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Efficient audits by pooling independent projects: Separation vs. conglomeration[1]

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Abstract

Within a costly state verification model with endogenous audit and commitment, the paper proposes a rationale for joint financing based on the reduction of audit costs. Joint financing dominates separate financing when the incentive effects brought about by optimally chosen variable intensity audits, with the worst outcomes audited intensively and the intermediate ones residually, outweigh the cost of joint financing. This is represented by the extra-deadweight loss due to the unnecessary audit that a successful project may undergo when jointly financed. The result always holds when joint financing involves coinsurance gains -a successful project bails out a failing one- but may also hold under contagion -a succeeding project is dragged down by a failing one. Moreover, it is robust to the sequencing of audits. The paper derives a number of testable predictions relating the emergence of joint financing to project returns, investment cost, bankruptcy costs, quality of accounting standards and timing of audits.

Keywords: contracts, auditing, project finance, conglomerates.

JEL Nos: D82, D83, D86.

1 Introduction

Real life provides many examples of credit relationships in which the borrower's ex-post moral hazard can be deterred by monitoring: audit committees in firms, external audit companies, regulatory agencies, bankruptcy courts are all examples of organisations/institutions in charge of monitoring the firm at various stages of its life to protect investors from borrowers' opportunistic behaviour. Such activities are nonetheless costly and involve a deadweight loss. The aim of this paper is to investigate whether and how such activities affect the firms' organisational structure.

To this aim, we consider a borrower that needs to finance two uncorrelated risky projects through a competitive credit market. The outcome of each project, success or fail, is ex-post private information to the borrower, who, upon its realisation, sends a report to the lender. The lender can verify the truthfulness of the report with a costly audit. We ask when combining the two projects under one roof (with joint liability) leads to lower costs relative to the case of separate financing.

Prior literature has focused on a setting in which joint finance brings about coinsurance benefits - the conglomerate fails only if all projects fail - and a trade-off determining the dominance of joint over separate finance may emerge, for example because of the lower market discipline of the conglomerate (Boot and Schmeits (2000), Inderst and Müller (2003)), its reduced probability of refinancing (Faure-Grimaud and Inderst, 2005), or its lower tax benefits (Leland, 2007).

More recently, however, the literature has shown that, when the proceeds of a failing project are low enough to drag into bankruptcy a successful one, joint finance brings about contagion losses that outweigh the coinsurance gains, increasing the probability of default and the expected default costs (Banal-Estañol, Ottaviani and Winton, 2013). In such cases, separate financing becomes more attractive than joint financing.

In this paper we show that, in the presence of audit costs, joint finance always dominates separate financing under coinsurance, a result that is in line with that obtained by the literature. In addition, **we show that an optimally chosen audit strategy brings about a new and unexplored trade-off that makes joint financing dominate separate financing even in a scenario of contagion, a result that is novel in the literature. Such result is robust to the sequencing of audits.**

To illustrate our argument, we analyse three financing regimes, providing for each of them a detailed characterisation of the optimal contract: 1) individual project finance and random audit of fail reported outcomes; 2) joint finance with random audit only of two fail reports and coinsurance; 3) joint finance with random audit of both two and one fail reports and contagion (risk contamination).

The feasibility of each financing regime depends on the expected returns of the projects as compared with the investment and audit costs. The financing regime that will optimally emerge, *joint vs separate financing,* trades-off the cost and benefit of joint financing. The benefit of joint financing is in the incentive effects brought about by optimally chosen variable intensity audits, with the worst outcomes, less likely to occur, audited intensively and the intermediate ones residually. The cost of joint financing is that it may involve an extra-deadweight loss relative to separate financing due to the audit of succeeding projects that may occur under a single fail outcome. *The extent of this loss depends on the degree of information disclosure between the firm and the investors, that we capture with the quality of the firms' accounting standards, and on the sequencing of audits.*

To understand the drivers of our results, consider that both in individual and joint finance there must be some audit of each failed report to stop the borrower from always reporting the low revenue outcome. With stand-alone finance there are only two reports for each project, fail or success. To maximise the reporting incentives (and minimise the frequency of audit), it is optimal to audit a fail report stochastically, pledging to the lender the entire returns from failure, plus as much as necessary of the revenue of a successful project to meet its expected costs, leaving the residual revenue to the borrower. Under joint finance, there are three actual and reported states: zero, one and two successes. Again, to maximise the reporting incentives, it is optimal to pledge the entire returns from zero successes to the lender. Since this amount is, by assumption, insufficient to let the lender break even, it is necessary to pledge also part or all of the returns from one success. In this way, the borrower cross-pledges the return from one successful project and gives up the rent she could have obtained when this was financed as a stand-alone, thereby slackening the reporting constraints. The form of the optimal joint finance contract depends on whether the amount pledged under the zero and one success outcomes is sufficient to meet the lender's participation constraint, i.e., cover the investment and the expected audit cost. If it is, audit can be concentrated on reports of two fails, neglecting audits of single (and no) fail reports. Since single fail reports are never audited when jointly financed, despite the fail of one of the projects, there is a saving in expected audit cost relative to the case in which each project was financed as a stand-alone. We refer to this subsequently as a *joint finance contract with coinsurance.*

If the expected return from reports of zero and one successes is insufficient to meet the lender's participation constraint, the borrower must additionally pledge part of the returns from two successes. This implies that reports of one success and one fail are sometimes audited, with the contagious default of a succeeding project. We refer to this as a *joint finance contract with contagion* (risk contamination).

In this case, because upon one success the firm only reports the aggregate return, the succeeding project cannot be disentangled from the failing one. Thus, potentially both projects must be audited even though only one has failed. This implies that, besides the cost of auditing the failing project, an extra audit cost has to be incurred also for the audit of the successful one. Such cost would not have been incurred if the projects were financed as stand-alones. *Thus, joint financing involves an extra cost when it brings about contagion.* However, such cost might be offset by the cost saving due to the variable intensity audit. If this is the case, then joint financing still dominates separate financing, even under contagion. There is therefore a trade-off from joint financing under contagion: a saving arising from the reduced joint audit frequency and a cost arising from the unnecessary audit of the successful project.

The extent of such trade-off depends on the information that is disclosed upon a one success report. In particular, the higher the information disclosure, the lower the extra audit cost from joint financing and the more likely is joint over separate financing. In the extreme case in which information disclosure is maximum, i.e., upon a one success report the borrower also reports which project has succeeded and which project has failed, the extra cost from joint financing is zero. This is because only the project reported as failing is audited and joint financing comes only with benefits, those arising from endogenous audit. It follows that joint financing always dominates separate financing, even under contagion.

The extent of the above described trade-off also depends on the sequencing of audits. In particular, under no information disclosure, the extra cost from joint financing is higher when audits are simultaneous rather than sequential. This is because under a one success report and no information disclosure both projects must be audited and it is not possible for the lender to exploit the leakage of information coming from a sequential audit.

The above results allow us to formulate two novel predictions on the impact of the quality of accounting standards and the timing of audits on the emergence of conglomerates.

The idea that misreporting incentives can be controlled by costly audits started in the costly state verification literature (Townsend (1979), Gale and Hellwig (1985)) in a world with deterministic audits and a single project with continuous revenue outcomes. Here the solution is a standard debt contract. The range of possible audit strategies was extended in Border and Sobel (1985) and Mookherjee and Png (1989), who allow stochastic audit and show that generally the audit probabilities are interior and fall with the profitability of the state, with the highest revenue state not audited. An alternative cost-efficient information acquisition system has been studied by Menichini and Simmons (2014), who, still within a single project setting, show that by adding a layer of ex-ante information acquisition correlated with future project returns, audits become deterministic and targeted on the worst signal-state combinations. In the present paper, pooling projects together is yet another reason that may make deterministic audit emerge as the optimal solution in a commitment scenario.

The paper is also related to Diamond (1984), who, with multiple lenders financing several independent projects, shows the optimality of delegating auditing to an intermediary so as to eliminate wasteful duplication of monitoring. However, delegation creates the problem of controlling the incentive of the intermediary to misreport to lenders. This is solved using a standard debt contract between the intermediary and lenders that pays a fixed repayment to lenders and punishes the intermediary any time he fails to deliver it. In such circumstances, the risk of the intermediary failing is minimised by financing several projects at once, as the chance of them all failing falls as the number of projects rises. Thus, there is a pooling of risks across projects that drives the intermediary's default risk to zero as the number of projects rises. And it is partly the reduced risk of multiple fails coming from the grouping of projects together, which, within the CSV framework, implies a lower frequency with which audits occur, one of the drivers of our results. Such lowered audit frequency, although accompanied by a more intensive audit, has an overall net effect of a reduction in the expected audit cost as compared with single finance.

However, this reduced risk of bankruptcy is possible in Diamond only so long as pooling returns from one success and one failure covers the total debt (Diamond, 1996), i.e., when coinsurance is feasible. If this is infeasible, risk contamination may occur in the sense that a successful project may be driven bankrupt by a failing one. This possibility has been first uncovered by Winton (1999) and explored in Banal-Estañol, Ottaviani and Winton (2013).[1] Within a setting in which external financing is obtained through debt and default costs depend on total realised project returns, the authors show that losses from risk contamination may arise and separate

[1]The potential for risk contamination has also been analysed by Leland (2007), while Luciano and Nicodano (2014) have considered the possibility of mitigating such risk by introducing conditional guarantees which, preserving the guarantor's limited liability, do not trigger its default.

financing dominates joint financing. One of our contributions is to show that, with risk contamination, a novel trade-off emerges when optimal stochastic audit is used. In particular, the extra bankruptcy cost from risk contamination highlighted by Banal-Estañol, Ottaviani and Winton (2013) may be offset by the cost saving from the endogenous audit policy - with the worst outcomes, less likely to occur, audited intensively, and the intermediate ones audited residually. As a result, joint financing may still dominate single financing even under contagion.

The remainder of the paper is organised as follows. Section 2 lays out the model assumption. Section 3 develops a standard CSV model in which two individual projects are financed as stand-alones in the competitive banking sector. Section 4 considers the case of two independent projects to illustrate the basic role of joint financing in reducing the deadweight loss of audits, both in the case in which coinsurance gains and risk contamination losses between projects arise. Section 5 compares individual and joint financing in these two settings (Sections 5.1 and 5.2), both for the case of full information disclosure (Section 5.2.1) and no information disclosure (Section 5.2.2), and derives some novel testable predictions (Section 5.3). Section 6 discusses some robustness issues. Section 7 concludes. All the proofs, unless otherwise specified, are in the Appendix.

