Effects of Informal Family Care on Formal Health Care: Zero-Inflated Endogenous Count for Censored Response

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Whether informal family health care is a substitute or complement for formal health care has been debated in the literature. If it is a substitute, then there is a scope to reduce formal health care cost by promoting informal family health care. Using Korean survey data for the elderly of age 65 or higher, this paper estimates the effect of informal family health care on formal health care, where the former is measured by the number of family health care givers and the latter is measured by the (logarithm of) formal health care expenditure. This task, however, poses a number of difficulties. The first is that the number of the family care givers is an endogenous count regressor. The second is that there seem to be too many zeros in the count (85%). The third is that the response variable also has a non-trivial proportion of zeros (14%). This paper overcomes these problems by combining a semiparametric estimator for a censored response with the idea of “zero-inflated” counts. The resulting two-stage procedure avoids strong parametric assumptions and behaves well computationally. Our main empirical finding is that informal family health care has a large substitute effect for diabetics that is statistically significant and large in magnitude, but the other effects are statistically insignificant for our given data size of about 3000.

Key Words: informal health care, formal health care, count variable, zero-inflated Poisson, control function approach, censored model.

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1 Introduction

With low fertility rates prevailing in most developed countries, the populations age fast, and this entails a high demand for health care. If the health care cost is borne only by formal health care, then eventually there may be a point at which the health care system ceases to be sustainable. If formal health care can be replaced to some extent by informal family health care, then this may lead to a considerable reduction on the formal health care cost.

In the literature of health economics, there are studies that examined the effects of informal health care on formal health care, which often find that informal care substitutes for formal care. Although there are studies such as Charles and Sevak (2005) showing that informal care (measured by the dummy for any informal care) is a substitute for nursing home care (measured by the dummy for ever staying in nursing home), in the following, we briefly review three studies that are the most relevant to our paper: Van Houtven and Norton (2004), Bolin et al. (2008) and Bonsang (2009).

In Van Houtven and Norton (2004), informal care is the care hours provided by all children (their spouse and their children), and formal cares including nursing home care and outpatient care are of eight different types in total (mostly continuously distributed, but formal home care and outpatient surgery are binary). Only about 19% of the respondents received informal care. Van Houtven and Norton used U.S. data: 1998 Health and Retirement Survey (HRS) and 1995 Asset and Health Dynamics Among the Oldest-Old Panel Survey (AHEAD). Van Houtven and Norton found that informal care is mostly a substitute except for outpatient surgery.

In Bolin et al. (2008), nine formal care variables are used including formal home care, visits to doctors and hospitalization days. Their informal care (informal care hours from children and grandchildren) has the non-zero proportion ranging 19-40% across the countries in their 2004 European data “SHARE”. Bolin et al. found that informal care is a substitute for formal home care, but a complement to doctor and hospital visits, and that the effects vary depending on the region (i.e., informal care interacts with the region dummies).

In Bonsang (2009), informal care is the care hours provided by children of the respondent (a single-living elderly), and formal cares are paid domestic help (low-skilled) and nursing care (high-skilled); both formal cares are home cares. Using the 2004 European data SHARE, Bonsang (2009) found that informal care is a substitute for the low-skilled formal home care,
but a weak complement for the high-skilled formal home care, and that the substitution effect
decreases as the level of disability of the elderly person increases (i.e., informal care interacts
with the disability level).

In terms of methods, Van Houtven and Norton (2004), Bolin et al. (2008) and Bon-
sang (2009) used a ‘two-part approach’. But strictly speaking, the methods used there to
deal with endogenous regressors apply only when the endogenous regressors are continuously
distributed. Probably because of this restriction, least squares estimator (LSE) was used to
estimate the reduced form (RF) model for informal care that is an endogenous regressor for
formal care (the response variable). But the LSE is problematic because the informal care
variable includes too many zeros. Also, the response variable has a non-trivial proportion
of zeros. In short, both the main endogenous regressor of interest and the response vari-
able are not continuously distributed to allow linear models, but either discrete or mixed
(discrete/continuous).

One reason for the endogeneity of informal care is that both formal and informal cares
may be determined simultaneously. Another reason is that both cares may share common
factors—most notably, health status. But controlling for health status is troublesome, be-
because it may be influenced by both cares. Note that, as instruments for informal care,
distances to children, placement of daughters in the birth order, or the number of (female)
children have been used in the literature.

While there is no particularly good solution for the endogeneity problem, this paper
will show a two-stage procedure to overcome the problems of too many zeros in a non-
negative endogenous regressor (informal care) and a non-trivial proportion of zeros in the
response variable (formal care). For the RF estimation of the non-negative regressor, we
will be using ‘Quasi Poisson’ approach, and for too many zeros, we will be using the zero-
inflated Poisson idea of Lambert (1992). In a nutshell, our two-stage procedure is applicable
to censored models with non-negative endogenous regressors including count variables where
the endogenous regressors have too many zeros.

The rest of this paper is organized as follows. Section 2 shows the details of the two-stage
procedure. Section 3 applies the estimator to Korean data to estimate the effect of informal
care on formal care, where informal care is the number of care givers (thus a count). Finally,
Section 4 concludes. A word on notation before proceeding further: ‘a II b|c’ denotes the
independence between a and b given c.
2 Two-Stage Procedure

2.1 Model Assumptions

Suppose that \( y_1 \geq 0 \) is formal care, \( y_2 \geq 0 \) is informal care (a count), \( x_1 \) is a \( k_1 \times 1 \) exogenous regressor vector for the \( y_1 \) structural form (SF) equation, and \( x \) is the \( k \times 1 \) system exogenous regressor vector for \((y_1, y_2)\) that strictly includes \( x_1 \). That is, only \( x_1 \) in \( x \) affects \( y_1 \) “directly”, and \( x \) is the collection of the exogenous regressors affecting either \( y_1 \) or \( y_2 \).

Observed are \((x_i, y_{1i}, y_{2i}), \ i = 1, \ldots, N,\) which are iid across \( i \).

Our approach below applies not only to a count, also to a non-negative \( y_2 \). In view of the iid assumption, we will often omit the subscript \( i \).

Assume that the observed \( y_1 \) and \( y_2 \) are generated from its latent versions \( y_1^* \) and \( y_2^* \) as follows: for unknown parameters \( \gamma_y, \gamma_x, \alpha \) and \( \beta \), an error term \( u_i \) and a binary variable \( q_i \),

\[
\begin{align*}
y_{1i} &= \max(0, y_{1i}^*) \quad \text{with} \quad y_{1i}^* = \gamma_y y_{2i} + x_{1i}' \gamma_x + u_i \quad \text{and} \quad u|x \text{ is symmetric around } 0; \\
y_{2i} &= q_i y_{2i}^*, \quad P(q = 1|x_i) = \frac{\exp(x_i' \alpha)}{1 + \exp(x_i' \alpha)} \quad \text{and} \quad E(y_{2i}^*|q = 1, x_i) = \exp(x_i' \beta).
\end{align*}
\]

Here \( y_{1i}^* \) is modelled as censored at zero with its error term symmetric around zero. This symmetry assumption is to use symmetrically censored least squares estimator (SCL) of Powell (1986), and may be replaced by another semiparametric assumption if a different semiparametric censored model estimator as in Powell (1984) or Lee (1992) is used.

