UNIVERSITY of York

# *Discussion Papers in Economics*

## No. 22/07

# The Socially Optimal Loan Auditing with Multiple Projects

Peter J. Simmons
Nongnuch Tantisantiwong

Department of Economics and Related Studies
University of York
Heslington
York, YO10 5DD

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Peter Simmons
Department of Economics and Related Studies, University of York, York, YO10 5DD, UK
email address: peter.simmons@york.ac.uk

Nongnuch Tantisantiwong
Risk Management Group, Krungthai Bank, Bangkok, Thailand
email address: ntantisantiwong@googlemail.com

October 9, 2022

**Abstract**

This paper fills the gap in the literature by introducing an efficient, incentive compatible audit policy that can minimise the social loss created by the audit cost while maximising social welfare. We apply this within a loan auditing context, but the method is also applicable to any accounting and tax audit context. We explain why the loan contract design for finance of projects varies between different situations. Each project outcome is random and private information of its individual owner, but reported outcomes can be audited at a cost. Our framework simultaneously determines incentive compatible auditing policies, interest rates and default probabilities to yield an efficient contract design. We show how the socially best loan audit policy and repayments depend on the degrees of information asymmetry and risk correlation between projects, the number of agents in the agreement and the agents' perception of loan default.

Key words: Optimal contract, Incentive compatible audit policy, Heterogeneous and correlated risk, Welfare, Loan auditing

JEL Codes: D81, D82, G21

# 1 Introduction

In this paper, we derive the optimal audit policy which maximises expected social welfare and, at the same time, minimises the deadweight loss created by the audit cost for both one-to-one and multiple auditor-auditee settings. Our study provides a connection between report auditing in the accounting literature and welfare maximisation as well as incentive compatibility in the economic literature. In the accounting literature, the issue of audit arises from the facts that the information of each subsidiary of a holding company or each division within a multidivisional organisation can be private. There are spillover effects between these units such that the total organisation performance depends on that of its units and the rewards for each unit depend on the performance of all units (Healy, 1985). As a result, the principles of component auditing have developed. Similar issues arise in economics in the context of group lending where the joint liability for loan repayment is similar to the overall financial viability of the multidivisional firm depending on the net performance of its divisions. The project outcome can be private information to each borrower, so some incentive compatibility devices must be used to ensure truthful reporting of outcomes and repayment by individual borrowers. In both literatures, there are also some implications for how the umbrella organisation (the loan group and the central arm of the multidivisional firm) does or should behave (see, e.g., Besley and Coate, 1995; De Quidt et al., 2016; Feltham et al., 2016) although in the former case the rules are often taken as given.

Auditing the reported outcomes of individuals who declare low or defaulting returns is a common way to deal with asymmetric information problems between the principal and agents or between the lender and borrowers. Typically in the accounting literature, the audits are complete (every unit is audited for sure) and randomization in the audit policy is not widely considered (Demski, Patell, and Wolfson, 1984). In accounting knowing that each audit is costly, some researchers (e.g., Yim, 2009) suggest that the audit budget can be conserved by implementing audit on a random sample of projects with low or defaulting returns[1]. While accounting and economic literature also considers other

[1] Yim (2009) states "An overly-committed audit budget ties up resources that could have been allocated for better alternative uses" p. 2000.

incentive problems e.g. controlling or monitoring agents/borrowers choice of project (Stiglitz, 1990; Madajewicz, 2011) or choice of effort in carrying out the project (Laffont and Rey, 2003; Drymiotes, 2007), surprisingly little attention has been paid to incentive problems in loan auditing especially for loans with joint liability and positively correlated risk. In this paper, we fill the gap in the literature by introducing an efficient, incentive compatible audit policy that can minimise the social loss created by the audit cost while maximising social welfare. We apply this within a loan auditing context, but the method is also applicable to any accounting and tax audit context.

In economics, typically the incentive for borrowers to cheat is controlled by requiring individual borrowers to report their outcome to the auditor(s) and then focussing audit (usually at a random rate) on those reporting lower financial results. This is to control the incentive for successful borrowers to misreport their outcome, hiding some of their profit. If a report is found to be falsely low by audit, the borrower can be punished. Particularly, a cheating borrower who is discovered can have all his revenue confiscated. Knowing this, each borrower has an incentive to report their outcome truthfully. However, audit has a cost and in microfinance this cost, which include transport and communication, can be high. Some prior studies emphasise the effects of peer pressure in group lending to give incentives for repayment (e.g., Besley and Coate, 1995; De Quidt et al., 2016; Karlan, 2007)

Typically, group lending has some special empirical features. It is designed to assist economic and social development in poorer regions (Ahlin et al., 2011; IMF, 2005). Its advantages are seen as a way of enabling excluded borrowers to access finance because it reduces the problems of lack of collateral, asymmetric information (ex-ante adverse selection as well as ex-post moral hazard), and administration costs of a large number of small loans. Historically, one common pattern is for lenders to start with group lending comprised with loans of small size per capita for small scale projects[2]. A typical Grameen bank group loan in the 1990's had no more than five borrowers and often the first

[2]Many empirical studies of conventional bank lending find a positive relationship between loan size and interest rate (e.g., Godlewski and Weill, 2011) and that the larger loan size is associated with the lower default probability (e.g., Agarwal and Hauswald, 2010). On the other hand, Gonzalez (2010) finds that smaller loans have a higher operation cost and so lenders set a higher interest rate premium. Brick and Palia (2007) find too that the smaller borrower firms are charged a higher interest rate. In order to allow the interest rate charged to small borrowing to be low, microfinance institutions lend to groups of small borrowers and peer monitoring and peer pressure have been employed.

loan is to a group of just two borrowers. However, some groups have been much larger than this. For example, German rural credit cooperatives usually had 75-250 members (Ghatak and Guinnane, 1999). The Foundation for International Community Assistance (FINCA) establishes "village banks", a form of microlending, with group sizes between 10 and 50 members and claims that they have achieved a high repayment rate[3]. later, the loans evolve into increasing size of loans to individuals[4], or increasing interest rates for groups with low average loan sizes (Gonzalez, 2010). Using 124 institutions in 49 countries, Cull, Demirguc-Kunt and Morduch (2007) find that an increase in loan size is associated with lower cost, leading to a higher rate of return on assets for individual-based lenders. Other writers find a small group size is important in determining group loan success (see, e.g., Devereux and Fishe, 1993). An important issue in auditing practice is that the number of auditees or reports submitted impacts on the quality and cost of audit; a random audit policy is therefore adopted to reduce the number of audited reports. Motivated by this issue, we attempt to explain how the efficient audit probability can be set up and how it varies between individual and group loans and depends on the numbers of borrowers and reports submitted.

This paper makes two main contributions to the economic and accounting literature. First, we contribute to the literature by introducing the efficient, incentive compatible audit policy for both one-to-one and multiple auditor-auditee settings. Looking at loan auditing as an example, our framework allows an audit probability to be endogenous, unlike literature that has deterministic, exogenous audit and punishment large enough to ensure a fair return to lenders. Here, we study individual and group loans with costly state verification under risk neutrality with a loan outcome known only by the particular borrower. Borrowers have zero collateral and use the loan to generate risky returns which may be low enough to compel a borrower to default on repayment, but on average each loan is profitable and socially desirable. To prevent a high-output borrower falsely declaring that his output is low, some audit on lower reports combined with some punishment when these are discovered to be false is necessary. A report of a low defaulting return can be randomly audited at a fixed cost per

[3] Source: FINCA (www.finca.org)

[4] Vigenina and Kritikos (2004) find that the size of individual loans tends to be larger than that of group loans.

audit, similar to Yim (2009) who proposes a bounded simple random sampling rule for tax auditing.

Analogous to a principal/multi-agent model of Feltham et al. (2016) in which the principal either contracts with all parties directly or delegates part of the contracting authority to an agent, our framework has the lender perform the audit for individual loan contracts and two borrowers perform the audit in group loan contracts. In group lending, the joint liability or group expected surplus is an incentive for peer auditing. According to Stiglitz (1990), Madajewicz (2011) and Laffont and Rey (2003), monitoring is most efficiently done by fellow borrowers in the group rather than by lenders (see, e.g., Armendariz and Morduch, 2005; Madajewicz, 2011). A reason for this is that the group members are better informed than the lender (see, e.g., Brau and Woller, 2004; Agarwal and Hauswald, 2010[5]) and that the borrowers belong to a social group which may allow nonpecuniary social sanctions as well as pecuniary sanctions being imposed on cheats (see, e.g., Karlan, 2007 Besley and Coate, 1995; De Quidt et al., 2016). This helps prevent cheating, and audit by borrowers is therefore more efficient than audit by the lender; thus, each audit is cheaper if performed by a borrower than the lender (see, e.g., Stiglitz, 1990; Varian, 1990; Armendariz and Morduch, 2005; Assadi and Ashta, 2014). Hence, it makes sense for two of the borrowers to have special status. One is appointed as chief auditor and audits the reports of other defaulting borrowers. The second audits the outcome of the chief auditor when the latter declares default. This is a generalization of the setup first proposed by Banerjee, Besley and Guinnane (1994) and is also related to the idea of delegated audit of Diamond (1984).

Our framework shows that, with a group loan, the way incentive compatibility works depends crucially on how each borrower thinks his own report will affect the solvency of the group; the group is classified as (i) unprofitable, (ii) marginal and (iii) non-marginal depending on whether the group is expected to default if he cheats. The results highlight that the group size has strong implications for the nature of incentive compatibility constraints in a group loan. In other words, the efficient, incentive compatibility condition and the audit probability depend on the total number of agents (e.g., the number of taxpayers in the tax audit context) although the audit is randomly performed only on

[5] Agarwal and Hauswald (2010) documented that some private credit information is primarily local and that it is more likely that the bank will face non-payment of borrowers located farther away.

those reporting low or defaulting outcomes (i.e., tax payers reporting low income).

Second, to the best of our knowledge, we are among the first to show how the optimal incentive compatible audit policy, default and interest rates and social welfare vary with general degrees of correlation of outcomes between auditees. It is commonly known that most group loans are to borrowers who are similar to one another or live in the same area (Assadi and Ashta, 2014). Local shocks are likely to affect all borrowers, leading to positive correlation of revenues and reducing the possibilities for risk diversification between group members. Our framework emphasises the impact of correlation on the optimal group contract. To date, only a handful of studies have looked at the role of correlation of returns between borrowers (e.g., Ahlin and Townsend, 2007; Katzur and Lensink, 2012; Kurosaki and Khan, 2012)[6]. Goodstein, Hanouna, Ramirez and Stahel (2017) find that a higher delinquency rate in surrounding zip codes increases the probability of a strategic default and Varian (1990) notes that borrowers' homogeneity can make the lender worse off. How to model correlated risks is still an issue. The few existing studies on group lending take very specific models of correlation (see, for example, Sinn, 2013). Here, we introduce the use of the beta-binomial distribution in the analysis of group lending, as it allows for varying degrees of correlation. The positive correlation of returns induces a higher chance of many fails and many successes within a group as compared with independent risks, heightening the chance of group default.

For ease of exposition, the paper analyses how different group conditions (e.g., the size of group and the distribution of returns of individual borrowers) will result in different incentive compatible auditing contracts. We find that, with uncorrelated borrower risks, group loans with our group rules dominate individual loans in terms of social welfare with the two forces at work being risk diversification and a lower audit cost per project. Our results extend those of Baland, Somanathan and Wahhaj (2013) who allow for some self-finance and a variable loan size; under their assumptions, they find that the expected surplus for a risk neutral borrower in a group loan is highest when the default rate on the group loan is lowest. Our paper finds that, with correlation, the advantage of group lending in relation

[6] Most studies assume identical independent risks (iid) of each borrower; some studies analyse lending with independent but heterogeneous risks of borrowers (see, for example, Stiglitz, 1990).

to risk diversification disappears and that the audit probability increases with the degree of correlation between individuals' outcomes and group size. Our approach and findings should apply to other audit problems such as audit with correlated risk by setting a higher random audit probability in the financial statement or tax audit. We would expect more efficiency and lower cost through identifying the correlation between component successes, randomising the audit on components and adopting the audit policy that leads to the highest welfare of the organisations.

The rest of the paper is outlined as follows. The next section introduces a simple model for individual and group loans, describing the time line and audit mechanism for both individual lending (one-to-one setting) and group lending (multiple auditors-auditees setting). The section then derives the socially optimal audit probability for each loan form and each risk distribution. It also establishes the borrower's welfare for each loan form and analyses the relative social merits of individual and group lending based on welfare for different numbers of agents and different types of risks. In Section 3, the binomial distribution is used in simulation to reflect independent risk while the beta binomial distribution is used to reflect positively correlated risk of auditees' projects. In Section 4, assumptions such as the possibilities of collusion between auditors and auditees as well as dynamic settings (e.g., renegotiation or sequential group lending) are considered. Section 5 concludes and provides the implications of our findings to other auditing practices and policymaking. Technical proofs are relegated to the appendix.