2 The Model Assumptions

An entrepreneur/borrower has two investment projects with uncorrelated returns, each costing I, which can be funded from a risk neutral investor. Each project gives a random return ς, $\varsigma \in \{H, L\}$, with $H > I > L > 0$. Outcome H occurs with probability p, while outcome L with probability $1-p$. Each project is socially profitable, i.e., the expected return covers the investment cost: $pH + (1-p)L > I$. The return of each project is freely observable only to the entrepreneur and not to the investor. Once the return is realised, the entrepreneur reports the projects' outcome to the investor. Because of output unobservability, the borrower has an incentive to report L on each. But since $I > L$, the only way for the investor to recoup the investment cost on a single project is to carry out an audit. This has a cost $c > 0$ per project and its result is observable and verifiable.

The possible ex-post outcomes vary with how projects are grouped in their financing. With stand-alone projects there are only two outcomes to the contract on each project, $\varsigma = \{L, H\}$. With the two projects jointly financed in a single contract four outcomes are possible: two successes, with probability p^2, two failures, with probability $(1-p)^2$, one success and one failure, with probability $2p(1-p)$. Thus,

$\varsigma \in \{LL, HL, LH, HH\}$.

Following the outcome ς, with single finance, the borrower sends a report $\sigma^S \in \{0, 1\}$, corresponding to the number of projects which succeed. In particular, $\sigma^S = 0$ denotes a report of zero successes, while $\sigma^S = 1$ denotes a report of one success. With joint finance, the borrower sends a report $\sigma^J \in \{0, 1, 2\}$, with $\sigma^J = 0$ denoting a report of zero successes, $\sigma^J = 1$ of one success (and one failure), and $\sigma^J = 2$ of two successes.

When the report involves the success of just one of the two projects ($\sigma^J = 1$), an insitutional framework governs the minimum information the entrepreneur must convey to the lender. This may require the entrepreneur to report the return on each project (full information disclosure) or to report just the aggregate return on the combined projects (no information disclosure). In particular, under full information disclosure, the report of one success ($\sigma = 1$) specifies also which of the two projects succeeded. This matters for the design of the audit policy as it allows the lender to target audit on the failing project, saving an unnecessary audit of the succeeding one. Under no information disclosure, the report of the aggregate return of the two projects does not allow the lender to target audit on the failing project. However, by using sequential audit, the lender can still save an unnecessary audit. Indeed, if by choosing one project at random and auditing it the lender finds it is a fail, he knows from the report that the second project is a success. Thus, no audit of the second project is necessary. If the first audit reveals a success, then there is a risk that the entrepreneur cheated on the second project, thus calling for a further audit. *We capture these scenarios by a categorical variable s reflecting the regulation in force, $s = 1$ for full information disclosure and $s = 1/2$ for no information disclosure. Because the full information disclosure is a special case of the one with partial information disclosure, in the following we carry out the analysis keeping the degree of information disclosure as a parameter.*

For each financing regime, following a report σ^i, $i = \{S, J\}$, a costly audit may occur to verify the truthfulness of the report. This is observable and the result of it verifiable, and it is designed so that the entrepreneur has the incentive to truthfully report the outcome ς.

Under single finance, an audit of each project may occur with probability m_0^S following a report of no success ($\sigma^S = 0$).

Under joint finance, an audit of one or both projects may occur following a report of no success ($\sigma^J = 0$) or one success ($\sigma^J = 1$). Upon a report $\sigma^J = 0$, projects may be audited sequentially: m_0^J, $m_{0,i}$, $i = \{L, H\}$, where m_0^J denotes the first stage audit probability following a report of zero successes and $m_{0,i}$ denotes the second stage audit probability conditional on the first stage audit discovering $i = \{L, H\}$.

Upon a report $\sigma^J = 1$, the number of projects that may be audited depends on the degree of information disclosure. *For a generic* s, *since it is not possible to disentangle the succeeding project form the failing one, both projects must be audited* sequentially, i.e., one, with probability m_1, and then, depending on the outcome of the first audit, possibly the other, with probability $m_{1,H}$.[2]

3 Single finance

When each project is funded as a stand-alone, a contract specifies repayments and the probability with which an audit will occur. Let m_0^S be the probability of auditing a report $\sigma = 0$. Let R_1 be the repayment due following a report $\sigma^S = 1$, $R_{0|\varsigma}$ be the repayment due following a report $\sigma = 0$, and an audit which reveals that the state is $\varsigma \in \{L, H\}$, and $R_{0|.}$ be the repayment with report $\sigma^S = 0$, but no audit.

The contract has commitment so that in the play of the game an audit must actually occur even though the lender knows that a fail report must be truthful. All repayments are non-negative and the borrower has limited liability.

The sequence of events is as follows, with the corresponding game tree sketched in Fig. 1.

1. A financing contract is offered and, if accepted, the borrower is committed to the investment.

2. Nature (N) chooses the project outcome, $\varsigma = \{H, L\}$. This is only observed by the borrower (A), who makes a report σ^S to the investor (P).

3. If $\sigma^S = 1$ is reported, there is no audit. If $\sigma^S = 0$ is reported, the investor can audit with probability m_0^S to discover the true project outcome, or not audit with probability $1 - m_0^S$.

4. Conditional on the report and audit decisions, repayments are made as described.

[2]Notice that there is no second stage audit after the first reveals a fail, i.e., $m_{1,L} = 0$.

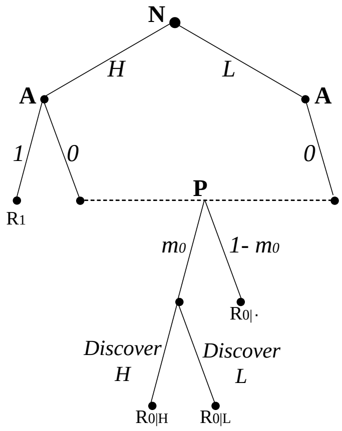

Fig. 1. Game tree with project finance

The contract $\mathcal{P}^S$ sets repayments $R_{0|H}$, R_1, $R_{0|\cdot}$, $R_{0|L}$, and monitoring probability m_0^S to

$$\max EP^S = p\left(H - R_1\right) + (1-p)\left[m_0^S\left(L - R_{0|L}\right) + \left(1 - m_0^S\right)\left(L - R_{0|\cdot}\right)\right] \tag{1}$$

$$\text{st } pR_1 + (1-p)\left[\left(1 - m_0^S\right) R_{0|\cdot} + m_0^S\left(R_{0|L} - c\right)\right] \geq I \tag{2}$$

$$R_1 \leq m_0^S R_{0|H} + \left(1 - m_0^S\right) R_{0|\cdot} \tag{3}$$

$$0 \leq R_1, R_{0|H} \leq H \text{ and } 0 \leq R_{0|\cdot}, R_{0|L} \leq L \tag{4}$$

where (1) is the borrower's expected profit per-project, (2) is the participation constraint, ensuring that the lender breaks even in expected terms on each project, (3) is the truth-telling constraint, ensuring that upon a high state the borrower prefers to report truthfully rather than cheating and be audited with probability m_0^S, and (4) the limited liability conditions.

The solution to programme P^S is described in Proposition 1:

Proposition 1 *The optimal contract when each project is funded as a stand-alone has:*

(i) *maximum punishment for detected false low state report:* $R_{0|H} = H$;

(ii) *zero low state return for the borrower:* $R_{0|L} = R_{0|\cdot} = L$;

(iii) *random audit of low state reports,* m_0^S :

$$m_0^S = \frac{I - L}{p\left(H - L\right) - (1-p)\,c} < 1; \tag{5}$$

(iv) *lender repayment following a high state report equal to* $R_1 = \frac{(H-L)I-(1-p)L(H-L+c)}{p(H-L)-(1-p)c} < H$, *and expected return to the borrower equal to* *the expected return net of the expected audit cost:*

$$EP^S = pH + (1-p)\,L - I - (1-p)\,m_0^S c > 0. \tag{6}$$

From 5, the condition that guarantees that the single finance contract is feasible, is[3]

$$pH + (1-p)\,L - I \geq (1-p)\,c. \qquad \text{(Condition 1)}$$

This condition can be represented in the the space of $H-L$ and $I-L$ by a linear function with intercept $\frac{(1-p)\,c}{p}$ and slope $\frac{1}{p}$ (Fig. 2). The line from the origin instead represents the locus of exogenous parameters where $NPV = 0$:

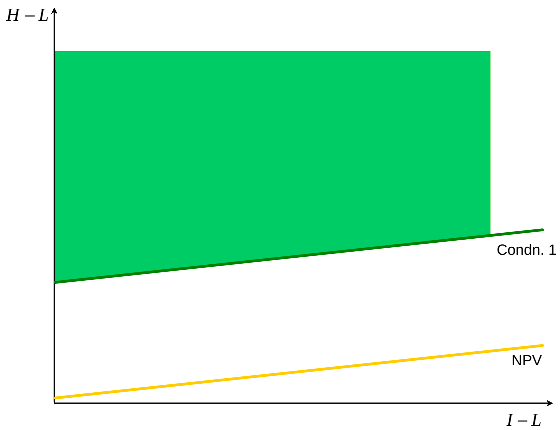

Fig. 2. Area of feasible single finance contracts

The intuition behind these results is the following. When Condition 1 does not hold, the expected revenue from the project cannot cover the investment cost. Thus, no contract is signed, despite the project having positive NPV. If Condition 1 does hold, the frequency of audit is positive (since if $m_0^S = 0$, from (3), $R_1 = R_{0|\cdot} \leq L$ and there is insufficient revenue to meet the investment cost). The deadweight loss of audit is minimised by raising $R_{0|H}$ up to H and reducing the audit probability until the incentive constraint (3) holds with equality. In addition, low state repayments, whether audited or not, are set to give zero surplus to the borrower: $R_{0|\cdot} = R_{0|L} = L$. However, since $R_1 < H$, the borrower gets a rent in the high state (expected returns (6) are positive).

[3] If Condition 1 does not hold, the expected return to the borrower is negative and financing does not occur.

4 Joint financing

When two projects with independent ex-post private returns are jointly financed four possible outcomes may arise: $\varsigma \in \{LL, HL, LH, HH\}$. The borrower reports to the lender the number of successes, $\sigma^J \in \{0,1,2\}$, and, possibly, when reporting a single success, which project has succeeded, according to the degree of information disclosure s.

Any report the borrower makes must be feasible in that she has to have funds to make the appropriate repayment. Conditional on the report, the lender can audit at the random rate specified in the contract. The contract has to list an audit strategy that overcomes the temptations to cheat in the report. We assume there is commitment in the contract, so the lender has to carry through the audit policy even knowing that this will never catch a cheat.

Because reports must be feasible, a report of two successes ($\sigma^J = 2$) must be truthful. So, it will not be audited. Let R_2 be the repayment due following such a report.