Since \( x \) appears for \( q \) and \( y_{2i}^* \), the \( q \) and \( y_{2i}^* \) equations should be regarded as a RF. This RF view is necessary because \( y_1 \) does not appear for the \( q \) and \( y_{2i}^* \) equations, and also because \( E(y_{2i}^*|q = 1, x) = \exp(x' \beta) \) is adopted, not the more “structural” \( E(y_{2i}^*|x) = \exp(x' \beta) \). There are two views on RF’s as noted in Lee (2012). One view is that there is a SF for \( y_2 \) with \( y_1 \) and “\( x_2 \)” as the regressors, and substituting the \( y_1 \) SF and then solving the equation for \( y_2 \) yields the \( y_2 \) RF with \( x \) on the right hand side. The problem with this view is that it is not clear whether the equation is solvable for \( y_2 \) or not, and if so, whether the solution is unique (and stable). The other view on RF is to take \( E(y_2|x) \) as the \( y_2 \) RF, and use a parametric function for \( E(y_2|x) \) as an approximation if desired. The problem with this view is that no information/structure can be imposed on \( E(y_2|x) \) and the parametric form may be ad hoc.

Some further remarks about the model are in order. First, a sample selection model holds for \( y_{2i}^* \) because \( y_{2i}^* \) is observed only when \( q = 1 \); the binary ‘selection variable’ \( q \) is assumed
to follow the logit model whereas \( y_2^* \) given \( q = 1 \) is posited to have an exponential regression function. Second, the key implication of the selection model for \( y_2 \) is

\[
E(y_2|x) = P(q = 1|x)E(y_2^*|q = 1, x) = \frac{\exp(x'\alpha)}{1 + \exp(x'\alpha)} \exp(x'\beta).
\]

Third, the expression ‘too many zeros” may be formally defined as

\[
E[(y_2 - \frac{\exp(x'\alpha)}{1 + \exp(x'\alpha)} \exp(x'\beta))^2] < E[(y_2 - \exp(x'\beta))^2];
\]

i.e., the logit function improving the \( \exp(x'\beta) \) prediction of \( y_2 \) is defined as “too many zeros in \( y_2 \)”. Fourth, it may be better to model \( y_1 \) also as a sample selection model rather than as the censored model which is a special case of selection model, but the censored model is adopted for simplicity because dealing with a sample selection model is difficult—this would not matter much though as the proportion of zeros is low for \( y_1 \) in our data (14%).

Define \( 1[A] = 1 \) if \( A \) holds and 0 otherwise, and call \( y_2^* = 0 \) ‘participation zero’. As done in Lee (2011), it is helpful to compare three different models for \( q \) in relation to the participation zero possibility:

Model 1 : \( q = 1[y_2^* > 0] \) where \( y_2 = qy_2^* = 0 \) implies \( y_2^* = 0 \);

Model 2 : \( q \) determined by some variables (and \( y_2^* \)) with participation 0 possible;

Model 3 : \( q \) determined by some variables (and \( y_2^* \)) with participation 0 impossible.

Model 1 is the ‘corner solution model’ in which case \( y_2 \) becomes also a zero-censored model as \( y_1 \) is. Model 2 is relevant if \( q = 1 \) is only an “attempt/try” for an activity and \( y_2^* \) is a “performance” in the activity following the attempt/try. Model 3 is relevant if \( q = 1 \) is having the actual activity and \( y_2^* \) is the degree of the activity with zero ruled out.

For instance, \( q = 1 \) may be an attempt/try to export, where \( y_2^* = 0 \) is possible even if one tries (\( q = 1 \)). Instead of attempt/try, one may define \( q = 1 \) as actually exporting and \( y_2^* \) as the actual export volume that cannot be zero. Which one between Models 2 and 3 to adopt may depend on what is available in the data. If a variable \( q \) for ‘whether one desires to export or not’ is available in the data along with the export volume including zero, then \( y_2 = qy_2^* \) is the observed export volume with \( y_2^* = 0 \) possible. If only the actual export volume including zero without such a variable for \( q \) is available, then one has no choice but to set \( q = 1[y_2 > 0] \) with participation zero ruled out. In our data, since there is no separate variable for \( q \), we will set \( q = 1[y_2 > 0] \) to adopt Model 3.
One may wonder ‘why not adopt Model 1 that looks simpler than Model 3’. The answer is that there is really no difference between Model 1 and Model 3 for our empirical analysis. To see the point, suppose $y_2^* = x'\alpha + v_2$ with $v_2$ being logistic independently of $x$ and Model 1 holds. Then

$$q = 1[y_2^* > 0] = 1[x'\alpha + v_2 > 0] \implies E(q|x) = \frac{\exp(x'\alpha)}{1 + \exp(x'\alpha)} \quad \text{and}$$
$$E(y_2^*|q = 1, x) = E(y_2^*|y_2^* > 0, x) = x'\alpha + E(v_2|v_2 > -x'\alpha, x) \neq \exp(x'\alpha).$$

In this case, the exponential model is only an approximation for $x'\alpha + E(v_2|v_2 > -x'\alpha, x)$, and consequently we need to allow different parameters $\alpha$ for $E(q|x)$ and $\beta$ for $E(y_2^*|q = 1, x)$ as when Model 3 is adopted.

### 2.2 First Stage To Obtain Control Function

In our two-stage procedure, the first stage consists of two parts: estimating $\alpha$ in the logit model for $E(q|x)$ and estimating $\beta$ in the exponential model for $E(y_2^*|q = 1, x)$. For the latter, one can use Quasi-Poisson (QPOI) maximum likelihood estimator (MLE): maximize the usual Poisson likelihood function with $q = 1$ attached to use the “sandwich-form” asymptotic variance. That is, the QPOI maximand is

$$\frac{1}{N} \sum_i q_i \{y_2 x'_i b - \exp(x_i b)\}$$

and the asymptotic variance matrix is

$$E^{-1}\{q xx' \exp(x'\beta)\} \cdot E[q xx' \{y - \exp(x'\beta)\}^2] \cdot E^{-1}\{q xx' \exp(x'\beta)\}.$$ 

Denoting the first-stage estimators as $\hat{\alpha}$ and $\hat{\beta}$, the second-stage is estimating $\gamma_y$ and $\gamma_x$ for the $y_1$ SF allowing for the endogeneity of $y_2$ in the $y_1$ SF. As reviewed in Lee (2012), there are several different methods to deal with an endogenous regressor in a limited dependent variable (LDV) model—the LDV model is the zero-censored model for $y_1$ in our case. Among those methods, the most convenient for our empirical analysis is ‘control function (CF)’ approach, because many interaction terms between $y_2$ and elements of $x$ will be allowed. With the endogeneity of $y_2$ removed by a CF, we can freely allow such interaction terms, which is complicated in the other approaches for the $y_2$ endogeneity. Specifically, a residual $\hat{v}_2$ for $y_2$ is obtained from the first stage, and it is used as an extra regressor in the $y_1$ SF. Not just $\hat{v}_2$, but also $\hat{v}_2^2$ and $\hat{v}_2^3$ can be used if including those terms removes the $y_2$ endogeneity.
better by accounting for the additive part of \( u \) that depends on \( v_2 \). Then \((\hat{v}_2, \hat{v}_2^2, \hat{v}_2^3)\) becomes the CF, and the \( y_2 \) endogeneity can be tested by looking at whether their coefficients are all zero or not. Going further than \((\hat{v}_2, \hat{v}_2^2, \hat{v}_2^3)\), higher order terms or interaction terms between \( \hat{v}_2 \) may be used as well.

