UNIVERSITY of York

Discussion Papers in Economics

No. 25/04

Monetary Policy, Financing Constraints, and Rational Asset Price Bubbles

Junming Chen

Department of Economics and Related Studies
University of York
Heslington
York, YO10 5DD

MONETARY POLICY, FINANCING CONSTRAINTS, AND RATIONAL ASSET PRICE BUBBLES†

JUNMING CHEN*

Abstract

This paper studies the issue of "should monetary policy lean against rational asset price bubbles" by establishing an analytically tractable New Keynesian model with endogenous capital accumulation. Rational bubbles may exist in equilibrium because of the extra liquidity they generate for financially constrained firms when a lumpy investment opportunity arrives. Under certain conditions, bounded bubble-driven fluctuations (in output) may emerge via both supply-side and demand-side mechanisms. The monetary policy analyses of the model do not strongly favor a leaning-against-the-bubble strategy and emphasize a special overreaction risk that it may suffer from relative to its conventional counterpart.

JEL Classification: E12, E22, E32, E44, E52, E63, G12

Keywords: rational asset price bubbles, monetary policy, financing constraints, New Keynesian model

† This version: November 2025. This paper is a development of two chapters in my PhD thesis, "Monetary Policy and Rational Asset Price Bubbles in a New Keynesian Framework: A Theoretical Analysis", submitted at the University of York, 2025. I am grateful for the contributions, at various stages and aspects, of my supervisor, Professor Neil Rankin. I also thank Professor Paulo Santos Monteiro, Professor Peter Smith, and participants in the 2025 Durham-York-Newcastle Centre for Macroeconomic Policy Workshop for helpful comments. Responsibility for the final product is mine alone.

* Dept of Economics and Related Studies, University of York, York, YO10 5DD, UK. Email: junming.chen@york.ac.uk; cjm_hz@126.com.

It has now been nearly two decades since the 2008 financial crisis. Yet, whether the objective of financial stability, of which one crucial consideration is large volatility of asset prices usually viewed as a sign of speculative "bubbles" in the market,[1] should be taken into account in the monetary policy formulation or not remains an unsettled issue of recurrent debate.[2]

Aiming to provide some rigorous theoretical foundations for studying this policy question, I develop a New Keynesian (NK, for short) model with endogenous capital accumulation, which permits potential *rational* bubble-driven fluctuations (in output) via both supply-side and demand-side mechanisms in equilibrium, so that meaningful model-based study of their possible interactions with alternative monetary interventions can be conducted. In order to understand in depth the economic forces at work, the established model is sparing enough that analytical solutions are mostly available. It is hoped that the transmission mechanism and the consequences of various types of monetary policy interventions for the bubbly economy will then be clear, and that policy suggestions based on these results will in turn be fairly transparent as well. Also, given that the NK framework remains the workhorse framework deployed for modern monetary policy analysis, discoveries from the present work are expected to generate insights regarding restoring the NK paradigm as a suitable one for deliberations about monetary policy in the post-crisis era.

I conduct the analysis in this paper in two broad steps. First, I build up a benchmark version of the model without nominal rigidities, which permits the potential equilibrium existence of rational bubble, and I analyze in depth its long and short run general equilibrium impacts on the economic system. Second, I incorporate nominal rigidities into the model which then gives rise to the non-neutrality of monetary policy and hence enables me to conduct discussions of monetary policy in the presence of rational bubbles in a tractable manner.

There are two key assumptions in the present framework. First, individual firms are assumed to be subject to uninsurable idiosyncratic investment shocks a là Kiyotaki and Moore (2019), such that firms cannot make capital investment whenever they want. This assumption may be thought of as capturing the lumpiness of firm-level investment and it generates ex post heterogeneity

[1] The work by LeRoy and Porter (1981) and Shiller (1982) identifies that large scale of volatility of actual observed asset prices is hardly explained by their market fundamental. Tirole (1985) points out that this phenomenon may suggest the possible existence of bubbles in the asset prices.

[2] For example, Bernanke and Gertler (1999, 2001) insist that central banks should focus on stabilizing inflation and the output gap but ignoring asset market developments, while many others, including Borio and Lowe (2002) and Taylor (2014) argue that the achievement of low and stable inflation is not a guarantee for financial stability and call for monetary policy to pay more attention to asset price movements.

among firms. Second, the firms are financially constrained, such that they have no access to any external means of financing, but they can purchase a type of bubbly asset. The combination of these two ingredients then creates room for rational asset price bubbles to emerge in equilibrium, because in that environment, the bubbles may serve as a financing vehicle in generating extra liquidity for firms with an investment opportunity and thus may be valuable to their rational holders even though they are intrinsically worthless.

The parsimonious setup allows me to obtain a set of closed-form solutions of the (baseline) model, the key one of which clearly demonstrates that rational bubbles in this type of financing-constraint environment are not required to grow at the rate of interest in equilibrium, because they also benefit their holders, the firms, by providing extra liquidity when an investment opportunity comes, thus generating a "*liquidity premium*" component in the bubble pricing equation. As a result, a rational bubble will not be ruled out by standard transversality conditions of infinitely lived agents. Furthermore, three mutually exclusive regions of parameter space are identified, only one of which (the one with a sufficiently low arrival rate of an investment opportunity) permits a possible bubbly equilibrium.

After characterizing the existence and potential multiplicity of (zero-inflation) steady states with or without a bubble, I proceed to analyze the equilibrium properties of the (full) model around such a steady state and the role of alternative monetary policies in shaping them. In particular, I study two categories of interest rate rules: a strict (zero) inflation targeting rule and a Taylor-type simple rule with a potential leaning-against-the-bubble (LAB, for brevity) component, to assess the implications of the policy responding explicitly to the bubble in different scenarios.[3]

Several findings of interest emerge from the study of the model built up in this paper.

First, and most importantly, the economy turns out to be always exposed to bounded bubble fluctuations if it is bubbleless at a steady state and when the arrival rate of an investment opportunity is low enough. The bubble fluctuations display a property of intrinsic persistence due to their rational nature: a random one-period innovation in the market value of the bubble could have a prolonged impact on the equilibrium dynamics of the system. Furthermore, if nominal rigidities are absent, the degree of persistence of a bubble-driven fluctuation is algebraically shown

[3] I provide some discussions about welfare-relevant "optimal" monetary policy in the body of the paper, but I choose not to formally investigate it. Specific rationale behind this practice is given in Section IV below.

to be positively related to the arrival rate of an investment opportunity: the scarcer it is, the less persistent the bubble and the resulting output fluctuations are. This is because in that situation the liquidity premium commanded by a bubble is higher, which in turn entails a lower required growth rate for the bubble price in equilibrium. The bubble impact on output is also shown to be positively related to the arrival rate of an investment opportunity.

Second, an increase in the size of the bubble generally enhances the financing capacity of an investing firm, leading to an expansion of capital investment and thus output as well. A decline in the implied market value of a unit of capital, Tobin's q, also occurs and reflects an improvement in the efficiency of the economy in allocating resources during the course of a bubble-led boom.

Third, in the presence of nominal rigidities, LAB is indispensable in achieving the ambitious goal of hitting a strict inflation target, especially when the economy is inefficient in terms of resource allocation but is bubbleless at a steady state. However, this is a theoretical benchmark, and in a more practical policy deliberation, adopting an explicit LAB motive in a Taylor-type simple interest rate rule may risk causing an economic downturn in the face of a bubble boom, the responsibility for which lies critically in the asymmetric feedback mechanism between the evolution of the rational bubble and the policy rate. This type of "overreaction risk", on the other hand, is not very likely to occur in a policy regime with conventional feedback only on the output gap and inflation.

Fourth, contrary to the finding about the economy fluctuating near a bubbleless steady state, when the system fluctuates around a bubbly steady state, adopting a LAB strategy appears to be neither necessary nor effective for the monetary authority as far as insulating the system from bubble-driven fluctuations is concerned, under either a strict inflation targeting or a simple interest rate rule regime. This outcome may be due to the fact that the two determinants of the law of motion of the bubble are then affected in opposite ways by interest rate variations, a novel feature which appears to be different from that in the related literature on rational bubbles with monetary policy analysis.

Contribution to the literature. In the related literature, an asset "bubble" is typically defined as the difference between the observed price of the asset and the expected discounted value of the dividends it generates (i.e., the "fundamental" value of the asset).[4] A notable feature is that a considerable amount of effort in the research field of monetary policy analysis with asset bubbles is devoted to asset price volatility caused by "irrational" bubbles (e.g., Bernanke and Gertler 1999, 2001, Carlstrom and Fuerst 2007, Nisticò 2012), but little to its counterpart, the one studied in the present paper, "rational" bubbles, especially in a NK framework. By "rational", is meant that the bubble component of an asset price is in equilibrium consistent with rational expectations, individual optimization, and market clearing. "Irrational bubbles", on the contrary, are not required to be so.[5] Therefore, the general equilibrium properties of rational and irrational bubbles could be quite different, which in turn implies that relevant monetary policy implications could potentially be different. In order to have a more comprehensive understanding of the issue at hand, paying more attention to monetary policy analysis with rational bubbles is thus worthwhile.

There are two dominant types of scenarios that can give rise to a rational bubble in the existing theoretical models.[6] The most celebrated one is "dynamic inefficiency", which stems from the work by Samuelson (1958) and Tirole (1985). This class of models features a (two-period) overlapping generations (OLG) structure with the economy growing over time, where cohorts who born later are richer than their predecessors either because they have higher productivity or because they are larger and can produce more at scale. Rational bubbles that grow at the rate of interest in this type of situation serve as a method of intertemporal transfer of wealth that makes all agents in the economy better off. Based on this mechanism, Galí (2021) investigates the interaction between monetary policy and rational bubbles in a NK model with perpetual-youth OLG of the type in Blanchard (1985). Although inspiring and elegant, bubble fluctuations could only have an impact in his model mainly as aggregate demand shifters. In other words, unlike the present model, potential supply-side impact of rational bubbles on an economy and the associated monetary policy implications are absent in Galí (2021).

[4] See, e.g., Tirole (1985) and Hirano and Toda (2024, 2025) for formal definition and characterization of asset price bubbles. Recent literature also adopts a "risk premium" approach towards defining a "bubble". See, e.g., Caballero and Simsek (2020).

[5] For surveys on models of rational or irrational bubbles, see Brunnermeire and Oehmke (2013), Miao (2014), Martín and Ventura (2018), Barlevy (2018), Hirano and Toda (2024).

[6] See also, Santos and Woodford (1997) for conditions that allow rational bubbles to arise in general equilibrium. Hirano and Toda (2025) also provide more recent views of the theory of rational bubbles.

Another prevailing approach of modelling rational bubbles features financial frictions, where bubbles generally arise to relax the financing constraints faced by economic agents.[7] The reason that a rational bubble emerges in the present model puts it in this category, and its spirit is closely related to that of Wang and Wen (2012). By extending Miao, Wang, and Xu (2015), Dong, Miao, and Wang (2020) and Ikeda (2021) study monetary policy implications of this type of bubble in a NK framework. However, their models are analytically intractable, leading to some ambiguity about the general equilibrium mechanisms at work and about the interactions between their proposed monetary policies and the rational bubbles. Additionally, an important feature of actual speculative bubble episodes is absent in Dong, Miao, and Wang (2020) and Ikeda (2021), namely, scenarios where a no-bubble economic state is the starting point. My research in this paper therefore also complements their work in these two regards.

There are models investigating the monetary policy implications of rational bubbles but which abstract from NK elements, examples being Biswas, Hanson, and Phan (2020) and Asriyan et al. (2021), among some others. While analyses in these models are mostly conducted with an OLG structure, which are arguably too stylized for potential quantitative policy analysis, the channel through which the effects of monetary policy are transmitted is distinct from that highlighted in models with NK features.

I. The Baseline Model without Nominal Rigidities

The structure of the baseline version of the model is closely related to Wang and Wen (2012), which features an infinite-horizon production economy populated by two types of economic agents: a unit mass of perfectly competitive firms producing homogeneous consumption goods and a unit mass of identical infinitely lived households who supply labor inelastically and trade the shares of the firms.

However, crucially for the model's distinctive features relative to Wang and Wen (2012),[8] here firms are assumed to be subject to a type of independent idiosyncratic investment shock a là Kiyotaki and Moore (2019), which is uninsurable. On the other hand, firms can invest in a type of

[7] Examples of this type also include Bewley (1983), Aiyagari (1994), Kocherlakota (2009), Miao, Wang, and Zhou (2015), Hirano and Yanagawa (2017), Miao and Wang (2018) in an infinite-horizon framework; Martín and Ventura (2012, 2016) in a 2-period OLG framework.

[8] Specifically, in Wang and Wen (2012), individual firms are subject to an idiosyncratic investment-specific productivity shock which follows a defined distribution; also, the overall supply of bubbly assets varies overall time. There are other differences between my model and theirs, but they are not in the central concerns here.

intrinsically worthless bubbly asset,[9] whose overall supply outstanding is constant and equal to one. These modifications enable me to obtain a full set of closed form solutions of the model without undermining the key mechanisms concerning the general equilibrium implications of rational bubbles of this type.

To streamline the analysis, it is also assumed that there is no aggregate uncertainty about fundamentals. Time is discrete and indexed by $t = 0,1,...$

A. Firms

In period t, a typical individual firm $j \in [0,1]$ produces homogeneous consumption goods Y_{jt} by hiring labor $N_{jt} \geq 0$ at a competitive real wage rate W_t with physical capital $K_{jt} \geq 0$, according to the Cobb-Douglas production function

$$(1) \qquad Y_{jt} = K_{jt}^{\alpha} N_{jt}^{1-\alpha},$$

where $\alpha \in (0,1)$. Solving the static optimal labor demand problem yields the familiar results that $N_{jt} = \left[(1-\alpha)/W_t\right]^{1/\alpha} K_{jt}$ and $Y_{jt} = \left[(1-\alpha)/W_t\right]^{(1-\alpha)/\alpha} K_{jt}$, which in turn imply that in aggregate, employment and output depend only on the aggregate capital stock. As a consequence, the cross-section distribution of individual capital stock does not need to be tracked. The gross operating profit for the firm also turns out to be proportional to the capital stock and is given by

$$(2) \qquad Y_{jt} - W_t N_{jt} = R_{kt} K_{jt} = \alpha Y_{jt},$$

with $R_{kt} \equiv \alpha \left[(1-\alpha)/W_t\right]^{(1-\alpha)/\alpha}$.