# 2 A Simple Model

## 2.1 Assumptions

### 2.1.1 Distribution of risk

In this section, we introduce a simple framework to derive the optimal incentive compatible audit policy that can minimise the social loss created by the audit cost while maximising social welfare. In our framework, there are a risk neutral lender, who has access to a safe interest rate ($r$), and $n$ risk-neutral borrowers. Each borrower $i$ has a project requiring finance of $B$. In common with much of the literature (see, e.g., Stiglitz, 1990; Banerjee et al.1994, Laffont, 2003; Laffont and Rey, 2003; Ahlin and Townsend 2007), we assume that each project $i$ yields one of two returns; with some probability $p_i$, the project succeeds and yields high revenue of $H$, and with probability $1 - p_i$ the project fails and yields low revenue of $L$. A set of success probabilities $(p_1 \,..p_n)$ is a sample draw from a distribution $f(p_1 \,..p_n)$ in which each $p_i$ has an identical mean of $\overline{p}$. In general, the random probabilities of successes $(p_1 \,..p_n)$ may be mutually correlated. Define the number of successes as $k$ and the joint distribution of risk as $g(k, p_1..p_n)$. The distribution of the outcomes is common knowledge. Let $C_k$ be any permutation of $k$ integers for $k \in [0, n]$; for example, if $n = 5$ and $k = 4$, there are five permutations: $\{1, 2, 3, 4\}, \{1, 2, 3, 5\}, \{1, 2, 4, 5\}, \{1, 3, 4, 5\}, \{2, 3, 4, 5\}$. The number of elements $i \in C_k$ is actually $\left( \begin{array}{c} n \\ k \end{array} \right)$. The conditional density of the number of successes being $k$ is $h(k|p_1, ...p_n) = \Sigma_{C_k} \Pi_{i \in C_k} p_i \Pi_{i \notin C_k}(1 - p_i)$. So

$$\begin{aligned} g(k, p_1..p_n) &= h(k|p_1..p_n)f(p_1..p_n) \\ &= \Sigma_{C_k} \Pi_{i \in C_k} p_i \Pi_{i \notin C_k}(1 - p_i)f(p_1..p_n) \end{aligned}$$

It follows that[7]

$$\begin{aligned} E_k(k|p_1...p_n) &= \Sigma p_i \\ E_p E_k(k|p_1...p_n) &= n\overline{p} \end{aligned}$$

Even if individual project risks are identically distributed, success on different projects can be correlated. Allowing (especially positively) correlated risks is empirically important. Given that group

[7] As stated above the contract is written on the basis of the individual risks being a draw from the distribution $f(p_1, ...p_n)$. We are particularly interested in the distribution of the number of successes on the $n$ projects. We give some general results; let $(p_1...p_n)$ be a sample from an arbitrary multivariate distribution with mean vector $(\overline{p}..\overline{p})$.

For $n = 1$, it is just the binomial case $E(1|p) = p$. For $n = 2$,

$$\begin{aligned} E_k(k|p_1 p_2) &= 2\Pr(k = 2) + 1 * \Pr(k = 1) + 0\Pr(k = 0) \\ &= 2p_1 p_2 + [p_1(1 - p_2) + p_2(1 - p_1)] = p_1 + p_2 \end{aligned}$$

Suppose that it is true for $n - 1$: $E_k(k|p_1...p_{n-1}) = \Sigma_1^{n-1} p_i$; then, for $n$ borrowers, we have the recurrence rule:

$$\begin{aligned} E_k(k|p_1...p_n) &= p_n(1 + E_k(k|p_1...p_{n-1}) + (1 - p_n)E_k(k|p_1...p_{n-1}) \\ &= p_n + E_k(k|p_1...p_{n-1}) \end{aligned}$$

Therefore,

$$\begin{aligned} E_k(k|p_1...p_n) &= \Sigma p_i \\ E_p E_k(k|p_1...p_n) &= n\overline{p} \end{aligned}$$

loans are typically to geographically close borrowers, the idea of common systemic risk on a group loan is attractive. Due to the localised nature of group lending, group members may face some common risks and thus the success of projects may be correlated (see, e.g., Varian, 1990). The dependence of outcomes violates the independence assumption of the binomial distribution. The tail probabilities (especially the lower tail) are likely to be higher than with independent risks, since downside catastrophic risk is probably more common in the developing economy context in which group lending occurs. Particular events such as extreme weather (e.g., droughts and floods), geological events (e.g., earthquakes) and economic and political events (e.g., commodity price shocks and revolution) are likely to cause common high downside risk in the borrower group. Similarly, good shocks are likely to be correlated across borrowers if they are localised. That is why we model an exante situation in which the risks facing different borrowers are initially random variables and positively correlated.

#### 2.1.2 Auditing

Only the borrower can costlessly see the outcome of his project. The outcome of a project can be revealed to the lender and all other borrowers only by costly audit which will then reveal publicly the revenue of audited borrowers. With individual loans on multiple projects, there is a separate contract between each borrower and the lender which is set to give the lender a fair return allowing for borrower default and the cost of audit. Each borrower only has an interest in his own contract, so audit of defaulting outcomes must be performed by the lender.

With a single group loan financing the projects of all individual borrowers, all borrowers report their project outcome to the group which then organises any audits and makes a group repayment to the lender. Any remaining group surplus is distributed equally between successful borrowers and truthful fails. The group contract again ensures that, with truthful reports, the lender receives the fair return on the total group loan. Unlike individual loans, there is joint liability for the group loan amongst borrowers. So long as the total group revenue can cover the fair repayment on the group loan, successful borrowers bail out defaulting borrowers to avoid seizure of all group revenue. Thus, each borrower has an incentive to audit his fellow borrowers to ensure that they truthfully report successful outcomes. There are strong arguments in the literature that the group is better informed about the situation of group members than the lender and is also in a position to impose tougher sanctions for truth-telling on group members than the lender (see, e.g., Assadi and Ashta, 2014; Everett; 2015). In Stiglitz (1990) and Karlan (2007), strong emphasis is put on the role of the group of borrowers in enforcing truthful revelation and repayment through peer pressure. In such cases, it is the group of borrower(s), not the lender, who pay the audit cost. In Banerjee et al. (1994), one group member is designated as the auditor, but most group loans are to symmetrically placed borrowers, each with a risky project. To operationalise this in our framework, two group members have an audit role set by the group collectively: the first borrower receives all $n-1$ reports from the other borrowers and audits each of the fail reports, and the second audits only the reported fail outcome of the first auditor.

We assume that the lender can audit fail reports at a cost of $C$ per borrower[8] and borrowers can audit a failed report at the cost per borrower of $c$ ($\leq C$). In common with the literature, the audit probability is selected to ensure truthful reporting and all revenues of detected false reporters are seized, satisfying the revelation principle (see, e.g., Townsend, 1979; Gale and Hellwig, 1985) and maximum punishment (see, e.g., Border and Sobel, 1987)[9].

#### 2.1.3 Lending and repayment

In our framework, the timing is as follows:

(i) Initially, each borrower receives finance for a risky project. The individual or group repayments for solvent agents are agreed. The lender does not know $p_i$ for each borrower $i$ but knows the distribution from which $p_i$ is a draw. The amount of repayment is set to give the lender a zero expected excess return above the safe rate. The rule that, if default occurs with either an individual or group loan, the lender seizes all the discovered assets of the borrowers is agreed[10]. The audit probability on

[8] Vigenina and Kritikos (2004) find that the size of individual loans tends to be larger than that of group loans. Using 124 institutions in 49 countries, Cull et al. (2007) find that an increase in loan size is associated with lower cost especially for individual-based lenders - but only upto a point. In addition, Cull et al. (2009) report that the institutions with the higher cost per unit lent charges the higher interest rate. Similarly, Brick and Palia (2007) find that smaller loans have a higher operation cost and so lenders set a higher interest rate premium. This is reinforced by Gonzalez (2010).

[9] It is well known that applying maximum punishment on false defaulters minimises the risk of false reporting and helps attain incentive compatibility under risk neutrality (see, for example, Border and Sobel, 1987; Besley and Coate, 1995).

[10] The projects in this framework are differentiated only by the chance of success. The setup is equivalent to one in which the project yields 0 if it fails, but the borrowers have to post collateral of $L$ that can then be seized if the project

an agent who declares a fail ($m$) is also set in the initial contract.

(ii) Each borrower executes the project, observes his revenue outcome, and reports either a success or fail to the lender and other borrowers. In particular, with individual loan contracts between a borrower and the single lender, all borrowers report their outcome to the lender. With a group loan, borrowers report their outcome to the group. These individual borrower reports are made simultaneously.

(iii) The auditor(s) carry out random audits on reported fails. With any loan form and any audit arrangements, if there are $k$ reported successes, $n-k$ audits must be undertaken each with probability $m$. With individual loans, the lender performs the audit and a detected cheating borrower loses his whole revenue to the lender. With a group loan, audit by two group members is socially preferred to audit by the lender (see Appendix A). The results of these audits become public knowledge to the lender and all borrowers[11]. If a borrower is audited and is found to have cheated, the group seizes all the revenue of that borrower and denies him any share of group surplus.

(iv) Based on truthful reports, payments are made out of the project(s) revenues after auditing. For an individual loan, the lender seizes the revenue of a fail following the audit. For a group loan, the group either repays and divides up its remaining surplus equally between all borrowers or defaults in which case the lender seizes the group revenue.

#### 2.1.4 Social desirability condition

We assume social desirability conditions of the project: $H > (1+r)B + C > L$ and

$$\overline{p}H + (1-\overline{p})L > (1+r)B + (1-\overline{p})C > (1+r)B + (1-\overline{p})c \tag{1}$$

where $\overline{p}$ is the mean probability of success for each project. Therefore, only successes can repay their own loan in full, and each loan has a positive net expected social return even with the high audit cost. Multiplying the first inequality of Eq. (1) by $n$ yields

$$n\overline{p}H + (n-n\overline{p})L > (1+r)nB + (n-n\overline{p})C \tag{2}$$

fails and the borrower defaults.

[11] Since audit results are public to all including the lender, the group itself or its auditors cannot cheat.

As shown above, the mean number of successes in a population of $n$ borrowers: $E_pE_k(k|p_1..p_n) = n\overline{p}$, so Eq.(2) can be rewritten as

$$E_pE_k(k|p_1..p_n)H + (n - E_pE_k(k|p_1..p_n))L > (1+r)nB + (n - E_pE_k(k|p_1..p_n))C \tag{3}$$

That is, on average each project is profitable whatever happens to the other projects, and at the mean number of successes the group revenues are sufficient to repay the loan. There is the usual audit commitment problem for the auditor (the lender in this case); he has to commit to pay the cost to discover the borrower assets even though he knows that the borrowers are truthfully reporting. If $L > C$, then the lender can always cover the audit cost.

## 2.2 Individual loans

The lender directly contracts with each borrower, setting the required repayment from a successful borrower ($P_I$) and the audit probability on a loan which reports a failure. The borrower reports the outcome to the lender. If fail is reported, he is audited with the agreed probability $m_I$. A successful borrower makes the repayment to the lender, a truthful failed borrower pays his entire revenue $L$ and a detected cheating successful borrower loses his whole revenue $H$ to the lender. The lender sets the terms of each loan identically on each borrower. With an assumption that the lender knows only the distribution of $p_i$ ($\bar{p}$ is the mean of $p_i$ in the marginal distribution), the repayment per loan $P_I$ is set to give the lender a non-negative expected surplus above the risk-free interest rate ($r$) on each individual loan[12]:

$$P_I\bar{p} + L(1-\bar{p}) \geq (1+r)B + m_IC(1-\bar{p})$$

where $C$ is the audit cost. Since on average individual projects fail with probability $1 - \overline{p}$, this is also the expected default rate of an individual loan. Given our assumptions on returns and loan size,

[12] If the lender treats each borrowers $i$ loan in isolation from other loans, the marginal distribution of $p_i$ is used to compute the repayment on each loan, but the mean of the marginal distribution is again $\overline{p}$.

The lender sets the common repayment per loan to just give a zero expected surplus on the $n$ loans in total

$$P_IE_pE_k(k|p_1..p_n) + L(n - E_pE_k(k|p_1..p_n)) = (1+r)nB + m_IC(n - E_pE_k(k|p_1..p_n))$$

$E_pE_k(k|p_1..p_n) = n\overline{p}$ so the repayment is set identically for $n$ borrowers.

borrowers who fail must default, but successful borrowers should repay. Borrower surplus is

$$\begin{aligned} S_I &= (H - P_I) \text{ with probability } p_i \\ &= 0 \text{ with probability } 1 - p_i \end{aligned}$$

Then, the expected surplus of the $i^{th}$ borrower is

$$ES_I = \bar{p}(H - P_I)$$

If the successful borrower reports fail and the loan is audited (with probability $m_I$), all his revenue is confiscated. As a result, a successful borrower will truthfully report if

$$(1 - m_I)(H - L) \leq H - P_I \tag{4}$$

The individual loan contract solves the problem of choosing $m_I$ and $P_I$ to maximize the total welfare between the lender and borrower subject to the lender at least breaking even in expected value terms and a constraint on the borrower which requires truth-telling behaviour (Eq. (4)). We note that if the lender participation constraint binds, the lender's surplus is zero and the total welfare is equal to the expected individual borrower surplus per loan.

The contract problem is

$$\begin{aligned} \max_{m_I, P_I} ES_I &= \bar{p}(H - P_I) \\ \text{s.t. lender participation constraint} &: \overline{p}P_I \geq (1 + r)B + (m_I C - L)(1 - \overline{p}) \\ \text{incentive compatibility} &: (1 - m_I)(H - L) \leq H - P_I \end{aligned}$$

Both the lenders participation constraint and the incentive compatibility constraint must bind. If the participation constraint is slack, $P_I$ can be reduced which slackens the incentive constraint and raises the objective. If the incentive constraint is slack, $m_I$ can be reduced which slackens the participation constraint allowing $P_I$ to also be reduced. Jointly solving the two binding constraints for the variables

$P_I$ and $m_I$ yields the solutions:

$$
\begin{aligned}
m_I &= \frac{(1+r)B - L}{\bar{p}(H-L) - C(1-\bar{p})} && (5)\\
P_I &= \frac{(1+r)B + (m_I C - L)(1-\bar{p})}{\bar{p}} \\
&= (H-L)\frac{(1+r)B - L}{\bar{p}(H-L) - C(1-\bar{p})} + L && (6)
\end{aligned}
$$

With social desirability $\overline{p}H + (1-\overline{p})L > (1+r)B + (1-\overline{p})C$, the optimal audit probability is strictly between zero and one. The required repayment $P_I$ reflects the feasibility of the project with the first term's denominator reflecting the audit cost transferred from the lender. With $m_I < 1$, Eq. (6) also shows that at the optimum $P_I$ is set to be equivalent to the lender's expected gain from loan auditing: $m_I H + (1 - m_I)L$. Knowing $P_I$, we can find the interest rate ($R_I$) required for an individual loan as $P_I$ is equal to $(1 + R_I)B$. Eq. (6) is consistent with many empirical studies of conventional bank lending which document a positive relationship between loan size and repayment (interest rate) (see, e.g., Godlewski and Weill, 2011). The optimal expected surplus of the $i^{th}$ borrower is then

$$
\begin{aligned}
ES_I &= \bar{p}(H - P_I) \\
&= \bar{p}(H-L)\left[1 - \frac{(1+r)B - L}{\bar{p}(H-L) - C(1-\bar{p})}\right] > 0
\end{aligned}
$$

It shows that the higher the audit probability, the lower the expected surplus of the borrower and the total welfare.

