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How Does the Effectiveness of Fiscal Deficit Stimulus Depend on the Expected Path of Debt Stabilisation? Unorthodox Outcomes When Monetary Policy Follows a Simple Taylor Rule.

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Abstract

Staggered prices and finitely-lived agents create scope for a debt-financed tax cut to raise output. We study analytically how the impact multiplier depends on whether debt is expected gradually to return to its original level or else to rise to a permanently higher level, and on the speed of this. Under a simple Taylor Rule, the first debt path raises, but the second *lowers*, output on impact. With the first debt path, the multiplier is also probably hill-shaped in debt persistence. However, even a short-lived initial exogenous nominal interest-rate peg makes the multiplier probably positive with both debt paths.

Keywords

fiscal deficit, staggered prices, finitely-lived agents, overlapping generations, output multiplier, debt persistence, debt gradualism, Taylor Rule, temporary nominal interest-rate peg

JEL Classification

E62, E63, H62

1. Introduction

The first decades of the twenty-first century have seen occasional significant returns to Keynesian policies of expanding or contracting fiscal deficits as a way of boosting or dampening aggregate demand and thereby also output and inflation. Expansions were witnessed, for example, during the 2008 global financial crisis and the 2020 Covid pandemic; and contractions during the 2022 global inflation surge after the Russian invasion of Ukraine. However, particularly in the case of expansions, fiscal stimulus needs to be followed by fiscal stabilisation, to prevent an explosive path of government debt. In a world of forward-looking agents, the question then arises as to how the expected path of debt which is triggered by the short-run deficits influences the effectiveness of the stimulus. In this paper we seek to explore this question theoretically.

To do this, we need a modelling framework which gives clear scope for fiscal deficit stimulus to 'work', which means not only some stickiness in nominal prices, but also avoidance of Ricardian Equivalence so that government debt is not neutral in its effects. Although fiscal stimulus in the form of tax-financed government spending has received much attention within the Dynamic New Keynesian framework, fiscal stimulus in the form of debt-financed tax cuts has until recently been fairly neglected. This, however, is beginning to change, especially with the application of 'HANK' (Heterogeneous Agent New Keynesian) models to the question.[1] HANK models produce non-Ricardian behaviour by appealing to credit constraints, preventing some agents from borrowing and lending as freely as they would like. Another acknowledged mechanism causing non-Ricardian behaviour is presence of overlapping generations (OLGs), where private agents' lives are shorter than the infinite life of the government, causing them to ignore taxes which are expected to arrive after their death. Recent work on HANK models has emphasised similarities between these frameworks, with it being argued that a substantial proportion of the empirically-relevant features of consumption behaviour found in sophisticated HANK models can be captured by the simple, analytically tractable, 'perpetual

[1] Examples of recent contributions of this kind are Angeletos et al. (2024), Auclert et al. (2024) and Wolf (2025).

youth' OLG model (Blanchard (1985))[2]. In the present paper, we use the perpetual youth structure because of its potential for yielding analytical insights, but these similarities show that it can be interpreted as capturing not just life-cycle effects on consumption, but also – rather more incompletely – effects of financial frictions.

Another key consideration which frames our analysis is that it is well established that the monetary policy regime matters, even though our primary interest is in fiscal policy. In the era of monetary policy conducted via interest-rate control, we would ideally like to use a simple, neutral, rule for the interest rate. The best known is the Taylor Rule (Taylor (1993)), although this exists in various versions. We indeed take a simple version of the Taylor Rule as our baseline case. This is because it is familiar, it has empirical support, and it is robust in the sense that it can be applied both in sticky-price and flexible-price economies, which is important in case of any doubt about the degree of price stickiness. By contrast, a number of recent studies of fiscal stimulus have assumed as their baseline case a regime of pegging the real interest rate exogenously. This can be helpful analytically, but it is not encountered as a practical real-world rule and it is not a robust rule, in the sense that it is infeasible under fully flexible prices. It is also infeasible if it is taken to mean fixing the real interest rate forever at its pre-shock value under conditions in which the level of debt changes permanently, because the latter necessitates a higher long-run equilibrium real rate. A Taylor Rule, with its defining characteristic that the nominal interest rate is a function of the current inflation rate, hence seems to be the obvious rule to study as an initial, basic, general-purpose, monetary policy description.

A feature of our approach which enables analytical solutions is that we parameterise the path of government debt so that it follows a simple, exogenous, geometrically-declining – or, alternatively, geometrically-rising – trajectory, governed by a speed-of-adjustment parameter, μ. We study two types of fiscal deficit shock. The first involves a 'temporary-but-persistent' increase in debt, in which, after an initial deficit, fiscal surpluses are then run so that debt returns asymptotically to its pre-shock level. The second involves a 'permanent-but-gradual'

[2] This point in particular is made by Angeletos et al. (2024). They argue that the pattern of 'intertemporal marginal propensities to consume' (a concept introduced in Auclert et al. (2024)) which is implied by fully-featured HANK models constructed to match the data, can be quite well approximated by the perpetual-youth OLG model. They also point out that the perpetual-youth model is formally equivalent to a particular stylised but tractable HANK model, as Farhi and Werning (2019) have shown.

increase, in which at first repeated deficits are run, but these gradually turn into small surpluses so that debt stabilises at a new, higher, level.[3] A key point of interest is to know how μ, which represents either the expected 'persistence' or the expected 'gradualism' in the time path of debt, affects the impact multiplier on output of the fiscal shock. In the Dynamic New Keynesian literature, the study of temporary-but-persistent shocks to various policy variables is commonplace, but the study of 'permanent-but-gradual' shocks is less so. Hence a particular focus will be on the difference which the second type of debt path makes.

As the paper's title indicates, the most striking findings from this exploration are that under a simple Taylor Rule the outcomes of certain variations in the expected path of debt are not what orthodox Keynesian reasoning would lead one to expect. A debt-financed tax cut associated with a permanent-but-gradual increase in debt causes a *recession* on impact, rather than a boom. On the other hand, a debt-financed tax cut associated with a temporary-but-persistent increase in debt causes a boom on impact, but greater debt persistence (i.e. slower subsequent fiscal consolidation) is very likely to affect the size of the boom *non-monotonically*, such that the size of the impact multiplier first increases, then peaks, and then declines, as persistence increases.[4] This implies that there exists an 'impact-maximising' speed of fiscal consolidation.

To see how more orthodox results might arise and to understand better the reasons for the outcomes under a simple Taylor Rule, we proceed to modify the monetary policy rule. A large body of empirical work finds that, in practice, nominal-interest-rate setting is subject to inertia. A plausible generalisation of the simple Taylor Rule regime is thus to introduce an initial phase, of exogenous length, in which the nominal interest rate remains pegged at its pre-shock value, with the Taylor Rule itself only kicking in at the end of this phase. This could alternatively represent a phase during which the rate is stuck at its 'zero lower bound', a situation which has received much attention in recent years. We are still able to solve the model analytically in this regime. We find that a peg for as few as two periods may be sufficient to reverse the recession outcome under a permanent-but-gradual debt increase, converting it into a boom. This reversal

[3] See Figure 1 for illustrations of these two paths.
[4] To isolate the intertemporal redistribution role of taxation, we consider only lump-sum taxes.

will occur for sure if the debt increase is not too gradual. We further find that a peg for as few as two periods is sufficient to ensure that the boom resulting from a temporary-but-persistent debt increase, even though it could still be 'hill-shaped' in the degree of persistence, is now much more likely to be always increasing in persistence, and will be so for sure if the Taylor Rule is very strongly counter-inflationary. The general lesson is therefore that obtaining orthodox Keynesian results from fiscal deficit stimulus in a reliable way depends critically on there being a delay in the application of the Taylor Rule.

Our analysis shares a number of elements with some papers in the literature, especially Angeletos et al. (2024). These authors also construct a Dynamic New Keynesian model with perpetual-youth agents and use it to study the effects of fiscal deficits and government debt. Although their main aim is to consider whether deficits can be 'self-financing', they obtain, among other results, that the impact multiplier of a debt-financed tax cut is always increasing in debt persistence. We too obtain this for a temporary-but-persistent debt increase if we follow Angeletos et al.'s monetary rule of holding the real interest rate constant. Our contribution is hence especially to explore other monetary policy regimes, and moreover to do so by analytical methods; and also to study permanent-but-gradual debt increases, which Angeletos et al. do not do. Our paper also overlaps with Auclert et al. (2024), who, like us, particularly pay attention to the impact multiplier. Their paper is more empirically-focused than ours and it presents a general framework for applying a large variety of 'HANK' models to the fiscal stimulus question, with no attention given to OLG models. Auclert et al. also do not consider permanent-but-gradual debt increases. However some of the results which we obtain for temporary-but-persistent debt increases are consistent with some of their numerically-derived results for such cases. For these, our paper can be seen as contributing analytical proofs and a more explicit set of microfoundations.

In the older literature, the first paper to study fiscal policy in a perpetual-youth economy with Calvo-style price setting (Calvo (1983)) was Leith and Wren-Lewis (2000). Woodford (2011) presented many interesting analytical results for tax-financed government spending increases in a Dynamic New Keynesian framework, especially through the device of allowing for geometric decay of the initial shock, which we also adopt here. Our paper can also be seen as an extension of Ascari and Rankin (2013) to consider a greater variety of debt paths. Albonico et al. (2021), too, build on that paper with a particular focus on how the steady state level of debt affects the multiplier.

In what follows, the structure of the model is laid out in Section 2. The effects of a deficit with alternative paths for stabilising debt are studied under a simple Taylor Rule in Section 3. Section 4 then extends the monetary regime to include a temporary exogenous peg of the nominal interest rate. Section 5 concludes.

2. The Structure of the Model

Since our aim is not to try to 'match the data' of real-world fiscal policy, but rather to understand underlying theoretical linkages in as enlightening a way as possible, we use the simplest model incorporating the key features which we wish to study, namely finitely-lived agents and staggered price setting. Specifically, households are assumed to have a constant probability of death, following Blanchard (1985); while firms are assumed to have a constant probability of being allowed to adjust prices, following Calvo (1983). We embed these in a Dynamic New Keynesian model of a closed economy of a type familiar from the work of Woodford (2003), Galí (2015) and many others. As befits our interest mainly in short-run, business-cycle, behaviour, we abstract from capital and its accumulation and treat labour as the only productive input.

(i) Household behaviour

The constant population of households is normalised to one. Every period, each household has a constant probability of death equal to 1-q. In aggregate, this is also the proportion who die every period, and every period they are replaced by 1-q newborn. 's' denotes the birth period of a household. Households are born with zero financial assets, but may accumulate assets or debt during their lifetimes. As in Blanchard (1985), there is a competitive insurance market such that in equilibrium each household receives a gross annuity of 1/q on its financial assets if it remains alive, but its assets pass to the insurance company if it dies.[5] We shall not consider aggregate uncertainty, so that all financial assets are perfect substitutes from the holders' viewpoints and hence pay the same nominal and real rates of return.

A household's optimisation problem as of period n may be expressed as:

$$\text{maximise} \quad \Sigma_{t=n}^{\infty}(\beta q)^{t-n}[\ln C_{s,t} \; + \; \psi \ln(1-L_{s,t})]$$

$$\text{subject to} \quad C_{s,t} + F_{s,t+1} \; = \; (1/q)(1+r_{t-1})F_{s,t} \; + \; (W_t / P_t)L_{s,t} \; + \; \Pi_t / P_t \; - \; \tau_t$$

$$\text{for } t = n,...,\infty, \tag{1}$$

where $0 < \beta < 1$, $0 < q \leq 1$, $\psi > 0$, $s \leq n$. To enable aggregation across households of different ages we need their preferences to be homothetic over composite consumption and leisure, and here we ensure this by a simple log-linear utility function.[6] In the budget constraint, $F_{s,t}$ is the household's real financial assets at the start of period t, exclusive of the real interest (at rate r_{t-1}) due on them.[7] W_t is the money wage earned in the competitive, flexible-wage, labour market. Each household also receives an equal share of firms' profits, Π_t/P_t, and is subject to a lump-sum tax, τ_t. Although this is a monetary economy and thus contains nominal variables, we will treat it as 'cashless', i.e. households' demand for real money balances is assumed to be sufficiently small that as a reasonable approximation it can be neglected.

The first-order conditions which result from solving this problem yield a 'labour supply' function and an Euler equation for optimal intertemporal consumption choice for the individual household. Since they are linear in ($C_{s,t}$, $L_{s,t}$) with coefficients independent of s, they have

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[5] Here our formal treatment follows Blanchard's microfoundations, but, as indicated in the Introduction, alternative microfoundations which would yield the same aggregate behaviour are proposed by Farhi and Werning (2019, Sect. IIIA). In their account, 1-q represents the probability of an idiosyncratic negative shock to an agent's discount factor, rather than physical death. It is assumed that agents cannot borrow. They hence face non-negativity constraints on their financial wealth and the shock is taken to be large enough that this constraint binds. Nevertheless, an insurance scheme against such shocks is still assumed to exist.

[6] As discussed in Ascari and Rankin (2007), such a function implies that some old households will have negative labour supply. This is an unattractive feature, microeconomically speaking. However we accept it here since it is not critical to the macroeconomic properties of the model. It allows us to use a simpler specification than would otherwise be necessary.

[7] The real and nominal interest rates between periods t and t+1 are related by $1+r_t \equiv (1+i_t)P_t / P_{t+1}$.

equivalents in terms of aggregate variables. Denoting the latter by dropping the '*s*' subscript (so $X_t \equiv \Sigma_{s=-\infty}^{t}(1-q)q^{t-s}X_{s,t}$, for any variable $X_{s,t}$ [8]), the aggregate relationships are:

$$L_t \;=\; 1 \;-\; \psi P_t C_t / W_t\,, \tag{2}$$

$$C_{t+1} \;=\; \beta(1+r_t)C_t \;-\; (1+\psi)^{-1}(1/q-1)(1-\beta q)(1+r_t)F_{t+1}\,. \tag{3}$$

As in Blanchard (1985), the 'aggregate' version of the consumption Euler equation contains F_{t+1}, unlike the 'individual' version. This is a compositional effect due to the fact that the set of households over which C_{t+1} is an aggregate is not the same as the set over which C_t is an aggregate: some of the latter die and are replaced between t and t+1.[9] It can be seen to drop out if households are infinitely-lived (q = 1). For a given expectation of C_{t+1}, it implies a positive effect of households' financial wealth on current consumption. Since, in equilibrium, this wealth will consist of the stock of government debt, this is the channel through which an increase in debt raises aggregate consumption.

