Inter-DRG Resource Allocation in a Prospective Payment System: A Stochastic Kernel Approach*

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Abstract

This paper empirically investigates the resource distribution dynamics across Diagnosis Related Groups (DRGs) of elective surgery patients, in a continuing Prospective Payment System (PPS). Existing econometric literature has mainly focussed on the impact of PPS on average Length of Stay (LOS) concluding that the average LOS has declined post PPS. There is little literature on the distribution of this decline across DRGs, in a PPS. The present paper helps fill this gap. It models the evolution over time of the empirical distribution of LOS across DRGs. The empirical distributions are estimated using a non parametric “stochastic kernel approach” based on Markov Chain theory. The results for inlier episodes suggest that resource redistribution will increase capacity and expected number of admissions for DRGs having increasing waiting times. In addition, adjustments in relative cost weights are perceived as price signals by hospitals leading to a change in their casemix. The results for high outlier patients reveal that improved quality of care is one of the factors causing reduction in high outlier episodes.
1 Introduction and Background

The empirical analysis of the impact of prospective payment systems (PPS) or case-mix funding on resource allocation in hospitals has received widespread attention in recent years. Under a PPS, hospitals are paid a lump sum per admission. Such a system is assumed to be better than alternative funding mechanisms such as cost reimbursement system where hospitals are paid the actual cost incurred for each patient episode (Newhouse, 1996). The main objective of introducing PPS was to improve hospital efficiency.

The case-based payment system, also known as case-mix funding, was introduced in Victorian public hospitals in 1993. Prior to this public hospitals were funded on a global budget basis or “historical plus” system. Under the global budget basis each hospital annually negotiated its budget with the Health Department. The budget was adjusted upwards to account for inflation and new programs in the hospital, and then slightly adjusted downwards in anticipation of productivity savings. The output was measured in bed-days (number of inpatient hospital days) and number of separations (number of inpatient discharges). Under such a funding regime, distinction between hospitals on the basis of complexity of patients treated was limited. After the introduction of case-mix funding, hospitals were funded on the basis of volume adjusted for complexity.

The main objective of introducing case-mix funding in Victoria was to achieve significant reductions in health expenditures while maintaining service outputs. In a case-mix funding regime, the admitted patients output is counted by coding hospital separations into DRGs. The DRGs are determined on the basis of diagnosis codes for patient episodes. These codes are based on the ICD 10 classification and hospitals can enter up to 24 diagnostic codes for each patient episode. Each DRG is assigned a weight based on its relative resource consumption. Each patient separation is multiplied by the corresponding DRG weight and this weighted throughput is used for counting the output for funding purposes. In addition this formula is further adjusted for unusually long stay or short stay patients. Such patients are determined by setting low and high trim points for each DRG.

In Victoria, the trim points are set at one-third and three times the average Length of
Stay (LOS) for a particular DRG. Patients with LOS between the trim points are called inliers and those with LOS above and below the trim points are called high and low outliers respectively. Low outlier cases receive a fraction of inlier case weight. On the other hand, hospitals receive some additional weight for high outliers for each day above the high trim point. It should be noted that the daily additional weight is related to the inlier weight and the average length of stay but the relationship varies for different DRGs and is subject to ceiling and floor restrictions. Such an adjustment enables each adjusted outlier separation to be expressed in terms of equivalent inlier weights termed Weighted Inlier Equivalent Separation (WIES). Thus WIES is the unit of counting admitted patient throughput under the case-mix formula.

In each funding round, the health department sets up a price per unit of WIES using a “top down” approach in which prices are determined on the basis of the amount allocated to health budget, rather than on the basis of a benchmark cost. Thus the case-mix formula is not a price setting mechanism and does not reflect actual costs of treatment. The DRG cost weights are determined from a series of specific cost weight studies commissioned by the health department. These studies are undertaken mostly by large metropolitan hospitals which have resources to undertake such a sophisticated costing exercise. Though the quality of cost weight studies is increasing, the system’s ability to fine tune the weights has been questioned because of the high volatility of DRG cost weights. The volatility in DRG cost weights between the funding rounds combined with temporal changes in the price per unit of WIES may transform a specific DRG from profitable (“lucrative”) to loss-making (“non-lucrative”). This might send price signals to the hospital to reduce the number of procedures in a particular DRG.

The funding regime of Victorian hospitals is not purely based on case-mix. In addition to WIES funding, hospitals also get additional funding in the form of fixed overhead costs, specified grants for teaching and research, performance and quality incentives and bonus funding for hospital demand management strategies. The health department encourages efficiency by setting WIES targets for each hospital and a public throughput 2% above this.

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1 LOS denotes average length of stay throughout the paper unless stated otherwise.
2 Therefore only large/metropolitan hospitals are effectively capable of reallocating the resources across DRGs, in response to changes in cost weights. Thus only such hospitals are used for empirical analysis in the present paper. These hospitals account for most of hospital separations in Victoria.
target is penalised by reductions in bonuses payable to the hospital. In addition to WIES targets hospitals also face targets relating to waiting times for emergency services, critical care and elective surgery under the Hospital Access Program (HAP).

In the short term, under the pressure of meeting multiple targets, hospitals face a real risk of bearing an operating deficit. Thus, in order to minimize such deficits, hospitals may try to maximise their revenue by reallocating resources within DRGs (particularly related to elective patients). Thus in the Victorian ‘fixed and variable’ case-mix funding regime hospitals’ behaviour reflects responses to fixed costs (infrastructure, salaries etc.), variable costs (WIES) and marginal cost (per diem expenses for high outliers) components. The payment system is not fully prospective and can at best be called a mixed payment system with most of the payment being prospective.

A PPS regime enables payers (health department) and providers (hospitals) to share the financial risks of patient care. Such a sharing mechanism is termed “supply side cost sharing” by Ellis and McGuire (1993). The theoretical models (see for example, Selden (1990); Newhouse (1996); Ellis and McGuire (1996); Siciliani (2006)) that underlie these works can be classified into three broad categories. Firstly, case-mix funding could lead to a moral hazard effect where hospitals have an incentive to change the intensity of services provided to a given set of patients. Secondly, the reimbursement mechanism could lead to a selection effect whereby hospitals have an incentive to change the severity of patients they see, and thirdly, in a competitive setup hospitals could change their market share by specialization (practice style effect).

It has been argued in the literature that a fully prospective payment system leads to the technically efficient production of health care with hospitals keeping the difference between the payment per episode and the cost of treating a patient. Thus, unlike a cost reimbursement system (where hospitals attempt to obtain maximum reimbursement by treating the patient with maximum intensity) hospitals will not have an incentive to over provide a service due to rent in factor prices (Newhouse 1996). Although PPS encourages hospitals to provide services efficiently, it also increases the likelihood of patient selection thereby denying some patients the treatment they desire. Thus, there exists a tradeoff between selection and production efficiency (Ma 1994). Such a “selection-efficiency” tradeoff has
been extensively discussed in a review by [Newhouse (1996)]. One of the major implications of such a tradeoff could be under servicing of high cost patients [Newhouse (1983, 1996); Selden (1990)]. Under a PPS the marginal revenue to treat a high cost patient is likely to be less than the marginal cost of treatment and thus hospitals will have an incentive to under service such patients. [Newhouse (1996) and Newhouse (2002)] discusses several theoretical frameworks addressing the issue of selection from supply side perspective of health care. However, he (1996) observes that such theoretical frameworks have limited capabilities in terms of empirical work. This is mainly because of their requirement for information on unobservable variables viz. information on a hospital manager’s utility function for effort, unobserved patient factors affecting cost, the physicians utility function and error variances in the prices etc. In addition, most of the assumptions in these models are untestable.