Firm j is subject to an uninsurable idiosyncratic investment shock of the kind proposed by Kiyotaki and Moore (2019), $\tau_{jt} \sim B(1,\eta)$, i.e., the shock follows a Bernoulli distribution

$$(3) \qquad \tau_{jt} = \begin{cases} 1, & \text{with prob. } \eta \\ 0, & \text{with prob. } (1-\eta) \end{cases},$$

[9] By "intrinsically worthless", it means that it does not deliver any payoff or direct utility. Thus, the bubble is a "pure" one. Pure bubbly assets may be thought of as "pieces of papers" (Tirole 1985), "paper money" (Samelson 1958, Kiyotaki and Moore 2019), or useless land (Miao, Wang, and Zhou 2015), cryptocurrencies (Dong, Xu, and Zhang 2022).

which is independently and identically distributed over time and across firms and is independent of aggregate shocks, such that the firm can only have an opportunity to install I_{jt} units of new capital from I_{jt} units of the consumption goods with a constant probability η in each period. Therefore, each period only a constant measure $\eta \in (0,1)$ of firms can possibly make capital investment, with η becoming a natural index of scarcity or lumpiness of firm-level investment in this context. Given the capital depreciation rate, $(1-\lambda)$, with $\lambda \in (0,1)$, the law of motion for capital stock accumulation at the beginning of period $t+1$ for the firm is then given by

(4) $$K_{jt+1} = \tau_{jt} I_{jt} + \lambda K_{jt}.$$

Firms face financing constraints such that they can neither borrow nor raise new equity to fund their lumpy investments.[10] This also implies that they cannot pay negative dividends, i.e.,

(5) $$D_{jt} \geq 0$$

for all t, j. Nonetheless, firms can invest in a bubbly asset at price $Q_t^B \geq 0$ at date t,[11] although they cannot short sell it, with

(6) $$Z_{jt+1} \geq 0$$

the quantity purchased of the bubbly asset by firm *j* at the end of period *t*. The firm *j*'s flow-of-funds (FOF, for short) constraint at period *t* is then given by

(7) $$D_{jt} + \tau_{jt} I_{jt} + Q_t^B \left(Z_{jt+1} - Z_{jt} \right) = R_{kt} K_{jt}.$$

The left-hand side of (7) is the firm's expenditure on dividends paid out, investment, and net bubbly asset purchases, while the right-hand side is the operational revenue income.

The individual firm *j*'s objective is to maximize the expected present value of the firm:

(8) $$V_{j0} = E_0 \sum_{t=0}^{\infty} \Lambda_{0,t} D_{jt} ,$$

[10] The motive for assuming the complete absence of external financing channel for firms is mainly for modelling convenience. There are several ways that may be used to justify this assumption in the literature. For example, there may be weak enforcement institutions such that firms cannot commit to making any future repayments of their borrowing, which then effectively prevents firms from issuing any debts (Martín and Ventura 2012); or it may be considered as an extreme case of borrowing restriction due to inalienable entrepreneurial skills for production in the style of Hart and Moore (1994) (see also, Hirano and Yanagawa 2017). On the other hand, it may be too costly for firms to engage in equity financing so that they choose not to issue any new equities to raise funds for investment (Miao, Wang, and Zhou 2015).

[11] The non-negativity on the bubble price comes from free disposal assumption of the asset.

with $\Lambda_{0,t} = \Lambda_{0,1} \times \Lambda_{1,2} \times ... \times \Lambda_{t-1,t}$ the stochastic discount factor between period 0 and *t*, subject to a sequence of the FOF constraints (7), the capital accumulation equation (4), and the non-negativity constraints for dividends and bubble holdings for $t = 0,1,...$, given $K_{j0} > 0$ and $Z_{j0} = 1$. Let q_t be the (implied) market value of one unit of capital, i.e., Tobin's (marginal) *q*. The following propositions then characterize the results derived from the firm's optimization problem, with proof being provided in Appendix A.

Proposition 1

If $q_t = 1$ *(for all t), then* $Q_t^B = 0$ *(for all t) in equilibrium. Firms are indifferent to the lumpy investment opportunity and the financial frictions do not actually matter.*

Proposition 2

If $q_t > 1$ *(for all t), then in equilibrium:* $Q_t^B \geq 0$ *and the law of motion for the rational bubble is given by*

(9) $$Q_t^B = E_t \left\{ \Lambda_{t,t+1} \left[1 + \eta \left(q_{t+1} - 1 \right) \right] Q_{t+1}^B \right\},$$

while for Tobin's q,

(10) $$q_t = E_t \left\{ \Lambda_{t,t+1} \left[R_{kt+1} + \lambda q_{t+1} + \eta (q_{t+1} - 1) R_{kt+1} \right] \right\}.$$

A firm j with an investment opportunity will optimally choose not to hold any bubbly assets but to use all its available resources to invest in capital, with the optimal investment rule given by

(11) $$I_{jt} = R_{kt} K_{jt} + Q_t^B Z_{jt}.$$

Firms without an investment opportunity will optimally choose to purchase the bubbly asset and to pay out dividends.

To have some intuition behind the above propositions, note first of all that in the present context, Tobin's *q* is a natural index measuring the efficiency of the model economy in terms of allocating resources. In the first-best situation, a firm would want to invest in capital up to the point where the marginal revenue of possessing capital equals its marginal cost, i.e., Tobin's *q* equals one. Therefore, if $q_t > 1$, it means that more investment in capital is profitable and thus desirable. But this additional amount of investment cannot be carried out by investing firms in that circumstance because of the presence of financing constraints. A rational bubble can then make a difference: by selling bubbly assets (to non-investing firms), firms with an investment opportunity can acquire additional funds to expand their investment, so that a bubble can be valuable to its rational holders even though it is intrinsically worthless, with $Q_t^B > 0$ being possible in equilibrium. Of course, the materialization of the additional liquidity benefits generated by the bubble relies on the collective faith of its investors (firms), because a bubble is valuable to someone only when other people find it valuable, so a no-bubble equilibrium always exists.

The pricing equation (9) also indicates formally that if the price of the bubble is strictly positive, then the return on it consists of not only the capital gains, but also the *liquidity premium* that is represented by the term, $\eta\left(q_{t+1}-1\right)$. In other words, unlike the kind of models studied in an OLG model where rational bubbles emerge because of dynamic inefficiency, the bubble in the present model is not required to grow at the rate of interest, because of the liquidity premium it provides: when an investment opportunity arrives in the next period with probability η, one unit of the bubble can be resold against its value Q_{t+1}^B, financing investment at a marginal cost equal to unity and generating marginal revenues measured by $q_{t+1} > 1$. Thus, the rational bubble will not be ruled out by the standard transversality conditions of the infinite-lived agents.

A similar narrative applies to the required return for a unit of capital. As indicated by (10), a unit of capital generates R_{kt+1} units of additional funds which can then be used to finance the lumpy investment, and which in turn produces additional expected return, i.e., the 'liquidity premium', measured by $\eta(q_{t+1}-1)R_{kt+1}$. These outcomes concerning the liquidity premium contained in the pricing equations for the bubble and Tobin's q are also consistent with those found in the relevant literature including Wang and Wen (2012), Miao and Wang (2018), Kiyotaki and Moore (2019).

On the other hand, if $q_t = 1$, i.e., Tobin's q equals one, it implies that the economy is efficient in allocating resources regardless of the occurrences of the idiosyncratic investment shock and the absence of external financing channels. As a consequence, bubbles are not valued by the economic agents and the equilibrium must be bubbleless.

B. Households

Households are identical and they supply labor inelastically, with labor endowment, N_t, equal to one. Households can trade riskless bonds which are in zero net supply in aggregate, and firms' equities. The net supply of each firm's shares is normalized to one.[12]

The optimization problem facing the representative household is to maximize expected lifetime utility, $E_0 \sum_{t=0}^{\infty} \beta^t \log C_t$, subject to a sequence of budget constraints

$$C_t + B_{t+1} + \int_0^1 Z_{jt+1}^{Sh} \left(V_{jt} - D_{jt} \right) dj = W_t N_t + \left(1 + r_{t-1} \right) B_t + \int_0^1 Z_{jt}^{Sh} V_{jt} dj \tag{12}$$

for $t = 0, 1, \ldots$. $\beta \equiv 1/(1+\rho) \in (0,1)$ is the subjective discount factor, C_t is the consumption for the household; B_{t+1} is the value of the one-period riskless bonds purchased at the end of period *t*, with r_t the (net) risk-free rate between period *t* and *t*+1; Z_{jt+1}^{Sh} is the individual firm *j*'s stock shares purchased by the household at the end of period *t*, with V_{jt} the (before dividends) market value of it. From the household's maximization problem, we have $1 = E_t \left[\left(1 + r_t \right) \Lambda_{t,t+1} \right]$, where

$$\Lambda_{t,t+1} \equiv \beta \frac{C_t}{C_{t+1}} \tag{13}$$

is defined as the stochastic discount factor between period *t* and *t*+1.

C. General Equilibrium

Competitive Equilibrium.— A competitive equilibrium consists of sequences of prices $\left\{ W_t, r_t, Q_t^B, V_{jt} \right\}_{t=0}^{\infty}$, and quantities $\left\{ I_{jt}, N_{jt}, K_{jt+1}, Z_{jt+1}, Y_{jt}, Z_{jt+1}^{Sh}, C_t, B_{t+1} \right\}_{t=0}^{\infty}$ for $j \in [0,1]$, such that:

(a) Given prices $\left\{ W_t, Q_t^B \right\}_{t=0}^{\infty}$, the sequence of quantities $\left\{ I_{jt}, N_{jt}, K_{jt+1}, Z_{jt+1} \right\}_{t=0}^{\infty}$ solves each individual firm *j*'s optimization problem (8) subject to (4)-(7);

[12] In the Supplemental Appendix A (Chen 2025), I provide an extension to this setup, where households are allowed to buy the bubbly asset as the firms do. It demonstrates that this alternative assumption does not alter any of the results derived here. The reason is that the the expected return on the bubble is too low to the households due to the presence of the liquidity premium, so that they will optimally choose not to hold any of the bubbly asset in equilibrium.

(b) Given prices $\{r_t, Q_t^B, V_{jt}\}_{t=0}^{\infty}$, the sequence of quantities $\{C_t, B_{t+1}, Z_{jt+1}^{Sh}\}_{t=0}^{\infty}$ maximizes the representative household's expected lifetime utility subject to (12);

(c) The markets for labor, riskless bonds, the bubbly asset, individual firms' shares, and consumption goods all clear, i.e., $\int_0^1 N_{jt} dj = N_t = 1$, $B_t = 0$, $\int_0^1 Z_{jt} dj = 1$, $\int_0^1 Z_{jt}^{Sh} dj = 1$ for $\forall j \in [0,1]$, and $C_t + \int_0^1 I_{jt} dj = \int_0^1 Y_{jt} dj$.

Aggregation and General Equilibrium.— Denote Ω^i with measure η the set of individual firms with an investment opportunity and Ω^s with measure $1-\eta$ the set of firms without an investment opportunity in each period. Define aggregate variables: $I_t \equiv \int_0^1 I_{jt} dj = \int_{j\in\Omega^i} I_{jt} dj$, $K_t \equiv \int_0^1 K_{jt} dj$, $N_t \equiv \int_0^1 N_{jt} dj$, $Y_t \equiv \int_0^1 Y_{jt} dj$. The goods market clearing condition then becomes $C_t + I_t = Y_t$.

In aggregation, we thus have: $N_t = \left[\left(1-\alpha\right)/W_t\right]^{1/\alpha} K_t$, $Y_t = \left[\left(1-\alpha\right)/W_t\right]^{(1-\alpha)/\alpha} K_t$, which implies that $Y_t = K_t^{\alpha} N_t^{1-\alpha} = K_t^{\alpha}$. Also, from (2), $W_t = \left(1-\alpha\right) Y_t / N_t = \left(1-\alpha\right) K_t^{\alpha}$, and

$$R_{kt} = \alpha \frac{Y_t}{K_t} = \alpha K_t^{\alpha-1}, \tag{14}$$

which turns out to be the marginal product of capital (MPK). The law of motion for aggregate capital stock is given by

$$K_{t+1} = I_t + \lambda K_t . \tag{15}$$

Given that the arrival of an investment opportunity is i.i.d. across firms and through time, those who turn out to have an investment opportunity must possess η proportion of total capital stock in the economy and of the total amount of the bubbly asset at the start of each period, i.e., $\int_{j\in\Omega^i} K_{jt}^i dj = \eta K_t$ and $\int_{j\in\Omega^i} Z_{jt}^i dj = \eta$. Therefore, for the case when Tobin's q is greater than one, aggregating (11) across all investing firms yields the equation for aggregate investment:

$$I_t = \eta R_{kt} K_t + \eta Q_t^B . \tag{16}$$

Equation (16) thus demonstrates that a positive bubble boosts aggregate investment by increasing directly firms' available funds.

Finally, notice that although the pricing equation for Tobin's q, (10), is derived under the assumption that $q_t > 1$ (see Appendix A), it still applies when $q_t = 1$.

Next, I characterize the steady states and general equilibrium dynamics of the baseline economy, which are among the key theoretical results of the model.

II. Steady States and Dynamics of the Baseline Model

A. Steady States

From the Euler equation of the household and definition (13), $1/(1+r) = \Lambda = \beta$ in a steady state (SS, for brevity). Define $q_t^B \equiv Q_t^B / K$ as the bubble-capital ratio with K the SS value of the aggregate capital stock. Then evaluating (10) in a SS yields

(17) $$R_k = \frac{(1-\lambda\beta)q}{\beta\left[(1-\eta)+\eta q\right]},$$

combining which with the SS version of (14) can always recover the SS value of aggregate capital stock and the other aggregate variables of interest.

We have already learnt that (from Proposition 1) when Tobin's q is equal to 1, the economy must be bubbleless. By setting $q = 1$, we can use (17) to obtain $R_k = R_k^* \equiv 1/\beta - \lambda$ in the SS.