For an LDV regressor such as \( y_2 \), it is not obvious which form of residual will be the best choice for CF. For a count regressor, there is no “natural” residual. To motivate our approach to this, consider generating a Poisson regressor \( y \) with parameter \( \exp(x'\xi + \varepsilon) \) where \( \varepsilon \) with \( \varepsilon \text{ II } x \) is related to \( u \) so that \( y \) becomes endogenous for \( y_1 \); e.g., \( u \) consists of \( \varepsilon \) and an additive error. To generate \( y \), many exponential random durations with the same parameter \( \exp(x'\xi + \varepsilon) \) should be generated first. Then the number of the exponential durations that can be fit into the unitary time interval is the desired \( y \)—after this, \( y_1 \) can be generated using \((x \text{ and } y) \) and \( u \) that depend on \( \varepsilon \). For the endogenous \( y \), at least the following two types of residuals can be thought of.

The ‘additive residual’ for \( y \) is \( y - \exp(x'\xi) \), from which it follows that

\[
E\{y - \exp(x'\xi) \mid x\} = E\{E\{y - \exp(x'\xi) \mid \varepsilon, x\} \mid x\} = E\{\exp(x'\xi)\varepsilon - \exp(x'\xi) \mid x\} = E[\exp(x'\xi) \cdot (\varepsilon^e - 1) \mid x] = 0
\]

which holds by rescaling \( \varepsilon \) such that \( e^e = 1 \) and including the constant scale factor in the intercept of \( x'\xi \). That is, using \( y - \exp(x'\xi) \) amounts to using \( \exp(x'\xi)(e^e - 1) \) as a CF in the \( y_1 \) SF. If \( \varepsilon \) is small, then \( \exp(x'\xi)(e^e - 1) \approx \exp(x'\xi)\varepsilon \). A better choice than the additive residual might be the multiplicative residual \( y\exp(-x'\xi) - 1 \), which leads to

\[
E\{y\exp(-x'\xi) - 1 \mid x\} = E[ E\{y_2\exp(-x'\xi) - 1 \mid \varepsilon, x\} \mid x] = E(e^e - 1\mid x) = 0.
\]

Hence, using \( y_2 \exp(-x'\xi) - 1 \) is analogous to using \( e^e - 1 \) as a CF in the \( y_1 \) SF. If \( \varepsilon \) is small, then \( e^e - 1 \approx \varepsilon \).

The main difference between the two residuals is that the additive residual carries the heteroskedasticity factor \( \exp(x'\xi) \) while the multiplicative residual does not. For \( y_2 = qy_2^* \), the two residuals are, respectively,

\[
y_2 - \frac{\exp(x'\alpha)}{1 + \exp(x'\alpha)}\exp(x'\beta) \quad \text{and} \quad y_2\{\frac{\exp(x'\alpha)}{1 + \exp(x'\alpha)}\exp(x'\beta)\}^{-1} - 1.
\]

For our empirical analysis, we will try both residuals, because which is better will be determined ultimately by how much endogeneity can be picked up by each type of residual; the more the better.
Since SCL in the second stage needs only the symmetry of \(u|x\), the only parametric assumption invoked in our two-stage procedure is the logit in the first-stage. Since there is no practical semiparametric estimator for binary responses, assuming logit does not seem so restrictive. If we desire to avoid even the logit assumption, then we may assume simply

\[ E(y_2|x) = \exp(x'\beta). \]

This will be also applied to our data later, and as it turns out, its performance is inferior to our two-stage procedure allowing for “zero inflation”.

### 2.3 Second Stage with Symmetrically Censored LSE (SCL)

In our two-stage procedure, the second-stage is SCL with a CF used as an extra regressor to remove the \(y_2\) endogeneity. Here we explain SCL first, pretending that \(y_2\) is exogenous for a while. To simplify notations, define

\[ w \equiv (y_2, x'_1)' \quad \text{and} \quad \gamma \equiv (\gamma_y, \gamma_x'). \]

to get \(y_{1i} = \max(0, w'_i\gamma + u_i)\).

Observe

\[ w'\gamma + u \geq 0 \iff u \geq -w'\gamma. \]

If \(w'\gamma > 0\), then the censoring of \(y_1\) at zero replaces the lower tail of \(u\) with a “mass” \(-w'\gamma\). The idea of SCL is to artificially replace the upper tail with \(w'\gamma\) to restore the symmetry of \(u\). This leads to a moment condition:

\[ E\{ 1[w'_\gamma > 0] \cdot (1[|u| < w'\gamma]|u + w'\gamma 1[|u| \geq w'\gamma]) \cdot w \} = 0. \]

A minimand with the moment condition as its asymptotic first order condition is

\[ \frac{1}{N} \sum_i \left[ \left( y_{1i} - \max(0.5y_{1i}, w'_i\gamma) \right)^2 + 1[y_{1i} > 2w'_i\gamma] \cdot \left( (0.5y_{1i})^2 - (\max(0, w'_i\gamma))^2 \right) \right] \]

and SCL is obtained by minimizing this for \(\gamma\).

If \(w'_i\gamma \simeq \infty \forall i\), then the minimand becomes the LSE minimand \(N^{-1} \sum_i (y_{1i} - w'_i\gamma)^2\); in fact, what is needed is only \(u > -w'\gamma (-w'\gamma\) being smaller than the lower support boundary of \(u|w\) for which \(w'\gamma \simeq \infty\) is sufficient. The second-order (Hessian) matrix of SCL is

\[ H \equiv E(1[|u| < w'\gamma]ww') \]
which becomes $E(ww')$ that is the second-order matrix of LSE when $|u| < w'\gamma$ always (implied by $w'\gamma \simeq \infty$). If the censoring proportion becomes small, then SCL becomes close to LSE, and in this sense, SCL is a natural estimator for a censored response with a small censoring proportion. The main advantage of SCL over MLE’s for the censored model is that SCL does not specify the distribution of $u|w$ and allows an unknown form of heteroskedasticity because the above moment condition does not require $u II w$.

Powell (1986) suggested an iterative scheme to get $\hat{\gamma}$. Start with an initial estimate $\hat{\gamma}_0$, say LSE, and then iterate the following until convergence:

$$\hat{\gamma} = \left( \sum_i 1[w_i'\hat{\gamma}_0 > 0] \cdot w_i w_i' \right)^{-1} \sum_i \{1[w_i'\hat{\gamma}_0 > 0] \min(y_{1i}, 2w_i'\hat{\gamma}_0) \cdot w_i \}.$$

This does not guarantee global convergence. Also the matrix to be inverted may not be invertible. If this problem occurs, then removing $1[w_i'\hat{\gamma}_0 > 0]$ in the inverted matrix may help. From our experience, however, this algorithm works well.

Going back to the case with endogenous $y_2$, let $v_2$ be either the additive or multiplicative residual from the $y_2$ RF. Then the second stage in our two-stage procedure is SCL with $w$ augmented by the CF $v_2$ (and $\hat{v}_2^2$ and $\hat{v}_3^2$). With the endogeneity of $y_2$ removed by the CF, SCL can be implement as above. The only modification needed is the asymptotic variance of SCL because the first stage estimation errors $\hat{\alpha} - \alpha$ and $\hat{\beta} - \beta$ affect the SCL asymptotic variance through $\hat{v}_2$, which is to be examined in detail in the following subsection. Our two-stage procedure works well computationally, because all estimators involved (logit, QPOI and SCL) converge well. This computational advantage should not be downplayed as it matters greatly in practice.

