mr B reads the novel. Therefore, cP_AaP_Ab. Mr B has the preference aP_BbP_Bc. There are no other individuals in this society.

Sen now assumes that $(c, a) \in D_A$ and $(b, c) \in D_B$. Sen writes that a liberal argument can be made for the case that given the choice between A reading the book and no one reading it, A's preference should be turned into a social preference. Likewise, B's preference should be made the social preference for the case between B reading the novel and nobody reading it. Therefore, we arrive at cPa and bPc, and aPb due to the Pareto principle. These preferences manifest a case of cyclicity of the social relation so that no social decision function exists.

Note that decisiveness in one direction only would have been enough to generate an impossibility result. From a conceptual point of view, however, decisiveness both ways is much closer to the idea that individuals have a certain degree of autonomy.

1.2 Gibbard's theory of alienable rights

Sen's negative result had a tremendous impact on the imagination of many researchers in social choice theory. How can this impossibility result be turned into a possibility theorem? It seems obvious that there are many ways to achieve this, among them the suggestions to introduce domain restrictions of individual preferences, to constrain the Pareto principle or to weaken the condition of liberalism. Of course, all three paths will successfully eliminate Sen's negative result – at some price, however. As a matter of fact, a bit further on we shall briefly consider the idea to weaken the libertarian claim. We shall not elaborate on the suggestion to constrain the Pareto rule though Sen's Lady Chatterley story may be a good case against applying the Pareto principle 'automatically'.

For reasons below where we shall focus on choice functions and the existence of non–empty choice sets, we shall slightly redefine what it means for an individual to be decisive. We have said above that whenever $(x, y) \in D_i$ and xP_iy, then xPy socially. We will now say that individual i is decisive over the pair (x, y), i.e., $(x, y) \in D_i$ and

xP_iy, if y will never be socially chosen when x is available and i strictly prefers x to y. Kelly (1988) speaks of exclusionary power in this context. In other words, whenever $(x, y) \in D_i$ and xP_iy, then y will not be an element of the choice set $C(S)$, the set of socially chosen states from S, where $S \in K$ denotes a set of implementable social states and K stands for the family of all finite non–empty subsets of the set X of social alternatives. Sen's original result would now assert that under conditions U, P, and L^* (respectively L), the set of socially chosen states can be the empty set. In other words, a social decision function does not exist for all profiles $(R_1 \ldots, R_n)$.

In what follows, we wish to discuss Gibbard's (1974) theory of alienable rights. In this approach the libertarian claim of an individual is not *directly* constrained, i.e., made dependent on some feature within the individual's own preference ordering but it is shown that under well–defined conditions, a person may find it advantageous to forgo certain assigned rights. Thus, in the case of Lady Chatterley's Lover, prude Mr A may be willing to waive his right to c over a, since given the unanimous preference for a over b, Mr B's right to b over c will make it impossible for him to obtain c anyway. By waiving his rights, he will be able to secure that a is socially chosen and not b which would be his worst outcome.

Let us motivate Gibbard's suggestion a bit more by presenting his own Angelina–Edwin case. Gibbard's starting point for this particular proposal to reconcile the exercise of individual rights with the Pareto principle is that 'there is a strong libertarian tradition of free contract, and on that tradition, a person's rights are his to use or bargain away as he sees fit' (1974, p. 397).

There are three persons, Angelina, Edwin and 'the judge'. Angelina would like to get married to Edwin but would settle for the judge who is happy with whatever she wants. Edwin would like to remain single but would rather marry Angelina than see her marry the judge. There are three alternatives: Edwin and Angelina get married (x); Angelina and the judge marry and Edwin remains single (y); both Edwin and Angelina (and, of course, the judge) remain single (z). Angelina has the preference xP_AyP_Az. Edwin has the ordering zP_ExP_Ey.

Gibbard argues that Angelina has a right to marry the judge instead of remaining single. So Angelina has a libertarian claim over the pair (y, z). Edwin has the right to remain a bachelor rather than marry Angelina. So he has a claim over the pair (z, x). And finally, Edwin and Angelina have a unanimous preference for x over y. Thus, the combination of the weak Pareto principle and the two libertarian claims leads to a preference cycle, viz. yPz, zPx, and xPy.