Most of the earlier econometric research on PPS has tried to estimate its effects by analysing the length of stay (LOS) data for different Diagnostic Related Groups (DRGs). For example, in making a case for selection effect, [Newhouse and Byrne (1988)] argue that some of the decline in LOS after PPS is caused by a shift of more severe cases to facilities not paid by PPS. Most of the empirical evidence is on the effect of a switch to PPS regime on LOS and the general consensus is that the average LOS per discharge at hospitals has declined after the introduction of PPS [Freiman et al. (1989); DesHarnais et al. (1990); Manton et al. (1993); Ellis and McGuire (1996); Norton et al. (2002)].

Although PPS led to a relative reduction in LOS for inpatients, potential imperfections in the funding regime could offset to a certain extent the very achievements of PPS. One such example is per diem payment for a patient with unusually long LOS. In PPS (without provision of per diem payment) the marginal revenue of keeping a patient in hospital for an additional day is zero whereas the marginal cost is positive. Then the hospital has no incentive to increase LOS of an episode unnecessarily in order to maximise funding. On the other hand, the introduction of per diem payment makes marginal revenue positive and if hospitals incur marginal cost lower than the marginal revenue, they will have an incentive to keep patients longer than required. It has been argued that such “imperfections” in hospital funding might introduce an incentive for hospitals to increase expenditure and

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3 Here high cost patients refer to the patients having relatively higher cost within the distribution of costs of an individual DRG.
maximize reimbursement thereby partially offsetting the anticipated effect of PPS (Norton et al., 2002). Moreover, even a continuing PPS regime might induce such behaviour by hospitals because of temporal changes in the relative DRG weights which are used for funding inpatient episodes in hospitals. For example, in Victoria, relative DRG weights are highly volatile up to the extent of +10% to -10% per year. Such volatility in DRG weights might lead to incentives for hospitals to assign patients to profit making DRGs (“DRG-creep”) or split a patient into several cases (“patient splitting”) in order to maximise funding for that particular inpatient episode.

Similarly, hospitals might redistribute their resources to more lucrative patients and discharge less lucrative patients “quicker and sicker” (which could result in increasing readmission rates). The waiting time incentives and bonus payments related to improved quality of care may also lead to a redistribution of resources across DRGs. It should be noted that the definition of a lucrative DRG might change in each time period (based on its relative weight) and thus resources need to be again redistributed to maximize reimbursement from the government. Hence, it is imperative to simultaneously consider the cross-sectional (across DRGs) and dynamic (over years) behaviour of resource allocation.

Thus, an empirical framework to test the distribution dynamics of resource allocation decisions across DRGs in a PPS is critical from a policy point of view. Such an analysis reflects on the efficacy of a case-mix policy and will be of interest to policy makers. For example, the conclusions from such an analysis might shed light on the perception that high cost patients are underserviced or that changes in the relative costs of DRGs is one of the factors responsible for resource allocation decisions across DRGs.

The main objective of this paper is to empirically investigate the resource distribution dynamics across elective surgery DRGs in a continuing PPS regime. As discussed earlier, the extant literature has mainly focussed on the impact of the introduction of PPS on average LOS by comparing the pre/post PPS scenarios and concluded that the average LOS has declined post PPS. However, there is little literature on the distribution of this decline in LOS across DRGs, in a continuing PPS regime. The present paper intends to

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4This argument is particularly valid for elective non-urgent episodes of patient care which are funded under PPS. Thus only such episodes are considered for empirical analysis in this paper.
help fill this gap. In a continuing PPS regime, hospitals might be less inclined to reduce LOS in “lucrative” DRGs and hence the overall reduction in LOS might be a result of a disproportionately higher reduction in less “lucrative” DRGs\textsuperscript{5}. This paper proposes an empirical framework to test for such a behaviour by explicitly analysing the temporal changes in the re-distribution of resources, in a continuing PPS regime.

The empirical methodology entails modelling the evolution over time of the empirical distribution of length of stay across DRGs. The main purpose of this approach is to record not only the mean and variance of the distribution but also the mobility of each DRG within the distribution. Thus, such an approach helps identify certain inter-DRG allocative patterns which might be induced by the continuing PPS regime. The empirical distributions are estimated using a “stochastic kernel approach” based on the Markov Chain theory. A stochastic kernel can be defined as a complete description of transitions from state ‘i’ to state ‘j’ in an empirical distribution. For example, the stochastic kernel related to an empirical distribution of average LOS in each DRG will help answer the following question: What is the probability that the proportion of LOS in a particular group of DRGs will increase, decrease or remain the same within the next two years? The paper uses monthly Victorian patient data (around one million patient episodes each year) with a time span of eight years (1998-99 to 2005-06).

The empirical analysis is done separately for inlier and outlier episodes. The main conclusions are: For inlier episodes, over a 2 year transition the DRGs with high quantile shares (with values above 75th percentile) of LOS are likely to have their shares reduced (transition of LOS shares from high to low), the DRGs with middle quantile shares of LOS are likely to have their shares unchanged (persistence of shares), the DRGs with low shares of LOS are likely to have their shares further reduced (transition of shares from low to lower). Thus, the distribution of LOS shares for inlier episodes shows the emergence of three peaks, the most prominent being in the middle of the distribution containing the largest number of DRGs. The empirical analysis further tests the hypothesis that a change in the relative cost of DRGs explains the stratification of inlier shares. This is done by using a conditional

\textsuperscript{5}It should be noted that \textit{ex-ante} hospitals would like to specialize in the “lucrative” DRG treatment \cite{Rauner et al., 2003} but \textit{ex-post} in a continuing PPS regime, even a specialist hospital will have an incentive to redistribute its resources in response to temporal changes in relative DRG weights.
approach where stochastic kernels are re-estimated using the share of inliers weighted by the relative cost of DRGs. The results reveal that relative cost weights are one of the factors explaining resource allocation decisions across DRGs. The trend in waiting times for DRGs around the three peaks is analysed to test the effect of waiting times on resource allocation. The results for inlier episodes suggest that resource redistribution will increase capacity and the expected number of admissions for DRGs with increasing waiting times. For high outlier episodes, over a 2 year transition the cross section of LOS distribution of DRGs converges to zero. Since hospital acquired adverse events are the main cause of high outlier episodes, the above trend indicates improvement in quality of care for these patients. The actual quality of care data for these DRGs confirms that improved quality of care is one of the factors causing a reduction in the high outlier episodes.

Thus, the empirical methodology used in this paper sheds further insight into the resource dynamics across DRGs, which was not possible using traditional econometric methods. The rest of the paper is organized as follows: Section 2 proposes an analytic framework to motivate the empirical analysis. Section 3 discusses the data and the methodology. The results are discussed in Section 4. Section 5 concludes.

2 Analytic Framework

As discussed in the previous section, in a continuing PPS regime, funding is at the DRG level and all hospitals or hospital groups get the same amount of money for treating patients from a particular DRG. The main objective of this paper is to analyse the resource distribution dynamics across DRGs in a PPS. Thus the analysis is done at the DRG level for the hospital sector as a whole. This is mainly because, specifically in our case where we focus on elective surgeries, a hospital level study may give misleading results regarding potential patient selection. For example, unobserved patient choice for elective treatment may be responsible for changes in DRGs’ share of LOS across hospitals which could be picked up as a selection effect in a hospital level analysis. On the other hand, if a particular DRGs’ share of resource use changes significantly at the hospital sector level, it indicates a potential selection effect or change in treatment intensity in response to a relative cost change for that DRG. It should be noted that the effect of change in patient profile on
resource use will be negligible as patient profile for elective DRGs has been almost constant in Victoria during the 8 year period considered by our analysis. A funding body will also be interested in the patterns of resource allocation within DRGs at the hospital sector level rather than at individual hospital level. A potential selection effect at the hospital sector level indicates that some patients might miss out on hospital treatment altogether, which is a critical policy issue in terms of adverse health outcomes for the population. The analysis of hospital level selection is certainly an avenue for research, but more appropriate for studies with goals other than those of this analysis.

