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Abstract

In this paper, we develop a unifying econometric framework for the analysis of heterogeneous panel data models that can account for both spatial dependence and unobserved common factors. To tackle the challenging issues of endogeneity caused by both the spatial lagged term and the correlation between regressors and factors, we propose to approximate common factors by cross-section averages of independent variables only, and deal with the spatial endogeneity via the instrumental variables. We develop the individual estimators as well as the Mean Group and the Pooled estimators, and establish their consistency and asymptotic normality. Monte Carlo simulations confirm that the finite sample performance of our proposed estimators are quite satisfactory. We demonstrate the usefulness of our approach with an application to a gravity model of bilateral trade flows for 91 pairs of 14 European Union (EU) countries, and find that the trade flows between the UK and EU members would fall substantially following a hard Brexit.

JEL Classification: C13, C15, C23.

Key Words: Cross Section Dependence, Heterogeneous Spatial Panel Data Model, Factor Model, Instrumental Variable Analysis.

1 Introduction

The increasingly globalised nature of the world economy and pervasive evidence of cross-section dependence (CSD) found in many empirical studies (e.g., Pesaran (2015); Mastromarco et al. (2016a,b)) have driven considerable interests in developing panel data models that can explain observed patterns of comovement among macroeconomic and financial variables. Two strands of work have been proposed in the literature. The first uses a spatial-based approach which aims to explain cross-sectional comovement by assuming that the behaviour of each spatial unit is related to the behaviour of a finite number of its neighbours. The second strand uses a factor-based approach which allows each individual to be affected by unobserved common factors with different intensities. Chudik et al. (2011) show that the common factor structure characterises strong CSD, as its influence does not die out as the number of cross-section units, N, grows, whilst the spatial dependence characterises weak CSD as its influence becomes negligible as N becomes large.

With the exception of a small number of studies, the existing literature adopts either a factor structure or a spatial dependence structure but not both. To date, limited progress has been made on the development of unified models incorporating both spatial dependence and common factors. Pesaran and Tosetti (2011) allow the error components in the panel model to be spatially dependent and contain unobserved common factors. They propose to use the common correlated effects (CCE) estimator advanced by Pesaran (2006). Mastromarco et al. (2016a) propose a technique for modelling stochastic frontier panels by combining the exogenously driven factor-based approach and an endogenous threshold regime selection mechanism advanced by Kapetanios et al. (2014). Bai and Li (2014) consider a static homogeneous spatial autoregressive panel data model with common shocks, and develop quasi maximum likelihood (QML) estimation. Using a similar modelling framework, Yang (2018) develops a consistent estimator that combines CCE and instrumental variable (IV) / generalised method of moments (GMM) estimation, see also Wang and Lee (2018). Furthermore, Bai and Li (2015) develop a bias-corrected principal components (PC) estimator for spatial dynamic panel data models with common shocks, see also Shi and Lee (2017), Lu (2017) and Kuersteiner and Prucha (2018).

All the aforementioned studies have approached a joint analysis of the spatial and the factor dependence under the simplifying assumption that the slope parameters are homogeneous. In practice, in a data rich environment, slope homogeneity is a very restrictive assumption, as the strength and even the direction of the spatial dependence between entities may vary over space and time. Slope heterogeneity allows the data to speak about the relative susceptibility of each spatial unit to external conditions. In the spatial literature, there are a few empirical studies advocating to use heterogeneous spatial coefficients. Examples include Behrens et al. (2012), who derive a gravity model of trade flows with heterogeneous spatial coefficients. Bailey et al. (2016) develop a multi-step estimation procedure that can distinguish CSD that is purely spatial from one that is due to common factors. They apply it to house price inflation data from Metropolitan

Statistical Areas in the US and find strong evidence of parameter heterogeneity. There are also a small number of theoretical papers that explicitly allow the spatial coefficient to be heterogeneous, see Aquaro et al. (2019), LeSage and Chih (2018), and Shin and Thornton (2020).

Following this research trend, our primary objective is to develop a unifying econometric framework for the estimation of heterogeneous panel data models that can jointly accommodate spatial and factor dependence. To achieve this objective, we will tackle the challenging issues for developing a consistent estimator in the presence of spatial heterogeneity as well as endogeneity caused by the spatial lagged term and the correlation between regressors and unobserved factors. In this paper, we follow Yang (2018), and propose to combine the CCE and IV estimation procedures. We first approximate unobserved common factors by the CCE approach, and then deal with the spatial endogeneity using the IV method. However, unlike almost all the studies in the CCE literature that use the cross section averages (CSA) of both dependent and independent variables, denoted by $\bar{y}_t$ and $\bar{\boldsymbol{x}}_t$, as factor proxies, we propose to use $\bar{\boldsymbol{x}}_t$ only.

In the literature on the spatial panel data models with interactive effects, most studies rely heavily on the assumption that the independent variables share the same factors as the dependent variable (e.g., Pesaran and Tosetti (2011); Bai and Li (2014, 2015); Lu (2017); Yang (2018)), which is rather restrictive and untestable. Following the macroeconomic and financial forecasting literature, e.g., Boivin and Ng (2006), Bai and Ng (2008), Kelly and Pruitt (2015), we allow the dependent and independent variables to be influenced by different sets of unobserved factors. In this general case, we develop consistent estimation directly by controlling the factor components, that are correlated with the independent variables through using $\bar{\boldsymbol{x}}_t$ only. Furthermore, as has been almost neglected in the literature, the traditional use of $\bar{y}_t$ as factor proxy in the CCE estimation could cause bias estimation due to its correlation with the idiosyncratic errors. Such correlation would be non-negligible unless N is large and can be even stronger in spatial panel data models, depending on the magnitude of the spatial coefficient and the sparsity/density of the spatial weighting matrix. Since we propose to use only $\bar{\boldsymbol{x}}_t$ as factor proxies, the factors specific to the dependent variable may be left unaccounted for. This could lead to efficiency loss, but does not affect the consistency of the proposed estimators. We show, through Monte Carlo simulations, that the efficiency gain from the use of the additional $\bar{y}_t$ does not offset the increase in bias in the sense that the root mean squared error (RMSE) for the estimator using $(\bar{y}_t, \bar{\boldsymbol{x}}_t)$ as factor proxies is similar to that for the estimator using $\bar{\boldsymbol{x}}_t$ only. Another motivation for using $\bar{\boldsymbol{x}}_t$ only is that the corresponding CCE estimator can be easily applied to nonlinear spatial panel data model with unobserved factors.[1]

We show that the individual parameters can be consistently estimated by applying the de-factored IVs directly to the original regressions. The resulting estimators are $\sqrt{T}$-consistent and follows asymptotic normal distributions. We also derive the Mean Group and the Pooled estimators, and establish that they are $\sqrt{N}$-consistent and asymptotic

[1] For example, Boneva and Linton (2017) apply the CCE using $\bar{\boldsymbol{x}}_t$ only as factor proxies to the binary choice model.

normally distributed. We then provide the nonparametric variance estimators, which are robust to the presence of heteroskedastic and serially-correlated disturbances as well as the parameter heterogeneity. These robust variance estimators provide valid inference even for models with different sets of unobserved factors. In the special case where the parameters are homogeneous, we show that the Pooled estimator is $\sqrt{NT}$-consistent. However, our results are different from Yang (2018), who assumes that the dependent and independent variables share the same common factors.

Via extensive Monte Carlo simulations, we investigate the finite sample properties of our proposed estimators using different combinations of factors approximations and (internal) IVs. Overall, we find that the Mean Group or Pooled estimator using $\bar{\boldsymbol{x}}_t$ only as factor proxies and de-factored first-order spatial lagged term of the independent variables ($\tilde{\boldsymbol{X}}^*$) as instruments, outperforms all the other alternatives considered. In particular, we find that the estimator using ($\bar{y}_t$, $\bar{\boldsymbol{x}}_t$) as factor proxies is outperformed by our proposed estimator in terms of bias, especially when N is small. The size of our proposed estimator using $\bar{\boldsymbol{x}}_t$, is close to the 5% nominal level in almost all simulation cases unless when both N and T are small. In summary, we recommend to use our proposed estimator utilising only $\bar{\boldsymbol{x}}_t$ as factor proxies in practice.

We apply our method to bilateral trade data from 14 European Union (EU) countries. One of the mos important policy issues about Brexit, which has been extensively debated in the UK, is what is the nature and right magnitude of EU trade union impact on trade flows between the UK and EU member countries. We find that in the event of a hard Brexit, the trade flows between the UK and EU members would fall as substantially as 35%.

The rest of the paper is organised as follows. Section 2 describes the model and assumptions. Section 3 develops the asymptotic theory for the individual, the Mean Group and the Pooled estimators. Section 4 presents the finite sample performance of the proposed estimators via Monte Carlo simulations. Section 5 provides an empirical application to an analysis of bilateral trade flows for 91 pairs of 14 EU countries. Section 6 concludes and discusses some extensions. The mathematical proofs are relegated to the Appendix. Lemmas used for the proofs of the main theorems and additional Monte Carlo simulation results can be found in the Online Supplement.

Notations. We use C to denote a finite positive constant, whose value may change from place to place. For an $N \times N$ real matrix, $\boldsymbol{A} = (a_{ij})$, $\|\boldsymbol{A}\| = \sqrt{tr(\boldsymbol{A}\boldsymbol{A}')}$ denotes the Euclidean (or Frobenius) norm. The row or the column sum of $\boldsymbol{A}$ is uniformly bounded in absolute value if $\|\boldsymbol{A}\|_\infty = \underset{1\leq i\leq N}{max} \sum_{j=1}^{N} |a_{ij}| < C$ or $\|\boldsymbol{A}\|_1 = \underset{1\leq j\leq N}{max} \sum_{i=1}^{N} |a_{ij}| < C$ for all N. We use $vec(\boldsymbol{A})$ to denote the vectorisation operator that stacks the columns of $\boldsymbol{A}$ into a column vector, $\lambda_{\max}(\boldsymbol{A})$ and $\lambda_{\min}(\boldsymbol{A})$ to denote the largest and the smallest eigenvalue of $\boldsymbol{A}$, respectively, and $tr(\boldsymbol{A})$ to denote the trace of $\boldsymbol{A}$. We put a dot in the subscript for a matrix(vector) whenever necessary to distinguish matrix (vector), say $\boldsymbol{A}_{1.}$, from $\boldsymbol{A}_{.1}$. The symbol $\otimes$ denotes the Kronecker product of matrices, and $(N,\ T) \xrightarrow{j} \infty$ denotes N and T tend to infinity jointly.

2 The Model and Assumptions

Consider the following heterogeneous spatial autoregressive panel data model with unobserved common factors (HSAR-F):

$$\begin{aligned} y_{it} &= \rho_i y_{it}^* + \boldsymbol{x}_{it}'\boldsymbol{\beta}_i + \boldsymbol{\gamma}_{1i}'\boldsymbol{f}_{1t} + \boldsymbol{\gamma}_{2i}'\boldsymbol{f}_{2t} + \varepsilon_{it} \\ &= \rho_i y_{it}^* + \boldsymbol{x}_{it}'\boldsymbol{\beta}_i + \boldsymbol{\gamma}_{yi}'\boldsymbol{f}_{yt} + \varepsilon_{it}, \end{aligned} \tag{1}$$

where y_{it} is the dependent variable of the i-th spatial unit at time t, $y_{it}^* = \sum_{j=1}^{N} w_{ij} y_{jt}$ is its spatial lagged variable with w_{ij} being the (i,j)-th entry of the $N \times N$ spatial weighting matrix, $\boldsymbol{W}$, ρ_i is the spatial autoregressive parameter, measuring the strength of heterogeneous spatial dependence or spillover among individuals, $\boldsymbol{x}_{it} = (x_{it,1},\ x_{it,2},\ \ldots,\ x_{it,k})'$ is a $k \times 1$ vector of independent variables and $\boldsymbol{\beta}_i$ is a $k \times 1$ vector of heterogeneous slope parameters, $\boldsymbol{f}_{1t}$ and $\boldsymbol{f}_{2t}$ are $r_1 \times 1$ and $r_2 \times 1$ vectors of unobserved factors affecting y_{it} with $\boldsymbol{\gamma}_{1i}$ and $\boldsymbol{\gamma}_{2i}$ being the corresponding factor loadings, $\boldsymbol{f}_{yt} = (\boldsymbol{f}_{1t}', \boldsymbol{f}_{2t}')'$ is an $r_y \times 1$ vector with $r_y = r_1 + r_2$ with factor loading vector specified by $\boldsymbol{\gamma}_{yi} = (\boldsymbol{\gamma}_{1i}', \boldsymbol{\gamma}_{2i}')'$, and ε_{it} is the idiosyncratic disturbance.

Next, we consider the following data generating process (DGP) for $\boldsymbol{x}_{it}$:

$$\boldsymbol{x}_{it} = \boldsymbol{\Gamma}_{1i}'\boldsymbol{f}_{1t} + \boldsymbol{\Gamma}_{2i}'\boldsymbol{f}_{3t} + \boldsymbol{v}_{it} = \boldsymbol{\Gamma}_{xi}'\boldsymbol{f}_{xt} + \boldsymbol{v}_{it}, \tag{2}$$

where $\boldsymbol{f}_{1t}$ was defined in (1), $\boldsymbol{f}_{2t}$ is an $r_3 \times 1$ vector of unobserved factors affecting $\boldsymbol{x}_{it}$ only, $\boldsymbol{f}_{xt} = (\boldsymbol{f}_{1t}', \boldsymbol{f}_{3t}')'$ is an $r_x \times 1$ vector with $r_x = r_1 + r_3$, $\boldsymbol{\Gamma}_{1i}$ and $\boldsymbol{\Gamma}_{2i}$ are $r_1 \times k$ and $r_3 \times k$ matrices of factor loadings and $\boldsymbol{\Gamma}_{xi} = (\boldsymbol{\Gamma}_{1i}', \boldsymbol{\Gamma}_{2i}')'$, and $\boldsymbol{v}_{it} = (v_{it,1},\ v_{it,2},\ \ldots,\ v_{it,k})'$ is a $k \times 1$ vector of idiosyncratic errors.

The model given by (1) and (2) is quite general. It can accommodate both weak and strong CSD through the spatial lagged term and common factors. Furthermore, it allows the dependent and independent variables to be partly influenced by different factors. It is easy to see that y_{it} and $\boldsymbol{x}_{it}$ share the common factors, $\boldsymbol{f}_{1t}$, but they also have their own specific factors, $\boldsymbol{f}_{2t}$ and $\boldsymbol{f}_{3t}$, respectively. This generalisation is more practical given that the factors are latent and we do not know, a priori, whether the dependent and independent variables share the same factors. More importantly, through slope heterogeneity, it allows the data to speak about the relative susceptibility of each spatial unit to external conditions. Our model encompasses several existing models, e.g., Pesaran (2006), Bai (2009), Aquaro et al. (2019), Bai and Li (2014) and Yang (2018).

To develop consistent estimation for the $(k+1) \times 1$ vector of parameters, $\boldsymbol{\theta}_i = (\rho_i, \boldsymbol{\beta}_i')'$, we should address two sources of endogeneity in (1), in which $\boldsymbol{x}_{it}$ are correlated with unobserved factors, $\boldsymbol{f}_{1t}$ whilst the spatial lagged term, y_{it}^*, is correlated with both unobserved factors, $\boldsymbol{f}_{yt}$ and idiosyncratic errors, ε_{it}. Existing statistical methods to deal with endogeneity have been developed separately for either the spatial dependence or a factor structure, but not both. The spatial endogeneity can be resolved by using QML estimation (Lee (2004)) or IV/GMM estimation (Kelejian and Prucha (1998, 1999). The unobserved common factors can be proxied by either PC estimates (Stock and Watson (2002); Bai (2003, 2009)) or CSA of the data (Pesaran (2006)). Recently, a few studies have attempted to combine both approaches. For example, Yang (2018) proposes to estimate a homogeneous static spatial panel data model with common shocks through combining CCE and IV/GMM analysis, while Bai and Li (2015) and Shi and Lee (2017) develop a bias-corrected QML-PC estimator for the homogeneous dynamic spatial panel data model with common shocks.

In this paper, we extend Yang (2018) and propose to use a variant of CCE estimation combined with IV analysis for consistent estimation of $\boldsymbol{\theta}_i$. Write the model (1) in matrix form:

$$\boldsymbol{y}_{.t} = \boldsymbol{\rho} \boldsymbol{W} \boldsymbol{y}_{.t} + \boldsymbol{B} \boldsymbol{\chi}_t + \boldsymbol{\gamma}_y \boldsymbol{f}_{yt} + \boldsymbol{\varepsilon}_{.t}, \tag{3}$$

where

$$\underset{(N\times 1)}{\boldsymbol{y}_{.t}} = \begin{bmatrix} y_{1t} \\ \vdots \\ y_{Nt} \end{bmatrix}, \underset{(N\times N)}{\boldsymbol{\rho}} = \begin{bmatrix} \rho_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \rho_N \end{bmatrix}, \underset{(N\times Nk)}{\boldsymbol{B}} = \begin{bmatrix} \boldsymbol{\beta}_1' & \dots & \boldsymbol{0} \\ \vdots & \ddots & \vdots \\ \boldsymbol{0} & \dots & \boldsymbol{\beta}_N' \end{bmatrix}, \underset{(k\times 1)}{\boldsymbol{\beta}_i} = \begin{bmatrix} \beta_{i1} \\ \vdots \\ \beta_{ik} \end{bmatrix},$$

$$\underset{(N\times r_y)}{\boldsymbol{\gamma}_y} = \begin{bmatrix} \boldsymbol{\gamma}_{y1}' \\ \vdots \\ \boldsymbol{\gamma}_{yN}' \end{bmatrix}, \underset{(Nk\times 1)}{\boldsymbol{\chi}_t} = \begin{bmatrix} \boldsymbol{x}_{1t} \\ \vdots \\ \boldsymbol{x}_{Nt} \end{bmatrix}, \quad \underset{(k\times 1)}{\boldsymbol{x}_{it}} = \begin{bmatrix} x_{it,1} \\ \vdots \\ x_{it,k} \end{bmatrix} \text{ and } \underset{(N\times 1)}{\boldsymbol{\varepsilon}_{.t}} = \begin{bmatrix} \varepsilon_{1t} \\ \vdots \\ \varepsilon_{Nt} \end{bmatrix}.$$

and then stack (3) over t:

$$\boldsymbol{y} = (\boldsymbol{I}_T \otimes \boldsymbol{\rho} \boldsymbol{W}) \boldsymbol{y} + (\boldsymbol{I}_T \otimes \boldsymbol{B}) \boldsymbol{\chi} + (\boldsymbol{I}_T \otimes \boldsymbol{\gamma}_y) \boldsymbol{f}_y + \boldsymbol{\varepsilon}, \tag{4}$$

where

$$\underset{(NT\times 1)}{\boldsymbol{y}} = \begin{bmatrix} \boldsymbol{y}_{.1} \\ \vdots \\ \boldsymbol{y}_{.T} \end{bmatrix}, \underset{(NTk\times 1)}{\boldsymbol{\chi}} = \begin{bmatrix} \boldsymbol{\chi}_1 \\ \vdots \\ \boldsymbol{\chi}_T \end{bmatrix}, \underset{(Tr_y\times 1)}{\boldsymbol{f}_y} = \begin{bmatrix} \boldsymbol{f}_{y1} \\ \vdots \\ \boldsymbol{f}_{yT} \end{bmatrix} \text{ and } \boldsymbol{\varepsilon} = \begin{bmatrix} \boldsymbol{\varepsilon}_{.1} \\ \vdots \\ \boldsymbol{\varepsilon}_{.T} \end{bmatrix}.$$

First, to describe how to eliminate the common factors, $\boldsymbol{f}_y$ from (4), we suppose they are observable momentarily and define the following idempotent matrices:

$$\tilde{\boldsymbol{M}}_{\boldsymbol{f}_y} = \boldsymbol{M}_{\boldsymbol{f}_y} \otimes \boldsymbol{I}_N \quad \text{and} \quad \boldsymbol{M}_{\boldsymbol{f}_y} = \boldsymbol{I}_T - \boldsymbol{F}_y (\boldsymbol{F}_y' \boldsymbol{F}_y)^{-1} \boldsymbol{F}_y', \tag{5}$$

where $\boldsymbol{F}_y = (\boldsymbol{f}_{y1}, \ \boldsymbol{f}_{y2}, \ \dots, \ \boldsymbol{f}_{yT})'$ is a $T \times r_y$ matrix. It is easily seen that

$$\tilde{\boldsymbol{M}}_{\boldsymbol{f}_y} (\boldsymbol{I}_T \otimes \boldsymbol{\gamma}_y) \boldsymbol{f}_y = vec(\boldsymbol{\gamma}_1 \boldsymbol{F}_y' \boldsymbol{M}_{\boldsymbol{f}_y}') = \boldsymbol{0}, \tag{6}$$

$$(\boldsymbol{M}_{\boldsymbol{f}_y} \otimes \boldsymbol{I}_N)(\boldsymbol{I}_T \otimes \boldsymbol{B}) \boldsymbol{\chi} = (\boldsymbol{I}_T \otimes \boldsymbol{B})(\boldsymbol{M}_{\boldsymbol{f}_y} \otimes \boldsymbol{I}_N) \boldsymbol{\chi}, \tag{7}$$

and

$$\begin{aligned} \tilde{\boldsymbol{M}}_{\boldsymbol{f}_y} (\boldsymbol{I}_T \otimes \boldsymbol{\rho} \boldsymbol{W}) \boldsymbol{y} &= (\boldsymbol{M}_{\boldsymbol{f}_y} \otimes \boldsymbol{I}_N)(\boldsymbol{I}_T \otimes \boldsymbol{\rho} \boldsymbol{W}) \boldsymbol{y} \\ &= (\boldsymbol{I}_T \otimes \boldsymbol{\rho} \boldsymbol{W})(\boldsymbol{M}_{\boldsymbol{f}_y} \otimes \boldsymbol{I}_N) \boldsymbol{y}, \end{aligned} \tag{8}$$

in which the equalities suggest that $\tilde{\boldsymbol{M}}_{\boldsymbol{f}_y} (\boldsymbol{I}_T \otimes \boldsymbol{\rho} \boldsymbol{W}) \boldsymbol{y}$ can be regarded as either the de-factored spatial lagged term or the spatial lagged term of the de-factored $\boldsymbol{y}$.

Therefore, through pre-multiplying (4) by $\tilde{\boldsymbol{M}}_{\boldsymbol{f}_y}$, we can derive the following de-factored regression:

$$\tilde{\boldsymbol{y}}_0 = (\boldsymbol{I}_T \otimes \boldsymbol{\rho W})\tilde{\boldsymbol{y}}_0 + (\boldsymbol{I}_T \otimes \boldsymbol{B})\tilde{\boldsymbol{\chi}} + \tilde{\boldsymbol{\varepsilon}}_0, \tag{9}$$

where $\tilde{\boldsymbol{y}}_0 = (\boldsymbol{M}_{\boldsymbol{f}_y} \otimes \boldsymbol{I}_N)\boldsymbol{y}$ is de-factored $\boldsymbol{y}$, and $\tilde{\boldsymbol{\chi}}$ and $\tilde{\boldsymbol{\varepsilon}}_0$ are defined similarly. It is clear that the transformed model (9) is a heterogeneous spatial autoregressive panel data model for the de-factored data, to which we can apply the IV method to consistently estimate the parameters.

Next, we describe how to obtain the internal IVs. Rewriting (9) as

$$(\boldsymbol{I}_T \otimes (\boldsymbol{I}_N - \boldsymbol{\rho W}))\tilde{\boldsymbol{y}}_0 = (\boldsymbol{M}_{\boldsymbol{f}_y} \otimes \boldsymbol{I}_N)(\boldsymbol{I}_T \otimes \boldsymbol{B})\boldsymbol{\chi} + \tilde{\boldsymbol{\varepsilon}}_0, \tag{10}$$

and assuming that $\boldsymbol{I}_N - \rho \boldsymbol{W}$ is invertible (see Assumption 4) yield

$$\begin{aligned}\tilde{\boldsymbol{y}}_0 &= (\boldsymbol{I}_T \otimes (\boldsymbol{I}_N - \rho\boldsymbol{W})^{-1})(\boldsymbol{M}_{\boldsymbol{f}_y} \otimes \boldsymbol{I}_N)(\boldsymbol{I}_T \otimes \boldsymbol{B})\boldsymbol{\chi} + (\boldsymbol{I}_T \otimes (\boldsymbol{I}_N - \rho\boldsymbol{W})^{-1})\tilde{\boldsymbol{\varepsilon}}_0 \\ &= (\boldsymbol{M}_{\boldsymbol{f}_y} \otimes \boldsymbol{I}_N)(\boldsymbol{I}_T \otimes (\boldsymbol{I}_N - \rho\boldsymbol{W})^{-1})(\boldsymbol{I}_T \otimes \boldsymbol{B})\boldsymbol{\chi} + (\boldsymbol{I}_T \otimes (\boldsymbol{I}_N - \rho\boldsymbol{W})^{-1})\tilde{\boldsymbol{\varepsilon}}_0. \end{aligned} \tag{11}$$

The equalities in (11) imply that valid instruments can be constructed by (the higher orders of) the spatial terms of the de-factored regressors and also the de-factored components of (the high orders of) the spatial terms of regressors. Without loss of generality, we focus on the latter case for the convenience of the asymptotic analysis.

The above analysis is infeasible because common factors are unobserved. Here we follow Pesaran (2006) and develop a CCE-type estimator. Note that Yang (2018) also studies a spatial panel data model with interactive effects, but she considers only homogeneous coefficients, assumes that y_{it} and $\boldsymbol{x}_{it}$ share the same common factors and then uses the CCE estimator with $(\bar{y}_t, \bar{\boldsymbol{x}}_t')'$ as factor proxies. However, we show that the use of $\bar{y}_t$ may result in bias in the presence of spatial dependence, unless N is substantially large. To illustrate, consider the homogeneous panel data model with unobserved factors:

$$y_{it} = \boldsymbol{x}_{it}'\boldsymbol{\beta} + \boldsymbol{\gamma}_i'\boldsymbol{f}_t + \varepsilon_{it}. \tag{12}$$

To estimate $\boldsymbol{\beta}$, we follow Pesaran (2006) and augment it with $(\bar{y}_t, \bar{\boldsymbol{x}}_t')'$:

$$y_{it} = \boldsymbol{x}_{it}'\boldsymbol{\beta} + \boldsymbol{\eta}_{1i}'\bar{\boldsymbol{x}}_t + \eta_{2i}\bar{y}_t + \hat{\varepsilon}_{it}, \tag{13}$$

where $\bar{y}_t = \frac{1}{N}\sum_{i=1}^N y_{it}$, $\bar{\boldsymbol{x}}_t = \frac{1}{N}\sum_{i=1}^N \boldsymbol{x}_{it}$, and $\hat{\varepsilon}_{it} = \boldsymbol{\gamma}_i'\boldsymbol{f}_t - \boldsymbol{\eta}_{1i}'\bar{\boldsymbol{x}}_t - \eta_{2i}\bar{y}_t + \varepsilon_{it}$. Assuming that ε_{it} is independent of $\boldsymbol{f}_t$, $\boldsymbol{x}_{it}$ for all i and t, and ε_{jt} for $i \neq j$, we have

$$E(\bar{y}_t\varepsilon_{it}) = \frac{1}{N}\sum_{j=1}^N E[(\boldsymbol{x}_{jt}'\boldsymbol{\beta} + \boldsymbol{\gamma}_j'\boldsymbol{f}_t + \varepsilon_{jt})\varepsilon_{it}] = \frac{1}{N}\sum_{j=1}^N E(\varepsilon_{jt}\varepsilon_{it}) = \frac{\sigma_{\varepsilon_i}^2}{N} = O(\frac{1}{N}).$$

When N is small, the CCE estimators may suffer from finite sample bias duo to the "smearing effect" (Greene (2010)).

This issue will become more complicated in the presence of spatial dependence. Again for simplicity we consider the homogeneous model (12) from which we obtain

$$\begin{aligned} E(\bar{y}_t \varepsilon_{it}) &= \frac{1}{N} \boldsymbol{\iota}_N' E[(\boldsymbol{I_N} - \rho \boldsymbol{W})^{-1} (\boldsymbol{X}_{.t} \boldsymbol{\beta} + \boldsymbol{\gamma}' \boldsymbol{f}_t + \boldsymbol{\varepsilon}_{.t}) \varepsilon_{it}] \\ &= \frac{1}{N} \boldsymbol{\iota}_N' E((\boldsymbol{I_N} - \rho \boldsymbol{W})^{-1} \boldsymbol{\varepsilon}_{.t} \varepsilon_{it}) = \frac{1}{N} \sum_{r=0} \sum_{j=1}^{N} \rho^r w_{ji}^r E(\varepsilon_{it})^2 \\ &= \sum_{r=0} \sum_{j=1}^{N} \rho^r w_{ji}^r \frac{\sigma_{\varepsilon_i}^2}{N} = \frac{C \sigma_{\varepsilon_i}^2}{N}, \end{aligned} \tag{14}$$

where $\boldsymbol{\iota}_N$ is an $N \times 1$ vector of ones, $\boldsymbol{X}_{.t} = (\boldsymbol{x}_{1t}, \boldsymbol{x}_{2t}, \ldots, \boldsymbol{x}_{Nt})'$, $\boldsymbol{\gamma} = (\gamma_1, \gamma_2, \ldots, \gamma_N)$, w_{ji}^r is the (j,i)-th element of $\boldsymbol{W}^r$, and C is a constant which becomes larger as the magnitude of ρ rises and/or the spatial weighting matrix, $\boldsymbol{W}$, becomes more dense.

In this paper, we propose a simpler approach which uses only $\bar{\boldsymbol{x}}_t$ to approximate the unobserved factors. There have been only two studies using this simpler approach: Boneva and Linton (2017) apply the corresponding CCE estimator in a binary choice model; Chen and Yan (2019) propose a 3-step CCE estimator for a homogeneous panel data model with unobserved common factors. However, the related asymptotic theory has not been developed yet. As another contribution of this paper, we will fill this gap by developing rigorous asymptotic theory for our heterogeneous spatial panel data model with unobserved common factors, using only $\bar{\boldsymbol{x}}_t$ as proxy for factors.

The main motivation behinds the CCE estimation is to deal with the endogeneity issue caused by the correlation between regressors and unobserved factors. In this regard, similar to the control function approach, it is sufficient to control the factors that are correlated with $\boldsymbol{x}_{it}$, i.e., $\boldsymbol{f}_{xt}$ in (2), which can be approximated by $\bar{x}_t$ as shown next. [2] Let $\boldsymbol{V}_{.t} = (\boldsymbol{v}_{1t}, \boldsymbol{v}_{2t}, \ldots, \boldsymbol{v}_{Nt})'$ and vectorise (2):

$$vec(\boldsymbol{X}_{.t}') = \boldsymbol{\Gamma}_1' \boldsymbol{f}_{1t} + \boldsymbol{\Gamma}_2' \boldsymbol{f}_{3t} + vec(\boldsymbol{V}_{.t}') = \boldsymbol{\Gamma}_x' \boldsymbol{f}_{xt} + vec(\boldsymbol{V}_{.t}'), \tag{15}$$

where $\boldsymbol{\Gamma}_1 = (\boldsymbol{\Gamma}_{11}, \boldsymbol{\Gamma}_{12}, ..., \boldsymbol{\Gamma}_{1N})$, $\boldsymbol{\Gamma}_2 = (\boldsymbol{\Gamma}_{21}, \boldsymbol{\Gamma}_{22}, ..., \boldsymbol{\Gamma}_{2N})$ and $\boldsymbol{\Gamma}_x = (\boldsymbol{\Gamma}_{x1}, \boldsymbol{\Gamma}_{x2}, ..., \boldsymbol{\Gamma}_{xN})$, then pre-multiply both sides of it by $\boldsymbol{\Upsilon} = \frac{1}{N} \boldsymbol{\iota}_N' \otimes \boldsymbol{I}_k$ yields

$$\bar{\boldsymbol{x}}_t = \boldsymbol{\Upsilon} vec(\boldsymbol{X}_{.t}') = \boldsymbol{\Upsilon} \boldsymbol{\Gamma}_x' \boldsymbol{f}_{xt} + \boldsymbol{\Upsilon} vec(\boldsymbol{V}_{.t}') = \bar{\boldsymbol{\Gamma}}_x' \boldsymbol{f}_{xt} + \bar{\boldsymbol{v}}_t, \tag{16}$$

where

$$\bar{\boldsymbol{x}}_t = \frac{1}{N} \sum_{i=1}^{N} \boldsymbol{x}_{it}, \quad \bar{\boldsymbol{v}}_t = \boldsymbol{\Upsilon} vec(\boldsymbol{V}_{.t}') = \frac{1}{N} \sum_{i=1}^{N} \bar{\boldsymbol{v}}_{it}, \quad \bar{\boldsymbol{\Gamma}}_x = (\boldsymbol{\Upsilon} \boldsymbol{\Gamma}_x')' = \frac{1}{N} \sum_{i=1}^{N} \boldsymbol{\Gamma}_{xi}. \tag{17}$$

Assuming that $\bar{\boldsymbol{\Gamma}}_x$ has a full row rank, namely, Rank $(\bar{\boldsymbol{\Gamma}}_x) = r_x \leq k$ for all N including $N \to \infty$, then we have:

$$\boldsymbol{f}_{xt} = (\bar{\boldsymbol{\Gamma}}_x \bar{\boldsymbol{\Gamma}}_x')^{-1} \bar{\boldsymbol{\Gamma}}_x (\bar{\boldsymbol{x}}_t - \bar{\boldsymbol{v}}_t). \tag{18}$$

[2] If we could find other variables that are affected by either $\boldsymbol{f}_{yt}$ or $\boldsymbol{f}_{xt}$ or both, but they are not included in model (1), we may add CSA of those variables as proxy for factors like Chudik and Pesaran (2015).

Lemma 1 in the Online Supplement shows that $\bar{\boldsymbol{v}}_t$ converges to zero in quadratic mean as $N \to \infty$[3] such that $\boldsymbol{f}_{xt}$ can be approximated by the cross-sectional average, $\bar{\boldsymbol{x}}_t$:

$$\boldsymbol{f}_{xt} = (\boldsymbol{\Gamma}_{x0}\boldsymbol{\Gamma}'_{x0})^{-1}\boldsymbol{\Gamma}_{x0}\bar{\boldsymbol{x}}_t + o_p(1), \tag{19}$$

where $\bar{\boldsymbol{\Gamma}}_x \xrightarrow{p} \boldsymbol{\Gamma}_{x0} = E(\boldsymbol{\Gamma}_{xi})$.

Augmenting model (1) with $\bar{\boldsymbol{x}}_t$ only may leave $\boldsymbol{f}_{2t}$ unaccounted for and hence one may be concerned about efficiency loss. However, if the dimension of $\boldsymbol{f}_{2t}$ is larger than 1, the additional use of $\bar{y}_t$ does not always achieve efficiency gain because $\bar{y}_t$ cannot provide a good approximation to $\boldsymbol{f}_{2t}$.[4] Moreover, in this paper we do not impose any restrictions on the correlation between $\boldsymbol{f}_{2t}$ and $\boldsymbol{f}_{3t}$. In fact, when $\boldsymbol{f}_{2t}$ and $\boldsymbol{f}_{3t}$ are correlated, $\bar{\boldsymbol{x}}_t$ may provide some information for $\boldsymbol{f}_{2t}$ through its dependence on $\boldsymbol{f}_{3t}$.

To establish the asymptotic theory, we make the following assumptions:

Assumption 1. *The $r_a(= r_1+r_2+r_3)\times 1$ vector of unobserved factors $\boldsymbol{f}_{at} = (\boldsymbol{f}'_{1t}, \boldsymbol{f}'_{2t}, \boldsymbol{f}'_{3t})'$ are covariance stationary with zero mean and absolutely summable autocovariances, and distributed independently of the idiosyncratic errors, ε_{is} and $\boldsymbol{v}_{is}$, for all i, t, s.*

Assumption 2. *The idiosyncratic errors ε_{it} and $\boldsymbol{v}_{js}$ are distributed independently of each other, for all i, j, t, s. For each i, ε_{it} and $\boldsymbol{v}_{it}$ follow stationary linear processes with absolutely summable autocovariances: $\varepsilon_{it} = \sum_{l=0}^{\infty} a_{il}\epsilon_{i,t-l}$ and $\boldsymbol{v}_{it} = \sum_{l=0}^{\infty} \boldsymbol{B}_{il}\boldsymbol{\zeta}_{i,t-l}$, where $(\epsilon_{it}, \boldsymbol{\zeta}'_{it})' \sim IID(0_{k+1}, \ \boldsymbol{I}_{k+1})$ with finite fourth-order moments. Denote $\sigma^2_{\varepsilon,i} = Var(\varepsilon_{it}) = \sum_{l=0}^{\infty} a^2_{il}$ and $\boldsymbol{\Omega}_{v,i} = Var(\boldsymbol{v}_{it}) = \sum_{l=0}^{\infty} \boldsymbol{B}_{il}\boldsymbol{B}'_{il}$. There exists a constant C such that $0 < \sigma^2_{\varepsilon,i} < C$ and $\|\boldsymbol{\Omega}_{v,i}\| < C$ for all i.*

Assumption 3. *The factor loadings, $\boldsymbol{\gamma}_{yi}$ and $\boldsymbol{\Gamma}_{xi}$, are independently and identically distributed across i, and independent of $\varepsilon_{jt}, \boldsymbol{v}_{jt}$, and $\boldsymbol{f}_{at}$, for all i, j, and t. Both $\boldsymbol{\gamma}_{yi}$ and $\boldsymbol{\Gamma}_{xi}$ have finite means, $\bar{\boldsymbol{\gamma}}_y$ and $\bar{\boldsymbol{\Gamma}}_{x0}$, and finite variances. For each i, $\boldsymbol{\gamma}_{yi} = \bar{\boldsymbol{\gamma}}_y + \boldsymbol{\mu}_{yi}$, where $\boldsymbol{\mu}_{yi} \sim IID(0, \ \boldsymbol{\Omega}_\mu)$ and $\boldsymbol{\Omega}_\mu$ is a positive definite matrix satisfying $\|\boldsymbol{\Omega}_\mu\| < C$. The matrix $\bar{\boldsymbol{\Gamma}}_x$ given by (17), has the full row rank for all N including $N \to \infty$.*

Assumption 4. *The $N \times N$ nonstochastic spatial weighting matrix, $\boldsymbol{W} = (w_{ij})$ with $w_{ii} = 0$, has bounded row and column sum norms, namely, $\|\boldsymbol{W}\|_\infty < C$ and $\|\boldsymbol{W}\|_1 < C$, respectively, and*

$$\rho_{sup} \equiv \sup_i |\rho_i| < \max\{1/\|\boldsymbol{W}\|_1, 1/\|\boldsymbol{W}\|_\infty\}. \tag{20}$$

Assumption 5. *The parameters, $\boldsymbol{\theta}_i = (\rho_i, \boldsymbol{\beta}'_i)'$ follow the random coefficient model:*

$$\boldsymbol{\theta}_i = \boldsymbol{\theta} + \boldsymbol{\xi}_i, \ \ \boldsymbol{\xi}_i \sim \ IID(0, \boldsymbol{\Omega}_\xi) \ for \ i = 1, ..., N, \tag{21}$$

where $\boldsymbol{\theta} = (\rho, \boldsymbol{\beta}')'$, $\|\boldsymbol{\theta}\| < C$, $\boldsymbol{\Omega}_\xi$ is a $(k+1) \times (k+1)$ positive definite matrix satisfying $\|\boldsymbol{\Omega}_\xi\| < C$. Moreover, $\boldsymbol{\xi}_i$ is distributed independently of $\boldsymbol{\gamma}_{yi}$, $\boldsymbol{\Gamma}_{xi}$, ε_{it}, $\boldsymbol{v}_{it}$ for all i and t.

[3]This also holds with unequal weights as long as it satisfies granaular condition in Assumption 5 in Pesaran (2006).

[4]This could explain why some CSD tests, (e.g., Pesaran (2015)) tend to reject the null of no cross-section dependence in many empirical applications of CCE analysis, for example, Mastromarco et al. (2016b), Ertur and Musolesi (2017) and Yang (2018).

Assumptions 1-3 and 5 are standard. In particular, we do not impose any restrictions on the relationship among unobserved factors which could even help in improving estimation efficiency as mentioned earlier. When using both $\bar{y}_t$ and $\bar{\boldsymbol{x}}_t$ as factor proxies, we require the means of factor loadings, $\boldsymbol{\gamma}_y$ and $\boldsymbol{\Gamma}_{x0}$, to be non-zero. If we use only $\bar{\boldsymbol{x}}_t$ as factor approximation, then we require only the weaker condition that $\boldsymbol{\Gamma}_{x0} \neq \boldsymbol{0}$. Furthermore, the rank condition, Rank $(\bar{\boldsymbol{\Gamma}}_x) = r_x \leq k$, is no more restrictive than the one used in Pesaran (2006) and Yang (2018), and it is easier to be satisfied when $r_2 \geq 1$.

Assumption 4 is also standard in the spatial literature. While bounded column and row sums make cross-section correlations manageable, the condition in (20) ensures that $\boldsymbol{S} = (\boldsymbol{I}_N - \boldsymbol{\rho W})$ is invertible, see Lemma 1 in Aquaro et al. (2019). This also implies that $\boldsymbol{S}^{-1}$ is uniformly bounded in row and column sums in absolute values as

$$\begin{aligned} \|\boldsymbol{S}^{-1}\|_1 = \|\mathrm{I}_N + \boldsymbol{\rho W} + (\boldsymbol{\rho W})^2 + \dots\|_1 &\leq 1 + \|\boldsymbol{\rho}\|_1\|\boldsymbol{W}\|_1 + \|\boldsymbol{\rho}\|_1^2\|\boldsymbol{W}\|_1^2 + \dots \\ &\leq \frac{1}{1-\rho_{sup}\|\boldsymbol{W}\|_1} < C, \end{aligned} \tag{22}$$

and similarly, $\|\boldsymbol{S}^{-1}\|_\infty < C$.