Upon a report of one success ($\sigma^J = 1$), *for a generic s, since it is not possible to disentangle the succeeding project from the failing one, both projects must be audited* sequentially. [4] So, faced with a report $\sigma^J = 1$, the lender has to select the first project to audit randomly at the endogenously chosen rate m_1, or not audit at all. In the case in which he does not audit, with probability $1 - m_1$, he demands a repayment $R_{1|\cdot}$ in total on the two projects. If he does audit and the first audit reveals a fail, then the lender stops auditing as, from the report received, he knows the other project must be a success, and gets a repayment $R_{1|L}$. If the first audit reveals a success, instead, the lender can go on to audit the second project at the endogenously chosen rate $m_{1,H}$, or not audit. If he does audit and discovers a success, he demands a repayment $R_{1|HH}$, while he demands a repayment $R_{1|HL}$ if discovers a failure. If he does not audit, with probability $1 - m_{1,H}$, he demands $R_{1|H\cdot}$ as a repayment.

Under a report of zero successes ($\sigma^J = 0$), both projects may be audited and the lender can randomly choose which one, if any, with probability 1/2 on each (by the principle of insufficient reason). Denote with m_0^J the probability to audit one of the two projects, and with $1 - m_0^J$ the probability of auditing neither. In cases in which the lender does not audit, he demands a repayment $R_{0|\cdot}$ and the game ends. When the lender does audit and discovers the outcome for the selected project, he can decide to go further and audit the remaining project, with probability $m_{0,i}$, $i \in \{L, H\}$,

[4] *In the robustness section we also consider the case of full information disclosure ($s = 1$), where, upon a report $\sigma^J = 1$, the borrower states which of the two projects has succeeded so that an audit can be concentrated on the reported failed project.*

where the second subscript denotes the outcome of the first audit, or to stop, with probability $1 - m_{0,i}$. Denote with $R_{0|ij}$, $i, j = \{L, H\}$, the repayment the lender gets upon receiving a report of zero successes when he audits both projects and discovers the true state to be i on the first and j on the second, and with $R_{0|i\cdot}$, the repayments in case he audits just one project and discovers the true state to be i, but does not audit the other.

The sequence of events is as follows:

1. A financing contract is offered and, if accepted, the borrower is committed to the investment.
2. Nature (N) chooses the projects' outcome, $\varsigma = \{LL, HL, LH, HH\}$. This is only observed by the borrower (A), who makes a report σ^J to the investor (P).
3. If $\sigma^J = 2$ is reported, there is no audit. If $\sigma^J = 1$ is reported, since there is no information disclosure, the investor can audit the first project with probability m_1 or not audit with probability $1 - m_1$. Conditional on having audited the first project and having discovered a success, the investor can audit the second with probability $m_{1,H}$, or not audit, with probability $1 - m_{1,H}$. If $\sigma^J = 0$ is reported, the investor can audit the first project with probability m_0^J or not audit with probability $1 - m_0^J$. Conditional on having audited the first project, the investor can audit the second with probability $m_{0,i}$, or not audit, with probability $1 - m_{0,i}$, $i \in \{L, H\}$.
4. Conditional on the report and audit decisions, repayments are made.

The corresponding game tree is sketched in Figure 3.

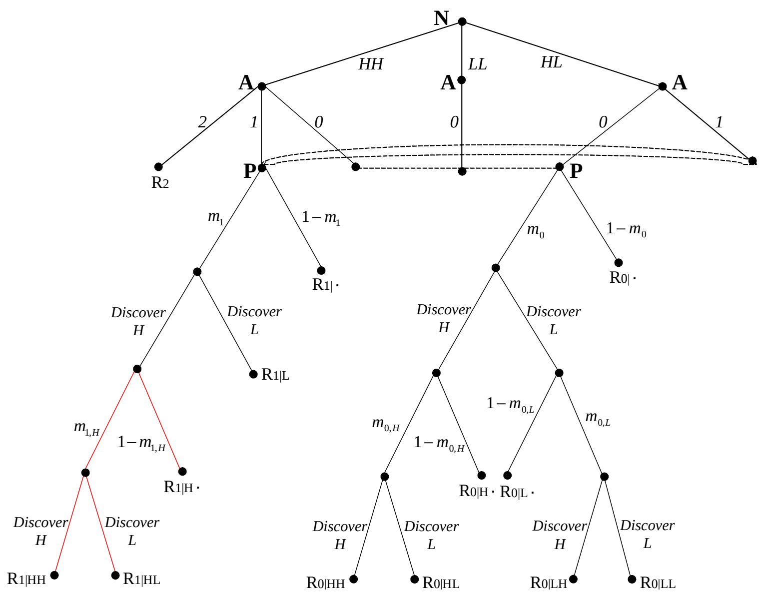

Fig. 3. Game tree with joint finance and no information disclosure

4.1 The contract problem

In this section we set up the contract problem under joint financing consisting in maximising the borrower's profits, subject to the lender getting a non-negative return, to the incentive constraints guaranteeing that the borrower does not cheat on the reports and to the limited liability conditions.

The borrower's joint payoff function with truthtelling is

$$
\begin{gathered}
EP^{J}(s) = p^{2}(2H - R_{2}) + 2p(1-p)\left\{H + L - (1-m_{1})R_{1|\cdot} - \right. \\
\left. m_{1}\left[sR_{1|L} + (1-s)\left(m_{1,H}R_{1|HL} + (1-m_{1,H})R_{1|H\cdot}\right)\right]\right\} + \\
(1-p)^{2}\left\{2L - \left(1-m_{0}^{J}\right)R_{0|\cdot} - m_{0}^{J}\left[m_{0,L}R_{0|LL} + (1-m_{0,L})R_{0|L\cdot}\right]\right\}.
\end{gathered} \tag{7}
$$

The participation constraint requires the expected return to the lender from financing both projects covers the joint loan costs and the expected audit costs:

$$
\begin{gathered}
EP^{L}(s) = p^{2}R_{2} + 2p(1-p)\left\{(1-m_{1})R_{1|\cdot} + \right. \\
\left. m_{1}\left[\left(sR_{1|L} + (1-s)\left(m_{1,H}\left(R_{1|HL} - c\right) + (1-m_{1,H})R_{1|H\cdot}\right)\right) - c\right]\right\} + \\
(1-p)^{2}\left\{\left(1-m_{0}^{J}\right)R_{0|\cdot} + m_{0}^{J}\left[m_{0,L}\left(R_{0|LL} - c\right) + (1-m_{0,L})R_{0|L\cdot} - c\right]\right\} \geq 2I.
\end{gathered} \tag{8}
$$

With two true successes, there are two ways of cheating. To declare zero successes, or to declare one. The incentive constraint that ensures that a borrower with two successes prefers to make a truthful report $\sigma^{J} = 2$ rather than a false report $\sigma^{J} = 0$ is:

$$
R_{2} \leq \left(1-m_{0}^{J}\right)R_{0|\cdot} + m_{0}^{J}\left[m_{0,H}R_{0|HH} + (1-m_{0,H})R_{0|H\cdot}\right] \tag{9}
$$

By constraint 9 the repayment due by reporting truthfully, R_2, is no higher than what is due by cheating and reporting two fails. To get this latter amount, consider that, in order to ascertain the truthfulness of the borrower's report, the lender has to audit both projects sequentially. Because the borrower has cheated, a first stage audit by the lender, which occurs with probability m_0^J, always reveals a success. Any second stage audit, which occurs with probability $m_{0,H}$, also reveals a success, and has associated repayment for the lender $R_{0|HH}$. If no second stage audit occurs, with probability $1 - m_{0,H}$, the associated repayment for the lender is $R_{0|H\cdot}$.

The incentive constraint that ensures that a borrower with two successes prefers to make a truthful report $\sigma^{J} = 2$ rather than a false report $\sigma^{J} = 1$ is:

$$
R_{2} \leq (1-m_{1})R_{1|\cdot} + m_{1}\left[m_{1,H}R_{1|HH} + (1-m_{1,H})R_{1|H\cdot}\right] \tag{10}
$$

By constraint 10, the repayment due by reporting truthfully, R_2, is no higher than what is due by cheating and reporting one success. To get this latter amount, **consider that, when reporting one success, the borrower reports just the aggregate return, and the truthfulness of the report can only be ascertained by auditing both projects sequentially.** In particular, because the borrower has cheated, a first stage audit by the lender, which occurs with probability m_1, always reveals a success. Any second stage audit, which occurs with probability $m_{1,H}$, also reveals a success, and an associated repayment for the lender $R_{1|HH}$. If no second stage audit occurs, with probability $1-m_{1,H}$, the associated repayment for the lender is $R_{1|H\cdot}$.

The incentive that a borrower with one success prefers to make a truthful report $\sigma^J = 1$ rather than a false report $\sigma^J = 0$ is:

$$(1-m_1)\,R_{1|\cdot} + m_1\left[sR_{1|L} + (1-s)\left(m_{1,H}R_{1|HL} + (1-m_{1,H})\,R_{1|H\cdot}\right)\right] \leq \left(1-m_0^J\right)R_{0|\cdot} + \\ + m_0^J\left[\frac{1}{2}\left(m_{0,H}R_{0|HL} + (1-m_{0,H})R_{0|H\cdot}\right) + \frac{1}{2}\left(m_{0,L}R_{0|LH} + (1-m_{0,L})R_{0|L\cdot}\right)\right]. \quad (11)$$

The expected compensation associated with a truthful report $\sigma^J = 1$ (left hand side of constraint 11) takes into account that a first audit of one of the projects can occur with probability m_1 and discover either a success or a fail, depending on the degree of information disclosure s. If $s = 1$, since the borrower discloses which is the failing project, an audit will certainly reveal a fail and there is no further audit. If $s = 1/2$, the lender chooses to audit one project at random and may either discover a fail (so the second project needs no audit) or a success, in which case the lender goes on to randomly audit the second project.

Last, the limited liability conditions are:

$$\begin{aligned} R_2, R_{1|HH}, R_{0|HH} &\leq 2H, \quad (12) \\ R_{1|\cdot}, R_{1|L}, R_{1|H\cdot}, R_{1|HL}, R_{0|H\cdot}, R_{0|HL}, R_{0|LH} &\leq H+L, \\ R_{0|\cdot}, R_{0|L\cdot}, R_{0|LL} &\leq 2L. \end{aligned}$$

Suming up, the contract problem $\mathcal{P}^J$ is to choose R_2, $R_{1|\cdot}$, $R_{1|H\cdot}$, $R_{1|HH}$, $R_{1|HL}$, $R_{1|L}$, $R_{0|H\cdot}$, $R_{0|HL}$, $R_{0|LH}$, $R_{0|\cdot}$, $R_{0|L\cdot}$, $R_{0|LL}$, $R_{0|HH}$, and monitoring probabilities m_0^J, $m_{0,H}$, $m_{0,L}$, m_1, $m_{1,H} \in [0,1]$ to maximise the objective function (7), subject to the participation constraint (8) being non-negative, to the incentive constraints (9), (10), and (11), and to the limited liability conditions (12).