This section formally quantifies the effect of the reimbursement system on the average resource use at a hospital sectoral level. This is done by extending the hospital level framework suggested by Ellis and McGuire (1996) to the DRG level. The framework decomposes the impact of the relative price changes of DRGs on average resource use for the hospital sector. Let us assume that a patient seeks elective treatment for DRG $i$. In a PPS, the patient might be subjected to an admittance criteria based on the relative price of DRG $i$: $A_i(p_i)$ where $A_i$ is the admittance criteria and $(p)$ is the relative price vector. For example, such an admittance criteria might be based on a decision rule where patients who are perceived to be high outliers will be avoided. Similarly, a patient might be subjected to different treatment intensity ($T_i(p_i)$) based on the reimbursement system. For example, if relative price of DRG $i$ is perceived to be low then the patient might be discharged “quicker and sicker” from the hospital. Similarly, patients having multiple diagnoses may be subjected to a classification-bias ($C_i(p_i)$) where the hospital sector would tend to place such patients in a higher relative price DRG. Thus the admittance criteria ($A$), treatment intensity ($T$ and classification bias ($C_i(p_i)$) jointly determine the number of patients treated $N_i$ and severity of patients $V_i$ in DRG $i$ which in turn yields the corresponding LOS. Formally:

$$N_i = N_i(A_i(p_i), T_i(p_i), C_i(p_i))$$

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6 For example, 5 year percentage change (from 2000 to 2005) in ratio of number of patients to total patients, for three main diagnostic categories are: 0.8% (Hip & Knee), 0.6% (Gynaecology) and 0.3% (procedures of digestive system).

7 We are thankful to an anonymous referee for this suggestion.

8 It should be noted that above factors are unobservables and data enables us to observe only number of patients treated in a particular DRG.
\[ V_i = V_i(A_i(p_i), T_i(p_i)) \] \hspace{1cm} (2)

\[ LOS_i = L_i(V_i(p_i), T_i(p_i), C_i(p_i)) \] \hspace{1cm} (3)

where

\[ N_i = \text{Number of patients treated in DRG } i \]

\[ V_i = \text{Severity of patients treated in DRG } i \]

\[ LOS_i = \text{Length of Stay for DRG } i \]

Denoting DRG \( i \)'s share of discharges as \( S_i \) the LOS across all DRGs is:

\[ LOS = \sum_i S_i \ LOS_i \] \hspace{1cm} (4)

Given Eqns. (1), (2) and (3), the total effect of reimbursement system on resource use can be decomposed into four components:

\[
\frac{dLOS}{dp} = \left[ \sum_i S_i \frac{\partial LOS}{\partial T_i} \frac{\partial T_i}{\partial p_i} \right] + \left[ \sum_i S_i \frac{\partial LOS}{\partial V_i} \frac{\partial V_i}{\partial p_i} \right] + \left[ \sum_i S_i \frac{\partial LOS}{\partial C_i} \frac{\partial C_i}{\partial p_i} \right] + \left[ \sum_i LOS_i \frac{\partial S_i}{\partial p_i} \right]
\] \hspace{1cm} (5)

The first term is a moral hazard effect, the second term is the selection effect, the third term is a classification-bias and fourth term is what we define as the redistribution effect.

This paper focusses on quantifying the redistribution effect by using the non-parametric approach of stochastic kernels. In particular we focus on the impact of relative price changes on the share of resource use \( (S_i) \) for a particular DRG.

### 3 Empirical Methodology

In order to test for the behavioural response of the hospital sector towards temporal changes in the relative cost weights of DRGs, the empirical framework focusses on the cross-DRG
distribution dynamics of a DRG’s share of inlier and outlier LOS relative to total LOS. As an illustration, consider Figure 1 where the vertical axis indexes increasing share of outliers in each DRG and the horizontal axis, time. Figure 1 records the densities corresponding to cross-DRG outlier distributions, over two time periods which correspond to two different funding rounds. Figure 1 represents a hypothetical scenario where in period $t$ most DRGs have medium levels of outlier shares and there are very few DRGs with very high or very low outlier shares.

As discussed earlier, in a PPS the relative cost weights of DRGs change over each funding round and thus the distribution of outliers across DRGs is likely to fluctuate. For example as illustrated in Figure 1, the outlier distribution for the same pool of DRGs changes shape in period $t + s$ and shows a pattern where the share of outliers for some DRGs have increased (for example as in DRG 1), some have decreased (for example as in DRG 3) and some have remained almost the same (for example as in DRG 2). In addition the period $t + s$ distribution also reveals a pattern of clustering where DRGs with very high outlier shares have clustered together, DRGs with medium outlier shares have clustered together and DRGs with very low outlier shares have clustered together.

Such an emerging pattern of clustering with multiple peaks is in contrast to the distribution in period $t$ and is termed as *stratification*. Since the underlying population in our analysis are DRGs stratification might result from selection effect, moral hazard, classification bias or redistribution effect. In this paper we focus on the redistribution effect where DRG shares are affected by relative cost changes.

---Insert Figure 1 about here---

The empirical analysis in this paper explores such patterns by analysing the *intradistribution dynamics* of cross-DRG distributions, using actual inpatient data. For example, Figure 1 indicates three types of intradistributional dynamics: i) Persistence: Some DRGs in period $t + s$ have almost the same inlier ratio as in period $t$ (e.g. DRG 2); ii) Churning or Mobility: Some DRGs which have a high ratio of inliers in period $t + s$ had a lower ratio of inliers in period $t$ (e.g. DRG 1) and some DRGs which have a lower ratio of inliers in period $t + s$ had a higher ratio of outliers in period $t$ (e.g. DRG; 3) iii) Clustering or stratification: DRGs have clustered into three distinct categories which has led some DRGs
which had a similar share in period $t$ to separate from each other and be part of different clusters in period $t + s$.

Thus Figure 1 represents an evolving pattern of inlier distributions across two time periods with mobility and stratification of the distribution happening simultaneously. The econometric framework in the present paper intends to capture such distributional dynamics. In addition, the empirical model will also project such distributional dynamics in future using the observed data. The econometrics of analysing the distribution dynamics directly was introduced in the economic growth literature by Quah (1997, 1990). Quah argued that the extant empirical techniques were incapable of capturing the intradistribution dynamics. For example, comparing the mean and variances of cross sectional distributions of DRGs over time will not shed any light on the stratification or mobility of the distribution.

Similarly, the comparison of time series behaviour of outlier shares in each DRG or subgroups of DRG will also be uninformative on distribution dynamics. Even the more sophisticated techniques of cross section and panel data regressions capture behaviour at the conditional mean and will not be useful for analysing the dynamic behaviour of distributions. The impact of a continuing PPS regime on the resource dynamics across DRGs can be best captured by analysing the mobility or churning of DRG distributions. Quah (1997) suggests the use of stochastic kernels for such analysis, which are discussed next.