A continuum of varieties of the consumption good is assumed, with varieties indexed by z, for all $z \in [0,1]$. A household's composite consumption, $C_{s,t}$, is a CES function of these varieties, where the constant elasticity of substitution is θ (> 1). When the household optimally allocates a given nominal expenditure, $I_{s,t}$, across varieties, this results in a demand function for an individual variety of:

$$C_{s,t}(z) \;=\; [P_t(z)/P_t]^{-\theta} I_{s,t}/P_t \qquad \text{where} \qquad P_t \;\equiv\; [\textstyle\int_0^1 (P_t(z))^{1-\theta}dz]^{1/(1-\theta)}\,. \tag{4}$$

The maximised value of $C_{s,t}$ is then $I_{s,t}/P_t$, P_t being the economy's consumer price index.

(ii) Firm behaviour

Each variety of good is produced by a firm which is its sole supplier, so that the market structure is one of monopolistic competition. Firm z produces output $Y_t(z)$ with the technology $Y_t(z) = [L_t(z)]^{\sigma}$ ($0 < \sigma \le 1$), where $L_t(z)$ is its labour input. The demand function which it faces

[8] Except for financial assets, where $F_t \equiv \Sigma_{s=-\infty}^{t-1}(1-q)q^{t-1-s}F_{s,t}$.

[9] In deriving (3), we make use of an individual household's consumption function, giving its consumption as an explicit linear function of its total lifetime human and non-human wealth.

is the aggregate across all consumers of the function in (4). We may write this as $Y_t(z) = [P_t(z)/P_t]^{-\theta} Y_t$, where $Y_t \equiv [\int_0^1 P_t(z)Y_t(z)dz]/P_t$ is aggregate real output. Since firm z is infinitesimal relative to the economy as a whole, it takes (Y_t, P_t) as given. In the labour market the firm acts as a perfect competitor, drawing on the economy-wide pool of labour.

Following Calvo (1983), a firm can only change its price upon receiving a random signal which arrives with probability 1-α in each period. The 'new' price, which is chosen by firms receiving such a signal in period t, is X_t. X_t maximises the expected discounted value of the firm's profits, taking account of the sequence of expected demand functions for its current and future output, the probabilistic constraint on when it can next adjust its price, and the current and expected future values of the wage. This is by now a well-understood problem, so we shall not dwell on its details here. Its solution is the following expression:

$$X_t = \left[\frac{\theta}{\theta - 1} \frac{1}{\sigma} \frac{\Sigma_{j=0}^{\infty} \alpha^j \Delta_{t,t+j} W_{t+j} P_{t+j}^{\theta/\sigma - 1} Y_{t+j}^{1/\sigma}}{\Sigma_{j=0}^{\infty} \alpha^j \Delta_{t,t+j} P_{t+j}^{\theta - 1} Y_{t+j}} \right]^{1/[1-\theta+\theta/\sigma]}, \tag{5}$$

where $\Delta_{t,t+j} \equiv (1+r_t)^{-1} ... (1+r_{t+j-1})^{-1}$ ($\equiv$ 1 if $j = 0$). Given Calvo-style price-staggering, the price index in any period can be written:

$$P_t = [\Sigma_{j=0}^{\infty} (1-\alpha)\alpha^j X_{t-j}^{1-\theta}]^{1/(1-\theta)}. \tag{6}$$

(iii) Behaviour of the monetary and fiscal authorities

The rules governing monetary and fiscal policy will be specified below. Here we simply note that the instrument of monetary policy in any period t is taken to be the nominal interest rate, i_t; while the instrument of fiscal policy in period t is taken to be the stock of government debt, D_{t+1} (being the stock at the end of period t).

The government's budget constraint is:

$$D_{t+1} = (1+r_{t-1})D_t - \tau_t. \tag{7}$$

D_t is government debt measured in units of the composite consumption good. We treat the value of government debt as being indexed to P_t rather than fixed in nominal terms, which means we abstract from the possibility of the government reducing its real debt burden by unexpected price level increases. We also confine attention to lump-sum taxation in order to focus on redistribution rather than incentive effects. We furthermore omit government spending since the role of this has been much studied elsewhere. (7) thus serves to determine the level of taxation, τ_t, in any period, given the inherited value of D_t and the chosen value of D_{t+1}. Moreover we assume that the government always obeys a 'No-Ponzi-Game' condition, so that it satisfies the intertemporal budget constraint:

$$D_t \;=\; \Sigma_{j=0}^{\infty}\Delta_{t,t+j}\tau_{t+j}\,. \tag{8}$$

(iv) Market equilibrium conditions

Since there is no investment or government spending, the goods market equilibrium condition (both for individual goods z and the composite good) is simply $Y_t = C_t$. In the financial asset market, the only available asset is government debt, so the equilibrium condition is $F_{t+1} = D_{t+1}$. In the labour market, an individual firm's demand for labour, taking its output $Y_t(z)$ as given, can be considered to be its inverted production function. Firms' outputs must satisfy their individual demand functions. If we use these to substitute out firms' outputs from their inverted production functions and then aggregate across firms, we obtain the left-hand side of the following equation:

$$Y_t^{1/\sigma}P_t^{\theta/\sigma}\Sigma_{j=0}^{\infty}(1-\alpha)\alpha^j X_{t-j}^{-\theta/\sigma} \;=\; 1 \;-\; \psi P_t Y_t \,/\, W_t\,. \tag{9}$$

The right-hand side is aggregate labour supply, from (2). Hence (9) imposes labour market equilibrium, implicitly determining the wage, W_t.

In summary, the complete set of equilibrium conditions, applying in every period t, is (3) (with $C_t = Y_t$, $F_{t+1} = D_{t+1}$), (5), (6) and (9), together with rules determining i_t and D_{t+1} to be specified below. The government budget constraint (7) is in general not needed as an extra condition, since we will treat D_{t+1} as the exogenously-chosen fiscal policy instrument, leaving (7) to determine τ_t residually.

(v) The zero-inflation steady state equilibrium

In the long run the economy will converge on a zero-inflation steady state, so it is helpful to understand the properties of the latter. In a zero-inflation steady state all the effects of price staggering disappear, so the equilibrium should be the same as under fully flexible prices.

Steady-state output is determined by the price and wage sector, given by (5), (6) and (9). Setting variables to time-invariant values here, we obtain:

$$Y = \left[1 + \frac{\theta}{\theta - 1}\frac{\psi}{\sigma}\right]^{-\sigma}. \tag{10}$$

(10) shows that steady state output is independent of any parameters of monetary policy. This is to be expected, given that price rigidities are only temporary. Perhaps more surprisingly, (10) also shows that output is independent of government debt, *D*. It might have been thought that higher government debt would raise households' demand for leisure and so reduce labour supply and thereby output. There is such an effect, but it turns out that it is cancelled out by the rise in the real interest rate, described below. This rise reduces the expected present value of households' time endowments measured in terms of goods, so that, overall, the exogenous component of households' total lifetime wealth is unchanged, and therefore labour supply and output are unchanged.[10]

The steady state real interest rate can be determined from the steady state version of the 'aggregate Euler equation', (3). Using the foregoing relationships and setting variables to time-invariant values, we obtain:

$$1 + r = \frac{1}{\beta - (1+\psi)^{-1}(1/q - 1)(1 - \beta q)D/Y}. \tag{11}$$

Since Y is independent of D, it is clear from this that r is increasing in D, provided $q < 1$. This is a standard property of a closed-economy overlapping-generations model: see, e.g., Diamond (1965). One way of explaining it is as follows. An increase in r causes households to choose a stronger upward 'tilt' to their desired lifetime consumption profiles. Since their lifetime profile of labour income is 'flat' and they are born with zero financial assets, to achieve this they need

[10] The same property for a somewhat similar economy was noted by Ganelli (2007).

to accumulate assets over their lifetimes, because it is the rising interest income on these which enables higher consumption later in life. Hence a rise in r increases the household sector's steady state demand for financial assets. If the supply of government debt increases, a rise in r is therefore necessary to clear the asset market. Note that when government debt is zero, $1+r = 1/\beta$, i.e. the gross real interest rate simply equals households' gross pure time preference rate. This is the same outcome as when agents are infinitely-lived ($q = 1$). It occurs because the desired time profile of consumption is then 'flat' and thus the same as that of labour income, so households have no need for financial wealth to match up the two profiles.

(vi) Log-linearisation and a partially-reduced form of the model

To study dynamic behaviour we take a log-linear approximation of the model. This is done around a steady state in which inflation and government debt are zero (the 'reference' steady state). The choice to log-linearise, and the choice of the point around which to do so, are not innocuous, because there are significant non-linearities in the model.[11] However it is necessary to do this to enable us to conduct an in-depth inspection of the underlying dynamic structure. The reference steady state used here is the one which yields the simplest structure and so which best facilitates a baseline analysis.

Three helpful reduced-form equations which result from this process are given by (12) - (14) below. Lower-case or 'hatted' symbols generally denote log-deviations of variables from their reference steady state values; e.g. $y_t \equiv \ln Y_t - \ln Y_R$ ('R' denoting the reference steady state). However since government debt is zero in the reference steady state and so its log-deviation is not well defined, we replace it by $d_t \equiv D_t / Y_R$. (See (10) for Y_R.) Likewise $\hat{\tau}_t \equiv \tau_t / Y_R$. For the interest rates, $(\hat{i}_t, \hat{r}_t)$ are log-deviations of their 'gross' values; while the inflation rate is $\pi_t \equiv p_t - p_{t-1}$, with $p_t \equiv \ln P_t$. (Hence $\hat{r}_t \equiv \hat{i}_t - \pi_{t+1}$.)

$$\pi_t = \beta\pi_{t+1} + \kappa y_t , \tag{12}$$

$$y_t = y_{t+1} - (\hat{i}_t - \pi_{t+1}) + \zeta d_{t+1} , \tag{13}$$

$$d_{t+1} = \beta^{-1} d_t - \hat{\tau}_t . \tag{14}$$

[11] For Calvo-style price staggering, this is particularly emphasised by Ascari (2004).

(ζ,κ) are the composite parameters $\zeta \equiv (1+\psi)^{-1}(1/q-1)(1/\beta-q)$ and $\kappa \equiv (1/\alpha-1)(1-\alpha\beta)[\sigma+(1-\sigma)\theta]^{-1}[1+(\sigma/\psi)(\theta-1)/\theta]$. Note that ζ, $\kappa > 0$, except when q = 1, which causes $\zeta = 0$. The first two equations are notably similar to their counterparts in the canonical New Keynesian models of Woodford (2003), Galí (2015) and others, with (12) being a 'New Keynesian Phillips Curve' (NKPC) equation, and (13) an 'expectational IS' equation. The only difference is the presence of government debt as a shift parameter in the IS equation. This was already discussed with reference to (3), of which (13) is the linearised counterpart. (14) is the linearised version of the government budget constraint, (7).

(vii) The 'natural' level of output and the 'output gap'

The 'natural' level of output is the value that output would take if prices were perfectly flexible. We can show that this is in fact the same as actual output in the zero-inflation steady-state equilibrium, and hence is also given by (10). Thus, under fully flexible prices, (10) applies in every period and not just in the steady state. This is not surprising, since in a zero-inflation steady state prices behave as if they are fully flexible. In our model, the natural level of output is therefore unaffected by the level of government debt. Defining the 'output gap' as the difference between actual output and its natural level, it then follows that debt has the same effects on output as on the output gap, in our framework. Hence in what follows we just refer to 'output'.

3. The Expected Debt Path's Influence on the Impact Effect of Fiscal Deficit Stimulus under a Simple Taylor Rule

(i) A temporary-but-persistent debt increase

Suppose that at t = 0 there is an unanticipated tax cut and hence a fiscal deficit causing government debt to increase, with government debt subsequently being gradually reduced to its old, pre-shock, level, via higher taxes. Figures 1(a) and 1(b) illustrate such time paths. Algebraically, d_{t+1} is given by:

$$d_{t+1} = \mu d_t + \varepsilon_t . \tag{15}$$

Recall that d_{t+1} is debt at the end of period t, and so non-predetermined in period t. In our experiment, $d_0 = 0$ and ε_t represents the one-off tax cut, so that $\varepsilon_0 > 0$, $\varepsilon_1 = \varepsilon_2 = ... = 0$. The parameter μ (where $0 \leq \mu < 1$) measures the degree of persistence of the debt increase. This policy hence represents a fiscal deficit which lasts just for period 0, and where fiscal consolidation starts in period 1, with the speed of such consolidation being inversely measured by μ. The corresponding level of taxation, using (14), is:

$$\hat{\tau}_t = -\varepsilon_t + (\beta^{-1} - \mu)d_t . \tag{16}$$

(16) shows that we could equivalently think of this as a policy which makes taxation an increasing linear function of the start-of-period debt stock, except in the impact period.

The convenience of this 'autoregressive' rule for debt is that, under a simple Taylor Rule, and given the completely 'forward-looking' structure of the rest of the economy, the perfect-foresight time path will be such that all endogenous variables converge on their steady state values at the same rate as debt, i.e. at the rate inversely measured by μ. This is because debt is the only predetermined state variable in the system, so that – provided determinacy of the equilibrium is satisfied – the perfect-foresight equilibrium values of other variables will be proportional (in terms of deviations from the steady state) just to government debt. It hence becomes straightforward to derive tractable analytical solutions.

In this section we assume that monetary policy follows a simple Taylor Rule (see Taylor (1993)). Although the literature contains various versions of Taylor Rules (or, more generally, of rules for setting the short-term nominal interest rate), a very standard and 'simple' one involves feedback just on the current inflation rate and output gap, and no additional source of dynamics such as time lags or a time-varying path for the 'intercept' term in the rule. In general, in log-linearised form such a rule may be written as:

$$\hat{i}_t = \bar{i} + \phi_\pi \pi_t + \phi_y y_t . \tag{17}$$

(ϕ_π, ϕ_y) are the feedback parameters, satisfying $\phi_\pi, \phi_y \geq 0$ plus further restrictions discussed below. $\bar{i}$ is the intercept term whose value is chosen to ensure a target level of steady-state inflation, which we take to be zero. This implies, as is well known, that $\bar{i}$ must be set equal to the steady-state real interest rate. Here, as noted, the steady-state real interest rate depends on the level of steady-state debt. In this sub-section, that level of debt is zero, which implies (see (11)) that the steady-state real interest rate is just $1/\beta - 1$; or, in log-deviation form, zero. Hence in the present case we set $\bar{i} = 0$.

