Ordinal and Cardinal Measures of Health Inequality: An Empirical Comparison

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Abstract: When measuring health inequality using ordinal data, analysts typically must choose between indices specifically based upon ordinal data and more standard indices using ordinal data which has been transformed into cardinal data. This paper compares inequality rankings across a number of different approaches and finds considerable sensitivity to the choice between ordinal and cardinal based indices. There is relatively little sensitivity to the ethical choices made by the analyst in terms of the weight attached to different parts of the distribution.

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1. Introduction

Since the vast majority of summary inequality indices are mean-based they require a cardinal measure of the outcome variable in question. While there are some health measures which are cardinal (e.g. body mass index) they are typically not comprehensive. More general health measures are nearly always categorical and ordinal rather than cardinal. Thus to obtain a summary measure of inequality it is necessary to either (a) employ an inequality measure which is specifically designed to deal with ordinal data or (b) to transform the ordinal measure into a cardinal measure and then employ a standard inequality index. [1] While there are examples of both (a) and (b) there is relatively little empirical comparison of the approaches using the same data. This short paper attempts to fill this gap by calculating inequality indices for Irish data for the 1994-2001 period using both approaches and formally comparing the inequality rankings for each measure.

It could be argued that since inequality measures specifically designed to deal with ordinal data are now available, analysts should always use such indices. However, it also seems fair to suggest that such measures are less well developed than their cardinal counterparts and most analysis up to now has employed the cardinal approach and thus an empirical comparison of the two approaches is warranted.

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[1] For examples of health inequality analysis using a specifically ordinal approach see Allinson and Foster (2004) and Abul Naga and Yalcin (2007). See Jones and van Doorslaer (2003) for a discussion of how to transform ordinal health data into a cardinal measure which can be used for the analysis of health inequality with respect to income. Note that in this paper we are examining "pure health inequality".

In section 2 of the paper we describe the different inequality measures which we calculate. Section 3 discusses the data used and presents the results while section 4 gives concluding comments.

2. Ordinal and Cardinal Measures of Inequality

Individual level data on health often comes in the form of a self-assessment of health (SAH). Individuals answer a question of the form: in general, how good would you say your health is? The possible answers are: very bad, bad, fair, good and very good (the exact wording can differ from survey to survey but it is generally of the above type). While this measure appears to give a good indicator for overall health (Idler and Benyamini, 1997) it is not cardinal, and with only five categories, it is not suited to the application of standard inequality indices such as the Gini coefficient. To obtain a summary inequality index from such data it is necessary to employ an index which is specifically designed to deal with ordinal data or else to transform the ordinal data into cardinal data and then use a standard index. We now briefly discuss both of these approaches in turn.

Allison and Foster (2004) present a methodology for analysing inequality when data is of a qualitative nature, as is the case with SAH. They show how bivariate comparisons of any two distributions of SAH can give a partial inequality ordering. Abul Naga and Yalcin (2007) build upon this by presenting a parametric family of inequality indices for qualitative data.

Suppose we have a measure of SAH with n different categories which can be clearly ordered $1,\ldots,\ n$. Let m denote the median category and let P_i denote the *cumulative* proportion of the population in category i, where $i=1,\ \ldots,\ n$. The inequality index proposed by Abul Naga and Yalcin (2007) is then

$$I_{\alpha,\beta} = \frac{\sum_{i<m} P_i^{\alpha} - \sum_{i\geq m} P_i^{\beta} + (n+1-m)}{k + (n+1-m)}$$

where $k = (m-1)\left(\frac{1}{2}\right)^{\alpha} - [1+(n-m)\left(\frac{1}{2}\right)^{\beta}]$ and α and β are parameters chosen by

the analyst and are ethical choices which essentially reflect the weight given to inequality above and below the median. For a given value of β, as $\alpha \to \infty$ less weight is given to disparities below the median, while similarly for given values of α as $\beta \to \infty$ less weight is given to disparities above the median. The case where $\alpha = \beta = 1$ is that where effectively equal weight is given to disparities above and below the median. Following Abul Naga and Yalcin (2007) we calculate $I_{1,1}(.), I_{1,4}(.), I_{1,\infty}(.), I_{4,1}(.)$ and $I_{\infty,1}(.)$.