In order to avoid Sen's impossibility result, the cycle has to be broken somewhere, but where? Gibbard argues that Edwin, of course, has the right to remain single, but he should think twice about exercising this right. 'He can bargain it away to keep

Angelina from marrying the judge' (p. 398). Though Edwin prefers z over x and has the chance to avoid x by exercising his right over (z, x), Angelina has a right over (y, z) and prefers y to z. This, however, means that when Edwin exercises his right, at the end of the day he will see Angelina wedded to the judge. And this would be worse for Edwin (we apologize for this phrase) than arriving at state x. Therefore, it will be to Edwin's own advantage to waive his right over (z, x) in favour of the Pareto preference xPy.

The idea behind this story is the source of Gibbard's general solution to which we now turn in detail. Let there be a rights–system $D = (D_1, \ldots, D_n)$ and a finite set $S = \{x, y, \ldots\}$ of feasible alternatives. We define a subset $W_i(R|S)$ of the set of protected pairs D_i for each individual i – let us call it the 'waiver set' of person i – by the following condition:
$(x, y) \in W_i(R|S)$ iff there exists a sequence $\{y^1, y^2, \ldots, y^\lambda\}$ of states in S such that

1. $y^1 \neq y$
2. $y^\lambda = x$
3. yR_iy^1 and
4. $\forall\, t \in \{1, 2, \ldots, \lambda - 1\}$, at least one of the following holds:
   $\forall\, j \in \{1, \ldots, n\} : y^tP_jy^{t+1}$,
   $\exists\, k \in \{1, \ldots, n\}, k \neq i : (y^t, y^{t+1}) \in D_k$ and $y^tP_ky^{t+1}$.

Note that given the set of feasible alternatives, the decision to waive or not to waive one's right(s) strongly depends on the preferences of the other persons. With the help of the idea of a sequence of alternatives in S, Gibbard formulates the following libertarian claim.

Condition GL (Gibbard's libertarian claim for alienable rights). For every preference profile $(R_1, \ldots, R_n)$, every $S \in K$, every $i \in N$, and every pair $(x, y) \in X$, if $(x, y) \in D_i$ and xP_iy and $(x, y) \notin W_i(R|S)$, and furthermore x is in S, then y is not an element of the choice set $C(S)$.

The following possibility result is then obtained.

THEOREM 4.2 (Gibbard's rights–waiving solution). There exists a collective choice rule that satisfies conditions U, P and GL.

We abstain from presenting a proof of this result. The basic idea should have become clear from the two illustrations above. The central role of the waiver set is to break a cycle whenever there is one. Gibbard's resolution scheme is quite mechanistic and places heavy demands on information holding and processing. The individual who has to decide whether to waive some right or not has to calculate sequences that comprise pairs of unanimous strict preferences and pairs over which other persons exercise their decisiveness. These sequences can be quite long. But Gibbard's scheme is effective, no doubt about that.

1.3 Conditional and unconditional preferences

We started this chapter by claiming that in a liberal society each person i has a protected or recognized private sphere D_i with at least one pair of personal alternatives over which this person is decisive. In the two illustrations by Sen and Gibbard, it was assumed that the spheres of the persons involved had exactly this property. In his paper from 1974, Gibbard was very formal when he specified the private spheres of the members of society. His idea was that social states can be decomposed into different components representing aspects within the recognized spheres of the individuals. More precisely, if X_i stands for the set of feature–alternatives x_i of person i, the state space X is given by the full Cartesian product $X_1 \times \ldots \times X_n$, if the society considered comprises n individuals. There could also be a set X_0 of public feature–alternatives which would represent aspects outside the private spheres of the individuals, but in the sequel we shall omit this component for the sake of simplicity. A particular social state x can then be written as $x = (x_1, \ldots, x_i, \ldots, x_n)$, where $x_i \in X_i$ for all i. Gibbard assumed that $|X_i| \geq 2$ for all $i \in \{1, \ldots, n\}$. Individual i's recognized private sphere can then be identified as $D_i = \{(x, y)|x_k = y_k$ for $k \neq i$, and $x_i \neq y_i\}$. The elements of D_i are called i–variants. They only differ with respect to the specification of i's personal component.