By solving Programme $\mathcal{P}^J$ two possible cases may arise. The first displays coinsurance, i.e., the proceeds of a succeeding project are used to save an unsuccessful one. This allows no audit reports of one fail, which would have instead been audited under single finance. The second case displays contagion, i.e., the failure of one project drags down a successful one. Thus, also reports of one success, which would not have been audited under single finance, are audited under joint finance, and, depending on the degree of information disclosure, the audit may involve one or both projects. The properties of the second-best contract are described in Proposition 2.

Proposition 2 *Suppose two identical and independent projects are jointly financed. The optimal joint second-best contract may display coinsurance or contagion. In either case it has:*

(i) maximum punishment for detected false reporting: $R_{0|HH} = R_{1|HH} = 2H$; $R_{0|H\cdot} = R_{0|HL} = R_{0|LH} = H + L$;

(ii) zero rent to the borrower in the lowest true state (both projects fail): $R_{0|L\cdot} = R_{0|\cdot} = R_{0|LL} = 2L$;

Moreover, when the second-best contract displays coinsurance, it has:

1. *deterministic audit of reports of zero successes at first stage or at second stage having discovered a cheat by first stage audit,* $m_0^J = m_{0,H} = 1$; *random audit of reports of zero successes at second stage having discovered a truthful report:*

$$m_{0,L} = \frac{4(I-L) + 2(1-p)^2 c - p(2-p)(H-L)}{p(2-p)(H-L) - 2(1-p)^2 c} \leq 1; \tag{13}$$

2. *repayments pooled in the top two states,* $R_{1|\cdot} = R_2 = 2L + \frac{2(I-L)(H-L)}{p(2-p)(H-L)-2(1-p)^2 c} < H + L$, *so that the borrower with at least one success is indifferent between truthfully reporting one or two successes;*

3. *no audit following a single fail report, i.e.,* $m_1 = m_{1,H} = 0$;

4. *borrower's expected return:*

$$EP^J = 2[pH + (1-p)L - I] - (1-p)^2 m_0^J (1 + m_{0,L}) c. \tag{14}$$

When the second-best contract displays contagion, it has:

5. *deterministic audit for reports of two fails:* $m_0^J = m_{0,L} = m_{0,H} = 1$;

6. *zero rent to the borrower reporting one success, whether audited or not:* $R_{1|\cdot} = R_{1|L} = R_{1|HL} = R_{1|H\cdot} = H + L$;

7. *random first stage auditing for single fail reports:*

$$m_1(s) = \frac{2(I-L) - p(2-p)(H-L) + 2(1-p)^2 c}{p[p(H-L) - 2(1-p)(2-s)c]} \tag{15}$$

and deterministic second stage auditing when first stage auditing has revealed a success , $m_{1,H} = 1$;

8. *repayment after a report of two successes higher than* $H + L$:

$$R_2(s) = 2H - 2(H-L)\frac{pH + (1-p)L - I - \{1 - p[s + p(1-s)]\}c}{p^2(H-L) - 2p(1-p)(2-s)c};$$

9. *borrower's expected return:*

$$\begin{aligned} EP^J(s) &= 2[pH + (1-p)L - I] - (1-p)^2 m_0^J (1 + m_{0,L}) c \\ &\quad -2p(1-p) m_1(s) [1 + (1-s) m_{1,H}] c. \end{aligned} \tag{16}$$

The intuition behind the results in Proposition 2 is the following. Common to coinsurance and contagion, maximum punishment and zero rent to the borrower in the lowest truthfully reported states (results (i) and (ii)) maximise the incentive for truth-telling whilst also keeping the observation cost as small as possible.

When the second-best contract displays coinsurance (results 1 to 4 in Proposition 2), a strictly positive probability of auditing a report of two fails is required since otherwise the borrower could always report no success and get away with cheating, leading to repayments which do not cover the investment cost. Moreover, since first stage audit m_0^J has two incentive effects, one working directly at the first stage and the other combining with $m_{0,L}$ at the second stage, first stage audit is a more powerful control on potential cheating than second stage audit. Thus $m_0^J = 1$ and $m_{0,L} \leq 1$.

The incentive to cheat between a report of one or two successes is controlled by pooling the repayments, $R_{1|\cdot} = R_2$. These repayments must be above $2L$, since otherwise there would be insufficient revenue to the lender to recoup the loans cost, and no higher than $H + L$, the highest revenue available if only one project succeeds. With flat repayments for one or two successes, audit of projects following one fail report is unnecessary since the borrower has no incentive to cheat: $m_1 = m_{1,H} = 0$.

These results are in line with those obtained by the early literature highligting the diversification benefits of joint financing (Lewellen, 1971; Diamond, 1984, among others), except that we allow for optimal random audits. However, as pointed out by some more recent literature (Leland, 2007; Banal-Estañol, Ottaviani and Winton, 2013), joint financing may also involve the contagious default of succeeding projects dragged into bankruptcy by the failing ones. When this is the case (results 5 to 9 of Proposition 2), additional revenues in excess of $H+L$ must be raised from the report of two successes to cover the investment plus audit cost of the two projects. But to ensure truthful reports of two successes, reports of only one success must sometimes be audited ($m_1 > 0$). The optimal audit probabilities are nevertheless decreasing in the profitability of the state. In particular, while a report of zero successes is audited deterministically ($m_0^J = m_{0,L} = m_{0,H} = 1$), a report of one success is audited with a lower intensity. This improves the incentive to truthfully declare one success instead of no successes and it is efficient as it minimises the audit cost. Indeed, since all intermediate repayments are equal to $H + L$, the borrower with only one successful project might have an incentive to report zero successes rather than one, so as to bet on the possibility of not being audited. To make sure that this does not happen, the lender always audits reports of zero successes ($m_0^J = m_{0,L} = m_{0,H} = 1$). Since the borrower gets zero anyway by reporting 0 or 1 success, he might then be indifferent between cheating and telling the truth. However, he is still better off by telling the truth because the audit costs are lower upon a 1 success report and so the ex ante profits are higher. Thus, audits are concentrated on the worst state report which is more likely to reflect cheating and on which strong audit will have more power in ensuring truthtelling, while intermediate state reports are audited residually.

As regards the intensity of audits following a report of one success, this varies depending on whether the first stage audit reveals a success or a fail. If it reveals a fail, the second project must be a success and its audit can be avoided, with no wasteful audit of a successful project. But if the first audit reveals a success, the second project must still be audited. Using a low probability of audit of the first project ($m_1 > 0$) but maximum probability of audit of the second ($m_{1,H} = 1$) gives the most powerful truthtelling incentive and economises on wasteful audit cost.

Proposition 2 has shown that the joint second-best contract may display coinsurance or contagion. Proposition 3 states conditions under which each scenario arises.

Proposition 3 *From 13, the joint finance contract with coinsurance arises when*

$$p\,(2-p)\,(H-L) - 2\,(I-L) \geq 2\,(1-p)^2\,c. \qquad \text{(Condition 2)}$$

From 15, the joint finance contract with contagion is feasible when

$$2\left\{pH+(1-p)L-I-(1-p)c\left[1+p(1-s)\right]\right\}\geq 0. \qquad \text{(Condition 3)}$$

Conditions 2 and 3 have the interpretation that the projects taken together are sufficiently profitable to cover the investment and certain audit cost of the bottom and the intermediate report, respectively. Indeed, Condition 2 can be obtained by assuming that in the participation constraint (8) the revenue from zero or one success outcomes ($2L$ and $H+L$) is sufficient to meet the investment cost plus certain audit of the lowest report. This allows there to be no audit of the intermediate report since there is a common repayment after one and two successes.

If Condition 2 is violated and the revenue from zero or one success outcomes is insufficient to meet the investment plus certain audit cost of the lowest report, then extra-resources must be raised from a two success outcome ($2H$), which implies that also the intermediate report must be audited to stop a borrower with two successes to report one. Condition 3 is obtained by assuming that collecting the revenue from zero, one and two successes outcomes ($2L$, $H+L$ and $2H$) yields enough expected revenue to cover the investment cost and the cost incurred by auditing deterministically all reports involving two fails or with a reported one fail at least one of the projects, depending on the degree of information disclosure, s.

Condition 2 can be represented in the the space of $H-L$ and $I-L$ by a linear function with intercept $2(1-p)^2 c/p(2-p)$ and slope $2/p(2-p)$ (Fig. 5).

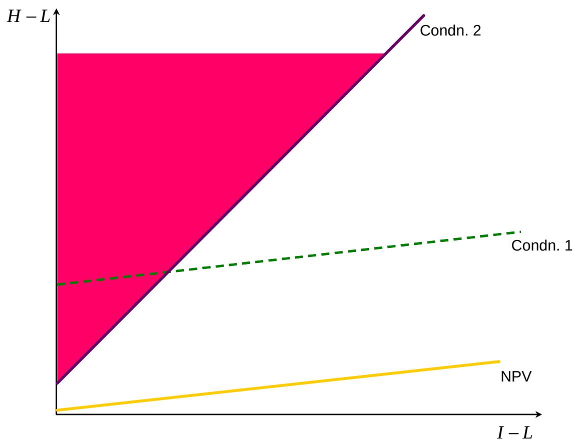

Fig. 5. Area of feasible joint finance contracts with coinsurance

Thus, a coinsurance contract arises whenever the high state return is sufficiently high relative to the investment cost, i.e., to the left of the Condition 2 line.

Condition 3 can be represented in the the space of $H - L$ and $I - L$ by a linear function with intercept $(1 + p(1 - s))(1 - p)c/p$ and slope $1/p$ (Fig. 6).

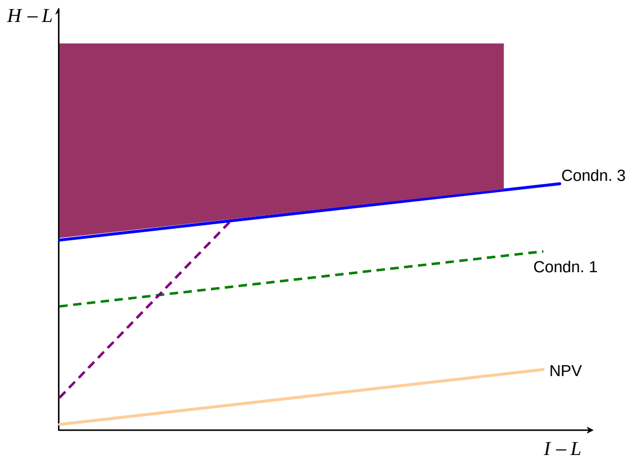

Fig. 6. Area of feasible joint finance contracts with contagion

A contagion contract can then arise whenever the investment cost is high relative to the high state return, i.e., above the Condition 3 line but below the Condition 2 line.