Under these policy rules, the complete model consists of (12), (13), (15) and (17). Formally it is a third-order difference equation system in (π_t, y_t, d_t). Of these, only d_t is predetermined, so for a determinate, bounded, perfect-foresight solution to exist we need exactly one of the system's three eigenvalues to lie 'inside the unit circle'. Given that d_t evolves autonomously, one of the eigenvalues is simply μ, the persistence parameter of debt, and this lies inside the unit circle by assumption. We therefore need the other two eigenvalues to lie outside it. This is the same as the standard determinacy requirement for the basic New Keynesian model without finite lifetimes or debt. It is straightforward to show that it holds under the well-known 'Taylor Principle' condition:

$$\phi_\pi + \phi_y(1-\beta)/\kappa > 1, \tag{18}$$

which requires a sufficient degree of combined inflation and output feedback.[12] Assuming that (18) is satisfied, as we do henceforth, the perfect foresight solution will be one in which the non-predetermined variables all move proportionally to d_t.

Rather than solve for the equilibrium using a standard method such as that in Blanchard and Kahn (1980), we here take an alternative approach which better reveals the forces at work. First, since the time path of y_{t+i} for $i \geq 0$ is bounded, and since $\beta \in (0,1)$, we may solve forward the NKPC equation, (12), to get:

$$\pi_t = \kappa \Sigma_{i=0}^{\infty} \beta^i y_{t+i} . \tag{19}$$

This reminds us that, as has often been noted, under the NKPC the current inflation rate depends on the discounted present value of current and future output gaps. Having argued above that all variables converge to their steady state values at the rate μ, we can furthermore say that $y_{t+i} = \mu^i y_t$. Using this in (19), we then more specifically obtain:

[12] See, for example, Woodford (2003), p. 254

$$\pi_t \;=\; \frac{\kappa}{1-\beta\mu} y_t \,. \tag{20}$$

This 'forward-integrated' version of the NKPC exhibits the short-run trade-off between inflation and output under our assumed policies.

Second, given that y_{t+i} tends to zero as i tends to infinity, we may also solve forward the expectational IS equation, (13), to get:

$$y_t \;=\; -\; \Sigma_{i=0}^{\infty} \hat{r}_{t+i} \;+\; \zeta\, \Sigma_{i=0}^{\infty} d_{t+1+i} \,. \tag{21}$$

A similar relationship has been noted by others. It says that current output depends on the sum of current and future real interest rate deviations. However in our model it also depends on the sum of current and future debt deviations: this is a novel feature introduced by the fact that agents are finitely-lived. (21) demonstrates very clearly the boost to current aggregate demand which is caused by the expected path of government debt. Again we can exploit the fact that variables such as $\hat{r}_{t+i}$ and $\hat{d}_{t+1+i}$ decay to zero at the rate μ, which enables us to write (21) more specifically as:

$$y_t \;=\; -\; \frac{1}{1-\mu} \hat{r}_t \;+\; \zeta \frac{1}{1-\mu} d_{t+1} \,. \tag{22}$$

This 'forward-integrated' version of the IS equation will later be very useful for understanding the channels through which a fiscal deficit stimulus acts on current output.

Third, we may rewrite the Taylor Rule, (17) (with $\bar{i}$ = 0) as an expression for the real interest rate, again using the fact that variables such as π_{t+1} decay to zero at the rate μ, so that $\pi_{t+1} = \mu\pi_t$:

$$\hat{r}_t \;=\; (\phi_\pi - \mu)\pi_t \;+\; \phi_y y_t \,. \tag{23}$$

It is now apparent that (20), (22) and (23) constitute a system of three 'static' equations in the three unknowns $(\pi_t, y_t, \hat{r}_t)$. Hence it is simple to derive explicit solutions for these three variables from it.

As mentioned, the initial debt-financed tax cut is given by ε_0, which also implies $d_1 = \varepsilon_0$. The effect of this on output in the impact period can readily be calculated using the method just described. This yields the multiplier:

$$\frac{dy_0}{d\varepsilon_0} = \frac{\zeta}{1-\mu+\phi_y+\kappa(\phi_\pi-\mu)/(1-\beta\mu)}. \tag{24}$$

It is clear that (24) is positive if $\phi_\pi > \mu$. In fact the sum of the third and fourth terms in the denominator is always positive if the determinacy condition (18) holds (see Appendix A), so that (24) is always positive. In other words, a one-period deficit and increase in debt, with debt then gradually being reduced to its pre-shock level, successfully stimulates current output under a simple Taylor Rule. This is probably what we would expect, given that, so long as $q < 1$ (and hence $\zeta > 0$), agents have finite lives. Finite lives mean that some of the expected future tax rises associated with the initial debt increase will occur after the death of agents currently alive, whence their perceived lifetime wealth increases and they therefore consume more, with this boost to aggregate demand then raising output in the face of 'sticky' prices. However, this does not give the full story of why output increases, as we discuss next.

From direct inspection of (24), it is not immediately obvious whether $dy_0/d\varepsilon_0$ is increasing or decreasing in μ, because μ affects the denominator of (24) in quite a non-linear way. However, a detailed analysis of this formula, conducted in Appendix A, reveals the following:

Proposition 1 *Under a simple Taylor Rule, the impact multiplier on output of a temporary-but-persistent government debt increase, where* μ $(0 \leq \mu < 1)$ *is the degree of persistence, is unambiguously positive. Its relationship to* μ *depends particularly on the size of the inflation feedback parameter,* ϕ_π *:*

(i) *if* $\phi_\pi < \beta^{-1} + (1-\beta)^2\beta^{-1}\kappa^{-1}$*, it is monotonically increasing in* μ*;*
(ii) *if* $\beta^{-1} + (1-\beta)^2\beta^{-1}\kappa^{-1} < \phi_\pi < \beta^{-1} + \beta^{-1}\kappa^{-1}$*, it is first increasing and then decreasing in* μ*;*
(iii) *if* $\beta^{-1} + \beta^{-1}\kappa^{-1} < \phi_\pi$*, it is monotonically decreasing in* μ*.*

There are hence three ranges of ϕ_π: low, intermediate and high, each one giving rise to qualitatively different behaviour. This is illustrated by the numerical examples in Figure 2. The Taylor Rule's output feedback parameter, ϕ_y, on the other hand, has no significant influence on how the multiplier varies with μ.

The impact multipliers on inflation and the real interest rate can be calculated as:

$$\frac{d\pi_0}{d\varepsilon_0} = \frac{\zeta}{\phi_\pi - \mu + \kappa^{-1}(1-\beta\mu)(1+\phi_y - \mu)}, \tag{25}$$

$$\frac{d\hat{r}_0}{d\varepsilon_0} = \frac{\zeta}{1 + (1-\mu)/[\phi_y + \kappa(\phi_\pi - \mu)/(1-\beta\mu)]}. \tag{26}$$

(25) and (26) are positive. This is obvious by inspection if $\phi_\pi > \mu$. Even if $\phi_\pi < \mu$, we can still show that it is true provided (18) holds. It is also obvious that $d\pi_0/d\varepsilon_0$ is always increasing in μ. It is less obvious that $d\hat{r}_0 / d\varepsilon_0$ is always increasing in μ, but again we can establish this formally by using (18).

Returning to the output multiplier, intuition for why debt persistence affects it non-monotonically can be gained from (22), the forward-integrated IS equation. For $t = 0$, this becomes:

$$y_0 = -\frac{1}{1-\mu}\hat{r}_0 + \zeta\frac{1}{1-\mu}d_1. \tag{27}$$

Recall that the variables here are all 'deviations', and so are zero in the absence of the policy shock. Having seen that the deficit raises the real interest rate, i.e. that $\hat{r}_0 > 0$, it is clear from (28) that there are conflicting forces acting on current output: an expansionary force via the term in d_1 (= ε_0), and a contractionary force via the term in $\hat{r}_0$. We know, however, that the net effect is positive. Now consider, for a given d_1, how higher μ alters the size of the impact on output. Suppose for the moment that, instead of following the simple Taylor Rule, monetary policy pegs the real interest rate permanently at its pre-shock value, so that $\hat{r}_0$ is held at 0.[13] Then (27) shows that higher μ would unambiguously raise the multiplier, because it would raise $\zeta/(1-\mu)$. This is the 'amplifying' effect of greater debt persistence. It arises simply because the slower is the government to raise taxes to bring debt back down, the greater is the boost to the expected lifetime net wealth of agents who receive the initial tax cut, and hence the greater

[13] As mentioned, such a policy is treated as the baseline monetary policy in Angeletos et al. (2024) and Auclert et al. (2024). In our framework it would not be implementable as a determinate equilibrium just by taking a special case of our simple Taylor Rule. It would need, for example, a Taylor Rule with an appropriately time-varying path for the 'intercept term'.

is the stimulus to aggregate demand. This amplifying effect is also clearly commented upon by Angeletos et al. (2024, p. 1365).

Next, go back to the case of a simple Taylor Rule, thus restoring the positive $\hat{r}_0$ to the picture. If $\hat{r}_0$ were unaffected by μ, an increase in μ would still raise the multiplier, since the factor 1/(1-μ) would be higher and would multiply an unchanged positive value of $-\hat{r}_0 + \zeta d_1$. However, we know from the discussion of (26) that higher μ in fact increases $\hat{r}_0$, i.e. it magnifies the impact effect on the real interest rate. This tends to lower (27) and so constitutes a 'dampening' effect of greater debt persistence on the multiplier. Depending on its strength, it may either dominate, or be dominated by, the amplifying effect. Proposition 1 says that the dampening effect will dominate when ϕ_π is large and/or when μ is large. To see why intuitively, consider again (23), the partial solution for the *real* interest rate implied by the Taylor Rule. For $t = 0$, it is:

$$\hat{r}_0 = (\phi_\pi - \mu)\pi_0 + \phi_y y_0 . \tag{28}$$

We noted in discussing (25) that higher μ unambiguously raises the impact effect on inflation, i.e. increases the size of π_0. This is mainly because it makes the short-run NKPC steeper, as can be seen from (20). Via (28), this boost to π_0 then augments the rise in the real interest rate at $t = 0$, especially if ϕ_π is large. The effect will also be stronger if μ is already close to 1: 1-$\beta\mu$ is then close to zero and a given increase in μ therefore has a proportionally larger effect on it, so that, via (20), it has a larger effect on π_0.[14]

The presence of both amplifying and dampening effects of greater persistence in the case of the deficit multiplier contrasts with the case of the government spending multiplier, where there is only a dampening effect. The dampening mechanism is similar: greater persistence means a greater expected future demand stimulus, and hence a stronger surge in current inflation. This then pushes up the real interest rate at $t = 0$ more strongly through the Taylor Rule. However there is no amplifying effect, because at a constant real interest rate, the speed

[14] (28) also shows that higher μ lowers the coefficient on π_0, which works to weaken the rise in $\hat{r}_0$; but it can be shown that the strengthening effect dominates under the mild condition that $\phi_\pi > \beta^{-1}$.

with which government spending returns to its old level is irrelevant for the impact multiplier.[15] In the case of government spending, the effects of higher expected future levels operate only via the real interest rate; whereas in the case of deficits and debt, they also operate directly through agents' lifetime net wealth. This is an important difference between the two types of fiscal stimulus, demonstrating the greater subtlety in the effects of deficits.

It is not our goal to conduct serious quantitative investigations, but some brief comments on the numbers involved in Figure 2 may be made. The most notable feature is that the set of ϕ_π values for which the multiplier is non-monotonic in μ is large. Given a set of broadly central values for the non-policy parameters,[16] it is non-monotonic for all ϕ_π in the range (1.02, 64.13). Such a range spans all the empirically likely values of ϕ_π, recalling that Taylor's own original rough estimate, based on U.S. data, was 1.5. Hence the non-monotonicity in the effect of debt persistence on the multiplier is widespread: it is the rule, rather than the exception.

(ii) A permanent-but-gradual debt increase

Following the initial budget deficit and increase in the debt, an alternative to bringing debt back to its old level over time is to stabilise it at a permanently higher level, but to do so gradually. Such a path allows the government to run a succession of deficits, rather than than switching abruptly from deficit to surplus in the period after the initial shock. See Figure 1(c) for an illustration. To represent this, we replace the rule (15) for the debt level by:

$$d_{t+1} = \mu d_t + (1-\mu)\bar{d} . \tag{29}$$

Here, $\bar{d}$ is the new steady-state level of debt, which we treat as a policy parameter, while μ (where $0 \leq \mu < 1$) now represents the degree of 'gradualism' in attaining that level, with higher μ corresponding to a slower approach of d_{t+1} to $\bar{d}$. The corresponding level of taxation is, using (14):

[15] This independence of the multiplier from the persistence of government spending under a constant real interest rate is noted by Woodford (2011): see p.8 there. He does not note that, under a simple Taylor Rule, the multiplier is lowered by persistence, although this can be inferred from his formulae. Christiano et al. (2011) do note the latter point: see p.89 there.

[16] $\alpha = 0.75$, $\theta = 15$, $\sigma = 0.5$, $\psi = 0.93$ (chosen to make $L_R = 1/3$), $q = 0.95$. The value of q implies an 'expected lifetime', or an 'expected time to the next binding borrowing constraint', of $1/(1\text{-}q) = 5$ years, taking a 'period' to be a quarter. Note that q only affects ζ, so that, qualitatively speaking, it is irrelevant for how the multiplier varies with μ.

$$\hat{\tau}_t = (\beta^{-1} - \mu)d_t - (1-\mu)\bar{d} . \tag{30}$$

Figure 1(d) depicts the time path of taxation. The tax cut can now persist for a number of periods, but it must eventually turn into a tax rise, in order to finance the interest on the permanently higher debt stock.

We again assume that the economy starts in the 'reference' steady state where debt is zero. In $t = 0$, it is then announced that the rule (29), with $\bar{d} > 0$, will be adopted from $t = 1$ onwards. This means that debt at the end of period 0, d_1, remains at zero, and the first tax cut occurs in $t = 1$, with the first increased level of debt being at the end of period 1, and given by $d_2 = (1-\mu)\bar{d}$. This timing makes the path of debt in the case of a permanent-but-gradual increase the exact 'mirror image' of its path in the case of a temporary-but-persistent increase, as can easily be seen by comparing Figures 1(a) and 1(c). Moreover if we set $\bar{d} = \varepsilon_0$, then the peak debt level anywhere along the time path is the same under the two policies.

Monetary policy continues to be given by a simple Taylor Rule of the form (17). However, since the debt increase is in this case permanent, the real interest rate in the new steady state also increases, to $\hat{r} = \zeta\bar{d}$ (as seen by setting $y_t = y_{t+1}$ in (13)). Faced with this, to ensure zero inflation in the new steady state, the intercept term in the Taylor Rule needs to increase, to $\bar{i} = \zeta\bar{d}$. We assume this adjustment occurs as soon as the new fiscal policy is announced. It would be possible to allow for delayed adjustment of the intercept term, but we wish to continue to study the simplest possible version of the Taylor Rule, with no time lags or time-variation in its parameters.