### 3.1 Stochastic Kernel

A stochastic kernel is a mapping which quantifies how distributions evolve over time. The distribution dynamics methodology (Quah 1997) assumes that the density distribution $\phi_{t+1}$ for the shares evolves according to a Markov process:

$$\phi_{t+n} = M \cdot \phi_t$$

where $M$ is an operator mapping the transition between the share distribution existing in time $t$ to the share distribution in time $t + n$. Thus a stochastic kernel is a continuous time variant of a discrete transition probability matrix (TPM). A TPM is an alternative way of quantifying distribution dynamics where, for example, outlier shares of DRGs are categorized in distinct discrete cells and then the observed transitions out of and into
these discrete cell are counted. However, Quah (1997) and Chung (1967) argue that such a discretisation of a continuous variable (share of inliers and outliers in our case) can distort the distribution dynamics and lead to misleading results. Thus any categorization of outlier shares into specific ranges will be arbitrary and such arbitrariness means that setting out different ranges of share of outliers might give different conclusions about the actual projection of the distribution in the future.

Quah (1997) further argues that instead of discretisation the number of distinct cells in a TPM should be allowed to tend to infinity and then to the continuum. The corresponding TPM with a continuum of rows and columns is termed as stochastic kernel. Thus a stochastic kernel can be formally defined as:

\[\text{Definition 1 Stochastic Kernel}\]

Let \(\mu\) and \(\nu\) be elements of \(B\) that are probability measures on \((R, \mathcal{R})\). A stochastic kernel relating \(\mu\) and \(\nu\) is a mapping \(M_{(\mu, \nu)}:\ (R, \mathcal{R}) \rightarrow [0,1]\) satisfying:

(i) \(\forall S \in R\), the restriction \(M_{(\mu, \nu)}(S, .)\) is a probability measure;

(ii) \(\forall A \in \mathcal{R}\), the restriction \(M_{(\mu, \nu)}(., A)\) is \(\mathcal{R}\) measurable;

(iii) \(\forall A \in \mathcal{R}\), we have \(\mu(A) = \int M_{(\mu, \nu)}(S, A)d\nu(S) M_{(\mu, \nu)}(., A)\).

where:

\((R, \mathcal{R})\) is the underlying state space with \(R\) being the real line and \(\mathcal{R}\) collection of its Borel sets.

\(B(R, \mathcal{R})\) denotes the Banach space of bounded finitely-additive set functions on the measurable space \((R, \mathcal{R})\) endowed with total variation norm:

\[\forall \mu \in B(R, \mathcal{R}): \quad |\mu| = \sup \sum_j |\mu(A_j)|\]

where the supremum in this definition is taken over all \(A_j: j = 1,2,\ldots,n\) finite measurable partitions of \(R\).

The main concept of stochastic kernel is defined by condition (iii). Taking initial period as \(t\), for a given LOS share \(S\) there is a fraction \(d\nu(S)\) of DRGs with shares close to \(S\). In

\(\text{For the technical derivation of stochastic kernel interested readers can refer to Section 4 in Quah (1997).}\)
period \( t + n \) part of DRGs contained in \( d\nu(S) \) will move to a subset \( A \subseteq R \). Normalising this fraction of DRGs by the total number of DRGs, we have the stochastic kernel given by \( M_{(\mu,\nu)}(S, A)d\nu(S) \). Stochastic kernels are generated by applying explicit laws of motion to the cross sectional distributions. They can be expressed as a conditional density which is estimated using an optimal bandwidth and kernel choice. See Appendix A.1 for a detailed discussion on estimation of stochastic kernels.

In order to estimate the conditional density, the issue of optimal bandwidth and kernel choice is crucial (Pagan and Ullah 1999). Quah (2004) reports a measure of relative efficiency based on the cross validation criteria of minimum integrated least square error, where the bandwidth is permitted to vary with the kernel \( K \) using the method suggested by Silverman (1986). According to this criterion the Epanechnikov kernel turns out to be optimal but the other kernels also achieve efficiencies close to it (Quah, 2004). We use Epanechnikov for our empirical analysis. The bandwidth, or the smoothing parameter, is calculated using the methodology suggested by Silverman (1986). The choice of bandwidth is not arbitrary but data dependent, and thus the stochastic kernel estimation is robust to bandwidth selection. See Appendix A.2 for a detailed discussion on kernel choice and bandwidth selection.

The assumptions of a Markov process underlying the stochastic kernel estimation used in our analysis are noteworthy. We assume that the transition probability of DRG share from value \( i \) to \( j \), say in response to a unit change in relative price of DRG, is constant over time. In other words, the behavioral response of hospital sector to a continuing PPS remains consistent over time. This is a plausible assumption as there has been no significant structural shift in PPS in Victoria in the 8 years considered for our analysis. Such an assumption of time homogeneity is common to Markov models used in economics and health economics applications even for time spans longer than that used here (For example see Quah (1997); Norton (1992); Craig and Sendi (2002)).

In addition, by definition the Markov property which states that given the entire past history the present state depends only on the penultimate state, holds. Since the empirical analysis is done at the DRG level the data is aggregated over all hospitals. Such an aggregation could lead to a bias at the hospital sector level. Thus the empirical analysis is
restricted to major teaching and suburban hospitals to ensure homogeneity among hospitals. These hospitals account for most of the elective surgical episodes in Victoria. In fact for the so called tertiary DRGs all the patients are treated in these hospitals. These hospitals belong to the same payment group under the case-mix funding regime, are research active and have the same technological inputs for treating patients.

The stochastic kernel technique is best suited for our analysis as: i) it allows us to trace the distribution dynamics of DRGs within a distribution which is not possible to estimate using a parametric method ii) it can be applied in the settings of patient level analysis where detailed data is not available for confidentiality reasons and iii) it allows for conditioning of covariates on the distribution to analyse the impact of variables on the distribution dynamics which helps answer interesting policy questions. For example, in this paper we analyse the impact of waiting time incentives and quality of care reforms on distribution dynamics. The results obtained by applying the stochastic kernel mapping on the current distribution can be displayed in a three dimensional diagram or a two-dimensional contour map. The empirical methodology in this paper applies stochastic kernel mapping to quantify the evolution of distribution of DRG’s shares of outliers and inliers by using inpatient data in Victorian hospitals.

3.2 Data

The empirical study uses Australian patient level Victorian Admitted Episodes Dataset (VAED) with a time span of eight years (1998-99 to 2005-06). VAED data is most appropriate for the analysis as this dataset is used for health services planning, policy formulation and case-mix funding purposes. The dataset consists of over one million patient episodes per year with detailed information on length of stay, diagnosis, patient origin etc. As discussed earlier only large hospitals have resources to undertake sophisticated case-mix costing exercises and thus mostly these hospitals will respond to a change in DRG cost weights. Thus our analysis is restricted to large teaching and suburban hospitals. The data is further refined by removing emergency cases.\footnote{Emergency admissions are not included in the analysis because hospital care for such patients has very little discretionary capacity and thus no scope for resource redistribution. On the other hand, public hospital systems have genuine discretion for non-urgent elective surgery patients \cite{Brook2008} which...}
The patients’ diagnosis in the data is coded by Australian Refined Diagnosis Related Group (ARDRG) which are a slight modification of standard DRG codes based on ICD-10 AM classification. The DRG code definitions reported in VAED are adjusted for DRG reclassification and are consistent throughout the eight years. The variable “w12_ifs” classifies each patient episode into a low outlier (coded as “L”), inlier (coded as “I”) or high outlier (”H”). Outlier patient episodes are determined by setting low and high trim points for each DRG. The empirical analysis is done at DRG level.