### 2.3 Group loan

The group has a fixed size $n$ with a known distribution of the number of successes $k$. $E_p E_k(k|p_1..p_n)$ is the mean number of successes from a population of size $n$ (Hereinafter $E(k|n)$). The realized outcome of any one borrower is private information to that borrower unless the borrower is audited. Each borrower makes a report of his outcome to the group. If audited, the true outcome of the borrower is known by all borrowers and the lender. Audit per borrower costs $c$. Following from Eq.(1) and Eq. (3), the $n$ individual projects are socially desirable in the sense that $E(k|n)(H-L)+nL-(1+r)nB-c(n-E(k|n)) > 0$. That is, the mean group revenue covers the investment cost of the lender plus the cost of auditing each of the mean number of fails.

Recall the time line in Section 2.1.3, loans to individuals are made, and the lender only knows the exogenous individual loan size, the group size and the distribution of the number of successes. The loan contract sets the repayment required from the group ($P$) and the probability with which each reported fail will be audited ($m$)[13]. The group repayment and $m$ are set so that the lender gets a non-negative expected surplus above the safe rate, allowing for joint liability between borrowers, for group default when many borrowers fail and also for the audit cost on reported fails.

The group undertakes and pays for the audit. There are two reasons for this. First, as well documented the audit cost per borrower may be lower for a group member because of better information. Second and more fundamentally, part of the costs of incentive compatibility are internalised if the group audits. A successful borrower knows that if he cheats, he will reduce the group revenue and his share of the surplus of a nondefaulting group because of his expected audit.

With truthful reporting, group revenue received is

$$kH + (n-k)L - cm(n-k) = k(H - L + cm) + n(L - cm)$$

when there are $k$ successes. If the revenue received by the group is lower than the required group repayment, the group defaults and the lender seizes all the group revenue; otherwise, the group repays the amount set in the group loan contract. There will be a critical number of successes $k^*$, the lowest number of successes which allows the group to repay (See proof in Appendix B). If $k < k^*$, the group defaults and pays all its revenue $k(H - L + cm) + n(L - cm)$ to the lender. So conditional on the group defaulting, the average return to the lender from all the defaulting states is $E^*(k|k < k^*, n)(H - L + cm) + n(L - cm)$ where $E^*(.|k < k^*)$ is the mean of the truncated distribution of the number of successes, i.e., $E(k|k < k^*, n)/\Pr(k < k^*, n)$. $\Pr(k < k^*, n)$ is the default probability on the group loan. Hence, in expected terms the lender receives

(i) $P$ with probability $\Pr(k \geq k^*, n)$

[13] To ensure truthful reports on all projects requires each reported fail to have a positive probability of audit ($m > 0$). This is costly and decreases the group's surplus. The audit probability is optimally set at its lowest level ensuring truthful reporting.

(ii) $E^*(k|k < k^*, n)(H - L + cm) + n(L - cm)$ with probability $\Pr(k < k^*, n)$.

The lender has to at least break even on the group loan in expected terms. Thus, $P$ and $k^*$ are jointly determined by

$$\Pr(k \geq k^*, n)P + E(k|k < k^*, n)(H - L + cm) + n(L - cm)\Pr(k < k^*, n) \geq (1 + r)nB$$

$$k^*(H - L + cm) + n(L - cm) \geq P$$

The surplus of the group ($S_G$) depends on $k$ :

$$\begin{aligned} S_G &= kH + (n - k)L - cm(n - k) - P \text{ if } k \geq k^* \\ &= 0 \text{ if } k < k^* \end{aligned}$$

Hence,

$$ES_G = E(k|k \geq k^*, n)(H - L + cm) + \Pr(k \geq k^*, n)[n(L - cm) - P]$$

which is shared equally between the truthful reporting borrowers.

### 2.3.1 Group incentive compatibility

Each borrower has to decide what he should report: a success or a fail, knowing the audit probability $m$ but not at this stage knowing the outcome of other borrowers. If they are truly a fail, the best the borrower can do is "reporting a fail and paying $L$ to the lender/group"; such a borrower still has a chance of a share of the surplus if enough other borrowers have succeeded and the group does not default.

If they are a success, what should they report? Their payoff depends on whether the group defaults after their report. In the case that successes tell the truth, they report $H$ and get (i) a share of the surplus if the group does not default or (ii) nothing if the group does default and has no surplus to distribute. Alternatively, they could cheat, report a fail, keep $H$ for themself and give the group $L$. If they are audited, their cheating is discovered for sure and they lose not only $H$ but also any right to a share of the group surplus if the group does not default. If they are not audited, then they gain $H - L$ directly and, if the group does not default even after their cheating, they also get a share of the surplus coming from the other $n-1$ borrowers. Hence, the best report for a success to make depends on the reports of other borrowers which set whether the group defaults or not.

In deciding whether to cheat in his report, a successful borrower has to assess whether the group will default if he either tells the truth or cheats. His decision depends on information/beliefs he has about the outcomes of other borrowers. He knows the distribution of the number of successes and that the audit probability is incentive compatible for all borrowers. Hence, he assumes rationally that all other borrowers report truthfully[14] and thus that expected reported group revenue from the other $n-1$ borrowers is $E(k|n-1)(H-L)+(n-1)L-cm(n-1-E(k|n-1))$. Here, $E(k|n-1)$ is the mean number of successes out of $n-1$ borrowers.

With truthful reporting, this successful borrower expects reported group revenue will be

$$\begin{aligned} G_T^e &= (E(k|n-1)+1)H+(n-1-E(k|n-1))L-cm(n-1-E(k|n-1)) \\ &= (E(k|n-1)+1)(H-L+cm)+n(L-cm) \end{aligned}$$

Note that $E(k|n)$ can be any real number between 0 and $n$, but $k^*$ and $k$ must be integers. Instead, suppose the successful borrower cheats, paying just $L$ to the group and retaining $H-L$ for himself. He knows there is a chance he will be audited and the expected cost of this $cm$ has to be added into his view of group revenue if he cheats. If he is audited, he loses $H-L$ (in addition to $L$ that has already paid to the group loan) and the right to a share of the group surplus (if any). However, if not audited, he keeps $H-L$ and gets a share of the group surplus if any. If he cheats and may be audited with probability $m$, he thinks group revenue will be

$$\begin{aligned} G_c^e &= E(k|n-1)(H-L)+(n-1)L+L-cm(n-E(k|n-1)) \\ &= E(k|n-1)(H-L+cm)+n(L-cm) \end{aligned}$$

Of course, $G_c^e < G_T^e$ (in fact $G_c^e = G_T^e - (H-L+cm)$). It shows that a successful borrower expects group revenue to be higher if he truthfully declares a success rather than cheats.

[14] Rai and Sjöström (2004) document that a contract inducing truthful revelation about the success of individual projects can dominate in both individual and group lending.

If $k^* > E(k|n-1)+1 > E(k|n-1), G_T^e < P$; a successful borrower thinks that the group will default even if he tells the truth. He also expects the group to default if he cheats. We call this an unprofitable group. If $E(k|n-1) < k^* \leq E(k|n-1)+1$ , then $G_C^e < P \leq G_T^e$ ; he thinks the number of successes for the other $n-1$ borrowers will be $E(k|n-1)$ and believes that, on average, the group will be solvent if he truthfully reports and that the group will be insolvent if he cheats. We call this group a marginal group. If $k^* \leq E(k|n-1)$, a successful borrower thinks that on average the group will be solvent whatever he reports. We call this a non-marginal group. This gives us three cases:

(i) unprofitable group: $G_c^e < G_T^e < P$ with $k^* > E(k|n-1)+1$

(ii) marginal group: $G_c^e < P \leq G_T^e$ with $E(k|n-1) < k^* \leq E(k|n-1)+1$

(iii) nonmarginal group: $P \leq G_c^e < G_T^e$ with $k^* \leq E(k|n-1)$

To work out his own return from his report, the individual borrower has to judge if the group will be solvent or insolvent after he either cheats or tells the truth since this determines if the group has any surplus to distribute. Thus, incentive compatibility requires the expected gain from unaudited cheating to be lower than the expected gain from telling the truth:

$$(1-m)[H-L+\frac{\max(0,G_c^e-P)}{n}] \leq \frac{\max(0,G_T^e-P)}{n}$$

The group contract sets $m, P$ and $k^*$ to maximize the expected surplus per borrower whilst ensuring that the lender participation and the incentive compatibility constraints are satisfied. Thus, the contract problem is

$$\max_{m,P,k^*} E(k \geq k^*)(H-L+cm)/n + \Pr(k \geq k^*)[L-cm-P/n] \tag{7}$$

$$\text{s.t. } \Pr(k \geq k^*, n)P + E(k|k < k^*, n)(H-L+cm) + n(L-cm)\Pr(k < k^*, n) \geq (1+r)nB \tag{8}$$

$$k^*(H-L+cm) + n(L-cm) \geq P \tag{9}$$

$$(1-m)[H-L+\frac{\max(0,G_c^e-P)}{n}] \leq \frac{\max(0,G_T^e-P)}{n} \tag{10}$$

Eq. (8) and Eq. (9) are the lender's participation constraints. Eq. (10) is the incentive compatibility constraint. Notice that the precise way in which incentive compatibility controls truth-telling is determined endogenously through the contracts choice of $m, P, k^*$; hence, the form of the group (non-marginal, marginal or unprofitable) is also endogenous. We can combine the conditions in Eq. (8) and Eq. (9) yielding

$$[\Pr(k \geq k^*, n)k^* + E(k|k < k^*, n)]\,(H - L + cm) + n(L - cm) \geq (1 + r)nB$$

Even with cheating controlled, the lender receives a nonnegative expected return. Optimally, the lenders participation constraint in Eq. (8) must bind. If it were slack, then $P$ could be reduced which would raise the objective in Eq. (7), slacken Eq. (9) and slacken or have no effect on Eq. (10) depending on whether it is an unprofitable group. Similarly, Eq. (10) must bind optimally since otherwise $m$ could be reduced which would raise the objective. With a binding lender's participation constraint in Eq. (8), the group repayment in terms of $k^*$ and $m$ is

$$P = \frac{(1 + r)nB - (H - L + cm)E(k|k < k^*, n) - n(L - cm)\Pr(k < k^*, n)}{\Pr(k \geq k^*)} \tag{11}$$

Substituting out $P$, a non-defaulting group has surplus

$$\begin{aligned} S_G &= k(H - L + mc) + n(L - cm) - P \\ &= k(H - L + mc) + \frac{E(k|k < k^*, n)(H - L + mc)}{\Pr(k \geq k^*)} + \frac{n(L - cm) - (1 + r)nB}{\Pr(k \geq k^*)} \end{aligned}$$

Hence,

$$\begin{aligned} ES_G &= E(k|k \geq k^*)(H - L + mc) + E(k|k < k^*, n)(H - L + mc) + n(L - cm) - (1 + r)nB \\ &= E(k|n)(H - L) - mc(n - E(k|n)) - (1 + r)nB + nL \end{aligned}$$

It shows that the investment cost of the lender is transferred to the group of borrowers, and that the audit creates a deadweight loss.

### 2.3.2 Incentive compatible audit policies

As mentioned earlier, there are three possible cases: (i) an unprofitable group if $k^* > E(k|n-1)+1$; (ii) a marginal group if $E(k|n-1) < k^* \leq E(k|n-1)+1$ and (iii) a non-marginal group if $k^* \leq E(k|n-1)$.

The form of the incentive compatibility (IC) constraint differs between unprofitable, marginal and non-marginal groups:

$$
\begin{aligned}
(1-m_u)[H-L] &\leq 0 \text{ for an unprofitable group} \\
(1-m_m)[H-L] &\leq \frac{G_T^e - P}{n} \text{ for a marginal group} \\
(1-m_n)[H-L+\frac{G_c^e - P}{n}] &\leq \frac{G_T^e - P}{n} \text{ for a nonmarginal group}
\end{aligned}
$$

and hence the solutions for $m$, $P$ and $k^*$ will vary between these groups.

**(i) Unprofitable group:** $G_c^e < G_T^e < P$

Any successful borrower, who thinks the group will default regardless of his decision, should always cheat since he gets 0 from telling truth but has a chance of $H-L$ from cheating. Each of the successful borrowers thinks the same (they all use the expected return to form beliefs) and cheats, resulting in default of the group loan. In order to stop everyone cheating, the group has to set the audit probability $m_u = 1$.

**(ii) Marginal group** $G_c^e < P \leq G_T^e$

For a marginal group, we know $E(k|n-1) < k^* \leq E(k|n-1)+1$ and IC requires

$$(1-m_m)[H-L] \leq \frac{G_T^e - P}{n}$$

Appendix C shows that we must have the audit probability $m_m > 0$. Basically, in a marginal group if $m_m = 0$, the payoff from truth-telling (his share of the expected group surplus) is less than the gain from cheating, violating the above IC constraint. Therefore, to ensure truth-telling requires $m_m > 0$.