To solve the model under this new debt trajectory, we need to modify our previous method based on equations (20), (22) and (23). The derivation previously given for (22) ceases to be valid because $\hat{r}_{t+i}$ and $\hat{d}_{t+1+i}$ no longer decay to zero over time. Therefore we define new 'deviation' variables:

$$\tilde{r}_t \equiv \hat{r}_t - \hat{r} , \qquad \tilde{d}_t \equiv d_t - \bar{d} . \tag{31}$$

Here the real interest rate and government debt are measured relative to their 'new' steady-state values, rather than to their 'reference' steady-state values. $\tilde{r}_t$ and $\tilde{d}_t$ do decay to zero over time, and do so at the rate μ, with $\tilde{r}_{t+1} = \mu\tilde{r}_t$ (for $t \geq 0$), $\tilde{d}_{t+1} = \mu\tilde{d}_t$ (for $t \geq 1$). Using these redefined variables, we can now validly employ similar reasoning to that used earlier to derive counterparts of (22) and (23):

$$y_t \;=\; -\;\frac{1}{1-\mu}\tilde{r}_t \;+\; \zeta\frac{1}{1-\mu}\tilde{d}_{t+1}, \tag{32}$$

$$\tilde{r}_t \;=\; (\phi_\pi - \mu)\pi_t \;+\; \phi_y y_t\,. \tag{33}$$

These equations differ only from (22) and (23) in that $(\tilde{r}_t, \tilde{d}_{t+1})$ replace $(\hat{r}_t, \hat{d}_{t+1})$. (20), the forward-integrated NKPC equation, remains valid under the new policy. The model hence now consists of (20), (32) and (33), implicitly determining $(\pi_t, y_t, \tilde{r}_t)$. Apart from the redefinitions of variables, it is the same as the system used previously.

To obtain the impact effect of a permanent-but-gradual debt increase, we now set $t = 0$ in the system (20), (32) and (33). Note that, under the policy as defined, $\tilde{d}_1$ (the exogenous shock when the system is written this way), equals $-\bar{d}$. From the discussion above, it can then be seen that, mathematically speaking, this is identical to the system (20), (22) and (23) for $t = 0$ used previously, except for the crucial fact that the exogenous shock d_1 (= ε_0) in the latter system is now replaced by the exogenous shock $\tilde{d}_1$ $(= -\bar{d})$ in the former system. It thus follows that, if we also set $\bar{d} = \varepsilon_0$ so that the peak debt level is equal under the two policies, then the multiplier for a permanent-but-gradual debt increase is simply equal to minus the multiplier for a temporary-but-persistent debt increase, as given by (24). It is hence:

$$\frac{dy_0}{d\bar{d}} \;=\; -\frac{\zeta}{1-\mu+\phi_y+\kappa(\phi_\pi-\mu)/(1-\beta\mu)}\,. \tag{34}$$

We already observed that (24) was positive for all parameter values (given determinacy of equilibrium), so that, clearly, (34) is negative for all parameter values. The way in which $dy_0 / d\bar{d}$ varies with parameters such as μ and ϕ_π is obviously also the same as was analysed extensively earlier, but just with the opposite sign. We summarise this finding in the following proposition:

Proposition 2 *Under a simple Taylor Rule, the impact multiplier on output for a permanent-but-gradual debt increase,* $dy_0 / d\bar{d}$ *, as given by (34), is opposite in sign but equal in magnitude to the impact multiplier for a temporary-but-gradual debt increase,* $dy_0 / d\varepsilon_0$*, given by (24), when these policies are scaled to equate peak debt levels. It is thus negative for all parameter values.*

The fact that a permanent-but-gradual increase in government debt should cause a downturn, rather than an upturn, in the economy, seems to go completely against what might be expected, given the Keynesian foundations of price stickiness and government-debt-as-net-wealth which the economy is built on. Based on Proposition 2, a fiscal stimulus of an apparently fairly orthodox type will not only be unsuccessful at boosting output, but will actually have the opposite effect to the one desired. In fact the result implies that, by implementing an initial surplus rather than a deficit, the government can *reduce* its debt over time while also creating a boom. This would seem like an inviting 'win-win' situation for any government.

At this point it might be asserted that such a 'contractionary fiscal expansion' is a consequence of an arbitrary feature of the way we have specified the Taylor Rule. The fact that the intercept term is increased as soon as the fiscal expansion is announced might seem like a contractionary response of monetary policy, making it unsurprising that the overall impact effect on output is negative. However, leaving the intercept term permanently unchanged would not fundamentally alter the outcome that there is a recession. It would complicate the picture by creating positive steady state inflation, and in turn this would have a positive effect on steady state output. [17] But it would not alter, qualitatively, the time path of output relative to its eventual steady state value: output would still drop relative to this on impact, and then gradually rise towards it. Such a path would still be a path of recession, even though the benchmark against which recession is measured would have changed.

The question then remains of why, under a simple Taylor Rule, a permanent-but-gradual debt increase is contractionary, yet a temporary-but-persistent one is expansionary. The exact symmetry between the multipliers (34) and (24) implies that there must be systematic forces at

[17] This is due to the effect of discounting on the setting of staggered prices, and can be seen from $y = \kappa^{-1}(1-\beta)\pi$, which is the steady state version of the Phillips Curve equation, (12). Non-zero 'trend inflation' also causes other effects on steady state output once non-linearities are accounted for, as Ascari (1998, 2004) and Ascari and Sbordone (2014) have highlighted.

work. What really matters is how debt is expected to change between the end of the impact period and the long run. If it is expected to fall (as with a temporary-but-persistent increase), this is expansionary; but if it is expected to rise (as with a permanent-but-gradual increase), this is contractionary. Note that an expected falling or rising path for debt implies that the economy's 'natural' real interest rate (that which would prevail under fully flexible prices) is also falling or rising (respectively). This can be seen from the IS equation, (13), where under flexible prices the output gaps y_t and y_{t+1} are zero, so that the natural real interest rate is just given by ζd_{t+1}. Why booms or recessions emerge is because a simple Taylor Rule (i.e. one with time-invariant parameters) is not capable of making the actual real interest rate track the path of the natural real interest rate (or something approximating it) without generating a boom or a recession as a by-product. This can be understood by looking again at (23), where the Taylor Rule was rewritten as a rule for the real interest rate. If we reintroduce the intercept term, we obtain a version which holds for both the fiscal experiments we have considered:

$$\hat{r}_t = \bar{i} + (\phi_\pi - \mu)\pi_t + \phi_y y_t \qquad (35)$$

Note that the coefficient on inflation is most likely to be positive, like that on output.[18] We also know, from the short-run Phillips curve trade-off, (20), that along the transition path inflation and output must either both fall or both rise. Therefore in order to induce a path for the real interest rate which steadily falls over time, inflation and output also need to fall over time; and, conversely, in order to make the real interest rate steadily rise over time, inflation and output also need to rise over time. Now, the only way to obtain falling paths for inflation and output is if they both unexpectedly jump up on impact, i.e. if there is a boom, because inflation and output will then gradually decline as the boom subsides. Hence a boom *has* to be the response to the type of debt shock which implies a falling path for the natural real interest rate. Similarly, a recession has to be the response to the type of debt shock which sets the natural real interest rate on a rising path, because the only way to make the actual real interest rate gradually ascend over time is through steady increases in inflation and output as the economy recovers from a

[18] Even if $\phi_\pi < \mu$, the the argument below can still be sustained by appealing to the determinacy condition, (18).

surprise recession. The actual real rate does not *exactly* track the natural rate in these situations, except in the long run; but the responses of output and inflation help it to *roughly* track the natural rate, and so are part of the necessary adjustment process.

4. An Extension to a Temporary Nominal Interest Rate Peg

We now assume the nominal interest rate is exogenously pegged at its pre-shock value for T periods after the fiscal shock occurs (T also being exogenous), after which it once more obeys a simple Taylor Rule. This could be interpreted as a situation in which monetary policy is constrained by the 'zero lower bound' on the nominal interest rate for T periods, although we will not also incorporate an exogenous disturbance (such as to households' discount rate) which drives the economy to the zero lower bound. Moreover, unlike in many papers on such 'liquidity trap' situations[19], we will not assume that the fiscal intervention lasts for exactly the same length of time as the nominal interest rate peg; rather, we will study the same paths for government debt as before. This is because our aim is to understand the effects of deficits and debt when monetary and fiscal policy are not coordinated, with monetary policy just being governed by some basic type of rule. The 'temporary peg' could alternatively simply represent inertia in the setting of the nominal interest rate by the central bank, whether deliberate (such as due to caution about immediately negating the government's fiscal stimulus efforts), or involuntary (such as due to institutional constraints on decision making). Whatever the case, it is plausible that feedback from macroeconomic variables to the policy rate is not instantaneous, unlike what was assumed in Section 3. Another way of capturing this would be to introduce 'interest rate smoothing' into the Taylor Rule by adding the lagged interest rate as an extra feedback variable. The two-phase approach which we adopt here highlights more sharply the difference which nominal interest rate inertia makes to the transmission mechanism of deficits and debt.

[19] E.g. Christiano et al. (2011), Eggertsson (2011), Albonico et al. (2021).

(i) A temporary-but-persistent debt increase

The time path of debt is given by (15), as before. The nominal interest rate, on the other hand, is given by:

$$\hat{i}_t = \begin{cases} 0 & \text{for } t = 0,\ldots,T-1 \\ \phi_\pi \pi_t & \text{for } t = T,\ldots,\infty \end{cases} \tag{36}$$

As this shows, for $t \geq T$ the Taylor Rule (17) (with $\bar{i} = 0$) is restored, although for added simplicity we have set $\phi_y = 0$.

Private-sector behaviour remains determined by (12) and (13). Given the economy's completely 'forward-looking' structure, we know that from period T onwards output and inflation must take the same values as if the Taylor Rule were operative in all periods. It is only during the 'peg' phase that their values potentially differ from this. To solve for y_t and π_t in this phase, we hence use our previous, 'Taylor Rule', solutions for (y_T,π_T) as the 'terminal' conditions for the phase, and then solve the 'peg' system recursively. For example, for y_T, the terminal condition is:

$$y_T = \frac{\zeta}{1-\mu+\kappa(\phi_\pi-\mu)/(1-\beta\mu)} d_{T+1}, \tag{37}$$

which is just the formula (24) but written for $t = T$ and setting $\phi_y = 0$.[20] Since d_{T+1} decays geometrically to zero with T, then the larger is T, the closer is y_T to zero; and similarly for π_T.

During the 'peg' phase, the dynamics are governed by the system consisting of (12), (13) and (15), but with $\hat{i}_t$ imposed to be zero. The perfect foresight solution will now not involve all variables moving proportionally to d_t; rather, it will depend on all three of the system's eigenvalues. Of these eigenvalues, one will just be the persistence parameter of debt, μ, as before. The other two, however, will no longer both lie outside the unit circle, reflecting the fact that, as is well known, pegging the nominal interest rate exogenously leads to indeterminacy of the perfect foresight equilibrium if such a peg is permanent. In Appendix B, we show that one of these eigenvalues (λ_2, say), is still greater than 1; but the other (λ_1, say),

[20] Since the equation system is linear, the 'multiplier' (24) also holds directly in terms of 'deviations' of variables, which is how it is written here.

lies between 0 and 1. The fact that λ_1 lies inside the unit circle nevertheless does not cause a problem, except potentially in the limit as we let $T \to \infty$, as will be seen below. Essentially, we solve for $(\pi_t, y_t, \hat{r}_t)$ by 'running time backwards' from $t = T$ to $t = 0$, using (37) and its counterpart for π_T as the 'initial' conditions for this calculation.

More technical details about the above method are provided in Appendix B. The impact multiplier which results from it takes the form:

$$\left[\frac{dy_0}{d\varepsilon_0}\right]_T = \omega(\mu,T)\frac{\zeta}{D(\mu,\phi_\pi)} + [1-\omega(\mu,T)]\frac{\zeta}{D(\mu,0)}, \qquad (38)$$

where

$$\omega(\mu,T) \equiv \delta(\mu)[\mu/\lambda_1]^T + [1-\delta(\mu)][\mu/\lambda_2]^T, \quad (>0)$$

$$\delta(\mu) \equiv (\beta^{-1}-\lambda_1)(\lambda_2-\mu)/[(\lambda_2-\lambda_1)(\beta^{-1}-\mu)], \quad (>0)$$

$$D(\mu,\phi_\pi) \equiv 1-\mu+\kappa(\phi_\pi-\mu)/(1-\beta\mu). \quad (>0)$$

$D(\mu,\phi_\pi)$ is the same function as in the denominator of (24) (but with $\phi_y = 0$). $D(\mu,0)$ is consequently this function with ϕ_π set equal to zero. It can then be seen that (38), the deficit multiplier with a *T*-period nominal interest rate peg, is a weighted average[21] of two other multipliers. The first of these is $\zeta / D(\mu,\phi_\pi)$, which is just the multiplier under a permanent Taylor Rule, (24), already analysed in Section 3(i). The second is $\zeta / D(\mu,0)$, whose interpretation we discuss below.

It is of particular interest to know, first, how the multiplier (38) depends on *T*; and, second, how it depends on μ. Neither *T* nor μ affect (λ_1,λ_2), as is clear from the results in Appendix B. Starting with the dependence on *T*, (38) shows that *T* affects the multiplier by altering the weights $\omega(\mu,T)$ and 1- $\omega(\mu,T)$. It is immediately apparent that if both μ/λ_1 and μ/λ_2 are less than one, then as *T* goes from 0 to ∞, $\omega(\mu,T)$ goes from 1 to 0. (Appendix C moreover establishes that this decline is monotonic.) Now, $\mu/\lambda_2 < 1$ always holds, since $\lambda_2 > 1$; but both $\mu/\lambda_1 < 1$ and $\mu/\lambda_1 > 1$ are possible. We can also see that, if $\mu/\lambda_1 > 1$, then as *T* goes from 0 to

[21] This term is loose because the 'weight' $\omega(\mu,T)$ can sometimes exceed 1, as noted below.

∞, $\omega(\mu,T)$ will go from 1 to ∞ (again monotonically, as shown in Appendix C). Thus, under the nominal interest rate peg, $\mu = \lambda_1$ is a critical value of μ. If $\mu < \lambda_1$, the multiplier tends to a finite value, $\zeta / D(\mu,0)$, as the peg phase lengthens; but if $\mu > \lambda_1$, it becomes unbounded.