Gibbard formulated the following libertarian claim which is much stronger than Sen's original condition of liberalism.

Condition GL'. For every individual i and for all distinct social alternatives x and y, if x and y differ only with respect to something in i's recognized private sphere, then i is decisive over (x, y), i.e., if xP_iy and x is in S, y is not chosen from S [$y \notin C(S)$] and if yP_ix and y lies in S, $x \notin C(S)$.

Gibbard demands that person i be decisive over every pair of i–variants within the private sphere. Sen required decisiveness for at least one pair of alternatives. For his own libertarian claim, Gibbard could show the following

THEOREM 4.3 (Gibbard's conditional preferences). There exists no collective choice rule that satisfies conditions U and GL'.

Proof. Let us assume that there are two private features b and w that can be independently chosen by two individuals 1 and 2. We consider four states: $x = (bb)$, $y = (wb)$, $z = (bw)$, and $v = (ww)$, where in each tuple, the first (second) component refers to person 1(2). Thus $S = \{x, y, z, v\}$ and the private spheres of persons 1 and 2 are

$D_1 = \{(x, y), (z, v)\}$ and $D_2 = \{(x, z), (y, v)\}$. The preferences of the two persons are assumed to be

$$\begin{array}{llll} 1: & ww & 2: & bw \\ & bb & & wb \\ & bw & & ww \\ & wb & & bb \end{array}$$

According to condition GL', person 1 eliminates alternatives y and z from the choice set $C(S)$. Person 2 eliminates alternatives x and v from the choice set so that $C(S) = \emptyset$.

A closer look at the preferences of the two persons reveals that both persons have so–called conditional preferences. Given that person 2 picks w, person 1 prefers component w to feature b. Given that person 2 picks b, person 1 prefers b to w. Person 1 can be called a conformist, person 2 then is a non–conformist. Imagine that persons 1 and 2 are two young ladies who are invited to a dinner party. Let us suppose that each of the two owns a black dress and a white dress. Person 1 prefers that they both wear the same colour, be it white or black. Person 2 prefers that they wear different colours. It is assumed that both women take their decision independently. There is no coordination (could there be one?).

Perhaps a possibility theorem for conditional preferences is too much to ask for. Gibbard examined so–called unconditional preferences next.

Definition 4.1 (Unconditional Preferences). Individual i has unconditional preferences with respect to his or her recognized private sphere D_i if for all $(x, y) \in D_i$, whenever xP_iy, then $(x_i, z)P_i(y_i, z)$ for all z, where (x_i, z) is short for $(z_1, \ldots, z_{i-1}, x_i, z_{i+1}, \ldots, z_n)$ and (y_i, z) stands for $(z_1, \ldots, z_{i-1}, y_i, z_{i+1}, \ldots, z_n)$.

The vector z comprises the personal features of all the other individuals in the society considered. Whenever person i has manifested a strict preference for x_i over y_i for some constellation z, this strict preference is required to hold for any constellation of features of the other individuals. A preference profile that satisfies the property of unconditionality will be given in the proof of the next theorem.

Earlier on in this chapter, we mentioned that at some point, we would discuss a weakening of the libertarian claim. Here is Gibbard's version.

Condition GL''. For every individual i and for all distinct social alternatives x and y, if $(x, y) \in D_i$ and xP_iy and furthermore, individual i's preferences are unconditional, then $[x \in S \rightarrow y \notin C(S)]$.

The reader should note that this is not a domain condition on preferences. It is a restriction on individual rights. The exercise of individual rights depends on the fulfilment of certain requirements within the individuals' preferences. Gibbard mentions that as long as the number of issues on which individuals are allowed to be decisive is as great as the number of individuals in the society, no consistency problem arises. However, the combination of condition GL'' and the weak Pareto rule yields another impossibility result.

THEOREM 4.4 (Gibbard's unconditional preferences). There exists no collective choice rule that satisfies conditions U, P and GL''.