5 Efficiency

In the following we contrast the efficiency of single and joint finance.

Under single financing, the expected profits obtainable from two stand-alone projects are twice the profits obtainable from each project as defined in Eq. 6. Under joint financing, expected profits are defined in Eq. 16.

For viable stand-alone finance Condition 1 must always hold. The detailed comparison of joint and single finance depends on whether Condition 2 or Condition 3 is satisfied, i.e., whether the joint contract involves coinsurance or contagion.

5.1 Efficiency under coinsurance

We first consider the case in which the joint second-best contract displays coinsurance (Condition 2 is satisfied). In this case, from Proposition 2, we know that reports of one success are never audited, $m_1 = m_{1,H} = 0$, and only reports of zero successes are audited with probability $m_0^J = 1$ and $m_{0,L}$ as defined in the proposition. By comparing the expected audit cost under separate and joint finance (Eqs. 6 and 14), we deduce that joint financing with coinsurance dominates project finance iff:

$$(1-p)\,c\underbrace{\frac{2\,(I-L)}{p\,(H-L)-(1-p)\,c}}_{2m_0^S} > (1-p)^2\,c\underbrace{\frac{4\,(I-L)}{p\,(2-p)\,(H-L)-2\,(1-p)^2\,c}}_{m_0^J\left(1+m_{0,L}\right)}.$$

The difference reduces to $\frac{2p^2(1-p)(H-L)(I-L)c}{[p(H-L)-(1-p)c]\left[p(2-p)(H-L)-2(1-p)^2c\right]}$, which is always positive. Thus, joint project financing with coinsurance has lower expected audit cost and dominates project finance.

To determine the driver of this result, we compare the audit probabilities in the two scenarios, $m_0^J\,(1+m_{0,L})-2m_0^S=\frac{2p(I-L)[p(H-L)-2(1-p)]c}{[p(H-L)-(1-p)c][p(2-p)(H-L)-2(1-p)^2c]}$. This difference is positive, thus indicating that there is more intensive audit under joint financing. It follows that the dominance of joint finance with coinsurance over project finance can be ascribed to the lower probability with which default occurs ($(1-p)^2$ under joint financing rather than $1-p$ under project financing), and thus the lower frequency with which audit is applied, along with the pooling of returns implied by Condition 2, that allows concentration of audit only on reports of two fails. This result in which an intensive audit is applied with a low frequency is reminiscent of Becker (1968) in which maximum deterrence is obtained at minimal cost by inflicting a high punishment with a sufficiently low probability.

We can thus state the following proposition:

Proposition 4 *Under coinsurance, joint financing always dominates single financing.*

This result is in line with that obtained by Diamond (1984) and also with Banal-Estañol, Ottaviani and Winton (2013) for the case in which coinsurance gains arise from joint financing.

5.2 Efficiency under contagion

We next consider the case in which the joint second-best contract displays contagion, i.e., it is not possible to meet the lender's participation constraint by pooling the top two returns and auditing only reports of zero successes, even deterministically ($m_0^J=m_{0,L}=1$). Extra-resources must thus be raised from the two success outcome, which implies that also reports of one success must be audited: $m_1, m_{1,H}>0$.

Overall, by comparing the expected audit cost under single and joint financing, (Eqs. 6 and 16), we get:

$$\underbrace{2m_0^S}_{\text{single}}(1-p)\,c-\left[\underbrace{(1-p)^2\,m_0^J\,(1+m_{0,L})+2p\,(1-p)\,m_1\,(s)\,[1+(1-s)\,m_{1,H}]}_{\text{joint}}\right]c, \tag{17}$$

which, using m_0^S as defined in (5), $m_0^J=m_{0,L}=m_{1,H}=1$ and $m_1\,(s)$ as defined in (15), can also be written as $\frac{2p(H-L)\{(1+(2-p)(1-s))[p(H-L)-(1-p)c]-(1+2(1-s))(I-L)\}(1-p)c}{[p(H-L)-(1-p)c][p(H-L)-2(1-p)(2-s)c]}$. The sign of this expression depends on the degree of information disclosure s. Focusing on the no information disclosure case ($s=1/2$), the difference in expected profits (17) reduces to $\frac{\{(2-p/2)[p(H-L)-(1-p)c]-2(I-L)\}2p(H-L)(1-p)c}{[p(H-L)-3(1-p)c][p(H-L)-(1-p)c]}$, whose sign, given that the denominator is positive, depends on the sign of the numerator. In particular, joint finance dominates single finance iff

$$\left(2-\frac{p}{2}\right)[p\,(H-L)-(1-p)c]-2\,(I-L)>0. \tag{18}$$

We can thus state Proposition 5.

Proposition 5 *Under contagion and no information disclosure ($s=1/2$), for given probability of success p and audit cost c, joint financing dominates single financing if Condition 18 holds.*

To disentangle the determinants of such result, notice that a novel trade off emerges under contagion. On one side there is a higher cost due to the audit of successful projects. On the other side, however, there is a saving in audit costs due to the optimally chosen random audit. But rather than driven by the lower probability with which default occurs, as in the coinsurance case, such saving is driven by the minimal audit of reports of one success. Indeed, unlike the case with coinsurance described in Section 5.1, the probability with which an audit occurs when $s=1/2$ is actually higher under joint than single financing (from (17) $(1-p)^2+2p\,(1-p)>1-p$). Given that a report of zero successes is audited with probability one ($m_0^J=m_{0,L}=1$), it turns out that in our setting the saving in expected audit costs relative to single financing may be ascribed to the random (and minimal) audit of reports of one success and one fail.

These results contrast with Banal-Estañol, Ottaviani and Winton (2013) who show that under contagion single financing dominates joint financing. This is to be ascribed to the different assumptions regarding the audit strategy. In Banal-Estañol, Ottaviani and Winton (2013), **audits are deterministic. In particular, any time the borrower cannot repay the loan in full, the corporation defaults**

and the ownership of the projects' realised returns is transferred to the creditor who is only able to recover a fraction of them. The default costs are then given by the fraction of returns that cannot be recovered and includes a fraction of the high state returns, a loss that would never occur if each project were financed separately.[5]

In our setting, audits are chosen optimally and are random. Thus, unlike deterministic audits, not any time the borrower is unable to repay the loan verification occurs. In addition, under joint financing audits are concentrated in states which are less likely to occur and minimal in intermediate states. The lower frequency of audits allows a saving in audit cost relative to the separate financing case that might offset the extra cost of auditing successful projects under one fail reports.

We use a graphical analysis to show the parameter space in which single finance dominates joint finance (Fig. 7). To do this, notice that, for the comparisons to be meaningful, both standalone finance and joint finance with contagion must be feasible, while joint finance with coinsurance must be infeasible. Thus, in the space of $H-L$ and $I-L$, since expression (18) must satisfy both Conditions 1 and 3, while Condition 2 must be violated, we are focusing on the area to the right of Condition 2 and to the left of Condition 3. The locus of exogenous parameters where single and joint finance are indifferent (pink line labelled indifference line) has intercept $(1-p)\,c/p$ and slope $4/p\,(4-p)$. Thus, for $H-L$ high relative to $I-L$, i.e., above the indifference line 18, joint finance is superior to single, for given p, c, while single finance is superior otherwise. Note that the value of L itself does not matter. It is the spread $H-I$ which is crucial since this controls the amount of cross pledging between the high and low return outcomes.

[5] However, their results also hold with a more general structure of default costs, provided there are not too extreme diseconomies of scale in default (Banal-Estañol and Ottaviani, 2013).

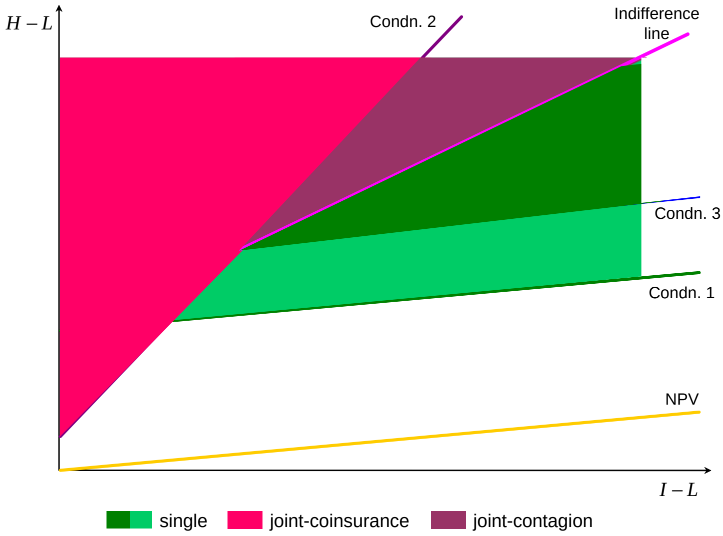

Fig. 7: Single vs joint finance

5.3 Testable predictions

Proposition 5 shows that the choice between single or joint financing is affected by four key parameters: the size of the investment cost I relative to the high state return H, the probability of success p, the quality of information s and the audit cost c.

In the following, we derive comparative statics predictions with respect to changes in the characteristics of the projects in order to investigate whether the joint financing area expands or shrinks as these characteristics vary.

We start by looking at the effects of a change in the probability of success, p. This affects both the intercepts and the slopes of Conditions 1, 2 and 3, as well as the indifference condition (18). In particular the intercepts increase and the slope decreases, widening the area where both joint and separate financing arise. This can be stated in the following prediction.

Prediction 1 *For higher probability of success, (i) both single and joint financing are feasible for a larger region of parameters and (ii) joint finance is optimal for a larger region of the remaining parameters.*

This prediction is consistent with a similar prediction obtained by Banal-Estañol, Ottaviani and Winton (2013). If the probability of success increases, the expected return pledgeable to creditors also increases and it becomes easier to finance projects.

Prediction 2 *Joint finance is more likely to arise when the investment cost is low or the return in case of success is high.*

Prediction 3 *For higher audit cost c, both single and joint financing are feasible for a smaller region of parameters and (ii) joint finance is optimal for a smaller region* *of the remaining parameters.*

Also this prediction is consistent with a similar one obtained by Banal-Estañol, Ottaviani and Winton (2013) and with evidence that merger activity is less likely in countries with weaker investor protection. Rossi and Volpin (2004), in particular, show that improvements in judicial efficiency and creditor rights significantly increase M&A activity. Along a similar line, Subramanian and Tung (2016) show that project financing is more frequent in countries with less efficient bankruptcy procedures and weaker creditor rights.