To characterise more precisely the multiplier's behaviour as T increases, we need to study $\zeta / D(\mu,0)$ carefully. This 'limit value' multiplier has some similarities to the 'Taylor Rule' multiplier, $\zeta / D(\mu,\phi_\pi)$, but also some significant differences. We reserve the details for Appendix C. The overall implications, however, can be summarised as:

Proposition 3 *Under a nominal interest rate peg for T periods followed by a simple Taylor Rule, as T increases the impact multiplier on output of a temporary-but-persistent debt increase, (38), always increases. If the debt persistence parameter, μ, is less than λ_1, the multiplier tends to a finite positive limit as $T \to \infty$; but if μ is greater than λ_1, it tends to infinity.*

The act of temporarily pegging the nominal interest rate therefore always enhances the stimulus given by a temporary-but-persistent increase in debt. This is not surprising, since it suppresses the immediate increase in the nominal interest rate which would otherwise occur. It is also not surprising that the multiplier has the potential to 'explode' as $T \to \infty$. The latter is a reflection of the fact, mentioned earlier, that if the nominal interest rate were pegged permanently, the system would have too many stable eigenvalues.[22] Perhaps less expected is the result that a sufficiently small μ can prevent the multiplier from exploding. Technically speaking, this is possible because, when we think of solving for output in the impact period by 'running time backwards' and treating period T as the 'starting' date, the 'initial' value of output is proportional to μ^T. This means that the greater is T, the more the 'initial' value of output is scaled down, and this effect can outweigh the explosive behaviour caused by λ_1.

Our particular interest is in how debt persistence affects the multiplier, so next we hold T constant and examine how an increase in μ affects (38). Having already seen that the 'Taylor Rule' multiplier can respond in a large variety of ways to higher μ, it is no surprise to find that in general the same is true for the multiplier with a temporary interest rate peg. However, rather than attempt an exhaustive taxonomy of cases, we will provide insight by centring our analysis

[22] Such an 'explosion' is the fiscal counterpart of the 'forward-guidance puzzle' in the literature on monetary policy: see, e.g., Del Negro et al. (2015).

on a special case. This is where the Taylor Rule which takes over in period T is assumed to implement a 'strict' zero inflation target, so that $\pi_t = 0$ from period T onwards. Zero inflation can be achieved by letting $\phi_\pi \to \infty$, as can be confirmed from (26). Given that letting $\phi_\pi \to \infty$ also reduces the 'Taylor Rule' multiplier on output, $\zeta / D(\mu, \phi_\pi)$, to zero, the first right-hand side ('RHS') term in (38) then disappears, leaving just the second RHS term. Admittedly this special case is extreme, because it involves an abrupt switch from a completely accommodating, to a completely non-accommodating, monetary policy at date T; but it is helpful in revealing the forces at work.

Being still more specific, first consider a one-period peg (T = 1) followed by a strict inflation target. In this case we can show that (38) reduces to:

$$\left[\frac{dy_0}{d\varepsilon_0}\right]_1^{\phi_\pi \to \infty} = \zeta \, . \tag{39}$$

The multiplier here just coincides with the coefficient on debt in the IS equation, (13). This is because future inflation and output are constrained to be zero (i.e. in the system comprising (12), (13) and (15), if we think of t as the impact period, π_{t+1} and y_{t+1} are zero), so that the multiplier for the impact period can be obtained just from the IS equation for that period. In this very special case, debt persistence, μ, is irrelevant for the multiplier.

When $T = 2$, we obtain:

$$\left[\frac{dy_0}{d\varepsilon_0}\right]_2^{\phi_\pi \to \infty} = [1 + (1+\kappa)\mu]\zeta \, . \tag{40}$$

This multiplier is not only larger, but it is now also clearly increasing in debt persistence. This is because, during the second period of the peg, inflation ('π_{t+1}') and output ('y_{t+1}') now rise, since in the second period debt is still above its pre-shock, zero, value, and the fixed nominal interest rate accommodates such rises. The anticipation of this in the impact period then enhances the impact boom, both because, at a given $\hat{i}_t$, it lowers the real interest rate and so further stimulates current consumption (the effect via $\hat{i}_t - \pi_{t+1}$ in the IS equation), and because it raises expected lifetime wealth (the effect via y_{t+1} in the IS equation).

For a general value of T followed by a strict inflation target, we obtain:

$$\left[\frac{dy_0}{d\varepsilon_0}\right]_T^{\phi_\pi \to \infty} = \zeta(\lambda_2-\lambda_1)^{-1}\sum_{t=0}^{T-1}[(1-\lambda_1)\lambda_2^{-t}-(1-\lambda_2)\lambda_1^{-t}]\mu^t , \tag{41}$$

(as derived in Appendix D). On the basis of this, we can assert:

Proposition 4 *Under a temporary nominal interest rate peg for at least two periods followed by a strict zero inflation target, the impact multiplier on output of a temporary-but-persistent debt increase, given by (41), is positive and monotonically increasing in* μ.

The proof follows directly from (41), noting that the term [.] there is unambiguously positive. Hence, under a nominal interest rate peg for at least two periods followed by a strict zero inflation target, it is robustly the case that greater debt persistence raises the multiplier, in contrast to the variety of effects that can occur under a permanent Taylor Rule. The reason why this holds for all $T > 1$ is that, as we increase T, the expansionary effects of expected future raised debt levels which we described above for the $T = 2$ case, and which work through higher expected future output and inflation occurring *within* the 'peg' phase, continue to accumulate.

Returning now to the general case of the multiplier (38) in which ϕ_π is finite rather than infinite, it is still possible that this is decreasing in μ for some parameter values. This is because the first RHS term in (38), which is once more non-zero, can still be decreasing in μ under some conditions. Nevertheless, we can show that for $T > 1$ the multiplier when μ is strictly positive is always greater than the multiplier when μ is zero. In other words, the 'average' effect of persistence is now sure to be strictly positive, even if the 'marginal' effect can still be negative for some parameter values. By contrast, under a permanent Taylor Rule, a positive average effect of persistence is not guaranteed: in particular, it is absent when ϕ_π is large. We summarise this as:

Proposition 5 *Under a temporary nominal interest rate peg for at least two periods which is followed by a simple Taylor Rule, the impact multiplier on output of a temporary-but-persistent debt increase,* $[dy_0 / d\varepsilon_0]_T$*, given by (38), is positive and always higher when* $\mu > 0$ *than when* $\mu = 0$.

To prove this, note that by Proposition 4, the second RHS term in (38) is positive and monotonically increasing in μ, provided $T > 1$. For $T > 1$, this term is therefore certainly higher when $\mu > 0$ than when $\mu = 1$. As regards the first RHS term in (38), this is the product of the functions $\omega(\mu,T)$ and $\zeta/D(\mu,\phi_\pi)$. By Proposition 1, $\zeta/D(\mu,\phi_\pi)$ is always strictly positive (for finite ϕ_π). Meanwhile, $\omega(\mu,T)$ is strictly positive for all $\mu \in (0,1]$ (shown in Appendix C), but – as can be seen from its definition in (38) – for $\mu = 0$ it is zero. It follows that the product of these functions is zero at $\mu = 0$ but strictly positive at $\mu > 0$. Hence the sum of both the RHS terms in (38) must be higher when $\mu > 0$ than when $\mu = 0$.

Our analysis so far, and especially the formula (41), implies that the longer the nominal interest rate is pegged, the more likely it is that the multiplier will be monotonically increasing in debt persistence. To reveal whether even a one- or two-period peg can have a substantial effect, in Figure 3 we graph some numerical examples for $T = 1$ and $T = 2$, assuming a 'mid-range' value for ϕ_π of 1.5. (The other parameter values are the same as in Figure 3.) Figure 7 illustrates that, while short periods of pegging make little difference when μ is below roughly 0.5, they do still make a substantial difference when μ is above this. In these examples, for μ values roughly in the range 0.5 to 0.9, when $T = 1$ or 2, greater persistence increases the multiplier by considerably more than it does when $T = 0$.

(ii) A permanent-but-gradual debt increase

We lastly consider the effect of temporary interest rate pegging when debt is increased in a permanent-but-gradual way, and so is once more given by (30). Monetary policy is again given by a temporary nominal interest rate peg followed by a simple Taylor Rule, as in (36), except that the Taylor Rule now includes an intercept term in order to ensure zero steady-state inflation. Using methods similar to those described in the preceding section, we can derive the following expression for the impact multiplier on output:

$$\left[\frac{dy_0}{d\bar{d}}\right]_T = [1-\omega(1,T)]\frac{\zeta}{D(1,0)} - [1-\omega(\mu,T)]\frac{\zeta}{D(\mu,0)} - \omega(\mu,T)\frac{\zeta}{D(\mu,\phi_\pi)}. \tag{42}$$

The functions appearing in this have the same definitions as in (38).

Previously, we found this fiscal measure produced a perverse, negative, multiplier, so a key question is whether a temporary nominal rate peg alters this. In general, the sign of (42) is ambiguous. To understand it, it is helpful again to begin with the extreme case in which the Taylor Rule is operated to hit a strict target of zero inflation. This implies letting $\phi_\pi \to \infty$, which reduces the third RHS term in (42) to zero. Looking at the other two terms, we note that the first term is just a special case of the second term, since it equals the second term evaluated at $\mu = 1$. Moreover, we have already argued that the second term (without its preceding minus sign) takes the more explicit form given by (41). Hence, as a function of μ, it is unambiguously increasing in μ, provided that $T > 1$. It then follows that, when $T > 1$ and $\mu < 1$, the difference of the first two terms, and thus the value of $[dy_0 / d\bar{d}]_T^{\phi_\pi \to \infty}$, must be positive. In other words, the temporary fixing of the nominal interest rate for two or more periods followed by a strict zero-inflation target completely reverses the sign of the impact effect of a permanent-but-gradual debt increase on output. It overturns the strange finding of a negative multiplier highlighted in Proposition 2, which arose under a permanent Taylor Rule. It thereby restores a more traditionally 'Keynesian' outcome of the kind which might be expected from a policy of deficit stimulus.

When the Taylor Rule is more moderate and does not impose immediate zero inflation, i.e. when ϕ_π is finite, then the third RHS term in (42) contributes a negative component. This can potentially dominate the overall sign of the multiplier, turning it negative. It will clearly do so if μ is sufficiently close to 1, since the difference of the first two terms in (42) then tends to zero. On the other hand, if μ is sufficiently close to zero, the third RHS term will itself be dominated because it then tends to zero as a result of $\omega(\mu,T)$ tending to zero (see the reasoning supporting Proposition 5). Hence, as μ goes from 0 to 1, there must exist a 'crossover' value of μ at which the multiplier switches from positive to negative. We encapsulate this result as:

Proposition 6 *Under a temporary nominal interest rate peg for at least two periods which is followed by a simple Taylor Rule, the impact multiplier on output of a permanent-but-gradual debt increase,* $[dy_0 / d\bar{d}]_T$ *,given by (42), is positive for low values of* μ *(the degree of 'gradualism' of the debt increase), but negative for high values of* μ*.*

At the end of Section 3 we noted that what really matters for the sign of the impact effect on output under a permanent simple Taylor Rule, is whether debt is expected to fall or to rise between the end of the impact period and the long run. In other words, the simple *levels* of debt are not what are important: it is their *expected changes ovcr time* that matter. In the current section, however, we see that if the Taylor Rule is temporarily suspended and the nominal interest rate is fixed at its pre-shock level for some interval, then the forces that affect current output seem to come closer to depending just on the simple levels of debt, because a permanent-but-gradual increase in debt, just like a temporary-but-persistent increase, may now provide stimulus, after all.

Insight into why this is so can be gained by looking again at the IS equation, (13). Rather than integrate it forward to infinity as done in (21), here we integrate it just over the interval of the peg, t = 0,...,T-1. We can moreover replace the real interest rate, $\hat{r}_t$, by $-\pi_{t+1}$, since $\hat{i}_t = 0$ implies $\hat{r}_t = -\pi_{t+1}$. This gives:

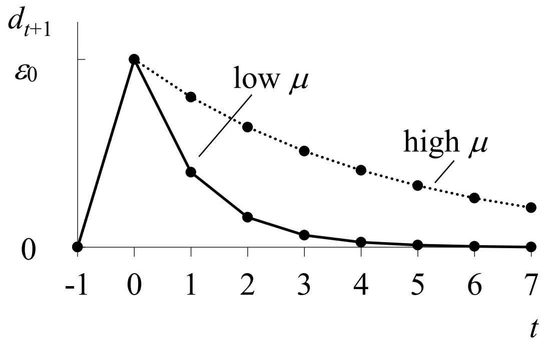
$$y_0 = \zeta\, \Sigma_{t=0}^{T-1} d_{t+1} + \Sigma_{t=0}^{T-1} \pi_{t+1} + y_T . \tag{43}$$

It is helpful to recall that all the variables are 'deviations' and that they would all be zero in the absence of shocks to debt. (43) holds for an *arbitrary* path of the debt levels, and so encompasses both the paths studied in this paper. Noting that the system is fully forward-looking, (43) shows that the effects of the debt path on y_0 can be divided into effects arising from the path of debt during the 'Taylor Rule' phase, i.e. $\{d_{T+1}, d_{T+2}, ...\}$, and effects arising from the path of debt during the 'peg' phase, i.e. $\{d_1, ..., d_T\}$. The former operate via y_T and via some fraction of $\Sigma_{t=0}^{T-1} \pi_{t+1}$; while the latter operate via $\zeta \Sigma_{t=0}^{T-1} d_{t+1}$ and via the remaining fraction of $\Sigma_{t=0}^{T-1} \pi_{t+1}$. Setting aside the effects via $\Sigma_{t=0}^{T-1} \pi_{t+1}$, the point to note is that the effect via $\zeta \Sigma_{t=0}^{T-1} d_{t+1}$ is unambiguously positive and just depends on the overall *levels* of debt during the peg phase: whether $\{d_1, ..., d_T\}$ are on a rising or falling path is of no importance. It represents the basic mechanism whereby government debt adds to 'net wealth', and thus increases aggregate demand. By contrast, under a simple Taylor Rule, higher debt operates in a more convoluted way and may not produce stimulus at all because, depending on the shape of the debt path, the effect via y_T could be either positive or negative. The effects on y_0 arising from the path of debt during either phase and which operate via the term $\Sigma_{t=0}^{T-1} \pi_{t+1}$, do not alter these basic considerations: they just enhance or reduce the other effects. In fact, if monetary policy were to peg the *real*, rather than the nominal, interest rate, over the interval t = 0,…,T-1, these additional effects would simply drop out.