While the following can serve as IVs in standard spatial literature, which consists of $\boldsymbol{X}_{.t}$ and their higher order spatial lags,

$$\underset{NT\times l}{\boldsymbol{Q}} = (\boldsymbol{Q}'_{.1},\ \boldsymbol{Q}'_{.2},\ \dots,\ \boldsymbol{Q}'_{.T})', \tag{23}$$

where $\boldsymbol{Q}_{.t}$ is an $N \times l$ matrix consisting of $(\boldsymbol{X}_{.t}, \boldsymbol{W}\boldsymbol{X}_{.t} \dots \boldsymbol{W}^r\boldsymbol{X}_{.t}, \dots)$ for $t = 1, \dots, T$ with $l \geq (k+1)$, de-factorisation are needed for them to be valid instruments for our model as discussed below (11). Specifically, defining the following matrices:

$$\underset{NT\times NT}{\tilde{\boldsymbol{M}}_{\bar{x}}} = \boldsymbol{M}_{\bar{x}} \otimes \boldsymbol{I}_N \quad \text{and} \quad \underset{T\times T}{\boldsymbol{M}_{\bar{x}}} = \boldsymbol{I}_T - \bar{\boldsymbol{X}}(\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}})^{-1}\bar{\boldsymbol{X}}', \tag{24}$$

where $\bar{\boldsymbol{X}} = (\bar{\boldsymbol{x}}_1, \bar{\boldsymbol{x}}_2,\ \dots, \bar{\boldsymbol{x}}_T)'$ are factor proxies, then we shall construct the $NT \times l$ matrix of instrumental variables, denoted as $\tilde{\boldsymbol{Q}} = (\boldsymbol{M}_{\bar{\boldsymbol{x}}} \otimes \boldsymbol{I}_N)\boldsymbol{Q}$ for our estimation below. Since we use only $\bar{\boldsymbol{x}}_t$ as factor approximation, the factors, $\boldsymbol{f}_{2t}$, specifically affecting y_{it}, may still be left unaccounted for. However, this does not affect the validity of the IVs, because $(\boldsymbol{M}_{\bar{\boldsymbol{x}}} \otimes \boldsymbol{I}_N)\boldsymbol{Q}$ is uncorrelated with $\boldsymbol{f}_2$ by Assumption 1 .

Finally, we introduce some standard identification conditions for individual coefficients, $\boldsymbol{\theta}_i$ as well as their mean counterparts, $\boldsymbol{\theta} = E(\boldsymbol{\theta}_i) = (\rho, \boldsymbol{\beta}')'$. For this purpose, define

$$\underset{T\times l}{\tilde{\boldsymbol{Q}}_{i0}} = \boldsymbol{M}_{\boldsymbol{f}_x}(\boldsymbol{I}_T \otimes \boldsymbol{b}'_i)\boldsymbol{Q},$$

where

$$\boldsymbol{M}_{\boldsymbol{f}_x} = \boldsymbol{I}_T - \boldsymbol{F}_x(\boldsymbol{F}'_x\boldsymbol{F}_x)^{-1}\boldsymbol{F}'_x, \qquad \boldsymbol{F}_x = (\boldsymbol{f}_{x1},\ \boldsymbol{f}_{x2},\ \dots,\ \boldsymbol{f}_{xT})',$$

and $\boldsymbol{b}_i = (0, \dots, 0, \underset{i}{1}, 0, \dots, 0)'$ is an $N \times 1$ unit vector. Also define

$$\underset{T\times(k+1)}{\boldsymbol{Z}_{i0}} = ((\boldsymbol{I}_T \otimes \boldsymbol{b}'_i\boldsymbol{G})(\boldsymbol{I}_T \otimes \boldsymbol{B})\boldsymbol{\chi}, \boldsymbol{X}_{i.}),$$

where $\boldsymbol{G} = \boldsymbol{W}\boldsymbol{S}^{-1}$ and $\boldsymbol{X}_{i.} = (\boldsymbol{x}_{i1}, \boldsymbol{x}_{i2}, \dots, \boldsymbol{x}_{iT})'$.

Assumption 6. *(i) For each i, the $l\times l$ matrix $\frac{\tilde{\boldsymbol{Q}}_{i0}'\tilde{\boldsymbol{Q}}_{i0}}{T}$ has a finite second order moment and there exists a non-stochastic and non-singular matrix $\boldsymbol{\Phi}_i$ such that $\boldsymbol{\Phi}_i = \underset{T\to\infty}{\text{plim}}\ \frac{\tilde{\boldsymbol{Q}}_{i0}'\tilde{\boldsymbol{Q}}_{i0}}{T}$. Moreover, $\boldsymbol{\Phi} = \lim_{N\to\infty}\frac{1}{N}\sum_{i=1}^N \boldsymbol{\Phi}_i$ exists and is non-singular.*

(ii) For each i, the $l\times(k+1)$ matrix $\frac{\tilde{\boldsymbol{Q}}_{i0}'\boldsymbol{Z}_{i0}}{T}$ has a finite second order moment and there exists a non-stochastic matrix $\boldsymbol{\Psi}_i$ with full column rank such that $\boldsymbol{\Psi}_i = \underset{T\to\infty}{\text{plim}}\ \frac{\tilde{\boldsymbol{Q}}_{i0}'\boldsymbol{Z}_{i0}}{T}$. Furthermore, $\boldsymbol{\Psi} = \lim_{N\to\infty}\frac{1}{N}\sum_{i=1}^N \boldsymbol{\Psi}_i$ exists with full column rank.

3 Asymptotic Theory

We develop individual estimators as well as the Mean Group and Pooled estimators for (1) and establish their asymptotic properties.

3.1 The Individual Estimator

Stacking equations (1) and (2) over t, we have:

$$\boldsymbol{y}_{i.} = \rho_i\boldsymbol{y}_i^* + \boldsymbol{X}_{i.}\boldsymbol{\beta}_i + \boldsymbol{F}_y\boldsymbol{\gamma}_{yi} + \boldsymbol{\varepsilon}_{i.} = \boldsymbol{Z}_i\boldsymbol{\theta}_i + \boldsymbol{F}_y\boldsymbol{\gamma}_{yi} + \boldsymbol{\varepsilon}_{i.}, \tag{25}$$
$$\boldsymbol{X}_{i.} = \boldsymbol{F}_1\boldsymbol{\Gamma}_{1i} + \boldsymbol{F}_3\boldsymbol{\Gamma}_{2i} + \boldsymbol{V}_{i.} = \boldsymbol{F}_x\boldsymbol{\Gamma}_{xi} + \boldsymbol{V}_{i.}, \tag{26}$$

where $\boldsymbol{y}_{i.} = (y_{i1}, y_{i2}, \dots, y_{iT})'$, $\boldsymbol{y}_i^* = (\boldsymbol{I}_T\otimes\boldsymbol{w}_i)\boldsymbol{y} = (y_{i1}^*, y_{i2}^*, ..., y_{iT}^*)'$, $\boldsymbol{w}_i = \boldsymbol{b}_i'\boldsymbol{W}$ is the i-th row of the spatial weighting matrix, $\boldsymbol{W}$, $\boldsymbol{Z}_i = (\boldsymbol{y}_i^*, \boldsymbol{X}_{i.})$, $\boldsymbol{X}_{i.} = (\boldsymbol{x}_{i1}, \boldsymbol{x}_{i2}, ..., \boldsymbol{x}_{iT})'$, $\boldsymbol{F}_1 = (\boldsymbol{f}_{11}, \boldsymbol{f}_{12}, ..., \boldsymbol{f}_{1T})'$, $\boldsymbol{F}_2 = (\boldsymbol{f}_{21}, \boldsymbol{f}_{22}, ..., \boldsymbol{f}_{2T})'$, $\boldsymbol{F}_3 = (\boldsymbol{f}_{31}, \boldsymbol{f}_{32}, ..., \boldsymbol{f}_{3T})'$, and $\boldsymbol{V}_{i.} = (\boldsymbol{v}_{i1}, \boldsymbol{v}_{i2}, ..., \boldsymbol{v}_{iT})'$.

To consistently estimate $\boldsymbol{\theta}_i = (\rho_i,\ \boldsymbol{\beta}_i')'$, we first pre-multiply both sides of (25) by $\boldsymbol{M}_{\bar{x}}$ defined in (24). This transformation is different from the infeasible one given by (9), in the sense that the factors, $\boldsymbol{f}_{2t}$, maynot be removed from (25). Nevertheless, augmenting (25) with $\bar{\boldsymbol{X}}$ only, we can consistently estimate $\boldsymbol{\theta}_i$ using IVs $\tilde{\boldsymbol{Q}}_i = \boldsymbol{M}_{\bar{x}}(\boldsymbol{I}_T\otimes\boldsymbol{b}_i')\boldsymbol{Q}$.[5] The individual estimators are given compactly by

$$\hat{\boldsymbol{\theta}}_i = \left(\boldsymbol{Z}_i'\boldsymbol{\Pi}_i\boldsymbol{Z}_i\right)^{-1}\boldsymbol{Z}_i'\boldsymbol{\Pi}_i\boldsymbol{y}_{i.}, \tag{27}$$

where $\boldsymbol{\Pi}_i = \tilde{\boldsymbol{Q}}_i(\tilde{\boldsymbol{Q}}_i'\tilde{\boldsymbol{Q}}_i)^{-1}\tilde{\boldsymbol{Q}}_i'$.

Using the facts that $\boldsymbol{Z}_i'\boldsymbol{M}_{\bar{x}}\tilde{\boldsymbol{Q}}_i = \boldsymbol{Z}_i'\tilde{\boldsymbol{Q}}_i = \boldsymbol{Z}_i'\boldsymbol{M}_{\bar{x}}(\boldsymbol{I}_T\otimes\boldsymbol{b}_i')\boldsymbol{Q}$, the above expression can be interpreted in three different ways and therefore, implies we can also estimate $\hat{\boldsymbol{\theta}}_i$ equally in three different ways: The first uses $\tilde{\boldsymbol{Q}}_i$ as IVs in the de-factored regression of $\boldsymbol{M}_{\bar{x}}\boldsymbol{y}_{i.}$ against $\boldsymbol{M}_{\bar{x}}\boldsymbol{Z}_i$; The second applies the IVs, $\tilde{\boldsymbol{Q}}_i$ directly to estimate (25); The third applies the IVs, $(\boldsymbol{I}_T\otimes\boldsymbol{b}_i')\boldsymbol{Q}$ to the de-factored regression. We prefer to employ the second option, which can be regarded as a one-step estimation procedure, where $\boldsymbol{M}_{\bar{x}}\boldsymbol{X}_{i.}$

[5]The instruments for the i-th individual regression can be expressed interchangeably as $(\boldsymbol{I}_T\otimes\boldsymbol{b}_i')(\boldsymbol{M}_{\bar{\boldsymbol{x}}}\otimes\boldsymbol{I}_N)\boldsymbol{Q} = (\boldsymbol{M}_{\bar{\boldsymbol{x}}}\otimes 1)(\boldsymbol{I}_T\otimes\boldsymbol{b}_i')\boldsymbol{Q} = \boldsymbol{M}_{\bar{x}}(\boldsymbol{I}_T\otimes\boldsymbol{b}_i')\boldsymbol{Q}$.

serve as IVs for $\boldsymbol{X}_{i.}$, and the remaining higher-order spatial terms in $\tilde{\boldsymbol{Q}}_i$ serves as IVs for $\boldsymbol{y}_{i.}^*$.

Substituting (25) in (27), we have:

$$\hat{\boldsymbol{\theta}}_i = \left(\frac{\boldsymbol{Z}_i'\boldsymbol{\Pi}_i\boldsymbol{Z}_i}{T}\right)^{-1}\frac{\boldsymbol{Z}_i'\boldsymbol{\Pi}_i(\boldsymbol{Z}_i\boldsymbol{\theta}_i + \boldsymbol{F}_1\gamma_{1i} + \boldsymbol{F}_2\gamma_{2i} + \varepsilon_{i.})}{T}, \tag{28}$$

and

$$\hat{\boldsymbol{\theta}}_i - \boldsymbol{\theta}_i = \left(\frac{\boldsymbol{Z}_i'\boldsymbol{\Pi}_i\boldsymbol{Z}_i}{T}\right)^{-1}\frac{\boldsymbol{Z}_i'\boldsymbol{\Pi}_i(\boldsymbol{F}_1\gamma_{1i} + \boldsymbol{F}_2\gamma_{2i} + \varepsilon_{i.})}{T}. \tag{29}$$

As shown in the Appendix, the right hand side of the above equation converges in probability to $\mathbf{0}$ and follows a normal distribution asymptotically.

Theorem 1. *Consider the heterogeneous spatial panel data model with common factors given by (1) and (2). Suppose that Assumptions 1- 4 and 6 hold. The individual coefficient estimator, $\hat{\boldsymbol{\theta}}_i$ defined in (27), is consistent for $\boldsymbol{\theta}_i$ as $(N,\ T) \xrightarrow{j} \infty$. Furthermore, as $(N,\ T) \xrightarrow{j} \infty$ and $T/N^2 \to 0$, we have*

$$\sqrt{T}(\hat{\boldsymbol{\theta}}_i - \boldsymbol{\theta}_i) \xrightarrow{d} N(0,\ \boldsymbol{\Omega}_i), \tag{30}$$

where

$$\boldsymbol{\Omega}_i = \left[\boldsymbol{\Psi}_i'\boldsymbol{\Phi}_i^{-1}\boldsymbol{\Psi}_i\right]^{-1}\left[\boldsymbol{\Psi}_i'\boldsymbol{\Phi}_i^{-1}\boldsymbol{\Sigma}_i\boldsymbol{\Phi}_i^{-1}\boldsymbol{\Psi}_i\right]\left[\boldsymbol{\Psi}_i'\boldsymbol{\Phi}_i^{-1}\boldsymbol{\Psi}_i\right]^{-1}, \tag{31}$$

$$\boldsymbol{\Sigma}_i = \lim_{T\to\infty} E\left(\frac{\tilde{\boldsymbol{Q}}_{i0}'\boldsymbol{\Omega}_{\varepsilon,i}\tilde{\boldsymbol{Q}}_{i0}}{T} + \frac{\tilde{\boldsymbol{Q}}_{i0}'\boldsymbol{F}_2\gamma_{2i}\gamma_{2i}'\boldsymbol{F}_2'\tilde{\boldsymbol{Q}}_{i0}}{T}\right), \tag{32}$$

with $\boldsymbol{\Omega}_{\varepsilon,i} = E(\boldsymbol{\varepsilon}_{i.}\boldsymbol{\varepsilon}_{i.}')$.

A consistent estimator for $\boldsymbol{\Omega}_i$, is given by

$$\hat{\boldsymbol{\Omega}}_i = \left(\frac{\boldsymbol{Z}_i\boldsymbol{\Pi}_i\boldsymbol{Z}_i}{T}\right)^{-1}\frac{\boldsymbol{Z}_i'\tilde{\boldsymbol{Q}}_i}{T}\left(\frac{\tilde{\boldsymbol{Q}}_i'\tilde{\boldsymbol{Q}}_i}{T}\right)^{-1}\hat{\boldsymbol{\Sigma}}_i\left(\frac{\tilde{\boldsymbol{Q}}_i'\tilde{\boldsymbol{Q}}_i}{T}\right)^{-1}\frac{\tilde{\boldsymbol{Q}}_i'\boldsymbol{Z}_i}{T}\left(\frac{\boldsymbol{Z}_i\boldsymbol{\Pi}_i\boldsymbol{Z}_i}{T}\right)^{-1}. \tag{33}$$

where $\hat{\boldsymbol{\Sigma}}_i$ can be constructed from the Newey and West (1987) robust estimator:[6]

$$\hat{\boldsymbol{\Sigma}}_i = \hat{\boldsymbol{\Sigma}}_{i,0} + \sum_{h=1}^{p_T}\left(1 - \frac{h}{p_T+1}\right)(\hat{\boldsymbol{\Sigma}}_{i,h} + \hat{\boldsymbol{\Sigma}}_{i,h}'), \tag{34}$$

in which

$$\hat{\boldsymbol{\Sigma}}_{i,h} = \frac{1}{T}\sum_{t=h+1}^{T}\hat{e}_{it}\hat{e}_{i,t-h}\tilde{\boldsymbol{z}}_{it}\tilde{\boldsymbol{z}}_{i,t-h}', \tag{35}$$

p_T is the bandwidth of the Bartlett kernel, and $\hat{\boldsymbol{e}}_i = \boldsymbol{M}_{\bar{x}}(\boldsymbol{y}_{i.} - \boldsymbol{Z}_i\hat{\boldsymbol{\theta}}_i) = (\hat{e}_{i1}, \hat{e}_{i2},\ \ldots, \hat{e}_{iT})'$, $\tilde{\boldsymbol{Z}}_i = M_{\bar{x}}\boldsymbol{Z}_i = (\tilde{\boldsymbol{z}}_{i1}, \tilde{\boldsymbol{z}}_{i2}, \ldots, \tilde{\boldsymbol{z}}_{iT})'$.

[6]Alternatively, one can also use Andrews (1991) type procedure.

3.2 The Mean Group Estimator

Consider the Mean Group estimator for $\boldsymbol{\theta} = E(\boldsymbol{\theta}_i)$ given by

$$\hat{\boldsymbol{\theta}}_{MG} = \frac{1}{N}\sum_{i=1}^{N}\hat{\boldsymbol{\theta}}_i. \tag{36}$$

To study its asymptotic property, we follow Assumption 5 and (29) and derive an expansion of (36) as follows:

$$\begin{aligned}
\hat{\boldsymbol{\theta}}_{MG} - \boldsymbol{\theta} &= \frac{1}{N}\sum_{i=1}^{N}(\hat{\boldsymbol{\theta}}_i - \boldsymbol{\theta}_i) + \frac{1}{N}\sum_{i=1}^{N}\boldsymbol{\xi}_i \\
&= \frac{1}{N}\sum_{i=1}^{N}\boldsymbol{\xi}_i + \frac{1}{N}\sum_{i=1}^{N}\left(\frac{\boldsymbol{Z}_i'\boldsymbol{\Pi}_i\boldsymbol{Z}_i}{T}\right)^{-1}\frac{\boldsymbol{Z}_i'\boldsymbol{\Pi}_i\boldsymbol{F}_1\gamma_{1i}}{T} \\
&+ \frac{1}{N}\sum_{i=1}^{N}\left(\frac{\boldsymbol{Z}_i'\boldsymbol{\Pi}_i\boldsymbol{Z}_i}{T}\right)^{-1}\frac{\boldsymbol{Z}_i'\boldsymbol{\Pi}_i\boldsymbol{F}_2\gamma_{2i}}{T} + \frac{1}{N}\sum_{i=1}^{N}\left(\frac{\boldsymbol{Z}_i'\boldsymbol{\Pi}_i\boldsymbol{Z}_i}{T}\right)^{-1}\frac{\boldsymbol{Z}_i'\boldsymbol{\Pi}_i\boldsymbol{\varepsilon}_{i.}}{T}.
\end{aligned} \tag{37}$$

It is shown in the Appendix that the first term dominates the rest.

Theorem 2. *Consider the heterogeneous spatial panel data model with common factors given by (1) and (2). Suppose that Assumptions 1-6 hold. Then, as $(N, T) \xrightarrow{j} \infty$ the mean group estimator $\hat{\boldsymbol{\theta}}_{MG}$ defined in (36), is consistent for $\boldsymbol{\theta}$ and*

$$\sqrt{N}\left(\hat{\boldsymbol{\theta}}_{MG} - \boldsymbol{\theta}\right) \xrightarrow{d} N\left(0, \boldsymbol{\Omega}_{MG}\right) \equiv N\left(0, \boldsymbol{\Omega}_{\xi}\right), \tag{38}$$

where $\boldsymbol{\Omega}_{\xi}$ is given by (21).

$\boldsymbol{\Omega}_{\xi}$ can be consistently estimated by the nonparametric estimator (e.g., Pesaran (2006)):

$$\hat{\boldsymbol{\Omega}}_{MG} = \hat{\boldsymbol{\Omega}}_{\xi} = \frac{1}{N-1}\sum_{i=1}^{N}(\hat{\boldsymbol{\theta}}_i - \hat{\boldsymbol{\theta}}_{MG})(\hat{\boldsymbol{\theta}}_i - \hat{\boldsymbol{\theta}}_{MG})'. \tag{39}$$

3.3 The Pooled Estimator

We now develop the Pooled estimator for $\boldsymbol{\theta} = E(\boldsymbol{\theta}_i)$. First, de-factor individual data through pre-multiplying $\boldsymbol{M}_{\bar{x}}$ and stack them as follows:

$$\underset{NT\times 1}{\tilde{\boldsymbol{y}}} = \begin{bmatrix}\tilde{\boldsymbol{y}}_1\\ \tilde{\boldsymbol{y}}_2\\ \vdots\\ \tilde{\boldsymbol{y}}_N\end{bmatrix} = \begin{bmatrix}\boldsymbol{M}_{\bar{x}}\boldsymbol{y}_{1.}\\ \boldsymbol{M}_{\bar{x}}\boldsymbol{y}_{2.}\\ \vdots\\ \boldsymbol{M}_{\bar{x}}\boldsymbol{y}_{N.}\end{bmatrix}, \qquad \underset{NT\times(k+1)}{\tilde{\boldsymbol{Z}}} = \begin{bmatrix}\tilde{\boldsymbol{Z}}_1\\ \tilde{\boldsymbol{Z}}_2\\ \vdots\\ \tilde{\boldsymbol{Z}}_{N.}\end{bmatrix} = \begin{bmatrix}\boldsymbol{M}_{\bar{x}}\boldsymbol{Z}_1\\ \boldsymbol{M}_{\bar{x}}\boldsymbol{Z}_2\\ \vdots\\ \boldsymbol{M}_{\bar{x}}\boldsymbol{Z}_N\end{bmatrix}, \tag{40}$$

where $\boldsymbol{y}_{i.}$ and $\boldsymbol{Z}_i$ are defined in (25). Then estimate the regression of $\tilde{\boldsymbol{y}}$ on $\tilde{\boldsymbol{Z}}$ using IVs specified by the following $NT \times l$ matrix:

$$\tilde{\boldsymbol{Q}} = \begin{bmatrix} \tilde{\boldsymbol{Q}}_1 \\ \tilde{\boldsymbol{Q}}_2 \\ \vdots \\ \tilde{\boldsymbol{Q}}_N \end{bmatrix} = \begin{bmatrix} \boldsymbol{M}_{\bar{x}}(\boldsymbol{I}_T \otimes \boldsymbol{b}_1')\boldsymbol{Q} \\ \boldsymbol{M}_{\bar{x}}(\boldsymbol{I}_T \otimes \boldsymbol{b}_2')\boldsymbol{Q} \\ \vdots \\ \boldsymbol{M}_{\bar{x}}(\boldsymbol{I}_T \otimes \boldsymbol{b}_N')\boldsymbol{Q} \end{bmatrix}. \tag{41}$$

The Pooled estimator for $\boldsymbol{\theta}$ is given by

$$\hat{\boldsymbol{\theta}}_P = (\tilde{\boldsymbol{Z}}'\tilde{\boldsymbol{Q}}(\tilde{\boldsymbol{Q}}'\tilde{\boldsymbol{Q}})^{-1}\tilde{\boldsymbol{Q}}'\tilde{\boldsymbol{Z}})^{-1}\tilde{\boldsymbol{Z}}'\tilde{\boldsymbol{Q}}(\tilde{\boldsymbol{Q}}'\tilde{\boldsymbol{Q}})^{-1}\tilde{\boldsymbol{Q}}'\tilde{\boldsymbol{y}}. \tag{42}$$

By (25), (40)-(42) and Assumption 5, we have:

$$\begin{aligned} \hat{\boldsymbol{\theta}}_P - \boldsymbol{\theta} = & \left\{ \left[\frac{1}{N}\sum_{i=1}^N \frac{\boldsymbol{Z}_i'\tilde{\boldsymbol{Q}}_i}{T} \right] \left[\frac{1}{N}\sum_{i=1}^N \frac{\tilde{\boldsymbol{Q}}_i'\tilde{\boldsymbol{Q}}_i}{T} \right]^{-1} \left[\frac{1}{N}\sum_{i=1}^N \frac{\tilde{\boldsymbol{Q}}_i'\boldsymbol{Z}_i}{T} \right] \right\}^{-1} \\ & \left[\frac{1}{N}\sum_{i=1}^N \frac{\boldsymbol{Z}_i'\tilde{\boldsymbol{Q}}_i}{T} \right] \left[\frac{1}{N}\sum_{i=1}^N \frac{\tilde{\boldsymbol{Q}}_i'\tilde{\boldsymbol{Q}}_i}{T} \right]^{-1} \left[\frac{1}{N}\sum_{i=1}^N \frac{\tilde{\boldsymbol{Q}}_i'(\boldsymbol{Z}_i\boldsymbol{\xi}_i + \boldsymbol{F}_1\boldsymbol{\gamma}_{1i} + \boldsymbol{F}_2\boldsymbol{\gamma}_{2i} + \boldsymbol{\varepsilon}_{i.})}{T} \right]. \end{aligned} \tag{43}$$

Theorem 3 below gives the consistency and asymptotic distribution for $\hat{\boldsymbol{\theta}}_P$.

Theorem 3. *Consider the heterogeneous spatial panel data model with common factors given by (1) and (2). Suppose that Assumptions 1-6 hold. Then, as $(N,T) \xrightarrow{j} \infty$, the Pooled estimator $\hat{\boldsymbol{\theta}}_P$ is consistent for $\boldsymbol{\theta}$ and*

$$\sqrt{N}\left(\hat{\boldsymbol{\theta}}_P - \boldsymbol{\theta}\right) \xrightarrow{d} N\left(0, \boldsymbol{\Omega}_P\right), \tag{44}$$

where

$$\boldsymbol{\Omega}_P = \left[\boldsymbol{\Psi}'\boldsymbol{\Phi}^{-1}\boldsymbol{\Psi}\right]^{-1}\left[\boldsymbol{\Psi}'\boldsymbol{\Phi}^{-1}\boldsymbol{\Sigma}_P\boldsymbol{\Phi}^{-1}\boldsymbol{\Psi}\right]\left[\boldsymbol{\Psi}'\boldsymbol{\Phi}^{-1}\boldsymbol{\Psi}\right]^{-1}, \tag{45}$$

$$\boldsymbol{\Sigma}_P = \lim_{N,T\to\infty} \frac{1}{N}\sum_{i=1}^N E\left(\frac{\tilde{\boldsymbol{Q}}_{i0}'\boldsymbol{Z}_{i0}}{T}\boldsymbol{\Omega}_\xi\frac{\boldsymbol{Z}_{i0}'\tilde{\boldsymbol{Q}}_{i0}}{T}\right), \tag{46}$$

We follow Pesaran (2006) and estimate $\boldsymbol{\Omega}_P$ by

$$\hat{\boldsymbol{\Omega}}_P = \left(\frac{\tilde{\boldsymbol{Z}}'\boldsymbol{\Pi}\tilde{\boldsymbol{Z}}}{NT}\right)^{-1}\left(\frac{\tilde{\boldsymbol{Z}}'\tilde{\boldsymbol{Q}}}{NT}\right)\left(\frac{\tilde{\boldsymbol{Q}}'\tilde{\boldsymbol{Q}}}{NT}\right)^{-1}\hat{\boldsymbol{\Sigma}}_P\left(\frac{\tilde{\boldsymbol{Q}}'\tilde{\boldsymbol{Q}}}{NT}\right)^{-1}\left(\frac{\tilde{\boldsymbol{Q}}'\tilde{\boldsymbol{Z}}}{NT}\right)\left(\frac{\tilde{\boldsymbol{Z}}'\boldsymbol{\Pi}\tilde{\boldsymbol{Z}}}{NT}\right)^{-1}, \tag{47}$$

where $\boldsymbol{\Pi} = \tilde{\boldsymbol{Q}}(\tilde{\boldsymbol{Q}}'\tilde{\boldsymbol{Q}})^{-1}\tilde{\boldsymbol{Q}}'$ and

$$\hat{\boldsymbol{\Sigma}}_P = \frac{1}{N}\sum_{i=1}^N\left(\frac{\tilde{\boldsymbol{Q}}_i'\boldsymbol{Z}_i}{T}(\hat{\boldsymbol{\theta}}_i - \hat{\boldsymbol{\theta}}_{MG})(\hat{\boldsymbol{\theta}}_i - \hat{\boldsymbol{\theta}}_{MG})'\frac{\boldsymbol{Z}_i'\tilde{\boldsymbol{Q}}_i}{T}\right). \tag{48}$$

Alternatively, $\hat{\boldsymbol{\Sigma}}_P$ can be constructed as

$$\hat{\boldsymbol{\Sigma}}_P = \frac{1}{N}\sum_{i=1}^N \frac{\tilde{\boldsymbol{Q}}_i'\hat{e}_i\hat{e}_i'\tilde{\boldsymbol{Q}}_i}{T^2} \quad \text{with} \quad \hat{e}_i = \boldsymbol{y}_{i.} - \boldsymbol{Z}_i\hat{\boldsymbol{\theta}}_P, \tag{49}$$

which is shown to be robust to heteroskedasticity, autocorrelation and parameter heterogeneity (e.g., Cui et al. (2019)).

3.4 The Homogeneous Model

We consider a special case where the parameters, $\boldsymbol{\theta}_i$, are homogeneous, i.e., $\boldsymbol{\Omega}_\xi = \mathbf{0}$ in Assumption 5. Consider the corresponding Pooled estimator, denoted as $\hat{\boldsymbol{\theta}}_P^*$. Similarly to (43), we have

$$\begin{aligned}\hat{\boldsymbol{\theta}}_P^* - \boldsymbol{\theta} = &\left\{\left[\frac{1}{N}\sum_{i=1}^{N}\frac{\boldsymbol{Z}_i'\tilde{\boldsymbol{Q}}_i}{T}\right]\left[\frac{1}{N}\sum_{i=1}^{N}\frac{\tilde{\boldsymbol{Q}}_i'\tilde{\boldsymbol{Q}}_i}{T}\right]^{-1}\left[\frac{1}{N}\sum_{i=1}^{N}\frac{\tilde{\boldsymbol{Q}}_i'\boldsymbol{Z}_i}{T}\right]\right\}^{-1} \\ &\left[\frac{1}{N}\sum_{i=1}^{N}\frac{\boldsymbol{Z}_i'\tilde{\boldsymbol{Q}}_i}{T}\right]\left[\frac{1}{N}\sum_{i=1}^{N}\frac{\tilde{\boldsymbol{Q}}_i'\tilde{\boldsymbol{Q}}_i}{T}\right]^{-1}\left[\frac{1}{N}\sum_{i=1}^{N}\frac{\tilde{\boldsymbol{Q}}_i'(\boldsymbol{F}_1\boldsymbol{\gamma}_{1i}+\boldsymbol{F}_2\boldsymbol{\gamma}_{2i}+\boldsymbol{\varepsilon}_{i.})}{T}\right].\end{aligned} \tag{50}$$

As shown in the following theorem, $\hat{\boldsymbol{\theta}}_P^*$ achieves a faster convergence rate than the Pooled estimator $\hat{\boldsymbol{\theta}}_P$ for the heterogeneous model.

Theorem 4. *Consider the spatial panel data model given by (1) and (2) but with homogeneous parameters, i.e., $\boldsymbol{\theta}_i = \boldsymbol{\theta}$ for all i. Suppose that Assumptions 1- 4 and 6 hold. Then, as $(N,\, T) \xrightarrow{j} \infty$, the Pooled estimator $\hat{\boldsymbol{\theta}}_P^*$ is consistent for $\boldsymbol{\theta}$, and if $T/N \to 0$, we have*

$$\sqrt{NT}(\hat{\boldsymbol{\theta}}_P^* - \boldsymbol{\theta}) \xrightarrow{d} N(0,\ \boldsymbol{\Omega}_P^*), \tag{51}$$

where

$$\boldsymbol{\Omega}_P^* = \left[\boldsymbol{\Psi}'\boldsymbol{\Phi}^{-1}\boldsymbol{\Psi}\right]^{-1}\left[\boldsymbol{\Psi}'\boldsymbol{\Phi}^{-1}\boldsymbol{\Sigma}_{P^*}\boldsymbol{\Phi}^{-1}\boldsymbol{\Psi}\right]\left[\boldsymbol{\Psi}'\boldsymbol{\Phi}^{-1}\boldsymbol{\Psi}\right]^{-1}, \qquad \boldsymbol{\Sigma}_{P^*} = \lim_{N\to\infty}\frac{1}{N}\sum_{i=1}^{N}\boldsymbol{\Sigma}_i, \tag{52}$$

$\boldsymbol{\Phi}$ and $\boldsymbol{\Psi}$ are defined in Assumption 6, and $\boldsymbol{\Sigma}_i$ is defined in (32).

We also suggest to estimate $\boldsymbol{\Sigma}_{P^*}$ by (49) but with $\hat{\boldsymbol{e}}_i^* = \boldsymbol{y}_{i.} - \boldsymbol{Z}_i\hat{\boldsymbol{\theta}}_P^*$ in place of $\hat{\boldsymbol{e}}_i$. Notice that our results in Theorem 4 are different from Theorem 1 in Yang (2018), who assumes that y_{it} and $\boldsymbol{x}_{it}$ share the same common factors. As is shown later via Monte Carlo simulations, our proposed variance estimator is robust to the presence of different factors in dependent and independent variables.

4 Monte Carlo Simulations

4.1 Design

In this section, we investigate the finite sample performance of the proposed Mean Group estimator and Pooled estimator. The DGP is specified as follows:

$$y_{it} = \rho_i y_{it}^* + \beta_{1i}x_{it1} + \beta_{2i}x_{it2} + \gamma_{1i}f_{1t} + \gamma_{2i}f_{2t} + \kappa_1\varepsilon_{it} \tag{53}$$
$$x_{itp} = \Gamma_{1ip}f_{1t} + \Gamma_{2ip}f_{3t} + \kappa_2 v_{itp}, \tag{54}$$

for $i = 1, 2, \ldots, N$, $t = 1, 2, \ldots, T$ and $p = 1, 2$.

We conduct four Monte Carlo experiments summarised in Table 1. In all of the experiments, we set the number of regressors as $k = 2$, and include two factors in both equations for y_{it} and $\boldsymbol{x}_{it}$. We consider both cases of $f_{2t} = f_{3t}$ and $f_{2t} \neq f_{3t}$.

Table 1: Various Experiment Settings

		Factors in (1) and (2)	
		Same	Different
Coefficients	Homogeneous	Experiment 1	Experiment 3
	Heterogeneous	Experiment 2	Experiment 4

In Experiments 1 and 2, we set $f_{2t} = f_{3t}$, and let $f_{2t} \neq f_{3t}$ in Experiments 3 and 4. The factors are generated by an AR(1) process:

$$f_{r,t} = \phi_{f_r} f_{r,t-1} + \xi_{f_{rt}}, \;\; r = 1, 2, 3; \;\; t = -49, -48, \ldots, T, \tag{55}$$

with $\phi_{f_r} = 0.5$ and $\xi_{f_{rt}} \sim IIDN(0, 1-\phi_{f_r}^2)$. The factor loadings are generated as $IIDN(0.5, 0.5)$, except for Γ_{1i2} and Γ_{2i1}, which are generated as $IIDN(0, 0.5)$ instead to avoid high collinearity among regressors. The first 50 observations are discarded as the burn-in sample.

In Experiments 1 and 3, we set the homogeneous parameters as[7]

$$\rho_i = \rho = 0.5 \text{ and } \beta_{ip} = \beta_p = p, \;\; i = 1, 2, \ldots, N; \;\; p = 1, 2.$$

In Experiments 2 and 4, we generate the heterogeneous parameters via

$$\begin{aligned} \rho_i &= \rho + \xi_{\rho_i}, \\ \beta_{ip} &= p + \xi_{\beta_{ip}}, \;\; p = 1, 2, \end{aligned} \tag{56}$$

where both ξ_{ρ_i} and $\xi_{\beta_{ip}}$ are generated as $IIDN(0, 0.04)$ for all i and p.

We allow the idiosyncratic errors to be both heteroskedastic and serially correlated. We first generate each component of $\boldsymbol{v}_{it}$ by an AR (1) process:

$$v_{itp} = \phi_{v_{ip}} v_{i,t-1,k} + \xi_{v_{ip}}, \;\; p = 1, 2; \;\; i = 1, 2, \ldots, N; \;\; t = -49, -48, \ldots, T, \tag{57}$$

where $\phi_{v_{ip}} = 0.5$ for all i and p, and $\xi_{v_{ip}} \sim IIDN(0, 1 - \phi_{v_{ip}}^2)$. For ε_{it}, the first half cross-section units are generated from an AR(1) process:

$$\varepsilon_{it} = \phi_{\varepsilon_i} \varepsilon_{i,t-1} + \sigma_i (1 - \phi_{\varepsilon_i}^2)^{0.5} \xi_{\varepsilon_{it}}, \;\;\; i = 1, 2, ... \lfloor N/2 \rfloor; \; t = -49, -48, \ldots, T, \tag{58}$$

whilst the second half cross-section units from an MA(1) process:

$$\varepsilon_{it} = \sigma_i (1 + \psi_{\varepsilon_i}^2)^{0.5} (\xi_{\varepsilon_{it}} + \psi_{\varepsilon_i} \xi_{\varepsilon_{it-1}}), \;\; i = \lfloor N/2 \rfloor + 1, \lfloor N/2 \rfloor + 2, ..., N; \;\; t = -49, -48, \ldots, T, \tag{59}$$

where $\phi_{\varepsilon_i} = \psi_{\varepsilon_i} = 0.5$ for all i, $\sigma_i^2 \sim IIDU(0.5, 1.5)$, $\xi_{\varepsilon_{it}} \sim IIDN(0, 1)$, and $\lfloor \bullet \rfloor$ denotes the integer part of a number. We also discard the first 50 observations of $\boldsymbol{f}_t$ and $\boldsymbol{v}_{it}$.

The parameter κ_1 in (53) and κ_2 in (54) are specified to control the noise-to-signal ratio, i.e., the proportion of the variance of the unobserved parts duo to the idiosyncratic error, and we set $\kappa_1 = 2$ and $\kappa_2 = 3$, without loss of generality.[8] As standard in the spatial literature, we use a row-normalised spatial weighting matrix, $\boldsymbol{W}$, with the h-ahead-and-h-behind neighbours specification, i.e., its elements are zero apart from those within h either side of the principle diagonal, which are $1/2h$. Here, we set $h = 2$, implying that most of the individuals have four neighbours.[9] Each experiment is replicated 1,000 times for each (N, T) pair with $N, T = 20, 50, 100$.

[7] In Online Supplement, we have also considered cases with $\rho = 0.2$ and 0.8.

[8] We have also considered other values and obtained the qualitatively similar results.

[9] In Online Supplement, we have also considered differing levels of spatial dependence with $h = 4$ and $h = 6$.

4.2 Estimation

We evaluate both the Mean Group estimator and the Pooled estimator in each experiment using the following estimation algorithm:[10]

- **Step 1: Construct the Factor Proxies.** For comparison, we consider the following four ways of factor approximation:

  Case 1: our proposed estimator using $\bar{\boldsymbol{x}}_t$ only as factor proxies.

  Case 2: use $(\bar{y}_t, \bar{\boldsymbol{x}}_t')'$ as factor proxies which is standard in the CCE literature.

  Case 3: use $(\bar{y}_t, \bar{y}_t^*, \bar{\boldsymbol{x}}_t')'$ as factor proxies. [11]

  Case 4: the infeasible estimator with using the true factors.

- **Step 2: De-factore the Data.** To deal with endogeneity caused by the correlation between regressors and unobserved factors, we project factors out by regressing $\boldsymbol{y}_{i.}$, $\boldsymbol{y}_i^*$ and $\boldsymbol{X}_{i.}$ on proxies for factors, $\hat{\boldsymbol{F}}$ constructed in Step 1 and store the resulting de-factored data.

- **Step 3: Construct the IVs.** To accommodate endogeneity due to the spatial lagged term, we apply the IV estimation by using the following three sets of instruments: $\tilde{\boldsymbol{X}}^*$, $\tilde{\boldsymbol{X}}^{2*}$, and $(\tilde{\boldsymbol{X}}^*, \tilde{\boldsymbol{X}}^{2*})$,[12] where $\tilde{\boldsymbol{X}}^{r*} = (\boldsymbol{M}_{\hat{\boldsymbol{F}}} \otimes \boldsymbol{I}_N)(\boldsymbol{I}_T \otimes \boldsymbol{W}^r)\boldsymbol{X}$, $\boldsymbol{M}_{\hat{\boldsymbol{F}}} = \boldsymbol{I}_T - \hat{\boldsymbol{F}}(\hat{\boldsymbol{F}}'\hat{\boldsymbol{F}})^{-1}\hat{\boldsymbol{F}}'$, and $\boldsymbol{X} = (\boldsymbol{X}_{.1}', \boldsymbol{X}_{.2}', \ldots, \boldsymbol{X}_{.T}')'$.

- **Step 4: Obtain the Estimators.** For the Mean Group estimator, we first obtain the individual estimators, $\hat{\boldsymbol{\theta}}_i = (\hat{\rho}_i, \hat{\boldsymbol{\beta}}_i')'$ for each i, through two stage least square (2SLS) using IVs constructed in Step 3, and then calculate their average $\hat{\theta}_{MG} = \frac{1}{N}\sum_{i=1}^N \hat{\boldsymbol{\theta}}_i$. For Pooled estimator, we first stack the de-factored data as in (40) and then estimate the model via 2SLS using IVs constructed in Step 3.

To evaluate the finite sample performance of the estimators, we consider the following four measures:

- **Bias** and **RMSE**

  The bias and RMSE for the j-th element of $\hat{\boldsymbol{\theta}}_{MG}$ or $\hat{\boldsymbol{\theta}}_P$, $\hat{\theta}^j, j = 1, 2, 3$, are defined as $\frac{1}{M}\sum_{m=1}^M(\hat{\theta}_m^j - \theta^j)$ and $\sqrt{\frac{1}{M}\sum_{m=1}^M(\hat{\theta}_m^j - \theta^j)^2}$, respectively, where M is the number of

[10] We have also investigated the performance of the naive estimator that completely ignores the existence of unobserved common factors in Online Supplements.