### 2.4 Asymptotic Distribution

With $w$ exogenous for $y_1$, the first- and second-order derivatives of the SCL minimand give the following asymptotic linear expansion of SCL:

$$\sqrt{N}(\hat{\gamma} - \gamma) = \frac{1}{\sqrt{N}} \sum_i H^{-1} \cdot 1[w_i'\gamma > 0] \min(1|u| < w_i'\gamma|u| + w_i'\gamma1|u| \geq w_i'\gamma)w_i + o_p(1)$$

$$= \frac{1}{\sqrt{N}} \sum_i H^{-1} \zeta_i + o_p(1), \quad \text{where} \quad \zeta_i \equiv 1[w_i'\gamma > 0] \min(1|u| < w_i'\gamma|u| + w_i'\gamma1|u| \geq w_i'\gamma)w_i.$$

From this, it follows that, with ‘$\sim$’ denoting convergence in law,

$$\sqrt{N}(\hat{\gamma} - \gamma) \sim N(0, H^{-1}E(\zeta\zeta')H^{-1}) \quad \text{where} \quad E(\zeta\zeta') = E\{1[w'\gamma > 0] \min(u^2, (w'\gamma)^2) \cdot ww'\}. $$
As already mentioned, the first-stage estimation errors $\hat{\alpha} - \alpha$ and $\hat{\beta} - \beta$ affect the SCL asymptotic variance through $\hat{v}_2$, which is discussed now.

Redefine $w$ and $\gamma$ as

$$w = (y_2, x_1', \hat{v}_2, \hat{v}_2^2, \hat{v}_3^2)'$$

and

$$\gamma = (\gamma_y, \gamma_x, \gamma_1, \gamma_2, \gamma_3)'$$

where $\hat{v}_2 = \hat{v}_2(\hat{\alpha}, \hat{\beta})$ that depends on $\hat{\alpha}$ and $\hat{\beta}$ is either the additive or multiplicative residual, and $(\gamma_1, \gamma_2, \gamma_3)$ is the coefficient vector for $(\hat{v}_2, \hat{v}_2^2, \hat{v}_3^2)$.

The presence of the first-stage estimators $\hat{\alpha}$ and $\hat{\beta}$ matters for the ‘gradient vector’ $\zeta$ in the above linear expansion of SCL, but not for the second-order matrix $H$. Hence write the asymptotic linear expansion as

$$\sqrt{N}(\hat{\gamma} - \gamma) = \frac{1}{\sqrt{N}} \sum_i H^{-1} \zeta_i(\alpha, \beta) + o_p(1)$$

$$= \frac{1}{\sqrt{N}} \sum_i H^{-1} \{\zeta_i(\alpha, \beta) + E(\zeta_{\alpha'}) \eta_{\alpha i} + E(\zeta_{\beta'}) \eta_{\beta i}\} + o_p(1)$$

where $\zeta_{\alpha'}$ and $\zeta_{\beta'}$ denote the derivatives of $\zeta(\alpha, \beta)$ for $\alpha$ and $\beta$, respectively, and $\eta_{\alpha i}$ and $\eta_{\beta i}$ are ‘influence functions’ for $\hat{\alpha}$ and $\hat{\beta}$:

$$\eta_{\alpha i} = \{E(ss^t)\}^{-1}s_i \text{ for logit score function } s_i = \{y_{2i} - \frac{\exp(x_i^t\alpha)}{1 + \exp(x_i^t\alpha)}\}x_i,$$

$$\eta_{\beta i} = [E\{qx \exp(x'\beta)\}]^{-1} q_i x_i \{y_{2i} - \exp(x_i^t\beta)\}.$$

Since the dimension of $\gamma$ is $(k_1 + 4) \times 1$ and the dimension of $\alpha$ and $\beta$ are both $k \times 1$, $\zeta_{\alpha'}$ and $\zeta_{\beta'}$ are $(k_1 + 4) \times k$ matrices, which can be obtained by numerical differentiation. See, e.g., Lee (2010) for more details on this way of accounting for the first-stage estimation errors.

From the asymptotic linear expansion taking into account $\hat{\alpha} - \alpha$ and $\hat{\beta} - \beta$, we get

$$\sqrt{N}(\hat{\gamma} - \gamma) \sim N(0, H^{-1}E(\lambda \lambda')H^{-1}) \text{ where } \lambda_i \equiv \zeta_i(\alpha, \beta) + E(\zeta_{\alpha'}) \eta_{\alpha i} + E(\zeta_{\beta'}) \lambda_i.$$

$E(\lambda \lambda')$ can be estimated consistently by replacing $(\alpha, \beta, \gamma)$ with $(\hat{\alpha}, \hat{\beta}, \hat{\gamma})$ and the expected values in $\lambda$ by the corresponding sample means. If $E(y_2|x) = \exp(x'\beta)$ is adopted, then the only required change is redefining $v_2$ without the logit probability and then removing $E(\zeta_{\alpha'}) \eta_{\alpha i}$ in $\lambda$. The endogeneity of $y_2$ can be tested using $(\hat{\gamma}_1, \hat{\gamma}_2, \hat{\gamma}_3)$, as their coefficients should be all zero under the null of $y_2$ exogeneity. Although we toiled to account for the first-stage estimation errors $\hat{\alpha} - \alpha$ and $\hat{\beta} - \beta$, they can be ignored for SCL under the null of $y_2$ exogeneity.
2.5 Details on Control Function

In practice, it may be enough for a CF to carry a significant estimate, and thus the results under $y_2$ exogeneity assumption differ much from those allowing $y_2$ endogeneity. But it would be more desirable to know what the CF looks like “underneath” and to justify it properly. Here we take a detailed look at the CF’s under more assumptions.

For an error $\varepsilon$ related to $u$ and a parameter vector $\tilde{\beta}$, make an extra assumption

$$E(y_2^*|q = 1, x, \varepsilon) = \exp(x'\tilde{\beta} + \varepsilon) \quad \text{and} \quad \varepsilon \sim \Pi(x, q).$$

This implies our earlier model assumptions

$$E(q = 1|x, \varepsilon) = P(q = 1|x) = \frac{\exp(x'\alpha)}{1 + \exp(x'\alpha)},$$

$$E(y_2^*|q = 1, x) = \int E(y_2^*|q = 1, x, \varepsilon) f(\varepsilon|x, q = 1)d\varepsilon = \exp(x'\tilde{\beta}) \int e^{\varepsilon}f(\varepsilon)d\varepsilon$$

$$= \exp(x'\tilde{\beta}) \exp\{\ln E(\varepsilon^2)\} = \exp(x'\tilde{\beta} + \ln E(\varepsilon^2)) = \exp(x'\tilde{\beta})$$

where $f(\varepsilon|x, q = 1)$ denotes the density of $\varepsilon|(x, = 1)$ and $\beta$ differs from $\tilde{\beta}$ only in that the intercept in $\beta$ equals the intercept in $\tilde{\beta}$ plus $\ln E(\varepsilon^2)$.

The reason for the extra assumption on $E(y_2^*|q = 1, x, \varepsilon)$ can be seen in

$$E\{y_2 - \frac{\exp(x'\alpha)}{1 + \exp(x'\alpha)} \exp(x'\beta) | x\} = E\{y_2 - \frac{\exp(x'\alpha)}{1 + \exp(x'\alpha)} \exp(x'\tilde{\beta}) | x\}$$

$$= E[\frac{\exp(x'\alpha)}{1 + \exp(x'\alpha)} \exp(x'\tilde{\beta})e^{\varepsilon} - \frac{\exp(x'\alpha)}{1 + \exp(x'\alpha)} \exp(x'\tilde{\beta}) | x]\}$$

$$= E[\frac{\exp(x'\alpha)}{1 + \exp(x'\alpha)} \exp(x'\tilde{\beta})E(\varepsilon^2) \frac{e^{\varepsilon}}{E(\varepsilon^2)} - \frac{\exp(x'\alpha)}{1 + \exp(x'\alpha)} \exp(x'\tilde{\beta}) | x]\}$$

$$= E[\frac{\exp(x'\alpha)}{1 + \exp(x'\alpha)} \exp(x'\tilde{\beta}) \cdot \frac{e^{\varepsilon}}{E(\varepsilon^2)} - 1 | x] = 0.$$