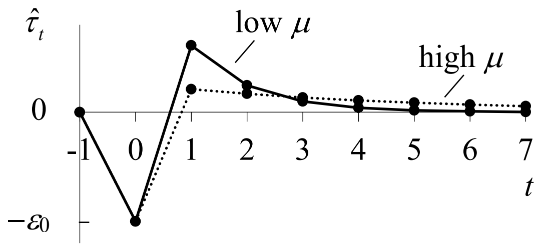
5. Conclusions

We have constructed what we consider to be the simplest possible model of staggered prices and finitely-lived, or occasionally credit-constrained, agents, in order to provide analytical insight into how the effectiveness of a deficit-financed tax cut as a tool of demand management depends on the expected path along which debt is stabilised. The standard way of representing monetary policy in recent decades has been via some version of a Taylor Rule. We find that the outcomes under a simple Taylor Rule may appear to contradict basic Keynesian thinking. First, a permanent-but-gradual debt increase depresses output on impact rather than stimulates it. Second, although output is stimulated by a temporary-but-persistent debt increase, the size of the impact is not necessarily amplified by delaying fiscal consolidation, and is likely to be maximised at some intermediate speed of consolidation. We further find that overturning these unorthodox outcomes can reliably be done, but this depends on there being inertia in the implementation of the Taylor Rule, such that the nominal interest rate is initially pegged for a while at its pre-shock value. It is likely that only a small amount of inertia is needed for this to occur.

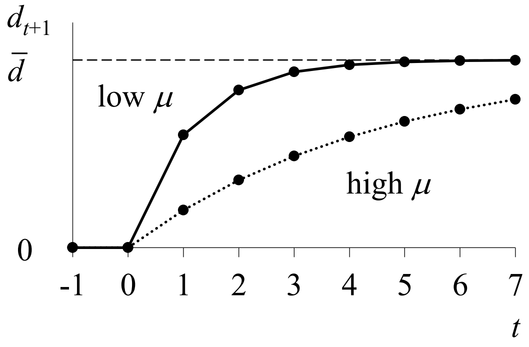
Despite the near-universality of Taylor-type rules in the academic literature, in practice interest-rate setting is not conducted according to any precise mathematical formula. At best, such rules are reasonable approximations of empirically-observed central bank behaviour, and there is much room for debate over the variables and magnitudes of feedback parameters which enter them. In this looser context, the practical implications of our analysis are that, if a central bank is known to react very promptly to macroeconomic conditions, then the use of fiscal deficits for demand management and thus output stabilisation may be risky, because the fiscal action can have a wide range of consequences depending on the imperfectly-known interest-rate rule. This is true even with a very orthodox rule such as the simple Taylor Rule. On the other hand, if the central bank is known to react sluggishly, then fiscal deficits may be effective because their impacts are more predictable. An aspect of policy which we have not explored in this paper is cooperation between the monetary authorities and the fiscal authorities. This is certainly worthy of attention. However, to address it satisfactorily requires us to answer further questions which it has been beyond our scope to tackle here, such as how welfare should be evaluated in a model with heterogeneous agents. We leave this and other important issues for future investigation.



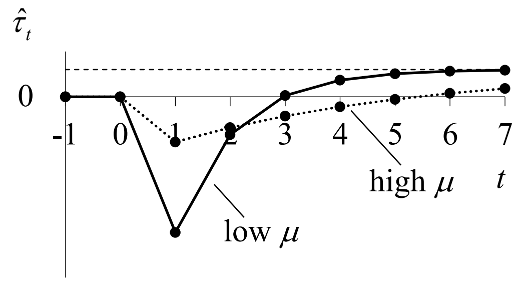
(a) Time paths of
temporary-but-persistent increases in debt



(b) Time paths of taxation accompanying
the debt paths in panel (a)



(c) Time paths of
permanent-but-gradual increases in debt



(d) Time paths of taxation accompanying
the debt paths in panel (c)

Figure 1

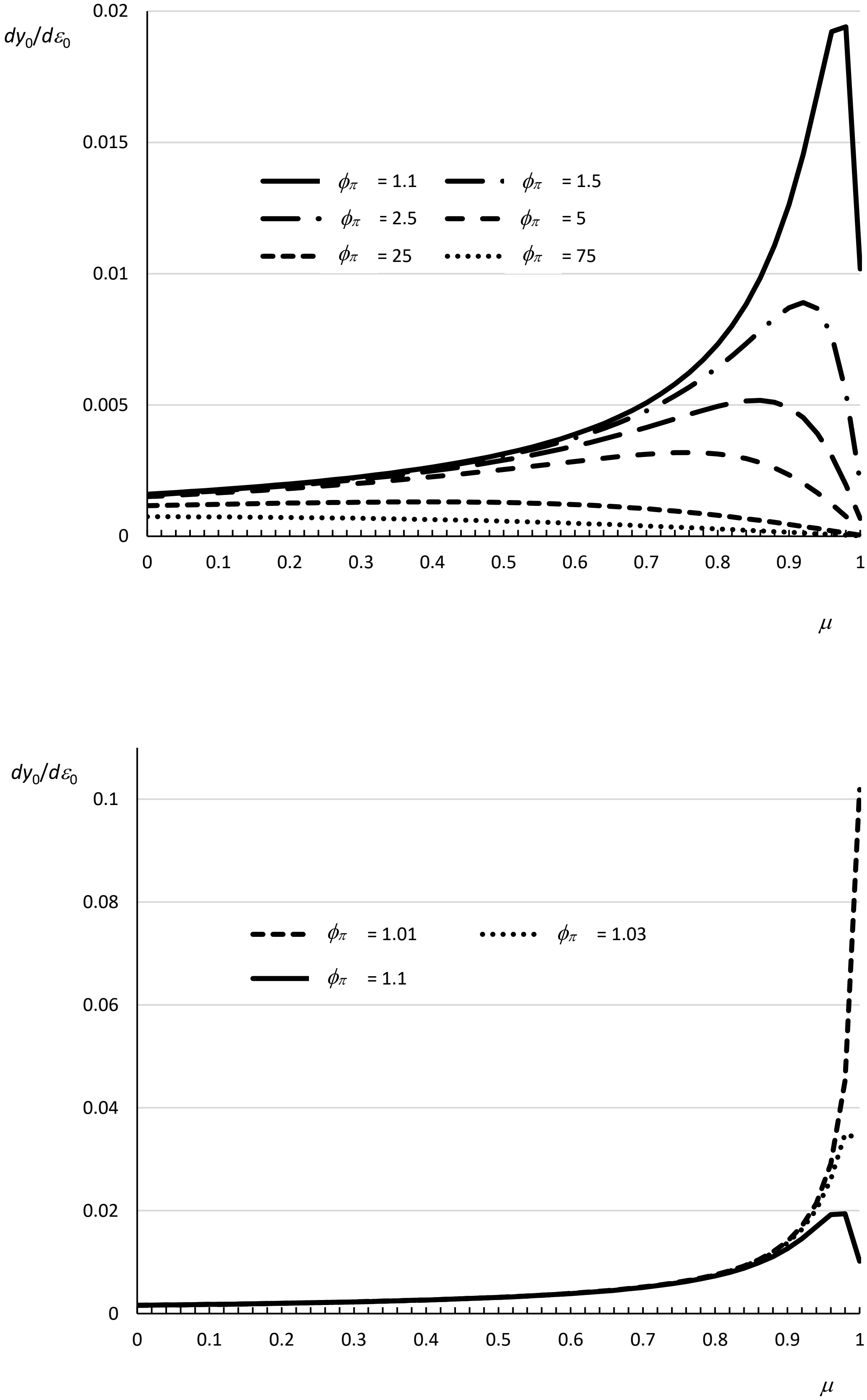


Figure 2 Numerical examples of the impact multiplier of a temporary-but-persistent increase in debt showing how it varies with μ and ϕ_π (with $\phi_y = 0$)

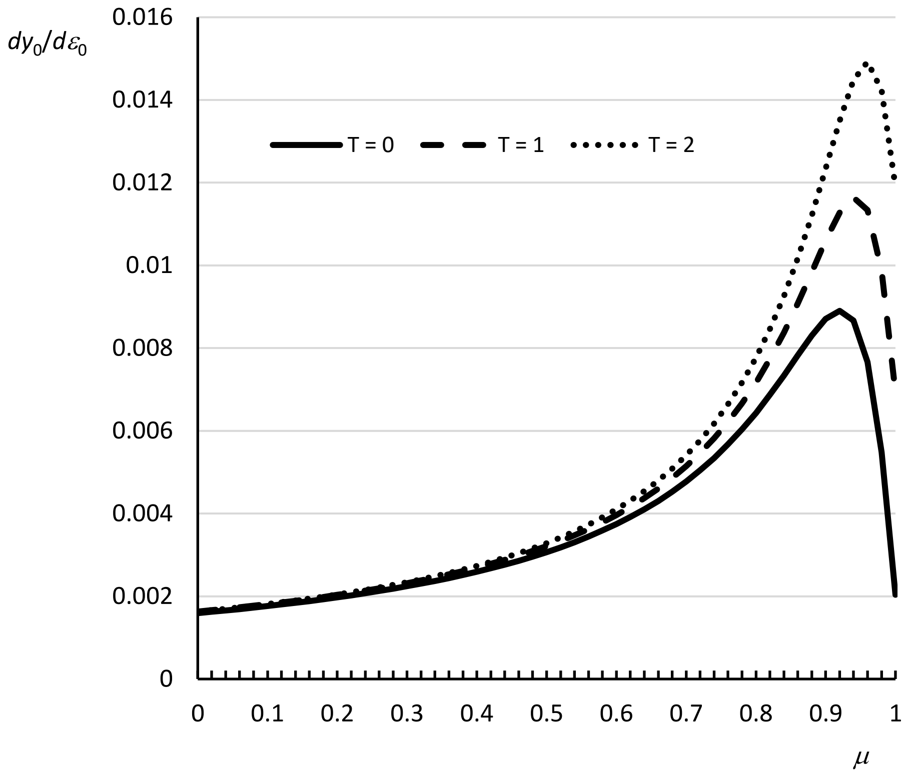


Figure 3 Numerical examples of the impact multiplier of a temporary-but-persistent increase in debt under an initial interest-rate peg, showing how it varies with μ and T (with $\phi_\pi = 1.5$, $\phi_y = 0$))

Appendix A Proof of Proposition 1

It is helpful to use the shorthand $D(\mu)$ for the function in the denominator of the multiplier $dy_0/d\varepsilon_0$, whose formula is given by (24). Hence $dy_0/d\varepsilon_0 = \zeta/D(\mu)$. $D(\mu)$ can also be slightly rearranged as:

$$D(\mu) = [1+\phi_y+\beta^{-1}\kappa-\mu] + \beta^{-1}\kappa(\phi_\pi-\beta^{-1})/(\beta^{-1}-\mu). \qquad \text{(A1)}$$

First, to prove that (24) is positive whenever the determinacy condition (18) holds, we need to show that (18) is necessary and sufficient for $D(\mu)$ to be positive. We can also write:

$$D(\mu) = 1-\mu + \Phi(\mu) \quad \text{where } \Phi(\mu) \equiv \phi_y + \beta^{-1}\kappa[1 + (\phi_\pi-\beta^{-1})/(\beta^{-1}-\mu)]. \qquad \text{(A2)}$$

Since $1-\mu \geq 0$ for all $\mu \in [0,1]$, then if we can show that $\Phi(\mu) \geq 0$ for all $\mu \in [0,1]$, it follows that $D(\mu) \geq 0$ for all $\mu \in [0,1]$. It is obvious that $\Phi(\mu) \geq 0$ when $\phi_\pi \geq \beta^{-1}$, so here we consider the case $\phi_\pi < \beta^{-1}$. In this case we note that $\Phi(\mu)$ is decreasing in μ. Next, observe that the determinacy condition (18) can also be re-expressed as the condition $\Phi(1) > 0$. When this holds, it therefore follows that $\Phi(\mu) > 0$ for all $\mu \in [0,1]$, which is what we desired to show.

Second, we address how (24) varies with μ. When $\phi_\pi < \beta^{-1}$, (A1) shows that $D(\mu)$ is always decreasing in μ and hence $dy_0/d\varepsilon_0$ is always increasing in μ. When $\phi_\pi > \beta^{-1}$, the situation is less immediately clear, because the linear term, [.], in (A1) is decreasing in μ, but the other, non-linear term, is increasing in μ. As asserted in Proposition 1, we can distinguish three possible cases. In Figure A1, we sketch the function $D(\mu)$ for each of these. The formula (A1) shows that $D(\mu)$ is the sum of a decreasing linear function, [.], and an increasing function which is a rectangular hyperbola, the latter having a horizontal asymptote which coincides with the horizontal axis and a vertical asymptote at $\mu = \beta^{-1}$. We are interested in the behaviour of $D(\mu)$ for values of μ between 0 and 1. Figure A1 shows that, roughly speaking, this depends on how 'tight' is the hyperbola to its asymptotes. If $\phi_\pi - \beta^{-1}$ (the numerator of the non-linear term) is small, the hyperbola lies close to its asymptotes and so $D(\mu)$ is downward-sloping for all $\mu \in [0,1]$; if $\phi_\pi - \beta^{-1}$ is large, the hyperbola lies far from its asymptotes, and so $D(\mu)$ is upward-sloping for all $\mu \in [0,1]$; but if $\phi_\pi - \beta^{-1}$ is of intermediate size, $D(\mu)$ is U-shaped for $\mu \in [0,1]$.

These three cases obviously result in the multiplier (24), i.e. $\zeta/D(\mu)$, being upward-sloping, downward-sloping, or hill-shaped, as a function of μ (respectively).

To define these three cases more precisely, consider the derivative of $D(\mu)$ with respect to μ:

$$D'(\mu) \;=\; -1 \;+\; \beta^{-1}\kappa(\phi_\pi - \beta^{-1})/(\beta^{-1}-\mu)^2 \,. \tag{A3}$$

Although the *sign* of $D'(\mu)$ is ambiguous when $\phi_\pi > \beta^{-1}$, it is clear that, as a function of μ, $D'(\mu)$ is unambiguously increasing. This is also apparent in Figure A1. It follows that if $D'(1) < 0$, then $D'(\mu) < 0$ for all $\mu \in [0,1]$ [case (a) in Figure A1]; if $D'(0) > 0$, then $D'(\mu) > 0$ for all $\mu \in [0,1]$ [case (c) in Figure A1]; and if $D'(0) < 0$ but $D'(1) > 0$, then $D'(\mu)$ goes from negative to positive as μ goes from 0 to 1 [case (b) in Figure A1]. Now, the above formula for $D'(\mu)$ implies:

$D'(0) \;=\; \kappa(\beta\phi_\pi - 1) \;-\; 1.$ Thus $D'(0) > 0$ if and only if $\phi_\pi > \beta^{-1} + \beta^{-1}\kappa$.

$D'(1) \;=\; \kappa(\beta\phi_\pi - 1)/(1-\beta)^2 \;-\; 1.$ Thus $D'(1) < 0$ if and only if $\phi_\pi < \beta^{-1} + \beta^{-1}\kappa(1-\beta)^2$.