On the other hand, if Tobin's q is greater than 1, then the economy could either be bubbleless or bubbly, as hinted by Proposition 2. Consider first the situation when $q > 1$ but $q^B = 0$. Combining the SS version of the aggregate investment equation (16) with that of (15) gives

(18) $$R_k = R_{k_l} \equiv \frac{1-\lambda}{\eta}.$$

Then equating (18) with (17) solves Tobin's q: $q = q_l \equiv \beta(1-\lambda)(1-\eta)/\left[\eta(1-\beta)\right]$. For $q > 1$, it implies that the condition $\eta_1 \equiv \beta(1-\lambda)/(1-\beta\lambda) > \eta (> 0)$ must be satisfied in this case.

If instead $q^B > 0$, we can solve for Tobin's q from the pricing equation of the bubble, (9):

(19) $$q = q_b \equiv 1 + \frac{1}{\eta}\left(\frac{1}{\beta} - 1\right),$$

which yields

(20) $$R_k = R_{k_b} \equiv (1-\lambda\beta)\left[1+\frac{1}{\eta}\left(\frac{1}{\beta}-1\right)\right]$$

by plugging (19) into (17). We can then make use of (20) and the SS version of (15)-(16) to obtain

(21) $$q^B = q_b^B \equiv \frac{1}{\eta}\left[(1-\eta)(1-\lambda\beta)-\left(\frac{1}{\beta}-1\right)\right].$$

Therefore, for $q_b > 1$ and $q^B > 0$, (19) and (21) imply that for a SS to be possibly bubbly, it must satisfy $\eta_2 \equiv 1-(1-\beta)/\left[\beta(1-\lambda\beta)\right] > \eta(>0)$.

The following proposition then identifies mutually exclusive regions of parameter space, each of which corresponds to different types of SS.

Proposition 3

<u>Case 1</u> *If $1-\lambda > \rho^2$, the parameter space of the model can be divided into three mutually exclusive regions for η, in which different types of SS prevail:*

(1a) Region 1, $\eta \in [\eta_1, 1)$. Only a bubbleless SS can exist, with $q^B = 0$, $q = q^* \equiv 1$, $R_k = R_k^*$, $K = K^*$, which corresponds to the economy's first-best allocation outcome.

(2a) Region 2, $\eta \in [\eta_2, \eta_1)$. Only a bubbleless SS can exist, with $q^B = 0$, $q = q_l > 1$, $R_k = R_{k_l}$, $K = K_l$.

(3a) Region 3, $\eta \in (0, \eta_2)$. Two SSs coexist, one of which is bubbleless with $q = q_l > 1$, $R_k = R_{k_l}$, $K = K_l$, while the other is bubbly with $q^B = q_b^B > 0$, $q = q_b > 1$, $R_k = R_{k_b}$, $K = K_b$. In addition, $q_l > q_b > q^* = 1$, $R_{k_l} > R_{k_b} > R_k^*$, and $K^* > K_b > K_l$.

<u>Case 2</u> *If $1-\lambda \le \rho^2$, the parameter space of the model is then divided into two mutually exclusive regions in neither of which a bubbly SS is possible:*

(1b) Region 1, $\eta \in [\eta_1, 1)$. Only the first-best SS exists.

(2b) Region 2, $\eta \in (0, \eta_1)$. Only a bubbleless SS can exist, with $q = q_l > 1$, $R_k = R_{k_l}$, $K = K_l$, where $R_{k_l} > R_k^*$ and $K^* > K_l$.

Proof: see Appendix B.

Figure 1 summarizes graphically the regions of parameter space with their associated SS(s).

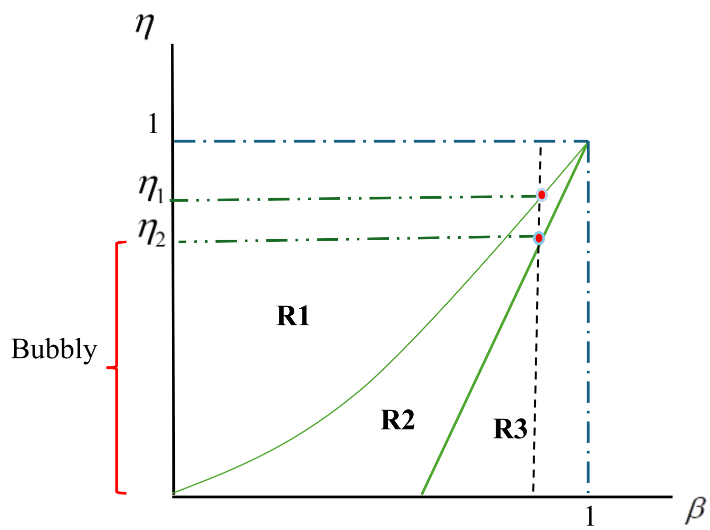

FIGURE 1. REGIONS OF PARAMETER SPACE

Note: R1: efficient and bubbleless; R2: inefficient and bubbleless; R3: inefficient and (possibly) bubbly (2 SSs). Note that Figure 1 covers case 2 of Proposition 3 as well: in that case, "R3" disappears with only the curve through the origin separating "R1" and "R2". Given a β value, only when the arrival rate of an investment opportunity lies below η_2 can a bubbly SS possibly exist.

Proposition 3 thus reveals the crucial role played by the arrival rate of an investment opportunity in admitting the existence of a bubbly equilibrium in the present framework: only when η is sufficiently low can a bubble emerge in equilibrium, i.e., when η lies in the region 3 of the parameter space in case 1. If instead, η is high enough so that it lies within the region 1 of either case 1 or 2, then the economy could reach the first-best allocation outcome by itself, even with the presence of the financial frictions. I discuss more thoroughly the economic narrative underlying these regions in Chen (2025), particularly in terms of the peculiar scenario where the economy is inefficient in allocating resources but where the bubble cannot emerge in equilibrium, i.e., when η lies in the region 2 of case 1 or 2 in Proposition 3.[13] In the Supplemental Appendix A2 (Chen 2025), I provide an extension to the model demonstrating that Proposition 3 still (largely) holds in an environment with multiple (instead of a single) bubble assets, which is the type of setup in

[13] To my knowledge, this phenomenon has not yet been properly addressed by relevant studies on rational bubbles with financing constraints. For instance, although Miao and Wang (2018) and Kiyotaki and Moore (2019) also recognize the existence of this additional region of parameter space, neither of them make analysis or explanation for it.

Miao, Wang, and Zhou (2015), Dong, Miao, and Wang (2020), and Ikeda (2021) among the relevant literature. It thus indicates that the (generic) conclusions drawn from the present framework should be robust to a wider range of relevant models in rational bubbles of this strand with more sophisticated settings.

B. Equilibrium Dynamics

In this subsection, I turn to analyze the equilibrium dynamics of the model economy in a neighborhood of a given SS characterized in Proposition 3. To make progress in this direction, I begin by log-linearizing the equilibrium conditions of the model around a SS. Throughout the analysis, $\hat{q}_t^B \equiv q_t^B - q^B$ denotes the deviation of the aggregate bubble-capital ratio from its SS value and $\hat{r}_t \equiv \log\left[(1+r_t)/(1+r)\right]$ is the log deviation of the gross real interest rate; for the remaining variables of interest, lowercase letters are used to define the log of the original variable while the "^" symbol on top of a variable are used to indicate the deviation from its SS value (except that $\hat{q}_t \equiv \log(q_t/q)$). When it is needed, I also continue to use subscript "*b*" to distinguish a SS value of a variable in a bubbly case, and "*l*" for that in a bubbleless but inefficient case, and to use the "*" symbol at the top right corner of a SS value of a variable to denote that in the most-efficient situation.

To streamline the discussion here, I focus on equilibria that may feature a bubble, i.e., when the economy is characterized by parameters of region 3 of case 1 in Proposition 3, where $1-\lambda > \rho^2$ and $\eta \in (0, \eta_2)$. I next analyze separately the circumstances of dynamics near a bubbleless SS and a bubbly SS in that bubbly economy. The full set of log-linearized equilibrium conditions are relegated to the Supplemental Appendix B1 (Chen 2025).

Around a Bubbleless SS.— When the economy features $\eta \in (0, \eta_2)$ and fluctuates around $q^B = 0$, the evolution of the bubble is described by

$$\hat{q}_t^B = \frac{(1-\eta)(1-\lambda\beta)\beta}{1-\beta} E_t \hat{q}_{t+1}^B. \tag{22}$$

It is noticeable from (22) that to a first-order approximation, the expected evolution of the bubble-capital ratio is autonomous and unaffected by changes in the interest rate or Tobin's q. Note also that $\eta \in (0, \eta_2)$ is equivalent to $\{(1-\beta)/[(1-\eta)(1-\lambda\beta)\beta]\} \in (0,1)$. Thus, as long as the economy lies within the region 3 of case 1 in Proposition 3, there exist bounded rational expectation solutions to (22) other than $\hat{q}_t^B = 0$ for all t,[14] which take the form:

$$\hat{q}_{t+1}^B = \frac{1-\beta}{(1-\eta)(1-\lambda\beta)\beta}\hat{q}_t^B + \varepsilon_{t+1}, \tag{23}$$

where $\varepsilon_{t+1} \equiv \hat{q}_{t+1}^B - E_t\hat{q}_{t+1}^B$ is thought of as the aggregate bubble innovation (the "bubble shock") in the context and as capturing actual speculative bubble episodes where mood swings in investor communities drive up or down the price for an asset on the sheer basis of expectations of future adjustments in the price of the same asset, regardless of the absence of any news concerning its fundamentals.[15] As shown in the Supplemental Appendix B2 (Chen 2025), the local dynamics of $(\hat{y}_t, \hat{q}_t^B)$ could then be jointly represented by

$$\begin{bmatrix} \hat{y}_{t+1} \\ E_t\hat{q}_{t+1}^B \end{bmatrix} = \begin{bmatrix} \lambda + \alpha(1-\lambda) & \alpha\eta \\ 0 & (1-\beta)/[(1-\eta)(1-\lambda\beta)\beta] \end{bmatrix} \begin{bmatrix} \hat{y}_t \\ \hat{q}_t^B \end{bmatrix}. \tag{24}$$

The system (24) has two eigenvalues, $\lambda_{b1} \equiv \lambda + \alpha(1-\lambda)$ and $\lambda_{b2} \equiv (1-\beta)/[(1-\eta)(1-\lambda\beta)\beta]$, both of which lie within the unit circle.[16] In other words, as has already been pointed out, the system in this case is always subject to bounded bubble(-driven) fluctuations. This may be somehow intuitive, given that the bubble is desirable in improving the economy's efficiency within this region but just depends on the "fragile" collective faith of the rational agents on it. This type of bubble fluctuations may be considered as a plausible representation of recurrent boom and bust episodes of aggregate bubbles that occur in the real economic world, with the bubbleless state being a resting point.

[14] This outcome also confirms that if the economy lies within the region 2 in Proposition 3, equilibria in a local area of the necessarily bubbleless SS must always be bubbleless, since when .. $\eta \in [\eta_2, \eta_1)$ .., $(1-\beta)/[(1-\eta)(1-\lambda\beta)\beta] > 1$. In other words, there does not exist a continuum of bubbly equilibria converging to the bubbleless SS in the region 2 economy.

[15] Galí (2021) considers similar type of bubble shocks.

[16] This outcome implies that the Blanchard-Kahn condition for determinacy of rational expectation solutions actually fails in this case, since the aggregate bubble-capital ratio is non-predetermined and should hence correspond to an unstable eigenvalue.

Let us assume that the system originally sits at its SS but is now hit by a "small" bubble shock, $\hat{q}_0^B = \varepsilon_0 > 0$ at some time $t = 0$, which lasts only for one period. The time paths for output and the aggregate bubble-capital ratio are then given by

(25) $$\hat{y}_t = \alpha\eta\left(\sum_{i=1}^{t}\left(\lambda_{b1}\right)^{t-i}\left(\lambda_{b2}\right)^{i-1}\right)\hat{q}_0^B,$$

(26) $$\hat{q}_t^B = \left(\lambda_{b2}\right)^t \hat{q}_0^B,$$

respectively. Notably, the evolutions of both the bubble and output are affected by η: the higher the arrival rate of an investment opportunity, the more persistent the bubble and the resulting output fluctuations are, as well as the larger is the initial bubble impact on output, which may be interpreted as the "*bubble multiplier*" and which is measured by $\alpha\eta$.[17]

The explanation for this phenomenon concerning persistence is twofold. First and foremost, the fact that a transitory bubble shock can generate prolonged fluctuations of the system is a consequence of the equilibrium requirement of the rational bubble that its capital gain matches the rate of interest (after deducting a liquidity premium), so that the initial bubble shock has an impact on the size of the bubble over time and thus on output as well. Second, a higher value of η implies a lower degree of inefficiency of the economy in equilibrium, given other things equal, which in turn implies a lower liquidity premium contained in the overall return on the bubble.[18] As a result, the bubble price must "grow" at a higher rate in order to be attractive enough in equilibrium alongside a higher arrival rate of an investment opportunity in the present context,[19] which contributes further to the persistence of the bubble fluctuation.

On the other hand, the bigger bubble multiplier on output given a higher value of η comes from the fact that the proportion of the bubbly asset that sold for investments is then higher, as indicated clearly by the aggregate investment equation (16), hence the same size of a bubble boom would induce a stronger "*crowding-in*" effect of the bubble on investment if the arrival rate of an

[17] Since output is predetermined in the baseline model, the impact of the bubble shock on output lags one period.

[18] This is not difficult to show formally. Intuitively, the higher the arrival rate of an investment opportunity, the closer the model economy is to the most efficient situation (i.e., when $\eta = 1$) and thus the less valuable the rational bubble is in terms of facilitating resources allocation, i.e., the lower liquidity premium it can command.

[19] In this specific context with asymptotically bubbleless equilibria, the size of the bubble actually shrinks over time. Therefore, it may be more accurate to say that the bubble price must "decline at a lower rate" with a higher value of the arrival rate of an investment opportunity.

investment opportunity is higher, resulting in a higher level of aggregate capital stock and thus output (one period later) on impact.