Prediction 4 *For a higher degree of information disclosure s, (i) single (joint) financing is feasible for a smaller (larger) region of parameters and (ii) joint finance is optimal for a larger range of parameters.*

This prediction, that, to the best of our knowledge, has not been formulated before, is consistent with the evidence by Rossi and Volpin (2004) showing that improvements in accounting standards significantly increase M&A activity. The channel identified by Rossi and Volpin (2004), through which higher disclosure increases M&A, goes through the identification of potential targets. Our results point to another channel, namely, the reduction in audit cost implied by an improved disclosure. It is also consistent with the evidence by Bris and Cabolis (2008) who show that the merger premium in cross-border mergers is positively related to the quality of the accounting standards in the acquiror's country.

6 Robustness

In this section we consider the relevance of the arguments to more general settings.

6.1 The optimal regulatory regime

Throughout the analysis, we have captured the degree of information disclosure between the firm and the investors using the categorical variable s. In the following, we determine which would be the optimal level of information disclosure, *if this were to be determined endogenously.* To this aim, we differentiate the profit function (16) with respect to s :

$$\frac{\partial EP(s)}{\partial s} = \frac{p\left[p(H-L) - 2(1-p)c\right] - 2\left[pH + (1-p)L - I - (1-p)c\right]}{\left[p(H-L) - (2-s)2(1-p)c\right]^2}.$$

The sign of the derivative depends on the sign of the term in the numerator, which coincides with the numerator of $m_1(s)$ (15). Since this is positive, the sign of the derivative is positive and optimally $s = 1$. Intuitively, under no information disclosure, there is an inefficiency due to the costly audit of successful projects. To minimise it, it is optimal for the firm to fully disclose information so as to allow lenders to target audit on failing projects.

6.2 Efficiency under contagion and full information disclosure

Having determined the optimal degree of information disclosure, we now replicate the efficiency analysis carried out in Section 5.2 assuming that $s = 1$, so as to investigate whether and to which extent this affects the results.

Under full information disclosure, the difference in audit costs (17) reduces to $\frac{p(H-L)[pH+(1-p)L-I-(1-p)c]2(1-p)c}{[p(H-L)-2(1-p)c][p(H-L)-(1-p)c]}$, which is strictly positive under Condition 1.[6] Thus, even in the case in which Condition 2 is violated and a coinsurance contract is infeasible, under full information disclosure joint finance dominates single finance.

Proposition 6 *Under contagion and full information disclosure, joint financing always dominates single financing.*

Intuitively, besides the cost saving due to the endogenous (and stochastic) audit probability, the possibility of disentangling the successful project from the failing one, targeting audits only on fail reports, rules out the extra inefficiency from investigating the successful project, and thus the monetary loss associated with contagion arising in the no information disclosure case. Thus, so long as audit of a single fail report is stochastic, joint financing always dominates single financing. In the extreme case in which the audit of a single success report is deterministic ($m_1 = 1$), the borrower is indifferent between financing the projects separately or jointly.

6.3 Simultaneous audit

In Section 5.2.2, we have shown that, under no information disclosure, the superiority of joint over single finance depends on a trade-off between the saving in audit cost due to random audit within an enlarged state space and the possible extra audit cost due to the unnecessary second stage audit of a successful project when the first stage audit has revealed a fail. Under simultaneous audits, lacking the leakage of information of

[6] Conditions 1 and 3 coincide under full information disclosure.

sequential audits, such extra audit cost is incurred with certainty. One may then think that the benefit of joint financing is offset by such extra-cost. To see whether this is the case, we compare the gains from joint finance with a required simultaneous audit of the projects and those from single finance. We find that joint finance may still dominate single finance, but that its advantage is reduced due to the impossibility of using the first audit to inform the second. In particular, under no information disclosure the difference between the expected profits under joint finance with simultaneous audit and single finance is equal to $\frac{2p(H-L)\{(3-p)[p(H-L)-(1-p)c]-3(I-L)\}(1-p)c}{[p(H-L)-4(1-p)c)][p(H-L)-(1-p)c)]}$, whose sign, given that the denominator is positive, depends on the sign of the numerator.[7]

We can portray this geometrically in an extension of Fig. 7 showing that the indifference line 18 at which single finance is as costly as joint finance shifts to the left. This allows us to derive a further testable prediction.

Prediction 5 *Under simultaneous audits, joint finance is optimal for a smaller region of parameters.*

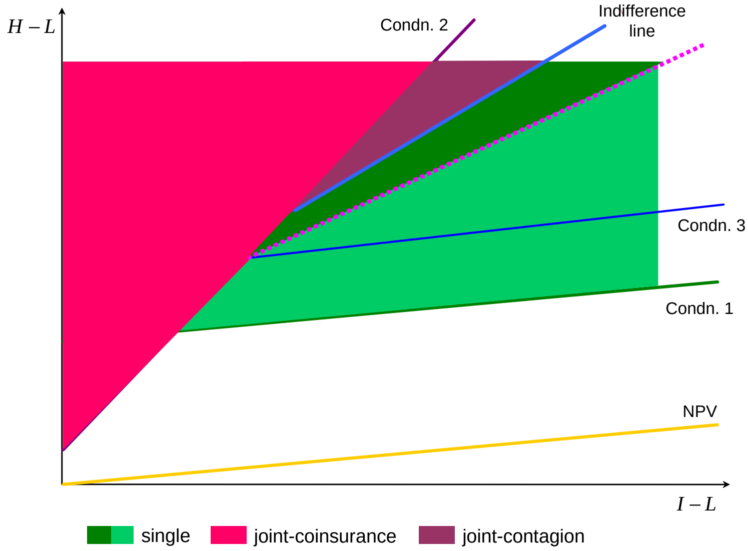

Fig. 8: Single vs joint finance under simultaneous audit

6.4 No commitment

We have assumed commitment, i.e., the lender carries out the audit strategy announced in the contract even though he knows that there is always truthtelling. If the lender is an intermediary in turn financed by shareholders, then shareholders can hold the lender to account to ensure audits are fulfilled. In a repeated contract setting, the lender could get away once with not carrying out his announced audit strategy. But in the next round the borrower should start anticipating that maybe if

[7]The full proof of the analysis is available upon request.

he cheats the lender will not monitor as stated in the contract. An alternative, even in a one shot contract, is that there is no commitment. After writing the contract, the lender can readjust his audit probability simultaneously with the borrower deciding what report to make. Typically this leads to a Nash equilibrium outcome of a non-cooperative game (Menichini and Simmons, 2006).

An alternative approach to ensuring commitment is to add a renegotiation proof constraint on the lender. Here the ex-ante contract is restricted to satisfy a renegotiation-proof constraint which removes any incentive for the ex-ante uninformed principal to change his action from that contracted once he has learnt the agent's action. For example, in our loan setting the ex-ante contract induces truthful borrower reports via the audit strategy, the lender knows the reports are truthful and so after receiving a low report has no incentive to audit. Generally in a loan contracting scenario this gives motivation for pooling repayments across true project outcomes in the ex-ante contract to prevent information revelation to the lender. This tends to favour aspects of a standard debt contract (Krasa & Villamil, 2000).

6.5 More than two projects

We have assumed only two projects and a single investor with sufficient funds to finance both of them. We found that joint finance increases the number of states allowing more precisely targeted audit. With more than two projects, the number of states will further increase, the monotonicity of audit probability and state should continue, and the superiority of joint over single project finance magnified. We conjecture some of the details of this. With n projects, if each is financially viable in the sense that Condition 1 holds, then any subset of the n projects can be jointly financed. The types of pooling of repayments across states is much more diverse with n projects. Choosing any k^* $(1 \leq k^* \leq n-1)$ the pooling of repayments for reports of $k \geq k^*$ at a level equal to the cash flows at k^*, taking all the reported revenue from audited states with $k < k^*$, may cover the joint investment cost nI and the certain audit cost of $k^* - 1$ projects. In the sense of Condition 2 above, for each such k^* we have a k^* pooling condition $PF(k^*)$ which should allow pooling of repayments above k^*. Then it should follow that if projects are k^*-feasible in this sense they are also $k^* + 1$-feasible.

The audit cost of such a joint finance contract is $\Pr(k < k^*)c$ and hence the lowest deadweight loss pooling contract will involve the lowest possible k^* allowing the maximum degree of pooling. This will be characterised by $R_k = kH + (n-k)L$ for $k \leq k^* - 1$ and to be incentive compatible must have positive audit chance below

k^*. And indeed, from the description above, we expect $m_k = 1, k < k^*$. With n projects the chance of extreme number of fails/succeses falls, eg., the chance of k fails is $(1-p)^k$, which falls with k. So one expects that reports of fewer successes will be audited more intensively. For $k > k^*$, $m_k = 0$ and $R_k = k^*H + (n-k^*)L$. But if no coinsurance contract is possible, i.e., if the equivalent of Condition 2 fails to hold, but the equivalent of Condition 1 still holds, the optimal joint finance contract cannot involve pooling.

This sketch extends the idea of pooling states and lowering expected audit cost to the multiproject case ($n > 2$). The close relation to a standard debt contract is clear.

7 Conclusion

The paper has proposed a rationale for joint financing based on the reduction of audit costs. We have shown that such reduction holds not only when joint financing generates coinsurance benefits, but also when it generates contagion costs. This is obtained through the enlargement of the reporting space, with an intensive audit of the collective worst outcomes, less likely to occur, and a lower or no intensive audit of the intermediate outcomes. As a result, for certain regions of the parameter space, the resulting optimal joint finance contract is a standard debt contract. The results are robust to the timing of audit -simultaneous rather than sequential- and, consistent with empirical evidence, stronger the better the quality of the accounting standards.

With several independent projects, the forces we identify should remain and the type of mechanism in which the reduced probability of a joint failure goes together with the highest audit frequency should lead to deterministic audit of the worst outcomes and no audit of the remaining ones, with a debt contract for the conglomerate emerging endogenously for a sufficiently large number of projects. Moreover, an increase in the correlation of returns should even favour joint financing through the audit cost saving to sequential audit, as knowing the outcome on one project provides information about the outcome of the other. We leave the development of these extensions to future research.