[11] Yang (2018) also suggests the possibility of including $\bar{y}_t^* (= \frac{1}{N}\sum_{i=1}^N y_{it}^*)$ as a factor approximation, but she does not provide any simulation evidence on its performance. Since it is easily seen that $\bar{y}_t^*$ is also endogenous, we would expect its performance to be be similar to or worse than Case 2. Additionally, there are two special situations in which $\bar{y}_t^*$ is equal to $\bar{y}_t$:

1. Each individual has H neighbours. After a row normalisation (with weight, $\frac{1}{H}$), we have:

$$\bar{y}_t^* = \frac{1}{N}\sum_{i=1}^N\sum_{j=1}^N w_{ij}y_{jt} = \frac{1}{N}\sum_{j=1}^N(\sum_{i=1}^N w_{ij}y_{jt}) = \frac{1}{N}\sum_{j=1}^N(H \times \frac{1}{H}y_{jt}) = \bar{y}_t.$$

2. The weighting matrix, W, is block diagonal, and within each block, every individual has a same number of neighbours. In this case, we would also have: $\bar{y}_t^* = \bar{y}_t$.

[12] For comparison, we also consider ordinary least square (OLS) estimator and case using $\tilde{\boldsymbol{X}}^{3*}$ as instruments. The results can be found in Online Supplement.

replications, θ^j is the j-th element of the true parameter vector $\boldsymbol{\theta}$, and $\hat{\theta}^j_m$ is its corresponding Mean Group or the Pooled estimator obtained in the m-th replication. Both the bias and RMSE results reported are multiplied by 100.

- **Size** and **Power**

  We evaluate the size of the estimators at the 5% significance level by

$$\text{Size}_{\hat{\theta}^j} = \frac{1}{M}\sum_{m=1}^{M} \mathbb{1}\left(\frac{|\hat{\theta}^j_m - \theta^j|}{\hat{\sigma}_{\hat{\theta}^j_m}} > 1.96\right),\ j = 1, 2, 3.$$

  where $\mathbb{1}(\bullet)$ is the indicator function, and $\hat{\sigma}^2_{\hat{\theta}^j_m}$ is the estimated variance of $\hat{\theta}^j_m$ computed using (39) or (49). When evaluating the power of the t-test, we consider the following alternative hypothesis:

$$(\rho, \beta_1, \beta_2)' = (0.45, 0.9, 1.9)'. \tag{60}$$

  Following the standard practice in the literature, e.g., Zhang and Boos (1994), we only report the size-adjusted power.

4.3 Simulation Results

To save space, we only report, in Table 2 to 5, the simulation results for Experiment 4, which considers the most general DGP with heterogeneous parameters and different factors in model (1) and (2). The results for Experiments 1 to 3 are reported in the Online Supplement, and are similar to those reported here.

[Table 2 to 5]

Bias and RMSE

Results on Bias and RMSE are reported in Table 2 for the Pooled estimator and in Table 3 for the Mean Group estimator. For the Pooled estimator that uses only $\bar{\boldsymbol{x}}_t$ as factor proxies, the biases are almost negligible even in small samples, say $(N, T) = (20, 20)$. Overall, we find that the Pooled estimator using the IV, $\tilde{\boldsymbol{X}}^*$ tends to outperform the estimator using $\tilde{\boldsymbol{X}}^{2*}$ or $(\tilde{\boldsymbol{X}}^*, \tilde{\boldsymbol{X}}^{2*})$. In particular, the difference between RMSEs for the estimator using $\tilde{\boldsymbol{X}}^*$ and that using $(\tilde{\boldsymbol{X}}^*, \tilde{\boldsymbol{X}}^{2*})$ is almost negligible. It can also be seen that the finite sample performance of the proposed Pooled estimator is very close to that of the infeasible estimator obtained by using the true factors. Furthermore, the Pooled estimator using $(\bar{y}_t, \bar{\boldsymbol{x}}_t')'$ as factor proxies is outperformed by our proposed estimator in the measure of bias, although its performance improves as N increases. When $\bar{y}_t$ is used as an additional factor proxy, its RMSEs are almost identical to those using $\bar{\boldsymbol{x}}_t$ only even in large samples. This is consistent with our theoretical conjecture as described in Section 2. Finally, the Pooled estimator using $(\bar{y}_t, \bar{y}^*_t, \bar{\boldsymbol{x}}_t')'$ as factor proxies is outperformed by our proposed estimator in terms of both bias and RMSE, although its performance gradually improves as N increases.

For the Mean Group estimator, we find that the estimator using a combination of $\bar{\boldsymbol{x}}_t$ and $\tilde{\boldsymbol{X}}^*$ tends to outperform the other estimators and its finite sample performance is quite close to that of the infeasible estimator.[13]

[13]Notice that all the Mean Group estimators using $(\tilde{\boldsymbol{X}}^*, \tilde{\boldsymbol{X}}^{2*})$ as instruments including the infeasible estimator, do not perform well even in the large samples. We conjecture that this may be due to the structure of the spatial weighting matrix which causes a high correlation between $\tilde{\boldsymbol{X}}^*$ and $\tilde{\boldsymbol{X}}^{2*}$.

In the Online Supplement, we also report the simulation results for the cases with $\rho = 0.8$ and $h = (4, 6)$. We find that the relative performance of the Pooled and the Mean Group estimators, using $(\bar{y}_t, \bar{\boldsymbol{x}}_t)$ as factor proxies, becomes worse. This is consistent with the theoretical implication given by (14). By contrast, the performance of our proposed estimator using $\bar{\boldsymbol{x}}_t$ only is still satisfactory.

Size and Power

We now turn to the size and power results of the Pooled and Mean Group estimators reported in Tables 4 and 5, respectively. The size of our proposed estimator using $\bar{\boldsymbol{x}}_t$, is close to the 5% nominal level in almost all cases except when both N and T are small.[14] The estimators using $(\bar{y}_t, \bar{\boldsymbol{x}}'_t)'$ or $(\bar{y}^*_t, \bar{y}_t, \bar{\boldsymbol{x}}'_t)'$ seem to exhibit some size distortion when N is small, and it does not disappear even as T increases. However, when N increases, their size becomes close to the nominal level. On the other hand, the power of our proposed estimator using $\bar{\boldsymbol{x}}_t$ is slightly higher than the other cases. All the tests considered are consistent as both N and T increase. Not surprisingly, the infeasible estimator has the best power performance, especially when both N and T are small.

In summary, we recommend to use our proposed estimator utilising $\bar{\boldsymbol{x}}_t$ only as factor proxies, as its finite sample performance is quite satisfactory. Furthermore, it outperforms the standard CCE estimator using $(\bar{y}_t, \bar{\boldsymbol{x}}_t)$ as factor approximation. The additional simulation results presented in the Online Supplement also confirm that our proposed estimator is robust to various DGP settings including heteroskedastic and serially correlated errors as well as different intensity of the spatial dependence.

5 Empirical Applications

Finding an appropriate trading relationship with the rest EU member countries after Brexit has been debated extensively in the UK parliament and is one of many factors that has made Brexit quite delayed. While positive effects of EU membership on trade flows is widely acknowledged, consensus is not reached in the literature on the right magnitude of this effect and the estimated results are quite heterogeneously. Through the estimation of gravity panel data model with fixed effects, Carrere (2006) uses bilateral trade data for 130 countries from 1962 to 1996 and find that EU membership increases intra-EU trade by an average of 104% over the period. Baier et al. (2008), which also estimate a gravity panel data model but use data for 96 countries from 1960 to 2000, obtain a smaller effect at around 60%, and similar results are also obtained by Ebell (2016) and Mayer et al. (2019). Even Lower results are presented in Hufbauer and Schott (2009), where the effect is estimated to be 31% by using data from 1976 to 2005, and Eicher and Henn (2011), where the effect is estimated to be 37% by using data from 1950 to 2000. Employing similar data set as Eicher and Henn (2011) but starting from 1970 and additionally using Bayesian Model Averaging to account for model uncertainty, Eicher et al. (2012) find EU membership increases bilateral trade by 51%.

While fixed effects are included in almost all of the above mentioned studies to control for multilateral resistance, that aims to capture the fact that the bilateral trade flows depend on the bilateral barriers as well as trade barriers across all trading partners (Anderson and Van Wincoop (2003)), this is unlikely to be enough considering that multilateral resistance

[14] One exception is observed for the Mean Group estimators using $\{\tilde{\boldsymbol{X}}^*, \tilde{\boldsymbol{X}}^{2*}\}$ as instruments. In this case the test tends to display the size distortions when T is small, which do not disappear even as N increases. See also footnote 13.

is generally unobserved, time-varying and bilateral heterogeneous (Mastromarco et al. (2016a)). Hence, in this section, we will carefully examine the impact of EU trade union on trade flows under the econometric framework which could deal with the multilateral resistance more appropriately.

Two approaches have been popular in the literature that aim to control the multilateral resistance through explicitly modelling the CSD across different spatial units. The spatial model, advanced by Behrens et al. (2012), proposes to capture the effect of the multilateral resistance through employing the spatial weighting matrix directly derived from economic theory. Next, the factor-based approach, proposed by Serlenga and Shin (2007, 2013), can control the multilateral resistance terms through unobserved common factors with heterogeneous loadings. While these two strands of methods are separately applied, it is now clear that they actually capture different forms (weak and strong) of CSD (Chudik et al. (2011)). Therefore, it is more sensible to estimate the gravity model of trade flows that accounts for both weak and strong CSD by incorporating the spatial-effects and common factors, jointly. Furthermore, unlike most studies that impose the parameters to be homogeneous, we allow all the parameters to be heterogeneous. Hence, we propose to estimate the following extended gravity model of Mastromarco et al. (2016a):

$$
\begin{aligned}
ln(trade_{it}) = {} & \rho_i ln(trade_{it})^* + \beta_{1i} ln(gdp_{it}) + \beta_{2i} ln(sim_{it}) + \beta_{3i} ln(rer_{it}) \\
& + \beta_{4i} ln(rfe_{it}) + \beta_{5i} eec_{it} + e_{it}, \qquad (61) \\
e_{it} = {} & \boldsymbol{\gamma}_{yi}' \boldsymbol{f}_{yt} + \varepsilon_{it}, \qquad (62)
\end{aligned}
$$

where $ln(trade_{it})$ is the logarithm of the sum of bilateral export and import flows measured in millions of US dollars at the 2000 price, $ln(trade_{it})^* = \sum_{j=1}^{N} w_{ij} ln(trade_{jt})$ is the spatial counterpart with w_{ij} the (i, j)-th element of the spatial weighting matrix, gdp_{it} is the sum of gross domestic product (GDP) of the country pairs at the 2000 dollar price, rer_{it} is the real exchange rate measured in the 2000 dollar price, sim_{it} is a similarity measure in terms of the size of the country pair constructed by

$$
sim_{it} = \left[1 - \frac{gdp_{i,ot}}{gdp_{it}}^2 - \frac{gdp_{i,dt}}{gdp_{it}}^2 \right], \qquad (63)
$$

where $gdp_{i,ot}(gdp_{i,dt})$ represents the GDP of the origin (destination) country, $rfe_{it} = |pgdp_{i,ot} - pgdp_{i,dt}|$ measures countries' difference in relative factor endowment where $pgdp_{i,ot}(pgdp_{i,dt})$ is per capita GDP. The dummy variable eec_{it}, equals to one when both countries of origin and destination belong to the European Economic Community.

We apply the proposed model to the dataset for 91 country-pairs out of 14 EU countries (Austria, Belgium-Luxemburg, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Netherlands, Portugal, Spain, Sweden and the UK) over the period 1960-2018 (59 years).

We first estimate the model without considering spatial effect, using the fixed effects (FE) estimator and the pooled CCE (CCEP) estimator by Pesaran (2006). In particular, we consider both CCEP estimators with and without using $\bar{y}_t$ as factor proxy, denoted as CCEP-XY and CCEP-X, respectively. The estimated results are presented in Table 6 and are quite different for different estimation methods. In particular, cce_{it} is estimated to increase bilateral trade by around 53% in the FE model, which is similar to those obtained in Mayer et al. (2019) and Eicher et al. (2012). This effect decreased substantially after controlling CSD through unobserved factors and estimation via CCEP. Surprisingly, although CCEP-XY and CCEP-X only differ in including or excluding $\bar{y}_t$ as an additional factor proxy, their estimation results are quite different. While the estimation in CCEP-X is 31.8% and is similar to Hufbauer and Schott (2009) and Eicher and Henn (2011), that in CCEP-XY is much lower and maybe underestimated due to endogeneity issue cause by $\bar{y}_t$.

[Table 6]

Next, we present the main results for the proposed methodology, that could joint model the spatial effects and unobserved factors. For comparison, we also provide the results for the spatial autoregressive model with autoregressive disturbances (SARAR). The SARAR is estimated by the QML estimation while our joint modelling approach is estimated by combining CCE and IV estimation. When approximating unobserved factors, we consider two cases, one with $\bar{\boldsymbol{x}}_t$ only, denoted as HSAR-X, and the other with $(\bar{y}_t, \bar{\boldsymbol{x}}_t')'$, denoted as HSAR-XY. As cee_{it} is a dummy variable, we exclude using its CSA as factor approximation. For the IVs, we use a subset of the first and second order spatial lagged term of independent variables which also excludes using cee_{it}. Following the convention in spatial literature, we construct the distance-based weighting matrix, $\boldsymbol{W}_{dis}$, by employing the inverse squared distance using the geographical coordinates of countries pair capitals. Furthermore, as a robustness check, we also consider the border-based spatial weighting matrix, $\boldsymbol{W}_{bor}$, on the basis of contiguity. The estimation results are summarised in Table 7.

[Table 7]

We first discuss the results of using $\boldsymbol{W}_{dis}$. The spatial coefficient produced by the SARAR is significant but positive at 0.051, which we expect to be negative. As argued in Mastromarco et al. (2016a), the spatial lagged term here is meant to measure the multilateral trade resistance. If the trade barriers between country q and country j ($q \neq i$ and $q \neq j$) are reduced, then the trade flows between country j and country q increase while the trade flows between the country i and j would decrease. Moreover, the SARAR suffers severely from CSD significantly presented in the residuals, implying that the model with the spatial effects only may be insufficient for controlling CSD in the gravity trade model. One the contrary, our approaches are statistically more satisfactory as the CD test reduces substantially albeit still significant at 5% level for both HSAR-X and HSAR-XY. Although the spatial coefficients for both HSAR-XY and HSAR-X are significantly negative, it is much larger in magnitude in the HSAR-XY estimation. It is also interesting to find that the estimation results for other parameters in HSAR-XY are qualitatively similar to CCEP-XY and those for HSAR-X are qualitatively similar to HSAR-X, except that the effects of rer_{it} and rfe_{it} are now both insignificant, which may be sensible considering that the final impacts of both rer_{it} and rfe_{it} would be ambiguous in the context of the impacts on inter-industry and intra-industry trade flows.

Next, we turn to the results obtained using the spatial weighting matrix, $\boldsymbol{W}_{bor}$. Except for HSAR-X, the estimation of spatial coefficient changes quite substantially for the other two models, which suggests the robustness of HSAR-X estimation to different weighting schemes. The results for other parameters are qualitatively similar to the previous findings.

In the model with the spatial effects, it is now standard to provide a summary measure of the direct, indirect and total effects of independent regressors on the dependent variable as follows (see LeSage and Pace (2009)):

- Average Direct Effect (ADE): The average direct effect of the p-th explanatory variables is the average of the diagonal elements of $(\boldsymbol{I}_N - \rho \boldsymbol{W})^{-1}\beta_p$, where β_p is the corresponding coefficient.
- Average Indirect Effect (AIE): The average indirect effect is given by the average row sum of the off-diagonal elements of $(\boldsymbol{I}_N - \rho \boldsymbol{W})^{-1}\beta_p$.
- Total Effects (TE): The total effects is the sum of ADE and AIE.

Overall, our proposed estimation model, HSAR-X produces sensible and the most robust results among all, we therefore only report the ADE, AIE and TE for HSAR-X in Table 8.[15]

[Table 8]

All the direct, indirect and total effects are statistically significant. The indirect effects are much smaller than the direct effects, and the signs of them are opposite due to the negative of the spatial coefficient. As a result, the total effects are smaller than the direct effect. The explanation why AIE takes an opposite sign to ADE is similar to the explanation behind the negative spatial coefficient. For example, the trade union membership boosts the bilateral trade flows between the corresponding country pairs directly, but at the same time, it tends to decrease the bilateral trade flows indirectly by reducing their trading demands with other countries. Moreover, the estimation results using the different spatial weighting matrices, $\boldsymbol{W}_{dis}$ and $\boldsymbol{W}_{bor}$, are qualitatively similar to each other, which again highlights the robustness of our approach.

To evaluate the potentially negative impact of Brexit on the UK economy, we focus simply on the effect of trade union on trade flows. From Table 8, we find that the direct and indirect effects are estimated at 42.68%[16] and -5.60% such that the total effect is 34.69% when using $\boldsymbol{W}_{dis}$ as weighting matrix, and they are 43.71%, -4.41% and 37.37%, respectively, when using $\boldsymbol{W}_{bor}$. The total effects here are lower than that simply estimated by FE effect model reported in Table 6 and some other studies, e.g. Eicher et al. (2012) and Mayer et al. (2019), but are similar to those reported in Hufbauer and Schott (2009) and Eicher and Henn (2011).

We then explore how these effects change after 2010 through recursive estimation. The results are plotted in Figure 1.

[Figure 1]

While the direct effects estimated using $\boldsymbol{W}_{dis}$ experienced an increase during 2011-2014, that estimated by using $\boldsymbol{W}_{bor}$ went through a decrease in the same time period. They both remained steady in recent years at around 0.35. On the contrary, the indirect effects estimated by $\boldsymbol{W}_{bor}$ decreased during 2011-2014, and that for $\boldsymbol{W}_{bor}$ increased meanwhile. Both of them also kept steady recently and were almost the same at around -0.05. It is interesting to find that both the changes of the total effects estimated by $\boldsymbol{W}_{dis}$ and $\boldsymbol{W}_{bor}$ are almost negligible in the past 8 years and the estimation results of using these two different weighting schemes are quite similar at around 0.3 with the latter being slightly smaller. These findings further confirm the robustness of our predictions on the reduction of trade flows between the UK and the EU members in the event of a hard Brexit.

6 Concluding Remarks

Limited progress has been made on the development of unified models incorporating both spatial dependence and common factors. A small number of studies have approached this issue, simply by assuming that the slope parameters are homogeneous. Following this research trend, we have developed a unifying econometric framework for the estimation of heterogeneous panel data models that can accommodate spatial and factor dependence, jointly. We propose to approximate common factors by cross-section averages of independent variables only, and deal with the spatial

[15] As stated in LeSage and Pace (2009), the significance of the direct and indirect effects for each regressor is not necessarily similar to that of the Pooled estimators reported above, because these effects are constructed by the product of spatial and slope coefficients, see also Elhorst (2014).

[16] $= e^{0.3554} - 1$

endogeneity via the instrumental variables method. We show that the individual parameters can be consistently estimated by applying the de-factored IVs directly to the original regressions. We also derive the Mean Group and the Pooled estimators, and establish that they follow asymptotic normal distributions. We then provide the nonparametric variance estimators, which are robust to the presence of heteroskedastic and serially-correlated disturbances as well as the parameter heterogeneity.

Monte Carlo simulations confirm that the finite sample performance of our proposed estimators is quite satisfactory. We demonstrate the usefulness of our approach with an application to a gravity model of bilateral trade flows for 91 pairs of 14 EU countries, and find that the trade flows between the UK and the rest of the EU members would fall substantially as large as 35%, following a hard Brexit.

We conclude by noting two avenues for future research. A natural avenue by which to develop a dynamic heterogeneous spatial panel data model that combines strong and weak CSD is to generalise our approach to accommodate the spatio-temporal dynamics, resulting in a framework that includes the spatial dynamic panel data model as a special case. Such an extension can shed further light on improving our understanding of the dynamic network of output connectedness, see Shin and Thornton (2020). Another important extension is the development of nonlinear/quantile heterogeneous panel data and quantile models incorporating both spatial dependence and common factors.

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Appendix A Proofs of Theorems

We provide the proofs for Theorems 1-4. All Lemmas used for the proofs can be found in the Online Supplement.

A.1 Proof of Theorem 1

A.1.1 Consistency

We first establish the consistency of the individual estimator, $\hat{\boldsymbol{\theta}}_i$ by proving that the right hand side (RHS) of (29) converges in probability to $\mathbf{0}$ as $(N,\ T) \xrightarrow{j} \infty$.

First, we analyse the property of $\frac{\boldsymbol{Z}_i'\boldsymbol{\Pi}\boldsymbol{Z}_i}{T}$, which is determined by $\frac{\tilde{\boldsymbol{Q}}_i'\tilde{\boldsymbol{Q}}_i}{T}$ and $\frac{\tilde{\boldsymbol{Q}}_i'\boldsymbol{Z}_i}{T}$. Using the definition of $\tilde{\boldsymbol{Q}}_i = \boldsymbol{M}_{\bar{x}}(\boldsymbol{I}_T \otimes \boldsymbol{b}_i')\boldsymbol{Q}$ and the result in Lemma 5(c), it is straightforward to show that

$$\frac{\tilde{\boldsymbol{Q}}_i'\tilde{\boldsymbol{Q}}_i}{T} = \frac{\tilde{\boldsymbol{Q}}_{i0}'\tilde{\boldsymbol{Q}}_{i0}}{T} + O_p(\frac{1}{N}) + O_p(\frac{1}{\sqrt{NT}}), \tag{A.1}$$

where $\tilde{\boldsymbol{Q}}_{i0} = \boldsymbol{M}_{\boldsymbol{f}_x}(\boldsymbol{I}_T \otimes \boldsymbol{b}_i')\boldsymbol{Q}$.

To analyse $\frac{\tilde{\boldsymbol{Q}}_i'\boldsymbol{Z}_i}{T}$, we notice from (4) that

$$\begin{aligned}
\boldsymbol{y} &= (\boldsymbol{I}_{NT} - \boldsymbol{I}_T \otimes \boldsymbol{\rho}\boldsymbol{W})^{-1}[(\boldsymbol{I}_T \otimes \boldsymbol{B})\boldsymbol{\chi} + (\boldsymbol{I}_T \otimes \boldsymbol{\gamma}_y)\boldsymbol{f}_y + \boldsymbol{\varepsilon}] \\
&= (\boldsymbol{I}_T \otimes (\boldsymbol{I}_N - \boldsymbol{\rho}\boldsymbol{W})^{-1})[(\boldsymbol{I}_T \otimes \boldsymbol{B})\boldsymbol{\chi} + (\boldsymbol{I}_T \otimes \boldsymbol{\gamma}_y)\boldsymbol{f}_y + \boldsymbol{\varepsilon}] \\
&= (\boldsymbol{I}_T \otimes \boldsymbol{S}^{-1})[(\boldsymbol{I}_T \otimes \boldsymbol{B})\boldsymbol{\chi} + (\boldsymbol{I}_T \otimes \boldsymbol{\gamma}_y)\boldsymbol{f}_y + \boldsymbol{\varepsilon}],
\end{aligned} \tag{A.2}$$

which implies that $\boldsymbol{Z}_i = (\boldsymbol{y}_i^*, \boldsymbol{X}_{i.})$ can be rewritten as:

$$\begin{aligned}
\boldsymbol{Z}_i &= ((\boldsymbol{I}_T \otimes \boldsymbol{w}_i)(\boldsymbol{I}_T \otimes \boldsymbol{S}^{-1})[(\boldsymbol{I}_T \otimes \boldsymbol{B})\boldsymbol{\chi} + (\boldsymbol{I}_T \otimes \boldsymbol{\gamma}_y)\boldsymbol{f}_y + \boldsymbol{\varepsilon}], \boldsymbol{X}_{i.}) \\
&= ((\boldsymbol{I}_T \otimes \boldsymbol{b}_i'\boldsymbol{W}\boldsymbol{S}^{-1})[(\boldsymbol{I}_T \otimes \boldsymbol{B})\boldsymbol{\chi} + (\boldsymbol{I}_T \otimes \boldsymbol{\gamma}_y)\boldsymbol{f}_y + \boldsymbol{\varepsilon}], \boldsymbol{X}_{i.}) \\
&= ((\boldsymbol{I}_T \otimes \boldsymbol{b}_i'\boldsymbol{G})[(\boldsymbol{I}_T \otimes \boldsymbol{B})\boldsymbol{\chi} + (\boldsymbol{I}_T \otimes \boldsymbol{\gamma}_y)\boldsymbol{f}_y + \boldsymbol{\varepsilon}], \boldsymbol{X}_{i.}) \\
&= ((\boldsymbol{I}_T \otimes \boldsymbol{b}_i'\boldsymbol{G})(\boldsymbol{I}_T \otimes \boldsymbol{B})\boldsymbol{\chi}, \boldsymbol{X}_{i.}) + ((\boldsymbol{I}_T \otimes \boldsymbol{b}_i'\boldsymbol{G})[(\boldsymbol{I}_T \otimes \boldsymbol{\gamma}_y)\boldsymbol{f}_y + \boldsymbol{\varepsilon}], 0) \\
&= \boldsymbol{Z}_{i0} + ((\boldsymbol{I}_T \otimes \boldsymbol{b}_i'\boldsymbol{G})[(\boldsymbol{I}_T \otimes \boldsymbol{\gamma}_1)\boldsymbol{f}_1 + ((\boldsymbol{I}_T \otimes \boldsymbol{\gamma}_2)\boldsymbol{f}_2 + \boldsymbol{\varepsilon}],\ 0),
\end{aligned} \tag{A.3}$$

where $\boldsymbol{Z}_{i0} = ((\boldsymbol{I}_T \otimes \boldsymbol{b}_i'\boldsymbol{G})(\boldsymbol{I}_T \otimes \boldsymbol{B})\boldsymbol{\chi}, \boldsymbol{X}_{i.})$. Hence

$$\frac{\tilde{\boldsymbol{Q}}_i'\boldsymbol{Z}_i}{T} = \frac{\tilde{\boldsymbol{Q}}_i'\boldsymbol{Z}_{i0}}{T} + \frac{\tilde{\boldsymbol{Q}}_i'(\boldsymbol{I}_T \otimes \boldsymbol{b}_i'\boldsymbol{G})[(\boldsymbol{I}_T \otimes \boldsymbol{\gamma}_1)\boldsymbol{f}_1 + (\boldsymbol{I}_T \otimes \boldsymbol{\gamma}_2)\boldsymbol{f}_2 + \boldsymbol{\varepsilon}],\ 0)}{T}. \tag{A.4}$$

By Lemma 8, the second term on RHS of (A.4) is negligible as $(N,T) \xrightarrow{j} \infty$. We thus focus on

$$\frac{\tilde{\boldsymbol{Q}}_i'\boldsymbol{Z}_{i0}}{T} = \left(\frac{\tilde{\boldsymbol{Q}}_i'(\boldsymbol{I}_T \otimes \boldsymbol{b}_i'\boldsymbol{G})(\boldsymbol{I}_T \otimes \boldsymbol{B})\boldsymbol{\chi}}{T}, \frac{\tilde{\boldsymbol{Q}}_i'\boldsymbol{X}_{i.}}{T}\right). \tag{A.5}$$

Using the definition of $\boldsymbol{Q}$ and Lemma 5(c), we have:

$$\frac{\tilde{\boldsymbol{Q}}_i'\boldsymbol{X}_{i.}}{T} = \frac{\tilde{\boldsymbol{Q}}_{i0}'\boldsymbol{X}_{i.}}{T} + O_p(\frac{1}{N}) + O_p(\frac{1}{\sqrt{NT}}), \tag{A.6}$$

and

$$\begin{aligned}
&\frac{\tilde{\boldsymbol{Q}}_i'(\boldsymbol{I}_T \otimes \boldsymbol{b}_i'\boldsymbol{G})(\boldsymbol{I}_T \otimes \boldsymbol{B})\boldsymbol{\chi}}{T} \\
=& \frac{\tilde{\boldsymbol{Q}}_i'(\boldsymbol{I}_T \otimes \boldsymbol{b}_i'\boldsymbol{G})(\boldsymbol{I}_T \otimes \boldsymbol{B})vec(vec(\boldsymbol{X}'_{.1}), vec(\boldsymbol{X}'_{.2}), ..., vec(\boldsymbol{X}'_{.T}))}{T} \\
=& \frac{\tilde{\boldsymbol{Q}}_i'(\boldsymbol{I}_T \otimes \boldsymbol{b}_i'\boldsymbol{G})vec(\boldsymbol{B}vec(\boldsymbol{X}'_{.1}), \boldsymbol{B}vec(\boldsymbol{X}'_{.2}), ..., \boldsymbol{B}vec(\boldsymbol{X}'_{.T}))}{T} \\
=& \frac{\tilde{\boldsymbol{Q}}_i'(\boldsymbol{I}_T \otimes \boldsymbol{b}_i'\boldsymbol{G})vec((\boldsymbol{X}_{1.}\boldsymbol{\beta}_1, \boldsymbol{X}_{2.}\boldsymbol{\beta}_2, ..., \boldsymbol{X}_{N.}\boldsymbol{\beta}_N)')}{T} \\
=& \frac{\tilde{\boldsymbol{Q}}_i'(vec(\boldsymbol{b}_i'\boldsymbol{G}(\boldsymbol{X}_{1.}\boldsymbol{\beta}_1, \boldsymbol{X}_{2.}\boldsymbol{\beta}_2, ..., \boldsymbol{X}_{N.}\boldsymbol{\beta}_N)')}{T} \\
=& \frac{\tilde{\boldsymbol{Q}}_i'(\boldsymbol{X}_{1.}\boldsymbol{\beta}_1, \boldsymbol{X}_{2.}\boldsymbol{\beta}_2, ..., \boldsymbol{X}_{N.}\boldsymbol{\beta}_N)\boldsymbol{G}'\boldsymbol{b}_i}{T} \\
=& \sum_{j=1}^{N} \frac{\tilde{\boldsymbol{Q}}_i'\boldsymbol{X}_{j.}\boldsymbol{\beta}_j\boldsymbol{G}_{ij}}{T} = \sum_{j=1}^{N} \boldsymbol{G}_{ij}\frac{\tilde{\boldsymbol{Q}}_i'\boldsymbol{X}_{j.}}{T}\boldsymbol{\beta}_j \\
=& \sum_{j=1}^{N} \boldsymbol{G}_{ij}\frac{\tilde{\boldsymbol{Q}}_{i0}'\boldsymbol{X}_{j.}}{T}\boldsymbol{\beta}_j + O_p(\frac{1}{N}) + O_p(\frac{1}{\sqrt{NT}}),
\end{aligned} \tag{A.7}$$

where $\boldsymbol{G}_{ij}$ is the ij-th element of $\boldsymbol{G}$. The forth equality follows from the fact that

$$\begin{aligned}
\underset{NT\times 1}{vec(\boldsymbol{B}vec(\boldsymbol{X}'_{.1}), \boldsymbol{B}vec(\boldsymbol{X}'_{.2}), ..., \boldsymbol{B}vec(\boldsymbol{X}'_{.T}))} &= \underset{NT\times 1}{vec}\left(\begin{bmatrix} \boldsymbol{\beta}_1'\boldsymbol{x}_{11} & \boldsymbol{\beta}_1'\boldsymbol{x}_{12} & \cdots & \boldsymbol{\beta}_1'\boldsymbol{x}_{1T} \\ \boldsymbol{\beta}_2'\boldsymbol{x}_{21} & \boldsymbol{\beta}_2'\boldsymbol{x}_{22} & \cdots & \boldsymbol{\beta}_2'\boldsymbol{x}_{2T} \\ \vdots & & & \\ \boldsymbol{\beta}_N'\boldsymbol{x}_{N1} & \boldsymbol{\beta}_N'\boldsymbol{x}_{N2} & \cdots & \boldsymbol{\beta}_N'\boldsymbol{x}_{NT} \end{bmatrix}\right) \\
&= vec\left(\begin{bmatrix} \boldsymbol{\beta}_1'\boldsymbol{X}_{1.}' \\ \boldsymbol{\beta}_2'\boldsymbol{X}_{2.}' \\ \vdots \\ \boldsymbol{\beta}_N'\boldsymbol{X}_{N.}' \end{bmatrix}\right),
\end{aligned} \tag{A.8}$$

and the last equality follows from Lemma 5(c) and the fact that $\boldsymbol{\beta}_i$ is uniformly bounded in probability and $\boldsymbol{G}$ has the bounded row and column norms.

Combining (A.6) and (A.7) and using Lemma 8, we can simplify (A.4) to

$$\frac{\tilde{\boldsymbol{Q}}_i'\boldsymbol{Z}_i}{T} = \frac{\tilde{\boldsymbol{Q}}_{i0}'\boldsymbol{Z}_{i0}}{T} + O_p(\frac{1}{N}) + O_p(\frac{1}{\sqrt{T}}), \tag{A.9}$$

which holds uniformly for all i. Using (A.1) and Assumption 6, and applying the Continuous Mapping Theorem, we have:

$$\frac{\boldsymbol{Z}_i'\boldsymbol{\Pi}_i\boldsymbol{Z}_i}{T} = \frac{\boldsymbol{Z}_{i0}'\boldsymbol{\Pi}_{i0}\boldsymbol{Z}_{i0}}{T} + O_p(\frac{1}{N}) + O_p(\frac{1}{\sqrt{T}}), \tag{A.10}$$

where $\boldsymbol{\Pi}_{i0} = \tilde{\boldsymbol{Q}}_{i0}'(\tilde{\boldsymbol{Q}}_{i0}'\tilde{\boldsymbol{Q}}_{i0}')^{-1}\tilde{\boldsymbol{Q}}_{i0}'$.

Second, to derive the asymptotic property of the remaining terms in RHS of (29), we only need to establish that as $(N,T) \xrightarrow{j} \infty$,

$$\frac{\tilde{\boldsymbol{Q}}_i'[\boldsymbol{F}_1\gamma_{1i} + \boldsymbol{F}_2\gamma_{2i} + \boldsymbol{\varepsilon}_{i.}]}{T} \to \boldsymbol{0},$$

which is proved in Lemma 5(a) and (b). Under (A.1), (A.9) and Assumption 6,

$$\frac{\boldsymbol{Z}_i'\boldsymbol{\Pi}_i[\boldsymbol{F}_1\gamma_{1i} + \boldsymbol{F}_2\gamma_{2i} + \boldsymbol{\varepsilon}_{i.}]}{T} \to \mathbf{0} \tag{A.11}$$

in probability as $(N,\ T) \xrightarrow{j} \infty$.

Combining (29), (A.10) and (A.11), and under Assumption 6, it is easily seen that $\hat{\boldsymbol{\theta}}_i$ is a consistent estimator of $\boldsymbol{\theta}_i$.

A.1.2 Asymptotic Normality

We turn to deriving the asymptotic distribution of $\hat{\boldsymbol{\theta}}_i$. First, multiplying both sides of (29) by $\sqrt{T}$:

$$\sqrt{T}(\hat{\boldsymbol{\theta}}_i - \boldsymbol{\theta}_i) = \left[\frac{\boldsymbol{Z}_i'\tilde{\boldsymbol{Q}}_i}{T}(\frac{\tilde{\boldsymbol{Q}}_i'\tilde{\boldsymbol{Q}}_i}{T})^{-1}\frac{\tilde{\boldsymbol{Q}}_i'\boldsymbol{Z}_i}{T}\right]^{-1}\frac{\boldsymbol{Z}_i'\tilde{\boldsymbol{Q}}_i}{T}(\frac{\tilde{\boldsymbol{Q}}_i'\tilde{\boldsymbol{Q}}_i}{T})^{-1}\frac{\tilde{\boldsymbol{Q}}_i'(\boldsymbol{F}_1\gamma_{1i} + \boldsymbol{F}_2\gamma_{2i} + \boldsymbol{\varepsilon}_{i.})}{\sqrt{T}}. \tag{A.12}$$

Using (A.1), (A.9) (A.10), Lemma 5 and Assumption 6, we have:

$$\begin{aligned}\sqrt{T}(\hat{\boldsymbol{\theta}}_i - \boldsymbol{\theta}_i) = &\left[\frac{\boldsymbol{Z}_{i0}'\tilde{\boldsymbol{Q}}_{i0}}{T}(\frac{\tilde{\boldsymbol{Q}}_i'\tilde{\boldsymbol{Q}}_{i0}}{T})^{-1}\frac{\tilde{\boldsymbol{Q}}_{i0}'\boldsymbol{Z}_{i0}}{T}\right]^{-1}\frac{\boldsymbol{Z}_{i0}'\tilde{\boldsymbol{Q}}_{i0}}{T}(\frac{\tilde{\boldsymbol{Q}}_{i0}'\tilde{\boldsymbol{Q}}_{i0}'}{T})^{-1}\frac{\tilde{\boldsymbol{Q}}_{i0}'[\boldsymbol{F}_2\gamma_{2i} + \boldsymbol{\varepsilon}_{i.}]}{\sqrt{T}} \\ &+ O_p(\frac{\sqrt{T}}{N}) + O_p(\frac{1}{\sqrt{N}}).\end{aligned} \tag{A.13}$$

The key here is to establish the asymptotic normality of $\frac{\tilde{\boldsymbol{Q}}_{i0}'e_{i.}}{\sqrt{T}} = \frac{\tilde{\boldsymbol{Q}}_{i0}'[\boldsymbol{F}_2\gamma_{2i}+\boldsymbol{\varepsilon}_{i.}]}{\sqrt{T}}$. From (23), we have $\boldsymbol{Q}_{i.} = (\boldsymbol{X}_{i.}, \boldsymbol{X}_{i.}^*, \boldsymbol{X}_{i.}^{2*}, ...)$, where $\boldsymbol{X}_{i.}^{r*} = (\boldsymbol{I}_T \otimes \boldsymbol{b}_i'\boldsymbol{W})\boldsymbol{X}$. Hence,

$$\frac{\tilde{\boldsymbol{Q}}_{i0}'(\boldsymbol{F}_2\gamma_{2i} + \boldsymbol{\varepsilon}_{i.})}{\sqrt{T}} = \begin{bmatrix} \frac{\boldsymbol{X}_{i.}'\boldsymbol{M}_{\boldsymbol{f}_x}(\boldsymbol{F}_2\gamma_{2i}+\boldsymbol{\varepsilon}_{i.})}{\sqrt{T}} \\ \frac{\boldsymbol{X}_{i.}^{*\prime}\boldsymbol{M}_{\boldsymbol{f}_x}(\boldsymbol{F}_2\gamma_{2i}+\boldsymbol{\varepsilon}_{i.})}{\sqrt{T}} \\ \vdots \end{bmatrix}. \tag{A.14}$$

We will establish only the asymptotic distribution of

$$\frac{\boldsymbol{X}_{i.}'\boldsymbol{M}_{\boldsymbol{f}_x}(\boldsymbol{F}_2\gamma_{2i} + \boldsymbol{\varepsilon}_{i.})}{\sqrt{T}}.$$

The asymptotic distributions of the other terms in (A.14) can be established in a similar manner.

By (26), we have

$$\frac{\boldsymbol{X}_{i.}'\boldsymbol{M}_{\boldsymbol{f}_x}(\boldsymbol{F}_2\gamma_{2i} + \boldsymbol{\varepsilon}_{i.})}{\sqrt{T}} = \frac{\boldsymbol{V}_{i.}'\boldsymbol{M}_{\boldsymbol{f}_x}(\boldsymbol{F}_2\gamma_{2i} + \boldsymbol{\varepsilon}_{i.})}{\sqrt{T}}. \tag{A.15}$$

Further, notice that

$$\frac{\boldsymbol{V}_{i.}'\boldsymbol{M}_{\boldsymbol{f}_x}(\boldsymbol{F}_2\gamma_{2i} + \boldsymbol{\varepsilon}_{i.})}{\sqrt{T}} = \frac{\boldsymbol{V}_{i.}'(\boldsymbol{F}_2\gamma_{2i} + \boldsymbol{\varepsilon}_{i.})}{\sqrt{T}} + \frac{\boldsymbol{V}_{i.}'\boldsymbol{F}_x(\boldsymbol{F}_x'\boldsymbol{F}_x)^{-1}\boldsymbol{F}_x(\boldsymbol{F}_2\gamma_{2i} + \boldsymbol{\varepsilon}_{i.})}{\sqrt{T}},$$

where the second term on RHS is negligible by Lemma 2(c) and Lemma 3(d). Hence,

$$\frac{\boldsymbol{V}_{i.}'\boldsymbol{M}_{\boldsymbol{f}_x}(\boldsymbol{F}_2\gamma_{2i} + \boldsymbol{\varepsilon}_{i.})}{\sqrt{T}} = \frac{1}{\sqrt{T}}\sum_{t=1}^{T}\boldsymbol{v}_{it}'(\boldsymbol{f}_{2t}'\gamma_{2i} + \varepsilon_{it}) + o_p(1). \tag{A.16}$$

The first term has zero mean, and by Assumption 3, $\boldsymbol{\gamma}_{2i}$ is bounded in probability. Furthermore, $\boldsymbol{f}_{2t}$, ε_{it} and $\boldsymbol{v}_{it}$ are mutually independent and have absolutely summable autocovariances. By applying the Central Limit Theorem (CLT) for linear processes with absolutely summable autocovariances, we have:

$$\frac{\boldsymbol{X}_{i.}'\boldsymbol{M}_{\boldsymbol{f}_x}\boldsymbol{F}_2\boldsymbol{\gamma}_{2i}}{\sqrt{T}} \xrightarrow{d} N(0, E(\frac{\boldsymbol{X}_{i.}'\boldsymbol{M}_{\boldsymbol{f}_x}\boldsymbol{F}_2\boldsymbol{\gamma}_{2i}\boldsymbol{\gamma}_{2i}{}'\boldsymbol{F}_2{}'\boldsymbol{M}_{\boldsymbol{f}_x}\boldsymbol{X}_{i.}}{T})), \tag{A.17}$$

$$\frac{\boldsymbol{X}_{i.}'\boldsymbol{M}_{\boldsymbol{f}_x}\boldsymbol{\varepsilon}_{i.}}{T} \xrightarrow{d} N(0, E(\frac{\boldsymbol{X}_{i.}'\boldsymbol{M}_{\boldsymbol{f}_x}\boldsymbol{\Omega}_{\varepsilon,i}\boldsymbol{M}_{\boldsymbol{f}_x}\boldsymbol{X}_{i.}}{T})). \tag{A.18}$$

Similarly, the asymptotic normality can be established for the rest terms in (A.14). By imposing $T/N^2 \to 0$ in (A.13), we complete the proof of Theorem 1.