Clearly, the two ranges of values specified here for ϕ_π give rise to cases (c) and (a), respectively. It can also be seen that the ranges are non-overlapping, with the threshold value of ϕ_π at which $D'(0) = 0$ lying above the threshold value at which $D'(1) = 0$. Hence there also exists an intermediate range of ϕ_π values, and it can be seen that, for ϕ_π within this range, $D'(0) < 0$ and $D'(1) > 0$, so that it gives rise to case (b). Thus we have derived the three outcomes given in Proposition 1.

Appendix B Derivation of the Multiplier (38)

The equation system consisting of (12), (13) (with $\hat{i}_t = 0$) and (15), in matrix form is:

$$\begin{bmatrix} \pi_{t+1} \\ y_{t+1} \\ d_{t+1} \end{bmatrix} = \begin{bmatrix} \beta^{-1} & -\beta^{-1}\kappa & 0 \\ -\beta^{-1} & 1+\beta^{-1}\kappa & -\zeta\mu \\ 0 & 0 & \mu \end{bmatrix} \begin{bmatrix} \pi_t \\ y_t \\ d_t \end{bmatrix} + \begin{bmatrix} 0 \\ -\zeta \\ 1 \end{bmatrix} \varepsilon_t \,. \tag{B1}$$

As stated in the main text, we denote the eigenvalues of this system as $(\lambda_1, \lambda_2, \lambda_3)$. Since the equation for d_{t+1} is independent, one eigenvalue ('λ_3') is just given by μ. The other two eigenvalues are the solutions of the quadratic characteristic equation:

$$\lambda^2 - (1+\beta^{-1}\kappa+\beta^{-1})\lambda + \beta^{-1} \; [\equiv f(\lambda)] = 0. \tag{B2}$$

The graph of $f(\lambda)$ is just an upright parabola. Evaluating $f(\lambda)$ at $\lambda = 0$ and $\lambda = 1$, we have $f(0) = \beta^{-1} > 0$ and $f(1) = -\beta^{-1}\kappa < 0$. Hence one solution for λ ('λ_1') must lie in (0,1) and the other solution ('λ_2') must lie in $(1,\infty)$. This establishes, as asserted in the main text, that the system as a whole has two eigenvalues in the unit circle, and one outside it. We may also note that the product of the eigenvalues, $\lambda_1\lambda_2$, equals the constant term in (B2), i.e. β^{-1}. Having seen that $0 < \lambda_1 < 1$, it then follows that $\lambda_2 > \beta^{-1}$, which further refines our calculation of the range in which λ_2 must lie.

The general solution to the equation system (ignoring for now the deficit shock ε_t, which is only non-zero in $t = 0$) is:

$$\begin{bmatrix} \pi_t \\ y_t \\ d_t \end{bmatrix} = \begin{bmatrix} A_1 v_{\pi 1} & A_2 v_{\pi 2} & A_3 v_{\pi 3} \\ A_1 & A_2 & A_3 \\ A_1 v_{d1} & A_2 v_{d2} & A_3 v_{d3} \end{bmatrix} \begin{bmatrix} \lambda_1^t \\ \lambda_2^t \\ \lambda_3^t \end{bmatrix}. \tag{B3}$$

Here, $[v_{\pi 1} \quad 1 \quad v_{d1}]'$ is the eigenvector associated with λ_1, where we have normalised the second element to unity; and similarly for the eigenvectors associated with λ_2 and λ_3. (A_1, A_2, A_3) are coefficients of integration to be determined by the boundary conditions. Using the coefficient matrix in (B1) to solve for the elements of these eigenvectors (but leaving (λ_1,λ_2) as implicitly determined by (B2)), we obtain:

$$\begin{bmatrix} \pi_t \\ y_t \\ d_t \end{bmatrix} = \begin{bmatrix} A_1 \dfrac{\beta^{-1}\kappa}{\beta^{-1}-\lambda_1} & A_2 \dfrac{\beta^{-1}\kappa}{\beta^{-1}-\lambda_2} & A_3 \dfrac{\beta^{-1}\kappa}{\beta^{-1}-\mu} \\ A_1 & A_2 & A_3 \\ 0 & 0 & A_3 \dfrac{(1-\mu)(\beta^{-1}-\mu)-\beta^{-1}\kappa\mu}{\zeta\mu(\beta^{-1}-\mu)} \end{bmatrix} \begin{bmatrix} \lambda_1^t \\ \lambda_2^t \\ \mu^t \end{bmatrix}. \tag{B4}$$

As explained in the main text, (π_T, y_T, d_T) are the values in the first period of the 'Taylor Rule' phase of the solution. d_T is given by $\mu^{T\text{-}1}\varepsilon_0$. y_T is given by (37) (with $d_{T+1} = \mu^T\varepsilon_0$), and π_T by a similar formula. Using the notation already introduced (see (38)), we may abbreviate (37) as $y_T = \zeta\mu^T\varepsilon_0/D(\mu,\phi_\pi)$, and write a similar formula for π_T. To obtain (A_1, A_2, A_3), we now set $t = T$ in (B4) and then substitute these (π_T, y_T, d_T) values into it. Doing this, the third equation in (B4) directly yields:

$$A_3 = \frac{\zeta\varepsilon_0}{1-\mu-\beta^{-1}\kappa\mu/(\beta^{-1}-\mu)}. \tag{B5}$$

It may be noticed that this formula (without the factor ε_0) is similar to the multiplier on output under the Taylor Rule, given by (24). In fact, if we set $\phi_\pi = \phi_y = 0$ in (24), we obtain exactly (B5). Using the notation which was introduced in (38), we may hence write A_3 as:

$$A_3 = \zeta\varepsilon_0 / D(\mu,0). \tag{B6}$$

Having solved for A_3, we may then substitute the solution back into the system just described to obtain solutions for A_1 and A_2. This yields:

$$A_1 = \left(\frac{\mu}{\lambda_1}\right)^T \frac{(\beta^{-1}-\lambda_2)/(\beta^{-1}-\mu) - 1}{(\beta^{-1}-\lambda_2)/(\beta^{-1}-\lambda_1) - 1}\left[\frac{1}{D(\mu,\phi_\pi)} - \frac{1}{D(\mu,0)}\right]\zeta\varepsilon_0, \tag{B7}$$

$$A_2 = \left(\frac{\mu}{\lambda_2}\right)^T \frac{1 - (\beta^{-1}-\lambda_1)/(\beta^{-1}-\mu)}{1 - (\beta^{-1}-\lambda_1)/(\beta^{-1}-\lambda_2)}\left[\frac{1}{D(\mu,\phi_\pi)} - \frac{1}{D(\mu,0)}\right]\zeta\varepsilon_0. \tag{B8}$$

(B4), in combination with (B6)-(B8), provides a full set of solutions for variables during $t = 1,\ldots,T\text{-}1$. In $t = 0$, and only in $t = 0$, there is the complication that ε_t is non-zero, so it is not immediate that these solutions also apply in $t = 0$, because we ignored ε_t when solving (B1). However, a careful inspection of the difference equations which apply specifically in $t = 0$ (which we shall not spell out here) shows that (B4), in combination with (B6)-(B8), does still yield valid solutions for (π_0, y_0). Hence, setting $t = 0$ in (B4), y_t in the impact period is given by $y_0 = A_1+A_2+A_3$, or:

$$y_0 = \left\{ \left(\frac{\mu}{\lambda_1}\right)^T \frac{(\beta^{-1}-\lambda_2)/(\beta^{-1}-\mu) - 1}{(\beta^{-1}-\lambda_2)/(\beta^{-1}-\lambda_1) - 1} + \left(\frac{\mu}{\lambda_2}\right)^T \frac{1 - (\beta^{-1}-\lambda_1)/(\beta^{-1}-\mu)}{1 - (\beta^{-1}-\lambda_1)/(\beta^{-1}-\lambda_2)} \right\}$$

$$\times \left[\frac{1}{D(\mu,\phi_\pi)} - \frac{1}{D(\mu,0)} \right] \zeta\varepsilon_0 + \frac{1}{D(\mu,0)} \zeta\varepsilon_0 \quad \text{(B9)}$$

This (without the factor ε_0) reproduces the solution for $[dy_0/d\varepsilon_0]_T$ given in (38). The term {.} here is what we denote as '$\omega(\mu,T)$' in (38). To see this, rewrite the term {.} as:

$$\left(\frac{\mu}{\lambda_1}\right)^T \frac{(\beta^{-1}-\lambda_1)/(\beta^{-1}-\mu) - (\beta^{-1}-\lambda_1)/(\beta^{-1}-\lambda_2)}{1 - (\beta^{-1}-\lambda_1)/(\beta^{-1}-\lambda_2)} + \left(\frac{\mu}{\lambda_2}\right)^T \frac{1 - (\beta^{-1}-\lambda_1)/(\beta^{-1}-\mu)}{1 - (\beta^{-1}-\lambda_1)/(\beta^{-1}-\lambda_2)}$$

(B10)

Note that the coefficients on $(\mu/\lambda_1)^T$ and $(\mu/\lambda_2)^T$ sum to 1. Defining the coefficient on $(\mu/\lambda_1)^T$ as $\delta(\mu)$, (B10) can therefore be expressed more compactly as:

$$\delta(\mu)[\mu/\lambda_1]^T + [1-\delta(\mu)][\mu/\lambda_2]^T,$$

which is the same as $\omega(\mu,T)$ in (38). To derive the version of the formula for $\delta(\mu)$ given in (38), the coefficient on $(\mu/\lambda_1)^T$ in (B10) can be simplified to:

$$\frac{(\beta^{-1}-\lambda_1)[(\beta^{-1}-\lambda_2)/(\beta^{-1}-\mu) - 1]}{\lambda_1-\lambda_2} = \frac{(\beta^{-1}-\lambda_1)(\lambda_2-\mu)}{(\lambda_2-\lambda_1)(\beta^{-1}-\mu)},$$

which is the same as $\delta(\mu)$ in (38).

Appendix C Proof of Proposition 3

In the main text, we asserted that as T goes from 0 to ∞, then if $\mu/\lambda_1 < 1$, $\omega(\mu,T)$ goes monotonically from 1 to 0; and if $\mu/\lambda_1 > 1$, $\omega(\mu,T)$ goes monotonically from 1 to ∞. It is easy to see the values of $\omega(\mu,T)$ at $T = 0$ and as $T \to \infty$ just from the formulae in (38). To characterise behaviour for intermediate values of T, note also that $\delta(\mu) > 0$, which follows from what was shown about the values of λ_1 and λ_2 in Appendix B. We also need to know the sign of $1 - \delta(\mu)$. This is less obvious, but $1 - \delta(\mu)$ can be written as:

$$1-\delta(\mu) \;=\; (\lambda_2-\beta^{-1})(\lambda_1-\mu)/[(\lambda_2-\lambda_1)(\beta^{-1}-\mu)]\,. \tag{C1}$$

Appendix B also showed that $\lambda_2 > \beta^{-1}$. Hence $1 - \delta(\mu) > 0$ when $\mu/\lambda_1 < 1$, but $1 - \delta(\mu) < 0$ when $\mu/\lambda_1 > 1$. In the light of these signs of $\delta(\mu)$ and $1 - \delta(\mu)$ and the formula for $\omega(\mu,T)$ in (38), our assertions about monotonic transitions then follow. It also follows that $\omega(\mu,T) > 0$ for all $\mu \in (0,1]$.

In the main text, we also noted that, as $T \to \infty$, then when $\mu < \lambda_1$ the multiplier (38) tends to the finite value, $\zeta/D(\mu,0)$. To see the properties of this 'limit value' multiplier, let us first write out the function $D(\mu,0)$ explicitly:

$$D(\mu,0) \;=\; [1+\beta^{-1}\kappa-\mu] \;-\; \beta^{-2}\kappa\,/\,(\beta^{-1}-\mu)\,. \tag{C2}$$

This is just a special case of the function $D(\mu)$ in (A1) in Appendix A, obtained by setting $\phi_\pi = \phi_y = 0$ in the latter. [The notation used in (38) and here slightly extends the notation in (A1), to acknowledge the dependence on ϕ_π.] $D(\mu)$ was sketched in Figure A1 for various values of ϕ_π, assuming $\phi_\pi > \beta^{-1}$. Figure C1(a) provides a similar sketch of $D(\mu,0)$. Comparing (C2) and (A1), $D(\mu,0)$ contains the same decreasing linear component, [.], as in $D(\mu)$, and it also contains a non-linear component which is a rectangular hyperbola with the same asymptotes as its counterpart in $D(\mu)$, except that in $D(\mu,0)$ this non-linear component is subtracted from, rather than added to, the term [.]. This means that $D(\mu,0)$ is not positive for all values of μ between 0 and 1, unlike $D(\mu)$. As Figure C1(a) shows, $D(\mu,0)$ is decreasing in μ, being strictly positive at $\mu = 0$, but reaching zero at $\mu = \lambda_1$, and thereafter becoming negative. To show that it reaches zero at precisely $\mu = \lambda_1$, multiply the expression in (C2) by $\beta^{-1} - \mu$ and set the result to zero. If μ in this equation is then set equal to λ_1, this reproduces the characteristic equation (B2) in Appendix B, so that when $\mu = \lambda_1$, $D(\mu,0)$ must indeed equal zero.

In Figure C1(b), we convert the graph of $D(\mu,0)$ to a graph of $\zeta/D(\mu,0)$. It is clear that $\zeta/D(\mu,0)$ must be positive and increasing in μ for $\mu < \lambda_1$ and must tend to infinity as $\mu \to \lambda_1$. As μ passes through λ_1, $\zeta/D(\mu,0)$ jumps discontinuously to minus infinity, after which it rises but remains negative. We can also easily see that $\zeta/D(\mu,0)$ for $\mu < \lambda_1$ is strictly greater than $\zeta/D(\mu)$ (i.e. the multiplier (24)). This is because, as (A1) shows, $D(\mu)$ is unambiguously increasing in ϕ_π and ϕ_y, so that setting these parameters to zero, as was done in obtaining $D(\mu,0)$, implies that $D(\mu,0) < D(\mu)$.