3.3 Variables

The allocation of resources across DRGs is captured by two types of DRG specific variables: i) Share: LOS of outliers/inliers in a particular DRG relative to LOS of outliers/inliers in all DRGs; ii) Proportion: LOS of outliers/inliers in a particular DRG relative to total LOS (including both outlier and inlier episodes) in all DRGs. Thus for high outliers, the share of each DRG is calculated by dividing the total LOS of high outliers in that DRG to total LOS of high outliers across all DRGs. Thus for DRG $i$ the share of outlier is:

$$Sh_{HO}^i = \frac{LOS_{HO}^i}{\sum_{i=1}^{N} LOS_{HO}^i}$$

where $LOS_{HO}^i$ denotes total LOS in high outlier episodes of DRG $i$.

Thus this share represents the LOS distribution of high outlier episodes in a particular DRG relative to LOS distribution of high outlier episodes in all DRGs. Similarly the share of inliers ($Sh_{I}^i$) can be defined as:

$$Sh_{I}^i = \frac{LOS_{I}^i}{\sum_{i=1}^{N} LOS_{I}^i}$$

where $LOS_{I}^i$ denotes total LOS in inlier episodes of DRG $i$. The proportions for high outlier episodes $Pr_{HO}^i$ and inlier episodes $Pr_{I}^i$ for a particular DRG are defined as:

$$Pr_{HO}^i = \frac{LOS_{HO}^i}{\sum_{i=1}^{N} LOS_i}$$

makes this subgroup of patients appropriate to test our hypotheses of resource dynamics.

11The analysis for low outlier episodes is not done in our study as there are very few DRGs with low outlier episodes which makes it almost impossible to generate a balanced panel of DRGs over eight years.
where $LOS_{i}^{HO}$ denotes total LOS in high outlier episodes of DRG $i$ and $LOS_{i}$ total LOS (including inliers and outliers) of DRG $i$.

$$Pr_{i}^{l} = \frac{LOS_{i}^{l}}{\sum_{i=1}^{N} LOS_{i}}$$

$LOS_{i}^{l}$ denotes total LOS in inlier episodes of DRG $i$ and $LOS_{i}$ total LOS (including inliers and outliers) of DRG $i$.

The main advantage of using proportions and shares is that they neutralize the effect of overall reduction in LOS after the introduction of case-mix funding regime. It is to be noted that the LOS has on an average decreased by 10 percent after the introduction of case-mix funding (AIHW, 2005). The use of proportions and shares in empirical analysis ensures that the patterns of resource allocation across DRGs capture only the redistribution of reduction in LOS. In addition, as evident from Eq (5) theoretically, shares are crucial in determining the impact of reimbursement system on resource use.

The extant studies on the effect of PPS have mainly focussed on its impact on average LOS. The empirical analysis in the present paper takes a step further and sheds light on the distribution of LOS changes across DRGs. The advantage of using shares and proportions is that it will shed more light on how these changes in average LOS have been distributed across DRGs. The empirical analysis uses a balanced panel of 177 DRGs over 96 months. The summary statistics of the panel data are presented in Table 1.

--- Insert Table 1 about here ---

The empirical methodology involves stochastic kernel mapping of above defined shares and proportions. Such a mapping will reveal the direct distribution dynamics of outlier and inlier episodes in a continuing PPS. In addition, based on the actual observed data, a projection of distribution is made for a horizon of 24 months.

4 Results

The stochastic kernel mapping is done separately for inlier patient episodes and high outlier patient episodes. In addition, the stochastic kernel for inlier shares is re-estimated by
conditioning it on the relative cost weights of DRGs. This is done in order to test if relative cost weights explain the changes in the distribution dynamics of resource allocation across DRGs.

4.1 Inliers

The upper half of Figure 2 shows the stochastic kernel and the corresponding contour plot\textsuperscript{12} for 24-month transitions in the share of inliers data. The choice of a 24 month transition is only for convenience and clarity as transitions probabilities across different states of inlier shares are independent of time (time homogeneity assumption of the Markov Process). The main results of the stratification of the distribution are robust to the choice of time horizon. However, to plot a stochastic kernel one has to choose a time horizon. We have chosen a 24 month time horizon for two reasons: i) In Victoria, the cost study to calculate relative cost weights used in period ‘t’ is done in period ‘t-1’ using the patient cost data from period ‘t-2’. For example, the Victorian Health Department engaged KPMG to conduct 2000-01 Victorian cost weight study of 1999-2000 inpatients. The relative cost weights derived from this study were used subsequently for funding purposes; ii) the department frequently uses two years of data to calculate average costs for the purpose of weight calculations. This is done to increase the statistical reliability of weights and to reduce the impact of unexplained cost variations (Victorian Department of Human Services \textit{2006}).

The stochastic kernel could be traced by picking any point on the axis marked ‘Period t’ and extending it parallel to the axis marked ‘Period t + 24’. Thus the stochastic kernel is a probability density function and the projection traced out 24 months ahead is nonnegative and integrates to unity. The projection is analogous to the row of a discrete transition probability matrix where probabilities in different states sum up to 1. Thus stochastic kernel mapping can be used to trace the share of inliers over a 24 month period.

Figure 2 shows how the cross sectional distribution at time \( t \) evolves into that at \( t + 24 \). The distribution will show the behaviour of persistence if most of the graph in Figure 2 was concentrated along the 45-degree diagonal. This would mean that inlier shares in

\textsuperscript{12}The econometric analysis is done using the tsrf shell provided by Danny Quah.
the distribution remain where they began. However, if say share of inliers in the DRGs change drastically i.e. DRGs with a high share of inliers in period $t$ become DRGs with a low share of inliers in period $t + 24$ (high to low transition) and DRGs with low share of inliers become those with a high share of inliers in period $t + 24$ (low to high transition), the stochastic kernel mapping will rotate 90 degrees counter-clockwise from that 45-degree diagonal.

Figure 2 shows a multiple peaks feature in the distribution of inlier shares. The shares of inliers have stratified and the stochastic kernel rises towards three local maxima. This is reflected by an emergence of three peaks which is clear from the corresponding contour plot of the kernel.

—Insert Figure 2 about here—

The lines on the contour plot connect points at the same height on the three-dimensional kernel. The contour plot further reveals the peak at the lower quantile of shares (peak 1) shifts left of the 45 degree line whereas the peaks on the middle quantile (peak 2) and higher quantile (peak 3) remain on the 45 degree line. The emergence of peak 2 is a significant finding. Its main interpretation is that over a 24 month horizon the DRGs with middle quantile inlier shares will increase. Most of the portion of probability mass remains clustered around the main diagonal. However, the two dips on the principal ridge of the distribution (across the 45 degree line) in Figure 2 indicate that portions of the cross section do transit from low to middle level, and high to middle level, thereby contributing to the formation of peak 2. In addition the middle portion of the cross section shows a behaviour of persistence over a 24 month time horizon. Peak 1 has a lower number of DRGs compared to peak 2 and is located slightly to the left of the 45 degree line. This indicates that DRGs with lower shares of inliers in period ‘$t$’ will have their shares decreased in period ‘$t+24$’. Peak 3 has the least number of DRGs which means that over a 24 month horizon the number of DRGs with inlier shares in the higher quantile range in period ‘$t$’ will decrease. The intensity of the graph (marked by darker shades) reveals that variation of shares in DRGs around peak 1 is the lowest and around peak 3 is the highest.