Proof. We assume again that there are four social states: $x = (bb)$, $y = (wb)$, $z = (bw)$, and $v = (ww)$. Also, the private spheres are the same as before: $D_1 = \{(x, y), (z, v)\}$; $D_2 = \{(x, z), (y, v)\}$. According to condition U, we can admit the following preferences for persons 1 and 2.

$$\begin{array}{llll} 1: & ww & 2: & bb \\ & bw & & bw \\ & wb & & wb \\ & bb & & ww \end{array}$$

The orderings of both individuals satisfy the property of unconditionality. Therefore, due to condition GL'', alternatives z and v are eliminated by person 2, x and z are eliminated by person 1 and the Pareto condition (appropriately redefined in terms of choices) prohibits the choice of y. Thus, $C(S) = \emptyset$.

1.4 Conditional and unconditional preferences again: matching pennies and the prisoners' dilemma

Let us have a second look at Theorems 4.3 and 4.4, from a different perspective so to speak (see Gaertner (1993) for further details). In connection with the proof of Theorem 4.3, we presented an illustration where two young ladies were, independent of each other, choosing their personal or private features, viz. b or w. The four social states x, y, z and v obviously are the outcome of these independent choices. Given the conditional preferences of the two women used in the proof of Theorem 4.3, we can depict this situation via the matrix in Figure 4.1.

1 \ 2	w	b
w	4 , 2	1 , 3
b	2 , 4	3 , 1

Figure 4.1

The entries in the four cells of the matrix are meant to represent the ordinal preferences of the two women (no cardinality is involved here). The first number refers to person 1, the second number to person 2. The reader should check that the numbers in the cells match the rankings postulated in the proof of Theorem 4.3. The matrix representation in Figure 4.1 is well known from non–cooperative game theory. It depicts the 'matching pennies' case and it is common knowledge that no Nash equilibrium in pure strategies exists for such a situation.

Since this is a book on social choice and not on non–cooperative game theory, a few explanations may be in order at this point. We wish to define a game form (a) as a set N of n players; (b) a set S_i of (pure) strategies for each player $i \in N$ with $s_i \in S_i$, and $s = (s_1, \ldots, s_n)$ as a vector of strategies with $s \in \prod_{j=1}^{n} S_j$; (c) a set X of feasible outcomes; and (d) an outcome function h which maps $\prod_j S_j$ into X (exactly one outcome is specified for each element of $\prod_j S_j$). In the situation given by Figure 4.1, there are four strategy vectors, viz. $(w, w), (w, b), (b, w)$, and (b, b), and four outcome vectors, corresponding to these four strategy vectors. We define a strategy vector $\hat{s} = (\hat{s}_1, \ldots, \hat{s}_n)$ as a (non–cooperative) Nash equilibrium, if for each player i with $s_i \in S_i$, the following holds: $h(\hat{s}) R_i h(\hat{s}_1, \ldots, \hat{s}_{i-1}, s_i, \hat{s}_{i+1}, \ldots, \hat{s}_n)$.
A Nash equilibrium represents a strategy vector where each and every player has given the best response to the strategies of all the other players so that there is a situation of simultaneous best responses. Obviously, in the 'matching pennies' game, at each cell in the matrix either player 1 or player 2 can do better by switching to the other strategy. This means that there is no Nash equilibrium in pure strategies. There is, however, a mixed–strategy equilibrium which is based on the idea that positive probabilities are attached to the pure strategies.

Obviously, the objectives of the conformist and the nonconformist do not match – there is no equilibrium strategy for the two 'players'. What does this mean within our context of social choices? The nonexistence of an equilibrium point (in pure strategies) apparently translates into the Gibbardian result that the choice set is empty. But the 'game' that the two women play would have an outcome or result in real life, wouldn't it?

Let us now examine Theorem 4.4 and the preference structure used in the proof. In this situation, we obtain the matrix in Figure 4.2.

$1 \setminus 2$	w	b
w	4 , 1	2 , 2
b	3 , 3	1 , 4

Figure 4.2

In this case where both persons display unconditional preferences, a dominant strategy Nash equilibrium exists. Person 1 prefers her personal feature w to b independently of whether person 2 chooses w or b. And person 2 prefers her private aspect b to w independently of what person 1 does. Both persons obviously possess a dominant strategy which yields an equilibrium point (viz. wb with the outcome vector (2,2)) that is Pareto inferior to the strategy combination bw with its outcome (3,3). This is the 'classical' structure of the prisoners' dilemma game. The inefficiency of the Nash equilibrium translates within the Gibbardian social choice context into a clash between condition GL'' and the Pareto condition with the result that, again, the choice set is empty. In this situation as well, there will be an outcome in real life. Don't you agree?