The major result of this section can hence be summarized as below:

Proposition 4

(1) If $1-\lambda > \rho^2$ *and* $\eta \in (0, \eta_2)$, *then the bubbleless SS is locally stable, i.e., bounded rational bubble fluctuations are now possible. This implies that bounded fluctuations in the price of the bubbly asset, driven purely by random jumps in expectations, are possible; and that these cause persistent fluctuations in output by affecting the availability of finance for investment and thus affecting the capital stock.*

(2) The degree of persistence of the bubble fluctuations is endogenously determined by the stable eigenvalue λ_{b2}. *Other things being equal, the higher the arrival rate of an investment opportunity, the more persistent the bounded bubble(-driven) fluctuations and the larger the bubble multiplier on output.*

<u>A Numerical Example</u> To provide a numerical illustration of the equilibrium dynamics near the bubbleless SS in the face of a bubble shock, I adopt plausible calibrated values at quarterly frequency which are consistent with the relevant literature for the model parameters, with $\beta = 0.99, \alpha = 0.4, \lambda = 0.97$. By setting $\eta = 0.05$, we have $\lambda_{b1} = 0.982$ and $\lambda_{b2} = 0.268$. The impulse responses of the model near the bubbleless SS to a one-period bubble shock with $\hat{q}_0^B = \varepsilon_0 = 1\%$ is depicted in Figure 2.

Under flexible prices, the bubble shock affects the economy through a supply side mechanism. Specifically, investment, capital, and output increase in the face of the bubble shock, as the investing firms are now acquiring more funds from selling the bubble. Tobin's q declines on impact as the marginal product of capital decreases because of a rise in the future capital stock. On the other hand, consumption is crowded out at the early stage of the bubble shock, since the increase in aggregate output does not catch up with the higher goods' demand for investment;[20] as a result, the real interest rate needs to go up to clear the bond market.[21] As time evolves, investment declines

[20] This is also true when labor supply is elastic, i.e., even when output could adjust on impact.

[21] This contributes further to the declining Tobin's q on impact.

very quickly as the bubble shrinks at a fast speed of λ_{b2} in this example, while output builds up but just declines gradually (reflected by the high λ_{b1}),[22] with consumption picking up and the real interest rate declining accordingly.

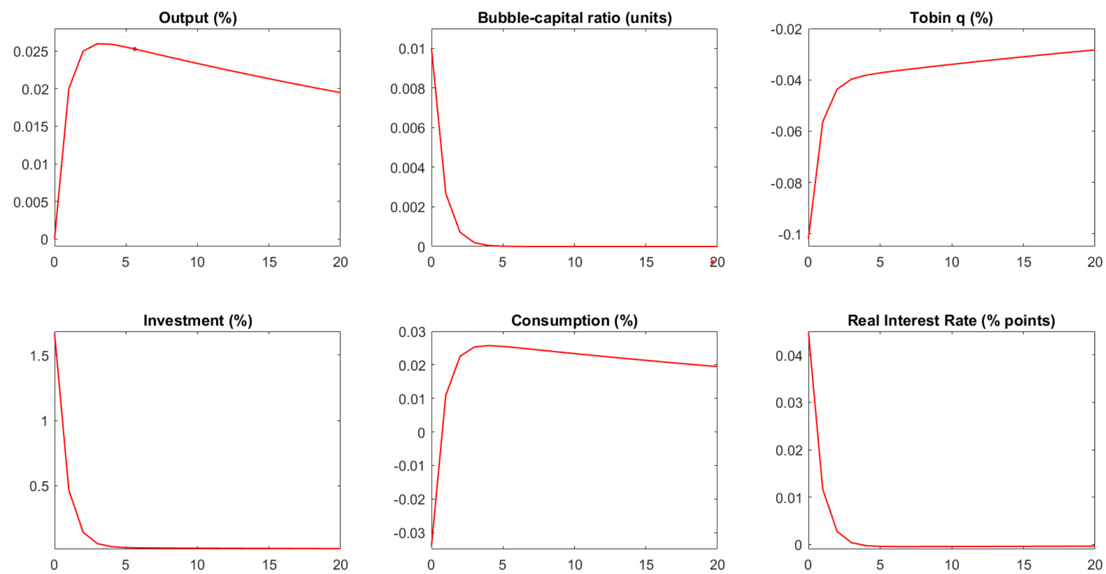

FIGURE 2. IMPULSE RESPONSES OF THE BASELINE MODEL TO 1% UNITS OF POSITIVE BUBBLE SHOCK (QUARTERLY)

Around a Bubbly SS.— When the economy fluctuates around a bubbly SS, the representative expectational difference equations system of the model is of third order. Although algebraic outcome is unavailable in this case, in numerical simulations with a wide range of theoretically valid calibrated values for the model parameters, it turns out that the system now always has one real and stable eigenvalue but two conjugate complex unstable eigenvalues, implying that bounded bubble fluctuations cannot occur around a bubbly SS.[23]

III. The Full Model with Nominal Rigidities and a Monetary Authority

This section presents the full model, where I incorporate New Keynesian (NK, for short) features into the baseline model, so that monetary factors are non-neutral for the real economy and hence meaningful monetary policy discussions can be conducted. The major ingredients of it are inherited that from the baseline model, except that now a retail sector is also introduced into the

[22] In this numerical experiment, output peaks at the fourth period (if the impact period is considered as the first period).

[23] Because in this case the Blanchard-Kahn condition for the determinacy of rational expectation solutions to the system is satisfied, given that only output is predetermined while the bubble (ratio) and Tobin's q are non-predetermined.

framework to incorporate monopolistic competition and nominal stickiness, while there is the central bank setting the nominal interest rate, which will be specified in Section IV.

A. The Structure of the Full Model

Wholesale Firms.— The core spirit of the setup of the wholesale firm sector is essentially the same as of that in Section I. A, but now a typical firm $j \in [0,1]$ produces homogeneous wholesale goods which are then sold to retailers at nominal price, P_t^W, in a competitive market. The static optimal labor demand problem of the firm becomes

$$\text{(27)} \qquad \begin{aligned} &\max_{N_{jt}} \frac{P_t^W}{P_t} Y_{jt} - W_t N_{jt} \\ &s.t.\ Y_{jt} = K_{jt}^{\alpha} N_{jt}^{1-\alpha} \end{aligned},$$

which yields $R_{kt} = \alpha \left(P_t^W / P_t \right)^{1/\alpha} \left[(1-\alpha) / W_t \right]^{(1-\alpha)/\alpha}$, where P_t is the aggregate nominal price index which will be defined below.

The remaining setup of the wholesale firms is the same as that in Section I. A, thus Propositions 1 and 2 still apply here in the setting of the full model.

Households.— Households, on the other hand, now supply labor elastically through a Walrasian labor market. A representative household seeks to maximize expected lifetime utility $E_0 \sum_{t=0}^{\infty} \beta^t \left[\log C_t - N_t^{1+\varphi} / (1+\varphi) \right]$, with $\varphi > 0$, subject to a sequence of period budget constraints

$$\text{(28)} \qquad C_t + \frac{B_{t+1}^n}{P_t} + \int_0^1 Z_{jt+1}^{Sh} \left(V_{jt} - D_{jt} \right) dj = W_t N_t + \frac{(1+i_{t-1}) B_t^n}{P_t} + \int_0^1 Z_{jt}^{Sh} V_{jt} dj + D_t^R$$

for $t = 0,1,\ldots$. C_t is the household's period t consumption measured in real terms, B_{t+1}^n is the nominal value of the one-period nominally riskless bonds purchased at the end of period *t*, i_t is the (net) risk-free nominal interest rate between period *t* and *t*+1, D_t^R is the profits received from (owning) the retail firms.[24] Solving the optimization problem of the household yields the first-order-conditions $W_t = C_t N_t^{\varphi}$ and $1 = E_t \left[\Lambda_{t,t+1} (1+i_t) P_t / P_{t+1} \right]$.

[24] Thus, I assume that each of the households receives an equal share of retailers' overall profits.

Retailers.— The setup for the retail sector is relatively standard in a NK model with endogenous capital accumulation.[25] At each period *t*, a monopolistically competitive retailer $i \in [0,1]$ purchases wholesale goods from the wholesale firms and differentiates them at no cost into retail good, $Y_t(i)$, which is sold at the nominal price $P_t(i)$. Retail goods are then transformed into final goods Y_t^f according to the CES production function, $Y_t^f = \left(\int_0^1 Y_t(i)^{(\epsilon-1)/\epsilon} di\right)^{\epsilon/(\epsilon-1)}$, where $\epsilon > 1$. Households and wholesale firms purchase these final goods (for consumption and investment, respectively), with aggregate demand $Y_t^f = C_t + \int_0^1 I_{jt} dj$. Given this CES aggregation technology, the demand curve facing an individual retailer is given by

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} Y_t^f, \quad \text{where } P_t \equiv \left(\int_0^1 P_t(i)^{1-\epsilon} di\right)^{1/(1-\epsilon)} \tag{29}$$

is the aggregate nominal price index.

As in Calvo (1983), retailers are assumed only to be able to change their prices in any period with constant probability $1-\xi$. As a result, under the sticky price setting, the evolution of the aggregate price index satisfies

$$P_t = \left[(1-\xi)\left(P_t^*\right)^{1-\epsilon} + \xi\left(P_{t-1}\right)^{(1-\epsilon)}\right]^{1/(1-\epsilon)}, \tag{30}$$

with P_t^* denoting the price chosen by retailers setting their price in period *t*.[26]

When an opportunity of resetting price arrives at time *t*, the retailer selling good *i* chooses the price P_t^* which maximizes its expected present value of profits

$$\sum_{k=0}^{\infty} \xi^k E_t \left\{ \Lambda_{t,t+k} Y_{t+k|t} \left(\frac{P_t^* - P_{t+k}^W}{P_{t+k}} \right) \right\} \tag{31}$$

subject to the demand schedule: $Y_{t+k|t} = \left(P_t^* / P_{t+k}\right)^{-\epsilon} Y_{t+k}^f$, with $Y_{t+k|t}$ indicating the output in period $t+k$ of the retailer that last resets its price in period *t*. The first-order condition for the retailer's optimization problem (31) is given by

25 See, e.g., Bernanke, Gertler, and Gilchrist 1999.

26 Since all retailers choose their price in period *t* face the same optimization problem (and the same constraints), the individual index can be omitted here.

$$(32) \qquad \sum_{k=0}^{\infty} \xi^k E_t \left\{ \Lambda_{t,t+k} Y_{t+k|t} \left(\frac{P_t^*}{P_{t+k}} - \frac{\epsilon}{\epsilon - 1} \frac{P_{t+k}^W}{P_{t+k}} \right) \right\} = 0 ,$$

which then yields the optimal price setting rule

$$(33) \qquad P_t^* = \frac{\epsilon}{\epsilon - 1} \frac{\sum_{k=0}^{\infty} \xi^k E_t \left\{ \Lambda_{t,t+k} P_{t+k}^{\epsilon-1} P_{t+k}^W Y_{t+k} \right\}}{\sum_{k=0}^{\infty} \xi^k E_t \left\{ \Lambda_{t,t+k} P_{t+k}^{\epsilon-1} Y_{t+k} \right\}} .$$

Combining (33) with (30) and log-linearizing around a zero-inflation steady state (ZISS, for short), after some manipulation then yields a version of the New Keynesian Phillips curve (NKPC, for brevity)

$$(34) \qquad \pi_t = \beta E_t \pi_{t+1} - \frac{(1-\xi)(1-\beta\xi)}{\xi} \hat{\mu}_t^p ,$$

where $\pi_t \equiv p_t - p_{t-1}$ is the inflation (rate) with $p_t \equiv \log P_t$, and $\hat{\mu}_t^p \equiv \mu_t^p - \mu^p$ with $\mu_t^p \equiv \log\left(P_t / P_t^W\right)$ the average (log) price markup (rate) and $\mu^p \equiv \log\left[\epsilon / (\epsilon - 1)\right]$ the (log) optimal markup (rate) under flexible prices.

B. General Equilibrium

When it comes to general equilibrium, the earlier analysis in the baseline model carries through, with modifications that $N_t = \left[P_t^W (1-\alpha)/(P_t W_t)\right]^{1/\alpha} K_t$, $Y_t = \left[P_t^W (1-\alpha)/(P_t W_t)\right]^{(1-\alpha)/\alpha} K_t$. Also, aggregating (29) across retailers and applying wholesale goods market clearing condition, gives

$$(35) \qquad \int_0^1 Y_t(i) di = \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} Y_t^f di = \Delta_t^P Y_t^f = Y_t ,$$

with $\Delta_t^P \equiv \int_0^1 \left(P_t(i)/P_t\right)^{-\epsilon} di \geq 1$ a measure of price dispersion across individual retailers. In a local area of a ZISS, it can be shown that up to a first-order approximation, $\Delta_t^P \simeq 1$.[27] Thus, in an approximate sense, $Y_t^f = C_t + I_t \simeq Y_t = K_t^\alpha N_t^{1-\alpha}$. Also, $W_t = (1-\alpha) P_t^W Y_t / (P_t N_t)$, and labor market equilibrium implies that $(1-\alpha) P_t^W / P_t = N_t^{1+\varphi} C_t / Y_t$. Moreover,

[27] See, e.g., Galí 2015, Chapter 3 Appendix 3.4.

(36) $$R_{kt} = \alpha \frac{P_t^W Y_t}{P_t K_t} = \alpha \frac{P_t^W}{P_t} K_t^{\alpha-1} N_t^{1-\alpha}$$

turns out to be the marginal revenue product of capital.

C. Steady States

Recalling the reasoning of Section II, the derivations of SS values of R_k, q and q^B are (jointly) determined solely by the pricing equations for the bubbly asset and Tobin's *q*, together with the investment equation. In other words, the SS values of these three price variables are unaffected by the newly introduced NK features in the model. Therefore, the conclusions drawn from Proposition 3 still apply here.[28] Also, in a ZISS, $1/(1+r) = \Lambda = \beta$ with r the real interest rate along the SS, and $P^W/P = (\epsilon - 1)/\epsilon$. Derivations of the values of other aggregate variables of interest along a ZISS are referred to the Supplemental Appendix C1 (Chen 2025).

IV. Monetary Policy in a Bubbly Environment

In this section, I finally undertake the task of studying the equilibrium dynamics in a local area of a given ZISS of the full model, particularly one in which emergence of rational bubble(-driven) fluctuations is possible, and the role that alternative monetary policy interventions may play in shaping or eliminating those fluctuations. In order to do so, I maintain the assumption that the arrival rate of an investment opportunity of the economy satisfies the condition for a potential existence of a rational bubble in equilibrium, i.e., $\eta \in (0, \eta_2)$ with $1 - \lambda > \rho^2$ throughout the analysis of this section. Fundamental shocks are also excluded from considerations, so that the discussions can be focused on bubble-driven ones.