The slight anti clockwise rotation of peak 1 indicates that DRGs with a lower share of inliers in period ‘$t$’ will have their shares further decreased in period ‘$t+24$’. Transition of
LOS shares from low to lower values could be mainly due to a reduction in the number of patients in these DRGs. Given an almost constant patient profile in the last 8 years, this reduction in inpatient numbers could be a result of a change in the intensity of treatment for patients, i.e. patients in DRGs which have low average LOS might be treated as same day patients and such a behaviour might be contributing towards a consistent annual increase of 5 percent in same day separations in Victorian hospitals. Although the funding for same day patients is lower than those of inliers, hospitals save on the fixed costs associated with an inlier multi day admission and hence could find a same day episode relatively profitable.

On the other hand the reduction in inlier shares from ‘low to lower’ for patients in DRGs with a higher average LOS indicates a potential selection effect where these patients are left out of elective hospital treatment altogether. The intradistribution dynamics of the proportion of inliers is presented in the lower half of Figure 2. The proportion of inliers show similar trends as the share of inliers.

We next focus on the factors which explain such distribution dynamics of the inlier shares. The stratification in inlier shares (specifically generation of peak 2) implies that hospitals will redistribute resources for elective surgeries in a way that the share of LOS increases for some DRGs whose value of LOS shares lies between peak 1 and 2 and decreases for DRGs whose share value lies between peak 2 and 3. Such behaviour of Victorian hospital sector could be a result of two factors: i) the government’s policy of targeting select DRGs to reduce waiting times; ii) Hospitals’ policies of maximizing revenue from government funding.

4.2 Impact of waiting times on resource redistribution

The interaction between waiting times and resource allocation can be tested by comparing the changes in waiting times for the three groups of DRGs concentrated on the three peaks of the distribution. Based on unconditional stochastic kernel plotted in Figure 3, the peaks are formed at share values of 0.3%, 0.5% and 1%. Table 2 reports the change in waiting times over a two year rolling window for three categories of DRGs (first: with a share less than 0.3%, second with a share between 0.3 and 0.5 % and third with a share between 0.5 and 1%). Last column shows average change for all DRGs.
One interesting trend that emerges from the above table is the decrease or below average increase in waiting times for DRGs having medium share values (between 0.3-0.5) and an increase in waiting time for the other two categories of DRGs. Theory linking resource allocation and waiting times (Iversen, 1993) argues that for every production capacity there is an optimal waiting time maximising the number of admissions. An increase in wait has two opposing effects with respect to the expected number of admissions: i) the capacity utilization effect: it permits capacity utilization and accordingly number of admissions to increase and ii) it pulls resources away from medical treatment to more administrative tasks thereby reducing number of admissions. Iversen (1993) argues that the first effect dominates the second for moderate waiting times which is the case in our study. Our main finding of stratification in inlier shares is that over a 24 month horizon DRGs shares around peak 2 (with decreasing waiting times) show a behaviour of persistence whereas shares of DRGs around peak 1 and peak 3 are likely to decrease.

Thus there are two main implications of our finding: i) Resources are redistributed in such a way that LOS shares for DRGs experiencing decreasing waiting times (around peak 2) will remain almost the same and ii) DRGs experiencing increasing waiting times (around peak 1 and peak 3) will have LOS shares reduced which will increase capacity utilization and expected number of admissions for these DRGs. Such a trend indicates that hospitals will increase capacity and number of admissions for DRGs having increasing waiting times. Victorian PPS allows funding for this increased capacity through an “additional throughput pool” where access to additional revenue is dependent on whether patients on waiting list are treated (Street and Duckett, 1996). The aim of such a policy is to promote allocative efficiency through the waiting list initiative. The methodology used in this paper and subsequent finds allow to test the response of hospitals to such policy and confirm that waiting time incentive is one of the factors responsible for redistribution of resources across DRGs.
4.2.1 Impact of relative costs of DRGs

Ideally, detailed data on the costing of each patient episode can shed light on the profitability of treating patients in a particular DRG which can be subsequently used to check if resources are being transferred from non profitable to profitable DRGs. Unfortunately, such data is not commonly available for research purposes because of confidentiality issues. Therefore, we use relative cost weights of DRGs as a proxy to test if these are responsible for the stratification of inlier shares. One main contribution of this study is that the proposed methodology outlined in this paper provides a framework to analyse the impact of covariates on distribution dynamics by conditioning the stochastic kernel. The impact of relative cost weights of DRGs on the distribution of inlier shares is discussed next.

The evidence of stratification of inlier shares in the above section was obtained by using unconditional distribution dynamics. The obvious next step would be to do a conditional analysis which sheds more light on the underlying factors that explain the distributional dynamics of inliers. In particular we want to condition on the relative cost weights of DRGs to test if these affect resource redistribution across DRGs. The data on relative cost weights of DRGs is taken from various issues of Victoria - Public Hospitals Policy and Funding Guidelines published by the Department of Health Victoria (Victoria Department of Human Services 2006). These guidelines report relative cost weights for each DRG which are used to calculate the weighted inlier equivalent separation (WIES) for each inlier episode. For example an inlier episode in DRG A with relative cost weight of 0.5 will have a WIES of 0.5 and an inlier episode in DRG B with relative cost weight of 1.5 will have a WIES of 1.5. The funding body decides a dollar value of unit WIES in each funding round. The product of this dollar value to WIES is the relative price of DRG. Thus if a hospital treats exactly the same number of patients with the same number of LOS in DRG A and B, it will get more funding for patients in DRG B as they have higher WIES relative to DRG A. In order to condition on the relative cost of DRGs we weight the DRG shares and proportions used in the unconditional analysis by the corresponding WIES. The conditional analysis uses 75 DRGs which are a subset of DRGs used in the unconditional analysis. The sample of DRGs was restricted to 75 because of a lack of consistent data on cost weights for other DRGs.
The stochastic kernel and corresponding contour maps for weighted shares are reported in Figure 3. The unconditional plots of stochastic kernels and corresponding contour map for the same 75 DRGs are also reported to enable a direct comparison between conditional and unconditional distribution dynamics. The unconditional plots again confirm evidence of stratification in inlier shares as shown by the emergence of three peaks out of which two peaks are prominent. The stochastic kernel on the weighted shares will describe how relative costs of DRGs will alter the cross sectional distribution of shares. Thus, to test our hypothesis that change in relative costs of DRGs results in the stratification of inlier shares, we need to observe if the stochastic kernel transforming the unconditional distribution to conditional one removes the stratification or polarization of inlier shares. The conditioned stochastic kernel reported in Figure 3 confirms that the stratification is removed after the inlier shares are weighted by relative DRG costs. Thus relative costs are one of the determinants of resource allocation across DRGs. The results for proportions show similar trend and are skipped here.

The above result that resource distribution responds to relative cost weights has significant policy implications. The PPS funding formula in Victoria is a method by which health funds are distributed, rather than a price setting mechanism which reflects the actual cost (Victoria, 2001). Such a funding mechanism allows for disparity between cost of treatment and the funding received for a particular DRG, which can potentially make some DRGs loss making. Our results show that hospitals are responding to changes in the relative cost weights. The change in relative weights is a result of an annual adjustment of DRG weights based on Cost Weight Studies which identify the average cost of DRGs with consequent adjustment of weights to reflect relative average costs. In Victoria, PPS is used to fund public not-for-profit hospitals.

The fundamental feature of Victorian PPS is subsidiarity. The principal of subsidiarity means that services provided to any patient should be decided as close as possible to the interface between that patient and hospital and casemix is entirely neutral about the relative cost advantages or disadvantage of particular forms of care. Such a principle implicitly expects that variation in cost weights should not be perceived as a price signal and
hospitals should not adjust their casemix to avoid loss making DRGs. Our results show that the Victorian public hospital sector is indeed redistributing its resources (changing their casemix) in response to changes in the relative cost weights. The main policy implication of such a trend is that patients from loss making DRGs may face adverse health outcomes which will affect the hospital system’s social responsibility of providing adequate care to the community.