1.5 The game form approach to rights

On the following pages we wish to discuss whether Sen's conception of individual rights as well as Gibbard's stronger notion correspond to our intuitive idea of what it means for an individual to have and exercise a right. Remember that according to Sen's definition every individual (or at least two individuals) is (are) decisive over a pair of distinct social states as long as these states differ only with respect to some private aspect. Decisiveness over a pair (x, y) for individual i means that if xP_iy, then the social preference is xPy or, formulated in terms of choices, if xP_iy, then y will be eliminated from the choice set for society, i.e., $y \notin C(S)$. Do individuals really have the power to eliminate social states from further consideration? Except for some very special cases to which we will turn towards the end of this section, this exclusionary power does not correspond to our intuitive conception of what it means for an individual to have a right. Let us be more specific.

In Sen's as well as in Gibbard's formulation, individual rights are seen as restrictions on social choice. These constraints on social choice are linked to the individuals' preferences over some pairs of social states. Gaertner, Pattanaik and Suzumura (GPS for short) have proposed an alternative formulation of rights in terms of game forms. In their approach from 1992, individual rights are formulated by specifying the admissible strategies or actions of each player and the complete freedom of each player to pick any of the permissible strategies and the obligation not to choose a non–admissible strategy (for more details on this, see Fleurbaey and Gaertner (1996)). In the case of the two young ladies, choosing black or choosing white were both admissible strategies. There were no non–admissible strategies. Under the GPS proposal, the exercise of particular rights determines *particular features* of a social state.

An early forerunner of this idea was Nozick (1974) who argued that 'individual rights are co–possible; each person may exercise his rights as he chooses. The exercise of these rights fixes some features of the world. Within the constraints of these fixed features, a choice can be made by a social choice mechanism based on a social ordering, if there are any choices left to make!' (p. 166). Obviously, Nozick, unlike Sen, did not see individual rights linked to individual preferences as constraints on social choice. Under Nozick's conception, the individual's act of choice from among several available options fixes only some features of the social states. And it is this and not the individual's preferences over some pairs of social states that imposes a constraint on social choice.

We consider the following example. The reader will remember the conformist and the non–conformist from the proof of Theorem 4.3. Each woman has two dresses, a white one and a black one. Both women are completely ignorant of each other's preferences, and when each woman makes her choice (either white or black), she has absolutely no clue as to what the other will do. We assume again that all the other aspects which shape a social state have already been determined. For simplicity reasons, we disregard these aspects altogether. The preference orderings of the conformist (person 1) and the non–conformist (person 2) then are as before:

$$\begin{array}{llcll} 1: & ww & \quad & 2: & bw \\ & bb & & & wb \\ & bw & & & ww \\ & wb & & & bb \end{array}$$

According to Sen's condition of liberalism and the idea of an individual's recognized private sphere, at least one of the following cases (a) and (b) and also at least one of the cases (c) and (d) must be satisfied:

(a) $(ww, bw) \in D_1$ and $(bw, ww) \in D_1$;

(b) $(bb, wb) \in D_1$ and $(wb, bb) \in D_1$;

(c) $(bw, bb) \in D_2$ and $(bb, bw) \in D_2$;

(d) $(wb, ww) \in D_2$ and $(ww, wb) \in D_2$.

Given these personal spheres and given the potential decisiveness of the two women, Sen's formulation of individual rights as well as Gibbard's formulation run into several difficulties.

Difficulty 1. Suppose that case (a) holds. Let the two young ladies freely choose their dresses without knowing the other's preferences and her actual choice. GPS argue that given such ignorance, each woman may apply the 'maximin principle' (in this case, choosing that colour which avoids the worst outcome) and therefore, 1 picks b and 2 chooses w. Clearly, this result is inconsistent with Sen's condition of liberalism and person 1's decisiveness over the pair (ww, bw), for Sen's condition would declare bw as socially dispreferred. In the choice function formulation, bw would be eliminated from the set of chosen elements. Given that bw is the strategy combination of the two women's free choices of their respective dresses, 'very few people would be willing to say that there was a violation of the right of any individual' (1992, p. 165) to choose her dress. And GPS go on saying that 'the fact that each individual is free to choose ... without any external constraint seems to capture the entire intuitive content of our conception of the right under consideration' (p. 165).