We must also have $m_m < 1$ optimally. Otherwise, the left-hand side of IC is 0 and $G_T^e - P > 0$; it would be possible to reduce $m_m$, raise borrower surplus, and still satisfy IC:

$$\frac{\partial ES_G}{\partial m} = -c[n - E(k|n)] < 0$$

So, in fact the group must have $0 < m_m < 1$. For a marginal group which requires audit, $m_m$ solves

$$(1-m_m)[H-L] = \frac{(G_T^e - P)_{m=m_m}}{n} \tag{12}$$

Appendix D shows that

$$m_m = \frac{n((1+r)B - L) + [\Pr(k \geq k^*)(n-1-E(k|n-1)) - E(k|k<k^*,n)](H-L)}{c[\Pr(k \geq k^*)(E(k|n-1)+1) + E(k|k<k^*,n) - n] + n\Pr(k \geq k^*)(H-L)}$$

**(iii) Nonmarginal group:** $P \leq G_c^e < G_T^e$

Incentive compatibility (IC) requires

$$(1-m_n)[H - L + \frac{(G_c^e - P)_{m=m_n}}{n}] \leq \frac{(G_T^e - P)_{m=m_n}}{n}$$

Since optimally IC binds, using the expressions for $G_c^e - P$ and $G_T^e - P$ generates a convex quadratic function of $m$ (see Appendix E):

$$\begin{aligned} F(m_n) \quad = \quad & \frac{n - E(k|n-1)\Pr(k \geq k^*) - E(k|k < k^*)}{n\Pr(k \geq k^*)} cm_n^2 \\ & + \left[ \frac{(1+r)B - L}{\Pr(k \geq k^*)} - \frac{(H-L)}{n}\left( n + E(k|n-1) + \frac{E(k|k < k^*)}{\Pr(k \geq k^*)} \right) - \frac{c}{n} \right] m_n \\ & + \frac{(H-L)(n-1)}{n} = 0 \end{aligned}$$

The first term is positive because $(n - E(k|n-1))\Pr(k \geq k^*) + E(n-k|k < k^*) > 0$. The slope at $m_n = 0$ is negative because the second term is negative (see proof in Appendix E). The intercept (the last term) is positive for all $n > 1$. Thus, $F(m_n)$ is convex and there are two positive roots $(\lambda_1, \lambda_2)$ as shown in Figure 1. Incentive compatibility with efficient audit requires the lowest $m$ ensuring that $F(m_n)$ is non-positive; hence, the lower root of the quadratic gives the required audit probability.

[Insert Figure 1 about here]

The root must be less than unity since if $m_n = 1$, the IC constraint becomes $G_T^e - P \geq 0$. The nonmarginal group has $G_T^e > P$, so the IC constraint is slack and reducing $m_n$ marginally still satisfies IC but raises group surplus with truth-telling. Hence, any $m_n$ for a nonmarginal group must be in $(0,1)$.

## 2.4 Group lending vs individual lending

Appendix F derives the difference in interest rates between individual and group loans and shows that

$$R_I - R_G = \frac{nm_I(H-L)+nL}{nB} + \frac{E[G(k|k<k^*,n)]-(1+r)nB}{\Pr(k \geq k^*,n)nB} \tag{13}$$

where $R_I$ is the individual loan's interest rate and $R_G$ is the group loan's interest rate for a group size $n$. The first term in Eq. (13) is the lender's expected gain from loan auditing relative to the total size of $n$ individual loans assuming that all projects reporting fail. When $k < k^*$, the group revenue is less than the repayment required which is set to break even with the lender's investment cost i.e. $E[G(k|k < k^*, n)] < (1 + r)nB$; therefore, the second term is negative. The numerator of the second term reflects the lender's expected maximum loss from group's default. As the size of the second term increases, the gap between $R_I$ and $R_G$ becomes smaller. If group lending has a ratio of the expected maximum loss to non-default probability higher than the lender's expected gain from auditing $n$ individual loans, then $R_G$ can be higher than $R_I$. That is, whether $R_G$ is less or greater than $R_I$ depends on various factors such as the distribution of $p_i$, the group size $n$, the loan size $B$ and the values of $H - L$, the safe rate $r$, the group audit cost $c$, and optimal audit probabilities for both individual and group lending.

The ranking of the welfare (expected surplus per borrower) of individual and group loans is identical to that by the expected audit cost:

$$\begin{aligned} ES_G - nES_I &= E(k|n)(H-L) - mc(n - E(k|n)) + nL - n\bar{p}H - n(L - m_I C)(1-\bar{p}) \\ &= (n - n\bar{p})(m_I C - mc) \end{aligned}$$

where $m$ is the audit probability varying with whether the group is unprofitable, marginal or non-marginal. Because $n > n\bar{p}$, whether the group loan's expected surplus is greater than the individual loan's depends on the difference between the expected audit cost per borrower of both loan forms. Thus, the relative audit probabilities of an individual loan and a group loan play a large role in their relative efficiency.

**Proposition 1** *In an unprofitable group, $m$ must be equal to 1. In this case, the borrower's surplus* *would be greater in a group loan if*

$$c < \frac{(1+r)B - L}{\bar{p}(H-L) - C(1-\bar{p})} C$$

**Proposition 2** *For a marginal group, $m_m > m_I$ iff*

$$\frac{\bar{p}H + (1-\bar{p})L - (1+r)B - C(1-\bar{p})}{(1+r)B - L} > \frac{(G_T^e - P)_{m=1}/n}{(H-L) - (G_T^e - P)_{m=0}/n} \tag{14}$$

*The proof is in Appendix D.*

The left-hand side of Eq. (14) in Proposition 2 is the profit rate on an individual loan with truth-telling and $m_I = 1$. Its numerator is the expected social revenue net of the audit cost and its denominator is the social cost of the project, so the ratio measures the social profit rate on an individual loan. While $m = 1$ ensures truth-telling, borrowers are tempted to cheat if $m = 0$. Thus, the ratio on the right-hand side of Eq. (14) is the borrower's gain from truth-telling with $m = 1$ relative to the net gain from cheating with $m = 0$. If the net gain from cheating is large enough to make the condition in Eq. (14) hold, $m_m$ will need to be sufficiently high to deter cheating.

**Proposition 3** *For a nonmarginal group, $m_n > m_I$ iff*

$$c < \frac{(n-1-nm_I)(H-L)}{m_I} - (G_c^e - P)_{m=m_I},$$

*equivalent to*

$$(1-m_I)\left[(H-L) + \frac{(G_c^e - P)_{m=m_I}}{n}\right] > \frac{(G_T^e - P)_{m=m_I}}{n} \tag{15}$$

*where at the individual loan optimum*

$$m_I = \frac{(1+r)B - L}{p(H-L) - C(1-p)}$$

*See the detail in Appendix E.*

Proposition 3 shows that if the group applies $m = m_I$ and the gain from truth-telling is lower than that from non-audited cheating, more cheats are encouraged until all successes cheat as they know that the group will not default no matter whether they tell the truth or lie. This indicates that the audit probability applied is too low. To avoid this, the group should raise the audit probability, resulting in $m_n > m_I$.

# 3 Simulation

The above comparisons of the features of optimal individual and group loans are implicit. In order to identify the loan form with the highest welfare, we choose a parametric form for the distribution of $p_i$ which allows for a wide range of the positive correlation of risks between borrowers, its mean and its variance. In Section 2, the distribution of $k$ was in a general form $g(k, p_1..p_n) = h(k|p_1..p_n)f(p_1..p_n)$. $h(k|p_1..p_n)$ is a binomial distribution, but the restriction we add is that $f(p_1..p_n)$ is the product of beta distributions. That is, the probability of $(p_1..p_n)$ is the probability of a sample of size $n$ drawn from a beta distribution with positive parameters $\alpha$ and $\beta$, i.e., $B(\alpha, \beta)$ where

$$B(\alpha,\beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)} = \frac{(\alpha-1)!(\beta-1)!}{(\alpha+\beta-1)!}$$

As the parameters $\alpha$ and $\beta$ vary, the values of mean, variance and correlation between the chance of success for different borrowers differ. With positive $\alpha$ and $\beta$, the correlation must always be positive. Keeping $\bar{p}$ constant, the lower are both $\alpha$ and $\beta$, the higher are the correlation and the variance of risks (see Figure 2).

[Insert Figure 2 about here]

Each borrower receives a draw of a chance of success $p_i$ from the beta distribution; given this, the actual number of successes then follows a binomial distribution $h(k|p_1..p_n)$[15]. This is called the beta binomial[16] which can cover a variety of skewness situations and degrees of correlation between project outcomes. Here, the number of successes coming from the sample drawn for $n$ borrowers has density

$$g(k|p_1..p_n) = h(k|p_1..p_n)B(\alpha,\beta) \tag{16}$$

The mean number of successes of the beta binomial-distributed probability is $n\alpha/(\alpha+\beta)$.

[15] Here, $X_i$ is the outcome for the $i^{th}$ borrower ($X_i = 0, 1$ with 1 being success), the conditional distribution $X_i|p_i$ is Bernouilli($p_i$) and the marginal distribution of $p_i$ is Beta($\alpha, \beta$). Thus the joint probability of $(X_i, p_i)$ =Bernouilli($p_i$)Beta($\alpha, \beta$). Recall that the Bernoulli($p$) density is $p^k(1-p)^{1-k}, k = 0, 1$; Bernouilli($p$) has mean $p$ and variance $p(1-p)$. The mean of the beta-distributed probability $\bar{p}$ is $\alpha/(\alpha+\beta)$ and its variance is $\rho\bar{p}(1-\bar{p}) = \alpha\beta/[(\alpha+\beta)^2(1+\alpha+\beta)]$ (Moraux, 2010). The correlation between the binary outcomes across any two individual projects is $\rho = 1/(1+\alpha+\beta)$.

[16] The correlated binomial distribution can be viewed as a special case of heterogeneous distribution where risks are heterogeneous but correlated.

For the simulations, we take a range of values of $\alpha$ and $\beta$ yielding combinations of the probability of success $\bar{p}$ and correlations $\rho$ (e.g., $\alpha = 0.132, \beta = 0.198$ yields $\bar{p} = 0.4$ and $\rho = 0.75$, and $\alpha = 0.5, \beta = 0.5$ yields $\bar{p} = 0.5$ and $\rho = 0.5$). We also provide the simulation result for the case of zero correlation which is equivalent to the binomial case with identical and independent risks (iid). For the individual loan, the mean probability of success from the binomial and betabinomial are identical. In each case, we take $H = 10, B = 4, r = 0.1$ and vary the combinations of group size ($n$), the revenue of a failed project or "collateral" value ($L = 2, 3$), audit cost for individual loans ($C = 0.1, 0.2$) and audit cost for group loans ($c = 0.1, 0.2$) to simulate the impact of the variation of $H - L$ (gain from cheating), audit costs ($c, C$) and group size on the interest rate, default rate, welfare and thus the optimal lending form (individual loans correspond to $n = 1$).

The optimal $m, P$ and $k^*$ are obtained for each scenario. These variables depend on each other as well as the parameters set above; $k^*$ determines whether the group is unprofitable, marginal or nonmarginal. The simulation results in Figures 3-7 show six selected cases[17]:

(i) $L = 2, c = 0.1, C = 0.2, \rho = 0, \bar{p} = 0.5$ (our benchmark case);

(ii) $L = 2, c = 0.1, C = 0.2, \rho = 0.5, \bar{p} = 0.5$ (higher correlation compared to the benchmark case);

(iii) $L = 2, c = 0.1, C = 0.2, \rho = 0, \bar{p} = 0.4$ (lower mean probability of success compared to the benchmark case);

(iv) $L = 2, c = 0.1, C = 0.2, \rho = 0.75, \bar{p} = 0.4$ (higher correlation and lower mean probability of success compared to the benchmark case);

(v) $L = 2, c = C = 0.2; \rho = 0.5, \bar{p} = 0.5$ (no cost advantage between borrowers' audit and lenders' audit);

(vi) $L = 3, c = 0.1, C = 0.2, \rho = 0, \bar{p} = 0.4$ (higher expected project return[18] compared to the benchmark case);

For each combination of parameters, we contrast the optimal outcomes for the group loan ($n > 1$) with those of an individual loan ($n = 1$).

[17] The full simulation results are available from the authors upon request.
[18] It is also equivalent to a case with higher value of the collateral.

## 3.1 Interest and default rates

Figure 3 illustrates that the individual loan's interest rate ($R_I$) is higher than the group loan's interest rates for all group sizes ($R_G$) - that is, effectively repayment per borrower is lower with a group loan. The exceptions to this generalization is case (iv) where risk is high and highly correlated (e.g., $\bar{p} = 0.4$ and $\rho = 0.75$); the interest rate for the group with two borrowers (115.37%) is higher than that for the individual loan (105.84%). It also shows that the interest rate generally declines as $n$ increases, especially when the outcomes of projects are iid. Consequently, the default rate shown in Figure 4 declines but in an up-and-down pattern[19] as the group becomes larger, consistent with Baland et al. (2013). We, however, note that with iid risk the decrease in interest rates becomes insignificant once the group has more than five members.

For a given mean, higher correlation raises the risk for group loans and hence the interest rate and repayment required. This requires a higher number of successes for the group to be solvent (compare cases (iii) and (iv) with $\bar{p} = 0.4$ in Figure 5). As a result, the expected default rate is higher (compare cases (iii) and (iv) in Figure 4). For case (iv) where $\rho = 0.75, p = 0.4$, the individual loan's default rate is lower than that of the group loan[20]. In this case, the interest rate for a 2-borrower group loan is higher than that for an individual loan, causing the default rate in the group loan to be higher than the individual loan. Although the interest rate declines for a 3-borrower group, the default rate does not fall; the correlation raises the chance of a large number of simultaneous fails requiring high $k^*$ for the group to be solvent (see Figure 5). For the correlated groups with more than three borrowers, neither the interest rate nor the default rate drops significantly as the group becomes larger.

[Insert Figures 3-5 about here]

Figures 3 and 4 also highlight that not only do the interest rate and default rate fall with an increase in $n$ in iid cases, but they also fall with an increase in $L$ and $\bar{p}$ (compare (iii) with (vi) for $L$

[19] The graph shows a downward trend, but with an up-and-down pattern which is due to the discrete value of $k$.