That is, using the additive residual CF amounts to using

$$\frac{\exp(x'\alpha)}{1 + \exp(x'\alpha)} \exp(x'\tilde{\beta}) \cdot \{\frac{e^{\varepsilon}}{E(\varepsilon^2)} - 1\} \approx \frac{\exp(x'\alpha)}{1 + \exp(x'\alpha)} \exp(x'\tilde{\beta})\varepsilon \quad \text{if} \varepsilon \text{ is small}.$$ 

Analogously, using the multiplicative residual CF amounts to using $\{e^{\varepsilon}/E(\varepsilon^2)\} - 1 \approx \varepsilon \text{ if} \varepsilon \text{ is small}.$

In the above extra assumption, since we need to have $y_2$ exogenous once $\varepsilon$ is controlled, the relation of $\varepsilon$ to $u$ should be the only source for the $y_2$ endogeneity. A natural question to arise is how restrictive the assumption ‘$\varepsilon \sim \Pi(x, q)$’ is. Literally, it is restrictive in requiring that the $y_2$ endogeneity source $\varepsilon$ be independent of the selection equation $q$ as well as of
x. But "ε II (x, q)" does not necessarily imply "y^*_2 II q|x" that the 'outcome equation' y^*_2 and the selection equation q are independent given x—an assumption often invoked in practice—because y^*_2 has randomness sources other than ε. To see this point, think of generating an uniform random variable to use it (along with (x, ε)) to generate both y^*_2 and q; through the same uniform random variable, q and y^*_2 become related despite ε II (x, q).

2.6 Two-Part Approach in the Literature

It is helpful to compare our two-stage procedure to the two-part approach in the literature. The two-part approach assumed

first part : 1[y^*_1 > 0] = 1[γ_y y_2 + x'_1 γ_x + u > 0] and y_2 = x'δ + v
second part : y_1 = ξ_y y_2 + x'_1 ξ_x + e_i given y_1 > 0

where δ and ξ are parameters, and v and e are error terms.

For the first part, substitute y_2 = x'δ + v to obtain

1[y^*_1 > 0] = 1[γ_y(x'δ + v) + x'Sγ_x + u > 0] = 1[x'ψ + γ_y v + u > 0]

where ψ ≡ γ_yδ + Sγ_x and S consists of 0’s and 1’s such that x'_1 = x'S;

ψ is the RF parameters for 1[y^*_1 > 0] while (γ_y, γ_x) is the SF parameters. For the endogeneity of y_2 in the first part, a CF approach combined with minimum distance estimator (MDE) was used: the LSE residual ˆv for the y_2 equation is used along with x to obtain (ˆψ, ˆγ_y), and then (δ, γ_x) is estimated by MDE using ˆψ ≃ ˆγ_yδ + Sγ_x—simply imagine LSE of ˆψ on (ˆγ_y, S) to estimate (δ, γ_x).

Some remarks on the two-part approach are in order. First, (γ_y, γ_x) can be estimated in the 1[y^*_1 > 0] SF with ˆv controlled; no MDE is needed. Second, the linear model for y_2 is not plausible as y_2 has many zeros. Third, the second part of the two-part approach has been "sold" (relative to sample selection models) for a better prediction of y_1; hence the second part is not suitable to allow for endogenous regressors.

3 Empirical Analysis

Our data was drawn from the elderly of age 65 or above in ‘the Korean Longitudinal Study of Ageing’ for the year 2008. The information on the variables can be found in Table 1.
In Table 1, ‘formal’ is the annual medical and long-term care expenditure in about $1000—the other amounts in the table are all annual amounts in the same unit. The number of care givers is our informal family care variable, 85% of which are zeros. Table 1 also shows yearly informal care hours (‘care hours’) of which 85% are zeros again, but this variable will not be used for $y_2$—the estimation results with care hours as $y_2$ is mostly insignificant with no endogeneity of $y_2$ picked up by the CF’s.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean (SD)</th>
<th>Min,Max</th>
<th>Variable</th>
<th>Mean (SD)</th>
<th>Min,Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>formal ($1,000)</td>
<td>1.179 (2.34)</td>
<td>0, 48.4</td>
<td>age</td>
<td>74.6 (6.12)</td>
<td>65, 107</td>
</tr>
<tr>
<td># care givers</td>
<td>0.215 (0.58)</td>
<td>0, 4</td>
<td>male</td>
<td>0.425 (0.494)</td>
<td>0, 1</td>
</tr>
<tr>
<td>care hours</td>
<td>157 (619)</td>
<td>0, 8760</td>
<td>married</td>
<td>0.636 (0.481)</td>
<td>0, 1</td>
</tr>
<tr>
<td>fi. asset ($1,000)</td>
<td>4.88 (21.6)</td>
<td>0, 500</td>
<td>Seoul</td>
<td>0.137 (0.343)</td>
<td>0, 1</td>
</tr>
<tr>
<td>real est. ($1,000)</td>
<td>152 (222)</td>
<td>0, 2948</td>
<td>work</td>
<td>0.213 (0.409)</td>
<td>0, 1</td>
</tr>
<tr>
<td>own house</td>
<td>0.409 (0.49)</td>
<td>0, 1</td>
<td>kid-par ($1,000)</td>
<td>13.5 (28.2)</td>
<td>0, 866</td>
</tr>
<tr>
<td>fam.inc. ($1,000)</td>
<td>16.3 (21.0)</td>
<td>0, 700</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>pension ($1,000)</td>
<td>1.42 (4.44)</td>
<td>0, 94.9</td>
<td>nkids</td>
<td>3.99 (1.61)</td>
<td>0, 10</td>
</tr>
<tr>
<td>hi.bl. pressure</td>
<td>0.091 (0.288)</td>
<td>0, 1</td>
<td>nfem.kids</td>
<td>1.92 (1.40)</td>
<td>0, 8</td>
</tr>
<tr>
<td>diabetes</td>
<td>0.048 (0.215)</td>
<td>0, 1</td>
<td>nkids-co</td>
<td>0.412 (0.56)</td>
<td>0, 3</td>
</tr>
<tr>
<td>cancer/tumor</td>
<td>0.013 (0.114)</td>
<td>0, 1</td>
<td>nfem.kids-co</td>
<td>0.092 (0.30)</td>
<td>0, 3</td>
</tr>
<tr>
<td>chronic pulmo.</td>
<td>0.016 (0.127)</td>
<td>0, 1</td>
<td>nkids-act</td>
<td>2.61 (1.41)</td>
<td>0, 8</td>
</tr>
<tr>
<td>chronic liver</td>
<td>0.005 (0.073)</td>
<td>0, 1</td>
<td>nkids-act</td>
<td>0.765 (0.97)</td>
<td>0, 7</td>
</tr>
<tr>
<td>cardio disease</td>
<td>0.035 (0.183)</td>
<td>0, 1</td>
<td>nkids-30</td>
<td>0.597 (0.99)</td>
<td>0, 6</td>
</tr>
<tr>
<td>cerebral bl.vessel</td>
<td>0.038 (0.191)</td>
<td>0, 1</td>
<td>nkids-60</td>
<td>0.838 (1.18)</td>
<td>0, 6</td>
</tr>
<tr>
<td>mental disease</td>
<td>0.016 (0.125)</td>
<td>0, 1</td>
<td>nkids-120</td>
<td>0.768 (1.22)</td>
<td>0, 9</td>
</tr>
<tr>
<td>arthritis/rheuma.</td>
<td>0.195 (0.396)</td>
<td>0, 1</td>
<td># generations</td>
<td>1.48 (1.06)</td>
<td>0, 4</td>
</tr>
</tbody>
</table>