The inconsistency that we have just derived would also hold for cases (b), (c) and (d). We would only have to change the individuals' preference orderings and then use the hypothesis of maximin behaviour in a similar way as above. Since Sen's formulation of rights demands that at least two individuals be decisive over at least one pair, there will necessarily be a violation of Sen–type rights even though, from our intuitive point of view, no individual's right has been infringed. Maximin behaviour makes a lot of sense in the situation considered, but other choice rules (maximax behaviour of a risk-prone individual, for example) would have generated the same kind of inconsistency if we had modified the individual orderings appropriately.

Difficulty 2. What happens when we allow a person to be decisive over both pairs? Sen requires decisiveness over at least one pair, so decisiveness over both pairs is not excluded by him. Gibbard's condition GL' demands decisiveness over every pair of i–variants. This type of power leads to an inconsistency that clashes even more with our intuition.

Take person 1 and suppose that she is decisive both over the pair (ww, bw) and the pair (bb, wb). Given her preferences as stated above, person 1 would be able to eliminate strategy combination bw as well as wb. But this is rather strange. In the current situation where each of the two women is choosing either a white dress or a black dress, not knowing what the other person will do, person 1, under our intuitive conception, can either choose white or black. By picking w, person 1 can secure that the strategy combinations bb and bw will be excluded. By choosing b, she can make sure that the strategy combinations ww and wb will be eliminated. But person 1 has no power to ensure that bw and wb will be excluded. How on earth could she achieve this? Yet this is exactly the power that person 1 has under cases (a) and (b), given Sen's formulation of liberalism (slightly extended, of course, by appropriating two pairs of states) and given Gibbard's condition GL'.

To be more concrete, let us look at our example of the conformist and the non–conformist again. Gibbard's preference–based scheme of rights–assignment is unintuitive. In most rights systems, there is no legal claim to match another person in terms of colour (or habit) nor is there a right to be different. By eliminating bw and wb, person 1 would exactly have this power. She would be able to exclude non–conformity. An analogous argument would have applied if we had allowed person 2 to be decisive over two pairs. In the latter case, person 2 would have had the power to exclude conformity, given her preferences. All this sounds extremely unintuitive.

Difficulty 3. What happens when we allow both persons to be decisive over two pairs each? It would be very unnatural indeed if we granted this possibility to one of the two persons but not to the other. In this situation where cases (a)– (d) are satisfied, the choice set will be empty under the stated preferences, as we know from Gibbard's Theorem 4.3. On the other hand, there will, of course, be a social outcome after all. This, however, is tantamount to saying that the decisiveness of one of the two persons must have been broken inevitably. Again, there is a clash between our intuition and the Sen–Gibbard claim of rights–exercising. We do not know which outcome will come about eventually. Sure enough, it will be disappointing for one of the two women. But this is perfectly understandable given the preferences of the two young ladies. Actually, the game–theoretic analysis shows, as pointed out above, that there is no equilibrium in pure strategies in such a conflict of conformity vs. non–conformity. There is, however, a mixed–strategy equilibrium (where each of the two women chooses each of her two actions with non–zero probability), and this seems to make a lot of sense in the given conflict.

Let us reiterate that the main difference between the Sen–Gibbard approach and the GPS approach is that in the former the constraints on social choice are linked to the individuals' preferences over pairs of social states whereas in the latter the individuals merely choose aspects of social states that fall within their recognized private sphere. It is only in rather special cases that the choice of an aspect of a social state is linked to exactly one state. Let us assume that only two strategy combinations are available, say ww and bw. This means that person 1 has the choice between aspects w and b while person 2 has no choice at all. For her, only a white dress is available. Let us further assume that this information is also known to person 1. Then, if person 1 prefers ww to bw, the choice of aspect w automatically means that bw has been eliminated. Therefore, in such a case there is a direct link between person 1's preference and a constraint on social choice. Person 1 has exclusionary power. This argument would also apply in a case where the second woman, let's say, has already picked her colour and woman 1 has been informed of this choice.