[20] Baland et al. (2013) also documented that with bank and social sanctions, MFIs' lending may shift toward individual loan when there is a risk of strategic default by risk-neutral borrowers.

and compare (i) and (iii) for $\bar{p}$). This finding is consistent with the evidence of negative relationships between collateral and the interest rate in the literature (see, for example, Agarwal and Hauswald, 2010; Menkhoff, Neuberger and Rungruxsirivorn, 2012) and between collateral and default risk (see, for example, Jiménez, Salas and Saurina, 2006). Moreover, comparing cases (ii) and (v) in Figure 3 highlights that the group interest rate rises with an increase in the audit cost. When the group's audit cost ($c$) increases from 0.1 to 0.2, the interest rate shifts up.

## 3.2 The relative social merits of individual and group lending

As shown in Figure 5, the groups can be classified into three types: (i) an unprofitable group if $k^* > E(k|n-1)+1$, e.g., any $n$ with $\rho = 0.75, \bar{p} = 0.4$; (ii) a marginal group if $E(k|n-1) < k^* \leq E(k|n-1)+1$, e.g., any $n$ with $\rho = 0.5, \bar{p} = 0.5$ or $\rho = 0, \bar{p} = 0.4$ and (iii) a non-marginal group if $k^* \leq E(k|n-1)$, e.g., $n > 4$ with $\rho = 0, \bar{p} = 0.5$. In accordance with the type of group, Figure 6 plots the values of $m$ for $1 \leq n \leq 10$. The group should apply $m = 1$ if the mean and correlation of risk is high (the group is unprofitable). In other cases, optimally the value of $m$ is lower than 1. The higher the value of $\bar{p}$, the lower the value of $m$. In addition, the lender or borrowers can apply a lower $m$ as the collateral value ($L$) increases.

Given a group size $n$, nonmarginal groups can apply a lower $m$ compared to their marginal counterparts. With the same value of $c$ but lower $m$, the expected audit cost is lower and the expected surplus is higher in nonmarginal groups. So, for a fixed group size, the borrower would prefer to be in a group whose optimal policy makes it more profitable.

In contrast, the audit probability is higher in larger groups. As the group size increases, the interest rate and the default rate for group loans are typically lower and thus the excess number of successes ($n - k^*$) is higher. For a given case as $n$ increases, the gain from undetected cheating tends to increase due to the increase in the excess number of successes which allows the undetected cheat a share of the group surplus; to discourage cheating requires a sufficiently high probability that a fail will be audited. Similarly, group loans require a higher audit probability than individual loans as shown in Figure 6.

[Insert Figure 6 about here]

[Insert Figure 7 about here]

The expected audit cost in Figure 6 shows two main features of interest. Firstly, it is generally highest for individual loans largely due to the audit cost advantage of the group. Secondly, for group loans the 2-member group has the lowest expected audit cost. The audit cost in a group loan rises with group size both because the number of reported fails rises and because $m$ rises. Overall on balance when $c < C$, a 2-member group has the lowest expected audit cost and hence the highest expected surplus per borrower (see Figure 7). When the cost of audit by borrowers is as large as the cost of audit by the lender, the expected audit cost of an individual loan is below the per capita audit cost of a group loan (see case (v) with $c = C = 0.2$). In other words, the advantage of group lending over individual lending disappears if the audit costs per failed borrower are identical in the two loan forms.

The results indicate that, choosing $m$ optimally, a two-person group loan dominates individual loans exactly because of the lower policing cost of the group loan, consistent with the finding of Cason, Gangadharan and Maitra (2012)[21]. But if the audit costs are identical, then individual loans dominate group loans in welfare terms. Other indications are that the expected surplus per borrower rises with $\bar{p}$ and $L$ but falls slightly as the group becomes larger and the borrowers' outcomes have higher correlation.

To sum up, we know that, with risk neutrality and an audit cost advantage for the group, group loans dominate individual loans in terms of interest and default rates and indeed welfare (expected surplus per borrower). On interest and default rates, the only exception is when the group has two members with high individual risk which is also highly correlated among borrowers. With iid risk, the group loan with joint liability is often seen as having better risk diversification possibilities than individual loans because it has possibilities of cross subsidisation within the group, and largely because the chance of a high number of simultaneous fails and group default is low. This diversification gain

[21] With the subgame perfect equilibrium approach, Cason et al. (2012) reports their experimental finding that when peer-monitoring cost is lower than the monitoring cost of the lender, group lending has higher monitoring and lower default rate than individual lending.

should increase with the size of the group. However, the advantage can be dissipated since (i) with correlated risks, the chance of simultaneous failures is greater than independent risks and (ii) with asymmetric information and the need for costly audit, fail reports within the group must be audited.

We find that the benefit of a larger group loan from decreasing interest rates becomes insignificant if the group has more than five members with iid risk or three members with correlated risk. But in terms of expected surplus, the advantage of the group loan rests strongly on the group having a cost advantage in audit. Amongst different size groups, small groups ($n = 2$) are welfare preferable but the difference between $n = 2$ and $n = 10$ in welfare is small. With a slight fall in expected surplus per borrower as $n$ increases, the group's lower interest rate and default rate may be taken into consideration and the group could choose $2 \leq n \leq 5$ (except for the group which has low mean probability of success and outcomes are highly correlated). This finding supports the empirical evidence and arguments provided by the existing literature. For example, Devereux and Fishe (1993) find that a small group size is important in determining group loan success and a common group size is 3-5 members. The heuristic argument of Abbink, Irlenbusch and Renner (2006) is that 3 is sufficient to get reasonable risk diversification and 5 is an upper bound set by the requirement for high solidarity in the borrower group to police repayment by individuals. Therefore, on all these counts group loans to small groups tend to be preferred to individual loans unless there is high correlation and low $\overline{p}$.

If group lending does not have a cost advantage in auditing ($C = c$), group lending still has lower interest and default rates, but individual loans dominate group loans on welfare grounds. This may explain why individual loans are more common in urban areas where there is less information asymmetry (e.g., commercial banks have better information system about borrowers living nearby) and why group loans through cooperatives or self-help groups are more common in rural areas where transaction and information costs are higher for urban banks.

# 4 Extensions

Two common problems in the literature on costly state verification are handling the commitment and collusion problems. We largely sidestep these issues by assuming that audit and its outcome are public knowledge to the lender. This is consistent with empirical practice, e.g., bankruptcy law, filing of audited company accounts, etc.

With regard to the usual commitment problem, if an exante contract is incentive compatible with truth-telling ensured by an exante contracted audit probability, then auditors know that expost reports are truthful and hence a group or individual borrower which reports a fail must truly have failed. Hence, the auditor has no incentive to carry out the contracted costly audit but can save $cm$ by just not auditing.

In the literature, this is addressed either by adding a renegotiation constraint to the exante contract which ensures that the expost best action is for the auditor to carry out the contracted audit policy (see, e.g., Laffont, 2003; Gagnepain, Ivaldi and Martimort, 2013; Ali, Miller and Yang, 2016) or by abandoning the search for incentive compatibility and allowing the reporting and audit strategy to be determined expost in a noncooperative game between the auditor and the reporting borrowers. Typically, the latter involves a Nash equilibrium in the audit probability and the probability that the reporters cheat in their report (e.g., Khalil and Parigi, 1998; Krasa and Villamil, 2000; Phelan, 2017). In this context, the lack of commitment costs some loss of welfare; ultimately, it could bring borrowers and lenders to an impasse (i.e., borrowers know that they will actually never face audit and so they always cheat) and prevent any contract being signed.

In Gagnepain et al. (2013) and Ali et al. (2016), there is a long term contract covering several periods of project returns and repayments, uninformed parties gain information over time from the behaviour of informed parties and may wish to renegotiate the contractual arrangement in light of this. Renegotiation proofness nullifies this incentive. In contrast, we work with a single period model. The auditor initially is contracted to audit all fail reports at least stochastically. In order to ensure that the contracted audit is carried out expost, the renegotiation proof constraint in our context for an individual loan must give the auditor some gain from audit:

$$(1-p)(Y-Cm) \geq 0$$

i.e., there is a fail report wp $1-p$; auditing this generates $Y$ additional income for the auditor(s) but costs at most $Cm$. So long as the above condition holds, the auditor(s) gains from carrying out the audit. With a group loan and group audit, the two group auditors know that their audit costs will be reimbursed. As the audit result is common knowledge, the lender would know if some failed reports are not audited and thus no additional renegotiation constraint is necessary. In an extreme case, the group itself could expost not pay the auditors and collectively decide to report a group failure, but again audits and their outcomes are public to the lender; therefore, it is not a viable strategy for the group to falsely report group failure.

Collusion can only arise with a group loan and group audit. Collusion could conceivably be between the first auditing borrower and a non-audit borrower. For example, a successful borrower could pay part of his gain $H-L$ to the auditor in exchange for the auditor approving a report that he has failed. The auditor will only accept this if, given all the borrower reports, the auditor believes that the group is solvent even if this one borrower is allowed to cheat. However, even if this is so, the auditor will not accept the deal if the bribe is less than his loss of the group surplus share. Since all borrowers are in a symmetric position, all successes would offer this deal to the auditing borrower, but then for sure the auditor would lose surplus share.

The other possibility is that the two auditors could colllude if they are both successful. The two auditors could agree to allow both to report that the other has failed without any audit. However, this will be revealed to the lender who will see that two failed reports have not been audited.

# 5 Conclusion

We fill the gap in the economic literature by deriving the optimal audit probabilities, which minimize the audit cost while maximizing social welfare, for both one-to-one and multiple auditor-auditee settings. The information about individual project outcomes is asymmetric between auditor(s) and auditee(s) and individual outcomes can be correlated. Our framework is developed within a loan auditing context which has rarely been deeply studied, but the method is also applicable to any accounting and tax audit context.

Our framework contains some conceptual innovations. With asymmetric information and costly audit possibilities, incentive compatibility comes into play. The nature of the incentive compatibility restrictions varies across loans. There are alternative ways to achieve truth-telling; the incentive compatibility requirement makes the audit probability endogenous as part of the contract problem which depends on various factors such as the number of auditee(s), the risk level and the distribution of risk. Hence, the optimal audit probability of reported fails varies between groups of borrowers and between individual and group loans. We characterize the optimal contract forms in terms of exante welfare. The key contract variables are the interest rate, the probability of default and the audit probability. We derive some theoretical comparisons of these in different loan situations.

Our general framework also deals with one important feature of most group loan settings which are geographically concentrated, creating correlation between different borrower risks. While prior studies tend to take a specific value of correlation (e.g., 0.5) in their analysis, our framework allows any degrees of correlation between borrowers' risk, consistent with empirical studies using the proximity between borrowers to reflect the correlation of risk (e.g., Goodstein et al., 2017). To identify the best form of loan and the best size of group, we conduct some numerical simulations using a beta binomial distribution of individual project risks. This allows for varying degrees of positive correlation between individual borrower risks. Here, we find that the audit cost advantage, the degree of correlation and the chance of success on individual projects all play important roles in determining the best incentive compatible contract, the best type of loan and the optimal group size. Usually, small group loans (i.e., 2 members in the group) are the best form for welfare so long as the group has an audit cost advantage over the external lender. A group size of up to 5 members allows the benefits of lower interest and default rates. Our results are consistent with the finding of Bourjade and Schindele (2012) that the optimal group size is limited, and support the empirical evidence that group-lending institutions prefer providing loans to small groups (see, e.g., Devereux and Fishe, 1993; Abbink et al. 2006; Assadi and Ashta, 2014).

In common with much of the literature (e.g., Stiglitz, 1990; Banerjee et al., 1994; Laffont, 2003; Laffont and Rey, 2003; Ahlin and Townsend, 2007), we have used a simple setting of two state outcomes for each individual. Exceptions are Besley and Coate (1995), Border and Sobel (1987) and Mookherjee and Png (1989). We could extend the model to an arbitrary finite number of states. This would of course expand the number of incentive compatibility constraints and the ideas of unprofitable, marginal and non-marginal groups. With iid risks, a single continuous distribution could be used for each project. The mixture distribution idea underlying the beta-binomial distribution could also still be used, leading to a compound distribution of revenues. Again, there would be implications for analysing incentive compatibility.

We have restricted attention to ensuring good behaviour through static financial penalties on cheats. In the literature on group lending especially Karlan (2007), strong emphasis is put on the role of the group of borrowers in enforcing truthful revelation and repayment through peer pressure. A common device is to motivate self-policing of the group by refusing future loans to the group if any group member defaults (Sinn, 2013). This would require a repeated loans context. However, abstracting from these highlights the relationship between endogenous audit probability, default risk, interest rate and welfare for different group sizes and probability distributions of project returns.

We have also assumed risk neutrality of all parties to the contract which has been the most widely used assumption in the literature (see, e.g., Stiglitz, 1990; Banerjee et al.1994; Laffont 2003; Laffont and Rey, 2003; Ahlin and Townsend 2007; Sinn 2013; De Quidt et al., 2016). Further studies may look at risk aversion especially of borrowers which could add some dimensions.

Our work has some policy implications. Lenders may apply our approach to determine the right policy of auditing and the interest rate which allows them to achieve cost efficiency and a low default rate but still induces truth-telling by borrowers and leaves borrowers with the highest surplus. The adoption of excessive auditing is costly to both auditors and auditees and require a high interest rate.

On the other hand, insufficient auditing can induce cheating or, in some extreme cases, loan default. While individual and group lending can coexist, choosing the loan form appropriate for a particular economic condition as well as well-designed terms of the loan contract can reduce the probability of non-repayment and for the economy it can reduce the overall level of non-performing loans while increasing social welfare. Policymakers may impose some legislation on information provided by both lenders and borrowers or provide repayment guarantees or insurance for small, risky projects (raising $L$ in our framework); this will lower audit and monitoring costs as well as the interest rate to both individual and group loans and thus raise social welfare. In an economy where group lending is the best loan form, policymakers may provide a platform for borrowers from different areas to form groups. This will promote group lending with low risk correlation between projects and thus raise social welfare; in particular, lenders benefit from risk diversification and borrowers benefit from lower interest and default rates.