‘fi. asset’ is financial asset amount, and ‘real est.’ is real asset amount. ‘own house’ is the dummy for owning a house. ‘fam.inc.’ is household income, and pension is pension and other welfare receipt amount. ‘hi.bl. pressure’ is the dummy for high blood pressure. ‘cancer/tumor’ is the dummy for cancer or malign tumor. ‘chronic pulmo.’ is the dummy for chronic pulmonary disease. ‘chronic liver’ is the dummy for chronic liver disease. ‘cerebral bl.vessel’ is the dummy for cerebral blood vessel disease. ‘arthritis/rheuma.’ is the dummy
for arthritis or rheumatism. ‘male’ is the male dummy, ‘Seoul’ is the dummy for living in Seoul, and ‘work’ is the dummy for working. ‘kid-par’ is the transfer amount from children to the parents. ‘nkids’ is the number of children and ‘nfem.kids’ is the number of female children. ‘nkids-co’ is the number of children cohabiting with the respondent, and ‘nkids-act’ is the number of children economically active. ‘nkids-30’ is the number of non-cohabiting children living in 1-30 minutes’ distance by public transportation; nkids-60 and nkids-120 are analogously defined for 31-60 minutes and 61-120 minutes, respectively. ‘# generations’ is the number of generations living together.

To avoid extreme values in the amount variables, all amount variables are transformed with $\ln(\cdot + 1)$ so that 0 remains 0 after the transformation and positive values remain positive. Other than the variables in Table 1, self-reported health status is also available in five categories. But when health status was used for estimation, its coefficient was significantly positive, implying that health status is likely to be affected by formal/informal care, and thus it cannot be used as a regressor. Although the children-related variables can be used as instruments (IV) for $y_2$, there is no good IV for health status. Hence health status is dropped from the regressor list. By omitting health status, the $y_2$ endogeneity becomes more likely.

To appreciate the consequence of omitting health status, consider a linear model for positive health status $h$ and a linear $y_1$ SF with $h$ explicit:

$$h = \theta_1 y_1 + \theta_2 y_2 + \theta'_x x + \kappa \quad (\theta_1, \theta_2 > 0) \quad \text{and} \quad y_1 = \gamma_h h + \gamma_y y_2 + x'_1 \gamma_x + u \quad (\gamma_h < 0)$$

where $\theta$'s are parameters and $\kappa$ is an error term; ‘$\theta_1, \theta_2 > 0$’ means improving health with health care, and ‘$\gamma_h < 0$’ means lesser formal care for the healthier. Substitute the $h$ equation into the $y_1$ equation to obtain

$$y_1 = \gamma_h (\theta_1 y_1 + \theta_2 y_2 + \theta'_x x + \kappa) + \gamma_y y_2 + x'_1 \gamma_x + u = \gamma_h \theta_1 y_1 + (\gamma_h \theta_2 + \gamma_y) y_2 + \gamma_h \theta'_x x + x'_1 \gamma_x + (\gamma_h \kappa + u).$$

Solve this for $y_1$ to get

$$y_1 = \frac{1}{1 - \gamma_h \theta_1} \{(\gamma_h \theta_2 + \gamma_y) y_2 + \gamma_h \theta'_x x + x'_1 \gamma_x + (\gamma_h \kappa + u)}.$$

The interest is on the following effects of $y_2$ on $y_1$:

$\gamma_y$ (‘net effect’ with $h$ controlled) vs. $\gamma^*_y \equiv \frac{\gamma_h \theta_2 + \gamma_y}{1 - \gamma_h \theta_1}$ (‘gross effect’ with $h$ substituted out)
because only $\gamma^*_y$ is identified by dropping $h$ although the desired effect is $\gamma_y$—but one may “declare” that $\gamma^*_y$ is the desired effect. Since $1 - \gamma_h \theta_1 > 1$, the sign of the coefficient of $y_2$ depends on the sign of $\gamma_h \theta_2 + \gamma_y$ which consists of the net effect $\gamma_y$ of $y_2$ on $y_1$ and the ‘indirect effect’ $\gamma_h \theta_2 < 0$ of $y_2$ on $y_1$ through the improved health. Since $\gamma_h \theta_2 < 0$, $\gamma_y < 0$ implies $\gamma_h \theta_2 + \gamma_y < 0$; $\gamma_y > 0$, however, makes the sign of $\gamma_h \theta_2 + \gamma_y$ ambiguous. ‘$\gamma^*_y < 0$’ does not necessarily imply $\gamma_y < 0$; but $\gamma^*_y > 0$ implies $\gamma_y > 0$. Since $1 - \gamma_h \theta_1 > 1$ and $\gamma_h \theta_2 < 0$, the absolute magnitude of $\gamma^*_y$ is smaller than that of $\gamma_y$ when $\gamma_y > 0$; but when $\gamma_y < 0$, it is ambiguous.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Logit (t-value)</th>
<th>QPOI (t-value)</th>
</tr>
</thead>
<tbody>
<tr>
<td>financial asset</td>
<td>-0.034 (-1.53)</td>
<td>-0.012 (-1.35)</td>
</tr>
<tr>
<td>real estate</td>
<td>0.011 (0.26)</td>
<td>-0.007 (-0.41)</td>
</tr>
<tr>
<td>own hose</td>
<td>-0.245 (-1.63)</td>
<td>-0.107 (-1.84)</td>
</tr>
<tr>
<td>family income</td>
<td>0.057 (1.06)</td>
<td>0.031 (1.50)</td>
</tr>
<tr>
<td>pension</td>
<td>0.026 (0.91)</td>
<td>-0.012 (-1.25)</td>
</tr>
<tr>
<td>age</td>
<td>-0.068 (-0.45)</td>
<td>0.006 (0.10)</td>
</tr>
<tr>
<td>age2</td>
<td>0.109 (1.16)</td>
<td>0.000 (0.00)</td>
</tr>
<tr>
<td>male</td>
<td>0.661 (4.01)</td>
<td>0.029 (0.47)</td>
</tr>
<tr>
<td>married</td>
<td>0.119 (0.70)</td>
<td>-0.025 (-0.31)</td>
</tr>
<tr>
<td>Seoul</td>
<td>-0.707 (-3.68)</td>
<td>0.126 (1.91)</td>
</tr>
<tr>
<td>work</td>
<td>-0.820 (-3.80)</td>
<td>-0.109 (-1.40)</td>
</tr>
<tr>
<td>kid-par</td>
<td>-0.052 (-2.54)</td>
<td>-0.006 (-0.70)</td>
</tr>
<tr>
<td>nkids</td>
<td>0.225 (2.07)</td>
<td>0.024 (0.63)</td>
</tr>
<tr>
<td>nfem.kids</td>
<td>-0.180 (-1.63)</td>
<td>0.003 (0.09)</td>
</tr>
<tr>
<td>nkids-co</td>
<td>0.097 (0.60)</td>
<td>0.084 (1.36)</td>
</tr>
<tr>
<td>nfem.kids-co</td>
<td>0.349 (1.74)</td>
<td>0.057 (0.89)</td>
</tr>
<tr>
<td>nkids-act</td>
<td>-0.150 (-1.47)</td>
<td>-0.028 (-0.74)</td>
</tr>
<tr>
<td>nfem.kids-act</td>
<td>0.010 (0.08)</td>
<td>-0.127 (-2.75)</td>
</tr>
<tr>
<td>nkids-30</td>
<td>0.040 (0.60)</td>
<td>0.046 (2.05)</td>
</tr>
<tr>
<td>nkids-60</td>
<td>0.022 (0.41)</td>
<td>0.049 (2.43)</td>
</tr>
<tr>
<td>nkids-120</td>
<td>-0.033 (-0.55)</td>
<td>-0.009 (-0.42)</td>
</tr>
<tr>
<td># generations</td>
<td>0.227 (2.92)</td>
<td>0.050 (1.64)</td>
</tr>
</tbody>
</table>
Table 2 ‘Logit and Quasi-Poisson for $y_2$’ presents the estimates for the first-stage. Since most disease variables are highly significant but of no direct interest, we omit their results in Table 2 and in the remaining tables to simplify presentation; also omitted are the intercept estimates. In Table 2, $\text{age}^2/100$ (‘age2’) is used. The main variable of interest are the children-related variables as they are the IV’s for $y_2$ and thus should be significant in explaining $y_2$. ‘nkids’ and # generations are significant for logit, whereas nfem.kids-act, nkids-30 and nkids-60 are significant for QPOI.