There is yet another constellation with a direct link between individual preferences and the elimination of complete social states. It is the case where both women have a dominant strategy. If person 1, let's say, always chooses w irrespective of what person 2 does, and the latter always picks b irrespective of what person 1 decides and if this information is common knowledge, three of the four strategy combinations will be directly excluded. For each person there is an 'as if' choice situation. Each woman is choosing her aspect as if the other woman had already fixed hers. Person 2 knows that since woman 1 will never choose b, strategy combinations bw and bb will not come about. Woman 1 knows that person 2 will never pick w so that combinations ww and, again, bw will not occur. These inferences are perfectly plausible within the GPS formulation.

GPS emphasize that the game form formulation while corresponding to our intuition about individual rights does not heal the conflict between the exercise of those rights and the requirement of Pareto efficiency and that this problem appears to persist 'under virtually every plausible concept of individual rights that we can think of' (1992, p. 161). Particular domain conditions of the individuals' preferences would be able to achieve the compatibility of Pareto efficiency and the requirement of individual rights over private issues, given some notion of equilibrium, but these domain restrictions would, of course, need justification (see Gaertner (1993), and Gaertner and Krüger (1981)). Deb et al. (1997) have analysed the conflict between individual decisiveness over personal spheres and the Pareto principle in a general way, viz. for different notions of equilibria in games. Their first proposition, for example, states a necessary and sufficient condition for a dominant strategy equilibrium to have an outcome that is not Pareto efficient (see chapter 4.4 above).

1.6 A short summary

Not all societal issues should be resolved by using the majority rule or some variant of the latter. My decision in an n–member society to worship the Christian God as everybody else does in this community or to worship Buddha instead should entirely be taken by myself. There should be a certain degree of autonomy in decisions, at least as long as these decisions do no create severe negative externalities for others.

Can this issue of local decisiveness be dealt with within the Arrovian set–up of collective choice? Sen's answer to this question and that of many other scholars was in the affirmative. Sen clothed his idea of a certain degree of decisional autonomy in his condition of liberalism, added the requirements of Pareto efficiency and unrestricted domain of individuals' preferences, and came up with a surprise, an impossibility result. This negative finding stimulated many scholars, among them Sen himself, to look for a way out. Over the last three decades, there have been literally hundreds of serious contributions to this problem, ranging from restricting individual preferences over constraining the Pareto principle to modifying the condition of liberalism or autonomy.

Most of these proposals for resolution are interesting in themselves but do they conform to our intuitive idea of what it means for an individual to have *ad personam* rights over certain private matters and exercise them? In Sen's formulation, individual rights are viewed as restrictions on social choice. Gaertner et al. have proposed an alternative formulation of rights in terms of game forms. In this approach, individual rights are formulated by specifying the admissible strategies of each actor and the complete freedom of each actor to choose any of the permissible strategies and the obligation not to pick a non–admissible strategy. This formulation appears to be closer to what we observe in real life. It does not mean, however, that the game form analysis by itself will heal the conflict between the exercise of personal rights and the requirement of Pareto efficiency.

Recommended Reading:

Gaertner, W., Pattanaik, P. K. and Suzumura , K. (1992). 'Individual Rights Revisited'. *Economica*, 59: 161–77.

Gibbard, A. (1974). 'A Pareto–Consistent Libertarian Claim'. *Journal of Economic Theory*, 7: 388–410.

Sen, A. K. (1970b). *Collective Choice and Social Welfare,* chapt. 6. San Francisco, Cambridge: Holden–Day.

Historical Sources:

Nozick, R. (1974). *Anarchy, State and Utopia,* chapt. 7. New York: Basic Books.

Sen, A K. (1970a). 'The Impossibility of a Paretian Liberal'. *The Journal of Political Economy*, 78: 152–57.

More Advanced:

Deb, R., Pattanaik, P. K. and Razzolini, L. (1997). 'Game Forms, Rights, and the Efficiency of Social Outcomes'. *Journal of Economic Theory*, 72: 74–95.

Peleg, B. (1998). 'Effectivity Functions, Game Forms, Games, and Rights'. *Social Choice and Welfare*, 15: 67–80.