Finally, although falling outside the scope of this paper, our approach gives insight into generalising the study of other audit contexts and one would expect some of the forces we identify to generate similar findings. While here we can identify the best form of loan and the best size of group for a particular distribution of risk the borrowers face, a similar approach may be used to compare one-to-one and multiple auditor-auditee settings and determine the optimal number of auditees in the latter. The approach of determining the efficient, incentive compatible audit probability in other contexts such as component audit with correlated risk should be possible. With a higher degree of correlation between divisions' outcomes, the risk diversification benefit is diminished so more intensive random audit or peer review may be required. In practice, we would expect more efficient audit with lower audit cost and better (overall) firm performance through identifying the correlation between component successes, randomising the audit on components and adopting the audit policy that leads to the highest welfare of the organisations.

## Appendix A: Audit by the lender is more costly

Suppose that audit is done by the lender instead of the two assigned borrowers and that the same audit probability ($m$), repayment ($P$) and audit cost per audited loan ($C$) are imposed, there will be increments in the share of expected group surplus (due to the lender paying the audit cost) but the increment from cheating is higher than from truth-telling since with cheating there is an extra audit to do. More cheating will be induced if $m$ is the same as when the audit is done by the borrowers. To avoid this situation, the lender has to raise the audit probability ($m$) resulting in expected audit cost ($Cm$) which, from the lender's constraint, will require higher $P$ from borrowers to cover the incremental audit cost.

Proof: With the group loan's audit by two borrowers, the lender's constraint is

$$\Pr(k \geq k^*, n)P + E(k|k < k^*, n)(H - L + cm) + n(L - cm)\Pr(k < k^*, n) \geq (1 + r)nB \tag{17}$$

where $k^*$ is the minimum required number of successes which ensures the group has enough revenue to repay the loan of $nB$. With the group loan's audit by the lender, the lender's constraint becomes

$$\Pr(k \geq k^*, n)P + E(k|k < k^*, n)(H - L) + n(L)\Pr(k < k^*, n) \geq (1 + r)nB + E(Cm(n - k)|k < k^*, n)$$

which can be rewritten as

$$\Pr(k \geq k^*, n)P + E(k|k < k^*, n)(H - L + Cm) + n(L - Cm)\Pr(k < k^*, n) \geq (1 + r)nB \tag{18}$$

Eq. (18) is identical to the constraint in Eq. (17) if audit is done by the borrowers with the group loan's audit cost per borrower equal to $C$. With loans being audited by the lender, the expected group revenue if the successful borrower tells the truth $G^e_{TL}$ is

$$G^e_{TL} = G^e_T + cm(n - 1 - E(k|n - 1))$$

where $G^e_T$ is the expected group revenue after auditing when the audit is done by two borrowers (so the audit cost here is $c$) and the successful borrower tells truth. Likewise, the expected group revenue, if the successful borrower cheats, is

$$G^e_{cL} = G^e_c + cm(n - E(k|n - 1))$$

where $G_c^e$ is the expected group revenue after auditing when the audit is done by two borrowers and the successful borrower cheats. That is, the group saves an audit cost per audited loan ($c$) if the audit is conducted by the lender and the total amount of saving in the group with a borrower cheating is greater than without this cheating. Each successful borrower has more incentive to cheat; this can result in the group defaulting and the lender receiving just the low return $nL$ (as reported $k$ will tend to 0). To avoid this, the lender has to raise the audit probability to $\acute{m}$ - that is, the expected cost per audit in the lender's constraint increases to $C\acute{m}$, requiring a higher $P$ to cover a higher cost. As the group of borrowers will be required to repay more with the audit by the lender, the required number of successes and the expected default rate increase resulting in a lower share of surplus if the group does not default or a higher chance the borrower gains nothing because of the default of his group. Thus, the borrowers will be worse off if the audit is undertaken by the lender.

## Appendix B: Group lending is feasible

Under our assumptions about project returns and cost, there is always a unique smallest $k^*$[22]. With zero successes, the group cannot afford to repay and hence must default. If all group members succeed, then group revenue is $G(n|n) = nH > (1+r)nB$ and the group can certainly afford to repay. Group revenue is increasing in the number of successes; hence, there must be a smallest critical number of successes $k^*$ above which the group can repay and below which the group defaults. This just requires $H > (1+r)B + C > L$. To see this formally, the lender at least breaks even on the group loan in expected terms

$$(1+r)nB \leq \Pr(k \geq k^*, n)P + E(k|k < k^*, n)(H - L + cm) + n(L - cm)\Pr(k < k^*, n)$$

and $P \leq G(k^*)$ where $G(k^*) = k^*(H - L + cm) + n(L - cm)$

$$\begin{aligned}(1+r)nB &\leq \Pr(k \geq k^*, n)[(H - L + cm)k^* + n(L - cm)] + E((H - L + cm)k + n(L - cm)|k < k^*, n) \\ &\leq n(L - cm) + (H - L + cm)[\Pr(k \geq k^*, n)k^* + E(k|k < k^*, n)] = F(k^*) \qquad (19)\end{aligned}$$

[22] In some cases, $k^*$ may not exist - that is, even with $n$ successes, there may not be enough group revenue.

$F(k^*) \leq E(G(k|n))$ because $E(G(k|n)) = n(L - cm) + (H - L + cm)E(k|n)$ and

$$[\Pr(k \geq k^*, n)k^* + E(k|k < k^*, n)] < E(k|n)$$

If $k^* = 0, F(k^*) = n(L - cm) < (1 + r)nB$, violating the condition in Eq. (19); hence $k^* > 0$. If $k^* = n$,

$$\begin{aligned} F(k^*) &= n(L - cm) + (H - L + cm)[\Pr(n)n + E(k|k < n, n)] \\ &= n(L - cm) + (H - L + cm)E(k|n) = E(G(k|n)) > (1 + r)nB. \end{aligned}$$

Therefore, $F(k^*)$ is increasing in $k^*$. Applying the maximum audit cost ($m = 1$) yields the social desirability condition. As a result, so long as the social desirability condition holds, there is a unique minimal $k^*$.

## Appendix C: Audit probability in a marginal group must be positive

If $m_m = 0$, the successful borrower could cheat; if he does, he expects the group to default and his expected total gain is $H - L$. If he tells the truth, he gets an equal share of the group surplus with $1 + E(k|n - 1)$ successes:

$$\frac{(1 + E(k|n - 1))}{n}(H - L) + \frac{E(k|k < k^*, n)](H - L)}{n \Pr(k \geq k^*)} + \frac{L - (1 + r)B}{\Pr(k \geq k^*)}$$

since

$$G_T^e - P = \frac{[\Pr(k \geq k^*)(E(k|n - 1) + 1) + E(k|k < k^*, n)](H - L + cm_m)}{\Pr(k \geq k^*)} + \frac{n(L - cm_m - (1 + r)B)}{\Pr(k \geq k^*)}$$

and

$$(G_T^e - P)_{m=0} = \frac{[\Pr(k \geq k^*)(E(k|n - 1) + 1) + E(k|k < k^*, n)](H - L) + n(L - (1 + r)B)}{\Pr(k \geq k^*)}$$

His gain from truth-telling as compared with cheating at $m_m = 0$ is

$$\begin{aligned}
\frac{(G_T^e - P)_{m=0}}{n} - (H-L) &= \frac{[\Pr(k \geq k^*)(E(k|n-1)+1-n) + E(k|k<k^*,n)](H-L)}{n\Pr(k \geq k^*)} + \frac{L-(1+r)B}{\Pr(k \geq k^*)} \\
&= \frac{[\Pr(k \geq k^*)E(k|n-1) + E(k|k<k^*,n)](H-L) + n(L-(1+r)B)}{n\Pr(k \geq k^*)} \\
&\quad + \frac{1-n}{n}(H-L) \\
&= \frac{(G_c^e - P)_{m=0}}{n} - \frac{n-1}{n}(H-L) < 0
\end{aligned}$$

as $G_c^e < P$ for the marginal group. To prevent this, we need $m_m > 0$.

## Appendix D: The optimal audit probability for a marginal group loan

For a marginal group,

$$(1-m_m)[H-L] \leq \frac{(G_T - P)_{m=m_m}}{n}$$

We can then derive an explicit solution for $m_m$:

$$n(1-m_m)[H-L] = G_T - P \tag{20}$$

$$\begin{aligned}
n(1-m_m)[H-L] &= \frac{[\Pr(k \geq k^*)(E(k|n-1)+1) + E(k|k<k^*,n)](H-L+cm_m)}{\Pr(k \geq k^*)} \\
&\quad + \frac{n(L-cm_m) - (1+r)nB}{\Pr(k \geq k^*)}
\end{aligned}$$

$$\begin{aligned}
(1+r)nB - [\Pr(k &\geq k^*)(E(k|n-1)+1-n) + E(k|k<k^*,n)](H-L) - nL \\
&= m_m[\Pr(k \geq k^*)n(H-L) + c\,(\Pr(k \geq k^*)(E(k|n-1)+1) + E(k|k<k^*,n) - n)]
\end{aligned}$$

$$m_m = \frac{(1+r)nB - [\Pr(k \geq k^*)(E(k|n-1)+1-n) + E(k|k<k^*,n)](H-L) - nL}{\Pr(k \geq k^*)n(H-L) + c\,(\Pr(k \geq k^*)(E(k|n-1)+1) + E(k|k<k^*,n) - n)}$$

We rewrite $m_m$ as

$$\begin{aligned}
m_m &= \frac{(1+r)nB - [\Pr(k \geq k^*)(E(k|n-1)+1-n) + E(k|k<k^*,n)](H-L) - nL}{\Pr(k \geq k^*)n(H-L) + c\,(\Pr(k \geq k^*)(E(k|n-1)+1) + E(k|k<k^*,n) - n)} \\
&= \frac{(1+r)nB - nL + [n\Pr(k \geq k^*) - X](H-L)}{n\Pr(k \geq k^*)(H-L) - c[n-X]}
\end{aligned}$$

where

$$X = E(k|k < k^*, n) + \Pr(k \geq k^*)(E(k|n-1) + 1)$$

At $m = 0$,

$$(G_T^e - P)_{m=0} = \frac{X(H-L)}{\Pr(k \geq k^*, n)} - \frac{(1+r)nB - nL}{\Pr(k \geq k^*, n)}$$

and at $m = 1$,

$$\begin{aligned}
(G_T^e - P)_{m=1} &= \frac{X(H-L)}{\Pr(k \geq k^*, n)} - \frac{(1+r)nB - nL}{\Pr(k \geq k^*, n)} - \frac{c(n-X)}{\Pr(k \geq k^*, n)} \\
(G_T^e - P)_{m=1} &= (G_T^e - P)_{m=0} - \frac{c(n-X)}{\Pr(k \geq k^*, n)}
\end{aligned}$$

So

$$\begin{aligned}
m_m &= \frac{(1+r)nB - nL + n\Pr(k \geq k^*)(H-L) - X(H-L)}{n\Pr(k \geq k^*)(H-L) - c[n-X]} \\
&= \frac{\Pr(k \geq k^*)\left[n(H-L) - G_T^e - P)_{m=0}\right]}{\Pr(k \geq k^*)\left[n(H-L) + (G_T^e - P)_{m=1} - (G_T^e - P)_{m=0}\right]} \\
&= \frac{(G_T^e - P)_{m=0} - n(H-L)}{(G_T^e - P)_{m=0} - n(H-L) - (G_T^e - P)_{m=1}} \\
&= \frac{1}{1 - \frac{(G_T^e - P)_{m=1}}{(G_T^e - P)_{m=0} - n(H-L)}}
\end{aligned}$$

Compare $m_m$ with $m_I$:

$$m_I = \frac{(1+r)B - L}{\bar{p}(H-L) - C(1-\bar{p})},$$

$m_m$ would be greater than $m_I$ if

$$\begin{aligned}
\frac{1}{1 - \frac{(G_T^e - P)_{m=1}}{(G_T^e - P)_{m=0} - n(H-L)}} &> \frac{(1+r)B - L}{\bar{p}(H-L) - C(1-\bar{p})} \\
\bar{p}(H-L) - C(1-\bar{p}) &> ((1+r)B - L)\left(1 - \frac{(G_T^e - P)_{m=1}}{(G_T^e - P)_{m=0} - n(H-L)}\right) \\
\bar{p}H + (1-\bar{p})L - C(1-\bar{p}) &> (1+r)B - \frac{(G_T^e - P)_{m=1}}{(G_T^e - P)_{m=0} - n(H-L)}((1+r)B - L) \\
\frac{\bar{p}H + (1-\bar{p})L - (1+r)B - C(1-\bar{p})}{(1+r)B - L} &> \frac{(G_T^e - P)_{m=1}/n}{(H-L) - (G_T^e - P)_{m=0}/n} \qquad (21)
\end{aligned}$$

where

$$(G_T^e - P)_{m=1} = (G_c^e - P)_{m=0} + (H-L) - \frac{c(n - \Pr(k \geq k^*)(E(k|n-1)+1) - E(k|k < k^*, n))}{\Pr(k \geq k^*, n)}$$

Note that, from social desirability, $0 > L-(1+r)B > C(1-\bar{p})-\bar{p}(H-L)$ and $\bar{p}(H-L)-C(1-\bar{p}) > 0$.

The sign of the left-hand side of Eq. (14) depends on the social desirability condition. The numerator of the right-hand side is the borrower's share of surplus and the denominator is the difference between the borrower's gain from cheating and his share of surplus if telling the truth.