A Appendix

Proof of Proposition 1 Using maximum punishment ($R_{0|H} = H$) in the optimisation problem $\mathcal{P}^P$ and forming a Lagrangian with multiplier λ and μ, the FOC's wrt R_1, $R_{0|\cdot}$, $R_{0|L}$ and m_0^S are

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial R_1} &: (\lambda - 1) p - \mu \geq 0, R_1 \leq H \\
\frac{\partial \mathcal{L}}{\partial m_0^S} &: (1-p)\left(R_{0|\cdot} - R_{0|L}\right)(1-\lambda) - \lambda (1-p) c + \mu \left(H - R_{0|\cdot}\right) \geq 0, m_0^S \leq 1 \\
\frac{\partial \mathcal{L}}{\partial R_{0|L}} &: (\lambda - 1) m_0^S (1-p) \geq 0, R_{0|L} \leq L \\
\frac{\partial \mathcal{L}}{\partial R_{0|\cdot}} &: \left(1 - m_0^S\right)\left[(\lambda - 1)(1-p) + \mu\right] \geq 0, R_{0|\cdot} \leq L
\end{aligned}$$

1. $\lambda > 1$.
   Suppose $\lambda = 1$. Then by $\frac{\partial \mathcal{L}}{\partial R_1}$, $\mu = 0$. By $\frac{\partial \mathcal{L}}{\partial m_0^S}$, this implies $-\lambda (1-p) c \leq 0$, a contradiction, since $\frac{\partial \mathcal{L}}{\partial m_0^S} \geq 0$.

2. $R_{0|L} = R_{0|\cdot} = f_L$.
   By $\lambda > 1$, $\frac{\partial \mathcal{L}}{\partial R_{0|L}}, \frac{\partial \mathcal{L}}{\partial R_{0|\cdot}} > 0$ and $R_{0|L} = R_{0|\cdot} = f_L$.

3. $R_1 < H$
   Using $R_{0|L} = R_{0|\cdot} = L$, $R_{0|H} = H$ and $m_0^S = \frac{R_1 - L}{H - L}$ from the incentive constraint, the contract problem becomes to choose R_1 to max $p(H - R_1) \mid$ st $pR_1 + (1-p)\left(L - \frac{R_1 - L}{H-L} c\right) = I$. The objective function is decreasing in R_1, while the participation constraint is increasing in it, provided Condition 1 holds ($\frac{\partial PC}{\partial R_1} = \frac{1}{H-L}\left[p(H-L) - (1-p)c\right]$). R_1 is then obtained by solving the participation constraint, giving $R_1 = \frac{(H-L)I - (1-p)L(H-L+c)}{p(H-L)-(1-p)c}$. Substituting out in m_0^S, gives m_0^S (5). For $m_0^S < 1$, $pH + (1-p)L - I - (1-p)c > 0$, which certainly holds under Condition 1. This in turn implies from (3) that $R_1 < H$. The expected return to the borrower (6) is obtained using the solutions to the programme set out above in the objective function.

■

Proof of Proposition 2

1. Maximum punishment for false reports
   From programme $\mathcal{P}^{\text{seq}}$ we see that the punishment repayments $R_{1|HH}$, $R_{0|HH}$, $R_{0|H\cdot}$, $R_{0|HL}$, $R_{0|LH}$ only enter the incentive constraints. So, by setting maximum punishment, the right hand side of these increases and either m_0^J or m_1, $m_{1,H}$ or both can be reduced. For example if $R_{0|HH} < 2H$, then we can increase $R_{0|HH}$ and reduce m_0^J keeping $m_0^J m_{0,H} R_{0|HH}$ constant. This raises the right hand side of (9) because it raises $\left(1 - m_0^J\right) R_{0|\cdot}$ and slackens (8) due to the decreased frequency of the audit cost $m_0 c$. In turn this allows a reduction in R_2. Similar arguments apply to increases in $R_{0|H\cdot}$ and $m_{0,H}$ keeping $(1 - m_{0,H}) R_{0|H\cdot}$ constant and variations in $R_{1|HH}$ (increase) and m_1 (decrease) keeping $m_1 m_{1,H} R_{1|HH}$ constant, in $R_{0|HL}$ (increase) and m_0^J (decrease) keeping $m_0^J m_{0,H} R_{0|HL}$ constant, and $R_{0|LH}$ (increases) and $m_{0,L}$ (decreases) keeping

$m_0^J m_{0,L} R_{0|LH}$ constant. Thus, $R_{1|HH} = R_{0|HH} = 2H$, $R_{0|H\cdot} = R_{0|HL} = R_{0|LH} = H + L$. Given these, $m_{0,H}$ only enters the right hand side of (9) and is increasing in it. So we can set $m_{0,H} = 1$.

2. $R_{0|L\cdot} = R_{0|\cdot} = R_{0|LL} = 2L$.
If $R_{0|L\cdot} < 2L$ and $R_2 > 0$ we can reduce R_2 and raise $R_{0|L\cdot}$ so that $p^2 R_2 + (1-p)^2 m_0^J (1 - m_{0,L}) R_{0|L\cdot}$ stays constant, leaving both the objective function and the participation constraint unchanged. This slackens the incentive constraints, allowing a reduction in m_0. Similarly, we can reduce R_2 and raise $R_{0|\cdot}$ so that $p^2 R_2 + (1-p)^2 \left(1 - m_0^J\right) R_{0|\cdot}$ stays constant, leaving both the objective function and the participation constraint unchanged, while slackening the incentive constraints. We know $R_2 > 2L > 0$ since if $R_2 \leq 2L$ there is insufficient revenue to recoup the investment cost. Hence such reductions in R_2 are always possible. The result is $R_{0|L\cdot} = R_{0|\cdot} = 2L$. $R_{0|LL}$ only appears in the objective function and the participation constraint. Using $R_{0|\cdot} = R_{0|L\cdot} = 2L$, we have that lowering R_2 and raising $R_{0|LL}$ so as to keep $p^2 R_2 + (1-p)^2 m_0 m_{0,L} R_{0|LL}$ constant leaves both the objective and the participation constraint unchanged, while slackening the first and second incentive constraint. So, also $R_{0|LL} = 2L$.

3. $m_0 > 0$
Using the results of points 1 and 2, the contract problem becomes ($\mathcal{P}^{\text{seq}'}$):

$$\max EP^J(s) = p^2 (2H - R_2) + 2p(1-p) \left\{ H + L - (1 - m_1) R_{1|\cdot} - \right.$$
$$\left. m_1 \left[s R_{1|L} + (1-s) \left(m_{1,H} R_{1|HL} + (1 - m_{1,H}) R_{1|H\cdot} \right) \right] \right\},$$

$$EP^L(s) = p^2 R_2 + 2p(1-p) \left\{ (1 - m_1) R_{1|\cdot} + \right.$$
$$\left. m_1 \left[\left(s R_{1|L} + (1-s) \left(m_{1,H} \left(R_{1|HL} - c \right) + (1 - m_{1,H}) R_{1|H\cdot} \right) \right) - c \right] \right\} +$$
$$(1-p)^2 \left[2L - m_0^J (1 + m_{0,L}) c \right] \geq 2I.$$

$$R_2 \leq 2\left(1 - m_0^J\right) L + 2 m_0^J H \tag{19}$$
$$R_2 \leq (1 - m_1) R_{1|\cdot} + m_1 \left[2 m_{1,H} H + (1 - m_{1,H}) R_{1|H\cdot} \right] \tag{20}$$

$$(1 - m_1) R_{1|\cdot} + m_1 \left[s R_{1|L} + (1-s) \left(m_{1,H} R_{1|HL} + (1 - m_{1,H}) R_{1|H\cdot} \right) \right] \leq$$
$$\left(1 - m_0^J\right) R_{0|\cdot} + \frac{m_0^J}{2} \left[H + L + m_{0,L} (H + L) + (1 - m_{0,L}) R_{0|L\cdot} \right]. \tag{21}$$

If $m_0 = 0$, the first incentive constraint would give $R_2 \leq 2L$, which is less than $2I$. So we must have $m_0 > 0$. Moreover, constraint (21) must be binding. If not, it would be possible to lower $m_{0,L}$ slackening the participation constraint, thus allowing a reduction in R_2.

4. $m_0 = 1$, $m_1 = m_{1,H} = 0$
The variables are R_2, $R_{1|\cdot}$, $R_{1|H\cdot}$, $R_{1|HL}$, $R_{1|L}$, m_0, $m_{0,L}$, m_1, $m_{1,H}$. We know that $R_2 > 2L$ to provide sufficient expected revenue to repay the debt. Moreover, $m_{0,L} \geq 0$. So we can eliminate these two variables from the binding participation constraint and the binding incentive constraint (21), obtaining

$$m_{0,L} = \frac{2\left[sR_{1|L}+(1-s)\left(m_{1,H}R_{1|HL}+\left(1-m_{1,H}\right)R_{1|H\cdot}\right)\right]m_1}{m_0(H-L)} - \frac{m_0(H-L)-2(1-m_1)R_{1|\cdot}+4L}{m_0(H-L)},$$

$$\begin{aligned} R_2 \quad = \quad & \frac{-2(1-p)[p(H-L)-(1-p)c]\left\{m_1\left[sR_{1|L}+(1-s)\left(m_{1,H}R_{1|HL}+\left(1-m_{1,H}\right)R_{1|H\cdot}\right)\right]+(1-m_1)R_{1|\cdot}\right\}}{p^2(H-L)} \\ & +\frac{2(1-p)\left[1+(1-s)m_{1,H}\right]m_1c}{p} + \frac{2\left(I-(1-p)^2L\right)}{p^2} - \frac{4(1-p)^2Lc}{p^2(H-L)}. \end{aligned}$$

Substituting them out in the objective function (obj) and in the incentive constraints 19 and 20 (IC_1, IC_2) leaves the variables $R_{1|\cdot}$, $R_{1|L}$, $R_{1|H\cdot}$, $R_{1|HL}$, m_0, m_1, $m_{1,H}$. Starting from any feasible position in the variables, we can locally vary all the variables in ways which keep each constraint unchanged (thus requiring $dIC_1 = dIC_2 = 0$) and see which directions of change will improve the objective function $(dobj)$. This requires the variations to satisfy

$$\begin{aligned} dIC_i = & \frac{\partial IC_i}{\partial R_{1|\cdot}}dR_{1|\cdot} + \frac{\partial IC_i}{\partial R_{1|H\cdot}}dR_{1|H\cdot} + \frac{\partial IC_i}{\partial R_{1|HL}}dR_{1|HL} + \frac{\partial IC_i}{\partial R_{1|L}}dR_{1|L} \\ & + \frac{\partial IC_i}{\partial m_0}dm_0 + \frac{\partial IC_i}{\partial m_1}dm_1 + \frac{\partial IC_i}{\partial m_{1,H}}dm_{1,H} = 0, \;\; i = 1, 2 \end{aligned}$$

We use this to express local variations in $R_{1|\cdot}$, $R_{1|L}$ in terms of the variations in $R_{1|H\cdot}$, $R_{1|HL}$, m_0, m_1, $m_{1,H}$. Finally, we see the effect on the objective function:

$$dobj = \frac{\partial obj}{\partial R_{1|L}}dR_{1|L} + \frac{\partial obj}{\partial R_{1|\cdot}}dR_{1|\cdot} + \frac{\partial obj}{\partial R_{1|L}}dR_{1|HL} + \frac{\partial obj}{\partial R_{1|\cdot}}dR_{1|H\cdot} + \frac{\partial obj}{\partial m_0}dm_0 + \frac{\partial obj}{\partial m_1}dm_1 + \frac{\partial obj}{\partial m_1}dm_{1,H}.$$