4.3 High Outliers

The stochastic kernel mapping and corresponding contour plot for high outlier shares are presented in the upper half of Figure 4. The graph reveals the stochastic kernel is not positive for all shares and hence there is no one to one correspondence in probabilities for some share points. The overall trend shows that over a 24 month period, most of the DRGs with a positive share of high outliers in period ‘t’ will have very low or zero share of high outliers in period ‘t+24’. Though there are some peaks on the contour plot which are randomly distributed over the distribution, most of the distribution is concentrated around the 0-value of the period ‘t+24’ axis– extending parallel to the period ‘t’ axis. This indicates that over a 24 month horizon the cross sectional distribution of DRGs will converge to zero. The main conclusion from this result is that the case-mix funding regime is inducing hospitals to reduce high outlier episodes in elective surgery patients. It has been demonstrated in the literature on Victorian patients (Moje et al., 2006) that it is hospital-acquired injury which leads elective high-outlier patients to stay longer in a hospital. Our results show that over any 24 month horizon high outlier patients will reduce drastically to almost zero. Such a trend might indicate that hospitals are improving quality of care for such patients. We test this hypothesis by using actual quality data measured by the rate of adverse events using methodology suggested by Ehsani et al. (2006) and Jackson et al. (2006). Table 3 reports rate of adverse events for high outlier patients for a rolling window of 24 months starting from year 1999.

—Insert Table 3 about here—

The above trends show that quality of care (the inverse of rate of adverse events) has indeed increased in most cases with an average increase of 3% over the last 7 years.
The mapping using proportions of high outlier episodes and the corresponding contour maps are presented in lower half of Figure 4. The intradistribution dynamics of proportions is similar to those of shares.

---Insert Figure 4 about here---

As discussed earlier, a high outlier episode is funded on a per diem basis once patient’s LOS is three times more than the average LOS. However, the per diem payment rate in Victoria is determined by a fraction of the inlier weight and does not reflect the actual cost of treating a high outlier patient. Thus, even though the marginal revenue of keeping a patient is positive in a high outlier case, it might be below the marginal cost and hospitals might prefer to under service such patients. Unfortunately, it is not possible to do a conditional analysis on high outlier episodes at this stage because of lack of availability of data on some variables required to calculate high outlier DRG cost weights.

5 Conclusions

This paper has analysed the patterns of shares and proportions of high outlier and inlier episodes across DRGs from the perspective of distribution dynamics. The empirical analysis has captured a stratification behaviour in shares of inlier episodes and uni-polarization in the case of high outliers. For inlier episodes, over a 2 year transition, DRGs with high quantile shares of LOS are likely to have their shares reduced (transition of LOS shares from high to low), DRGs with middle quantile shares of LOS will be likely to have their shares unchanged (persistence of shares), DRGs with low shares of LOS will likely to have their shares further reduced (transition of shares from low to lower). For high outlier episodes, over a 2 year transition, the cross section LOS distribution of DRGs converges to zero. This indicates that in the current case-mix funding regime, the probability of inpatients episodes turning into high outlier episodes will decrease to almost zero.

The methodological contribution of the present paper is that it has outlined an empirical methodology to identify patterns of inter-DRG resource allocation in a continuing PPS. The application of stochastic kernel (a non parametric approach) to capture the evolution of the whole distribution has an advantage over standard regression methods which only provide
a picture of the behaviour of the conditional mean. Further, the panel data econometric methods control for and thus absorb heterogeneity into “individual effects” which would not allow us to explain differences across DRGs.

Three main findings in our study which are significant from the policy perspective are:

i) The hospital sector is responding to waiting times by redistributing resources in such a way that the capacity and number of admissions will increase for DRGs having increasing waiting times; ii) The quality of care is improving for elective surgery patients thereby reducing the number of high outlier episodes and iii) The hospital sector is changing its casemix in response to changes in relative cost weights. The above findings reflect on the efficacy of casemix funding regime in Victoria. Specifically, promoting allocative efficiency through waiting list initiative and access to safe, high quality health care are two key priorities of the Victorian Government. Our findings show that the hospital sector is responding positively to waiting time and quality of care initiative. The third finding of hospital sector as a whole changing its casemix in response to variation in relative cost weights should be further explored as such a response could lead to adverse health outcomes for a group of patients which will affect the hospital system’s social responsibility of providing adequate care to the community.

The results from the present study could be useful to understand the trends in efficiency in Victorian hospitals. As Jacobs et al. (2006) argue stand alone efficiency analysis treats a DMU as a black box and explains little as to why a particular level of efficiency is observed. The main inferences from our study could be used as an additional information by policy makers to understand the underlying factors that could be causing changes in efficiency levels.
References


Newhouse, J. (1983). Two prospective difficulties with prospective payment of hospitals, or, it’s better to be a resident than a patient with a complex problem. *J Health Econ* 2(3), 269–74.


Table 1: Summary Statistics: Balanced Panel of DRGs

<table>
<thead>
<tr>
<th>Variables</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Proportions (%)</strong></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Inliers</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>overall</td>
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<td>0.494</td>
<td>0.000</td>
<td>4.311</td>
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<tr>
<td>between</td>
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<td>0.026</td>
<td>2.794</td>
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<tr>
<td>within</td>
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<td>-1.577</td>
<td>3.953</td>
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<tr>
<td><strong>High Outliers</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>overall</td>
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<td>0.119</td>
<td>0.000</td>
<td>3.204</td>
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<td>between</td>
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<td>0.000</td>
<td>0.957</td>
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<tr>
<td>within</td>
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<td>-0.914</td>
<td>2.882</td>
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<tr>
<td><strong>Shares (%)</strong></td>
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<td></td>
<td></td>
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<tr>
<td><strong>Inliers</strong></td>
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<td>4.970</td>
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<td>between</td>
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<td><strong>High Outliers</strong></td>
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<td>within</td>
<td>0.832</td>
<td>-9.082</td>
<td>19.227</td>
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</table>

Sample Size: 16992
(Number of DRGs: 177
Time Span: 96 Months (July 1998 to June 2006)
Table 2: Inlier Patients: Change in Waiting Times

<table>
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<tr>
<th>Time Period</th>
<th>DRG Shares →</th>
<th>&lt;0.3</th>
<th>0.3-0.5</th>
<th>0.5-1.0</th>
<th>Average Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>2002-2004</td>
<td>ΔWait. Time (%)</td>
<td>8.32</td>
<td>4.70</td>
<td>1.15</td>
<td>4.75</td>
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<tr>
<td>2003-2005</td>
<td>ΔWait. Time (%)</td>
<td>29.82</td>
<td>6.63</td>
<td>8.16</td>
<td>18.96</td>
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<tr>
<td>2004-2006</td>
<td>ΔWait. Time (%)</td>
<td>17.95</td>
<td>-23.34</td>
<td>25.04</td>
<td>11.72</td>
</tr>
</tbody>
</table>
Table 3: High Outlier Patients: Rate of Adverse Events

<table>
<thead>
<tr>
<th>Time Period</th>
<th>Change in Rate of adverse events (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1999-2001</td>
<td>-7.89</td>
</tr>
<tr>
<td>2000-2002</td>
<td>-4.73</td>
</tr>
<tr>
<td>2001-2003</td>
<td>-2.30</td>
</tr>
<tr>
<td>2002-2004</td>
<td>2.36</td>
</tr>
<tr>
<td>2003-2005</td>
<td>1.05</td>
</tr>
<tr>
<td>2004-2006</td>
<td>-7.45</td>
</tr>
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</table>
Outlier distributions