A.2 Proof of Theorem 2

A.2.1 Consistency

Under Assumption 5, it is easily seen that

$$\frac{1}{N}\sum_{i=1}^{N}\boldsymbol{\xi}_i \xrightarrow{p} \mathbf{0}, \tag{A.19}$$

by the weak law of large numbers (WLLN). In Lemma 6, we derive:

$$\frac{1}{N}\sum_{i=1}^{N}\left(\frac{\boldsymbol{Z}_i'\boldsymbol{\Pi}_i\boldsymbol{Z}_i}{T}\right)^{-1}\frac{\boldsymbol{Z}_i'\boldsymbol{\Pi}_i\boldsymbol{F}_1\boldsymbol{\gamma}_{1i}}{T} = O_p(\frac{1}{\sqrt{NT}}) + O_p(\frac{1}{N}), \tag{A.20}$$

$$\frac{1}{N}\sum_{i=1}^{N}\left(\frac{\boldsymbol{Z}_i'\boldsymbol{\Pi}_i\boldsymbol{Z}_i}{T}\right)^{-1}\frac{\boldsymbol{Z}_i'\boldsymbol{\Pi}_i\boldsymbol{\varepsilon}_{i.}}{T} = O_p(\frac{1}{\sqrt{NT}}), \tag{A.21}$$

$$\frac{1}{N}\sum_{i=1}^{N}\left(\frac{\boldsymbol{Z}_i'\boldsymbol{\Pi}_i\boldsymbol{Z}_i}{T}\right)^{-1}\frac{\boldsymbol{Z}_i'\boldsymbol{\Pi}_i\boldsymbol{F}_2\boldsymbol{\gamma}_{2i}}{T} = O_p(\frac{1}{\sqrt{NT}}) + O_p(\frac{1}{N}). \tag{A.22}$$

Applying these results, we prove that $\hat{\boldsymbol{\theta}}_{MG}$ is a consistent estimator for $\boldsymbol{\theta}$ as $(N,\ T) \xrightarrow{j} \infty$.

A.2.2 Asymptotic Normality

We multiply both side of (37) by $\sqrt{N}$:

$$\begin{aligned}\sqrt{N}(\hat{\boldsymbol{\theta}}_{MG} - \boldsymbol{\theta}) &= \frac{1}{\sqrt{N}}\sum_{i=1}^{N}\boldsymbol{\xi}_i + \sqrt{N}\left[\frac{1}{N}\sum_{i=1}^{N}\left(\frac{\boldsymbol{Z}_i'\boldsymbol{\Pi}_i\boldsymbol{Z}_i}{T}\right)^{-1}\frac{\boldsymbol{Z}_i'\boldsymbol{\Pi}_i(\boldsymbol{F}_1\boldsymbol{\gamma}_{1i} + \boldsymbol{F}_2\boldsymbol{\gamma}_{2i} + \boldsymbol{\varepsilon}_{i.})}{T}\right] \\ &= \frac{1}{\sqrt{N}}\sum_{i=1}^{N}\boldsymbol{\xi}_i + \frac{1}{\sqrt{N}}\sum_{i=1}^{N}\left(\frac{\boldsymbol{Z}_i'\boldsymbol{\Pi}_i\boldsymbol{Z}_i}{T}\right)^{-1}\frac{\sqrt{N}\boldsymbol{Z}_i'\boldsymbol{\Pi}_i\boldsymbol{F}_1\boldsymbol{\gamma}_{1i}}{T} \\ &+ \frac{1}{\sqrt{N}}\sum_{i=1}^{N}\left(\frac{\boldsymbol{Z}_i'\boldsymbol{\Pi}_i\boldsymbol{Z}_i}{T}\right)^{-1}\frac{\boldsymbol{Z}_i'\boldsymbol{\Pi}_i\boldsymbol{F}_2\boldsymbol{\gamma}_{2i}}{T} + \frac{1}{\sqrt{N}}\sum_{i=1}^{N}\left(\frac{\boldsymbol{Z}_i'\boldsymbol{\Pi}_i\boldsymbol{Z}_i}{T}\right)^{-1}\frac{\boldsymbol{Z}_i'\boldsymbol{\Pi}_i\boldsymbol{\varepsilon}_{i.}}{T}.\end{aligned} \tag{A.23}$$

Using (A.20), (A.21) and (A.22), we obtain:

$$\frac{1}{N}\sum_{i=1}^{N}\left(\frac{\boldsymbol{Z}_i'\boldsymbol{\Pi}_i\boldsymbol{Z}_i}{T}\right)^{-1}\frac{\sqrt{N}\boldsymbol{Z}_i'\boldsymbol{\Pi}_i\boldsymbol{F}_1\gamma_{1i}}{T} = O_p(\frac{1}{\sqrt{T}}) + O_p(\frac{1}{\sqrt{N}}), \tag{A.24}$$

$$\frac{1}{N}\sum_{i=1}^{N}\left(\frac{\boldsymbol{Z}_i'\boldsymbol{\Pi}_i\boldsymbol{Z}_i}{T}\right)^{-1}\frac{\sqrt{N}\boldsymbol{Z}_i'\boldsymbol{\Pi}_i\boldsymbol{F}_2\gamma_{2i}}{T} = O_p(\frac{1}{\sqrt{T}}) + O_p(\frac{1}{\sqrt{N}}), \tag{A.25}$$

$$\frac{1}{N}\sum_{i=1}^{N}\left(\frac{\boldsymbol{Z}_i'\boldsymbol{\Pi}_i\boldsymbol{Z}_i}{T}\right)^{-1}\frac{\sqrt{N}\boldsymbol{Z}_i'\boldsymbol{\Pi}_i\boldsymbol{\varepsilon}_{i.}}{T} = O_p(\frac{1}{\sqrt{T}}). \tag{A.26}$$

Then, as $(N,T)\xrightarrow{j}\infty$, we can show that

$$\sqrt{N}(\hat{\boldsymbol{\theta}}_{MG}-\boldsymbol{\theta}) = \frac{1}{\sqrt{N}}\sum_{i=1}^{N}\boldsymbol{\xi}_i + O_p(\frac{1}{\sqrt{T}}) + O_p(\frac{1}{\sqrt{N}}) = \frac{1}{\sqrt{N}}\sum_{i=1}^{N}\boldsymbol{\xi}_i + o_p(1), \tag{A.27}$$

Under Assumption 5, we have:

$$\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\boldsymbol{\xi}_i \xrightarrow{d} N(\boldsymbol{0},\boldsymbol{\Omega}_{\xi}), \tag{A.28}$$

which completes the proof of Theorem 2.

A.3 Proof of Theorem 3

Using (43), (A.1) and (A.9), and under Assumption 6, it is straightforward to show that the asymptotic property of $\hat{\boldsymbol{\theta}}_P$ is determined by

$$\frac{1}{N}\sum_{i=1}^{N}\frac{\tilde{\boldsymbol{Q}}_i'(\boldsymbol{Z}_i\boldsymbol{\xi}_i + \boldsymbol{F}_1\gamma_{1i} + \boldsymbol{F}_2\gamma_{2i} + \boldsymbol{\varepsilon}_{i.})}{T}. \tag{A.29}$$

We can easily establish the consistency of $\hat{\boldsymbol{\theta}}_P$ by combining the results in(A.9), Assumption 6 and Lemma 7.

To derive its asymptotic distribution, we use results in Lemma 7 and obtain:

$$\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\frac{\tilde{\boldsymbol{Q}}_i'\boldsymbol{F}_1\gamma_{1i}}{T} = \frac{1}{\sqrt{N}}\sum_{i=1}^{N}\frac{\boldsymbol{Q}'(\boldsymbol{I}_T\otimes\boldsymbol{b}_i)\boldsymbol{M}_{\bar{x}}\boldsymbol{F}_1\gamma_{1i}}{T} = O_p(\frac{1}{\sqrt{N}}) + O_p(\frac{1}{\sqrt{T}}) = o_p(1), \tag{A.30}$$

$$\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\frac{\tilde{\boldsymbol{Q}}_i'\boldsymbol{\varepsilon}_{i.}}{T} = \frac{1}{\sqrt{N}}\sum_{i=1}^{N}\frac{\boldsymbol{Q}'(\boldsymbol{I}_T\otimes\boldsymbol{b}_i)\boldsymbol{M}_{\bar{x}}\boldsymbol{\varepsilon}_{i.}}{T} = O_p(\frac{1}{\sqrt{T}}) = o_p(1), \tag{A.31}$$

$$\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\frac{\tilde{\boldsymbol{Q}}_i'\boldsymbol{F}_2\gamma_{2i}}{T} = \frac{1}{\sqrt{N}}\sum_{i=1}^{N}\frac{\boldsymbol{Q}'(\boldsymbol{I}_T\otimes\boldsymbol{b}_i)\boldsymbol{M}_{\bar{x}}\boldsymbol{F}_2\gamma_{2i}}{T} = O_p(\frac{1}{\sqrt{T}}) + O_p(\frac{1}{\sqrt{N}}) = o_p(1). \tag{A.32}$$

As a result, we now focus on

$$\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\frac{\tilde{\boldsymbol{Q}}_i'\boldsymbol{Z}_i\boldsymbol{\xi}_i}{T} = \frac{1}{\sqrt{N}}\sum_{i=1}^{N}\frac{\tilde{\boldsymbol{Q}}_{i0}'\boldsymbol{Z}_{i0}\boldsymbol{\xi}_i}{T} + o_p(1) = \frac{1}{\sqrt{N}}\sum_{i=1}^{N}\frac{\boldsymbol{Q}'(\boldsymbol{I}_T\otimes\boldsymbol{b}_i)\boldsymbol{M}_{\boldsymbol{f}_x}\boldsymbol{Z}_{i0}}{T}\boldsymbol{\xi}_i + o_p(1), \tag{A.33}$$

where the first equality follows from (A.9). Notice under Assumption 5 that $\boldsymbol{\xi}_i$ is independent of all the other variables, including $\boldsymbol{\xi}_j$, and has finite second order moment. Therefore,

$$var(\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\frac{\boldsymbol{Q}'(\boldsymbol{I}_T\otimes\boldsymbol{b}_i)\boldsymbol{M}_{\boldsymbol{f}_x}\boldsymbol{Z}_{i0}}{T}\boldsymbol{\xi}_i)=\frac{1}{N}\sum_{i=1}^{N}E\left\{\frac{\boldsymbol{Q}'(\boldsymbol{I}_T\otimes\boldsymbol{b}_i)\boldsymbol{M}_{\boldsymbol{f}_x}\boldsymbol{Z}_{i0}}{T}\boldsymbol{\xi}_i\boldsymbol{\xi}_i'\frac{\boldsymbol{Z}_{i0}'\boldsymbol{M}_{\boldsymbol{f}_x}(\boldsymbol{I}_T\otimes\boldsymbol{b}_i')\boldsymbol{Q}}{T}\right\}$$
$$=\frac{1}{N}\sum_{i=1}^{N}E\left\{\frac{\boldsymbol{Q}'(\boldsymbol{I}_T\otimes\boldsymbol{b}_i)\boldsymbol{M}_{\boldsymbol{f}_x}\boldsymbol{Z}_{i0}}{T}\Omega_{\boldsymbol{\xi}}\frac{\boldsymbol{Z}_{i0}'\boldsymbol{M}_{\boldsymbol{f}_x}(\boldsymbol{I}_T\otimes\boldsymbol{b}_i')\boldsymbol{Q}}{T}\right\}$$
$$\leq\frac{\lambda_{max}(\boldsymbol{\Omega}_{\boldsymbol{\xi}})}{N}\sum_{i=1}^{N}E\left\{\frac{\boldsymbol{Q}'(\boldsymbol{I}_T\otimes\boldsymbol{b}_i)\boldsymbol{M}_{\boldsymbol{f}_x}\boldsymbol{Z}_{i0}}{T}\frac{\boldsymbol{Z}_{i0}'\boldsymbol{M}_{\boldsymbol{f}_x}(\boldsymbol{I}_T\otimes\boldsymbol{b}_i')\boldsymbol{Q}}{T}\right\},\tag{A.34}$$

which is finite under Assumption 5 and 6. The above inequality follows from a spectral decomposition of $\boldsymbol{\Omega}_{\boldsymbol{\xi}}$. Then by CLT, as $(N,\ T)\xrightarrow{j}\infty$, we have:

$$\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\frac{\boldsymbol{Q}'(\boldsymbol{I}_T\otimes\boldsymbol{b}_i)\boldsymbol{M}_{\boldsymbol{f}_x}\boldsymbol{Z}_{i0}}{T}\boldsymbol{\xi}_i\xrightarrow{d}N\left(0,\frac{1}{N}\sum_{i=1}^{N}E\left\{\frac{\boldsymbol{Q}'(\boldsymbol{I}_T\otimes\boldsymbol{b}_i)\boldsymbol{M}_{\boldsymbol{f}_x}\boldsymbol{Z}_{i0}}{T}\Omega_{\boldsymbol{\xi}}\frac{\boldsymbol{Z}_{i0}'\boldsymbol{M}_{\boldsymbol{f}_x}(\boldsymbol{I}_T\otimes\boldsymbol{b}_i')\boldsymbol{Q}}{T}\right\}\right).\tag{A.35}$$

which completes the proof of Theorem 3, given (43), (A.1), (A.9) and Assumptions 6.

A.4 Proof of Theorem 4

It is straightforward to establish the consistency of $\boldsymbol{\theta}_P^*$, following the similar steps employed in proving Theorem 3. Here we focus on its asymptotic distribution which, given (50), (A.1), (A.9) and Assumptions 6, is determined by

$$\sqrt{\frac{T}{N}}\sum_{i=1}^{N}\frac{\tilde{\boldsymbol{Q}}_i'(\boldsymbol{F}_1\gamma_{1i}+\boldsymbol{F}_2\gamma_{2i}+\boldsymbol{\varepsilon}_{i.})}{T},\tag{A.36}$$

Using the definition of $\boldsymbol{Q}$, we have:

$$\sqrt{\frac{T}{N}}\sum_{i=1}^{N}\frac{\boldsymbol{X}_{i.}'\boldsymbol{M}_{\bar{x}}(\boldsymbol{F}_1\gamma_{1i}+\boldsymbol{F}_2\gamma_{2i}+\boldsymbol{\varepsilon}_{i.})}{T}.\tag{A.37}$$

We will analyse the three terms in (A.37), separately.

Under Assumption 3, we have: $\gamma_{1i}=\bar{\gamma}_1+\mu_{1i}$. Since $\boldsymbol{M}_{\bar{x}}$ is the projection matrix for $\bar{\boldsymbol{X}}$, we have: $\bar{\boldsymbol{X}}'\boldsymbol{M}_{\bar{x}}=0$. Then, the first term in (A.37) can be written as

$$\begin{aligned}\sqrt{\frac{T}{N}}\sum_{i=1}^{N}\frac{\boldsymbol{X}_{i.}'\boldsymbol{M}_{\bar{x}}\boldsymbol{F}_1\gamma_{1i}}{T}&=\sqrt{\frac{T}{N}}\sum_{i=1}^{N}\frac{\boldsymbol{X}_{i.}'\boldsymbol{M}_{\bar{x}}\boldsymbol{F}_1(\bar{\gamma}_1+\mu_{1i})}{T}\\&=\sqrt{NT}\frac{\bar{\boldsymbol{X}}'\boldsymbol{M}_{\bar{x}}\boldsymbol{F}_1\bar{\gamma}_1}{T}+\sqrt{\frac{T}{N}}\sum_{i=1}^{N}\frac{\boldsymbol{X}_{i.}'\boldsymbol{M}_{\bar{x}}\boldsymbol{F}_1\mu_{1i}}{T}\\&=\sqrt{\frac{T}{N}}\sum_{i=1}^{N}\frac{\boldsymbol{X}_{i.}'\boldsymbol{M}_{\bar{x}}\boldsymbol{F}_1}{T}\mu_{1i}\\&=\sqrt{\frac{T}{N}}\sum_{i=1}^{N}\frac{\boldsymbol{\Gamma}_{xi}'\boldsymbol{F}_x'\boldsymbol{M}_{\bar{x}}\boldsymbol{F}_1}{T}\mu_{1i}+\sqrt{\frac{T}{N}}\sum_{i=1}^{N}\frac{\boldsymbol{V}_{i.}'\boldsymbol{M}_{\bar{x}}\boldsymbol{F}_1}{T}\mu_{1i},\end{aligned}\tag{A.38}$$

where the last equality follows from (26). Define $\bar{\boldsymbol{\Gamma}}_x^+ = (\bar{\boldsymbol{\Gamma}}_x \bar{\boldsymbol{\Gamma}}_x')^{-1}\bar{\boldsymbol{\Gamma}}_x$. Then, by (18) and following the same arguments in proving Theorem 1 in Westerlund and Urbain (2015), we have:

$$\sqrt{\frac{T}{N}}\sum_{i=1}^{N}\frac{\boldsymbol{\Gamma}_{xi}'\boldsymbol{F}_x'\boldsymbol{M}_{\bar{x}}\boldsymbol{F}_1}{T}\mu_{1i} = \sqrt{\frac{T}{N}}\sum_{i=1}^{N}\boldsymbol{\Gamma}_{xi}'\bar{\boldsymbol{\Gamma}}_x^{+\prime}\frac{\bar{\boldsymbol{V}}'\bar{\boldsymbol{V}}}{T}\mu_{1i} + O_p(\frac{\sqrt{T}}{N^{3/2}}) + O_p(\frac{1}{N}) + O_p(\frac{1}{\sqrt{NT}}), \tag{A.39}$$

$$\sqrt{\frac{T}{N}}\sum_{i=1}^{N}\frac{\boldsymbol{V}_{i.}'\boldsymbol{M}_{\bar{x}}\boldsymbol{F}_1}{T}\mu_{1i} = \sqrt{\frac{T}{N}}\frac{1}{T}\sum_{i=1}^{N}\boldsymbol{V}_i'\bar{\boldsymbol{V}}\bar{\boldsymbol{\Gamma}}_x^{+}\mu_{1i} + O_p(\frac{\sqrt{T}}{N}) + O_p(\frac{1}{\sqrt{N}}). \tag{A.40}$$

By Lemma 2(a) and under Assumption 3, (A.39) is dominated by the first term on the RHS and is of order $\sqrt{T/N}$, which is also the case for (A.40) by further using Lemma CA4 in Westerlund and Urbain (2015). As a result, we have

$$\sqrt{\frac{T}{N}}\sum_{i=1}^{N}\frac{\boldsymbol{\Gamma}_{xi}'\boldsymbol{F}_x'\boldsymbol{M}_{\bar{x}}\boldsymbol{F}_1}{T}\mu_{1i} = O_p(\sqrt{\frac{T}{N}}), \tag{A.41}$$

$$\sqrt{\frac{T}{N}}\sum_{i=1}^{N}\frac{\boldsymbol{V}_{i.}'\boldsymbol{M}_{\bar{x}}\boldsymbol{F}_1}{T}\mu_{1i} = O_p(\sqrt{\frac{T}{N}}), \tag{A.42}$$

which are negligible if we require $T/N \to 0$.

For the second term in (A.37), let $\boldsymbol{H} = \boldsymbol{F}_x\bar{\boldsymbol{\Gamma}}_x$. Then, using (16), we have:

$$\bar{\boldsymbol{x}}_t' = \boldsymbol{f}_{xt}'\bar{\boldsymbol{\Gamma}}_x + \bar{\boldsymbol{v}}_t',$$

which can be written as

$$\bar{\boldsymbol{X}} = \boldsymbol{F}_x\bar{\boldsymbol{\Gamma}}_x + \bar{\boldsymbol{V}} = \boldsymbol{H} + \bar{\boldsymbol{V}}, \tag{A.43}$$

where $\bar{\boldsymbol{X}} = (\bar{\boldsymbol{x}}_1, \bar{\boldsymbol{x}}_2, \ \ldots, \bar{\boldsymbol{x}}_T)'$, $\bar{\boldsymbol{V}} = (\bar{\boldsymbol{v}}_{.1}, \bar{\boldsymbol{v}}_{.2}, \ \ldots, \bar{\boldsymbol{v}}_{.T})'$. In Lemma 4(d), we derive:

$$\begin{aligned}
\sqrt{\frac{T}{N}}\sum_{i=1}^{N}\frac{\boldsymbol{X}_{i.}'\boldsymbol{M}_{\bar{x}}\boldsymbol{F}_2}{T}\gamma_{2i} &= \sqrt{\frac{T}{N}}\sum_{i=1}^{N}\frac{\boldsymbol{X}_{i.}'\boldsymbol{M}_{f_x}\boldsymbol{F}_2}{T}\mu_{2i} + O_p(\frac{1}{\sqrt{N}}) + O_p(\frac{\sqrt{T}}{N}) \\
&= \sqrt{\frac{T}{N}}\sum_{i=1}^{N}\frac{\boldsymbol{V}_{i.}'\boldsymbol{M}_{f_x}\boldsymbol{F}_2}{T}\mu_{2i} + O_p(\frac{1}{\sqrt{N}}) + O_p(\frac{\sqrt{T}}{N}) \\
&= \sqrt{\frac{T}{N}}\sum_{i=1}^{N}\frac{\boldsymbol{V}_{i.}'\boldsymbol{F}_2}{T}\mu_{2i} + O_p(\frac{1}{\sqrt{T}}) + O_p(\frac{1}{\sqrt{N}}) + O_p(\frac{\sqrt{T}}{N}),
\end{aligned} \tag{A.44}$$

where the first equality follows from the fact that $\bar{\boldsymbol{X}}'\boldsymbol{M}_{\bar{x}} = 0$ and μ_{2i} is independent and identically distributed. The second equality follows form the fact that $\boldsymbol{X}_{i.} = \boldsymbol{F}_x\boldsymbol{\Gamma}_{xi} + \boldsymbol{V}_{i.}$ (see (26)) and $\boldsymbol{M}_{f_x}$ is an idempotent matrix. The last equality follows from Lemma 2(c) and Lemma 3(d).

Under Assumption 1 and 2, $\boldsymbol{v}_{it}$ is independent of $\boldsymbol{f}_{2t}$, and both of them have finite second order moments. By Assumption 3, μ_{2i} is independently and identically distributed. Hence, as $N, T \xrightarrow{j} \infty$ and $T/N^2 \to 0$, we have:

$$\frac{1}{\sqrt{NT}}\sum_{i=1}^{N}\boldsymbol{X}_{i.}'\boldsymbol{M}_{\bar{x}}\boldsymbol{F}_2\gamma_{2i} \xrightarrow{d} N(0, \ \frac{1}{N}\sum_{i=1}^{N}E(\frac{\boldsymbol{X}_{i.}'\boldsymbol{M}_{f_x}\boldsymbol{F}_2\boldsymbol{\Omega}_{\mu_2}\boldsymbol{F}_2'\boldsymbol{M}_{f_x}\boldsymbol{X}_{i.}}{T}). \tag{A.45}$$

For the third term in (A.37), first, we have

$$\begin{aligned}
&\frac{1}{\sqrt{TN}}\sum_{i=1}^{N} \boldsymbol{X}_{i.}'\boldsymbol{M}_{\bar{x}}\boldsymbol{\varepsilon}_{i.} - \frac{1}{\sqrt{TN}}\sum_{i=1}^{N}\boldsymbol{X}_{i.}'\boldsymbol{M}_{f_x}\boldsymbol{\varepsilon}_{i.} \\
&= \frac{1}{\sqrt{TN}}\sum_{i=1}^{N}\Big\{\boldsymbol{X}_{i.}'\bar{\boldsymbol{X}}(\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}})^{-1}\bar{\boldsymbol{X}}'\boldsymbol{\varepsilon}_{i.} - \boldsymbol{X}_{i.}'\boldsymbol{H}(\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}})^{-1}\bar{\boldsymbol{X}}'\boldsymbol{\varepsilon}_{i.} + \boldsymbol{X}_{i.}'\boldsymbol{H}(\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}})^{-1}\bar{\boldsymbol{X}}'\boldsymbol{\varepsilon}_{i.} - \boldsymbol{X}_{i.}'\boldsymbol{H}(\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}})^{-1}\boldsymbol{H}'\boldsymbol{\varepsilon}_{i.} \\
&\qquad + \boldsymbol{X}_{i.}'\boldsymbol{H}(\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}})^{-1}\boldsymbol{H}'\boldsymbol{\varepsilon}_{i.} - \boldsymbol{X}_{i.}'\boldsymbol{H}(\boldsymbol{H}'\boldsymbol{H})^{-1}\boldsymbol{H}'\boldsymbol{\varepsilon}_{i.}\Big\} \\
&= \frac{1}{\sqrt{TN}}\sum_{i=1}^{N}\Big\{\boldsymbol{X}_{i.}'\bar{\boldsymbol{V}}(\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}})^{-1}\bar{\boldsymbol{X}}'\boldsymbol{\varepsilon}_{i.} + \boldsymbol{X}_{i.}'\boldsymbol{H}(\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}})^{-1}\bar{\boldsymbol{V}}'\boldsymbol{\varepsilon}_{i.} + \boldsymbol{X}_{i.}'\boldsymbol{H}[(\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}})^{-1}\boldsymbol{\varepsilon}_{i.} - (\boldsymbol{H}'\boldsymbol{H})^{-1}]\boldsymbol{H}'\boldsymbol{\varepsilon}_{i.}\Big\} \\
&= \frac{1}{\sqrt{TN}}\sum_{i=1}^{N}\Big\{\boldsymbol{X}_{i.}'\bar{\boldsymbol{V}}(\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}})^{-1}\bar{\boldsymbol{V}}'\boldsymbol{\varepsilon}_{i.} + \boldsymbol{X}_{i.}'\bar{\boldsymbol{V}}(\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}})^{-1}\boldsymbol{H}'\boldsymbol{\varepsilon}_{i.} + \boldsymbol{X}_{i.}'\boldsymbol{H}(\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}})^{-1}\bar{\boldsymbol{V}}'\boldsymbol{\varepsilon}_{i.} \\
&\qquad + \boldsymbol{X}_{i.}'\boldsymbol{H}[(\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}})^{-1} - (\boldsymbol{H}'\boldsymbol{H})^{-1}]\boldsymbol{H}'\boldsymbol{\varepsilon}_{i.}\Big\}. \qquad \text{(A.46)}
\end{aligned}$$

Then, we follow the same logic in (Westerlund and Urbain, 2015, p26), and finally obtain:

$$\frac{1}{\sqrt{TN}}\sum_{i=1}^{N}\boldsymbol{X}_{i.}'\boldsymbol{M}_{\bar{x}}\boldsymbol{\varepsilon}_{i.} = \frac{1}{\sqrt{TN}}\sum_{i=1}^{N}\boldsymbol{X}_{i.}'\boldsymbol{M}_{f_x}\boldsymbol{\varepsilon}_{i.} + O_p(\sqrt{\frac{T}{N}}) \qquad \text{(A.47)}$$

where

$$\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\frac{\boldsymbol{X}_{i.}'\boldsymbol{M}_{f_x}\boldsymbol{\varepsilon}_{i.}}{\sqrt{T}} \xrightarrow{d} N(0, \frac{1}{N}\sum_{i=1}^{N}E(\frac{\boldsymbol{X}_{i.}'\boldsymbol{M}_{f_x}\boldsymbol{\Omega}_{\varepsilon,i}\boldsymbol{M}_{f_x}\boldsymbol{X}_{i.}}{T})). \qquad \text{(A.48)}$$

as $(N,T) \xrightarrow{j} \infty$. By requiring $T/N \to 0$ and using the Slutsky's Theorem, we complete the proof of Theorem 4.

Table 2: The Finite Sample Performance of Alternative Pooled Estimators under Experiment 4

		Bias (× 100)									RMSE (× 100)								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
		Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																	
$\bar{\boldsymbol{x}}_t$	20	0.03	-0.2	-0.11	-0.05	-0.23	0.1	-0.03	0.01	-0.04	4.22	5.99	6.04	2.62	3.64	3.68	1.95	2.67	2.72
	50	-0.05	0.06	0.01	0.04	-0.08	0.05	0.08	-0.04	-0.01	2.41	3.66	3.82	1.57	2.36	2.48	1.14	1.67	1.73
	100	-0.02	0.11	0.04	0.02	-0.01	0.03	0.04	-0.04	0.00	1.69	2.63	2.66	1.06	1.6	1.67	0.82	1.17	1.21
		Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	0.21	-0.19	-0.1	0.04	-0.23	0.09	-0.14	0.02	-0.02	5.63	6.03	6.04	3.39	3.63	3.71	2.46	2.68	2.74
	50	-0.11	0.08	0.02	0.05	-0.09	0.04	0.05	-0.03	-0.01	2.93	3.7	3.83	1.94	2.36	2.48	1.4	1.67	1.76
	100	0.00	0.11	0.04	0.01	-0.01	0.03	0.01	-0.03	0.01	1.98	2.62	2.69	1.26	1.61	1.69	0.97	1.17	1.24
		Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	0.09	-0.2	-0.11	-0.02	-0.23	0.09	-0.04	0.01	-0.04	4.1	5.99	6.03	2.57	3.64	3.68	1.91	2.67	2.72
	50	-0.04	0.07	0.01	0.05	-0.08	0.04	0.08	-0.04	-0.01	2.34	3.67	3.81	1.56	2.36	2.47	1.13	1.67	1.74
	100	-0.01	0.11	0.04	0.02	-0.01	0.03	0.04	-0.04	0.00	1.64	2.62	2.66	1.03	1.6	1.67	0.81	1.17	1.22
		Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																	
$(\bar{y}_t, \bar{\boldsymbol{x}}_t')'$	20	-0.14	-0.14	-0.19	-0.27	-0.17	0.09	-0.23	0.03	-0.03	4.2	6.02	6.02	2.61	3.6	3.64	1.93	2.69	2.71
	50	-0.08	0.07	0.00	0.02	-0.07	0.07	0.04	-0.04	-0.01	2.43	3.69	3.87	1.56	2.35	2.47	1.16	1.67	1.73
	100	-0.03	0.08	0.05	0.01	-0.01	0.02	0.03	-0.04	0.00	1.69	2.68	2.65	1.06	1.6	1.67	0.82	1.15	1.21
		Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	-0.24	-0.11	-0.13	-0.4	-0.15	0.11	-0.51	0.05	0.03	5.68	6.06	5.98	3.4	3.59	3.67	2.47	2.69	2.73
	50	-0.17	0.08	0.02	0.01	-0.07	0.06	-0.01	-0.03	0.00	2.92	3.73	3.88	1.92	2.36	2.47	1.42	1.67	1.75
	100	0.00	0.08	0.05	0.00	-0.02	0.02	0.00	-0.04	0.01	1.98	2.67	2.7	1.26	1.6	1.68	0.97	1.15	1.24
		Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	-0.14	-0.14	-0.18	-0.29	-0.17	0.09	-0.28	0.03	-0.02	4.05	6.02	6	2.56	3.6	3.64	1.89	2.69	2.71
	50	-0.09	0.07	0.00	0.02	-0.07	0.06	0.03	-0.04	-0.01	2.35	3.7	3.86	1.54	2.35	2.47	1.15	1.67	1.73
	100	-0.02	0.08	0.05	0.01	-0.01	0.02	0.03	-0.04	0.00	1.64	2.67	2.66	1.03	1.6	1.66	0.81	1.15	1.21
		Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																	
$(\bar{y}_t, \bar{y}_t^{*}, \bar{\boldsymbol{x}}_t')'$	20	0.38	-1.04	-1.79	0.19	-0.98	-1.5	0.25	-0.77	-1.57	4.3	6.34	6.55	2.63	3.78	4.07	1.94	2.81	3.16
	50	0.09	-0.19	-0.61	0.17	-0.39	-0.55	0.19	-0.33	-0.59	2.47	3.83	4.06	1.58	2.42	2.55	1.17	1.7	1.85
	100	0.05	-0.04	-0.24	0.07	-0.17	-0.28	0.11	-0.19	-0.3	1.74	2.77	2.76	1.07	1.62	1.72	0.82	1.16	1.26
		Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	0.44	-1.04	-1.79	0.27	-0.99	-1.52	0.29	-0.78	-1.58	4.14	6.34	6.52	2.59	3.77	4.07	1.9	2.81	3.16
	50	0.1	-0.19	-0.61	0.19	-0.39	-0.55	0.2	-0.33	-0.6	2.4	3.83	4.06	1.56	2.42	2.54	1.17	1.7	1.85
	100	0.07	-0.04	-0.25	0.08	-0.17	-0.28	0.11	-0.19	-0.3	1.68	2.77	2.76	1.04	1.62	1.71	0.82	1.16	1.26
		Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	0.39	-1.07	-1.77	0.19	-1.01	-1.51	0.26	-0.79	-1.58	4.34	6.4	6.55	2.68	3.8	4.08	1.96	2.84	3.17
	50	0.09	-0.18	-0.62	0.19	-0.4	-0.54	0.19	-0.33	-0.6	2.45	3.86	4.07	1.57	2.44	2.54	1.17	1.7	1.86
	100	0.06	-0.04	-0.24	0.08	-0.17	-0.28	0.11	-0.19	-0.3	1.7	2.78	2.76	1.05	1.63	1.72	0.82	1.17	1.26
		Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																	
Infeasible	20	0.05	0.07	-0.03	-0.03	-0.22	0.07	0.00	-0.01	-0.03	3.22	5.74	5.77	2.15	3.47	3.62	1.58	2.55	2.59
	50	-0.05	0.08	-0.07	0.06	-0.04	0.01	0.06	-0.03	-0.02	2.06	3.56	3.73	1.37	2.3	2.4	1.06	1.6	1.66
	100	0.00	0.04	-0.01	0.02	0.00	0.03	0.03	-0.05	0.00	1.5	2.51	2.52	0.96	1.58	1.63	0.76	1.14	1.19
		Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	0.12	0.07	-0.04	0.01	-0.24	0.06	-0.07	0.01	0.00	3.8	5.75	5.84	2.42	3.46	3.63	1.79	2.56	2.63
	50	-0.09	0.09	-0.06	0.05	-0.05	0.01	0.04	-0.03	-0.01	2.43	3.57	3.77	1.6	2.31	2.4	1.24	1.61	1.68
	100	0.04	0.03	-0.02	0.05	-0.01	0.02	0.01	-0.05	0.01	1.72	2.49	2.55	1.11	1.58	1.64	0.87	1.15	1.21
		Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	0.08	0.07	-0.04	-0.01	-0.23	0.06	-0.01	0.00	-0.03	3.12	5.74	5.75	2.1	3.46	3.6	1.56	2.55	2.58
	50	-0.04	0.08	-0.07	0.06	-0.05	0.01	0.05	-0.03	-0.02	2.02	3.56	3.72	1.36	2.3	2.39	1.06	1.6	1.66
	100	0.01	0.04	-0.01	0.03	0.00	0.03	0.03	-0.05	0.01	1.47	2.5	2.51	0.94	1.58	1.62	0.75	1.14	1.19

Table 3: The Finite Sample Performance of Alternative Mean Group Estimators under Experiment 4

		Bias (× 100)									RMSE (× 100)								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
$\bar{\boldsymbol{x}}_t$	Use only $\tilde{\boldsymbol{X}}^*$ as Instruments																		
	20	0.05	-0.3	-0.22	-0.1	-0.21	0.16	-0.11	0.02	0.00	5.12	6.57	6.74	2.8	3.86	3.84	2.03	2.7	2.76
	50	0.08	0.06	-0.02	0.03	-0.04	0.09	0.05	-0.04	-0.01	2.81	4.15	4.24	1.62	2.5	2.55	1.15	1.67	1.74
	100	-0.01	0.16	0.04	-0.01	0.00	0.02	0.02	-0.04	0.00	1.97	2.94	3.03	1.11	1.69	1.75	0.84	1.18	1.25
	Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																		
	20	0.93	-0.41	-0.37	-0.17	-0.18	0.19	-0.18	-0.01	0.02	8.78	7.69	7.83	3.93	3.93	3.97	2.71	2.74	2.8
	50	0.21	0.1	-0.1	0.09	-0.04	0.06	0.03	-0.04	0.01	4.08	4.49	4.87	2.04	2.51	2.59	1.46	1.68	1.79
	100	0.12	0.08	-0.04	0.03	-0.01	-0.01	-0.01	-0.04	0.01	2.76	3.06	3.22	1.35	1.7	1.79	1	1.19	1.27
	Use $\tilde{\boldsymbol{X}}^*$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																		
	20	2.24	-0.52	-0.58	0.7	-0.29	0.00	0.3	-0.02	-0.08	4.89	6.51	6.64	2.77	3.85	3.83	2.01	2.7	2.75
	50	1.71	-0.17	-0.5	0.65	-0.14	-0.1	0.35	-0.08	-0.1	3.11	4.1	4.27	1.7	2.49	2.54	1.19	1.67	1.75
	100	1.54	-0.09	-0.46	0.54	-0.09	-0.16	0.29	-0.09	-0.09	2.38	2.88	3.01	1.19	1.68	1.75	0.88	1.18	1.25
$(\bar{y}_t, \bar{\boldsymbol{x}}_t')'$	Use only $\tilde{\boldsymbol{X}}^*$ as Instruments																		
	20	0.00	-0.4	-0.26	-0.29	-0.15	0.15	-0.29	0.03	-0.01	4.72	6.56	6.62	2.76	3.78	3.82	1.98	2.71	2.74
	50	0.02	0.14	0.01	0.01	-0.03	0.1	0.01	-0.04	0.00	2.74	4.21	4.2	1.6	2.48	2.53	1.16	1.66	1.74
	100	-0.01	0.13	0.02	-0.01	0.00	0.02	0.02	-0.05	0.01	1.94	2.9	2.98	1.09	1.68	1.74	0.83	1.16	1.24
	Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																		
	20	0.16	-0.3	-0.2	-0.49	-0.1	0.21	-0.49	0.03	0.04	8.81	7.1	7.3	3.73	3.81	3.93	2.61	2.74	2.77
	50	-0.07	0.15	0.1	0.05	-0.04	0.07	-0.03	-0.04	0.01	4.49	5.02	5.05	1.98	2.49	2.56	1.45	1.68	1.78
	100	0.06	0.07	0.06	0.02	0.00	-0.01	-0.01	-0.05	0.02	3.23	3.13	3.44	1.32	1.69	1.76	0.98	1.16	1.26
	Use $\tilde{\boldsymbol{X}}^*$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																		
	20	0.67	-0.44	-0.34	-0.1	-0.17	0.11	-0.21	0.02	-0.03	4.4	6.51	6.57	2.67	3.76	3.81	1.92	2.71	2.73
	50	0.88	0.01	-0.26	0.31	-0.07	0.01	0.14	-0.06	-0.04	2.73	4.18	4.2	1.6	2.48	2.52	1.16	1.66	1.74
	100	0.93	-0.03	-0.28	0.31	-0.05	-0.09	0.16	-0.07	-0.04	2.04	2.86	2.97	1.1	1.68	1.73	0.83	1.16	1.24
$(\bar{y}_t, \bar{y}_t^*, \bar{\boldsymbol{x}}_t')'$	Use only $\tilde{\boldsymbol{X}}^*$ as Instruments																		
	20	0.45	-1.52	-2.29	0.1	-1.21	-1.89	0.14	-1	-2.03	4.94	7.02	7.49	2.77	3.98	4.41	1.98	2.91	3.45
	50	0.14	-0.27	-0.76	0.13	-0.44	-0.71	0.13	-0.44	-0.79	2.83	4.37	4.53	1.64	2.56	2.64	1.17	1.72	1.94
	100	0.06	-0.06	-0.39	0.04	-0.21	-0.37	0.08	-0.25	-0.4	2.03	3.03	3.12	1.11	1.72	1.81	0.84	1.18	1.31
	Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																		
	20	1.05	-1.3	-2.46	0.43	-1.25	-2.04	0.47	-1.09	-2.15	10.94	8.27	8.4	3.73	4.05	4.57	2.62	2.98	3.57
	50	0.16	-0.3	-0.84	0.33	-0.49	-0.81	0.24	-0.47	-0.86	5.23	4.68	5.05	2.04	2.58	2.71	1.48	1.75	2.02
	100	0.16	-0.14	-0.45	0.13	-0.22	-0.42	0.12	-0.27	-0.43	2.78	3.22	3.38	1.36	1.73	1.85	0.99	1.19	1.34
	Use $\tilde{\boldsymbol{X}}^*$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																		
	20	1.21	-1.58	-2.4	0.4	-1.24	-1.97	0.32	-1.03	-2.08	4.69	6.97	7.45	2.72	3.99	4.43	1.95	2.92	3.48
	50	1.09	-0.43	-1.07	0.47	-0.5	-0.82	0.3	-0.47	-0.84	2.88	4.34	4.57	1.67	2.56	2.67	1.2	1.73	1.97
	100	1.08	-0.23	-0.72	0.38	-0.26	-0.49	0.24	-0.28	-0.45	2.17	3	3.15	1.13	1.72	1.83	0.85	1.19	1.33
Infeasible	Use only $\tilde{\boldsymbol{X}}^*$ as Instruments																		
	20	0.1	-0.01	0.03	-0.01	-0.19	0.14	-0.05	0.00	-0.01	3.64	6.07	6.38	2.2	3.58	3.74	1.61	2.58	2.62
	50	0.02	0.06	-0.15	0.05	0.00	0.03	0.02	-0.04	-0.01	2.3	3.83	4.06	1.42	2.39	2.43	1.05	1.62	1.67
	100	-0.01	0.02	-0.02	0.00	0.02	0.00	0.01	-0.06	0.01	1.76	2.77	2.88	0.99	1.64	1.68	0.77	1.15	1.21
	Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																		
	20	0.21	-0.06	-0.02	0.01	-0.2	0.15	-0.09	0.00	0.03	4.61	6.34	6.64	2.57	3.63	3.79	1.84	2.61	2.67
	50	-0.04	0.06	-0.08	0.04	0.01	0.02	0.01	-0.04	0.00	3.04	3.98	4.44	1.64	2.41	2.44	1.24	1.62	1.7
	100	0.02	0.00	-0.04	0.06	0.01	-0.02	-0.01	-0.06	0.02	2.16	2.86	3.01	1.14	1.65	1.71	0.89	1.15	1.24
	Use $\tilde{\boldsymbol{X}}^*$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																		
	20	0.91	-0.17	-0.35	0.28	-0.25	0.01	0.08	-0.02	-0.06	3.57	6.08	6.32	2.16	3.58	3.71	1.59	2.58	2.61
	50	0.82	-0.1	-0.51	0.33	-0.05	-0.1	0.16	-0.07	-0.07	2.32	3.83	4.08	1.42	2.38	2.42	1.05	1.62	1.67
	100	0.82	-0.13	-0.39	0.3	-0.04	-0.13	0.14	-0.08	-0.05	1.84	2.74	2.87	1	1.64	1.67	0.78	1.15	1.21