As a preliminary step, we need to derive the log-linearized equilibrium conditions around a ZISS for the non-policy block of the system, which can be specified by (22) for $q^B = 0$, or

(37) $$\hat{q}_t^B = E_t \hat{q}_{t+1}^B + \eta \beta q_b q_b^B E_t \hat{q}_{t+1} - q_b^B \hat{r}_t$$

for $q^B > 0$. For the pricing for Tobin's *q*,

[28] Note, however, the most efficient situation, i.e., when Tobin's *q* equals one, in the full model does not lead to the "first-best" allocation outcome in the baseline economy, because of the presence of the monopoly power of retailers. Also, it is now not so straightforward that SS capital stock is still monotonically decreasing in the marginal (revenue) product of capital due to the fact that labor supply is now elastic. But this point is not difficult to prove (see the Supplemental Appendix C2 (Chen 2025)).

(38) $$\hat{q}_t = \beta(\eta R_k + \lambda)E_t\hat{q}_{t+1} + (1-\lambda\beta)E_t\hat{r}_{kt+1} - \hat{r}_t \,.$$

For the accumulation of aggregate capital stock,

(39) $$(1-\lambda)\hat{i}_t = \hat{k}_{t+1} - \lambda\hat{k}_t \,.$$

When the economy is inefficient in allocating resources, the log-linearized aggregate investment equation is given by

(40) $$(1-\lambda)\hat{i}_t = \eta R_k(\hat{r}_{kt} + \hat{k}_t) + \eta\hat{q}_t^B \,.$$

Also,

(41) $$E_t\hat{y}_{t+1} - \frac{I}{Y}E_t\hat{i}_{t+1}^R = \hat{y}_t - \frac{I}{Y}\hat{i}_t^R + \frac{C}{Y}\hat{r}_t \,,$$

which is obtained from combining the Euler equation of the household and the goods market clearing condition, while

(42) $$\left(1+\frac{Y(1-\alpha)}{C(1+\varphi)}\right)\hat{y}_t = \left(\alpha+\frac{1-\alpha}{1+\varphi}\right)\hat{k}_t + \left(\frac{1-\alpha}{1+\varphi}\right)\hat{r}_{kt} + \left(\frac{I}{C}\frac{1-\alpha}{1+\varphi}\right)\hat{i}_t^R \,,$$

which is derived from combining the aggregate production function, the marginal revenue product of capital, and the labor market equilibrium relationship. Finally, for the inflation equation,

(43) $$\pi_t = \beta E_t\pi_{t+1} + \frac{(1-\xi)(1-\beta\xi)}{\xi}\left(\hat{k}_t - \hat{y}_t + \hat{r}_{kt}\right).$$

The notation used here is in line with that used in Section II. B. Also, $\hat{i}_t \equiv \log\left[(1+i_t)/(1+r)\right]$ denotes the deviation of the net nominal interest rate, with $\hat{r}_t \equiv \hat{i}_t - E_t\pi_{t+1}$ that of the net real interest rate. The log-deviation of investment is now denoted by $\hat{i}_t^R \equiv \log(I_t/I)$ in order to distinguish it from that of the nominal interest rate.

With prices being sticky, the above equilibrium conditions must be complemented with a monetary policy rule in order to close the model. In what follows, I analyze two sets of interest rate rules: one of which is a "strict-inflation-targeting" (SIT, for short) rule aiming to fully stabilize aggregate prices despite the possible emergence of bubble fluctuations in the system, while the other one is a "simple Taylor-type interest rate rule" (SIR, for short) with a potential leaning-against-the-bubble (LAB, for short) component, which is considered to be a more practical policy.

It is worth noting that the rationale for not assessing the design of monetary policies by some "welfare-maximization" criterion but instead mainly through the lens of stabilizing the model economy is twofold. Firstly, given that inflation has recently been a practically crucial concern of many central banks around the world, investigating the potential of a monetary policy and the role of a LAB strategy in ensuring price stability may hence be of special interest. Secondly, a special property of the present model is that any equilibrium featuring a bubble must be inefficient. In other words, the achievement of the first-best allocation outcome requires the exclusion of the rational bubble altogether. (Numerical) calculations of some type of welfare-optimized policy may also be highly sensitive to specific model assumptions. Given that the main purpose for building the present model is to understand the principal qualitative, instead of quantitative, underlying theoretical linkages between monetary interventions and the evolutionary dynamics of the rational bubble, I believe the present approach would be of generic pedagogical value and be more robust to different policy objectives with various economic backdrops.

To facilitate the analysis of the SIT rule, a corresponding "gap" system, in which endogenous variables are expressed in terms of the gap between the actual and "*natural*" values of them, will need to be derived beforehand. In accordance with the definition by Woodford (2003, Chapter 5), a "natural" level of an economic variable is the one that would prevail when price is assumed to be flexible *now and in the future*, given all predetermined and exogenous state variables.[29] Note that the above specified equilibrium relationships among the variables of interest under sticky prices also apply to their flexible price counterparts, except that with flexible prices, the inflation equation is not valid, and $\hat{\mu}_t^p = 0$ so that $\hat{r}_{kt} = \hat{y}_t - \hat{k}_t$. Throughout I cap a variable with a tilde to denote the "gap" between the actual and natural value of it, where the latter is indicated by a superscript "*n*" of the variable. Much of the derivation of the "gap" system in the present section follows the approach of Woodford (2003, Chapter 5), thus most of the details of this aspect are deferred to the Supplemental Appendix C3 (Chen 2025).

[29] Note again that the defined economy's natural equilibrium level is generally inefficient, both because of the presence of the monopoly power and of the rational bubble in the present full model.

I study the circumstances where the full model economy fluctuates near an inefficient but bubbleless SS in subsection A and that around a bubbly SS in subsection B.

A. Fluctuations near a Bubbleless ZISS

As has been pointed out in Section II. B, when the economy is near a bubbleless SS, the fact that $\lambda_{b2} \equiv (1-\beta)/\left[(1-\eta)(1-\lambda\beta)\beta\right] \in (0,1)$ always holds when $\eta \in (0,\eta_2)$ implies that there exist other bounded rational expectation solutions to (22) than the no-bubble-deviation one, taking the form

$$\hat{q}^B_{t+1} = \lambda_{b2}\hat{q}^B_t + \varepsilon_{t+1}, \tag{44}$$

which is a typical stationary AR(1) process. $\varepsilon_{t+1} \equiv \hat{q}^B_{t+1} - E_t\hat{q}^B_{t+1}$ is the rational bubble innovation and is assumed to be independent of monetary interventions, with $E_t\varepsilon_{t+1} = 0$. More broadly speaking, this scenario with bounded bubble fluctuations near a bubbleless SS may be considered as a plausible representation of a boom and a subsequent bust of bubble episodes similar to those that are apparent in real economies, with the no-bubble state being the resting point.[30]

This outcome together with the fact that monetary policy is incapable of affecting the bubble fluctuations themselves (to a first-order approximation) in this circumstance suggests that the system is always exposed to intrinsically persistent bounded bubble fluctuations, whose degree of persistence is endogenously determined by λ_{b2}. Nevertheless, monetary policy may still be able to influence the impacts of the bubble fluctuations on output or inflation through affecting the aggregate demand of the economy, as will be demonstrated in what follows, and based on which I assess the desirability of a LAB strategy.

A Strict-inflation-targeting (SIT) Rule.— As a fundamental concept underlying the discussions of the SIT rule, note that the real interest rate value that is consistent with the flexible-price allocation is called the *Wicksellian Natural Rate of Interest* by Woodford (2003) and is denoted by $\hat{r}^n_t$. Since in the present case the model economy is subject to bounded bubble fluctuations, the rational expectation solution to the system in a given period then depends not only on the (actual existing)

[30] Therefore, my approach to model "bubble shocks" is distinct from that proposed by Miao, Wang, and Xu (2015), Dong, Miao, and Wang (2020), or Ikeda (2021), in which a different type of "exogenous sentiment shock" is assumed to hit their systems only around bubbly SSs.

capital stock in that period, but also on the size of the bubble-capital ratio as well. With flexible prices and given that $\hat{k}_t^n = \hat{k}_t$ in any *current* period *t* by definition, the equilibrium natural rate of interest could be expressed as

$$\hat{r}_t^n = N_r^{k^n} \hat{k}_t + N_r^{q^{B^n}} \hat{q}_t^B , \tag{45}$$

with $N_r^{k^n}$ and $N_r^{q^{B^n}}$ some endogenously determined coefficients.

Now consider an interest-rate feedback rule with a time-varying intercept term of the form

$$\hat{i}_t = \bar{i}_t + \phi_y \tilde{y}_t + \phi_\pi E_t \pi_{t+1}, \quad (\phi_\pi \geq 1) , \tag{46}$$ [31]

with the short-term nominal interest rate i_t being the monetary policy instrument and $\bar{i}_t = \hat{r}_t^n$. Therefore, by proposing a policy rule such as (46), the central bank is assumed to target exactly one-for-one the natural rate of interest which varies due to changes in the capital stock and in the size of the bubble. Equivalently, this implies that in addition to systematic responses to variations in the output gap and in the rate of inflation expected in the next period, the authority also commits to systematic responses to endogenous variations in the capital stock and in the bubble of this special sort.

On the other hand, the fact that the expectational aggregate bubble-capital ratio evolves autonomously and is independent of interest rate changes in the case of the model economy fluctuating near a bubbleless SS implies that the actual time path of the rational bubble is identical to its flexible-price counterpart. Consequently, $\tilde{q}_t^B \equiv \hat{q}_t^B - \hat{q}_t^{B^n} = 0 \ \forall t$.

As shown in the Supplemental Appendix C3 (Chen 2025), we also have the following relationship:

$$\pi_t = \beta E_t \pi_{t+1} + \psi_3 (\psi_2 - 1) \tilde{y}_t , \tag{47}$$

where $\psi_2 \equiv (1+\varphi) C_l / \left[(1-\alpha) Y_l \right] + 1$ and $\psi_3 \equiv (1-\xi)(1-\beta\xi)/\xi$. With the proposed SIT rule (46), the local equilibrium dynamics of the "gap" system of the full model in the present case could then be represented by

[31] The policy coefficient on expected inflation is assumed always no less than one, so that the real interest rate does not decrease in the face of rising expected inflation.

(48) $$E_t\tilde{\mathbf{x}}_{t+1} = \mathbf{A}\tilde{\mathbf{x}}_t ,$$

where $\tilde{\mathbf{x}}_t \equiv \left[\tilde{q}_t, \tilde{y}_t, \pi_t\right]'$ and

$$\mathbf{A} \equiv \begin{bmatrix} \beta & (1-\lambda\beta)\psi_2 & 1-\phi_\pi \\ 0 & \psi_1 & (1-\phi_\pi)(\psi_1 - 1) \\ 0 & 0 & \beta \end{bmatrix}^{-1} \begin{bmatrix} 1 & (1-\lambda\beta)\psi_2\left[\alpha(\psi_2 - 1) + 1 - \lambda\right] + \phi_y & 0 \\ 0 & \psi_1 + \alpha\psi_2 + (\psi_1 - 1)\phi_y & 0 \\ 0 & \psi_3(1-\psi_2) & 1 \end{bmatrix},$$

with $\psi_1 \equiv 1 - Y_l(1-\alpha)/\left[I_l(1+\varphi)\right]$.

Since the system represented by (48) is purely forward looking, to guarantee a unique bounded rational expectation solution $\tilde{\mathbf{x}}_t = [0,0,0]'$ for all t,[32] all eigenvalues of the matrix $\mathbf{A}$ need to lie outside the unit circle. When this is true, inflation would be entirely stabilized. This in turn implies that $i_t = r_t^n \ \forall t$ according to the policy rule, i.e., the instrument rate must then match one-for-one the natural interest rate in equilibrium. The economic system tracks the natural level of allocation as a consequence of the successful implementation of the SIT rule.

Examine here a special situation where the policy feedback on the expected inflation is set to be equal to one, i.e., $\phi_\pi = 1$. In that case, the policy rule (46) can be written in terms of the (actual) real rate and involves no "direct" response to expected inflation. It is hence possible to solve the local equilibrium dynamics of the system without reference to the inflation equation (47), so that (48) degenerates into a second-order difference equation system,

(49) $$\begin{bmatrix} E_t\tilde{q}_{t+1} \\ E_t\tilde{y}_{t+1} \end{bmatrix} = \mathbf{B}\begin{bmatrix} \tilde{q}_t \\ \tilde{y}_t \end{bmatrix},$$

where

$$\mathbf{B} \equiv \begin{bmatrix} \dfrac{1}{\beta} & \dfrac{1}{\beta}\left\{\Psi_1 - \Psi_2\dfrac{(\psi_1 + \alpha\psi_2)}{\psi_1} + \phi_y\left[1 - \Psi_2\left(1 - \dfrac{1}{\psi_1}\right)\right]\right\} \\ 0 & 1 + \dfrac{\alpha\psi_2}{\psi_1} + \phi_y\left(1 - \dfrac{1}{\psi_1}\right) \end{bmatrix},$$

[32] Recall that all exogenous fundamental shocks are excluded in the present discussion.

with $\Psi_1 \equiv (1-\lambda\beta)\psi_2\left[\alpha(\psi_2-1)+1-\lambda\right]$ and $\Psi_2 \equiv (1-\lambda\beta)\psi_2$. $\left[\tilde{q},\tilde{y}_t\right]=[0,0]$ $\forall t$ is the only valid bounded solution to (49) if and only if both eigenvalues of the coefficient matrix **B** lie outside the unit circle, which requires that

(50) $$\phi_y > \frac{\alpha\psi_2}{1-\psi_1} \equiv \phi_y^*\left(>0\right).$$

If this is satisfied, then given the relationship between the inflation and the output gap implied by (47), $\pi_t = 0$ $\forall t$, i.e., full price stability can hence be guaranteed.

As a numerical illustration, given $\beta=0.99, \alpha=0.4, \lambda=0.97, \varphi=5$, $\xi=0.75$, $\epsilon=11$ and $\eta=0.05$, it turns out that $N_r^{k^n}=-0.0072$ and $N_r^{q^{B^n}}=0.0404$ in (45), which implies that if the zero-inflation target is successfully implemented, a positive bubble shock equal to the SS capital stock would drive the *Wicksellian* rate as well as the nominal rate up by around 4%. Also, the threshold value ϕ_y^* is around 0.79.