Figure 1: Distributional Dynamics

Increasing share of outliers
Figure 2: Inliers: Shares and Proportions
Figure 3: Inliers: Shares conditional on Relative DRG Costs

***: The axes rescaled by a factor of 10.
Figure 4: Outliers: Shares and Proportions
A.1 Estimating Stochastic Kernels

For estimation purposes, stochastic kernel can be written as (Arbia et al., 2005):

\[ \phi_{t+n}(S) = \int_0^{\infty} f_n(S'|S) \phi_t(S) dS \]  \hspace{1cm} (A-1)

where \( S \) is the share in period \( t + n \) and \( S' \) is the share in period \( t \). \( f_n(S'|S) \) is the conditional density which describes the probability that a DRG moves to a specific state of share, given the share in period \( t \). Thus a stochastic kernel can be expressed as a conditional density and its estimator can be derived from the estimation of conditional density. A nonparametric estimator for the conditional density as proposed by Rosenblatt (1956) is given by:

\[ \hat{f}_n(S'|S) = \frac{\hat{g}_n(S', S)}{\hat{h}_n(S')} \]  \hspace{1cm} (A-2)

where the estimator for the joint density \( \hat{g}_n(S', S) \) is given by:

\[ \hat{g}_n(S', S) = \frac{1}{Ja} \sum_{j=1}^{J} K \left( \frac{||S' - S'_j||}{a} \right) \left( \frac{||S - S_j||}{b} \right) \]  \hspace{1cm} (A-3)

and the estimator for the marginal density \( \hat{h}_n(S') \) is given by:

\[ \hat{h}_n(S') = \frac{1}{Ja} \sum_{j=1}^{J} K \left( \frac{||S' - S'_j||}{a} \right) \]  \hspace{1cm} (A-4)

where \( a \) and \( b \) are bandwidth parameters controlling the smoothness of fit, \( K \) is the chosen kernel function and \( ||S' - S'_j|| \) and \( ||S - S_j|| \) are the Euclidian metrics. Substituting Eqs (9) and (10) in (8) the conditional density estimator can be rewritten as:

\[ \hat{f}_n(S'|S) = \frac{1}{b} \sum_{j=1}^{J} w_i(S') K \left( \frac{||S - S_j||}{b} \right) \]

where

\[ w_i(S') = K \left( \frac{||S' - S'_j||}{a} \right) / \sum_{j=1}^{J} K \left( \frac{||S' - S'_j||}{a} \right) \]

The above kernel estimator is the Nadaraya-Watson kernel regression estimator. It shows that a conditional density can be obtained by the sum of \( J \) kernel functions in \( S \) space weighted by the \( w_i(S') \) in \( S' \) space.
A.2 Kernel Choice and Bandwidth Selection

Throughout this appendix $f$ is kernel estimate, $K$ is kernel and $h$ is bandwidth. Given that we have used general weight function estimate as the kernel estimate for any $x$, the expected value and variance of such an estimate is given by:

$$E[\hat{f}(x)] = \int \frac{1}{h} K\left(\frac{x - y}{h}\right) f(y) dy$$  \hspace{1cm} (A-5)

$$\text{var}[\hat{f}(x)] = \frac{1}{n} \int \frac{1}{h^2} K\left(\frac{x - y}{h}\right)^2 f(y) dy - \left\{ \int \frac{1}{h} K\left(\frac{x - y}{h}\right) f(y) dy \right\}^2$$  \hspace{1cm} (A-6)

The expected value of $\hat{f}$ is a smoothed version of the true density, obtained by convolving $f$ with the kernel scaled by the bandwidth. Thus the density estimate is of the form: smoothed version of true density + random error. The smoothed density depends on the choice of parameters through kernel and band width choice. Such a choice is critical for the discrepancy of density estimator $\hat{f}$ from the true density $f$. Most widely used measure of such a discrepancy which reflects the global accuracy of $\hat{f}$ as an estimator of $f$ is the mean integrated square error (MISE) defined as:

$$\text{MISE}(\hat{f}) = E \int (\hat{f}(x) - f(x))^2 dx$$  \hspace{1cm} (A-7)

which can be rewritten as:

$$\text{MISE}(\hat{f}) = \int (E\hat{f}(x) - f(x))^2 dx + \int \text{var}\hat{f}(x) dx$$

the sum of the squared bias ($\text{bias}_h$) and the variance at $x$. Assuming kernel $K$ is symmetric function satisfying:

$\int K(t) dt = 1$, $\int tK(t) dt = 0$, and $\int t^2K(t) dt = k_2 \neq 0$ and replacing $y = x - ht$ the integrated square bias is given by:

$$\int \text{bias}_h(x)^2 dx \approx \frac{1}{4} h^4 k_2 \int f''(x)^2 dx$$

and the variance is given by:

$$\int \text{var}\hat{f}(x) dx \approx n^{-1} h^{-1} \int K(t)^2 dt$$
Given the bias and variance, approximate value of MISE will be

\[
MISE \approx \frac{1}{4} h^4 k_2 \int f''(x)^2 dx + n^{-1} h^{-1} \int K(t)^2 dt
\]  

(A-8)

The above equation reveals that there is a trade-off between the bias and variance terms: the bias can be reduced at the expense of increasing the variance, and vice versa, by adjusting the amount of smoothing or the bandwidth. Suppose \( h \) is chosen to minimise MISE. Choosing very low \( h \) will eliminate the bias but increase the integrated variance. On the other hand choosing high value of \( h \) will reduce the random variation (measured by variance) but will increase the bias or the systematic error. Thus choice of bandwidth implies a trade-off between random and systematic error.

Silverman (1986) shows that the optimal value of bandwidth \( h \) that minimises approximate MISE is given by:

\[
h_{opt} = k_2^{-2/5} \left\{ \int K(t)^2 dt \right\}^{1/5} \left\{ \int f''(x)^2 dx \right\}^{-1/5} n^{-1/5}
\]  

(A-9)

It should be noted that the optimal value of bandwidth depends on the unknown kernel density estimate. We first discuss the choice of this unknown kernel.

**Choice of kernel:**

Substituting the value of \( h_{opt} \) (Eq. A-9) back into formula for approximate MISE (Eq. A-8) shows that if \( h \) is chosen optimally, then the approximate value of MISE will be

\[
\frac{5}{4} C(K) \left\{ \int f''(x)^2 dx \right\}^{1/5} n^{-4/5}
\]

where the constant \( C(K) \) is given by:

\[
k_2^{2/5} \left\{ \int K(t)^2 dt \right\}^{4/5}
\]

Above formulae show that minimising MISE means choosing kernel \( K \) which minimises \( C(K) \) which in turn reduces to minimising \( \int K(t) dt \). The kernel which minimises this term is the Epanechnikov kernel.
Choice of Bandwidth

As discussed earlier, Least square cross-validation method, a completely automatic method based on the dataset on hand, is widely used for choosing bandwidth. It is based on minimizing integrated square error which for any estimator \( \hat{f} \) can be written as:

\[
\int (\hat{f} - f)^2 = \int \hat{f}^2 - 2 \int \hat{f}f + \int f^2
\]

Since last term of above equation does not depend on \( \hat{f} \), the optimal choice of bandwidth corresponds to the choice which minimises \( \int \hat{f}^2 - 2 \int \hat{f}f \). The basic principle of least square cross-validation method is to construct an estimate of this term from the data themselves and then to minimise this estimate over \( h \) to give the choice of bandwidth. Thus the optimal value of bandwidth depends on the data set used and is not chosen subjectively.