Table 4: The Size and Power for Alternative Pooled Estimators under Experiment 4

		Size									Power								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
								Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments											
$\bar{\boldsymbol{x}}_t$	20	0.082	0.067	0.069	0.053	0.067	0.063	0.069	0.063	0.061	0.22	0.4	0.36	0.41	0.73	0.7	0.6	0.92	0.94
	50	0.048	0.054	0.061	0.055	0.063	0.065	0.045	0.044	0.058	0.53	0.77	0.73	0.85	0.98	0.97	0.99	1	1
	100	0.053	0.055	0.049	0.04	0.044	0.043	0.049	0.048	0.047	0.83	0.97	0.95	1	1	1	1	1	1
								Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	0.069	0.074	0.073	0.064	0.063	0.062	0.076	0.062	0.069	0.2	0.38	0.36	0.27	0.72	0.7	0.44	0.92	0.93
	50	0.053	0.054	0.065	0.069	0.063	0.061	0.062	0.044	0.057	0.38	0.78	0.71	0.66	0.98	0.98	0.91	1	1
	100	0.045	0.054	0.051	0.044	0.044	0.053	0.058	0.05	0.051	0.73	0.97	0.96	0.97	1	1	1	1	1
								Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	0.076	0.069	0.07	0.054	0.067	0.062	0.075	0.064	0.063	0.25	0.4	0.37	0.43	0.73	0.7	0.64	0.92	0.94
	50	0.053	0.054	0.067	0.066	0.061	0.064	0.05	0.044	0.055	0.53	0.77	0.73	0.85	0.98	0.97	0.99	1	1
	100	0.051	0.055	0.049	0.038	0.045	0.045	0.048	0.047	0.049	0.85	0.97	0.96	0.99	1	1	1	1	1
								Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments											
$(\bar{y}_t, \bar{\boldsymbol{x}}_t')'$	20	0.091	0.086	0.091	0.073	0.071	0.077	0.088	0.083	0.077	0.2	0.35	0.39	0.43	0.71	0.67	0.59	0.93	0.93
	50	0.068	0.063	0.078	0.069	0.067	0.078	0.059	0.055	0.069	0.5	0.75	0.7	0.86	0.99	0.97	0.99	1	1
	100	0.06	0.057	0.047	0.05	0.053	0.066	0.058	0.054	0.06	0.83	0.95	0.97	1	1	1	1	1	1
								Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	0.085	0.089	0.085	0.087	0.067	0.078	0.087	0.083	0.084	0.18	0.33	0.39	0.28	0.76	0.67	0.43	0.93	0.93
	50	0.056	0.064	0.074	0.08	0.068	0.069	0.069	0.058	0.069	0.38	0.73	0.71	0.68	0.98	0.97	0.9	1	1
	100	0.055	0.058	0.052	0.062	0.046	0.055	0.067	0.048	0.058	0.71	0.96	0.96	0.96	1	1	1	1	1
								Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	0.089	0.086	0.093	0.073	0.071	0.078	0.089	0.083	0.077	0.22	0.34	0.39	0.46	0.72	0.66	0.65	0.93	0.93
	50	0.064	0.064	0.076	0.068	0.068	0.079	0.07	0.055	0.067	0.53	0.74	0.72	0.86	0.98	0.98	0.99	1	1
	100	0.057	0.057	0.044	0.052	0.053	0.064	0.057	0.053	0.059	0.84	0.95	0.97	1	1	1	1	1	1
								Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments											
$(\bar{y}_t, \bar{y}_t^*, \bar{\boldsymbol{x}}_t')'$	20	0.095	0.086	0.093	0.077	0.084	0.092	0.067	0.085	0.101	0.19	0.36	0.34	0.44	0.75	0.65	0.59	0.93	0.95
	50	0.062	0.068	0.078	0.068	0.076	0.076	0.064	0.064	0.079	0.47	0.73	0.69	0.84	0.99	0.96	0.98	1	1
	100	0.062	0.063	0.05	0.046	0.053	0.061	0.057	0.053	0.061	0.79	0.95	0.96	1	1	1	1	1	1
								Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	0.096	0.088	0.091	0.078	0.086	0.094	0.08	0.085	0.1	0.24	0.35	0.35	0.42	0.75	0.65	0.63	0.93	0.95
	50	0.066	0.069	0.077	0.083	0.077	0.075	0.063	0.065	0.081	0.5	0.73	0.71	0.84	0.99	0.97	0.98	1	1
	100	0.058	0.064	0.05	0.046	0.056	0.062	0.058	0.054	0.064	0.82	0.95	0.96	1	1	1	1	1	1
								Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	0.1	0.072	0.097	0.091	0.081	0.079	0.098	0.097	0.124	0.14	0.44	0.33	0.34	0.76	0.69	0.48	0.95	0.95
	50	0.06	0.067	0.051	0.066	0.058	0.066	0.064	0.069	0.069	0.35	0.74	0.79	0.66	1	0.99	0.94	1	1
	100	0.057	0.059	0.062	0.057	0.066	0.067	0.062	0.064	0.058	0.54	0.95	0.96	0.93	1	1	1	1	1
								Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments											
Infeasible	20	0.073	0.078	0.069	0.075	0.073	0.069	0.063	0.074	0.078	0.35	0.37	0.47	0.59	0.8	0.69	0.82	0.95	0.95
	50	0.053	0.059	0.072	0.06	0.06	0.07	0.059	0.046	0.054	0.69	0.82	0.71	0.94	0.99	0.98	1	1	1
	100	0.063	0.053	0.057	0.048	0.06	0.054	0.057	0.053	0.055	0.9	0.98	0.97	1	1	1	1	1	1
								Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	0.082	0.083	0.072	0.074	0.067	0.068	0.074	0.074	0.08	0.28	0.37	0.41	0.52	0.8	0.68	0.7	0.95	0.95
	50	0.061	0.051	0.07	0.066	0.062	0.066	0.077	0.042	0.055	0.53	0.81	0.72	0.83	0.99	0.98	0.97	1	1
	100	0.053	0.051	0.057	0.056	0.061	0.056	0.063	0.058	0.058	0.81	0.98	0.96	0.99	1	1	1	1	1
								Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	0.078	0.079	0.065	0.079	0.07	0.071	0.062	0.075	0.077	0.38	0.36	0.47	0.61	0.8	0.7	0.83	0.95	0.95
	50	0.054	0.057	0.069	0.063	0.062	0.069	0.064	0.046	0.054	0.69	0.81	0.72	0.93	0.99	0.99	1	1	1
	100	0.058	0.052	0.057	0.054	0.059	0.057	0.052	0.053	0.057	0.91	0.98	0.97	1	1	1	1	1	1

Table 5: The Size and Power for Alternative Mean Group Estimators under Experiment 4

		Size									Power								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
								Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments											
$\bar{\boldsymbol{x}}_t$	20	0.054	0.061	0.063	0.057	0.065	0.064	0.061	0.055	0.053	0.2	0.3	0.31	0.38	0.67	0.65	0.56	0.92	0.95
	50	0.052	0.049	0.054	0.05	0.06	0.066	0.041	0.04	0.045	0.43	0.7	0.65	0.82	0.97	0.96	0.99	1	1
	100	0.058	0.053	0.045	0.04	0.05	0.055	0.046	0.047	0.045	0.67	0.92	0.93	0.99	1	1	1	1	1
								Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	0.069	0.054	0.052	0.059	0.061	0.061	0.066	0.052	0.049	0.13	0.28	0.31	0.28	0.65	0.64	0.37	0.92	0.94
	50	0.052	0.047	0.067	0.057	0.054	0.054	0.056	0.04	0.058	0.29	0.63	0.53	0.64	0.98	0.97	0.89	1	1
	100	0.05	0.046	0.047	0.044	0.043	0.057	0.053	0.045	0.047	0.51	0.9	0.9	0.95	1	1	1	1	1
								Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	0.104	0.059	0.062	0.07	0.065	0.06	0.073	0.052	0.052	0.24	0.31	0.32	0.43	0.68	0.65	0.59	0.92	0.95
	50	0.107	0.048	0.064	0.076	0.064	0.061	0.061	0.042	0.047	0.48	0.7	0.65	0.84	0.98	0.97	0.99	1	1
	100	0.147	0.051	0.048	0.067	0.048	0.049	0.076	0.047	0.043	0.76	0.92	0.94	0.99	1	1	1	1	1
								Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments											
$(\bar{y}_t, \bar{\boldsymbol{x}}_t')'$	20	0.066	0.071	0.056	0.07	0.073	0.075	0.072	0.06	0.065	0.2	0.29	0.36	0.39	0.7	0.62	0.63	0.94	0.95
	50	0.064	0.061	0.055	0.057	0.066	0.07	0.051	0.053	0.068	0.4	0.69	0.66	0.84	0.97	0.96	0.99	1	1
	100	0.064	0.053	0.05	0.041	0.052	0.054	0.054	0.045	0.058	0.74	0.92	0.93	1	1	1	1	1	1
								Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	0.063	0.063	0.065	0.067	0.068	0.078	0.084	0.062	0.066	0.15	0.27	0.31	0.31	0.67	0.63	0.4	0.92	0.94
	50	0.053	0.051	0.065	0.071	0.069	0.058	0.073	0.046	0.069	0.32	0.64	0.58	0.65	0.97	0.97	0.91	1	1
	100	0.054	0.052	0.057	0.051	0.054	0.057	0.056	0.051	0.057	0.51	0.88	0.89	0.95	1	1	1	1	1
								Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	0.065	0.075	0.056	0.075	0.074	0.072	0.072	0.061	0.066	0.22	0.28	0.37	0.41	0.69	0.62	0.64	0.93	0.95
	50	0.074	0.056	0.06	0.073	0.069	0.062	0.059	0.051	0.067	0.46	0.7	0.64	0.84	0.98	0.97	0.99	1	1
	100	0.093	0.05	0.051	0.051	0.053	0.056	0.069	0.047	0.056	0.8	0.94	0.92	1	1	1	1	1	1
								Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments											
$(\bar{y}_t, \bar{y}_t^*, \bar{\boldsymbol{x}}_t')'$	20	0.072	0.077	0.081	0.067	0.076	0.095	0.065	0.079	0.106	0.2	0.3	0.29	0.38	0.72	0.6	0.61	0.94	0.93
	50	0.054	0.06	0.068	0.058	0.074	0.068	0.052	0.063	0.082	0.4	0.66	0.63	0.83	0.97	0.97	0.98	1	1
	100	0.063	0.047	0.052	0.048	0.054	0.063	0.056	0.048	0.064	0.68	0.9	0.9	0.99	1	1	1	1	1
								Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	0.069	0.066	0.074	0.069	0.074	0.099	0.079	0.084	0.11	0.15	0.28	0.27	0.28	0.66	0.59	0.41	0.94	0.91
	50	0.057	0.052	0.068	0.07	0.079	0.066	0.072	0.061	0.082	0.3	0.61	0.56	0.63	0.98	0.97	0.89	1	1
	100	0.045	0.056	0.046	0.048	0.06	0.069	0.059	0.048	0.067	0.54	0.85	0.87	0.95	1	1	1	1	1
								Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	0.076	0.077	0.081	0.075	0.078	0.098	0.075	0.083	0.108	0.19	0.28	0.31	0.38	0.71	0.6	0.63	0.94	0.94
	50	0.079	0.06	0.066	0.079	0.075	0.067	0.068	0.062	0.084	0.45	0.67	0.61	0.81	0.97	0.97	0.99	1	1
	100	0.104	0.05	0.054	0.061	0.059	0.067	0.073	0.05	0.07	0.74	0.92	0.91	1	1	1	1	1	1
								Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments											
Infeasible	20	0.064	0.06	0.057	0.066	0.062	0.065	0.061	0.061	0.07	0.26	0.34	0.39	0.55	0.75	0.67	0.83	0.95	0.95
	50	0.049	0.044	0.052	0.057	0.067	0.065	0.054	0.047	0.06	0.54	0.74	0.69	0.91	0.99	0.97	1	1	1
	100	0.061	0.052	0.044	0.046	0.055	0.056	0.052	0.047	0.055	0.8	0.95	0.94	1	1	1	1	1	1
								Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	0.069	0.059	0.047	0.063	0.063	0.066	0.065	0.065	0.067	0.23	0.35	0.39	0.52	0.73	0.66	0.7	0.95	0.94
	50	0.046	0.052	0.06	0.054	0.065	0.057	0.063	0.043	0.055	0.43	0.7	0.67	0.81	0.98	0.98	0.96	1	1
	100	0.059	0.041	0.053	0.058	0.062	0.053	0.055	0.05	0.057	0.69	0.96	0.9	0.98	1	1	1	1	1
								Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	0.081	0.06	0.053	0.063	0.061	0.069	0.068	0.06	0.069	0.28	0.36	0.4	0.56	0.74	0.68	0.83	0.95	0.95
	50	0.072	0.049	0.058	0.065	0.068	0.064	0.054	0.043	0.06	0.54	0.74	0.7	0.93	0.99	0.98	1	1	1
	100	0.1	0.047	0.047	0.057	0.059	0.057	0.064	0.049	0.056	0.85	0.95	0.94	1	1	1	1	1	1

Table 6: The Panel Data Estimation Results without Spatial Effects

	gdp	sim	rfe	rer	eec	CD test
FE	1.909***	0.967***	0.018	0.034***	0.536***	−4.068
	(0.019)	(0.027)	(0.012)	(0.004)	(0.038)	[0.000]
CCEP-XY	1.261***	0.910***	−0.001	−0.037*	0.147***	-3.101
	(0.252)	(0.209)	(0.014)	(0.021)	(0.027)	[0.001]
CCEP-X	0.982***	1.025***	−0.015	0.012	0.318***	-2.473
	(0.218)	(0.134)	(0.029)	(0.009)	(0.039)	[0.013]

Notes: Figures in () indicate standard errors. ***,** and * denote that the coefficients are significant at 1, 5 and 10%. Figures in [] indicate the p-value of the CD test proposed by Pesaran (2015), that tests the validity of the null of weak CSD in residuals. FE stands for the fixed effects estimator. CCEP-XY is the common correlated effects Pooled estimator obtained from the regression, (61) augmented with CSA of the dependent variable and regressors, whereas CCEP-X augmented with CSA of the regressors only.

Table 7: The Panel Data Estimation Results with Spatial Effects

	W_{dis}			W_{bor}		
	SARAR	HSAR-XY	HSAR-X	SARAR	HSAR-XY	HSAR-X
ρ	0.051*	−0.642***	−0.258**	−.008	−0.127*	−0.227**
	(0.031)	(0.106)	(0.131)	(0.012)	(0.076)	(0.119)
gdp	1.850***	0.986***	1.055***	1.901***	0.878***	1.047***
	(0.047)	(0.239)	(0.263)	(0.022)	(0.238)	(0.261)
sim	1.021***	1.046**	1.088**	1.036***	0.975***	1.095***
	(0.048)	(0.173)	(0.209)	(0.044)	(0.160)	(0.168)
rfe	−0.002	0.009	−0.001	0.006	−0.034	−0.018
	(0.007)	(0.026)	(0.033)	(0.007)	(0.024)	(0.031)
rer	−0.033***	−0.026	0.015	−0.006***	−0.007	0.016
	(0.004)	(0.010)	(0.011)	(0.003)	(0.010)	(0.010)
eec	0.225***	0.129***	0.353***	0.212***	0.117***	0.357***
	(0.027)	(0.038)	(0.083)	(0.018)	(0.028)	(0.077)
CD test	-83.742	-2.456	-2.162	88.62	-2.445	-2.089
	(0.000)	(0.014)	(0.031)	(0.000)	(0.014)	(0.037)

Notes: W_{dis} and W_{bor} are population-based and border-based spatial weighting matrices. SARAR stands for the estimator obtained by spatial autoregressive model with autoregressive disturbances. HSAR-XY and HSAR-X stand for the estimators obtained by the model proposed in this paper that use $(\bar{y}_t, \bar{\boldsymbol{x}}_t')'$ and $(\bar{\boldsymbol{x}}_t)$ as factor proxies, respectively. See also the footnotes to Table 6.

Table 8: Average Direct, Indirect and Total effects of Regressors

	W_{dis}			W_{bor}		
	ADE	AIE	TE	ADE	AIE	TE
gdp	1.0625***	−0.1721***	0.8903***	1.0627***	−0.1322***	0.9305***
	(0.0706)	(0.0155)	(0.0597)	(0.0698)	(0.0118)	(0.0615)
sim	1.0947***	−0.1773***	0.9173***	1.1120***	−0.1384***	0.9736***
	(0.0436)	(0.0126)	(0.0378)	(0.0273)	(0.0092)	(0.0247)
rfe	−0.0097***	0.0015***	−0.0081***	−0.0179***	0.0022***	−0.0156***
	(0.0011)	(0.0002)	(0.0001)	(0.0010)	(0.0001)	(0.0009)
rer	−0.0155***	−0.0002***	0.0130***	0.0160***	−0.0020***	0.0140***
	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)
eec	0.3554***	−0.0576***	0.2978***	0.3626***	−0.0451***	0.3175***
	(0.0071)	(0.0036)	(0.0066)	(0.0061)	(0.0029)	(0.0056)

Notes: The ADE of the p-th explanatory variables measuring the average direct effect of this variable on the dependent variable calculated by averaging the diagonal elements of $(\boldsymbol{I}_N - \rho \boldsymbol{W})^{-1}\beta_p$, where β_p is the corresponding coefficient. The AIE of the p-th explanatory variables measuring the average indirect effect of this variable on the dependent variable calculated by averaging the row sum of the off diagonal elements of $(\boldsymbol{I}_N - \rho \boldsymbol{W})^{-1}\beta_p$. The TE, sum of ADE and AIE, measuring the total effects of any explanatory variables on the dependent variable. Standard errors reported in () have been evaluated via the bootstrap with 1,000 replications, see (Elhorst, 2014, p. 25). See also the footnotes to Table 6.

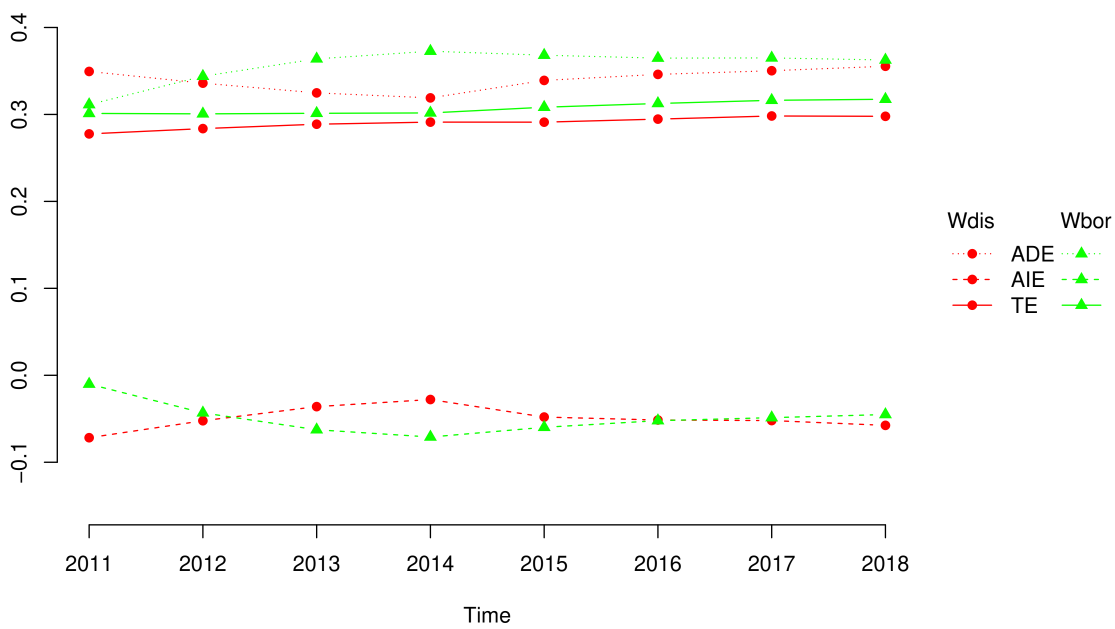

Figure 1: The impacts of cee_{it} on bilateral trade flows after 2010

Online Supplement to: Estimation and Inference in Heterogeneous Spatial Panel Data Models with a Multifactor Error Structure

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This Online Supplement contains two sections. Section S1 summarises the Lemmas used for the proofs of theorems in the main paper. Section S2 reports additional Monte Carlo results, complementing the results already reported in the main paper.

S1 Lemmas

Lemma 1. *Under Assumption 2, for all t, we have*

(a) $E(\bar{\boldsymbol{v}}_t) = 0, Var(\bar{\boldsymbol{v}}_t) = O(\frac{1}{N})$, *and hence* $\bar{\boldsymbol{v}}_t \xrightarrow{q.m.} 0$, *as* $N \to \infty$,

(b) $E\|\bar{\boldsymbol{v}}_t\|^2 = O(\frac{1}{N})$, $E\|\bar{\boldsymbol{v}}_t\| = O(\frac{1}{\sqrt{N}})$,

where $\bar{\boldsymbol{v}}_t = \boldsymbol{\Upsilon} vec(\boldsymbol{V}'_{.t}), \boldsymbol{\Upsilon} = \frac{1}{N}\boldsymbol{\iota}'_N \otimes \boldsymbol{I}_k$ *and q.m. means convergence in quadratic mean, which implies convergence in probability.*

Proof. This lemma shows that using only $\bar{\boldsymbol{x}}_t$ as factor proxies also works and the approximation error is negligible as $N \to \infty$, which is the counterpart of Lemma 1 in Pesaran (2006) and Yang (2018).

(a) By $\bar{\boldsymbol{v}}_t = \boldsymbol{\Upsilon} vec(\boldsymbol{V}'_{.t}) = \frac{1}{N}\sum_{i=1}^N \boldsymbol{v}_{it}$, we have $E(\bar{\boldsymbol{v}}_t) = \frac{1}{N}\sum_{i=1}^N E\boldsymbol{v}_{it} = 0$ under Assumption 2. Further, since $\boldsymbol{v}_{it}$ is independent of $\boldsymbol{v}_{jt}$ for any $i \neq j$, the variance of $\bar{\boldsymbol{v}}_t$ is

$$Var(\bar{\boldsymbol{v}}_t) = Var(\frac{1}{N}\sum_{i=1}^N \boldsymbol{v}_{it}) = \frac{1}{N^2}\sum_{i=1}^N var(\boldsymbol{v}_{it}) = \frac{1}{N^2}\sum_{i=1}^N \boldsymbol{\Omega}_{v,i} = O(\frac{1}{N}),$$

where we have used the fact that by Assumption 2, $\|\boldsymbol{\Omega}_{v,i}\| < C$ for all i.

The last statement is readily established by the definition of convergence in quadratic mean.

(b) The results can be simply proved by

$$E\|\bar{\boldsymbol{v}}_t\|^2 = E[tr(\frac{1}{N^2}\sum_{j=1}^N\sum_{i=1}^N \boldsymbol{v}_{jt}\boldsymbol{v}'_{it})] = tr(\frac{1}{N^2}\sum_{i=1}^N var(\boldsymbol{v}_{it})) = O(\frac{1}{N}),$$

where the last equality follows from Assumption 2. Then by Cauchy- Schwartz's Inequality,

$$E\|\bar{\boldsymbol{v}}_t\| \leq (E\|\bar{\boldsymbol{v}}_t\|^2)^{1/2} = O(\frac{1}{\sqrt{N}}).$$

■

Lemma 2. *Under Assumptions 1 and 2, for all i, we have*

(a) $\dfrac{\bar{\boldsymbol{V}}'\bar{\boldsymbol{V}}}{T} = O_p(\dfrac{1}{N})$, (b) $\dfrac{\boldsymbol{F}_a'\bar{\boldsymbol{V}}}{T} = O_p(\dfrac{1}{\sqrt{NT}})$,

(c) $\dfrac{\boldsymbol{V}_{i.}'\boldsymbol{F}_a}{T} = O_p(\dfrac{1}{\sqrt{T}})$, $\dfrac{\boldsymbol{\varepsilon}_{i.}'\boldsymbol{F}_a}{T} = O_p(\dfrac{1}{\sqrt{T}})$,

(d) $\dfrac{\boldsymbol{\varepsilon}_{i.}'\bar{\boldsymbol{V}}}{T} = O_p(\dfrac{1}{\sqrt{NT}})$, $\dfrac{\boldsymbol{V}_{i.}'\bar{\boldsymbol{V}}}{T} = O_p(\dfrac{1}{N}) + O_p(\dfrac{1}{\sqrt{NT}})$,

(e) $\dfrac{\boldsymbol{X}_i'\bar{\boldsymbol{V}}}{T} = O_p(\dfrac{1}{N}) + O_p(\dfrac{1}{\sqrt{NT}})$,

where $\bar{\boldsymbol{V}} = (\bar{\boldsymbol{v}}_1, \bar{\boldsymbol{v}}_2, \ldots, \bar{\boldsymbol{v}}_T)'$ is of dimension $T \times k$ with components $\bar{\boldsymbol{v}}_t = \boldsymbol{\Upsilon} vec(\boldsymbol{V}_{.t}') = \frac{1}{N}\sum_{i=1}^N \boldsymbol{v}_{it}$, $\boldsymbol{F}_a = (\boldsymbol{f}_{a1}, \boldsymbol{f}_{a2}, \ldots, \boldsymbol{f}_{aT})'$, $\boldsymbol{V}_{i.} = (\boldsymbol{v}_{i1}, \boldsymbol{v}_{i2}, \ldots, \boldsymbol{v}_{iT})'$, $\boldsymbol{\varepsilon}_{i.} = (\varepsilon_{i1}, \varepsilon_{i2}, \ldots, \varepsilon_{iT})'$ and $\boldsymbol{X}_{i.} = (\boldsymbol{x}_{i1}, \boldsymbol{x}_{i2}, \ldots, \boldsymbol{x}_{iT})'$.

Proof. The results here correspond to $(A.10) - (A.12)$ in Lemma 2 of Pesaran (2006) and can be proved similarly based on Lemma 1. Hence, we only give proof for (d) and (e).

(d) By definition, we have

$$\frac{\boldsymbol{\varepsilon}_{i.}'\bar{\boldsymbol{V}}}{T} = \frac{1}{T}\sum_{t=1}^T \varepsilon_{it}\bar{\boldsymbol{v}}_t' = \frac{1}{T}\sum_{t=1}^T \varepsilon_{it}(\frac{1}{N}\sum_{j=1}^N \boldsymbol{v}_{qt}') = \frac{1}{NT}\sum_{t=1}^T\sum_{j=1}^N \varepsilon_{it}\boldsymbol{v}_{qt}',$$

which is a k dimensional row vector. Clearly, for each of its element, $\frac{1}{NT}\sum_{t=1}^T\sum_{j=1}^N \varepsilon_{it}v_{qt,p}$, $p = 1, 2, ..., k$, its mean is zero and its variance is

$$\begin{aligned}
Var(\frac{1}{NT}\sum_{t=1}^T\sum_{j=1}^N \varepsilon_{it}v_{qt,p}) &= \frac{1}{N^2T^2}\sum_{t=1}^T\sum_{s=1}^T\sum_{j=1}^N\sum_{q=1}^N E(\varepsilon_{it}\varepsilon_{is})E(v_{qt,p}v_{hs,p}') \\
&= \frac{1}{N^2T^2}\sum_{t=1}^T\sum_{s=1}^T\sum_{j=1}^N E(\varepsilon_{it}\varepsilon_{is})E(v_{qt,p}v_{qs,p}') \\
&= \frac{1}{N^2T^2}\sum_{t=1}^T(\sum_{s=1}^T E(\varepsilon_{it}\varepsilon_{is}))(\sum_{j=1}^N E(v_{qt,p}v_{qs,p}')) \\
&= O(\frac{1}{NT}).
\end{aligned}$$

where the first equality follows from the independence of ε_{it} and $v_{qs,p}$ for any i, j, t, s, p, the second equality from the fact that $v_{qt,p}$ and $v_{hs,p}$ are independent for $j \neq q$, and the last equality from the absolutely summable autocovariance property of ε_{it} and the existence of second moment of $v_{qt,p}$.

The second result can be proven similarly by noticing that the existence of additional term $O_p(\frac{1}{N})$ is cause by the correlation between $\boldsymbol{V}_{i.}$ and $\bar{\boldsymbol{V}}$.

(e) According to (26) that $\boldsymbol{X}_{i.} = \boldsymbol{F}_x\boldsymbol{\Gamma}_{xi} + \boldsymbol{V}_{i.}$, we then have $\frac{\boldsymbol{X}_{i.}'\bar{\boldsymbol{V}}}{T} = \boldsymbol{\Gamma}_{xi}'(\frac{\boldsymbol{F}_x'\bar{\boldsymbol{V}}}{T}) + \frac{\boldsymbol{V}_{i.}'\bar{\boldsymbol{V}}}{T}$. Noticing that $\boldsymbol{F}_a = (\boldsymbol{F}_1, \boldsymbol{F}_2, \boldsymbol{F}_3) = (\boldsymbol{F}_y, \boldsymbol{F}_3) = (\boldsymbol{F}_x, \boldsymbol{F}_2)$ and using results in (b) (d) and the assumption that $\|\boldsymbol{\Gamma}_{xi}\| < C$, the result here follows readily. ■

Lemma 3. *Let $\boldsymbol{H} = \boldsymbol{F}_x\bar{\boldsymbol{\Gamma}}_x$. Under Assumptions 1-3,*

(a) $\frac{\boldsymbol{H}'\boldsymbol{H}}{T} = O_p(1)$, *(b)* $\frac{\boldsymbol{H}'\bar{\boldsymbol{V}}}{T} = O_p(\frac{1}{\sqrt{NT}})$, *(c)* $\frac{\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}}}{T} = O_p(1)$,

(d) $\frac{\bar{\boldsymbol{X}}'\boldsymbol{F}_x}{T} = O_p(1)$, $\frac{\bar{\boldsymbol{X}}'\boldsymbol{F}_2}{T} = O_p(\frac{1}{\sqrt{T}})$

(e) $\frac{\bar{\boldsymbol{X}}'\boldsymbol{V}_{i.}}{T} = O_p(\frac{1}{N}) + O_p(\frac{1}{\sqrt{T}})$, *(f)* $\frac{\bar{\boldsymbol{X}}'\boldsymbol{X}_{i.}}{T} = O_p(1)$,

(g) $\frac{\boldsymbol{H}'\boldsymbol{X}_i}{T} = O_p(1)$, *(h)* $\frac{\bar{\boldsymbol{X}}'\boldsymbol{\varepsilon}_{i.}}{T} = O_p(\frac{1}{\sqrt{T}})$.

Proof. (a) Under Assumption 3, each element of $\bar{\boldsymbol{\Gamma}}_x = \frac{1}{N}\sum_{i=1}^N \boldsymbol{\Gamma}_{xi}$ is bounded in probability and so does $\frac{\boldsymbol{F}_a'\boldsymbol{F}_a}{T} = O_p(1)$ under Assumption 1. As a result,

$$\frac{\boldsymbol{H}'\boldsymbol{H}}{T} = \bar{\boldsymbol{\Gamma}}_x'\frac{\boldsymbol{F}_x'\boldsymbol{F}_x}{T}\bar{\boldsymbol{\Gamma}}_x = O_p(1).$$

(b) As the elements of $\bar{\boldsymbol{\Gamma}}_x$ are bounded in probability and by Lemma 2 (b), $\frac{\boldsymbol{F}_a'\bar{\boldsymbol{V}}}{T} = O_p(\frac{1}{\sqrt{NT}})$, it follows directly that

$$\frac{\boldsymbol{H}'\bar{\boldsymbol{V}}}{T} = \bar{\boldsymbol{\Gamma}}_x'\frac{\boldsymbol{F}_x'\bar{\boldsymbol{V}}}{T} = O_p(\frac{1}{\sqrt{NT}}).$$

(c) Since $\bar{\boldsymbol{X}} = \boldsymbol{H} + \bar{\boldsymbol{V}}$ (see (A.43)), and by Lemma 2(a) and Lemma 3(a)(b), we have

$$\frac{\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}}}{T} = \frac{\boldsymbol{H}'\boldsymbol{H}}{T} + \frac{\bar{\boldsymbol{V}}'\bar{\boldsymbol{V}}}{T} + \frac{\boldsymbol{H}'\bar{\boldsymbol{V}}}{T} + \frac{\bar{\boldsymbol{V}}'\boldsymbol{H}}{T} = O_p(1) + O_p(\frac{1}{N}) + O_p(\frac{1}{\sqrt{NT}}) = O_p(1).$$

(d) Under Assumption 1, $\frac{\boldsymbol{F}_a'\boldsymbol{F}_a}{T} = O_p(1)$ and $\frac{\boldsymbol{F}_2'\boldsymbol{F}_x}{T} = O_p(\frac{1}{\sqrt{T}})$. Therefore, by Lemma 2(b), we would have

$$\begin{aligned}\frac{\bar{\boldsymbol{X}}'\boldsymbol{F}_x}{T} &= \bar{\boldsymbol{\Gamma}}_x'\frac{\boldsymbol{F}_x'\boldsymbol{F}_x}{T} + \frac{\bar{\boldsymbol{V}}'\boldsymbol{F}_x}{T} = O_p(1) + O_p(\frac{1}{\sqrt{NT}}) = O_p(1),\\ \frac{\bar{\boldsymbol{X}}'\boldsymbol{F}_2}{T} &= \bar{\boldsymbol{\Gamma}}_x'\frac{\boldsymbol{F}_x'\boldsymbol{F}_2}{T} + \frac{\bar{\boldsymbol{V}}'\boldsymbol{F}_2}{T} = O_p(\frac{1}{\sqrt{T}}) + O_p(\frac{1}{\sqrt{NT}}) = O_p(\frac{1}{\sqrt{T}}).\end{aligned}$$

(e) Similar to (d), this can be easily verified by Lemma 2(c)(d):

$$\frac{\bar{\boldsymbol{X}}'\boldsymbol{V}_{i.}}{T} = \bar{\boldsymbol{\Gamma}}_x'\frac{\boldsymbol{F}_x'\boldsymbol{V}_{i.}}{T} + \frac{\bar{\boldsymbol{V}}'\boldsymbol{V}_{i.}}{T} = O_p(\frac{1}{\sqrt{T}}) + [O_p(\frac{1}{N}) + O_p(\frac{1}{\sqrt{NT}})] = O_p(\frac{1}{N}) + O_p(\frac{1}{\sqrt{T}}).$$

(f) Recall that in (26), $\boldsymbol{X}_{i.} = \boldsymbol{F}_x\boldsymbol{\Gamma}_i + \boldsymbol{V}_{i.}$, then it follows that

$$\frac{\bar{\boldsymbol{X}}'\boldsymbol{X}_i}{T} = \frac{\bar{\boldsymbol{X}}'\boldsymbol{F}_x}{T}\boldsymbol{\Gamma}_i + \frac{\bar{\boldsymbol{X}}'\boldsymbol{V}_{i.}}{T} = O_p(1) + O_p(\frac{1}{N}) + O_p(\frac{1}{\sqrt{T}}) = O_p(1),$$

by Lemma 3(d)(e).

(g) Under Assumption 1 and Lemma 2(c), we have

$$\frac{\boldsymbol{H}'\boldsymbol{X}_{i.}}{T} = \bar{\boldsymbol{\Gamma}}_x'\frac{\boldsymbol{F}_x'\boldsymbol{F}_x}{T}\boldsymbol{\Gamma}_i + \bar{\boldsymbol{\Gamma}}_x'\frac{\boldsymbol{F}_x'\boldsymbol{V}_{i.}}{T} = O_p(1) + O_p(\frac{1}{\sqrt{T}}) = O_p(1).$$

(h) This can be easily verified according to Lemma 2(c)(d).

$$\frac{\bar{\boldsymbol{X}}'\boldsymbol{\varepsilon}_{i.}}{T} = \bar{\boldsymbol{\Gamma}}'_x \frac{\boldsymbol{F}'_x\boldsymbol{\varepsilon}_{i.}}{T} + \bar{\boldsymbol{\Gamma}}'_x \frac{\bar{\boldsymbol{V}}'\boldsymbol{\varepsilon}_{i.}}{T} = O_p(\frac{1}{\sqrt{T}}) + O_p(\frac{1}{\sqrt{NT}}) = O_p(\frac{1}{\sqrt{T}}).$$

■

Lemma 4. *Under Assumptions 1-3, for any i and j, (i could equal to j)*

(a) $\dfrac{\boldsymbol{X}'_{i.}\boldsymbol{M}_{\bar{x}}\boldsymbol{F}_x}{T} = O_p(\dfrac{1}{N}) + O_p(\dfrac{1}{\sqrt{NT}})$,

(b) $\dfrac{\boldsymbol{X}'_{i.}\boldsymbol{M}_{\bar{x}}\boldsymbol{X}_{j.}}{T} = \dfrac{\boldsymbol{X}'_{i.}\boldsymbol{M}_{f_x}\boldsymbol{X}_{j.}}{T} + O_p(\dfrac{1}{N}) + O_p(\dfrac{1}{\sqrt{NT}})$,

(c) $\dfrac{\boldsymbol{X}'_{i.}\boldsymbol{M}_{\bar{x}}\boldsymbol{\varepsilon}_{j.}}{T} = \dfrac{\boldsymbol{X}'_{i.}\boldsymbol{M}_{f_x}\boldsymbol{\varepsilon}_{j.}}{T} + O_p(\dfrac{1}{\sqrt{NT}})$,

(d) $\dfrac{\boldsymbol{X}'_{i.}\boldsymbol{M}_{\bar{x}}\boldsymbol{F}_2}{T} = \dfrac{\boldsymbol{X}'_{i.}\boldsymbol{M}_{f_x}\boldsymbol{F}_2}{T} + O_p(\dfrac{1}{N}) + O_p(\dfrac{1}{\sqrt{NT}})$.

Proof. All the results here can be proved similarly to Lemma 3 in Kapetanios et al. (2011), using results established in Lemma 2 and 3.

(a) Let $\boldsymbol{H} = \boldsymbol{F}_x\bar{\boldsymbol{\Gamma}}_x$, and $\boldsymbol{M}_{\boldsymbol{H}} = \boldsymbol{I}_T - \boldsymbol{H}(\boldsymbol{H}'\boldsymbol{H})^{-1}\boldsymbol{H}'$. Then according to (A.43), we have

$$\begin{aligned}
&\left\|\frac{\boldsymbol{X}'_{i.}\boldsymbol{M}_{\bar{x}}\boldsymbol{F}_1}{T} - \frac{\boldsymbol{X}'_{i.}\boldsymbol{M}_{\boldsymbol{H}}\boldsymbol{F}_1}{T}\right\| = \left\|\frac{\boldsymbol{X}'_{i.}\bar{\boldsymbol{X}}(\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}})^{-1}\bar{\boldsymbol{X}}'\boldsymbol{F}_1}{T} - \frac{\boldsymbol{X}'_{i.}\boldsymbol{H}(\boldsymbol{H}'\boldsymbol{H})^{-1}\boldsymbol{H}'\boldsymbol{F}_1}{T}\right\| \\
&= \left\|\frac{\boldsymbol{X}'_{i.}\bar{\boldsymbol{X}}(\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}})^{-1}\bar{\boldsymbol{X}}'\boldsymbol{F}_1}{T} - \frac{\boldsymbol{X}'_{i.}\boldsymbol{H}(\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}})^{-1}\bar{\boldsymbol{X}}'\boldsymbol{F}_1}{T} + \frac{\boldsymbol{X}'_{i.}\boldsymbol{H}(\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}})^{-1}\bar{\boldsymbol{X}}'\boldsymbol{F}_1}{T} - \frac{\boldsymbol{X}'_{i.}\boldsymbol{H}(\boldsymbol{H}'\boldsymbol{H})^{-1}\bar{\boldsymbol{X}}'\boldsymbol{F}_1}{T}\right. \\
&\left. + \frac{\boldsymbol{X}'_{i.}\boldsymbol{H}(\boldsymbol{H}'\boldsymbol{H})^{-1}\bar{\boldsymbol{X}}'\boldsymbol{F}_1}{T} - \frac{\boldsymbol{X}'_{i.}\boldsymbol{H}(\boldsymbol{H}'\boldsymbol{H})^{-1}\boldsymbol{H}'\boldsymbol{F}_1}{T}\right\| \\
&\leq d_1 + d_2 + d_3,
\end{aligned}$$

where

$$d_1 \equiv \left\|\frac{(\boldsymbol{X}'_{i.}\bar{\boldsymbol{X}} - \boldsymbol{X}'_{i.}\boldsymbol{H})(\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}})^{-1}\bar{\boldsymbol{X}}'\boldsymbol{F}_1}{T}\right\| \leq \left\|\frac{\boldsymbol{X}'_{i.}\bar{\boldsymbol{V}}}{T}\right\| \left\|(\frac{\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}}}{T})^{-1}\frac{\bar{\boldsymbol{X}}'\boldsymbol{F}_1}{T}\right\| = O_p(\frac{1}{N}) + O_p(\frac{1}{\sqrt{NT}})$$

by the sub-multiplicity property of norm (i.e., $\|AB\| \leq \|A\| \cdot \|B\|$) and Lemma 2(e) and Lemma 3(c)(d),

$$\begin{aligned}
d_2 &\equiv \left\|\frac{\boldsymbol{X}'_{i.}\boldsymbol{H}[(\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}})^{-1} - (\boldsymbol{H}'\boldsymbol{H})^{-1}]'\bar{\boldsymbol{X}}'\boldsymbol{F}_1}{T}\right\| \\
&\leq \left\|\frac{\boldsymbol{X}'_{i.}\boldsymbol{H}}{T}(\frac{\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}}}{T})^{-1}\right\| \left\|-\frac{\bar{\boldsymbol{V}}'\bar{\boldsymbol{V}}}{T} - \frac{\boldsymbol{H}'\bar{\boldsymbol{V}}}{T} - \frac{\bar{\boldsymbol{V}}'\boldsymbol{H}}{T}\right\| \left\|(\frac{\boldsymbol{H}'\boldsymbol{H}}{T})^{-1}\frac{\bar{\boldsymbol{X}}'\boldsymbol{F}_1}{T}\right\| = O_p(\frac{1}{N}) + O_p(\frac{1}{\sqrt{NT}})
\end{aligned}$$

by Continuous Mapping Theorem, Lemma 2(a) and Lemma 3(a)(b)(c)(d)(g)[1], and

$$d_3 \equiv \left\|\frac{\boldsymbol{X}'_{i.}\boldsymbol{H}(\boldsymbol{H}'\boldsymbol{H})^{-1}(\bar{\boldsymbol{X}}'\boldsymbol{F}_1 - \boldsymbol{H}'\boldsymbol{F}_1)}{T}\right\| \leq \left\|\frac{\boldsymbol{X}'_{i.}\boldsymbol{H}}{T}(\frac{\boldsymbol{H}'\boldsymbol{H}}{T})^{-1}\right\| \left\|\frac{\bar{\boldsymbol{V}}'\boldsymbol{F}_1}{T}\right\| = O_p(\frac{1}{\sqrt{NT}})$$

by Lemma 2(b) and Lemma 3(a)(g).