The above outcomes thus confirm that the strict-inflation target could be successfully carried out with a credible threat of the central bank adjusting the instrument rate if the actual allocation deviates from the natural one, which merely requires finite policy response coefficients, but conditional on the authority reacting precisely to any variations in the capital stock or in the size of the bubble in order to match one-for-one the natural rate of interest. In this sense, LAB is indispensable in attaining full price stability under the proposed SIT rule.

A Simple Interest Rate (SIR) Rule.— In this section, I adopt an alternative simple Taylor-type interest rate rule of the form

(51) $$\hat{i}_t = \phi_y\hat{y}_t + \phi_\pi\pi_t + \phi_q\hat{q}_t^B$$

to investigate if deploying LAB strategy is advisable or not in a more pragmatic setting (i.e., there is now no time-varying intercept term in (51)) with regards to attaining the goal of economic stability in a bubbly world. The SIR rule specified by (51) combines the conventional stabilization mechanisms which are parameterized by ϕ_y and ϕ_π, with a potential LAB mechanisms which is parameterized by $\phi_q \geq 0$. (22), (38)-(43) and (51) then jointly describe the equilibrium behavior of $\hat{k}_t, \hat{q}_t^B, \hat{q}_t, \hat{y}_t, \hat{\imath}_t^R, \hat{r}_{kt}$ and π_t in the neighborhood of a given bubbleless SS with the representative expectational difference equation system being fifth order.[33]

As a demonstration, three scenarios with different degrees of policy response to variations in the size of the bubble are examined: $\phi_q = 0, 0.01, 0.06$, while $\phi_y = 0.125$, $\phi_\pi = 1.5$ for all the three circumstances. Given the same set of calibrated values for the model parameters as in the above section, the impulse responses of the full model for these three cases with 1% units of one-period positive bubble shock hitting the system at some time $t = 0$ is depicted in Figure 3.

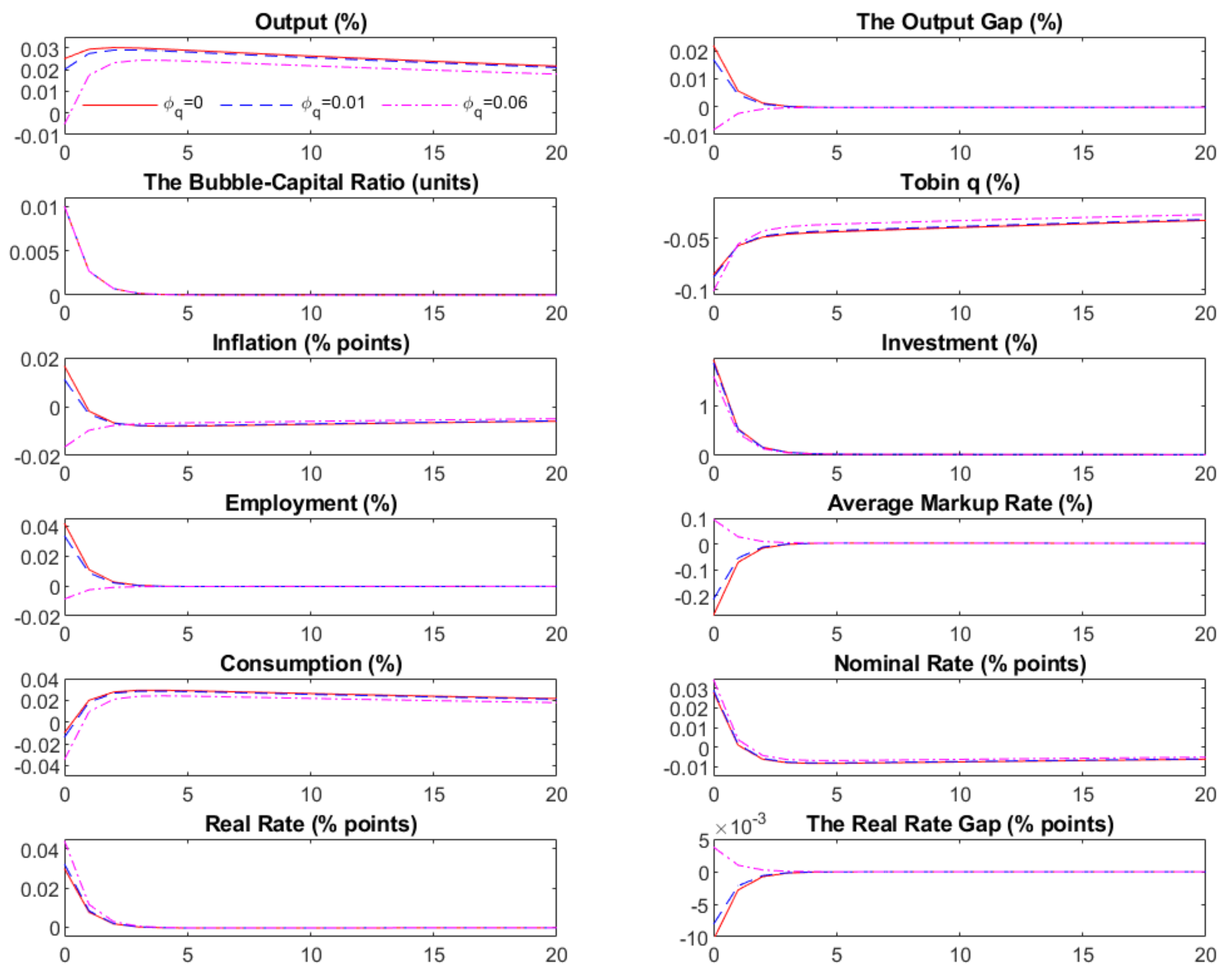

FIGURE 3. IMPULSE RESPONSES OF THE FULL MODEL TO 1% POSITIVE BUBBLE SHOCK (QUARTERLY)

A prominent feature in Figure 3 is that even a minor increase in the strength of the policy response to fluctuations in the size of the bubble appears to have drastic impact on the dynamics

[33] Note that the capital stock and the bubble-capital ratio here are treated as pre-determined variables.

of the system: compared to the conventional policy (i.e., $\phi_q = 0$), the commitment to a tiny response to the bubble with $\phi_q = 0.01$ is able to dampen the bubble impact on inflation and the output gap in an effective way. However, if a more aggressive LAB stance is deployed with the bubble coefficient increasing to 0.06, then both actual output and the output gap decrease in the face of the positive bubble shock, companied by deflation on impact, indicating a potential risk of the LAB policy "overreacting". But this type of overreaction does not happen if the authority sticks to a conventional policy regime and completely ignores the developments in the bubble size.[34]

To explain in more depth the specific patterns emerging under different policy regimes, note first that if prices were fully flexible in the present context, an appreciation in the bubble price would squeeze out consumption as well as drive up the (real) interest rate on impact, similar to what has been analyzed in Section II. B. When consumption declines, labor supply tends to increase due to the income effect, while the labor demand by wholesale firms would be unaffected at the beginning by the bubble shock, as the wholesale goods' price in real term then remains constant and capital input is predetermined. As a result, equilibrium employment would go up (with a lower equilibrium real wage), leading to a higher aggregate output level. Nonetheless, due to the high degree of inelasticity of labor supply, expansions in employment and thus output alongside the bubble boom would be rather mild (on impact) in the flexible price circumstance.

With price stickiness, however, the responsive dynamics of the system to the positive bubble shock could be quite different, as the real interest rate is now affected by monetary factors. If the policy feedback on deviations in output or inflation is accommodating and the actual real rate of interest is lower than the natural rate of interest on impact,[35] as in the first case where the bubble coefficient is set to zero in the SIR rule (51), actual consumption would tend to be higher relative to its natural counterpart because of the intertemporal substitution effect. While the higher demand pressure for the final goods tends to be inflationary for the economy, the combination of the increasing goods' demand and the sticky retail prices leads to a stronger expansionary need for production, which implies a lower than the (average) desired markup rate level (for the retailers)

[34] For example, even when the inflation coefficient in the SIR is raised to as high as 15, neither (the) output (gap) nor inflation would turn out to be decreasing in the face of the positive bubble shock in numerical experiments unreported here, as long as the bubble coefficient remains zero. At the very least, actual output is unlikely to decline relative to the SS level even with more extreme conventional policy coefficients.

[35] In unreported numerical simulations, if the policy feedback on the conventional targets (i.e., output and inflation) is aggressive, then the responsive pattern could vary somehow.

and a higher labor demand from the wholesale firms.[36] As a result, both employment and real wage go up on impact in the labor market equilibrium, with then a higher than the natural rate of aggregate output level, i.e., a positive output gap in general equilibrium.[37]

Given the non-trivial demand side impact of the bubble boom on output in the presence of nominal rigidities, it thus seems that allowing the policy to respond additionally to developments in the size of the bubble could be helpful in terms of matching the actual real rate of interest to its natural counterpart and then controlling inflation through depressing aggregate demand. This does appear to be proved by the policy experiment with $\phi_q = 0.01$, as it is evident that the real interest rate gap and then the output gap is narrowed by the active policy response to the bubble shock.

However, what is crucial here is that in the present scenario, the evolution of the bubble is independent of the (real) interest rate changes, i.e., monetary interventions have *no* (first-order) impact on the movements in the size of the bubble. As a consequence, even when the policy response to variations in the bubble size turns out to be aggressive, i.e., when $\phi_q = 0.06$ in the current experimental context – recall that the bubble coefficient in the natural real interest rate determination equation (45) is only around 0.04 – the "too high" nominal instrument rate would not be automatically "revised" down as it would when it comes to policy feedback on deviations in output or inflation, because now the bubble itself remains unaffected. Furthermore, since the market would rationally anticipate a certain degree of deflation in the longer term given the expanded production capacity from additional investment in physical capital today, i.e., when the supply side impact of the bubble boom kicks in, the "overreaction" risk of the LAB policy tends to be exacerbated, as the real interest rate today would then end up being even higher, even though the policy adjustments to the declines in current output and aggregate price level that results from the far lower consumption needs for the final goods partly offset the perverse effects.

Therefore, it is evident that the lack of an "endogenous feedback mechanism" onto the size of the rational bubble contributes greatly to the overreaction risk of the LAB strategy, while this is not very likely to happen in a conventional policy regime. This is because if the interest rate rule

[36] Aggregate investment is also higher in equilibrium in this case, because of the higher internal funds possessed by investing firms, which is contributed by a higher marginal revenue product of capital (due to both the higher real wholesale price and the higher labor input) relative to the outcome under flexible prices.

[37] The persistent positive output gap in the transition periods in the present scenario is partly explained by the stronger aggregate demand induced by the persistently higher capital investments (relative to the flexible price case) boosted initially by the intrinsically prolonged bubble boom with price stickiness.

involves a direct response to variations in the size of the bubble, although the changes in the policy rate cannot shape the evolution of the bubble itself, the latter can still affect aggregate demand and output via the LAB policy. Thus, leaning against the bubble in the SIR rule may *not* be advisable in handling bubble-driven fluctuations, because it risks causing an unintentional recession in the face of a bubble boom.

Interestingly, the above conclusion echoes to some extent that drawn from Ikeda (2021) from the point of view of households' welfare maximization. Ikeda studies a similar type of rational bubble, even though his model setup and the type of "bubble shock" studied there are different from the present one. In Ikeda's (2021) numerical investigations, extra monetary policy feedback on bubble developments is generally counterproductive, as it tends to excessively curb real economic activity and damage the welfare of households severely as a consequence. As in my model, bubbles in Ikeda (2021) are also beneficial to the economy, since they relax borrowing constraints of investing firms and hence stimulate capital investment and thus aggregate output. The positive effect of the higher efficiency of the economic system resulting from a bubble boom on the welfare of the households tends to outweigh the induced negative impact of possible higher volatility of inflation or the output gap,[38] so that dampening the bubble-driven boom too much would do more harm than good to the overall welfare of the economy. In fact, if the bubble coefficient in a simple interest rate rule is not restricted to be non-negative, then welfare optimization may even require the central bank to inflate the size of the bubble from an initial (positive) bubble shock by reducing the interest rate called for by the LAB strategy, as numerically shown to be the case in Dong, Miao, and Wang (2020) in a bubbly economy which shares similar key features with Ikeda's (2021).[39]

B. Fluctuations around a Bubbly ZISS

Next, I turn to study the equilibrium dynamics in a neighborhood of a bubbly SS, where, unlike the case studied in the previous section, the bubble is no longer evolving autonomously and is affected by changes in the interest rate. Monetary policy may thus now be effective in attaining

[38] Since the former positive impact is presumably of first order, while the latter should be of second order – recall that in a bubbly economy of the type studied in the present paper as well as in Ikeda (2021), the corresponding SS must always be *inefficient*.

[39] This conclusion obviously challenges the conventional wisdom of "leaning against the wind" which calls for an increase in the instrument rate to restrain a bubble boom.

(full) price stability by means of completely eliminating potential bubble-driven fluctuations of the system.

A Strict-inflation-targeting (SIT) Rule.— If prices were fully flexible, the equilibrium of the economy fluctuating around a bubbly SS would be determinate and not subject to any "sunspot" bubble shocks.[40] Thus, the natural real rate of interest at any current date *t* depends only on the capital stock, which could be expressed as $\hat{r}_t^n = N_{r_b}^{k^n} \hat{k}_t$, with $N_{r_b}^{k^n}$ again some endogenously determined coefficient.

Suppose now the central bank deploys an interest rate rule of the form

$$\hat{i}_t = \bar{i}_t + \phi_y \tilde{y}_t + \phi_\pi E_t \pi_{t+1} + \phi_q \tilde{q}_t^B, \quad (\phi_\pi \geq 1) \tag{52}$$

with $\bar{i}_t = \hat{r}_t^n$, i.e., the authority adjusts its interest-rate operating target to the exact same extent as the natural real rate of interest is changed by variations in the capital stock. An explicit LAB component is also introduced, parameterized by $\phi_q \geq 0$ in (52), in addition to the conventional stabilization mechanisms based on the output gap and (expected) inflation.