The second approach to calculating inequality indices when dealing with qualitative data is to transform the ordinal variable into a cardinal variable and then calculate standard inequality indices. Van Doorslaer and Jones (2003) provide a review and assessment of the various approaches to such a transformation and we adopt two of the procedures they discuss, interval regression and re-scaled ordered probit. The former procedure involves using interval regression to obtain a mapping from the empirical distribution function (EDF) of what is regarded as a valid index of health (such as the McMaster Health Utility Index (HUI)) to SAH.[2] By mapping from the cumulative frequencies of SAH categories into an index of health such as the McMaster HUI it is possible to obtain upper and lower limits of the intervals for the SAH categories. These can then be used in an interval regression to obtain a predicted value of the index for all individuals. This approach also sidesteps the need to re-scale the cardinal variable as it is already expressed in the units of the HUI.

Comparisons which they carry out for measures of SAH in Canada suggest that this approach to cardinalisation outperforms other approaches. Research by van Doorslaer and Koolman (2004) and van Ourti et al (2006) indicates that the values of the health index obtained are not very sensitive to the cut-off points chosen (see also Lecluse and Cleemput, 2006, and Lauridsten et al, 2004). Hence it should be acceptable to use cut-off points from the Canadian HUI to calculate a cardinal index of health for other countries. Thus to construct our measure of health for Ireland we use the values from the EDF of the Canadian HUI which correspond to the cumulative frequencies of SAH.[3] These values are then used as the upper and lower bounds for an interval regression with the following independent variables: age, gender, education, marital status and principal economic status (i.e. at work, unemployed etc.).[4]

An alternative procedure to interval regression is to estimate an ordered probit of SAH with independent variables such as age, gender and education (we use the same independent variables as in the case of interval regression). We then take the linear prediction of this ordered probit and re-scale it so that it takes a value from zero to one.

The two procedures thus give us two cardinal variables for health and we calculate the Gini coefficient for both of the variables. Thus overall we have seven different indices of health inequality, the five versions of $I_{\alpha,\beta}$ and the Gini coefficients for the two cardinal variables and we can assess the sensitivity of inequality rankings to (a) the ethical choices embodied in different values for α and β and (b) the choice of an ordinal based inequality index or a standard cardinal based inequality index using transformed data.

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[2] For a discussion of the Health Utility Index see Feeney at al (1995) and Torrance et al (1995, 1996).
[3] These figures were kindly supplied by Andrew Jones.
[4] To save on space we do not present these results but they are available on request.

3. Data and Results

The data we use comes from five of waves of the Living in Ireland Survey (LII), 1994 and 1998-2001.[5] The LII survey is a nationally representative survey which was collected annually between 1994 and 2001 and which formed the Irish part of the European Community Household Panel Survey. It has been used extensively in a variety of studies on (amongst other issues) poverty, deprivation and education. We choose not to use the 1995-1997 data owing to some missing observations on education, a variable which is important in the calculation of the cardinal health measure. Note that as our principal source of concern here is the ranking of each year by inequality (i.e. which of the five years had the highest inequality, second highest etc) the choice of particular year is not of paramount importance. The issue of attrition is also of secondary importance as long as its impact for any given year across the different inequality measures is random. Thus we are effectively assuming that there is no reason to believe that attrition will impact differently upon the Gini coefficient for a cardinal measure compared to its effect on, say, the $I_{1,1}$ measure.

Table 1 provides the values of the cumulative frequencies for the years in question, while table 2 provides the values of the different indices and table 3 lists the Kendall rank correlation coefficients for the indices. In interpreting these tables there are a number of issues to bear in mind. First of all, there is little point in comparing the absolute values of the indices. Since the different indices embody different assumptions about the underlying nature of the variable (ordinal versus cardinal) and also about the relative importance attached to different parts of the distribution (the values of α and β), comparison of the absolute values of the indices is akin to

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[5] For an overview of the Living in Ireland Survey, see Watson (2004).

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comparing apples and oranges. What is of interest however is the ranking of the years (in terms on inequality) provided by the different indices, hence the concentration on the rank correlations. Secondly, we are examining the sensitivity of the indices to the two factors referred to above, the underlying nature of the variable and the ethical assumptions of the analyst. Finally, in interpreting the significance levels in table 3 bear in mind that the null hypothesis is that the rankings are independent. Thus a statistically significant p-value indicates that the rankings are not independent but instead are correlated.