[1] We also use the fact that $A^{-1} \pm B^{-1} = A^{-1}(B \pm A)B^{-1}$ in deriving the inequality.

Under Assumption 3, we can show that $\boldsymbol{M_H} = \boldsymbol{M}_{f_x}$, and hence we have

$$\|\frac{\boldsymbol{X}_{i.}'\boldsymbol{M}_{\bar{x}}\boldsymbol{F}_1}{T} - \frac{\boldsymbol{X}_{i.}'\boldsymbol{M}_{f_x}\boldsymbol{F}_1}{T}\| = \|\frac{\boldsymbol{X}_{i.}'\boldsymbol{M}_{\bar{x}}\boldsymbol{F}_1}{T}\| = O_p(\frac{1}{N}) + O_p(\frac{1}{\sqrt{NT}}).$$

(b) Similarly, we have

$$\begin{aligned}
&\left\|\frac{\boldsymbol{X}_{i.}'\boldsymbol{M}_{\bar{x}}\boldsymbol{X}_{j.}}{T} - \frac{\boldsymbol{X}_{i.}'\boldsymbol{M_H}\boldsymbol{X}_{j.}}{T}\right\| = \left\|\frac{\boldsymbol{X}_{i.}'\bar{\boldsymbol{X}}(\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}})^{-1}\bar{\boldsymbol{X}}'\boldsymbol{X}_{j.}}{T} - \frac{\boldsymbol{X}_{i.}'\boldsymbol{H}(\boldsymbol{H}'\boldsymbol{H})^{-1}\boldsymbol{H}'\boldsymbol{X}_{j.}}{T}\right\| \\
&= \left\|\frac{\boldsymbol{X}_{i.}'\bar{\boldsymbol{X}}(\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}})^{-1}\bar{\boldsymbol{X}}'\boldsymbol{X}_{j.}}{T} - \frac{\boldsymbol{X}_{i.}'\boldsymbol{H}(\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}})^{-1}\bar{\boldsymbol{X}}'\boldsymbol{X}_{j.}}{T} + \frac{\boldsymbol{X}_{i.}'\boldsymbol{H}(\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}})^{-1}\bar{\boldsymbol{X}}'\boldsymbol{X}_{j.}}{T} - \frac{\boldsymbol{X}_{i.}'\boldsymbol{H}(\boldsymbol{H}'\boldsymbol{H})^{-1}\bar{\boldsymbol{X}}'\boldsymbol{X}_{j.}}{T}\right. \\
&\quad \left. + \frac{\boldsymbol{X}_{i.}'\boldsymbol{H}(\boldsymbol{H}'\boldsymbol{H})^{-1}\bar{\boldsymbol{X}}'\boldsymbol{X}_{j.}}{T} - \frac{\boldsymbol{X}_{i.}'\boldsymbol{H}(\boldsymbol{H}'\boldsymbol{H})^{-1}\boldsymbol{H}'\boldsymbol{X}_{j.}}{T}\right\| \\
&\leq d_1^* + d_2^* + d_3^*,
\end{aligned}$$

where

$$d_1^* \equiv \left\|\frac{(\boldsymbol{X}_{i.}'\bar{\boldsymbol{X}} - \boldsymbol{X}_{i.}'\boldsymbol{H})(\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}})^{-1}\bar{\boldsymbol{X}}'\boldsymbol{X}_{j.}}{T}\right\| \leq \left\|\frac{\boldsymbol{X}_{i.}'\bar{\boldsymbol{V}}}{T}\right\|\left\|(\frac{\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}}}{T})^{-1}\frac{\bar{\boldsymbol{X}}'\boldsymbol{X}_{j.}}{T}\right\| = O_p(\frac{1}{N}) + O_p(\frac{1}{\sqrt{NT}})$$

by Lemma 2(e) and Lemma 3(c)(f),

$$\begin{aligned}
d_2^* &\equiv \left\|\frac{\boldsymbol{X}_{i.}'\boldsymbol{H}[(\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}})^{-1} - (\boldsymbol{H}'\boldsymbol{H})^{-1}]'\bar{\boldsymbol{X}}'\boldsymbol{X}_{j.}}{T}\right\| \\
&\leq \left\|\frac{\boldsymbol{X}_{i.}'\boldsymbol{H}}{T}(\frac{\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}}}{T})^{-1}\right\|\left\|-\frac{\bar{\boldsymbol{V}}'\bar{\boldsymbol{V}}}{T} - \frac{\boldsymbol{H}'\bar{\boldsymbol{V}}}{T} - \frac{\bar{\boldsymbol{V}}'\boldsymbol{H}}{T}\right\|\left\|(\frac{\boldsymbol{H}'\boldsymbol{H}}{T})^{-1}\frac{\bar{\boldsymbol{X}}'\boldsymbol{X}_{j.}}{T}\right\| = O_p(\frac{1}{N}) + O_p(\frac{1}{\sqrt{NT}})
\end{aligned}$$

by Lemma 2(a) and Lemma 3(a)(b)(c)(f)(g), and

$$d_3^* \equiv \left\|\frac{\boldsymbol{X}_{i.}'\boldsymbol{H}(\boldsymbol{H}'\boldsymbol{H})^{-1}(\bar{\boldsymbol{X}}'\boldsymbol{X}_{j.} - \boldsymbol{H}'\boldsymbol{X}_{j.})}{T}\right\| \leq \left\|\frac{\boldsymbol{X}_{i.}'\boldsymbol{H}}{T}(\frac{\boldsymbol{H}'\boldsymbol{H}}{T})^{-1}\right\|\left\|\frac{\bar{\boldsymbol{V}}'\boldsymbol{X}_{j.}}{T}\right\| = O_p(\frac{1}{N}) + O_p(\frac{1}{\sqrt{NT}})$$

by Lemma 2(e) and Lemma 3(a)(g).

As a result, we have

$$\|\frac{\boldsymbol{X}_{i.}'\boldsymbol{M}_{\bar{x}}\boldsymbol{X}_{j.}}{T} - \frac{\boldsymbol{X}_{i.}'\boldsymbol{M}_{f_x}\boldsymbol{X}_{j.}}{T}\| = O_p(\frac{1}{N}) + O_p(\frac{1}{\sqrt{NT}}).$$

(c) Similarly, we have

$$\begin{aligned}
&\left\|\frac{\boldsymbol{X}_{i.}'\boldsymbol{M}_{\bar{x}}\boldsymbol{\varepsilon}_{j.}}{T} - \frac{\boldsymbol{X}_{i.}'\boldsymbol{M_H}\boldsymbol{\varepsilon}_{j.}}{T}\right\| = \left\|\frac{\boldsymbol{X}_{i.}'\bar{\boldsymbol{X}}(\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}})^{-1}\bar{\boldsymbol{X}}'\boldsymbol{\varepsilon}_{j.}}{T} - \frac{\boldsymbol{X}_{i.}'\boldsymbol{H}(\boldsymbol{H}'\boldsymbol{H})^{-1}\boldsymbol{H}'\boldsymbol{\varepsilon}_{j.}}{T}\right\| \\
&= \left\|\frac{\boldsymbol{X}_{i.}'\bar{\boldsymbol{X}}(\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}})^{-1}\bar{\boldsymbol{X}}'\boldsymbol{\varepsilon}_{j.}}{T} - \frac{\boldsymbol{X}_{i.}'\boldsymbol{H}(\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}})^{-1}\bar{\boldsymbol{X}}'\boldsymbol{\varepsilon}_{j.}}{T} + \frac{\boldsymbol{X}_{i.}'\boldsymbol{H}(\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}})^{-1}\bar{\boldsymbol{X}}'\boldsymbol{\varepsilon}_{j.}}{T} - \frac{\boldsymbol{X}_{i.}'\boldsymbol{H}(\boldsymbol{H}'\boldsymbol{H})^{-1}\bar{\boldsymbol{X}}'\boldsymbol{\varepsilon}_{j.}}{T}\right. \\
&\quad \left. + \frac{\boldsymbol{X}_{i.}'\boldsymbol{H}(\boldsymbol{H}'\boldsymbol{H})^{-1}\bar{\boldsymbol{X}}'\boldsymbol{\varepsilon}_{j.}}{T} - \frac{\boldsymbol{X}_{i.}'\boldsymbol{H}(\boldsymbol{H}'\boldsymbol{H})^{-1}\boldsymbol{H}'\boldsymbol{\varepsilon}_{j.}}{T}\right\| \\
&\leq \tilde{d}_1 + \tilde{d}_2 + \tilde{d}_3,
\end{aligned}$$

where

$$\tilde{d}_1 \equiv \left\| \frac{(\boldsymbol{X}'_{i.}\bar{\boldsymbol{X}} - \boldsymbol{X}'_{i.}\boldsymbol{H})(\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}})^{-1}\bar{\boldsymbol{X}}'\boldsymbol{\varepsilon}_{j.}}{T} \right\| \leq \left\| \frac{\boldsymbol{X}'_{i.}\bar{\boldsymbol{V}}}{T} \right\| \left\| (\frac{\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}}}{T})^{-1} \right\| \left\| \frac{\bar{\boldsymbol{X}}'\boldsymbol{\varepsilon}_{j.}}{T} \right\| = O_p(\frac{1}{N\sqrt{T}}) + O_p(\frac{1}{\sqrt{NT}})$$

by Lemma 2(e) and Lemma 3(c)(h),

$$\tilde{d}_2 \equiv \left\| \frac{\boldsymbol{X}'_{i.}\boldsymbol{H}[(\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}})^{-1} - (\boldsymbol{H}'\boldsymbol{H})^{-1}]'\bar{\boldsymbol{X}}'\boldsymbol{\varepsilon}_{j.}}{T} \right\|$$

$$\leq \left\| \frac{\boldsymbol{X}'_{i.}\boldsymbol{H}}{T}(\frac{\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}}}{T})^{-1} \right\| \left\| -\frac{\bar{\boldsymbol{V}}'\bar{\boldsymbol{V}}}{T} - \frac{\boldsymbol{H}'\bar{\boldsymbol{V}}}{T} - \frac{\bar{\boldsymbol{V}}'\boldsymbol{H}}{T} \right\| \left\| (\frac{\boldsymbol{H}'\boldsymbol{H}}{T})^{-1} \right\| \left\| \frac{\bar{\boldsymbol{X}}'\boldsymbol{\varepsilon}_{j.}}{T} \right\| = O_p(\frac{1}{N\sqrt{T}}) + O_p(\frac{1}{\sqrt{NT}})$$

by Lemma 2(a) and Lemma 3(a)(b)(c)(g)(h), and

$$\tilde{d}_3 \equiv \left\| \frac{\boldsymbol{X}'_{i.}\boldsymbol{H}(\boldsymbol{H}'\boldsymbol{H})^{-1}(\bar{\boldsymbol{X}}'\boldsymbol{\varepsilon}_{j.} - \boldsymbol{H}'\boldsymbol{\varepsilon}_{j.})}{T} \right\| \leq \left\| \frac{\boldsymbol{X}'_{i.}\boldsymbol{H}}{T}(\frac{\boldsymbol{H}'\boldsymbol{H}}{T})^{-1} \right\| \left\| \frac{\bar{\boldsymbol{V}}'\boldsymbol{\varepsilon}_{j.}}{T} \right\| = O_p(\frac{1}{\sqrt{NT}})$$

by Lemma 2(d) and Lemma 3(a)(g).

As a result, we have

$$\| \frac{\boldsymbol{X}'_{i.}\boldsymbol{M}_{\bar{x}}\boldsymbol{\varepsilon}_{j.}}{T} - \frac{\boldsymbol{X}'_{i.}\boldsymbol{M}_{f_x}\boldsymbol{\varepsilon}_{j.}}{T} \| = O_p(\frac{1}{\sqrt{NT}}).$$

(d) Similarly, we have

$$\left\| \frac{\boldsymbol{X}'_{i.}\boldsymbol{M}_{\bar{x}}\boldsymbol{F}_2}{T} - \frac{\boldsymbol{X}'_{i.}\boldsymbol{M}_{\boldsymbol{H}}\boldsymbol{F}_2}{T} \right\| = \left\| \frac{\boldsymbol{X}'_{i.}\bar{\boldsymbol{X}}(\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}})^{-1}\bar{\boldsymbol{X}}'\boldsymbol{F}_2}{T} - \frac{\boldsymbol{X}'_{i.}\boldsymbol{H}(\boldsymbol{H}'\boldsymbol{H})^{-1}\boldsymbol{H}'\boldsymbol{F}_2}{T} \right\|$$

$$= \left\| \frac{\boldsymbol{X}'_{i.}\bar{\boldsymbol{X}}(\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}})^{-1}\bar{\boldsymbol{X}}'\boldsymbol{F}_2}{T} - \frac{\boldsymbol{X}'_{i.}\boldsymbol{H}(\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}})^{-1}\bar{\boldsymbol{X}}'\boldsymbol{F}_2}{T} + \frac{\boldsymbol{X}'_{i.}\boldsymbol{H}(\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}})^{-1}\bar{\boldsymbol{X}}'\boldsymbol{F}_2}{T} - \frac{\boldsymbol{X}'_{i.}\boldsymbol{H}(\boldsymbol{H}'\boldsymbol{H})^{-1}\bar{\boldsymbol{X}}'\boldsymbol{F}_2}{T} \right.$$
$$\left. + \frac{\boldsymbol{X}'_{i.}\boldsymbol{H}(\boldsymbol{H}'\boldsymbol{H})^{-1}\bar{\boldsymbol{X}}'\boldsymbol{F}_2}{T} - \frac{\boldsymbol{X}'_{i.}\boldsymbol{H}(\boldsymbol{H}'\boldsymbol{H})^{-1}\boldsymbol{H}'\boldsymbol{F}_2}{T} \right\|$$

$$\leq \breve{d}_1 + \breve{d}_2 + \breve{d}_3,$$

where

$$\breve{d}_1 \equiv \left\| \frac{(\boldsymbol{X}'_{i.}\bar{\boldsymbol{X}} - \boldsymbol{X}'_{i.}\boldsymbol{H})(\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}})^{-1}\bar{\boldsymbol{X}}'\boldsymbol{F}_2}{T} \right\| \leq \left\| \frac{\boldsymbol{X}'_{i.}\bar{\boldsymbol{V}}}{T} \right\| \left\| (\frac{\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}}}{T})^{-1} \frac{\bar{\boldsymbol{X}}'\boldsymbol{F}_2}{T} \right\| = O_p(\frac{1}{N\sqrt{T}}) + O_p(\frac{1}{\sqrt{NT}})$$

by Lemma 2(e) and Lemma 3(c)(d),

$$\breve{d}_2 \equiv \left\| \frac{\boldsymbol{X}'_{i.}\boldsymbol{H}[(\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}})^{-1} - (\boldsymbol{H}'\boldsymbol{H})^{-1}]'\bar{\boldsymbol{X}}'\boldsymbol{F}_2}{T} \right\|$$

$$\leq \left\| \frac{\boldsymbol{X}'_{i.}\boldsymbol{H}}{T}(\frac{\bar{\boldsymbol{X}}'\bar{\boldsymbol{X}}}{T})^{-1} \right\| \left\| -\frac{\bar{\boldsymbol{V}}'\bar{\boldsymbol{V}}}{T} - \frac{\boldsymbol{H}'\bar{\boldsymbol{V}}}{T} - \frac{\bar{\boldsymbol{V}}'\boldsymbol{H}}{T} \right\| \left\| (\frac{\boldsymbol{H}'\boldsymbol{H}}{T})^{-1} \frac{\bar{\boldsymbol{X}}'\boldsymbol{F}_2}{T} \right\| = O_p(\frac{1}{N\sqrt{T}}) + O_p(\frac{1}{\sqrt{NT}})$$

by Continuous Mapping Theorem, Lemma 2(a) and Lemma 3(a)(b)(c)(d)(g), and

$$\breve{d}_3 \equiv \left\| \frac{\boldsymbol{X}'_{i.}\boldsymbol{H}(\boldsymbol{H}'\boldsymbol{H})^{-1}(\bar{\boldsymbol{X}}'\boldsymbol{F}_2 - \boldsymbol{H}'\boldsymbol{F}_2)}{T} \right\| \leq \left\| \frac{\boldsymbol{X}'_{i.}\boldsymbol{H}}{T}(\frac{\boldsymbol{H}'\boldsymbol{H}}{T})^{-1} \right\| \left\| \frac{\bar{\boldsymbol{V}}'\boldsymbol{F}_2}{T} \right\| = O_p(\frac{1}{\sqrt{NT}})$$

by Lemma 2(b) and Lemma 3(a)(g).

As a result, we have

$$\| \frac{\boldsymbol{X}'_{i.}\boldsymbol{M}_{\bar{x}}\boldsymbol{F}_2}{T} - \frac{\boldsymbol{X}'_{i.}\boldsymbol{M}_{f_x}\boldsymbol{F}_2}{T} \| = \| \frac{\boldsymbol{X}'_{i.}\boldsymbol{M}_{\bar{x}}\boldsymbol{F}_2}{T} \| = O_p(\frac{1}{N}) + O_p(\frac{1}{\sqrt{NT}}).$$

■

Lemma 5. *Under Assumptions 1-4,*

(a) $\dfrac{\boldsymbol{Q}'(\boldsymbol{I}_T\otimes \boldsymbol{b}_i)\boldsymbol{M}_{\bar{x}}\boldsymbol{F}_1\boldsymbol{\gamma}_{1i}}{T}=O_p(\dfrac{1}{N})+O_p(\dfrac{1}{\sqrt{NT}}),$

(b) $\dfrac{\boldsymbol{Q}'(\boldsymbol{I}_T\otimes \boldsymbol{b}_i)\boldsymbol{M}_{\bar{x}}\boldsymbol{\varepsilon}_{i.}}{T}=\dfrac{\boldsymbol{Q}'(\boldsymbol{I}_T\otimes \boldsymbol{b}_i)\boldsymbol{M}_{\boldsymbol{f}_x}\boldsymbol{\varepsilon}_{i.}}{T}+O_p(\dfrac{1}{\sqrt{NT}})=O_p(\dfrac{1}{\sqrt{T}}),$

$\dfrac{\boldsymbol{Q}'(\boldsymbol{I}_T\otimes \boldsymbol{b}_i)\boldsymbol{M}_{\bar{x}}\boldsymbol{F}_2\boldsymbol{\gamma}_{2i}}{T}=\dfrac{\boldsymbol{Q}'(\boldsymbol{I}_T\otimes \boldsymbol{b}_i)\boldsymbol{M}_{\boldsymbol{f}_x}\boldsymbol{F}_2\boldsymbol{\gamma}_{2i}}{T}+O_p(\dfrac{1}{N})+O_p(\dfrac{1}{\sqrt{NT}})=O_p(\dfrac{1}{\sqrt{T}})+O_p(\dfrac{1}{N})$

(c) $\dfrac{\boldsymbol{Q}'(\boldsymbol{I}_T\otimes \boldsymbol{b}_i)\boldsymbol{M}_{\bar{x}}(\boldsymbol{I}_T\otimes \boldsymbol{b}_i')\boldsymbol{Q}}{T}=\dfrac{\boldsymbol{Q}'(\boldsymbol{I}_T\otimes \boldsymbol{b}_i)\boldsymbol{M}_{\boldsymbol{f}_x}(\boldsymbol{I}_T\otimes \boldsymbol{b}_i')\boldsymbol{Q}'}{T}+O_p(\dfrac{1}{N})+O_p(\dfrac{1}{\sqrt{NT}}),$

Proof. (a) We first take a column from $\boldsymbol{Q}$ and express it generically as

$$\boldsymbol{Q}_c=[(\boldsymbol{W}^r\boldsymbol{x}_{.1,p})',\ (\boldsymbol{W}^r\boldsymbol{x}_{.2,p})',\ \ldots,\ (\boldsymbol{W}^r\boldsymbol{x}_{.1,p})']',$$

where $r=0,1,2,\ldots,p=1,2,\ldots,k$, $\boldsymbol{x}_{.t,p}=(x_{1t,p},\ x_{2t,p},\ .\ ,\ x_{Nt,p})'$, and $\boldsymbol{W}^0\equiv\boldsymbol{I}_N$. Then we have

$$\begin{aligned}
&\frac{\boldsymbol{Q}_c'(\boldsymbol{I}_T\otimes \boldsymbol{b}_i)\boldsymbol{M}_{\bar{x}}\boldsymbol{F}_1\boldsymbol{\gamma}_{1i}}{T}=\frac{(vec(\boldsymbol{W}^r(\boldsymbol{x}_{.1,p},\boldsymbol{x}_{.2,p},...,\boldsymbol{x}_{.T,p})))'(\boldsymbol{I}_T\otimes \boldsymbol{b}_i)\boldsymbol{M}_{\bar{x}}\boldsymbol{F}_1\boldsymbol{\gamma}_{1i}}{T}\\
&=\frac{(vec(\boldsymbol{b}_i'\boldsymbol{W}^r(\boldsymbol{x}_{.1,p},\boldsymbol{x}_{.2,p},...,\boldsymbol{x}_{.T,p})))'\boldsymbol{M}_{\bar{x}}\boldsymbol{F}_1\boldsymbol{\gamma}_{1i}}{T}=\frac{\boldsymbol{b}_i'\boldsymbol{W}^r(\boldsymbol{x}_{.1,p},\boldsymbol{x}_{.2,p},...,\boldsymbol{x}_{.T,p})\boldsymbol{M}_{\bar{x}}\boldsymbol{F}_1\boldsymbol{\gamma}_{1i}}{T}\\
&=\frac{\boldsymbol{w}_i^r(\boldsymbol{x}_{.1,p},\boldsymbol{x}_{.2,p},...,\boldsymbol{x}_{.T,p})\boldsymbol{M}_{\bar{x}}\boldsymbol{F}_1\boldsymbol{\gamma}_{1i}}{T}=\frac{\boldsymbol{w}_i^r(\boldsymbol{x}_{1.,p},\boldsymbol{x}_{2.,p},...,\boldsymbol{x}_{N.,p})'\boldsymbol{M}_{\bar{x}}\boldsymbol{F}_1\boldsymbol{\gamma}_{1i}}{T}\\
&=\sum_{j=1}^N w_{ij}^r\frac{\boldsymbol{x}_{j.,p}'\boldsymbol{M}_{\bar{x}}\boldsymbol{F}_1}{T}\boldsymbol{\gamma}_{1i}\\
&=\sum_{j=1}^N w_{ij}^r\frac{\boldsymbol{x}_{j.,p}'\boldsymbol{M}_{\boldsymbol{f}_x}\boldsymbol{F}_1}{T}\boldsymbol{\gamma}_{1i}+O_p(\frac{1}{N})+O_p(\frac{1}{\sqrt{NT}})\\
&=O_p(\frac{1}{N})+O_p(\frac{1}{\sqrt{NT}}),
\end{aligned}$$

where $\boldsymbol{w}_i^r$ is the i-th row of $\boldsymbol{W}^r$. The second last equality follows from Lemma 4(a) and the fact that $\boldsymbol{W}^r$ has bounded row and column sums and that $\boldsymbol{\gamma}_{1i}$ is bounded in probability. The last equality follows from the fact that $\boldsymbol{M}_{\boldsymbol{f}_x}\boldsymbol{F}_1=\boldsymbol{0}$.

(b) Taking a column from $\boldsymbol{Q}$, and as in the proof of (a) we can show that

$$\begin{aligned}
&\frac{\boldsymbol{Q}_c'(\boldsymbol{I}_T\otimes \boldsymbol{b}_i)\boldsymbol{M}_{\bar{x}}\boldsymbol{\varepsilon}_{i.}}{T}=\frac{(vec(\boldsymbol{W}^r(\boldsymbol{x}_{.1,p},\boldsymbol{x}_{.2,p},...,\boldsymbol{x}_{.T,p})))'(\boldsymbol{I}_T\otimes \boldsymbol{b}_i)\boldsymbol{M}_{\bar{x}}\boldsymbol{\varepsilon}_{i.}}{T}\\
&=\frac{(vec(\boldsymbol{b}_i'\boldsymbol{W}^r(\boldsymbol{x}_{.1,p},\boldsymbol{x}_{.2,p},...,\boldsymbol{x}_{.T,p})))'\boldsymbol{M}_{\bar{x}}\boldsymbol{\varepsilon}_{i.}}{T}=\frac{\boldsymbol{b}_i'\boldsymbol{W}^r(\boldsymbol{x}_{.1,p},\boldsymbol{x}_{.2,p},...,\boldsymbol{x}_{.T,p})\boldsymbol{M}_{\bar{x}}\boldsymbol{\varepsilon}_{i.}}{T}\\
&=\frac{\boldsymbol{w}_i^r(\boldsymbol{x}_{.1,p},\boldsymbol{x}_{.2,p},...,\boldsymbol{x}_{.T,p})\boldsymbol{M}_{\bar{x}}\boldsymbol{\varepsilon}_{i.}}{T}=\frac{\boldsymbol{w}_i^r(\boldsymbol{x}_{1.,p},\boldsymbol{x}_{2.,p},...,\boldsymbol{x}_{N.,p})'\boldsymbol{M}_{\bar{x}}\boldsymbol{\varepsilon}_{i.}}{T}\\
&=\sum_{j=1}^N w_{ij}^r\frac{\boldsymbol{x}_{j.,p}'\boldsymbol{M}_{\bar{x}}\boldsymbol{\varepsilon}_{i.}}{T}=\sum_{j=1}^N w_{ij}^r\frac{\boldsymbol{x}_{j.,p}'\boldsymbol{M}_{\boldsymbol{f}_x}\boldsymbol{\varepsilon}_{i.}}{T}+O_p(\frac{1}{\sqrt{NT}})\\
&=\frac{\boldsymbol{Q}'(\boldsymbol{I}_T\otimes \boldsymbol{b}_i)\boldsymbol{M}_{\boldsymbol{f}_x}\boldsymbol{\varepsilon}_{i.}}{T}+O_p(\frac{1}{\sqrt{NT}}),
\end{aligned}$$

where the second last equality follows from Lemma 4(c) and the assumption that $\boldsymbol{W}$ has bounded row and column norms.

According to (26), we further have

$$\begin{aligned}
&\sum_{j=1}^{N} w_{ij}^{r} \frac{\boldsymbol{x}_{j.,p}' \boldsymbol{M}_{\boldsymbol{f}_x} \boldsymbol{\varepsilon}_{i.}}{T} = \sum_{j=1}^{N} w_{ij}^{r} \frac{\boldsymbol{v}_{j.,p}' \boldsymbol{M}_{\boldsymbol{f}_x} \boldsymbol{\varepsilon}_{i.}}{T} \\
&= \sum_{j=1}^{N} w_{ij}^{r} \frac{\boldsymbol{v}_{j.,p}' \boldsymbol{\varepsilon}_{i.}}{T} - \sum_{j=1}^{N} w_{ij}^{r} \frac{\boldsymbol{v}_{j.,p}' \boldsymbol{F}_x (\boldsymbol{F}_x' \boldsymbol{F}_x)^{-1} \boldsymbol{F}_x' \boldsymbol{\varepsilon}_{i.}}{T} \\
&= \sum_{j=1}^{N} w_{ij}^{r} \frac{\boldsymbol{v}_{j.,p}' \boldsymbol{\varepsilon}_{i.}}{T} + O_p(\frac{1}{T}),
\end{aligned}$$

where the last equality follows from Lemma 2(c) and $\boldsymbol{W}$ has bounded row and column norms.

It is easy to see that

$$E(\sum_{j=1}^{N} w_{ij}^{r} \frac{\boldsymbol{v}_{j.,p}' \boldsymbol{\varepsilon}_{i.}}{T}) = 0,$$

by Assumption 2,

$$\begin{aligned}
var(\sum_{j=1}^{N} w_{ij}^{r} \frac{\boldsymbol{v}_{j.,p}' \boldsymbol{\varepsilon}_{i.}}{T}) &= \frac{1}{T^2} \sum_{j=1}^{N} \sum_{l=1}^{N} E(w_{ij}^{r} w_{il}^{r} \boldsymbol{v}_{j.,p}' \boldsymbol{\varepsilon}_{i.} \boldsymbol{\varepsilon}_{i.}' \boldsymbol{v}_{l.,p}) \\
&= \frac{1}{T^2} \sum_{j=1}^{N} \sum_{l=1}^{N} w_{ij}^{r} w_{il}^{r} E(\boldsymbol{v}_{j.,p}' \boldsymbol{\Omega}_{\boldsymbol{\varepsilon},i} \boldsymbol{v}_{l.,p}),
\end{aligned}$$

where $\boldsymbol{\Omega}_{\varepsilon,i}$ is the variance-covariance matrix of $\boldsymbol{\varepsilon}_{i.}$. Since ε_{it} is stationary with absolutely summable autocovariances, $\boldsymbol{\Omega}_{\varepsilon,i}$ has bounded row and column norms. Then by Assumption 2, we have

$$\frac{1}{T}\left|E(\boldsymbol{v}_{j.,p}' \boldsymbol{\Omega}_{\varepsilon,i} \boldsymbol{v}_{l.,p})\right| \leq \frac{1}{T} \lambda_{\max}(\boldsymbol{\Omega}_{\varepsilon,i}) \left|E(\boldsymbol{v}_{j.,p}' \boldsymbol{v}_{l.,p})\right| \leq \frac{C}{T} \left|E(\boldsymbol{v}_{j.,p}' \boldsymbol{v}_{l.,p})\right| = \frac{C}{T} \left|\sum_{t=1}^{T} E(v_{jt,p} v_{lt,p})\right| = O(1).$$

Note that $\boldsymbol{W}$ has bounded row and column norms and so does $\boldsymbol{W}^r$. This means that $\sum_{j=1}^{N} \sum_{l=1}^{N} w_{ij}^{r} w_{il}^{r}$ is finite, and hence,

$$var(\sum_{j=1}^{N} w_{ij}^{r} \frac{\boldsymbol{v}_{j.,p}' \boldsymbol{\varepsilon}_{i.}}{T}) = O(\frac{1}{T}) \quad \text{and} \quad \sum_{j=1}^{N} w_{ij}^{r} \frac{\boldsymbol{v}_{j.,p}' \boldsymbol{\varepsilon}_{i.}}{T} = O_p(\frac{1}{\sqrt{T}}).$$

This proves the first result. By using the result in Lemma 4(a), we can similarly prove the second result.

(c) Similarly to (a) and (b), for any two different columns of $\boldsymbol{Q}$, $\boldsymbol{Q}_c$ and $\boldsymbol{Q}_s$, we have

$$\begin{aligned}
&\frac{\boldsymbol{Q}_c'(\boldsymbol{I}_T\otimes\boldsymbol{b}_i)\boldsymbol{M}_{\bar{x}}(\boldsymbol{I}_T\otimes\boldsymbol{b}_i')\boldsymbol{Q}_s}{T}\\
=&\frac{(vec(\boldsymbol{W}^r(\boldsymbol{x}_{.1,p},\boldsymbol{x}_{.2,p},...,\boldsymbol{x}_{.T,p})))'(\boldsymbol{I}_T\otimes\boldsymbol{b}_i)\boldsymbol{M}_{\bar{x}}(\boldsymbol{I}_T\otimes\boldsymbol{b}_i')vec(\boldsymbol{W}^r(\boldsymbol{x}_{.1,s},\boldsymbol{x}_{.2,s},...,\boldsymbol{x}_{.T,s}))}{T}\\
=&\frac{(vec(\boldsymbol{b}_i'\boldsymbol{W}^r(\boldsymbol{x}_{.1,p},\boldsymbol{x}_{.2,p},...,\boldsymbol{x}_{.T,p})))'\boldsymbol{M}_{\bar{x}}vec(\boldsymbol{b}_i'\boldsymbol{W}^r(\boldsymbol{x}_{.1,s},\boldsymbol{x}_{.2,s},...,\boldsymbol{x}_{.T,s}))}{T}\\
=&\frac{\boldsymbol{b}_i'\boldsymbol{W}^r(\boldsymbol{x}_{.1,p},\boldsymbol{x}_{.2,p},...,\boldsymbol{x}_{.T,p})\boldsymbol{M}_{\bar{x}}(\boldsymbol{x}_{.1,s},\boldsymbol{x}_{.2,s},...,\boldsymbol{x}_{.T,s})'\boldsymbol{W}^{r\prime}\boldsymbol{b}_i}{T}\\
=&\frac{\boldsymbol{w}_i^r(\boldsymbol{x}_{.1,p},\boldsymbol{x}_{.2,p},...,\boldsymbol{x}_{.T,p})\boldsymbol{M}_{\bar{x}}(\boldsymbol{x}_{.1,s},\boldsymbol{x}_{.2,s},...,\boldsymbol{x}_{.T,s})'\boldsymbol{w}_i^{r\prime}}{T}\\
=&\frac{\boldsymbol{w}_i^r(\boldsymbol{x}_{1.,p},\boldsymbol{x}_{2.,p},...,\boldsymbol{x}_{N.,p})'\boldsymbol{M}_{\bar{x}}(\boldsymbol{x}_{1.,s},\boldsymbol{x}_{2.,s},...,\boldsymbol{x}_{N.,s})\boldsymbol{w}_i^{r\prime}}{T}\\
=&\sum_{j=1}^N\sum_{l=1}^N w_{ij}^r w_{il}^r\frac{\boldsymbol{x}_{j.,p}'\boldsymbol{M}_{\bar{x}}\boldsymbol{x}_{l.,s}}{T}=\sum_{j=1}^N\sum_{l=1}^N w_{ij}^r w_{il}^r\frac{\boldsymbol{x}_{j.,p}'\boldsymbol{M}_{\boldsymbol{f}_x}\boldsymbol{x}_{l.,s}}{T}+O_p(\frac{1}{N})+O_p(\frac{1}{\sqrt{NT}}).
\end{aligned}$$

Again, the last line follows from Lemma 4(b) and the fact that $\boldsymbol{W}^r$ has bounded row and column sums. ■

Lemma 6. *Under Assumptions 1-4,*

(a) $$\frac{1}{N}\sum_{i=1}^N\left(\frac{\boldsymbol{Z}_i'\boldsymbol{\Pi}_i\boldsymbol{Z}_i}{T}\right)^{-1}\frac{\boldsymbol{Z}_i'\boldsymbol{\Pi}_i\boldsymbol{F}_1\gamma_{1i}}{T}=O_p(\frac{1}{\sqrt{NT}})+O_p(\frac{1}{N}),$$

(b) $$\frac{1}{N}\sum_{i=1}^N\left(\frac{\boldsymbol{Z}_i'\boldsymbol{\Pi}_i\boldsymbol{Z}_i}{T}\right)^{-1}\frac{\boldsymbol{Z}_i'\boldsymbol{\Pi}_i\boldsymbol{\varepsilon}_{i.}}{T}=O_p(\frac{1}{\sqrt{NT}}),$$

$$\frac{1}{N}\sum_{i=1}^N\left(\frac{\boldsymbol{Z}_i'\boldsymbol{\Pi}_i\boldsymbol{Z}_i}{T}\right)^{-1}\frac{\boldsymbol{Z}_i'\boldsymbol{\Pi}_i\boldsymbol{F}_2\gamma_{2i}}{T}=O_p(\frac{1}{\sqrt{NT}})+O_p(\frac{1}{N}).$$

Proof. (a) This result can be readily shown from (A.1), (A.9), Assumption 6 and Lemma 5 (a).