When the economy fluctuates around a bubbly SS with $q^B > 0$, the bubble evolves according to (37). Therefore, the actual bubble size is now generally different from its natural counterpart, implying that $\tilde{q}_t^B$ is not necessarily zero anymore, but given by

$$\tilde{q}_t^B = E_t \tilde{q}_{t+1}^B + \eta \beta q_b q^B E_t \tilde{q}_{t+1} - q^B \tilde{r}_t . \tag{53}$$

As shown in the Supplemental Appendix C3 (Chen 2025), under the proposed SIT rule (52), a purely forward-looking system for the determination of $\{\tilde{q}_t^B, \tilde{q}_t, \tilde{y}_t, \pi_t\}$ in the present case can then be written in the form

$$E_t \tilde{\mathbf{z}}_{t+1} = \mathbf{C}_b \tilde{\mathbf{z}}_t , \tag{54}$$

where $\tilde{\mathbf{z}}_t \equiv [\tilde{q}_t^B, \tilde{q}_t, \tilde{y}_t, \pi_t]'$, with $\mathbf{C}_b$ the corresponding coefficient matrix.

To ensure that $\tilde{\mathbf{z}}_t = [0,0,0,0]'$ for all *t* is the unique bounded rational expectation solution to (54) so that the aggregate price is always fully stabilized with potential bubble-driven fluctuations being

[40] Recall that in Section II. B, when the baseline economy fluctuates around a bubbly SS, my extensive numerical investigations failed to find any cases where it was impossible to have a sunspot bubble shock in the system with flexible prices.

completely ruled out, requires that all four eigenvalues of $\mathbf{C}_b$ lie outside the unit circle. In numerical experiments with the same set of calibrated values for the exogenous parameters as in Section IV. A, $N_{r_b}^{k^n} = -0.0125$, while when $(\phi_y, \phi_\pi, \phi_q)$ take the values of, e.g., $(0.01, 1.01, 0.5)$, $(0.01, 1.02, 0)$ or $(0.03, 1.01, 0)$, I find that these can guarantee the determinacy of the system. This shows that although the policy response to the gap in bubble movements is seemingly less effective (regarding the required strength of feedback on deviations of the variable to its target level) than those to conventional targets in terms of ensuring full price stability, it does its job anyway in this SIT setting, as far as expanding the determinacy regions of the policy parametric space is concerned.[41] However, it is clearly not indispensable to have LAB feedback, unlike the case when the economy fluctuates near a bubbleless SS.

A Simple Interest Rate (SIR) Rule.— Let us now turn to investigate the same form of SIR rule as that specified by (51) in terms of achieving the policy goal of economic stability by entirely eliminating potential bubble-driven fluctuations. In the present case, the equilibrium behavior of the system is jointly described by (37)-(43) plus (51) for $\hat{k}_t, \hat{q}_t^B, \hat{q}_t, \hat{y}_t, \hat{i}_t^R, \hat{r}_{kt}$ and π_t in a neighborhood of the bubbly SS, with the expectational linear difference equations system being again fifth order. Thus, to insulate the system from bounded bubble-driven fluctuations, it is necessary for the system to be determinate, i.e., four of its five eigenvalues need to be unstable.

It turns out that, in line with the finding in the above section with the SIT rule, although allowing the policy to respond additionally to variations in the size of the aggregate bubble-capital ratio plays a seemingly only weakly effective role in ensuring a unique solution to the system, it does appear to be helpful with regard to expanding the determinacy region of the parameter space. For instance, while muting the response to movements in the size of the bubble by setting $(\phi_y, \phi_\pi) = (0.01, 0.90)$ with $\phi_q = 0$ fails to insulate the system from sunspot shocks, re-deploying the LAB strategy by additionally setting $\phi_q \geq 1.68$ or by increasing slightly the policy coefficient on inflation to 1.01 (with $\phi_y = 0.01$, $\phi_q = 0$) restores the system to determinacy.

To better comprehend why LAB appears to be only weakly effective as far as ruling out bubble-driven fluctuations is concerned, it may be instructive to consider how the rational bubbles interact

[41] This outcome is thus contrary to that obtained by Nisticò (2012) where irrational bubble fluctuations are studied, part of which states that LAB policy generally reduces the probability of a system being determinate.

with changes in the (real) interest rate in the present scenario. Before the bubble shock is ruled out, an interest rate rise tends to directly drive up the expected future bubble prices according to the law of motion of the bubble around a bubbly SS. From this standpoint, it seems that setting the bubble coefficient $\phi_q > 0$ in the SIR rule should be effective in generating an explosive path given the special rational property of the bubble.

However, there is another determinant other than the rate of interest in the pricing equation of the bubble in the present framework, namely, the "liquidity premium" which is positively correlated with the market value of a unit of capital (in the future). The value of the latter is increasing in the rate of interest, meaning that a higher interest rate level would lead to a higher anticipated market value of a unit of capital and hence a higher liquidity premium commanded by the rational bubble. Notably, however, the future bubble price is *decreasing* in the liquidity premium. In other words, a rise in the interest rate would *indirectly* lead to a *lower* bubble price expected in the future through the channel of an increasing future Tobin's q. In fact, if we rewrite (37) by using (38),

$$
\begin{aligned}
E_t\hat{q}^B_{t+1} &= \hat{q}^B_t - \eta\beta q_b q^B_b E_t\hat{q}_{t+1} + q^B_b\hat{r}_t \\
&= \hat{q}^B_t - \frac{\eta q_b q^B_b}{\eta R_{k_b} + \lambda}\hat{q}_t + \frac{\eta q_b q^B_b(1-\lambda\beta)}{\eta R_{k_b} + \lambda}E_t\hat{r}_{kt+1} + q^B_b\left(1 - \frac{\eta q_b}{\eta R_{k_b} + \lambda}\right)\hat{r}_t,
\end{aligned}
\tag{55}
$$

it is then evident from the second line of (55) that the direct impact of real interest rate changes on the expected bubble price in the next period is partially offset by the opposite effect of the interest rate on the expected Tobin's q value. It is also not difficult to show algebraically that the higher the arrival rate of an investment opportunity, the larger the offsetting effect induced by the Tobin's q's response to variations in the real interest rate.

Therefore, the net impact of a higher policy rate on the future bubble price may be insignificant, because the two determinants of the pricing of the rational bubble tend to move in opposite directions in response to the real interest rate changes. Putting this into the context of the LAB policy, this implies that feedback on variations in the size of the bubble would be less effective in terms of inflating the bubble to the extent that fluctuations are then completely ruled out under the Blanchard-Kahn condition for determinacy of the system, since the endogenous rise in the interest rate called for by the LAB policy in the face of a bubble boom has a milder impact on the size of the bubble after initial bubble shock. This mechanism is a consequence of the special equilibrium requirement that the anticipated return on the rational bubble in the present model consists of not only capital gains but also a liquidity premium. This outcome is also different from monetary policy implications in situations where rational bubbles emerge because of dynamic inefficiency.[42]

V. Concluding Comments

As an effort to enhance the theoretical comprehension of the unsettled policy issue of "should central banks lean against asset price bubbles", the present paper is devoted to studying the conduct of monetary policies in an environment where *rational* asset price bubbles may emerge because of the existence of financing constraints.

To build up the foundations, I have established an analytically tractable infinite-horizon dynamic general equilibrium framework in which individual firms are subject to a type of uninsurable idiosyncratic investment shock a là Kiyotaki and Moore (2019) and financing constraints as well. The combination of these two factors creates an environment for a rational bubble to exist in equilibrium, because it may then serve as a financing tool in terms of transferring resources between firms with and without an investment opportunity, and thus may be valuable to the rational agents even though the bubble is intrinsically worthless. As a consequence, the bubble is *not* required to grow at the rate of interest in equilibrium, because of the extra "*liquidity premium*" commanded by it resulting from its contribution to the relaxation of firms' financing constraints. Additionally, and most importantly, when NK features are incorporated, the model enables the potential emergence of bubble-driven fluctuations in equilibrium via both demand- and supply-side mechanisms, so that meaningful model-based monetary policy analysis could go ahead.

It turns out that if the model economy starts at a bubbleless steady state, then under certain conditions it is subject to bounded bubble-driven fluctuations (in output and inflation). In that case, leaning against the bubble (LAB) is necessary in order to match exactly one-for-one the natural rate of interest if the central bank aims to fully stabilize aggregate prices. However, an additional LAB response may be harmful in a more practical setting where the central bank adopts a simple Taylor-type interest rate rule, in the sense that it may cause an economic recession in the face of a bubble boom. This is because changes in the policy rate cannot affect the trajectory of the bubble when the economy fluctuates around a bubbleless steady state, but the bubble can affect (the

[42] In Galí (2021), policy responses to variations in rational bubbles are apparently more effective than those to conventional feedback variables.

demand side of) the economy through the LAB policy. The asymmetric feedback mechanism between the bubble and the interest rate is thus responsible for the downside risk of the extra policy response to variations in the size of the bubble in this case.

On the other hand, if the model economy is originally bubbly, then a LAB strategy is neither necessary for achieving the strict inflation target nor very effective in completely ruling out bounded bubble(-driven) fluctuations in the economic system. A likely underpinning particularly for the latter phenomenon is that the two determinants of the pricing of the bubble tend to move in opposite directions in the face of interest rate changes, so that the net effect of the latter called for by a LAB policy on the evolution of the bubble may be insignificant.

Therefore, overall, regardless of the fact that macroeconomic instability caused by the rational bubble fluctuations display distinctive patterns, especially of being intrinsically persistent, the analyses of the paper do not demonstrate a strong theoretical support for adopting a LAB strategy in the conduct of monetary policy. Particularly considering the potential risk of overreacting and unintentionally dragging the economy into a recession in an episode of a bubble boom, a conventional policy regime with only inflation and the output gap stabilization mechanisms may still be more robust in practice.

Although the analytical tractability of the present framework enables the transparency of the driving factors underlying the conclusions, it is undeniable that the model may not accurately capture the true systematic structure or the nature of asset market volatilities in a real economy. One important caveat is that the type of rational bubble studied here is deterministic, i.e., the bubble will not burst with any probability once it exists, which may thus be arguably of less concern to actual policy makers. Instead, scenarios featuring an eventual collapse of speculative bubbles which may themselves be harmful to the efficiency of the economy, for instance, where the bubble exists due to the presence of agency problems, may also be important for central bankers, but the relevant policy deliberations may be very different to the ones suggested in this paper. Therefore, further research in these directions is also needed in terms of addressing the ultimate policy question in practice.

Those limitations notwithstanding, the study of the paper hopefully provides useful theoretical insights for the policy issue at hand, especially by pointing out a potential downside risk that is special to monetary policies with additional LAB component, but which is missing in the conventional policy regime.

APPENDIX

A. Proofs of Proposition 1 and 2

Let q_{jt} , γ_{jt} , and μ_{jt} denote respectively the Lagrangian multipliers associated with the period t constraints (4), (6), and (5). The optimization problem facing the individual firm j can then be expressed as

(A1) $$\max_{\{I_{jt},Z_{jt+1},K_{jt+1}\}_{t=0}^{\infty}} \mathcal{L}_0 = E_0\sum_{t=0}^{\infty}\left\{\Lambda_{0,t}\left[D_{jt}+\mu_{jt}D_{jt}+\gamma_{jt}Z_{jt+1}+q_{jt}\left(\tau_{jt}I_{jt}+\lambda K_{jt}-K_{jt+1}\right)\right]\right\},$$

with $D_{jt}=R_{kt}K_{jt}-\tau_{jt}I_{jt}-Q_t^B\left(Z_{jt+1}-Z_{jt}\right)$ for $t=0,1,2,\ldots$. The first-order conditions are given by

(A2) $$\partial\mathcal{L}_t/\partial I_{jt}=-\tau_{jt}(1+\mu_{jt})+\tau_{jt}q_{jt}=0\,,$$

(A3) $$\partial\mathcal{L}_t/\partial Z_{jt+1}=-(1+\mu_{jt})Q_t^B+\gamma_{jt}+E_t\left\{\Lambda_{t,t+1}\left(1+\mu_{jt+1}\right)Q_{t+1}^B\right\}=0\,,$$

(A4) $$\partial\mathcal{L}_t/\partial K_{jt+1}=-q_{jt}+E_t\left\{\Lambda_{t,t+1}\left[\left(1+\mu_{jt+1}\right)R_{kt+1}+\lambda q_{jt+1}\right]\right\}=0\,,$$

plus the complementary slackness conditions.

In what follows I use a method of guess and verify.[43] Given that the investment shock is independently and identically distributed across firms and over time and that the production technology is constant-return-to-scale, it is conjectured that the multipliers, $\left\{\mu_{jt},\gamma_{jt},q_{jt}\right\}$, rely only on the aggregate state in period t and idiosyncratic shocks, so that $\mu_{jt}^{i/s}=\mu_t^{i/s},\gamma_{jt}^{i/s}=\gamma_t^{i/s},q_{jt}^{i/s}=q_t^{i/s}$, and

(A5) $$E_t\mu_{jt+1}=\eta\mu_{jt+1}^{i}+\left(1-\eta\right)\mu_{jt+1}^{s}\equiv\bar{\mu}_{t+1}\,,$$

as well as $E_t\gamma_{jt+1}=\eta\gamma_{jt+1}^{i}+\left(1-\eta\right)\gamma_{jt+1}^{s}\equiv\bar{\gamma}_{t+1}$ and $E_tq_{jt+1}=\eta q_{jt+1}^{i}+\left(1-\eta\right)q_{jt+1}^{s}\equiv\bar{q}_{t+1}$, where the superscripts “*i*” and “*s*” indicate that for firms with and without investment opportunities at period t respectively. By the independence assumption between the idiosyncratic and aggregate shocks, (A2)-(A4) can therefore be rewritten as

[43] This approach is particularly inspired by Wang and Wen (2012).