Substituting in the variations in $dR_{1|\cdot}$ and $dR_{1|L}$ which ensure that IC_1 and IC_2 hold, we get:

$$\begin{aligned} \frac{dobj}{dm_0} &= \frac{2p^2(1-p)(H-L)c}{p(H-L)-(1-p)c} > 0 \\ \frac{dobj}{dm_1} &= -\frac{2p^2(1-p)\left[1+m_{1,H}(1-s)\right]c}{p(H-L)-(1-p)c} < 0 \\ \frac{dobj}{dm_{1,H}} &= -\frac{2p^2(1-p)m_1(1-s)(H-L)c}{p(H-L)-(1-p)c} < 0 \end{aligned}$$

Thus, the objective function can be increased by increasing m_0 and reducing m_1 and $m_{1,H}$.
The solution has $m_0 = 1$ and $m_1 = m_{1,H} = 0$, so long as the implied $R_{1|\cdot}$, $R_{1|L}$, $R_{1|H\cdot}$, $R_{1|HL}$, $R_2 \geq 0$, $R_2 < 2H$, $R_{1|L}$, $R_{1|\cdot}$, $R_{1|H\cdot}, R_{1|HL} \leq H + L$, $m_{0,L} \leq 1$, and there are sufficient revenues to repay the debt cost.
Using $m_0 = 1$ and $m_1 = m_{1,H} = 0$ in the incentive constraints (19) and (20), we get $R_{1|\cdot} \leq 2H$ and $R_2 \leq R_{1|\cdot}$.Because $R_{1|\cdot} \leq H + L < 2H$, constraint (19) is

always slack and can be ignored. Moreover, because of monotonicity of repayments, from constraint (20) we deduce that $R_2 = R_{1|\cdot}$. Last, because $m_1 = 0$, $R_{1|L}, R_{1|HL}$ and $R_{1|H\cdot}$ are never paid and can be set to any value between 0 and $H + L$.
Using $m_0 = 1$ and $m_1 = m_{1,H} = 0$ in the solved out values of $m_{0,L}$ and R_2 and using $R_2 = R_{1|\cdot}$, we get $m_{0,L} = \frac{4(I-L)-p(2-p)(H-L)+2(1-p)^2c}{p(2-p)(H-L)-2(1-p)^2c}$, $R_{1|\cdot} = \frac{2(I-L)(H-L)}{p(2-p)(H-L)-2(1-p)^2c} + 2L$, as reported in points 1 and 2 of the proposition.
We next verify that $m_{0,L} \leq 1$ and $R_{1|\cdot} \leq H+L$. For these we need $p\,(2-p)\,(H-L) - 2\,(I-L) - 2\,(1-p)^2\,c \geq 0$.
Substituting out $R_2 = R_{1|\cdot}$ derived above in the objective function (7) we get the expected profits (14) as reported in point 4 of the proposition.

5. If equating R_2 and $R_{1|\cdot}$ (so allowing $m_1 = m_{1,H} = 0$) and setting $R_{1|\cdot} = H + L$ and $m_{0,L} = m_0 = 1$ fails to raise the revenue to meet the participation constraint (i.e., $p\,(2-p)\,(H-L) - 2\,(I-L) - 2\,(1-p)^2\,c < 0$), then extra revenue must be raised from a two successes outcome, which in turn requires $m_1, m_{1,H} > 0$ and $R_2 > R_{1|\cdot} = H + L$.
The problem is to choose $R_2, R_{1|L}, R_{1|HL}, R_{1|H\cdot}$ and the minimal $m_1, m_{1,H}$ which allows the participation constraint to be satisfied. This will minimise the deadweight loss of audit whilst meeting the participation constraint. Setting $m_{0,L} = m_0 = 1$ and $R_{1|\cdot} = H + L$ and allowing for $R_2 > R_{1|\cdot}$ and $m_1, m_{1,H} > 0$ in problem $\mathcal{P}^{\text{seq}'}$, the contract problem becomes ($\mathcal{P}^{\text{weak}}$):

$$\max p^2\,(2H - R_2) + 2p\,(1-p)\,m_1\left\{H + L - sR_{1|L} - \right.$$
$$\left. + (1-s)\left(m_{1,H}R_{1|HL} + (1-m_{1,H})\,R_{1|H\cdot}\right)\right\}$$

$$\text{st } p^2R_2 + 2\,(1-p)^2\,(L-c) + 2p\,(1-p)\,\{(1-m_1)\,(H+L) + \tag{22}$$
$$+ m_1\left[sR_{1|L} + (1-s)\left(m_{1,H}\left(R_{1|HL} - c\right) + (1-m_{1,H})\,R_{1|H\cdot}\right) - c\right]\} \geq 2I$$

$$R_2 \leq 2H \tag{23}$$

$$R_2 \leq (1-m_1)\,(H+L) + m_1\left[2m_{1,H}H + (1-m_{1,H})\,R_{1|H\cdot}\right] \tag{24}$$

$$m_1\left[sR_{1|L} + (1-s)\left(m_{1,H}R_{1|HL} + (1-m_{1,H})\,R_{1|H\cdot}\right)\right] +$$
$$+ (1-m_1)\,(H+L) \leq H + L. \tag{25}$$

6. $R_{1|H\cdot} = R_{1|HL} = R_{1|L} = H + L$.
If $R_{1|H\cdot} < H + L$ and $R_2 > H + L$ we can reduce R_2 and raise $R_{1|H\cdot}$ so that $p^2R_2 + 2p\,(1-p)\,(1-m_{1,H})\,(1-s)\,R_{1|H\cdot}$ stays constant, i.e., both the objective function and the participation constraint are unchanged. This slackens the first incentive constraint, while not violating the third incentive constraint, which is satisfied when $R_{1|H\cdot}$ is evaluated at its highest value, $H + L$. Thus, $R_{1|H\cdot} =$

$H+L$. For a similar argument, $R_{1|L}, R_{1|HL}$ can be increased up to $H+L$, while reducing R_2 in a way to keep both the objective function and the participation constraint unchanged. This does not violate the third incentive constraint, which is still satisfied when $R_{1|L}, R_{1|HL}$ are evaluated at their highest value, $H+L$. In each case, the relaxation of the incentive constraints, especially (24), allows a reduction in m_1. Thus, $R_{1|H\cdot} = R_{1|HL} = R_{1|L} = H+L$.
Notice that if constraint (24) is satisfied, then certainly constraint (23) is. So we can ignore (23). Moreover, using, $R_{1|H\cdot} = R_{1|HL} = R_{1|L} = H+L$, constraint (25) is satisfied (it becomes: $H+L \leq H+L$) and can be dropped.
The contract problem can then be written as:

$$\max p^2 (2H - R_2)$$

$$\text{st } p^2 R_2 + 2p(1-p)\{(H+L) - m_1 [1 + (1-s) m_{1,H}] c\} \tag{26}$$
$$+2(1-p)^2 (L-c) = 2I$$

$$R_2 \leq (H+L) + m_1 m_{1,H} (H-L) \tag{27}$$

7. $m_{1,H} = 1$
The monitoring probabilities $m_1, m_{1,H}$ do not enter the objective function, but only the participation and the incentive constraint. We know that both $m_1, m_{1,H}$ must be positive. An increase in either m_1 or $m_{1,H}$ slackens the incentive constraint, but increases the expected audit cost in the participation constraint. However, such increase is lower when $m_{1,H}$ is increased rather than m_1

$$\begin{aligned} \frac{\partial PC}{\partial m_1} &= -1 - (1-s) m_{1,H}, \\ \frac{\partial PC}{\partial m_{1,H}} &= -(1-s) m_1. \end{aligned}$$

Thus, it is optimal to increase $m_{1,H}$ to the maximum, $m_{1,H} = 1$.
To determine the remaining variables notice that (27) must bind since otherwise m_1 could be reduced, allowing a reduction in R_2 without violating (26). Solving 27 for m_1 gives $m_1 = \frac{R_2 - H - L}{H-L}$, which, substituted out in the participation constraint 26, gives $R_2(s)$ and $m_1(s)$ as reported in points 7 and 8 of the propositions. We next verify that $m_1(s) \leq 1$ and $R_2 \leq 2H$. For these, we need $pH + (1-p)L \geq I + (1-p)c[1 + p(1-s)]$.
Last, substituting out R_2 in the objective function we get the expected profits (16) as reported in point 9 of the proposition.

■

Proof of Proposition 3 This follows from using the results from Proposition 2 that $m_{0,L} \leq 1$ (13) and $m_1(s) \leq 1$ (15). ■

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Figure 3a describes the game tree under joint finance and full information disclosure.

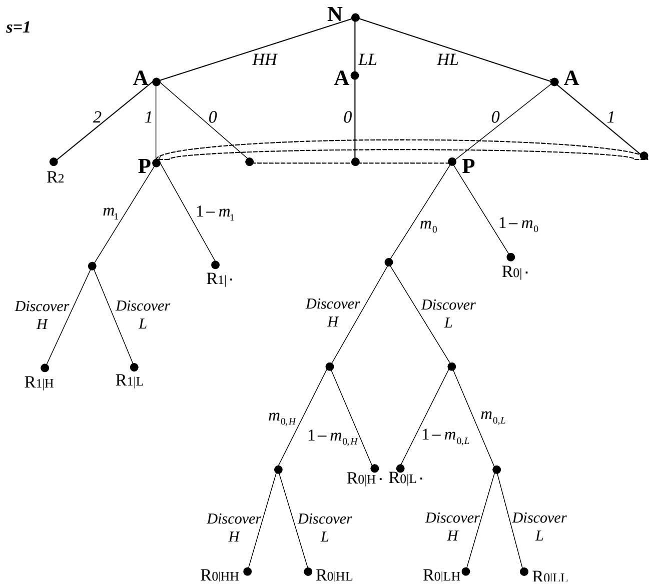

Fig. 3a. Game tree with joint finance and full information disclosure

****Upon full information disclosure ($s = 1$), the incentive constraint 10 has to be modified to take into account that, when reporting one success, the borrower also discloses which project has succeeded. This allows the lender to target audit on the reported failing one. In this latter case, $m_{1,H} = 0$ and the incentive constraint reads as

$$R_2 \leq (1 - m_1) R_{1|\cdot} + m_1 R_{1|H} \tag{28}$$

where $R_{1|H}$ is the repayment due upon an audit that reveals borrower cheating.

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