## Appendix E: The optimal audit probability for a non-marginal group loan

For a nonmarginal group,

$$(1-m_n)[H-L+\frac{(G_c-P)_{m=m_n}}{n}] \leq \frac{(G_T-P)_{m=m_n}}{n}$$

$m_n$ solves

$$\begin{aligned}(1-m_n)[H-L+\frac{(G_c-P)_{m=m_n}}{n}] &= \frac{(G_T-P)_{m=m_n}}{n}\\ (1-m_n)(H-L)-\frac{(G_T-G_c)_{m=m_n}}{n}-m_n\frac{(G_c-P)_{m=m_n}}{n} &= 0\end{aligned}$$

$G_T-G_c=H-L+cm$ for all $m$, so

$$\begin{aligned}(1-m_n)(H-L)-\frac{H-L+cm_n}{n}-m_n\frac{(G_c-P)_{m=m_n}}{n} &= 0\\ (1-m_n-\frac{1}{n})(H-L)-\frac{cm_n}{n}-m_n\frac{(G_c-P)_{m=m_n}}{n} &= 0 \qquad (22)\end{aligned}$$

Substituting

$$(G_c-P)_{m=m_n}=[E(k|n-1)+\frac{E(k|k<k^*,n)}{\Pr(k\geq k^*,n)}](H-L+cm_n)+\frac{n(L-cm_n)}{\Pr(k\geq k^*)}-\frac{(1+r)nB}{\Pr(k\geq k^*,n)}$$

into the left-hand side of Eq. (22):

$$\begin{aligned}&(1-m_n-\frac{1}{n})(H-L)-\frac{cm_n}{n}-m_n\frac{(G_c-P)_{m=m_n}}{n}\\ =\;&(1-m_n-\frac{1}{n})(H-L)-\frac{cm_n}{n}\\ &-\frac{m_n}{n}\{[E(k|n-1)+\frac{E(k|k<k^*,n)}{\Pr(k\geq k^*,n)}](H-L+cm_n)+\frac{n(L-cm_n)}{\Pr(k\geq k^*)}-\frac{(1+r)nB}{\Pr(k\geq k^*,n)}\}\\ =\;&\left(1-m_n-\frac{1}{n}-\frac{m_n}{n}[E(k|n-1)+\frac{E(k|k<k^*,n)}{\Pr(k\geq k^*,n)}]\right)(H-L)-\frac{m_nL}{\Pr(k\geq k^*)}\\ &-\left(\frac{1}{n}+\frac{m_n}{n}[E(k|n-1)+\frac{E(k|k<k^*,n)}{\Pr(k\geq k^*,n)}-\frac{n}{\Pr(k\geq k^*)}]\right)cm_n+\frac{m_n(1+r)B}{\Pr(k\geq k^*,n)}\end{aligned}$$

Therefore, IC for a nonmarginal group requires

$$\begin{aligned}
F(m_n) &= \frac{(n - E(k|n-1)\Pr(k \geq k^*) - E(k|k < k^*))cm_n^2}{n\Pr(k \geq k^*)} \\
&\quad + \frac{(1+r)nB - nL - (H-L)((n + E(k|n-1))\Pr(k \geq k^*) + E(k|k < k^*)) - c\Pr(k \geq k^*)}{n\Pr(k \geq k^*)} m_n \\
&\quad + \frac{(H-L)(n-1)}{n} \\
&= \frac{c(n - X + \Pr(k \geq k^*))}{n\Pr(k \geq k^*)} m_n^2 - ((H-L) + \frac{(G_c^e - P)_{m=0}}{n} + \frac{c}{n})m_n + \frac{(H-L)(n-1)}{n} = 0
\end{aligned}$$

For any $m$,

$$n - X + \Pr(k \geq k^*) = n - E(k|n-1)\Pr(k \geq k^*) - E(k|k < k^*)$$

$$\begin{aligned}
G_c^e - P &= \frac{[\Pr(k \geq k^*)E(k|n-1) + E(k|k < k^*, n)](H - L + mc)}{\Pr(k \geq k^*, n)} \\
&\quad + \frac{n(L - mc) - (1+r)nB}{\Pr(k \geq k^*, n)} \\
&= \frac{(X - \Pr(k \geq k^*, n))(H - L + mc)}{\Pr(k \geq k^*, n)} + \frac{n(L - mc) - (1+r)nB}{\Pr(k \geq k^*, n)}
\end{aligned}$$

If $m = 1$,

$$\begin{aligned}
(G_c^e - P)_{m=1} &= \frac{(X - \Pr(k \geq k^*, n))(H-L)}{\Pr(k \geq k^*, n)} + \frac{nL - (1+r)nB}{\Pr(k \geq k^*, n)} - \frac{c(n - X + \Pr(k \geq k^*, n))}{\Pr(k \geq k^*, n)} \\
&= (G_c^e - P)_{m=0} - \frac{c(n - X + \Pr(k \geq k^*, n))}{\Pr(k \geq k^*, n)}
\end{aligned}$$

Substituting

$$\frac{c(n - X + \Pr(k \geq k^*, n))}{\Pr(k \geq k^*, n)} = (G_c^e - P)_{m=0} - (G_c^e - P)_{m=1}$$

into $F(m_n)$ yields

$$F(m_n) = \frac{[(G_c^e - P)_{m=0} - (G_c^e - P)_{m=1}]}{n} m_n^2 - [\frac{(G_c^e - P)_{m=0} + n(H-L) + c}{n}]m_n + \frac{(H-L)(n-1)}{n}$$

The intercept and the first term are positive while the second term is negative. Thus,

$$\begin{aligned}
m_n &= \frac{(G_c^e - P)_{m=0} + n(H-L) + c}{2\left[(G_c^e - P)_{m=0} - (G_c^e - P)_{m=1}\right]} \\
&\quad - \frac{\sqrt{[(G_c^e - P)_{m=0} + n(H-L) + c]^2 - 4(H-L)(n-1)\left[(G_c^e - P)_{m=0} - (G_c^e - P)_{m=1}\right]}}{2\left[(G_c^e - P)_{m=0} - (G_c^e - P)_{m=1}\right]}
\end{aligned}$$

If $m_n > m_I$,

$$m_I < \frac{(G_c^e - P)_{m=0} + n(H-L) + c}{2\left[(G_c^e - P)_{m=0} - (G_c^e - P)_{m=1}\right]} - \frac{\sqrt{[(G_c^e - P)_{m=0} + n(H-L) + c]^2 - 4(H-L)(n-1)\left[(G_c^e - P)_{m=0} - (G_c^e - P)_{m=1}\right]}}{2\left[(G_c^e - P)_{m=0} - (G_c^e - P)_{m=1}\right]} \tag{23}$$

Rearraging Eq. (23) yields

$$\begin{aligned} & (G_c^e - P)_{m=0} + n(H-L) + c - 2m_I\left[(G_c^e - P)_{m=0} - (G_c^e - P)_{m=1}\right] \\ > \; & \sqrt{[(G_c^e - P)_{m=0} + n(H-L) + c]^2 - 4(H-L)(n-1)\left[(G_c^e - P)_{m=0} - (G_c^e - P)_{m=1}\right]} \end{aligned}$$

That is,

$$m_I^2\left[(G_c^e - P)_{m=0} - (G_c^e - P)_{m=1}\right] - m_I\left[(G_c^e - P)_{m=0} + n(H-L) + c\right] + (H-L)(n-1) > 0$$

where $(G_c^e - P)_{m=0} - (G_c^e - P)_{m=1} > 0$;

$$\begin{aligned} (H-L)(n-1) &> m_I((G_c^e - P)_{m=0} + n(H-L) + c) - m_I^2\left[(G_c^e - P)_{m=0} - (G_c^e - P)_{m=1}\right] \\ (H-L)(n-1) &> m_I((G_c^e - P)_{m=0} + n(H-L)) - m_I^2\left[(G_c^e - P)_{m=0} - (G_c^e - P)_{m=1}\right] + cm_I \end{aligned}$$

$$c < \frac{(H-L)(n-1-nm_I)}{m_I} - (G_c^e - P)_{m=0} + m_I\left[(G_c^e - P)_{m=0} - (G_c^e - P)_{m=1}\right] \tag{24}$$

So if the condition in Eq. (24) holds, $m_n > m_I$. Next, we can show that

$$\begin{aligned} & (G_c^e - P)_{m=0} - m_I\left[(G_c^e - P)_{m=0} - (G_c^e - P)_{m=1}\right] \\ = \; & (G_c^e - P)_{m=0} - m_I\frac{c(n - X + \Pr(k \geq k^*, n))}{\Pr(k \geq k^*, n)} \\ = \; & \frac{(X - \Pr(k \geq k^*, n))(H-L)}{\Pr(k \geq k^*, n)} + \frac{nL - (1+r)nB}{\Pr(k \geq k^*, n)} - m_I\frac{c(n - X + \Pr(k \geq k^*, n))}{\Pr(k \geq k^*, n)} \\ = \; & (G_c^e - P)_{m=m_I} \end{aligned}$$

Consequently, Eq. (24) can be rewritten as

$$\begin{aligned}
c &< \frac{(n-1-nm_I)(H-L)}{m_I} - (G_c^e - P)_{m=m_I} \\
n(H-L)+c &< \frac{(n-1)(H-L)}{m_I} - (G_c^e - P)_{m=m_I} \\
(n-1)(H-L) &> m_I\left[n(H-L) + (G_c^e - P)_{m=m_I}\right] + m_I c \\
(1-m_I)\left[n(H-L) + (G_c^e - P)_{m=m_I}\right] - m_I c &> -(n-1)(H-L) + \left[n(H-L) + (G_c^e - P)_{m=m_I}\right] \\
(1-m_I)\left[n(H-L) + (G_c^e - P)_{m=m_I}\right] &> (H-L) + m_I c + (G_c^e - P)_{m=m_I} \\
(1-m_I)\left[(H-L) + \frac{(G_c^e - P)_{m=m_I}}{n}\right] &> \frac{(G_T^e - P)_{m=m_I}}{n}
\end{aligned}$$

## Appendix F: Comparing individual and group interest rates

From Eq. (6), the required repayment of an individual loan is

$$1 + R_I = \frac{m_I(H-L)+L}{B}$$

From Eq. (11), the required repayment per borrower of a group loan is

$$1 + R_G = \frac{(1+r)nB - (H-L+cm)E(k|k<k^*,n) - n(L-cm)\Pr(k<k^*,n)}{\Pr(k \geq k^*)nB}$$

Thus, the difference between individual and group interest rates is

$$\begin{aligned}
R_G - R_I &= \frac{(1+r)nB - (H-L+cm)E(k|k<k^*,n) - n(L-cm)\Pr(k<k^*,n)}{\Pr(k \geq k^*)nB} - \frac{m_I(H-L)+L}{B} \\
&= \frac{(1+r)nB - E[G(k|k<k^*,n)]}{\Pr(k \geq k^*,n)nB} - \frac{m_I(H-L)+L}{B} \\
&= \frac{(1+r)nB - E[G(k|k<k^*,n)]}{\Pr(k \geq k^*,n)nB} - \frac{nm_I(H-L)+nL}{nB} \\
R_I - R_G &= \frac{nm_I(H-L)+nL}{nB} + \frac{E[G(k|k<k^*,n)] - (1+r)nB}{\Pr(k \geq k^*,n)nB}
\end{aligned}$$

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Figure 1: Audit probability for nonmarginal group ($m_n$)