(A6) $$\left(1+\mu_{jt}\right)\tau_{jt}=\tau_{jt}q_{jt};$$

(A7) $$\left(1+\mu_{jt}\right)Q_t^B=\gamma_{jt}+E_t\left\{\Lambda_{t,t+1}\left(1+\overline{\mu}_{t+1}\right)Q_{t+1}^B\right\};$$

(A8) $$q_{jt}=E_t\left\{\Lambda_{t,t+1}\left[\left(1+\overline{\mu}_{t+1}\right)R_{kt+1}+\lambda\overline{q}_{t+1}\right]\right\},$$

with

(A9) $$\mu_{jt}\geq 0, D_{jt}\geq 0 \quad \text{and} \quad \mu_{jt}D_{jt}=0,$$

(A10) $$\gamma_{jt}\geq 0, Z_{jt+1}\geq 0 \quad \text{and} \quad \gamma_{jt}Z_{jt+1}=0,$$

and the transversality conditions $\lim_{T\to\infty} E_t\left\{\Lambda_{t,t+T}q_{t+T}K_{jt+T}\right\}=0$, $\lim_{T\to\infty} E_t\left\{\Lambda_{t,t+T}Q_{t+T}^B Z_{jt+T+1}\right\}=0$. Note that from (A8), q_{jt} is independent of the individual firm's investment status, i.e., independent of the idiosyncratic investment shock. Thus, $q_{jt}^i=q_{jt}^s=q_{jt}=q_t=\overline{q}_t$. It is also noticeable that when imposing $\tau_{jt}=1$, (A6) implies that $1+\mu_{jt}^i=q_t\geq 1$. Let us consider the circumstances when q_t is equal to or greater than one sequentially.

<u>Case 1</u> Consider first the case when $q_t=1$ (for all *t*). Then from (A6), $\mu_{jt}^i=0$, hence $D_{jt}^i\geq 0$ according to the complementary slackness condition (A9). In this case it is immaterial to a firm whether it has an investment opportunity or not, such that the financial frictions do not actually matter. Suppose hence $D_{jt}^s\geq 0$ and $\mu_{jt}^s=0$ as well. Then given (A8) and (A5), $q_t=E_t\left\{\Lambda_{t,t+1}\left(R_{kt+1}+\lambda q_{t+1}\right)\right\}$. From (A7), $Q_t^B=\gamma_{jt}^{i/s}+E_t\left\{\Lambda_{t,t+1}Q_{t+1}^B\right\}$, which implies that $\gamma_{jt}^i=\gamma_{jt}^s=0$,[44] i.e., $Q_t^B=E_t\left\{\Lambda_{t,t+1}Q_{t+1}^B\right\}$. However, if this is true, in equilibrium the bubble will then be ruled out by the transversality condition, since $\lim_{T\to\infty} E_t\left\{\Lambda_{t,t+T}Q_{t+T}^B\right\}=Q_t^B=0$, where the fact that $\int_0^1 Z_{jt+T+1}dj\equiv 1$ is applied. Thus, if $q_t=1$ (for all *t*), then in equilibrium $Q_t^B=0$ (for all *t*).

<u>Case 2</u> If $q_t>1$ (for all *t*), then $\mu_{jt}^i>0$ according to (A6), which implies that $D_{jt}^i=0$. Suppose then $D_{jt}^s>0$, $\mu_{jt}^s=0$, while $Z_{jt+1}^i=0$ and $Z_{jt+1}^s>0$, i.e., firms with an investment opportunity

[44] Since the overall supply of the bubbly asset is assumed to be constant, it must be held by some firms for the bubble market to be cleared, so the multipliers cannot be both greater than zero according to the complementary slackness condition.

decide to use all their available resources to invest in capital by distributing zero dividends and selling all their bubbly asset, while firms without the investment opportunity choose to do the opposite. Then from (7), $I_{jt} = R_{kt}K_{jt}^{i} + Q_t^B Z_{jt}^{i}$. Meanwhile, for investing firms with $\tau_{jt} = 1$, $1 + \mu_{jt}^{i} = q_t$, and

(A11) $$\left(1+\mu_{jt}^{i}\right)Q_t^B = \gamma_{jt}^{i} + E_t\left\{\Lambda_{t,t+1}\left(1+\eta\mu_{jt+1}^{i}\right)Q_{t+1}^B\right\},$$

(A12) $$q_t = E_t\left\{\Lambda_{t,t+1}\left[\left(1+\eta\mu_{jt+1}^{i}\right)R_{kt+1} + \lambda q_{t+1}\right]\right\};$$

while for saving firms,

(A13) $$Q_t^B = E_t\left\{\Lambda_{t,t+1}\left(1+\eta\mu_{jt+1}^{i}\right)Q_{t+1}^B\right\},$$

with (A12) still applied. Rewrite (A13) and (A12) by making use of the fact that $1+\mu_{jt}^{i} = q_t$, we can then obtain the key pricing equations for the bubble and Tobin's q in Proposition 2. Q.E.D.

B. Proof of Proposition 3

For a bubbly SS to exist, it requires that $q > 1$ and $q^B > 0$, i.e., $1-(1-\beta)/[\beta(1-\lambda\beta)] \equiv \eta_2 > \eta$. In order for $\eta > 0$, it then requires that $\beta(2-\lambda\beta) > 1$, i.e.,

(B1) $$1-\lambda > \rho^2,$$

given that $\beta \equiv 1/(1+\rho)$. Furthermore, we have

(B2) $$\eta_1 - \eta_2 \equiv \frac{\beta(1-\lambda)}{1-\beta\lambda} - \left[1 - \frac{1-\beta}{\beta(1-\lambda\beta)}\right] = \frac{1}{1-\beta\lambda}\left[(1-\beta)\left(\frac{1}{\beta}-1\right)\right] > 0,$$

i.e., $\eta_1 > \eta_2$. Therefore, if (B1) is satisfied, η_1 and η_2 divide the parameter space of the model system into three mutually exclusive regions; or otherwise $\eta_2 \le 0$, there are only two regions of parameter space divided only by η_1, in both of which only a bubbleless SS can exist.

To prove that $q_l > q_b > q^* = 1$, $R_{k_l} > R_{k_b} > R_k^*$, and $K^* > K_b > K_l$ in region 3 of case 1 in Proposition 3, note first that according to (17), (B3) $$\left.\frac{\partial R_k}{\partial q}\right|_{given\,\eta} = \frac{\beta(1-\lambda\beta)(1-\eta)}{\beta^2\left[(1-\eta)+\eta q\right]^2} > 0\,.$$

Since for a given η within the region 3 of case 1, $R_{k_l} - R_{k_b} = \left[(1-\eta)(1-\lambda\beta)-(1/\beta-1)\right]/\eta > 0$, i.e., $R_{k_l} > R_{k_b}$ and thus $q_l > q_b$ according to (B3). Furthermore, given that $0<\eta<\eta_2<1$ in this scenario,

(B4) $$R_{k_b} = (1-\lambda\beta)\left[1+\frac{1}{\eta}\left(\frac{1}{\beta}-1\right)\right] > (1-\lambda\beta)\left[1+\left(\frac{1}{\beta}-1\right)\right] = \frac{1}{\beta}-\lambda = R_k^*,$$

while it is obvious that $q_b > 1 = q^*$. Providing diminishing marginal product of capital, we thus have $K^* > K_b > K_l$ for $R_{k_l} > R_{k_b} > R_k^*$.

To show that $R_{k_l} > R_k^*$ and thus $K^* > K_l$ in case 2 of Proposition 3, note that since $\eta \in (0,\eta_1)$ for the bubbleless and inefficient SS to exist,

(B5) $$R_{k_l} = \frac{1-\lambda}{\eta} > \frac{1-\lambda}{\eta_1} = \frac{1}{\beta}-\lambda = R_k^*,$$

while $q_l > q^* = 1$ ssumption. Q.E.D.

REFERENCES

Aiyagari, S. Rao. 1994. "Uninsured Idiosyncratic Risk and Aggregate Saving." *Quarterly Journal of Economics* 109: 659-684.

Asriyan, Vladimir, Luca Fornaro, Alberto Martin, and Jaume Ventura. 2021. "Monetary Policy for a Bubbly World." *Review of Economic Studies* 88: 1418-1456.

Barlevy, Gadi. 2018. "Bridging Between Policymakers' and Economists' Views on Bubbles." *Economic Perspectives* 42(4): 1-21.

Bernanke, Ben S., and Mark Gertler. 1999. "Monetary Policy and Asset Price Volatility." in *New Challenges for Monetary Policy*, Federal Reserve Bank of Kansas City, 77-128.

Bernanke, Ben S., and Mark Gertler. 2001. "Should central banks respond to movements in asset prices?" *American Economic Review* 91(2): 253-257.

Bernanke, Ben S., Mark Gertler, and Simon Gilchrist. 1999. "Financial Accelerator in a Quantitative Business Cycle Framework." In: John B. Taylor and Michael Woodford (eds.), *Handbook of Macroeconomics*, Amsterdam: Elsevier.

Bewley, Truman. 1983. “A Difficulty with the Optimum Quantity of Money.” *Econometrica* 51: 1485-1504.

Biswas, Siddhartha, Andrew Hanson, and Toan Phan. 2020. “Bubbly Recessions.” *American Economic Journal: Macroeconomics* 12(4): 33-70.

Blanchard, Olivier J. 1985. “Debt, Deficits, and Finite Horizons.” *Journal of Political Economy* 93(2): 223-247.

Borio, Claudio, and Philip Lowe. 2002. “Asset Prices, Financial and Monetary Stability: Exploring the Nexus.” BIS Working Paper no. 114.

Brunnermeier, Markus K., and Martin Oehmke. 2013. “Bubbles, Financial Crises, and Systemic Risk.” In: George M. Constantinides, Milton Harris, and Rene M. Stulz, (eds.), *Handbook of the Economics of Finance*, Vol. 2B, Amsterdam: Elsevier.

Caballero, Ricardo J., and Alp Simsek. 2020. “A Risk-Centric Model of Demand Recessions and Speculation.” *Quarterly Journal of Economics* 135(3): 1493-1566.

Carlstrom, Charles T., and Timothy S. Fuerst. 2007. “Asset Prices, Nominal Rigidities, and Monetary Policy.” *Review of Economic Dynamics* 10(2): 256-275.

Chen, Junming. 2025. “Monetary Policy and Rational Bubbles.” PhD Thesis, University of York.

Chen, Junming. 2025. “Supplemental Appendix to ‘Monetary Policy, Financing Constraints, and Rational Asset Price Bubbles’.” Available from the author upon request.

Dong, Feng, Jianjun Miao, and Pengfei Wang. 2020. “Asset Bubbles and Monetary Policy.” *Review of Economic Dynamics* 37: S68-S98.

Dong, Feng, Zhiwei, Xu, and Yu Zhang. 2022. “Bubbly Bitcoin.” *Economic Theory* 74: 973-1015.

Galí, Jordi. 2014. “Monetary Policy and Rational Asset Price Bubbles.” *American Economic Review* 104(3): 721-752.

Galí, Jordi. 2015. “Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework and its Applications.” (2nd ed.), Princeton; Oxford: Princeton University Press.

Galí, Jordi. 2021. “Monetary Policy and Bubbles in a New Keynesian Model with Overlapping Generations.” *American Economic Journal: Macroeconomics* 13(2): 121-167.

Hart, Oliver, and John Moore. 1994. “A Theory of Debt Based on the Inalienability of Human Capital.” *Quarterly Journal of Economics* 109: 841-879.

Hirano, Tomohiro, and Alexis Akira Toda. 2024. "Bubble Economics." *Journal of Mathematical Economics* 111: 102944.

Hirano, Tomohiro, and Alexis Akira Toda. 2025. "Bubble Necessity Theorem." *Journal of Political Economy* 133(1): 111-145.

Hirano, Tomohiro, and Noriyuki Yanagawa. 2017. "Asset Bubbles, Endogenous Growth, and Financial Frictions." *Review of Economic Studies* 84: 406-443.

Ikeda, Daisuke. 2021. "Monetary Policy, Inflation and Rational Asset Price Bubbles." *Journal of Money, Credit and Banking* 54(6): 1569-1603.

Kiyotaki, Nobuhiro, and John Moore. 2019. "Liquidity, Business Cycles, and Monetary Policy." *Journal of Political Economy* 127(6): 2926-2966.

Kocherlakota, Narayana R. 2009. "Bursting Bubbles: Consequences and Cures." Unpublished.

LeRoy, Stephen F., and Richard D. Porter. 1981. "The Present-Value Relation: Tests Based on Implied Variance Bounds." *Econometrica*, 49(3): 555-574.

Martín, Alberto, and Jaume Ventura. 2012. "Economic Growth with Bubbles." *American Economic Review* 102(6): 3033-3058.

Martín, Alberto, and Jaume Ventura. 2016. "Managing Credit Bubbles." *Journal of the European Economic Association* 14(3): 753–789.

Martín, Alberto, and Jaume Ventura. 2018. "The Macroeconomics of Rational Bubbles: A User's Guide." NBER Working Paper no. 24234.

Miao, Jianjun. 2014. "Introduction to Economic Theory of Bubbles." *Journal of Mathematical Economics* 53: 130-136.

Miao, Jianjun and Pengfei Wang. 2018. "Asset Bubbles and Credit Constraints." *American Economic Review* 108(9): 2590-2628.

Miao, Jianjun, Pengfei Wang, and Jing Zhou. 2015. "Asset bubbles, collateral, and policy analysis." *Journal of Monetary Economics* 76: S57-S70.

Miao, Jianjun, Pengfei Wang, and Zhiwei Xu. 2015. "A Bayesian DSGE Model of Stock Market Bubbles and Business Cycles." *Quantitative Economics* 6: 599–635.

Nisticò, Salvatore. 2012. "Monetary Policy and Stock-price Dynamics in a DSGE Framework." *Journal of Macroeconomics* 34(1): 126-146.

Samuelson, Paul A. 1958. "An Exact Consumption-loan Model of Interest with or without the Social Contrivance of Money." *Journal of Political Economy* 66(6): 467-482.

Santos, Manuel S., and Michael Woodford. 1997. "Rational Asset Pricing Bubbles." *Econometrica* 65(1): 19-57.

Shiller, Robert J. 1982. "Do Stock Prices Move Too Much to be Justified by Subsequent Changes in Dividends?" *American Economic Review* 71(3): 421-436.

Taylor, John B. 2014. "The Role of Policy in the Great Recession and the Weak Recovery." *American Economic Review* 104(5): 61-66.

Tirole, Jean. 1985. "Asset Bubbles and Overlapping Generations." *Econometrica* 53(6): 1499-528.

Wang, Pengfei, and Yi Wen. 2012. "Speculative Bubbles and Financial Crises." *American Economic Journal: Macroeconomics* 4(3): 184-221.

Woodford, Michael. 2003. "Interest and Prices," Princeton, NJ: Princeton University Press.