<table>
<thead>
<tr>
<th>Variables</th>
<th>SCL (tv)</th>
<th>CFEa (tv2, tv1)</th>
<th>CFEm (tv2, tv1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_2$</td>
<td>2.135 (2.40)</td>
<td>1.172 (0.98, 1.05)</td>
<td>1.757 (0.16, 1.71)</td>
</tr>
<tr>
<td>$y_2 \times \text{hi.bl. pressure}$</td>
<td>-0.275 (-2.08)</td>
<td>-0.162 (-1.11, -1.14)</td>
<td>-0.248 (-0.28, -1.81)</td>
</tr>
<tr>
<td>$y_2 \times \text{diabetes}$</td>
<td>-0.686 (-3.81)</td>
<td>-0.668 (-3.56, -3.68)</td>
<td>-0.673 (-0.52, -3.68)</td>
</tr>
<tr>
<td>$y_2 \times \text{mental disease}$</td>
<td>-0.605 (-1.88)</td>
<td>-0.461 (-1.42, -1.50)</td>
<td>-0.575 (-0.86, -1.77)</td>
</tr>
<tr>
<td>$y_2 \times \text{arthritis/rheuma}$</td>
<td>0.125 (0.83)</td>
<td>0.123 (0.79, 0.80)</td>
<td>0.133 (0.19, 0.88)</td>
</tr>
<tr>
<td>$y_2 \times \text{age}$</td>
<td>-0.026 (-2.32)</td>
<td>-0.020 (-1.65, -1.70)</td>
<td>-0.022 (-0.19, -1.78)</td>
</tr>
<tr>
<td>$y_2 \times \text{male}$</td>
<td>0.191 (1.21)</td>
<td>0.237 (1.41, 1.45)</td>
<td>0.201 (0.39, 1.28)</td>
</tr>
<tr>
<td>financial asset</td>
<td>0.047 (3.42)</td>
<td>0.046 (3.29, 3.31)</td>
<td>0.047 (2.81, 3.40)</td>
</tr>
<tr>
<td>real estate</td>
<td>0.159 (4.64)</td>
<td>0.159 (4.71, 4.76)</td>
<td>0.158 (3.81, 4.62)</td>
</tr>
<tr>
<td>own hose</td>
<td>-0.029 (-0.30)</td>
<td>-0.046 (-0.44, -0.44)</td>
<td>-0.032 (-0.28, -0.32)</td>
</tr>
<tr>
<td>family income</td>
<td>0.001 (0.03)</td>
<td>0.009 (0.27, 0.27)</td>
<td>0.001 (0.03, 0.05)</td>
</tr>
<tr>
<td>pension</td>
<td>0.068 (3.48)</td>
<td>0.068 (3.52, 3.52)</td>
<td>0.068 (3.03, 3.50)</td>
</tr>
<tr>
<td>age</td>
<td>0.378 (2.29)</td>
<td>0.348 (2.40, 2.43)</td>
<td>0.380 (1.44, 2.31)</td>
</tr>
<tr>
<td>age2</td>
<td>-0.262 (-2.43)</td>
<td>-0.239 (-2.49, -2.53)</td>
<td>-0.263 (-1.50, -2.45)</td>
</tr>
<tr>
<td>male</td>
<td>-0.136 (-1.15)</td>
<td>-0.115 (-0.93, -0.94)</td>
<td>-0.137 (-1.05, -1.16)</td>
</tr>
<tr>
<td>married</td>
<td>0.093 (0.91)</td>
<td>0.088 (0.86, 0.86)</td>
<td>0.091 (0.65, 0.89)</td>
</tr>
<tr>
<td>Seoul</td>
<td>-0.006 (-0.05)</td>
<td>-0.031 (-0.24, -0.25)</td>
<td>-0.006 (-0.04, -0.05)</td>
</tr>
<tr>
<td>work</td>
<td>-0.184 (-1.63)</td>
<td>-0.213 (-1.83, -1.84)</td>
<td>-0.187 (-1.46, -1.65)</td>
</tr>
<tr>
<td>kid-par</td>
<td>0.026 (1.78)</td>
<td>0.023 (1.48, 1.49)</td>
<td>0.026 (1.41, 1.77)</td>
</tr>
<tr>
<td>$\hat{v}_2$</td>
<td>0.414 (0.97, 1.10)</td>
<td>0.027 (0.03, 0.74)</td>
<td></td>
</tr>
<tr>
<td>$\hat{v}_2^2$</td>
<td>0.230 (1.85, 2.00)</td>
<td>-0.001 (0.00, -0.85)</td>
<td></td>
</tr>
<tr>
<td>$\hat{v}_2^3$</td>
<td>-0.069 (-2.06, -2.19)</td>
<td>0.000 (0.00, 1.10)</td>
<td></td>
</tr>
</tbody>
</table>
Table 3 presents the main estimation results where ‘tv’ stands for t-value, CFEa is the estimator with the additive error for CF, CFEm is the estimator with the multiplicative error for CF, and ‘tv2’ is the correct t-value taking into account the first-stage estimation errors whereas ‘tv1’ is the t-value ignoring the first-stage estimation errors (correct only under the null of $y_2$ exogeneity). For the sake of comparison, we show the SCL results ignoring the $y_2$ endogeneity in the first column, although we will not interpret the results.

Comparing CFEa and CFEm in Table 3, CFEm does not pick up the $y_2$ endogeneity as the CF ($\hat{v}_2, \hat{v}_2^2, \hat{v}_2^3$) are all insignificant—the Wald test for $H_0: \gamma_1 = \gamma_2 = \gamma_3 = 0$ is not rejected. In contrast, CFEa does pick up the $y_2$ endogeneity, which results in appreciable differences between SCL and CFEa in the estimates involving $y_2$. In the CFEa column, among the terms involving $y_2$, only the interaction term with diabetes is significant with a large effect estimate (67% reduction in formal health expenditure as $y_2$ goes up by 1); there is also weak evidences that $y_2$ interacts with mental disease, age and male.

Also notable in the CFEa column of Table 3 is that tv2 and tv1 are not much different: there is no reversal of statistical significance except for $v_2^2$ where tv2 is 1.85 while tv1 is 2.00. In contrast, tv2 and tv1 are much different in the CFEm column, particularly for the variables involving $y_2$ and $\hat{v}_2$. This might be due to the division of $y_2$ by the regression function for the multiplicative residual, as this might result in too big numbers and consequently some numerical instability. The poor performance of CFEm relative to CFEa is surprising, given the intuitive appeal of the multiplicative residual in the exponential model. This might be attributed to two factors: the just mentioned numerical instability, and $u$ containing the heteroskedastic factor present in the additive residual, but not in the multiplicative residual.