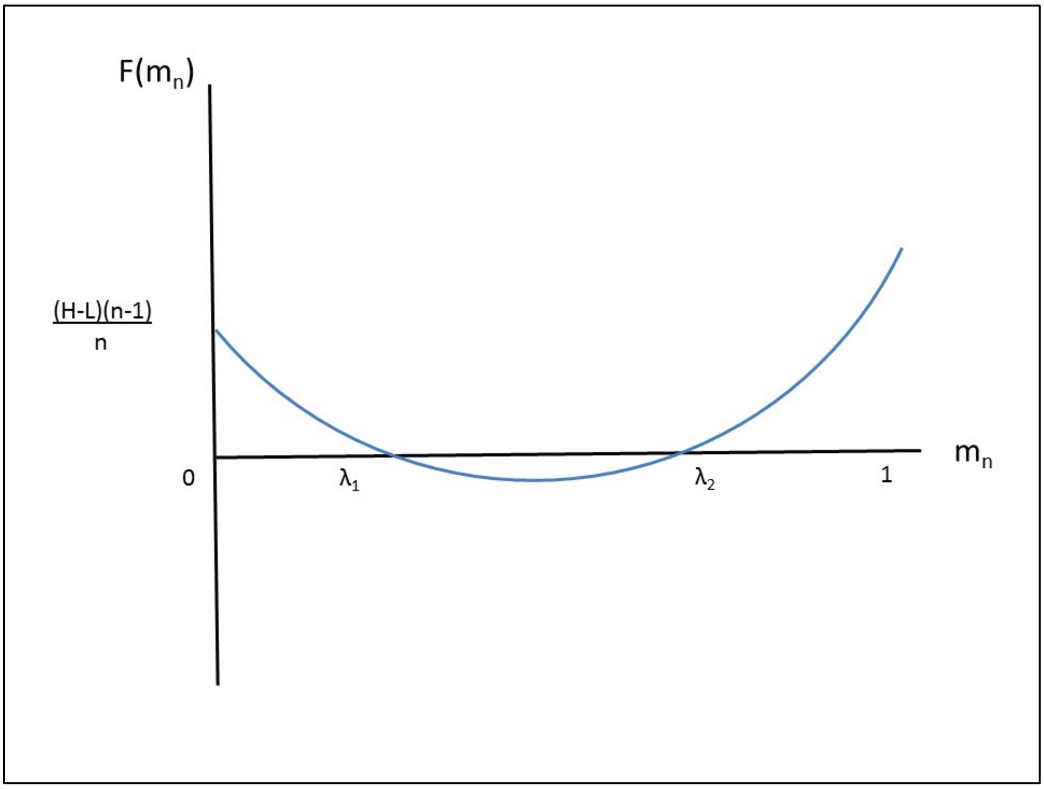



Figure 2: Correlation and Variance of $p_i$ with varying α and β

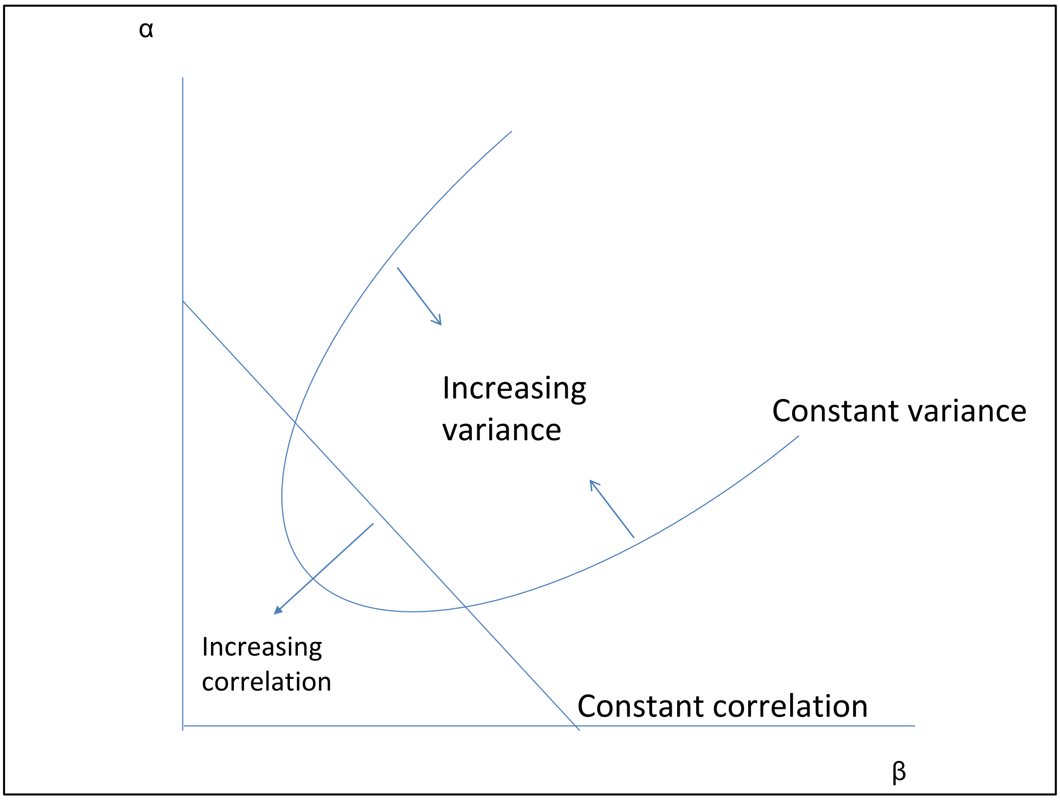


Figure 3: Interest rates for individual and group lending

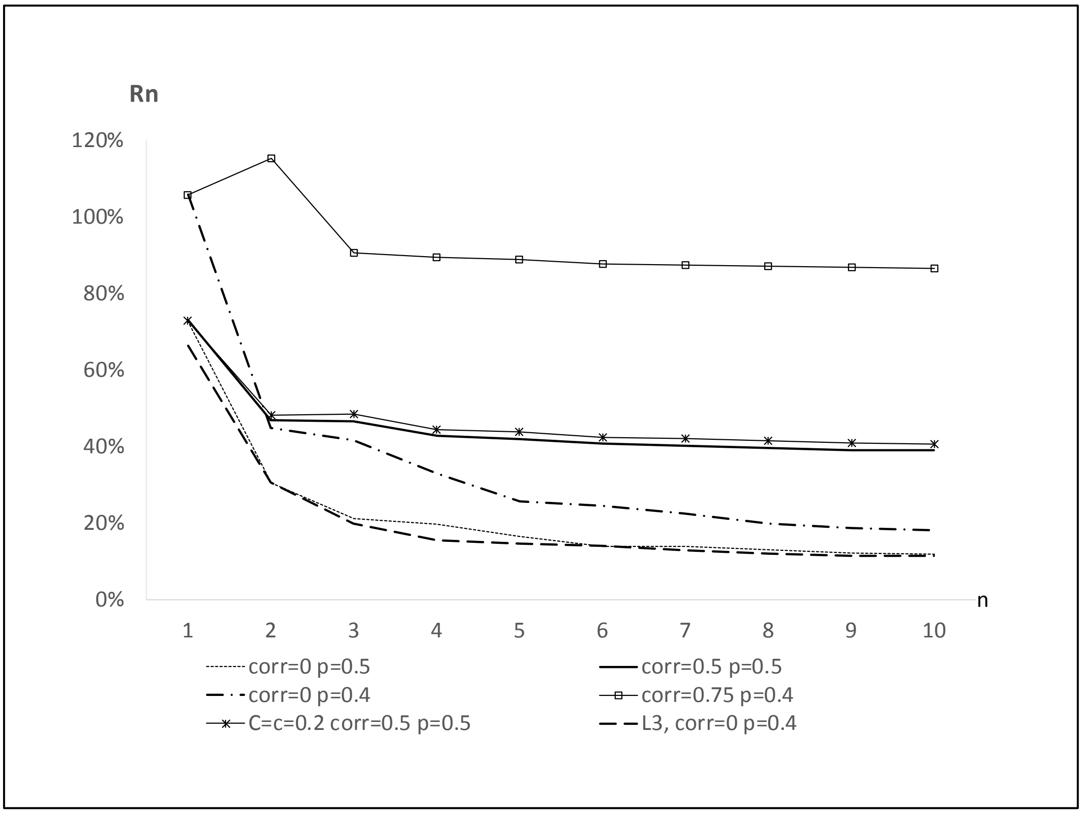



Figure 4: Probability of default for individual and group lending

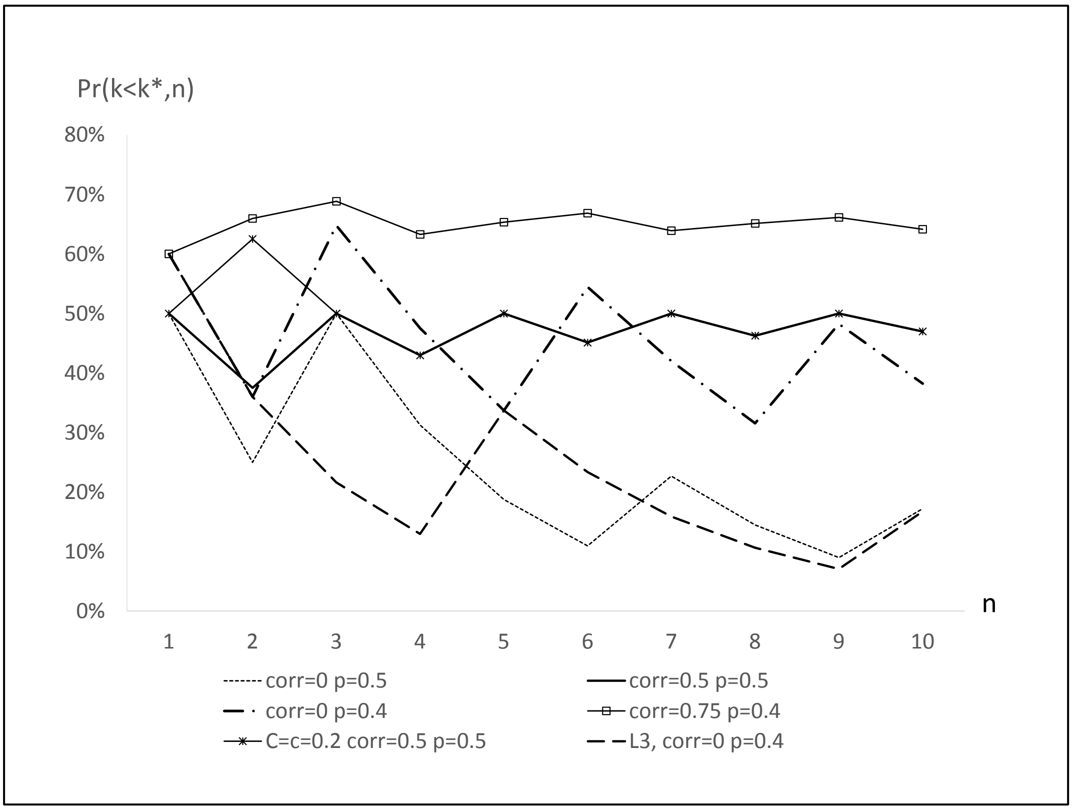
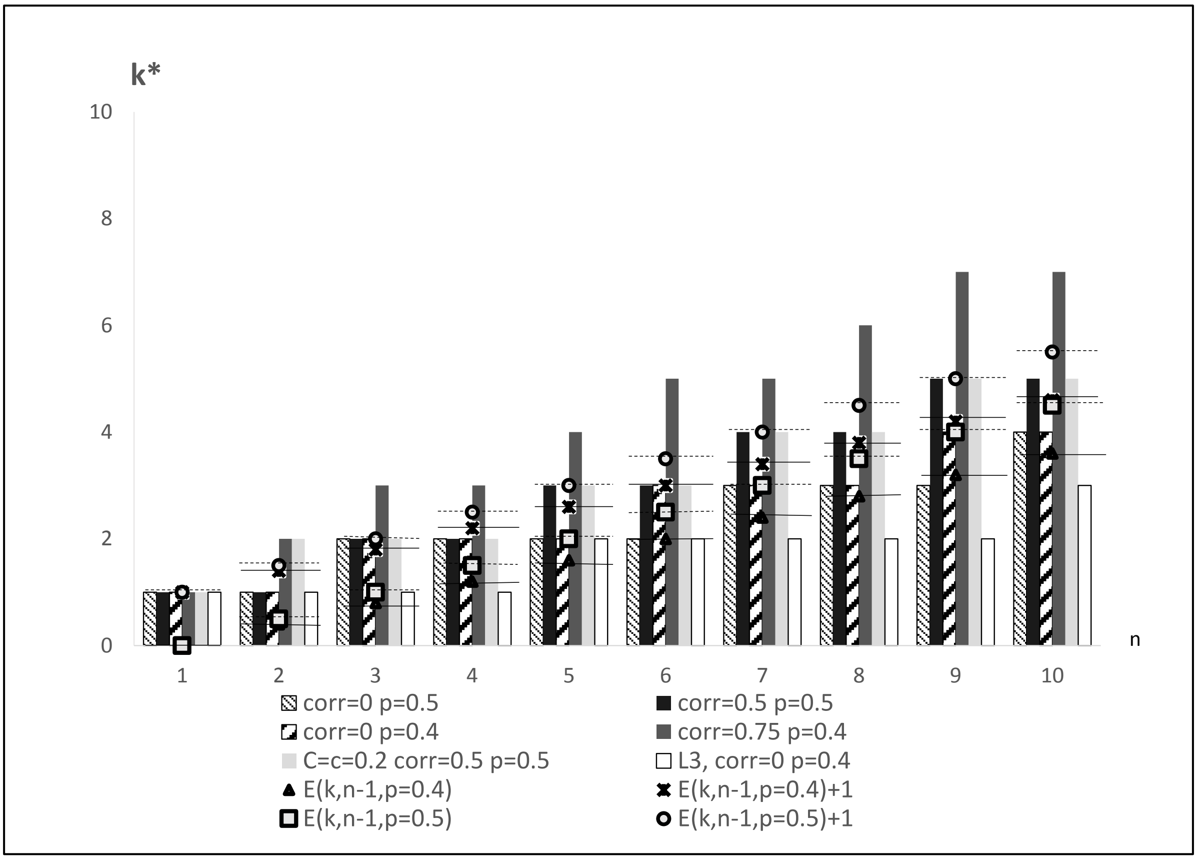


Figure 5: The minimum required number of successes for individual and group lending




Note: The horizontal lines with markers depict the means of successes shown in the legend

Figure 6: Audit probability and cost per borrower

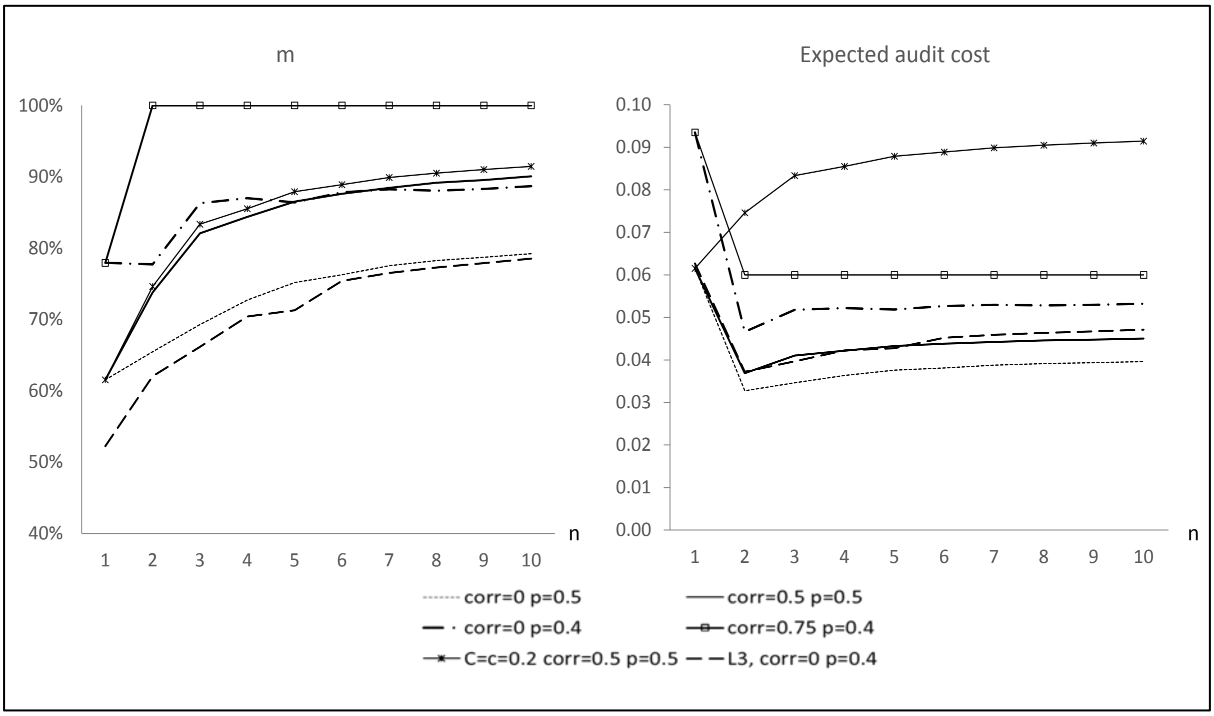


Figure 7: Expected surplus per borrower for individual and group lending