The first part of Proposition 3 concerns how the multiplier (38) changes as T increases. We now consider this for the cases $\mu < \lambda_1$ and $\mu > \lambda_1$ separately, bearing in mind the foregoing results. When $\mu < \lambda_1$, the weight $\omega(\mu,T)$ lies between 0 and 1 and decreases as T increases, so, in (38), the weight on $\zeta/D(\mu,\phi_\pi)$ decreases and the weight on $\zeta/D(\mu,0)$ increases. Since $\zeta/D(\mu,0)$ is larger, this unambiguously raises the multiplier. When $\mu > \lambda_1$, the 'weight' $\omega(\mu,T)$ is now greater than 1 and now increases as T increases, so, in (38), the weight on $\zeta/D(\mu,\phi_\pi)$ increases. Meanwhile, $\zeta/D(\mu,0)$ is now negative, but it is multiplied by a weight, 1 - $\omega(\mu,T)$, which is also negative, and which becomes more negative as T increases. Thus, when $\mu > \lambda_1$, both these effects work to ensure that the multiplier is once again increasing in T.

Appendix D Derivation of the Multiplier (41)

(41) is an alternative expression for $[1-\omega(\mu,T)]\zeta\,/\,D(\mu,0)$ in (38). The reason for deriving the seemingly more complicated formula (41) is that it is not easy to characterise the original expression as a function of μ by studying it as the ratio of the functions $1-\omega(\mu,T)$ and $D(\mu,0)$. $1-\omega(\mu,T)$ as a function of μ is very similar to $D(\mu,0)$. $D(\mu,0)$ is sketched in Figure C1(a). The graph of $1-\omega(\mu,T)$ has essentially the same shape, being decreasing in μ, and likewise starting at 1 when $\mu = 0$ and becoming negative when $\mu = \lambda_1$. Hence the properties of the function consisting of the ratio of these two functions are not immediately clear.

It is helpful first to re-express $D(\mu,0)$ and $\delta(\mu)$ (a component of $1-\omega(\mu,T)$), by treating their parameters as being (λ_1,λ_2), rather than (β,κ). We can do this by noting that the characteristic equation (B2) (see Appendix B) implies that $\beta^{-1} = \lambda_1\lambda_2$ and $1+\beta^{-1}\kappa+\beta^{-1} = \lambda_1+\lambda_2$. Some manipulation then gives:

$$D(\mu,0) \;=\; \frac{(\lambda_1-\mu)(\lambda_2-\mu)}{\lambda_1\lambda_2-\mu},$$

$$\delta(\mu) \;=\; \frac{(\lambda_1\lambda_2-\lambda_1)(\lambda_2-\mu)}{(\lambda_2-\lambda_1)(\lambda_1\lambda_2-\mu)}.$$

Using these in (38), we obtain:

$$\frac{1-\omega(\mu,T)}{D(\mu,0)}\zeta = \frac{(\lambda_2-\lambda_1)(\lambda_1\lambda_2-\mu)\ -\ [(1-\lambda_1)(\lambda_1-\mu)/\lambda_2^{T-1}\ -\ (1-\lambda_2)(\lambda_2-\mu)/\lambda_1^{T-1}]\mu^T}{(\lambda_2-\lambda_1)(\lambda_1-\mu)(\lambda_2-\mu)}\zeta$$

We now note that the right-hand side function is the ratio of two polynomial functions in μ. The order of the polynomial in the numerator is T+1, while that of the polynomial in the denominator is 2. It is clear that λ_1-μ and λ_2-μ are factors of the polynomial in the denominator. We can moreover easily verify that λ_1-μ and λ_2-μ are also factors of the polynomial in the numerator, by evaluating the numerator where $\mu = \lambda_1$ and where $\mu = \lambda_2$, respectively, showing that the numerator equals zero at both of these values. Hence we may proceed to divide out the numerator by, first, λ_1-μ, and then by λ_2-μ. These 'long division' exercises are tedious, so we shall not present the details here. The result, however, should be a polynomial of order T-1 in μ. This indeed is what can be observed in (41), which is the outcome of this calculation.

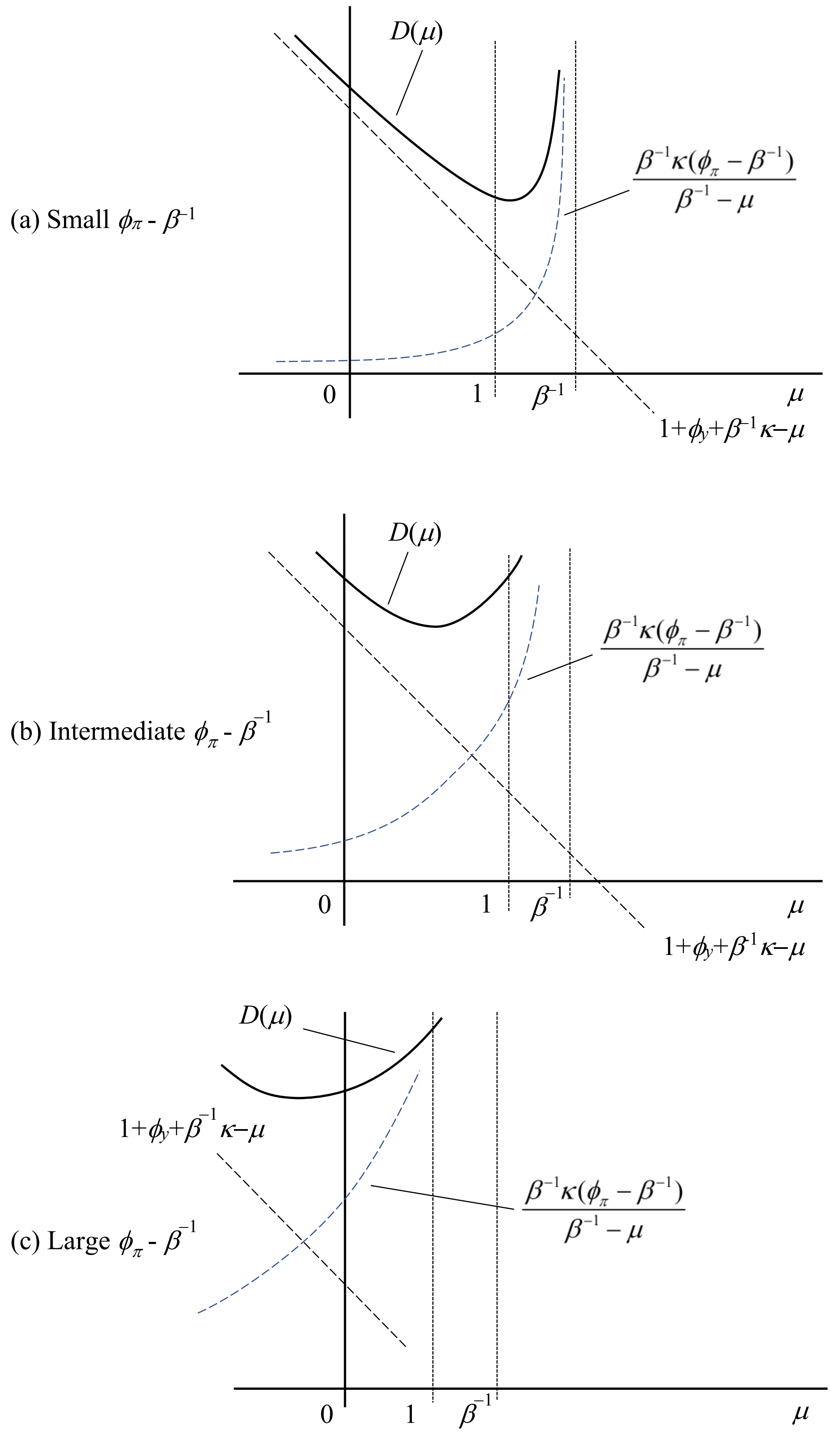


Figure A1 Variation of the denominator of $dy_0 / d\varepsilon_0$ with μ (for $\phi_\pi > \beta^{-1}$)

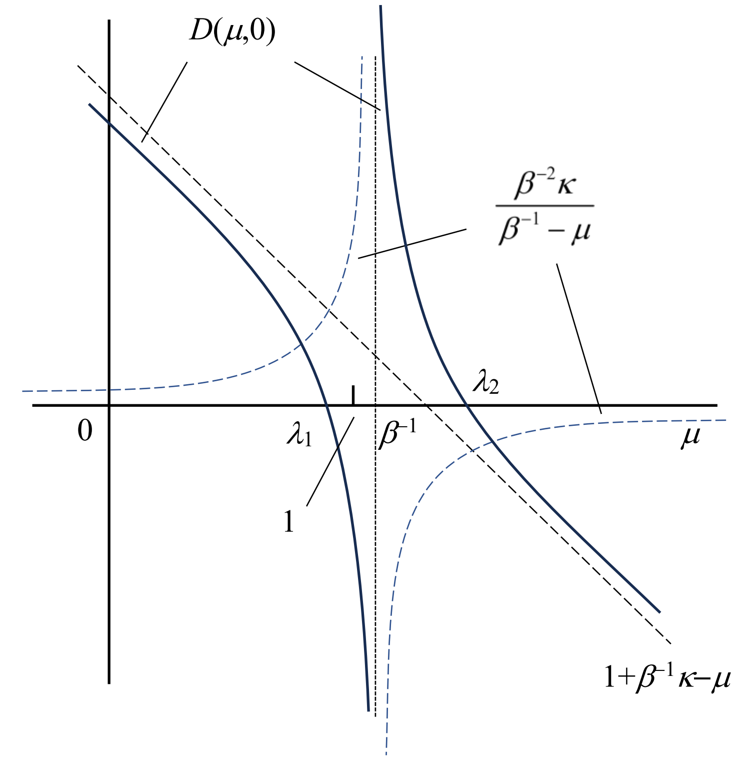


Figure C1(a) Variation of $D(\mu,0)$ with μ

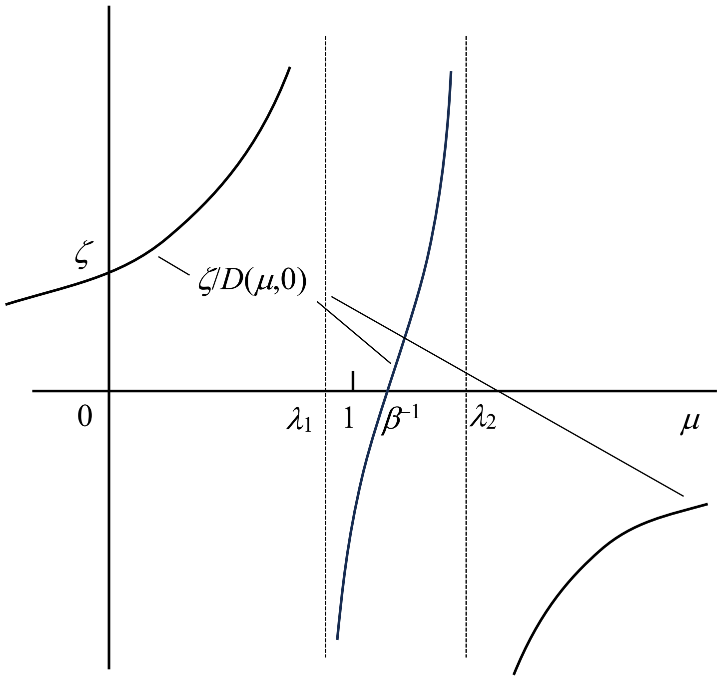


Figure C1(b) Variation of $\zeta/D(\mu,0)$ with μ

References

Albonico, A., Ascari, G. and Gobbi, A. (2021) 'The Public Debt Multiplier', *Journal of Economic Dynamics and Control* 132, 104204

Angeletos, G.-M., Lian, C. and Wolf, C.K. (2024) 'Can Deficits Finance Themselves?', *Econometrica* 92(5), 1351-1390

Ascari, G. (1998) 'On Superneutrality of Money in Staggered-Wage-Setting Models', *Macroeconomic Dynamics* 2, 383-400

Ascari, G. (2004) 'Staggered Prices and Trend Inflation: Some Nuisances', *Review of Economic Dynamics*, 7(3), 642-667

Ascari, G. and Rankin, N. (2007) 'Perpetual Youth and Endogenous Labour Supply: A Problem and a Possible Solution', *Journal of Macroeconomics* 29(4), 708-723

Ascari, G. and Rankin, N. (2013) 'The Effectiveness of Government Debt for Demand Management: Sensitivity to Monetary Policy Rules', *Journal of Economic Dynamics and Control* 37, 1544-1566

Ascari, G. and Sbordone, A.M. (2014) 'The Macroeconomics of Trend Inflation', *Journal of Economic Literature* 52(3), 679-739

Auclert, A., Rognlie, M., and Straub, L. (2024) 'The Intertemporal Keynesian Cross', *Journal of Political Economy* 132(12), 4068-4121

Blanchard, O.J. (1985) 'Debt, Deficits and Finite Horizons', *Journal of Political Economy* 93(2), 223-247

Blanchard, O.J. and Kahn, C.M. (1980) 'The Solution of Linear Difference Models under Rational Expectations', *Econometrica* 48(5), 1305-1311

Calvo, G. (1983) 'Staggered Prices in a Utility-Maximising Framework', *Journal of Monetary Economics* 12(3), 383-398

Christiano, L., Eichenbaum, M. and Rebelo, S. (2011) 'When Is the Government Spending Multiplier Large?', *Journal of Political Economy* 119(1), 78-121

Del Negro, M., Giannoni, M. and Patterson, C. (2015) 'The Forward Guidance Puzzle', Federal Reserve Bank of New York Staff Report No. 574

Diamond, P.A. (1965) 'National Debt in a Neoclassical Growth Model', *American Economic Review* 55(5), 1126-1150

Eggertsson, G.B. (2011) 'What Fiscal Policy Is Effective at Zero Interest Rates?', *NBER Macroeconomics Annual* 25(1), 59-112

Farhi, E. and Werning, I. (2019) 'Monetary Policy, Bounded Rationality and Incomplete Markets', *American Economic Review* 109(11), 3887-3928

Galí, J. (2015) *Monetary Policy, Inflation and the Business Cycle* (2nd ed.), Princeton NJ: Princeton University Press

Ganelli, G. (2007) 'Fiscal Policy Rules in an Overlapping-Generations Model with Endogenous Labour Supply', *Journal of Economic Dynamics and Control* 31, 1015-1036

Leith, C. and Wren-Lewis, S. (2000) 'Interactions between Monetary and Fiscal Policy Rules', *Economic Journal*, 110(462), 93-108

Taylor, J.B. (1993) 'Discretion versus Policy Rules in Practice', *Carnegie-Rochester Conference Series on Public Policy* 39, 195-214

Woodford, M. (2003) *Interest and Prices*, Princeton, NJ: Princeton University Press

Woodford, M. (2011), 'Simple Analytics of the Government Expenditure Multiplier', *American Economic Journal: Macroeconomics*, 3(1), 1-35

Wolf, C.K. (2025) 'Interest Rate Cuts versus Stimulus Payments: An Equivalence Result', *Journal of Political Economy* 133(4), 1235-1275