(b) According to (A.1), (A.9) and Lemma 5(b), we have

$$\begin{aligned}
&\frac{1}{N}\sum_{i=1}^N\left(\frac{\boldsymbol{Z}_i'\boldsymbol{\Pi}_i\boldsymbol{Z}_i}{T}\right)^{-1}\frac{\boldsymbol{Z}_i'\boldsymbol{\Pi}_i\boldsymbol{\varepsilon}_{i.}}{T}\\
=&\frac{1}{N}\sum_{i=1}^N\left\{\left(\frac{\boldsymbol{Z}_{i0}'\boldsymbol{\Pi}_{i0}\boldsymbol{Z}_{i0}}{T}\right)^{-1}\frac{\boldsymbol{Z}_{i0}'\tilde{\boldsymbol{Q}}_{i0}}{T}(\frac{\tilde{\boldsymbol{Q}}_{i0}'\tilde{\boldsymbol{Q}}_{i0}'}{T})^{-1}+O_p(\frac{1}{N})+O_p(\frac{1}{\sqrt{T}})\right\}\frac{\boldsymbol{Q}'(\boldsymbol{I}_T\otimes\boldsymbol{b}_i)\boldsymbol{M}_{\boldsymbol{f}_x}\boldsymbol{\varepsilon}_{i.}}{T}+O_p(\frac{1}{\sqrt{NT}}),
\end{aligned}$$

which is dominated by

$$\begin{aligned}
&\frac{1}{N}\sum_{i=1}^N\left(\frac{\boldsymbol{Z}_{i0}'\boldsymbol{\Pi}_{i0}\boldsymbol{Z}_{i0}}{T}\right)^{-1}\frac{\boldsymbol{Z}_{i0}'\tilde{\boldsymbol{Q}}_{i0}}{T}(\frac{\tilde{\boldsymbol{Q}}_{i0}'\tilde{\boldsymbol{Q}}_{i0}'}{T})^{-1}\frac{\boldsymbol{Q}'(\boldsymbol{I}_T\otimes\boldsymbol{b}_i)\boldsymbol{M}_{\boldsymbol{f}_x}\boldsymbol{\varepsilon}_{i.}}{T}\\
=&\frac{1}{N}\sum_{i=1}^N\left[\boldsymbol{\Psi}_i'\boldsymbol{\Phi}_i^{-1}\boldsymbol{\Psi}_i\right]^{-1}\boldsymbol{\Psi}_i'\boldsymbol{\Phi}_i^{-1}\frac{\boldsymbol{Q}'(\boldsymbol{I}_T\otimes\boldsymbol{b}_i)\boldsymbol{M}_{\boldsymbol{f}_x}\boldsymbol{\varepsilon}_{i.}}{T}(1+o_p(1))
\end{aligned}$$

by Assumption 6 and the Continuous Mapping Theorem. It is easy to see that

$$\frac{1}{N}\sum_{i=1}^{N}\left[\boldsymbol{\Psi}_i'\boldsymbol{\Phi}_i^{-1}\boldsymbol{\Psi}_i\right]^{-1}\boldsymbol{\Psi}_i'\boldsymbol{\Phi}_i^{-1}\frac{\boldsymbol{Q}'(\boldsymbol{I}_T\otimes\boldsymbol{b}_i)\boldsymbol{M}_{\boldsymbol{f}_x}\boldsymbol{\varepsilon}_{i.}}{T}$$

has zero mean and its variance is

$$\begin{aligned}
&var(\frac{1}{N}\sum_{i=1}^{N}\left[\boldsymbol{\Psi}_i'\boldsymbol{\Phi}_i^{-1}\boldsymbol{\Psi}_i\right]^{-1}\boldsymbol{\Psi}_i'\boldsymbol{\Phi}_i^{-1}\frac{\boldsymbol{Q}'(\boldsymbol{I}_T\otimes\boldsymbol{b}_i)\boldsymbol{M}_{\boldsymbol{f}_x}\boldsymbol{\varepsilon}_{i.}}{T})\\
&=\frac{1}{N^2}\sum_{i=1}^{N}\sum_{j=1}^{N}E(\left[\boldsymbol{\Psi}_i'\boldsymbol{\Phi}_i^{-1}\boldsymbol{\Psi}_i\right]^{-1}\boldsymbol{\Psi}_i'\boldsymbol{\Phi}_i^{-1}\frac{\boldsymbol{Q}'(\boldsymbol{I}_T\otimes\boldsymbol{b}_i)\boldsymbol{M}_{\boldsymbol{f}_x}\boldsymbol{\varepsilon}_{i.}}{T}\frac{\boldsymbol{\varepsilon}_{j.}'\boldsymbol{M}_{\boldsymbol{f}_x}(\boldsymbol{I}_T\otimes\boldsymbol{b}_j')\boldsymbol{Q}}{T}\boldsymbol{\Psi}_j\boldsymbol{\Phi}_j^{-1}\left[\boldsymbol{\Psi}_j'\boldsymbol{\Phi}_j^{-1}\boldsymbol{\Psi}_j\right]^{-1})\\
&=\frac{1}{N^2}\sum_{i=1}^{N}E(\left[\boldsymbol{\Psi}_i'\boldsymbol{\Phi}_i^{-1}\boldsymbol{\Psi}_i\right]^{-1}\boldsymbol{\Psi}_i'\boldsymbol{\Phi}_i^{-1}\frac{\boldsymbol{Q}'(\boldsymbol{I}_T\otimes\boldsymbol{b}_i)\boldsymbol{M}_{\boldsymbol{f}_x}\boldsymbol{\varepsilon}_{i.}}{T}\frac{\boldsymbol{\varepsilon}_{i.}'\boldsymbol{M}_{\boldsymbol{f}_x}(\boldsymbol{I}_T\otimes\boldsymbol{b}_i')\boldsymbol{Q}}{T}\boldsymbol{\Psi}_i\boldsymbol{\Phi}_i^{-1}\left[\boldsymbol{\Psi}_i'\boldsymbol{\Phi}_i^{-1}\boldsymbol{\Psi}_i\right]^{-1})\\
&\leq\frac{\lambda_{max}(\boldsymbol{\Omega}_{\varepsilon,i.})}{TN^2}\sum_{i=1}^{N}\left[\boldsymbol{\Psi}_i'\boldsymbol{\Phi}_i^{-1}\boldsymbol{\Psi}_i\right]^{-1}\boldsymbol{\Psi}_i'\boldsymbol{\Phi}_i^{-1}E(\frac{\boldsymbol{Q}'(\boldsymbol{I}_T\otimes\boldsymbol{b}_i)\boldsymbol{M}_{\boldsymbol{f}_x}(\boldsymbol{I}_T\otimes\boldsymbol{b}_i')\boldsymbol{Q}}{T})\boldsymbol{\Psi}_i\boldsymbol{\Phi}_i^{-1}\left[\boldsymbol{\Psi}_i'\boldsymbol{\Phi}_i^{-1}\boldsymbol{\Psi}_i\right]^{-1})\\
&=\frac{C}{TN^2}\sum_{i=1}^{N}\left[\boldsymbol{\Psi}_i'\boldsymbol{\Phi}_i^{-1}\boldsymbol{\Psi}_i\right]^{-1}\boldsymbol{\Psi}_i'\boldsymbol{\Phi}_i^{-1}E(\frac{\tilde{\boldsymbol{Q}}_{i0}'\tilde{\boldsymbol{Q}}_{i0}}{T})\boldsymbol{\Psi}_i\boldsymbol{\Phi}_i^{-1}\left[\boldsymbol{\Psi}_i'\boldsymbol{\Phi}_i^{-1}\boldsymbol{\Psi}_i\right]^{-1})\\
&=O_p(\frac{1}{NT}),
\end{aligned}$$

where the last step follows from Assumption 6 and the fact that $E(\frac{\tilde{\boldsymbol{Q}}_{i0}'\tilde{\boldsymbol{Q}}_{i0}}{T})$ is finite (by the finiteness of its second order moment and Jensen's Inequality). Therefore, we have the first result. The second result can be proved similarly. ■

Lemma 7. *Under Assumptions 1-4,*

(a) $\frac{1}{N}\sum_{i=1}^{N}\frac{\boldsymbol{Q}'(\boldsymbol{I}_T\otimes\boldsymbol{b}_i)\boldsymbol{M}_{\bar{x}}\boldsymbol{F}_1\boldsymbol{\gamma}_{1i}}{T}=O_p(\frac{1}{N})+O_p(\frac{1}{\sqrt{NT}})$,

(b) $\frac{1}{N}\sum_{i=1}^{N}\frac{\boldsymbol{Q}'(\boldsymbol{I}_T\otimes\boldsymbol{b}_i)\boldsymbol{M}_{\bar{x}}\boldsymbol{\varepsilon}_{i.}}{T}=\frac{1}{N}\sum_{i=1}^{N}\frac{\boldsymbol{Q}'(\boldsymbol{I}_T\otimes\boldsymbol{b}_i)\boldsymbol{M}_{\boldsymbol{f}_x}\boldsymbol{\varepsilon}_{i.}}{T}+O_p(\frac{1}{\sqrt{NT}})=O_p(\frac{1}{\sqrt{NT}})$,

$\frac{1}{N}\sum_{i=1}^{N}\frac{\boldsymbol{Q}'(\boldsymbol{I}_T\otimes\boldsymbol{b}_i)\boldsymbol{M}_{\bar{x}}\boldsymbol{F}_2\boldsymbol{\gamma}_{2i}}{T}=\frac{1}{N}\sum_{i=1}^{N}\frac{\boldsymbol{Q}'(\boldsymbol{I}_T\otimes\boldsymbol{b}_i)\boldsymbol{M}_{\boldsymbol{f}_x}\boldsymbol{F}_2\boldsymbol{\gamma}_{2i}}{T}+O_p(\frac{1}{N})+O_p(\frac{1}{\sqrt{NT}})=O_p(\frac{1}{\sqrt{NT}})+O_p(\frac{1}{N})$,

(c) $\frac{1}{N}\sum_{i=1}^{N}\frac{\boldsymbol{Q}'(\boldsymbol{I}_T\otimes\boldsymbol{b}_i)\boldsymbol{M}_{\bar{x}}(\boldsymbol{I}_T\otimes\boldsymbol{b}_i')\boldsymbol{Q}}{T}=\frac{1}{N}\sum_{i=1}^{N}\frac{\boldsymbol{Q}'(\boldsymbol{I}_T\otimes\boldsymbol{b}_i)\boldsymbol{M}_{\boldsymbol{f}_x}(\boldsymbol{I}_T\otimes\boldsymbol{b}_i')\boldsymbol{Q}'}{T}+O_p(\frac{1}{N})+O_p(\frac{1}{\sqrt{NT}})$.

Proof. (a) This result follows directly from Lemma 5(a).

(b) Using the same argument as in the proof of Lemma 5(b), we have

$$\frac{1}{N}\sum_{i=1}^{N}\frac{\boldsymbol{Q}'(\boldsymbol{I}_T\otimes\boldsymbol{b}_i)\boldsymbol{M}_{\bar{x}}\boldsymbol{\varepsilon}_{i.}}{T}=\frac{1}{N}\sum_{i=1}^{N}\frac{\boldsymbol{Q}'(\boldsymbol{I}_T\otimes\boldsymbol{b}_i)\boldsymbol{M}_{\boldsymbol{f}_x}\boldsymbol{\varepsilon}_{i.}}{T}+O_p(\frac{1}{\sqrt{NT}})$$
$$=\frac{1}{N}\sum_{i=1}^{N}\sum_{j=1}^{N}w_{ij}^r\frac{\boldsymbol{x}_{j.,p}'\boldsymbol{M}_{\boldsymbol{f}_x}\boldsymbol{\varepsilon}_{i.}}{T}+O_p(\frac{1}{\sqrt{NT}})$$
$$=\frac{1}{N}\sum_{i=1}^{N}\sum_{j=1}^{N}w_{ij}^r\frac{\boldsymbol{v}_{j.,p}'\boldsymbol{\varepsilon}_{i.}}{T}+O_p(\frac{1}{\sqrt{NT}}).$$

It is obvious that
$$E(\frac{1}{N}\sum_{i=1}^{N}\sum_{j=1}^{N}w_{ij}^r\frac{\boldsymbol{v}_{j.,p}'\boldsymbol{\varepsilon}_{i.}}{T})=0,$$
and
$$var(\frac{1}{N}\sum_{i=1}^{N}\sum_{j=1}^{N}w_{ij}^r\frac{\boldsymbol{v}_{j.,p}'\boldsymbol{\varepsilon}_{i.}}{T})=\frac{1}{N^2T^2}\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{l=1}^{N}\sum_{m=1}^{N}E(w_{ij}^r w_{lm}^r\boldsymbol{v}_{j.,p}'\boldsymbol{\varepsilon}_{i.}\boldsymbol{\varepsilon}_{l.}'\boldsymbol{v}_{m.,p})$$
$$=\frac{1}{N^2T^2}\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{m=1}^{N}w_{ij}^r w_{il}^r E(\boldsymbol{v}_{j.,p}'\boldsymbol{\Omega}_{\boldsymbol{\varepsilon},i}\boldsymbol{v}_{m.,p}),$$

where $\boldsymbol{\Omega}_{\varepsilon,i}$ is the variance-covariance matrix of $\boldsymbol{\varepsilon}_{i.}$. Since ε_{it} is stationary with absolutely summable autocovariances, $\boldsymbol{\Omega}_{\varepsilon,i}$ has bounded row and column norms. Then by Assumption 2, we have

$$\frac{1}{T}\left|E(\boldsymbol{v}_{j.,p}'\boldsymbol{\Omega}_{\varepsilon,i}\boldsymbol{v}_{l.,p})\right|\le\frac{1}{T}\lambda_{\max}(\boldsymbol{\Omega}_{\varepsilon,i})\left|E(\boldsymbol{v}_{j.,p}'\boldsymbol{v}_{l.,p})\right|\le C\frac{1}{T}\left|E(\boldsymbol{v}_{j.,p}'\boldsymbol{v}_{l.,p})\right|=C\frac{1}{T}\left|\sum_{t=1}^{T}E(v_{jt,p}v_{lt,p})\right|=O(1).$$

Since $\boldsymbol{W}$ has bounded row and column norms, so does $\boldsymbol{W}^r$. This means that $\sum_{j=1}^{N}\sum_{l=1}^{N}w_{ij}^r w_{il}^r$ is finite, and hence,

$$var(\frac{1}{N}\sum_{i=1}^{N}\sum_{j=1}^{N}w_{ij}^r\frac{\boldsymbol{v}_{j.,p}'\boldsymbol{\varepsilon}_{i.}}{NT})=O(\frac{1}{NT})\text{ and }\frac{1}{N}\sum_{i=1}^{N}\sum_{j=1}^{N}w_{ij}^r\frac{\boldsymbol{v}_{j.,p}'\boldsymbol{\varepsilon}_{i.}}{NT}=O_p(\frac{1}{\sqrt{NT}}),$$

which proves the first result. The second result can be proven similarly.

(c) This is a direct result of Lemma 5(c) ■

Lemma 8. *Under Assumptions 1-4,*

(a) $\dfrac{\boldsymbol{Q}'(\boldsymbol{I}_T\otimes\boldsymbol{b}_i)\boldsymbol{M}_{\bar{x}}(\boldsymbol{I}_T\otimes\boldsymbol{b}_i'\boldsymbol{G})(\boldsymbol{I}_T\otimes\boldsymbol{\gamma}_1)\boldsymbol{f}_1}{T}=O_p(\frac{1}{N})+O_p(\frac{1}{\sqrt{NT}}),$

(b) $\dfrac{\boldsymbol{Q}'(\boldsymbol{I}_T\otimes\boldsymbol{b}_i)\boldsymbol{M}_{\bar{x}}(\boldsymbol{I}_T\otimes\boldsymbol{b}_i'\boldsymbol{G})(\boldsymbol{I}_T\otimes\boldsymbol{\gamma}_2)\boldsymbol{f}_2}{T}=\dfrac{\boldsymbol{Q}'(\boldsymbol{I}_T\otimes\boldsymbol{b}_i)\boldsymbol{M}_{\boldsymbol{f}_x}(\boldsymbol{I}_T\otimes\boldsymbol{b}_i'\boldsymbol{G})(\boldsymbol{I}_T\otimes\boldsymbol{\gamma}_2)\boldsymbol{f}_2}{T}+O_p(\frac{1}{N})+O_p(\frac{1}{\sqrt{NT}})$
$=O_p(\frac{1}{N})+O_p(\frac{1}{\sqrt{T}}),$

(c) $\dfrac{\boldsymbol{Q}'(\boldsymbol{I}_T\otimes\boldsymbol{b}_i)\boldsymbol{M}_{\bar{x}}(\boldsymbol{I}_T\otimes\boldsymbol{b}_i'\boldsymbol{G})\boldsymbol{\varepsilon}}{T}=\dfrac{\boldsymbol{Q}'(\boldsymbol{I}_T\otimes\boldsymbol{b}_i)\boldsymbol{M}_{\boldsymbol{f}_x}(\boldsymbol{I}_T\otimes\boldsymbol{b}_i'\boldsymbol{G})\boldsymbol{\varepsilon}}{T}+O_p(\frac{1}{\sqrt{NT}})=O_p(\frac{1}{\sqrt{T}}).$

Proof. (a) Similarly to the proof of Lemma 5, we first take a column from $\boldsymbol{Q}$ and express it generically as

$$\boldsymbol{Q}_c = [(\boldsymbol{W}^r \boldsymbol{x}_{.1,p})',\ (\boldsymbol{W}^r \boldsymbol{x}_{.2,p})',\ \ldots,\ (\boldsymbol{W}^r \boldsymbol{x}_{.1,p})']',$$

where $r = 0, 1, 2, \ldots$, $p = 1, 2, \ldots, k$, $\boldsymbol{x}_{.t,p} = (x_{1t,p},\ x_{2t,p},\ .\ ,\ x_{Nt,p})'$, and $\boldsymbol{W}^0 \equiv \boldsymbol{I}_N$. Then we have

$$\begin{aligned}
&\frac{\boldsymbol{Q}_c'(\boldsymbol{I}_T \otimes \boldsymbol{b}_i)\boldsymbol{M}_{\bar{x}}(\boldsymbol{I}_T \otimes \boldsymbol{b}_i'\boldsymbol{G})(\boldsymbol{I}_T \otimes \boldsymbol{\gamma}_1)\boldsymbol{f}_1}{T} \\
=& \frac{(vec(\boldsymbol{W}^r(\boldsymbol{x}_{.1,p}, \boldsymbol{x}_{.2,p}, \ldots, \boldsymbol{x}_{.T,p})))'(\boldsymbol{I}_T \otimes \boldsymbol{b}_i)\boldsymbol{M}_{\bar{x}}(\boldsymbol{I}_T \otimes \boldsymbol{b}_i'\boldsymbol{G}\boldsymbol{\gamma}_1)vec(\boldsymbol{F}_1')}{T} \\
=& \frac{(vec(\boldsymbol{b}_i'\boldsymbol{W}^r(\boldsymbol{x}_{.1,p}, \boldsymbol{x}_{.2,p}, \ldots, \boldsymbol{x}_{.T,p})))'\boldsymbol{M}_{\bar{x}}vec(\boldsymbol{b}_i'\boldsymbol{G}\boldsymbol{\gamma}_1\boldsymbol{F}_1')}{T} \\
=& \frac{\boldsymbol{b}_i'\boldsymbol{W}^r(\boldsymbol{x}_{.1,p}, \boldsymbol{x}_{.2,p}, \ldots, \boldsymbol{x}_{.T,p})\boldsymbol{M}_{\bar{x}}\boldsymbol{F}_1\boldsymbol{\gamma}_1'\boldsymbol{G}'\boldsymbol{b}_i}{T} \\
=& \frac{\boldsymbol{w}_i^r(\boldsymbol{x}_{.1,p}, \boldsymbol{x}_{.2,p}, \ldots, \boldsymbol{x}_{.T,p})\boldsymbol{M}_{\bar{x}}\boldsymbol{F}_1\boldsymbol{\gamma}_1'\boldsymbol{G}'\boldsymbol{b}_i}{T} \\
=& \frac{\boldsymbol{w}_i^r(\boldsymbol{x}_{1.,p}, \boldsymbol{x}_{2.,p}, \ldots, \boldsymbol{x}_{N.,p})'\boldsymbol{M}_{\bar{x}}\boldsymbol{F}_1\boldsymbol{\gamma}_1'\boldsymbol{G}'\boldsymbol{b}_i}{T} \\
=& \sum_{j=1}^{N} w_{ij}^r \frac{\boldsymbol{x}_{j.,p}'\boldsymbol{M}_{\bar{x}}\boldsymbol{F}_1}{T}\boldsymbol{\gamma}_1'\boldsymbol{G}'\boldsymbol{b}_i = \sum_{j=1}^{N} w_{ij}^r \frac{\boldsymbol{x}_{j.,p}'\boldsymbol{M}_{\bar{x}}\boldsymbol{F}_1}{T}(\boldsymbol{\gamma}_1'\boldsymbol{G}')_i \\
=& \sum_{j=1}^{N} w_{ij}^r \frac{\boldsymbol{x}_{j.,p}'\boldsymbol{M}_{\boldsymbol{f}_x}\boldsymbol{F}_1}{T}(\boldsymbol{\gamma}_1'\boldsymbol{G}')_i + O_p(\frac{1}{N}) + O_p(\frac{1}{\sqrt{NT}}) \\
=& O_p(\frac{1}{N}) + O_p(\frac{1}{\sqrt{NT}}),
\end{aligned}$$

where $\boldsymbol{w}_i^r$ is the i-th row of $\boldsymbol{W}^r$ and $(\boldsymbol{\gamma}_1'\boldsymbol{G}')_i$ is the i-row of matrix $\boldsymbol{G}\boldsymbol{\gamma}_1$. The second last equality follows from Lemma 4(a) and the fact that $\boldsymbol{W}^r$ and $\boldsymbol{G}$ have bounded row and column sums and that $\boldsymbol{\gamma}_1$ is bounded in probability. The last equality follows from the fact that $\boldsymbol{M}_{\boldsymbol{f}_x}\boldsymbol{F}_1 = \boldsymbol{0}$.

(b) Taking a column from $\boldsymbol{Q}$, as in the proof of (a) we can show that

$$\begin{aligned}
&\frac{\boldsymbol{Q}_c'(\boldsymbol{I}_T \otimes \boldsymbol{b}_i)\boldsymbol{M}_{\bar{x}}(\boldsymbol{I}_T \otimes \boldsymbol{b}_i'\boldsymbol{G})(\boldsymbol{I}_T \otimes \boldsymbol{\gamma}_2)\boldsymbol{f}_2}{T} \\
=&\frac{(vec(\boldsymbol{W}^r(\boldsymbol{x}_{.1,p}, \boldsymbol{x}_{.2,p}, ..., \boldsymbol{x}_{.T,p})))'(\boldsymbol{I}_T \otimes \boldsymbol{b}_i)\boldsymbol{M}_{\bar{x}}(\boldsymbol{I}_T \otimes \boldsymbol{b}_i'\boldsymbol{G}\boldsymbol{\gamma}_2)vec(\boldsymbol{F}_2')}{T} \\
=&\frac{(vec(\boldsymbol{b}_i'\boldsymbol{W}^r(\boldsymbol{x}_{.1,p}, \boldsymbol{x}_{.2,p}, ..., \boldsymbol{x}_{.T,p})))'\boldsymbol{M}_{\bar{x}}vec(\boldsymbol{b}_i'\boldsymbol{G}\boldsymbol{\gamma}_2\boldsymbol{F}_2')}{T} \\
=&\frac{\boldsymbol{b}_i'\boldsymbol{W}^r(\boldsymbol{x}_{.1,p}, \boldsymbol{x}_{.2,p}, ..., \boldsymbol{x}_{.T,p})\boldsymbol{M}_{\bar{x}}\boldsymbol{F}_2\boldsymbol{\gamma}_2'\boldsymbol{G}'\boldsymbol{b}_i}{T} \\
=&\frac{\boldsymbol{w}_i^r(\boldsymbol{x}_{.1,p}, \boldsymbol{x}_{.2,p}, ..., \boldsymbol{x}_{.T,p})\boldsymbol{M}_{\bar{x}}\boldsymbol{F}_2\boldsymbol{\gamma}_2'\boldsymbol{G}'\boldsymbol{b}_i}{T} \\
=&\frac{\boldsymbol{w}_i^r(\boldsymbol{x}_{1.,p}, \boldsymbol{x}_{2.,p}, ..., \boldsymbol{x}_{N.,p})'\boldsymbol{M}_{\bar{x}}\boldsymbol{F}_2\boldsymbol{\gamma}_2'\boldsymbol{G}'\boldsymbol{b}_i}{T} \\
=&\sum_{j=1}^{N} w_{ij}^r \frac{\boldsymbol{x}_{j.,p}'\boldsymbol{M}_{\bar{x}}\boldsymbol{F}_2}{T}\boldsymbol{\gamma}_2'\boldsymbol{G}'\boldsymbol{b}_i \\
=&\sum_{j=1}^{N} w_{ij}^r \frac{\boldsymbol{x}_{j.,p}'\boldsymbol{M}_{\bar{x}}\boldsymbol{F}_2}{T}(\boldsymbol{\gamma}_2'\boldsymbol{G}')_i \\
=&\sum_{j=1}^{N} w_{ij}^r \frac{\boldsymbol{x}_{j.,p}'\boldsymbol{M}_{\boldsymbol{f}_x}\boldsymbol{F}_2}{T}(\boldsymbol{\gamma}_2'\boldsymbol{G}')_i + O_p(\frac{1}{N}) + O_p(\frac{1}{\sqrt{NT}}) \\
=&\frac{\boldsymbol{Q}_c'(\boldsymbol{I}_T \otimes \boldsymbol{b}_i)\boldsymbol{M}_{\boldsymbol{f}_x}(\boldsymbol{I}_T \otimes \boldsymbol{b}_i'\boldsymbol{G})(\boldsymbol{I}_T \otimes \boldsymbol{\gamma}_2)\boldsymbol{f}_2}{T} + O_p(\frac{1}{N}) + O_p(\frac{1}{\sqrt{NT}}),
\end{aligned}$$

where the second last equality follows from Lemma 4(a) and the assumption that $\boldsymbol{W}^r$ and $\boldsymbol{G}$ have bounded row and column sums norms and that $\boldsymbol{\gamma}_2$ is bounded in probability. The last equality follows from definition.

The second equality in (b) can be shown by noticing that

$$\frac{\boldsymbol{x}_{j.,p}'\boldsymbol{M}_{\boldsymbol{f}_x}\boldsymbol{F}_2}{T} = \frac{\boldsymbol{x}_{j.,p}'\boldsymbol{M}_{\boldsymbol{f}_x}\boldsymbol{F}_2}{T} = \frac{\boldsymbol{v}_{j.,p}'\boldsymbol{M}_{\boldsymbol{f}_x}\boldsymbol{F}_2}{T} = \frac{\boldsymbol{v}_{j.,p}'\boldsymbol{F}_2}{T} - \frac{\boldsymbol{v}_{j.,p}'\boldsymbol{F}_x(\boldsymbol{F}_x'\boldsymbol{F}_x)^{-1}\boldsymbol{F}_x'\boldsymbol{F}_2}{T}$$

and then using the same argument as in the proof of Lemma 5(b).

(c) This can be proven by using similar arguments as in the proofs of (a) and (b). Details are omitted here. ■

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S2 Supplementary Monte Carlo Simulations

This section provides supplementary Monte Carlo simulations results on the finite sample performance of our proposed estimators and alternative estimators. We first provide supplementary results for Experiment 4 in the main text. More specifically, additional results for the naive estimator, the OLS estimator and the estimator using $\tilde{\boldsymbol{X}}^{3*}$ as instruments will be reported in the Tables S1 – S4. We then provide Monte Carlo simulation results for the other three experiments listed in Table 1 of the main text and the results can be found in Tables S5 – S16. Finally, results for $\rho = 0.2$ (Tables S17 – S20), $\rho = 0.8$ (Tables S21 – S24), $h = 4$ (Tables S25 – S28), and $h = 6$ (Tables S29 – S32) are also provided.

S2.1 Supplementary Simulation Results for Experiment 4

This subsection provides complementary results for Experiment 4 by considering the naive estimator ignoring the unobserved factors, the OLS estimator ignoring the endogeneity issue cased by spatial lagged term and the estimator using $\tilde{\boldsymbol{X}}^{3*}$ as instruments.

The Bias and RMSE results for both the OLS estimator and the naive estimator are generally quite large and much worse than those reported in the main text. As a result, the sizes for these two estimators are severely distorted. The results for the estimator using $\tilde{\boldsymbol{X}}^{3*}$ as instruments are satisfactory and qualitatively similar to those using other set of IVs reported in Section 4.3 in the main text.

Table S1: The Finite Sample Performance of Alternative Pooled Estimators under Experiment 4 (Additional)

		Bias (× 100)									RMSE (× 100)								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
									OLS										
$\bar{\boldsymbol{x}}_t$	20	8.23	-0.93	-1.57	8.18	-1.06	-1.5	8.11	-0.83	-1.65	9.14	6.13	6.42	8.58	3.82	4.04	8.36	2.83	3.21
	50	7.02	-0.97	-2.04	7.14	-1.12	-2.04	7.16	-1.08	-2.1	7.43	3.82	4.37	7.31	2.61	3.23	7.28	1.99	2.74
	100	6.65	-0.96	-2.09	6.65	-1.09	-2.13	6.73	-1.13	-2.18	6.9	2.78	3.4	6.75	1.95	2.72	6.79	1.64	2.51
									Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments										
	20	0.07	-0.19	-0.1	0.02	-0.23	0.09	-0.05	0.01	-0.04	4.2	5.99	6.04	2.67	3.64	3.68	2	2.67	2.73
	50	-0.06	0.07	0.01	0.06	-0.08	0.04	0.06	-0.03	-0.01	2.4	3.67	3.81	1.61	2.36	2.47	1.17	1.67	1.74
	100	-0.02	0.11	0.04	0.01	-0.01	0.03	0.04	-0.04	0	1.66	2.62	2.66	1.06	1.6	1.67	0.83	1.17	1.22
									OLS										
$(\bar{y}_t, \bar{\boldsymbol{x}}_t')'$	20	2.67	-0.38	-0.67	2.66	-0.46	-0.47	2.62	-0.27	-0.59	4.64	6.03	6.08	3.62	3.62	3.68	3.21	2.7	2.77
	50	3.76	-0.49	-1.11	3.89	-0.64	-1.08	3.88	-0.6	-1.14	4.38	3.73	4.04	4.17	2.43	2.71	4.04	1.77	2.07
	100	4.17	-0.59	-1.29	4.16	-0.69	-1.33	4.18	-0.72	-1.36	4.44	2.72	2.95	4.28	1.75	2.13	4.25	1.36	1.83
									Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments										
	20	-0.27	-0.12	-0.15	-0.34	-0.16	0.1	-0.37	0.04	0	4.13	6.02	6	2.66	3.6	3.64	1.97	2.69	2.72
	50	-0.12	0.07	0.01	0.02	-0.07	0.06	0	-0.03	0	2.4	3.7	3.86	1.59	2.35	2.47	1.2	1.66	1.74
	100	-0.03	0.09	0.05	0	-0.01	0.03	0.03	-0.04	0	1.66	2.67	2.66	1.05	1.6	1.66	0.83	1.15	1.22
									OLS										
$(\bar{y}_t, \bar{y}_t^*, \bar{\boldsymbol{x}}_t')'$	20	3.18	-1.3	-2.31	3.12	-1.3	-2.12	3.11	-1.09	-2.2	4.99	6.39	6.74	3.98	3.87	4.34	3.62	2.92	3.52
	50	3.97	-0.78	-1.76	4.07	-0.98	-1.73	4.05	-0.92	-1.77	4.58	3.91	4.41	4.34	2.57	3.05	4.21	1.9	2.49
	100	4.28	-0.73	-1.62	4.26	-0.86	-1.65	4.27	-0.88	-1.69	4.55	2.85	3.19	4.38	1.84	2.37	4.35	1.45	2.09
									Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments										
	20	0.45	-1.04	-1.79	0.36	-1	-1.54	0.36	-0.78	-1.6	4.21	6.33	6.54	2.68	3.78	4.08	1.97	2.82	3.18
	50	0.11	-0.19	-0.61	0.23	-0.4	-0.57	0.21	-0.33	-0.6	2.45	3.84	4.06	1.61	2.42	2.55	1.21	1.7	1.86
	100	0.07	-0.04	-0.25	0.08	-0.17	-0.28	0.12	-0.19	-0.31	1.69	2.76	2.76	1.07	1.62	1.72	0.84	1.16	1.26
									OLS										
Infeasible	20	3.78	-0.6	-1.71	3.71	-0.96	-1.72	3.71	-0.73	-1.81	4.82	5.79	6.08	4.22	3.6	3.99	4.01	2.65	3.16
	50	3.76	-0.6	-1.8	3.88	-0.75	-1.75	3.85	-0.73	-1.77	4.22	3.61	4.15	4.1	2.42	2.99	3.98	1.75	2.43
	100	3.89	-0.66	-1.74	3.88	-0.7	-1.72	3.87	-0.75	-1.75	4.13	2.57	3.06	3.99	1.73	2.36	3.95	1.37	2.12
									Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments										
	20	0.06	0.07	-0.03	0.01	-0.23	0.05	-0.01	0	-0.03	3.17	5.74	5.76	2.15	3.46	3.6	1.59	2.56	2.6
	50	-0.06	0.09	-0.06	0.07	-0.05	0.01	0.04	-0.03	-0.01	2.07	3.55	3.73	1.41	2.3	2.39	1.08	1.6	1.66
	100	0.01	0.04	-0.01	0.03	0	0.03	0.03	-0.05	0	1.49	2.5	2.51	0.96	1.58	1.63	0.78	1.14	1.2
									OLS										
Naive	20	6.78	0.46	-3.21	6.71	0.32	-3.13	6.75	0.51	-3.22	7.52	5.88	6.83	7.09	3.72	4.92	6.97	2.8	4.27
	50	6.94	0.66	-3.19	7.1	0.48	-3.23	7.18	0.53	-3.31	7.35	3.71	4.98	7.27	2.4	4.12	7.3	1.82	3.8
	100	7.16	0.6	-3.27	7.27	0.52	-3.22	7.29	0.49	-3.29	7.43	2.65	4.27	7.38	1.81	3.7	7.36	1.33	3.55
									Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments										
	20	1.3	1.96	-0.76	1.15	1.95	-0.49	1.19	2.12	-0.57	3.81	6.21	5.96	2.68	4.23	3.77	2.13	3.51	2.82
	50	1.2	2.28	-0.6	1.31	2.1	-0.58	1.38	2.16	-0.64	2.78	4.38	3.82	2.12	3.2	2.59	1.87	2.82	1.92
	100	1.27	2.23	-0.67	1.39	2.16	-0.57	1.37	2.14	-0.61	2.29	3.53	2.78	1.86	2.8	1.87	1.68	2.5	1.42
									Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments										
	20	1.94	1.74	-1.04	1.76	1.75	-0.78	1.71	1.95	-0.82	4.73	6.15	6.04	3.34	4.11	3.84	2.72	3.4	2.9
	50	1.79	2.07	-0.89	1.95	1.91	-0.88	2.03	1.97	-0.94	3.62	4.26	3.91	2.8	3.05	2.67	2.57	2.66	2.06
	100	1.97	2.01	-0.98	2.1	1.95	-0.9	2.02	1.95	-0.91	3.14	3.33	2.88	2.63	2.63	2	2.36	2.33	1.59
									Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments										
	20	1.72	1.82	-0.96	1.61	1.8	-0.71	1.59	2	-0.77	4.09	6.14	5.98	3.01	4.14	3.79	2.47	3.42	2.86
	50	1.65	2.13	-0.82	1.8	1.95	-0.81	1.88	2.02	-0.87	3.24	4.27	3.85	2.57	3.09	2.64	2.35	2.69	2.01
	100	1.79	2.07	-0.9	1.94	2	-0.82	1.89	1.99	-0.85	2.83	3.38	2.82	2.4	2.66	1.95	2.2	2.37	1.54
									Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments										
	20	1.52	1.88	-0.86	1.35	1.88	-0.59	1.36	2.07	-0.65	3.89	6.18	5.97	2.78	4.18	3.78	2.25	3.47	2.84
	50	1.42	2.2	-0.71	1.53	2.04	-0.68	1.6	2.1	-0.74	2.95	4.33	3.84	2.29	3.15	2.6	2.07	2.76	1.96
	100	1.52	2.16	-0.78	1.63	2.09	-0.68	1.59	2.08	-0.71	2.5	3.45	2.8	2.08	2.74	1.9	1.89	2.44	1.47

Table S2: The Finite Sample Performance of Alternative Mean Group Estimators under Experiment 4 (Additional)

		Bias (× 100)									RMSE (× 100)								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
								OLS											
	20	8.65	-1.15	-1.81	8.43	-1.07	-1.53	8.36	-0.85	-1.7	9.58	6.58	6.87	8.84	3.94	4.17	8.61	2.85	3.24
	50	7.19	-1.01	-2.11	7.19	-1.1	-2.03	7.16	-1.09	-2.11	7.64	4.19	4.74	7.36	2.7	3.26	7.27	1.99	2.75
	100	6.69	-0.93	-2.15	6.63	-1.1	-2.15	6.69	-1.13	-2.18	6.95	2.95	3.66	6.73	2.02	2.78	6.75	1.64	2.52
$\bar{\boldsymbol{x}}_t$								Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments											
	20	0.08	-0.26	-0.19	-0.06	-0.2	0.16	-0.11	0.02	0	4.88	6.59	6.77	2.9	3.86	3.83	2.1	2.7	2.76
	50	0.02	0.09	0.02	0.06	-0.05	0.08	0.03	-0.03	0	2.88	4.13	4.31	1.66	2.49	2.55	1.18	1.66	1.75
	100	0.01	0.15	0.03	0	0	0	0.02	-0.04	0	1.96	2.92	3.01	1.11	1.67	1.75	0.85	1.18	1.25
								OLS											
	20	3.03	-0.67	-0.8	2.77	-0.48	-0.45	2.7	-0.28	-0.63	5.12	6.5	6.59	3.78	3.77	3.84	3.29	2.72	2.8
	50	3.95	-0.47	-1.18	3.95	-0.62	-1.08	3.89	-0.61	-1.15	4.65	4.18	4.35	4.23	2.54	2.75	4.05	1.77	2.09
	100	4.19	-0.57	-1.36	4.16	-0.7	-1.35	4.15	-0.72	-1.35	4.53	2.88	3.23	4.29	1.82	2.19	4.23	1.36	1.84
$(\bar{y}_t, \bar{\boldsymbol{x}}_t')'$								Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments											
	20	-0.12	-0.36	-0.21	-0.35	-0.14	0.18	-0.38	0.04	0.01	4.53	6.56	6.66	2.82	3.78	3.83	2	2.71	2.74
	50	-0.03	0.15	0.05	0.03	-0.03	0.09	-0.02	-0.04	0.01	2.8	4.18	4.26	1.62	2.48	2.53	1.19	1.66	1.75
	100	-0.02	0.12	0.03	-0.01	0	0	0.02	-0.05	0.01	1.9	2.91	2.96	1.09	1.67	1.72	0.84	1.16	1.25
								OLS											
	20	3.52	-1.81	-2.9	3.22	-1.58	-2.6	3.17	-1.36	-2.73	5.58	6.96	7.61	4.13	4.09	4.74	3.69	3.05	3.91
	50	4.1	-0.93	-2.01	4.12	-1.06	-1.95	4.05	-1.04	-1.99	4.82	4.39	4.86	4.39	2.71	3.21	4.2	1.96	2.67
	100	4.29	-0.77	-1.8	4.25	-0.92	-1.78	4.24	-0.94	-1.79	4.63	3.04	3.52	4.38	1.93	2.5	4.32	1.49	2.19
$(\bar{y}_t, \bar{y}_t^*, \bar{\boldsymbol{x}}_t')'$								Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments											
	20	0.43	-1.52	-2.27	0.22	-1.22	-1.93	0.23	-1.02	-2.06	4.79	7.04	7.53	2.83	3.99	4.41	2	2.92	3.48
	50	0.12	-0.29	-0.75	0.2	-0.46	-0.74	0.15	-0.45	-0.8	2.92	4.33	4.59	1.66	2.56	2.65	1.2	1.72	1.96
	100	0.08	-0.07	-0.4	0.06	-0.21	-0.39	0.1	-0.26	-0.41	1.96	3.04	3.09	1.11	1.7	1.8	0.84	1.18	1.32
								OLS											
	20	3.97	-0.77	-1.79	3.83	-0.96	-1.71	3.77	-0.75	-1.85	5.19	6.1	6.59	4.33	3.69	4.08	4.08	2.69	3.21
	50	3.91	-0.67	-1.92	3.94	-0.73	-1.76	3.86	-0.75	-1.77	4.43	3.87	4.47	4.16	2.48	3	3.99	1.77	2.44
	100	3.92	-0.7	-1.78	3.9	-0.71	-1.75	3.87	-0.76	-1.73	4.22	2.79	3.33	4.01	1.79	2.41	3.94	1.37	2.12
Infeasible								Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments											
	20	0.08	-0.01	0.03	0.02	-0.2	0.13	-0.05	0	0	3.61	6.12	6.36	2.22	3.58	3.72	1.62	2.58	2.63
	50	-0.02	0.04	-0.1	0.05	0	0.03	0.01	-0.04	0	2.33	3.82	4.08	1.44	2.39	2.41	1.06	1.61	1.67
	100	-0.01	0.02	-0.01	0.03	0.01	-0.01	0.01	-0.06	0.01	1.74	2.76	2.85	0.98	1.63	1.68	0.78	1.15	1.22
								OLS											
	20	7.05	0.2	-3.32	6.81	0.22	-3.13	6.81	0.38	-3.19	7.86	6.09	7.28	7.2	3.81	5	7.03	2.76	4.25
	50	7.1	0.49	-3.25	7.13	0.41	-3.17	7.13	0.41	-3.26	7.52	4.01	5.21	7.3	2.46	4.08	7.25	1.78	3.74
	100	7.22	0.56	-3.23	7.23	0.41	-3.18	7.21	0.37	-3.21	7.48	2.78	4.36	7.34	1.81	3.67	7.28	1.27	3.47
								Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments											
	20	1.35	1.8	-0.65	1.13	1.87	-0.41	1.09	2	-0.47	4.15	6.47	6.53	2.68	4.3	3.88	2.11	3.42	2.81
	50	1.22	2.16	-0.59	1.24	2.01	-0.49	1.29	2.02	-0.59	2.92	4.64	4.13	2.09	3.21	2.62	1.79	2.69	1.9
	100	1.21	2.19	-0.57	1.32	2.03	-0.53	1.27	1.98	-0.54	2.32	3.62	3.02	1.8	2.72	1.9	1.59	2.35	1.41
Naive								Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	2.01	1.65	-0.93	1.7	1.69	-0.64	1.64	1.83	-0.69	5.5	6.89	6.94	3.42	4.28	3.99	2.7	3.33	2.89
	50	1.96	1.98	-0.87	1.88	1.86	-0.78	1.93	1.84	-0.87	4.46	4.7	4.49	2.77	3.11	2.69	2.48	2.56	2.04
	100	1.92	1.96	-0.91	2.02	1.85	-0.84	1.91	1.81	-0.82	3.34	3.52	3.23	2.56	2.6	2.04	2.25	2.2	1.56
								Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments											
	20	1.76	1.72	-0.86	1.59	1.74	-0.61	1.53	1.88	-0.66	4.42	6.46	6.55	3.02	4.23	3.88	2.45	3.33	2.85
	50	1.68	2.01	-0.78	1.74	1.89	-0.71	1.78	1.88	-0.81	3.36	4.53	4.17	2.53	3.11	2.65	2.26	2.58	1.98
	100	1.75	2.05	-0.8	1.87	1.88	-0.77	1.8	1.84	-0.77	2.83	3.49	3.04	2.33	2.6	1.97	2.09	2.23	1.52
								Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	2.71	1.42	-1.27	1.78	1.67	-0.71	1.52	1.87	-0.66	4.64	6.35	6.57	3	4.2	3.91	2.37	3.33	2.84
	50	2.6	1.77	-1.21	1.93	1.83	-0.81	1.76	1.89	-0.81	3.68	4.41	4.25	2.56	3.08	2.68	2.19	2.59	1.98
	100	2.67	1.8	-1.22	2.03	1.84	-0.84	1.75	1.85	-0.76	3.32	3.33	3.16	2.4	2.57	2	2.02	2.23	1.51

Table S3: The Size and Power for Alternative Pooled Estimators under Experiment 4 (Additional)