Table 4 presents the estimation results under $E(y_2|x) = \exp(x^T\beta)$ which does away with logit. In Table 4, neither CFEa nor CFEm pick up the $y_2$ endogeneity in view of the t-values for the CF’s. As the result, the estimates and t-values of CFEa and CFEm are not much different from those of SCL under $y_2$ exogeneity. As in Table 3, tv2 and tv1 are little different in CFEa, whereas they are substantially different for CFEm, particularly for the variables involving $y_2$ and $\hat{v}_2$.

Although not shown, we also tried the ‘logit-only first stage’ just to see which part between logit and QPOI contributes more. The results for the mean squared error $N^{-1}\sum_i(y_{2i} - \hat{y}_{2i})^2$ where $\hat{y}_{2i}$ is the estimated $E(y_2|x)$ are, respectively, 0.284 (QPOI only), 0.283 (logit only), and 0.271 (both QPOI and logit as in the two-stage procedure). This shows that most
explanatory power for $y_2$ comes from its binary aspect and the positive values contribute only a little.

Table 4: SCL, CFE-additive and CFE-multiplicative for $y_1$: No Logit

<table>
<thead>
<tr>
<th>Variables</th>
<th>SCL (tv)</th>
<th>CFEa (tv2, tv1)</th>
<th>CFEm (tv2, tv1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_2$</td>
<td>2.135 (2.40)</td>
<td>1.816 (1.40, 1.57)</td>
<td>1.413 (0.55, 1.31)</td>
</tr>
<tr>
<td>$y_2 \times $hi.bl. pressure</td>
<td>-0.275 (-2.08)</td>
<td>-0.250 (-1.71, -1.80)</td>
<td>-0.223 (-0.49, -1.62)</td>
</tr>
<tr>
<td>$y_2 \times $diabetes</td>
<td>-0.686 (-3.81)</td>
<td>-0.674 (-3.67, -3.78)</td>
<td>-0.680 (-0.94, -3.63)</td>
</tr>
<tr>
<td>$y_2 \times $mental disease</td>
<td>-0.605 (-1.88)</td>
<td>-0.620 (-1.90, -1.88)</td>
<td>-0.550 (-1.03, -1.68)</td>
</tr>
<tr>
<td>$y_2 \times $arthritis/rheuma.</td>
<td>0.125 (0.83)</td>
<td>0.131 (0.82, 0.85)</td>
<td>0.146 (0.26, 0.96)</td>
</tr>
<tr>
<td>$y_2 \times $age</td>
<td>-0.026 (-2.32)</td>
<td>-0.025 (-2.00, -2.05)</td>
<td>-0.019 (-0.69, -1.48)</td>
</tr>
<tr>
<td>$y_2 \times $male</td>
<td>0.191 (1.21)</td>
<td>0.195 (1.15, 1.18)</td>
<td>0.216 (0.53, 1.37)</td>
</tr>
<tr>
<td>financial asset</td>
<td>0.047 (3.42)</td>
<td>0.047 (3.35, 3.37)</td>
<td>0.047 (3.13, 3.41)</td>
</tr>
<tr>
<td>real estate</td>
<td>0.159 (4.64)</td>
<td>0.159 (4.55, 4.60)</td>
<td>0.158 (4.49, 4.72)</td>
</tr>
<tr>
<td>own hose</td>
<td>-0.029 (-0.30)</td>
<td>-0.033 (-0.32, -0.33)</td>
<td>-0.032 (-0.30, -0.33)</td>
</tr>
<tr>
<td>family income</td>
<td>0.001 (0.03)</td>
<td>0.004 (0.11, 0.11)</td>
<td>0.002 (0.07, 0.07)</td>
</tr>
<tr>
<td>pension</td>
<td>0.068 (3.48)</td>
<td>0.069 (3.52, 3.53)</td>
<td>0.068 (3.23, 3.50)</td>
</tr>
<tr>
<td>age</td>
<td>0.378 (2.29)</td>
<td>0.362 (2.03, 2.08)</td>
<td>0.372 (1.77, 2.25)</td>
</tr>
<tr>
<td>age2</td>
<td>-0.262 (-2.43)</td>
<td>-0.250 (-2.11, -2.17)</td>
<td>-0.258 (-1.85, -2.38)</td>
</tr>
<tr>
<td>male</td>
<td>-0.136 (-1.15)</td>
<td>-0.132 (-1.08, -1.09)</td>
<td>-0.139 (-1.11, -1.18)</td>
</tr>
<tr>
<td>married</td>
<td>0.093 (0.91)</td>
<td>0.097 (0.94, 0.94)</td>
<td>0.090 (0.78, 0.88)</td>
</tr>
<tr>
<td>Seoul</td>
<td>-0.006 (-0.05)</td>
<td>-0.015 (-0.12, -0.12)</td>
<td>-0.009 (-0.06, -0.07)</td>
</tr>
<tr>
<td>work</td>
<td>-0.184 (-1.63)</td>
<td>-0.199 (-1.65, -1.70)</td>
<td>-0.185 (-1.58, -1.62)</td>
</tr>
<tr>
<td>kid-par</td>
<td>0.026 (1.78)</td>
<td>0.025 (1.58, 1.62)</td>
<td>0.026 (1.53, 1.75)</td>
</tr>
<tr>
<td>$\hat{\nu}_2$</td>
<td>0.183 (0.37, 0.50)</td>
<td>0.074 (0.10, 1.47)</td>
<td></td>
</tr>
<tr>
<td>$\hat{\nu}_2^2$</td>
<td>0.102 (0.92, 1.50)</td>
<td>-0.005 (0.00, -1.58)</td>
<td></td>
</tr>
<tr>
<td>$\hat{\nu}_2^3$</td>
<td>-0.027 (-0.80, -1.44)</td>
<td>0.000 (0.00, 1.76)</td>
<td></td>
</tr>
</tbody>
</table>

4 Conclusions

This paper examined whether informal health care can reduce formal health care, where the formal care $y_1$ is medical and long-term care expenditures (14% zeros) and the informal care $y_2$ is the number of family care givers (85% zeros). This task posed a number of diffi-
culties, because \( y_2 \) is an endogenous regressor that is a count variable with too many zeros, in addition to \( y_1 \) having a non-trivial proportion of zeros.

Facing the difficulties, we proposed a two-stage procedure where the first stage is estimating \( E(y_2|x) \) as the product of logit (using \( y_2 \) being positive or not) and an exponential regression function (using only positive \( y_2 \)’s)—the idea borrowed from ‘zero-inflated Poisson’. The second stage is applying a semi-parametric censored model estimator for \( y_1 \) with the endogeneity of \( y_2 \) removed by a control function (CF). Two types of CF’s were considered: one based on the additive residual \( y_2 - E(y_2|x) \), and the other based on the multiplicative residual \( \{y_2/E(y_2|x)\} - 1 \); the actual CF’s used were polynomial functions of these residuals.

Despite the intuitive appeal of the multiplicative residual as an exponential function appears, the additive residual CF approach performed much better than the multiplicative residual CF approach. Also, using only an exponential function for \( E(y_2|x) \) (i.e., ignoring the too-many zero problem) was tried, but the outcome was inferior to the procedure with both logit and exponential functions.

Our empirical result using Korean data for the elderly of age 65 and above showed that informal care is a substitute only for certain cases such as diabetes. There are weak evidences that informal care effect on formal care interacts also with mental disease, age and male. That is, as noted in the literature of informal and formal care trade-off, the effect of informal care on formal care is heterogeneous.