Purely eye-balling the indices in table 2, it is noticeable that for some indices the year-to-year variation is quite limited. This is particularly true of the case where $\alpha=1$ and of the Gini coefficient when using the interval regression approach. Expressing this in another way, the year-to-year variation in inequality is not very sensitive to the choice of β (i.e. the relative weight given to disparities above the median). Certainly on a year-by-year basis there appears to be greater sensitivity to the choice of α, the relative weight given to disparities below the median. The low value of the Gini coefficient for the interval regression approach reflects the fact that the distribution of the cardinal variable in this case is very tight.

Turning now to table 3, the correlation coefficients here essentially give us a summary of the extent to which the rankings of the years by inequality show some concordance with each other. Thus a coefficient of 0.8 between indices A and B effectively says that if index A ranks one year as higher than another there is an 80% chance that index B will also. Examining table 3 in some detail we first of all examine the correlations within the class of ordinal inequality measures (which are the correlations for the first five rows and columns). All of these coefficients have values above zero, even if not all of them are statistically significant. This suggests

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that the rankings by year are not very sensitive to the choices of α and β (in fact there is no sensitivity by rank to the choice of α). However, when turning to the correlations between the ordinal and cardinal based indices, we see that apart from the case where α=1, β=∞, there is practically no agreement by ranking between the indices and, in the case of the cardinal variable derived from the scaled ordered probit approach, some of the coefficients are negative. This reinforces how policy conclusions on health inequality may be very sensitive to the choice of underlying variable.

4. Conclusion

This paper has carried out an empirical comparison of health inequality measures applied to ordinal data. Two issues in particular have been examined: sensitivity to ethical choices made by the analyst in terms of the weight applied to different parts of the distribution and the choice between inequality measures designed specifically for ordinal data and measures which use ordinal data which has been transformed into cardinal data. Analysis using Irish data suggests that sensitivity, as measured by rank correlation coefficients, is considerably greater for the latter issue rather than the former. While it must be stressed that this result may be sensitive to the particular data set analysed, it serves as a reminder that choice of health inequality measure can matter empirically.

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Table 1: Cumulative Frequencies for Ordinal Health, 1994-2001

	Very Bad	Bad	Fair	Good	Very Good
1994	0.0076	0.0314	0.1851	0.5275	1
1998	0.0052	0.0276	0.1846	0.5394	1
1999	0.0046	0.027	0.1766	0.5279	1
2000	0.005	0.0268	0.1811	0.5502	1
2001	0.0062	0.0297	0.1864	0.5476	1

Table 2: Inequality Measures, 1994-2001

	$\alpha=1, \beta=1$	$\alpha=1, \beta=4$	$\alpha=1, \beta=\infty$	$\alpha=4, \beta=1$	$\alpha=\infty, \beta=1$	G(Int)	G(OP)
1994	0.348 (1)	0.470 (1)	0.490 (1)	0.689 (1)	0.945 (1)	0.037 (2)	0.133 (2)
1998	0.339 (3)	0.465 (2)	0.487 (3)	0.672 (3)	0.921 (3)	0.035 (3)	0.132 (4)
1999	0.340 (2)	0.464 (4)	0.483 (5)	0.688 (2)	0.944 (2)	0.035 (5)	0.121 (5)
2000	0.331 (5)	0.460 (5)	0.485 (4)	0.656 (5)	0.900 (5)	0.035 (4)	0.152 (1)
2001	0.337 (4)	0.465 (3)	0.489 (2)	0.660 (4)	0.905 (4)	0.038 (1)	0.133 (3)

Table 3: Kendall Rank Correlation Matrix

	$\alpha=1, \beta=1$	$\alpha=1, \beta=4$	$\alpha=1, \beta=\infty$	$\alpha=4, \beta=1$	$\alpha=\infty, \beta=1$	G(Int)	G(OP)
$\alpha=1, \beta=1$	1.000						
$\alpha=1, \beta=4$	0.600	1.000					
$\alpha=1, \beta=\infty$	0.200	0.600	1.000				
$\alpha=4, \beta=1$	1.000**	0.600	0.200	1.000			
$\alpha=\infty, \beta=1$	1.000**	0.600	0.200	1.000**	1.000		
G(Int)	0.000	0.400	0.800*	0.000	0.000	1.000	
G(OP)	-0.400	0.000	0.400	-0.400	-0.400	0.200	1.000

***: P value less than 0.01
**: P value less than 0.05
*: P value less than 0.1