		Size									Power								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
								OLS											
	20	0.659	0.078	0.1	0.912	0.08	0.093	0.981	0.068	0.12	0.18	0.4	0.34	0.31	0.71	0.7	0.41	0.94	0.95
	50	0.883	0.061	0.098	0.999	0.089	0.152	0.42	0.77	0.74	0.73	0.98	0.97	0.82	1	1			
	100	0.983	0.062	0.141	1	0.104	0.27	1	0.148	0.44	0.69	0.97	0.97	0.97	1	1	1	1	1
$\bar{\boldsymbol{x}}_t$								Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments											
	20	0.08	0.071	0.071	0.061	0.066	0.061	0.073	0.063	0.065	0.23	0.39	0.36	0.4	0.72	0.71	0.59	0.92	0.94
	50	0.056	0.051	0.066	0.063	0.06	0.063	0.046	0.043	0.053	0.53	0.78	0.73	0.82	0.98	0.98	0.98	1	1
	100	0.051	0.053	0.046	0.038	0.043	0.046	0.053	0.048	0.052	0.82	0.97	0.96	0.99	1	1	1	1	1
								OLS											
	20	0.167	0.088	0.1	0.255	0.072	0.081	0.363	0.084	0.089	0.26	0.34	0.39	0.37	0.76	0.68	0.62	0.94	0.94
	50	0.431	0.065	0.088	0.77	0.082	0.099	0.933	0.086	0.133	0.53	0.74	0.71	0.81	0.99	0.98	0.97	1	1
	100	0.779	0.067	0.084	0.985	0.075	0.141	1	0.097	0.234	0.86	0.96	0.97	0.99	1	1	1	1	1
$(\bar{y}_t, \bar{\boldsymbol{x}}_t')'$								Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments											
	20	0.089	0.084	0.096	0.076	0.072	0.077	0.088	0.086	0.078	0.22	0.35	0.39	0.41	0.72	0.66	0.63	0.93	0.93
	50	0.056	0.068	0.077	0.066	0.067	0.074	0.066	0.057	0.068	0.5	0.74	0.72	0.84	0.98	0.98	0.98	1	1
	100	0.053	0.058	0.045	0.052	0.051	0.06	0.062	0.051	0.061	0.84	0.96	0.97	1	1	1	1	1	1
								OLS											
	20	0.21	0.097	0.102	0.314	0.089	0.114	0.449	0.099	0.15	0.29	0.35	0.36	0.39	0.76	0.63	0.56	0.94	0.95
	50	0.472	0.066	0.099	0.809	0.093	0.137	0.951	0.103	0.189	0.5	0.73	0.71	0.8	0.99	0.97	0.97	1	1
	100	0.788	0.067	0.098	0.984	0.093	0.173	1	0.117	0.297	0.86	0.95	0.96	0.99	1	1	1	1	1
$(\bar{y}_t, \bar{y}_t^*, \bar{\boldsymbol{x}}_t')'$								Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments											
	20	0.087	0.09	0.09	0.09	0.088	0.095	0.083	0.086	0.101	0.2	0.36	0.34	0.38	0.76	0.63	0.63	0.93	0.94
	50	0.061	0.07	0.076	0.075	0.076	0.076	0.065	0.064	0.084	0.48	0.73	0.7	0.83	0.99	0.97	0.98	1	1
	100	0.054	0.061	0.045	0.052	0.055	0.063	0.064	0.056	0.065	0.81	0.95	0.96	1	1	1	1	1	1
								OLS											
	20	0.299	0.087	0.102	0.522	0.075	0.115	0.699	0.08	0.12	0.33	0.36	0.46	0.5	0.81	0.72	0.69	0.96	0.96
	50	0.538	0.063	0.103	0.842	0.089	0.149	0.963	0.083	0.205	0.69	0.8	0.74	0.91	0.99	0.98	0.99	1	1
	100	0.808	0.069	0.11	0.976	0.08	0.201	0.997	0.098	0.91 0.98	0.98		1 1	1		1 1	1		
Infeasible								Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments											
	20	0.076	0.077	0.072	0.084	0.071	0.073	0.071	0.074	0.073	0.32	0.37	0.47	0.59	0.8	0.7	0.84	0.95	0.96
	50	0.052	0.057	0.07	0.071	0.061	0.069	0.056	0.045	0.056	0.68	0.81	0.72	0.93	0.99	0.99	0.99	1	1
	100	0.058	0.053	0.058	0.05	0.06	0.055	0.064	0.056	0.053	0.9	0.98	0.97	1	1	1	1	1	1
								OLS											
	20	0.667	0.078	0.138	0.89	0.075	0.185	0.981	0.081	0.255	0.25	0.37	0.37	0.34	0.71	0.74	0.48	0.92	0.95
	50	0.906	0.076	0.187	0.998	0.067	0.304	1	0.063	0.475	0.48	0.78	0.77	0.75	0.98	0.97	0.83	1	1
	100	0.995	0.068	0.269	1	0.078	0.492	1	0.072	0.753	0.62	0.97	0.95	0.92	1	1	0.99	1	1
								Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments											
	20	0.154	0.091	0.084	0.159	0.112	0.071	0.151	0.152	0.063	0.26	0.38	0.38	0.44	0.67	0.76	0.74	0.87	0.95
	50	0.185	0.115	0.067	0.226	0.155	0.08	0.286	0.238	0.082	0.53	0.77	0.77	0.79	0.98	0.97	0.96	1	1
	100	0.223	0.16	0.072	0.335	0.252	0.09	0.446	0.407	0.074	0.73	0.95	0.95	0.96	1	1	1	1	1
								Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	0.206	0.096	0.088	0.215	0.096	0.075	0.227	0.141	0.068	0.22	0.35	0.36	0.36	0.69	0.72	0.59	0.87	0.95
Naive	50	0.249	0.105	0.068	0.348	0.133	0.084	0.435	0.207	0.1	0.32	0.77	0.76	0.64	0.98	0.98	0.84	1	1
	100	0.337	0.136	0.089	0.488	0.226	0.099	0.609	0.35	0.107	0.53	0.96	0.96	0.84	1	1	0.96	1	1
								Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments											
	20	0.199	0.093	0.085	0.224	0.104	0.075	0.228	0.145	0.064	0.27	0.38	0.4	0.4	0.67	0.74	0.71	0.88	0.94
	50	0.266	0.104	0.065	0.363	0.14	0.086	0.436	0.217	0.1	0.41	0.77	0.77	0.71	0.98	0.97	0.9	1	1
	100	0.357	0.139	0.077	0.53	0.23	0.101	0.625	0.36	0.099	0.63	0.96	0.95	0.9	1	1	0.99	1	1
								Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	0.176	0.097	0.084	0.177	0.108	0.072	0.196	0.151	0.063	0.28	0.39	0.38	0.46	0.67	0.76	0.73	0.88	0.95
	50	0.225	0.113	0.067	0.286	0.146	0.084	0.358	0.23	0.091	0.48	0.76	0.77	0.77	0.98	0.97	0.94	1	1
	100	0.296	0.15	0.079	0.43	0.246	0.089	0.528	0.389	0.08	0.68	0.95	0.95	0.95	1	1	1	1	1

Table S4: The Size and Power for Alternative Mean Group Estimators under Experiment 4 (Additional)

		Size									Power								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
									OLS										
	20	0.6	0.079	0.076	0.897	0.073	0.085	0.985	0.073	0.115	0.2	0.3	0.33	0.29	0.7	0.65	0.42	0.93	0.95
	50	0.842	0.059	0.104	0.996	0.093	0.135	1	0.095	0.244	0.41	0.71	0.6	0.75	0.98	0.98	0.85	1	1
	100	0.964	0.061	0.118	1	0.093	0.256	1	0.148	0.42	0.65	0.94	0.94	0.97	1	1	1	1	1
$\bar{\boldsymbol{x}}_t$									Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments										
	20	0.057	0.057	0.061	0.063	0.064	0.058	0.067	0.051	0.055	0.19	0.29	0.33	0.37	0.67	0.67	0.54	0.92	0.95
	50	0.055	0.047	0.058	0.053	0.057	0.062	0.043	0.041	0.053	0.41	0.68	0.64	0.82	0.98	0.97	0.99	1	1
	100	0.049	0.055	0.045	0.045	0.046	0.049	0.047	0.048	0.047	0.72	0.92	0.93	0.99	1	1	1	1	1
									OLS										
	20	0.133	0.072	0.063	0.231	0.074	0.08	0.347	0.065	0.073	0.21	0.28	0.38	0.36	0.69	0.63	0.56	0.94	0.95
	50	0.405	0.061	0.07	0.758	0.08	0.087	0.935	0.073	0.117	0.49	0.72	0.65	0.83	0.98	0.97	0.97	1	1
	100	0.708	0.053	0.085	0.98	0.074	0.134	0.998	0.099	0.223	0.77	0.95	0.93	0.99	1	1	1	1	1
$(\bar{y}_t, \bar{\boldsymbol{x}}_t')'$									Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments										
	20	0.058	0.065	0.06	0.074	0.072	0.073	0.073	0.06	0.066	0.19	0.28	0.35	0.35	0.67	0.65	0.64	0.93	0.94
	50	0.056	0.055	0.062	0.058	0.069	0.067	0.055	0.052	0.067	0.43	0.72	0.65	0.83	0.98	0.97	0.98	1	1
	100	0.046	0.043	0.054	0.047	0.049	0.057	0.057	0.048	0.056	0.77	0.93	0.92	0.99	1	1	1	1	1
									OLS										
	20	0.152	0.078	0.095	0.282	0.083	0.113	0.429	0.093	0.158	0.2	0.29	0.33	0.38	0.72	0.62	0.55	0.94	0.92
	50	0.415	0.065	0.096	0.783	0.097	0.131	0.95	0.109	0.214	0.47	0.69	0.63	0.82	0.97	0.97	0.97	1	1
	100	0.713	0.059	0.103	0.98	0.096	0.19	0.999	0.119	0.315	0.77	0.92	0.91	0.99	1	1	1	1	1
$(\bar{y}_t, \bar{y}_t^*, \bar{\boldsymbol{x}}_t')'$									Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments										
	20	0.058	0.073	0.083	0.082	0.075	0.098	0.065	0.083	0.105	0.19	0.29	0.26	0.36	0.69	0.61	0.64	0.94	0.93
	50	0.063	0.058	0.068	0.068	0.075	0.063	0.052	0.061	0.082	0.4	0.67	0.61	0.8	0.98	0.96	0.98	1	1
	100	0.044	0.05	0.047	0.055	0.053	0.057	0.06	0.049	0.066	0.76	0.91	0.92	0.99	1	1	1	1	1
									OLS										
	20	0.261	0.061	0.066	0.485	0.067	0.097	0.678	0.071	0.122	0.3	0.37	0.39	0.52	0.78	0.67	0.72	0.96	0.96
	50	0.47	0.054	0.087	0.835	0.077	0.124	0.955	0.075	0.2	0.64	0.73	0.7	0.9	0.99	0.98	0.99	1	1
	100	0.746	0.05	0.109	0.977	0.078	0.192	0.996	0.102	0.319	0.86	0.97	0.95	1	1	1	1	1	1
Infeasible									Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments										
	20	0.066	0.061	0.054	0.072	0.063	0.071	0.06	0.06	0.069	0.29	0.36	0.4	0.55	0.74	0.67	0.84	0.95	0.95
	50	0.054	0.045	0.059	0.056	0.063	0.063	0.045	0.044	0.055	0.51	0.75	0.68	0.91	0.99	0.98	1	1	1
	100	0.06	0.047	0.044	0.045	0.06	0.059	0.058	0.049	0.057	0.81	0.95	0.95	1	1	1	1	1	1
									OLS										
	20	0.638	0.06	0.111	0.877	0.066	0.154	0.98	0.069	0.237	0.28	0.35	0.35	0.41	0.69	0.7	0.56	0.92	0.95
	50	0.898	0.064	0.158	0.999	0.064	0.278	1	0.064	0.469	0.47	0.7	0.64	0.74	0.98	0.97	0.85	1	1
	100	0.991	0.052	0.228	1	0.067	0.466	1	0.06	0.718	0.61	0.96	0.92	0.95	1	1	0.99	1	1
									Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments										
	20	0.124	0.069	0.061	0.13	0.086	0.06	0.132	0.127	0.059	0.22	0.34	0.33	0.48	0.64	0.73	0.71	0.9	0.95
	50	0.146	0.087	0.055	0.203	0.136	0.075	0.242	0.202	0.065	0.41	0.68	0.65	0.78	0.97	0.97	0.95	1	1
	100	0.179	0.115	0.061	0.3	0.231	0.073	0.389	0.352	0.069	0.66	0.94	0.93	0.96	1	1	1	1	1
									Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments										
	20	0.165	0.077	0.069	0.186	0.087	0.068	0.198	0.118	0.069	0.19	0.33	0.32	0.37	0.61	0.7	0.61	0.91	0.94
Naive	50	0.202	0.088	0.065	0.278	0.123	0.071	0.396	0.186	0.082	0.25	0.67	0.64	0.6	0.98	0.97	0.82	1	1
	100	0.257	0.095	0.08	0.434	0.192	0.091	0.566	0.3	0.091	0.46	0.93	0.89	0.82	1	1	0.98	1	1
									Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments										
	20	0.168	0.074	0.068	0.192	0.082	0.063	0.201	0.118	0.071	0.22	0.35	0.34	0.44	0.63	0.73	0.7	0.9	0.95
	50	0.214	0.085	0.061	0.307	0.125	0.075	0.401	0.185	0.08	0.37	0.69	0.68	0.7	0.97	0.97	0.9	1	1
	100	0.305	0.11	0.072	0.471	0.207	0.087	0.596	0.306	0.086	0.57	0.95	0.91	0.91	1	1	0.99	1	1
									Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments										
	20	0.2	0.068	0.068	0.205	0.085	0.068	0.202	0.12	0.067	0.23	0.32	0.34	0.48	0.65	0.73	0.71	0.9	0.95
	50	0.273	0.079	0.07	0.331	0.123	0.077	0.39	0.185	0.081	0.41	0.69	0.66	0.78	0.97	0.97	0.93	1	1
	100	0.438	0.094	0.085	0.528	0.198	0.091	0.598	0.313	0.086	0.67	0.95	0.93	0.94	1	1	1	1	1

S2.2 Supplementary Results for Experiments 1 – 3

This subsection provides complete results for Experiments 1 – 3. The specifications of the data generating processes can be found in Section 4.1 of the main text. All the results here are satisfactory and qualitatively similar to those for Experiment 4.

Table S5: The Finite Sample Performance of Alternative Pooled Estimators under Experiment 1

		Bias (× 100)									RMSE (× 100)								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
		OLS																	
	20	7.67	-1.11	-1.55	7.6	-0.7	-1.55	7.56	-0.78	-1.37	8.47	5.8	6.18	7.93	3.67	3.93	7.74	2.69	3.02
	50	6.13	-0.81	-1.87	6.19	-0.88	-1.94	6.14	-0.93	-1.83	6.5	3.82	4	6.33	2.43	2.94	6.23	1.86	2.46
	100	5.41	-0.93	-1.75	5.45	-0.94	-1.77	5.43	-0.9	-1.74	5.61	2.68	3.07	5.53	1.8	2.43	5.47	1.39	2.09
		Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																	
	20	-0.09	-0.41	-0.13	0.06	0.03	-0.07	0.02	-0.04	0.02	3.97	5.68	5.93	2.47	3.57	3.56	1.64	2.54	2.64
	50	0.01	0.08	-0.1	-0.02	0.03	-0.11	-0.01	-0.03	-0.01	2.25	3.71	3.47	1.36	2.26	2.19	1.01	1.6	1.61
	100	-0.03	-0.06	-0.02	0.02	-0.06	0	0.01	-0.01	0.03	1.63	2.53	2.51	0.98	1.54	1.66	0.7	1.06	1.13
		Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	-0.06	-0.43	-0.11	-0.04	0.04	-0.05	-0.07	-0.03	0.02	5.14	5.7	5.94	3.33	3.57	3.6	2.24	2.56	2.65
$\bar{\boldsymbol{x}}_t$	50	-0.04	0.09	-0.09	0.02	0.02	-0.12	0.01	-0.03	-0.01	2.92	3.73	3.53	1.72	2.26	2.2	1.24	1.59	1.61
	100	-0.01	-0.07	-0.02	0	-0.05	0	0.04	-0.02	0.02	1.94	2.53	2.56	1.21	1.55	1.67	0.84	1.06	1.15
		Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																	
	20	-0.13	-0.41	-0.12	-0.01	0.03	-0.06	-0.01	-0.04	0.01	3.96	5.67	5.93	2.5	3.57	3.57	1.69	2.55	2.64
	50	0.03	0.08	-0.11	0.03	0.02	-0.12	0.01	-0.03	-0.01	2.28	3.71	3.47	1.37	2.25	2.19	1	1.59	1.61
	100	-0.03	-0.07	-0.02	0.02	-0.06	0	0.01	-0.01	0.03	1.64	2.52	2.52	1	1.54	1.66	0.7	1.05	1.13
		Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	-0.05	-0.42	-0.14	0.05	0.03	-0.07	0.01	-0.04	0.02	3.86	5.67	5.92	2.41	3.57	3.56	1.6	2.55	2.64
	50	0.01	0.08	-0.1	-0.01	0.02	-0.11	0	-0.03	-0.01	2.2	3.71	3.47	1.32	2.25	2.19	0.98	1.59	1.61
	100	-0.02	-0.07	-0.02	0.02	-0.06	0	0.02	-0.02	0.03	1.57	2.52	2.52	0.97	1.54	1.66	0.68	1.06	1.13
		OLS																	
	20	2.54	-0.65	-0.61	2.66	-0.22	-0.61	2.61	-0.31	-0.45	4.4	5.85	5.98	3.43	3.64	3.63	3.03	2.58	2.7
	50	3.72	-0.44	-1.16	3.81	-0.54	-1.24	3.78	-0.6	-1.12	4.3	3.8	3.76	4.01	2.33	2.53	3.89	1.7	1.97
	100	4.12	-0.74	-1.34	4.11	-0.74	-1.31	4.09	-0.68	-1.32	4.4	2.65	2.94	4.21	1.71	2.11	4.14	1.26	1.74
		Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																	
	20	-0.29	-0.4	-0.13	-0.17	0.04	-0.07	-0.19	-0.04	0.03	4.09	5.82	5.96	2.43	3.64	3.56	1.68	2.56	2.66
	50	-0.06	0.11	-0.07	-0.04	0.03	-0.11	-0.04	-0.03	0.01	2.33	3.77	3.55	1.36	2.27	2.21	1.01	1.6	1.62
	100	-0.04	-0.08	-0.02	0.02	-0.08	0.02	0.01	-0.02	0.02	1.67	2.57	2.6	0.98	1.55	1.67	0.69	1.05	1.13
		Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
$(\bar{y}_t, \bar{\boldsymbol{x}}_t')'$	20	-0.45	-0.41	-0.09	-0.45	0.07	0	-0.48	-0.01	0.02	5.24	5.85	5.95	3.3	3.64	3.61	2.28	2.58	2.66
	50	-0.1	0.11	-0.06	-0.03	0.02	-0.11	-0.04	-0.03	0.01	2.98	3.79	3.61	1.72	2.26	2.21	1.25	1.6	1.61
	100	-0.03	-0.08	-0.02	-0.01	-0.07	0.03	0.03	-0.02	0.01	1.99	2.57	2.65	1.23	1.56	1.68	0.83	1.05	1.15
		Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																	
	20	-0.43	-0.4	-0.1	-0.34	0.06	-0.03	-0.35	-0.02	0.03	4.06	5.82	5.95	2.45	3.64	3.57	1.73	2.57	2.65
	50	-0.03	0.1	-0.08	-0.01	0.02	-0.12	-0.04	-0.03	0.01	2.37	3.77	3.55	1.36	2.26	2.2	1	1.6	1.61
	100	-0.04	-0.08	-0.02	0.01	-0.08	0.03	0	-0.02	0.02	1.68	2.57	2.61	1	1.56	1.67	0.7	1.05	1.13
		Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	-0.29	-0.4	-0.13	-0.21	0.05	-0.06	-0.25	-0.03	0.01	3.95	5.82	5.95	2.37	3.63	3.56	1.64	2.56	2.65
	50	-0.05	0.11	-0.07	-0.04	0.03	-0.11	-0.04	-0.03	0.01	2.28	3.77	3.55	1.32	2.26	2.2	0.98	1.6	1.61
	100	-0.03	-0.08	-0.02	0.02	-0.08	0.02	0.01	-0.02	0.02	1.62	2.57	2.6	0.97	1.55	1.67	0.67	1.05	1.13
		OLS																	
	20	3	-1.51	-2.32	3.12	-1.03	-2.23	3.09	-1.11	-2.05	4.73	6.18	6.73	3.81	3.81	4.32	3.47	2.81	3.42
	50	3.9	-0.79	-1.8	3.98	-0.87	-1.89	3.95	-0.91	-1.74	4.47	3.85	4.1	4.18	2.45	2.94	4.06	1.84	2.38
	100	4.21	-0.89	-1.66	4.2	-0.9	-1.63	4.18	-0.84	-1.63	4.49	2.79	3.14	4.3	1.79	2.33	4.23	1.35	1.99
		Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																	
	20	0.11	-1.22	-1.77	0.3	-0.74	-1.63	0.29	-0.81	-1.44	4.26	6.1	6.57	2.48	3.75	4.02	1.71	2.7	3.09
	50	0.06	-0.21	-0.66	0.1	-0.28	-0.7	0.11	-0.33	-0.57	2.37	3.78	3.71	1.39	2.32	2.36	1.03	1.64	1.72
	100	0.02	-0.21	-0.3	0.09	-0.22	-0.27	0.08	-0.16	-0.26	1.71	2.68	2.66	1	1.57	1.7	0.7	1.07	1.17
		Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	0.2	-1.24	-1.79	0.35	-0.74	-1.64	0.32	-0.81	-1.45	4.12	6.1	6.56	2.42	3.74	4.03	1.67	2.7	3.09
$(\bar{y}_t, \bar{y}_t^{*}, \bar{\boldsymbol{x}}_t')'$	50	0.1	-0.22	-0.67	0.12	-0.28	-0.71	0.13	-0.33	-0.57	2.33	3.78	3.72	1.35	2.31	2.35	1	1.63	1.72
	100	0.04	-0.21	-0.3	0.09	-0.22	-0.27	0.09	-0.16	-0.27	1.66	2.67	2.67	0.99	1.57	1.7	0.68	1.07	1.17

Table S5 (Continued)

		Bias (× 100)									RMSE (× 100)								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																			
	20	0.26	-1.25	-1.8	0.37	-0.75	-1.64	0.37	-0.82	-1.46	4.19	6.11	6.57	2.48	3.74	4.03	1.74	2.71	3.09
	50	0.16	-0.23	-0.69	0.18	-0.29	-0.73	0.17	-0.33	-0.58	2.42	3.78	3.72	1.4	2.31	2.37	1.02	1.64	1.72
	100	0.04	-0.21	-0.3	0.09	-0.22	-0.27	0.1	-0.16	-0.27	1.72	2.67	2.68	1.03	1.58	1.7	0.71	1.07	1.17
Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																			
	20	0.15	-1.23	-1.79	0.31	-0.74	-1.63	0.29	-0.82	-1.44	4.37	6.19	6.56	2.52	3.77	4.03	1.73	2.72	3.09
	50	0.08	-0.23	-0.66	0.11	-0.27	-0.72	0.12	-0.32	-0.58	2.37	3.82	3.72	1.37	2.32	2.36	1.01	1.64	1.72
	100	0.03	-0.21	-0.3	0.09	-0.22	-0.27	0.09	-0.16	-0.27	1.68	2.68	2.67	0.99	1.57	1.7	0.69	1.08	1.17
OLS																			
	20	4.11	-1.38	-1.55	4.1	-0.73	-1.68	4.1	-0.82	-1.54	5.01	5.76	6.01	4.48	3.55	3.88	4.29	2.56	3.01
	50	4.19	-0.68	-1.57	4.29	-0.76	-1.7	4.25	-0.83	-1.58	4.64	3.66	3.83	4.45	2.35	2.76	4.33	1.77	2.24
	100	4.32	-0.77	-1.58	4.33	-0.84	-1.56	4.32	-0.81	-1.54	4.55	2.57	3.01	4.42	1.74	2.27	4.36	1.31	1.91
Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																			
	20	-0.07	-0.61	0	-0.01	0.06	-0.09	0.02	-0.03	0.04	3.3	5.62	5.8	2.03	3.49	3.48	1.39	2.41	2.59
	50	-0.06	0.09	-0.04	0	0.03	-0.11	-0.01	-0.04	-0.01	2.16	3.59	3.47	1.27	2.23	2.17	0.92	1.56	1.58
	100	-0.04	0.02	-0.03	0.03	-0.07	0	0.02	-0.03	0.02	1.57	2.48	2.54	0.95	1.53	1.66	0.66	1.03	1.12
Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																			
	20	0.01	-0.62	-0.04	-0.06	0.07	-0.08	-0.06	-0.01	0.07	3.92	5.66	5.85	2.47	3.49	3.53	1.67	2.43	2.59
Infeasible	50	-0.07	0.09	-0.04	0.03	0.03	-0.12	-0.02	-0.04	0	2.64	3.6	3.53	1.55	2.23	2.18	1.12	1.55	1.58
	100	-0.02	0.01	-0.03	0.01	-0.06	0.01	0.04	-0.03	0.01	1.82	2.48	2.59	1.15	1.54	1.66	0.79	1.04	1.14
Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																			
	20	-0.05	-0.62	-0.02	-0.04	0.06	-0.08	0	-0.02	0.05	3.28	5.63	5.81	2.02	3.49	3.48	1.39	2.42	2.58
	50	-0.04	0.09	-0.05	0.04	0.03	-0.12	0	-0.04	-0.01	2.15	3.59	3.47	1.27	2.23	2.17	0.92	1.55	1.57
	100	-0.03	0.01	-0.03	0.02	-0.06	0	0.02	-0.03	0.02	1.56	2.48	2.55	0.97	1.54	1.65	0.66	1.03	1.12
Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																			
	20	-0.03	-0.62	-0.02	-0.01	0.06	-0.09	0	-0.02	0.05	3.21	5.63	5.79	1.98	3.48	3.48	1.35	2.41	2.58
	50	-0.05	0.09	-0.04	0.01	0.03	-0.12	-0.01	-0.04	-0.01	2.11	3.59	3.47	1.24	2.23	2.16	0.9	1.56	1.57
	100	-0.03	0.01	-0.03	0.03	-0.07	0	0.02	-0.03	0.02	1.51	2.48	2.55	0.93	1.53	1.65	0.64	1.03	1.12
OLS																			
	20	7.98	-0.21	-1.87	8.05	0.24	-1.83	8.12	0.13	-1.61	8.52	5.48	6.07	8.3	3.56	4.13	8.26	2.67	3.23
	50	8.38	0.14	-1.88	8.48	0.2	-1.95	8.5	0.13	-1.85	8.65	3.62	4.01	8.6	2.34	3.01	8.58	1.73	2.55
	100	8.52	0.07	-1.85	8.64	0.08	-1.84	8.68	0.14	-1.81	8.69	2.53	3.19	8.71	1.59	2.51	8.73	1.19	2.22
Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																			
	20	3.01	1.15	0.63	3.16	1.65	0.71	3.2	1.55	0.98	4.44	5.66	5.75	3.87	3.94	3.74	3.55	3.14	2.97
	50	3.31	1.54	0.64	3.34	1.64	0.66	3.39	1.56	0.77	4.02	4	3.57	3.64	2.91	2.38	3.57	2.37	1.93
	100	3.42	1.46	0.7	3.47	1.52	0.78	3.5	1.6	0.8	3.86	3	2.7	3.67	2.25	1.89	3.61	2.02	1.5
Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																			
	20	4.34	0.74	-0.06	4.38	1.28	0.07	4.47	1.17	0.3	5.78	5.56	5.75	5.11	3.78	3.71	4.83	2.96	2.79
Naive	50	4.75	1.12	-0.1	4.8	1.21	-0.09	4.84	1.15	0.02	5.48	3.82	3.55	5.11	2.66	2.28	5.02	2.09	1.74
	100	4.9	1.04	-0.05	4.99	1.09	0.01	5.04	1.16	0.02	5.32	2.79	2.61	5.18	1.96	1.7	5.16	1.67	1.26
Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																			
	20	3.92	0.87	0.13	4.06	1.38	0.22	4.15	1.26	0.47	5.2	5.57	5.72	4.69	3.82	3.67	4.46	2.99	2.81
	50	4.42	1.22	0.06	4.49	1.31	0.07	4.51	1.24	0.19	5.04	3.85	3.49	4.75	2.7	2.27	4.67	2.15	1.74
	100	4.57	1.14	0.12	4.66	1.18	0.17	4.69	1.26	0.2	4.96	2.82	2.59	4.84	2.02	1.7	4.8	1.75	1.27
Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																			
	20	3.43	1.02	0.4	3.55	1.53	0.5	3.6	1.43	0.77	4.75	5.61	5.72	4.21	3.88	3.7	3.93	3.07	2.88
	50	3.78	1.4	0.39	3.81	1.5	0.42	3.85	1.43	0.53	4.44	3.93	3.53	4.1	2.82	2.31	4.02	2.27	1.83
	100	3.9	1.33	0.46	3.97	1.38	0.53	4	1.46	0.55	4.31	2.92	2.63	4.15	2.15	1.79	4.11	1.9	1.37

Table S6: The Finite Sample Performance of Alternative Mean Group Estimators under Experiment 1

		Bias ($\times$ 100)									RMSE ($\times$ 100)								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
	OLS																		
	20	8.03	-1.08	-1.73	7.88	-0.75	-1.57	7.79	-0.8	-1.39	9.01	6.24	6.77	8.24	3.87	4.05	7.97	2.76	3.05
	50	6.29	-0.86	-2	6.25	-0.88	-1.94	6.19	-0.94	-1.84	6.72	4.16	4.44	6.41	2.49	3.01	6.27	1.87	2.5
	100	5.5	-0.98	-1.81	5.48	-0.98	-1.79	5.44	-0.88	-1.73	5.74	2.96	3.35	5.56	1.87	2.47	5.48	1.4	2.09
	Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																		
	20	-0.12	-0.24	-0.12	0.08	0.02	0.01	0	-0.02	0.05	4.93	6.33	6.65	2.73	3.84	3.73	1.78	2.64	2.69
	50	0.08	0.05	-0.17	-0.02	0.03	-0.07	-0.01	-0.02	-0.01	2.65	4.17	4.03	1.44	2.35	2.32	1.04	1.61	1.68
	100	-0.04	-0.1	-0.01	0.02	-0.09	-0.01	0.03	0	0.04	1.84	2.86	2.87	1.04	1.6	1.73	0.72	1.09	1.16
	Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																		
	20	0.56	-0.35	0.15	0	-0.03	0.04	-0.12	-0.01	0.02	9.92	7.32	7.42	3.86	3.86	3.88	2.48	2.65	2.72
$\bar{\boldsymbol{x}}_t$	50	0.28	0.03	-0.25	0	0.04	-0.09	0.02	-0.03	-0.02	4.52	4.56	4.78	1.89	2.39	2.35	1.29	1.61	1.67
	100	0.07	-0.16	-0.06	0.01	-0.09	0.03	0.04	0	0.04	2.62	3.07	3.09	1.31	1.64	1.75	0.86	1.09	1.18
	Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																		
	20	-0.14	-0.3	-0.08	0.05	0.02	0.02	-0.04	-0.03	0.04	5.02	6.36	6.63	2.79	3.81	3.74	1.85	2.63	2.69
	50	0.13	0.08	-0.17	0.03	0.03	-0.09	0.01	-0.02	-0.02	2.68	4.17	4.04	1.47	2.35	2.31	1.03	1.61	1.67
	100	-0.04	-0.1	-0.02	0.01	-0.09	0.01	0.02	0.01	0.04	1.88	2.87	2.87	1.05	1.62	1.72	0.72	1.08	1.16
	Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																		
	20	1.97	-0.48	-0.52	0.81	-0.06	-0.14	0.42	-0.06	0.02	4.91	6.23	6.61	2.71	3.81	3.73	1.78	2.63	2.68
	50	1.51	-0.15	-0.59	0.53	-0.05	-0.24	0.28	-0.07	-0.1	2.92	4.11	4.02	1.5	2.35	2.31	1.04	1.61	1.67
	100	1.18	-0.29	-0.39	0.47	-0.17	-0.15	0.25	-0.03	-0.04	2.1	2.85	2.86	1.11	1.61	1.73	0.73	1.08	1.16
	OLS																		
	20	2.84	-0.64	-0.74	2.83	-0.27	-0.62	2.71	-0.33	-0.47	5.01	6.35	6.53	3.66	3.81	3.75	3.13	2.66	2.72
	50	3.9	-0.52	-1.31	3.92	-0.56	-1.27	3.86	-0.61	-1.15	4.6	4.19	4.2	4.14	2.4	2.62	3.97	1.73	2.03
	100	4.22	-0.78	-1.42	4.16	-0.78	-1.33	4.13	-0.67	-1.31	4.55	2.95	3.29	4.26	1.78	2.16	4.18	1.28	1.76
	Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																		
	20	-0.3	-0.31	-0.16	-0.16	0.02	-0.01	-0.22	-0.04	0.04	4.7	6.48	6.59	2.62	3.83	3.72	1.76	2.64	2.69
	50	-0.02	0.03	-0.16	-0.04	0	-0.08	-0.03	-0.03	0	2.69	4.27	4.06	1.42	2.36	2.31	1.04	1.62	1.68
	100	-0.08	-0.11	-0.03	0.02	-0.11	0.02	0.02	0	0.03	1.87	2.91	3.02	1.03	1.61	1.73	0.71	1.08	1.16
	Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																		
	20	-0.28	-0.39	-0.01	-0.4	0.03	0.07	-0.5	-0.01	0.02	8.48	7.04	7.61	3.7	3.84	3.84	2.37	2.66	2.72
$(\bar{y}_t, \bar{\boldsymbol{x}}_t')'$	50	-0.1	0.1	-0.14	-0.04	0.02	-0.08	-0.01	-0.03	-0.01	3.87	4.58	4.38	1.87	2.39	2.35	1.29	1.62	1.68
	100	0.03	-0.1	-0.05	0	-0.11	0.05	0.03	0	0.03	2.85	3.27	3.22	1.31	1.65	1.77	0.85	1.09	1.18
	Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																		
	20	-0.39	-0.33	-0.11	-0.27	0.04	0.02	-0.34	-0.03	0.07	4.7	6.5	6.56	2.64	3.8	3.73	1.78	2.63	2.69
	50	0.01	0.06	-0.16	0.01	0	-0.09	-0.02	-0.03	0	2.72	4.28	4.07	1.43	2.35	2.3	1.02	1.62	1.67
	100	-0.06	-0.11	-0.04	0.01	-0.11	0.03	0.01	0	0.03	1.9	2.91	3.03	1.04	1.62	1.73	0.72	1.08	1.16
	Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																		
	20	0.46	-0.38	-0.28	0.02	0	-0.05	-0.15	-0.05	0.03	4.49	6.43	6.55	2.5	3.81	3.71	1.69	2.64	2.68
	50	0.85	-0.08	-0.42	0.26	-0.04	-0.17	0.12	-0.05	-0.04	2.7	4.23	4.04	1.4	2.35	2.3	1.01	1.62	1.67
	100	0.89	-0.26	-0.33	0.34	-0.16	-0.08	0.18	-0.03	-0.02	2	2.89	3	1.06	1.61	1.73	0.71	1.08	1.16
	OLS																		
	20	3.34	-1.75	-2.91	3.29	-1.32	-2.73	3.17	-1.37	-2.58	5.35	6.8	7.63	4.04	4.04	4.75	3.55	2.99	3.79
	50	4.05	-0.99	-2.16	4.08	-1	-2.13	4.01	-1.01	-1.98	4.74	4.3	4.7	4.3	2.56	3.18	4.12	1.91	2.61
	100	4.32	-1	-1.86	4.24	-1	-1.74	4.21	-0.88	-1.73	4.66	3.1	3.55	4.35	1.89	2.46	4.26	1.4	2.09
	Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																		
	20	0.15	-1.4	-2.27	0.25	-0.98	-2.04	0.21	-1.03	-1.88	4.9	6.88	7.44	2.64	3.97	4.4	1.79	2.85	3.35
	50	0.09	-0.4	-0.95	0.08	-0.41	-0.88	0.09	-0.4	-0.77	2.76	4.33	4.35	1.45	2.41	2.53	1.04	1.67	1.86
	100	-0.01	-0.32	-0.42	0.08	-0.31	-0.36	0.08	-0.19	-0.36	1.97	3.02	3.1	1.05	1.64	1.79	0.72	1.11	1.22
	Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																		
	20	0.8	-1.54	-2.41	0.51	-1.06	-2.09	0.46	-1.09	-2.01	9.46	8.29	8.36	3.76	4.03	4.57	2.36	2.91	3.45
$(\bar{y}_t, \bar{y}_t^{*}, \bar{\boldsymbol{x}}_t')'$	50	0.29	-0.48	-1.12	0.24	-0.42	-0.96	0.26	-0.45	-0.86	4.18	4.69	4.96	1.9	2.45	2.6	1.32	1.68	1.9
	100	0.27	-0.38	-0.55	0.12	-0.32	-0.37	0.16	-0.21	-0.39	3.17	3.45	3.42	1.34	1.69	1.83	0.87	1.11	1.25

Table S6 (Continued)

		Bias (× 100)									RMSE (× 100)								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
								Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments											
	20	0.25	-1.42	-2.26	0.33	-0.99	-2.04	0.26	-1.04	-1.91	4.85	6.89	7.42	2.66	3.95	4.41	1.79	2.86	3.36
	50	0.17	-0.39	-0.99	0.17	-0.42	-0.92	0.15	-0.42	-0.79	2.8	4.32	4.39	1.46	2.41	2.53	1.03	1.67	1.86
	100	0.04	-0.32	-0.45	0.08	-0.31	-0.35	0.09	-0.2	-0.36	1.99	3.03	3.13	1.06	1.65	1.79	0.72	1.11	1.22
								Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	1.02	-1.48	-2.42	0.54	-1.02	-2.1	0.37	-1.05	-1.93	4.73	6.84	7.46	2.58	3.96	4.42	1.76	2.86	3.37
	50	1.07	-0.54	-1.26	0.42	-0.46	-0.99	0.27	-0.43	-0.83	2.79	4.29	4.4	1.46	2.41	2.56	1.04	1.67	1.88
	100	1.04	-0.48	-0.76	0.41	-0.36	-0.47	0.25	-0.22	-0.42	2.13	3.02	3.14	1.1	1.65	1.81	0.73	1.11	1.24
Infeasible								OLS											
	20	4.35	-1.36	-1.61	4.23	-0.8	-1.75	4.22	-0.83	-1.6	5.44	6.14	6.55	4.65	3.75	3.99	4.42	2.61	3.08
	50	4.33	-0.68	-1.73	4.38	-0.8	-1.72	4.3	-0.83	-1.6	4.84	3.91	4.25	4.55	2.44	2.84	4.38	1.78	2.3
	100	4.42	-0.76	-1.64	4.37	-0.87	-1.58	4.34	-0.8	-1.53	4.7	2.85	3.3	4.46	1.8	2.31	4.39	1.33	1.92
								Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments											
	20	-0.08	-0.47	0.11	-0.06	0.04	-0.06	0.02	0	0.06	3.86	6.15	6.41	2.21	3.7	3.6	1.45	2.47	2.63
	50	-0.04	0.12	-0.13	0.01	-0.01	-0.08	-0.02	-0.03	-0.01	2.48	3.98	3.93	1.33	2.33	2.28	0.95	1.57	1.65
	100	-0.06	0.01	-0.02	0.03	-0.09	0	0.03	-0.01	0.03	1.78	2.78	2.89	0.99	1.59	1.7	0.68	1.06	1.15
								Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	-0.01	-0.54	0.21	-0.09	0.06	-0.06	-0.06	0.02	0.1	5.18	6.4	6.83	2.72	3.69	3.68	1.71	2.5	2.64
	50	-0.03	0.16	-0.09	0.03	0	-0.1	-0.02	-0.03	0	3.31	4.14	4.18	1.67	2.35	2.3	1.14	1.56	1.65
	100	0.02	0.02	-0.04	0.02	-0.08	0.02	0.04	-0.01	0.03	2.43	2.93	3.1	1.23	1.63	1.73	0.81	1.07	1.17
								Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments											
	20	-0.04	-0.49	0.14	-0.08	0.05	-0.06	-0.01	0.01	0.08	3.86	6.14	6.43	2.22	3.69	3.6	1.43	2.48	2.63
	50	-0.03	0.14	-0.12	0.05	-0.01	-0.11	-0.01	-0.03	-0.01	2.47	3.96	3.94	1.33	2.32	2.27	0.94	1.56	1.64
	100	-0.04	0.03	-0.02	0.02	-0.08	0.01	0.03	-0.01	0.03	1.77	2.8	2.89	1	1.6	1.7	0.68	1.07	1.15
								Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	0.91	-0.65	-0.25	0.26	-0.02	-0.19	0.15	-0.02	0.01	3.74	6.11	6.37	2.14	3.68	3.59	1.41	2.47	2.62
	50	0.88	-0.04	-0.46	0.34	-0.06	-0.21	0.14	-0.06	-0.06	2.49	3.92	3.93	1.33	2.32	2.27	0.92	1.56	1.64
	100	0.9	-0.15	-0.37	0.36	-0.15	-0.11	0.19	-0.04	-0.02	1.91	2.77	2.88	1.02	1.59	1.7	0.68	1.06	1.15
Naive								OLS											
	20	8.3	-0.21	-2.06	8.11	0.1	-1.92	8.14	0.04	-1.67	8.91	5.73	6.59	8.37	3.72	4.17	8.27	2.71	3.24
	50	8.44	0.07	-2.03	8.42	0.13	-1.98	8.4	0.06	-1.87	8.72	3.86	4.32	8.54	2.35	3.06	8.47	1.72	2.56
	100	8.55	-0.06	-1.93	8.56	-0.01	-1.84	8.54	0.1	-1.79	8.73	2.72	3.41	8.63	1.63	2.52	8.59	1.15	2.19
								Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments											
	20	3.12	1.23	0.68	3.07	1.56	0.72	3.13	1.47	0.97	4.82	6.05	6.28	3.85	4.09	3.8	3.49	3.14	2.94
	50	3.23	1.47	0.55	3.21	1.55	0.65	3.23	1.47	0.71	4.01	4.24	3.85	3.51	2.89	2.47	3.42	2.3	1.91
	100	3.3	1.34	0.7	3.33	1.39	0.75	3.33	1.52	0.77	3.83	3.14	2.93	3.53	2.19	1.92	3.45	1.94	1.47
								Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	4.44	0.84	0.12	4.27	1.21	0.11	4.37	1.12	0.33	6.28	6.05	6.62	5.07	3.94	3.79	4.75	3	2.78
	50	4.71	1.11	-0.18	4.65	1.18	-0.08	4.67	1.09	0.02	5.52	4.19	4	4.97	2.7	2.38	4.85	2.05	1.73
	100	4.8	0.93	-0.07	4.83	1	0.04	4.83	1.12	0.04	5.35	3.02	2.92	5.02	1.97	1.73	4.95	1.63	1.26
								Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments											
	20	4.1	0.95	0.21	4	1.3	0.25	4.09	1.2	0.48	5.56	5.96	6.29	4.66	3.96	3.71	4.41	3.01	2.79
	50	4.39	1.17	-0.01	4.36	1.25	0.08	4.37	1.17	0.16	5.02	4.1	3.82	4.62	2.71	2.35	4.52	2.09	1.74
	100	4.49	1.03	0.1	4.52	1.08	0.18	4.51	1.21	0.2	4.93	2.98	2.81	4.7	1.99	1.74	4.61	1.69	1.26
								Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	4.59	0.82	-0.07	3.88	1.32	0.3	3.78	1.29	0.63	5.8	5.88	6.25	4.51	3.97	3.71	4.1	3.05	2.82
	50	4.72	1.08	-0.18	4.11	1.31	0.19	3.96	1.28	0.35	5.26	4.06	3.8	4.37	2.74	2.36	4.12	2.16	1.78
	100	4.79	0.94	-0.05	4.27	1.14	0.29	4.09	1.32	0.4	5.16	2.94	2.8	4.44	2.03	1.77	4.19	1.77	1.31

Table S7: The Finite Sample Performance of Alternative Pooled Estimators under Experiment 2

		Bias ($\times$ 100)									RMSE ($\times$ 100)								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
		OLS																	
	20	7.65	-0.72	-1.45	7.6	-0.66	-1.51	7.63	-0.84	-1.49	8.5	5.85	5.9	8.03	3.72	4.03	7.89	2.88	3.1
	50	6.12	-0.97	-1.96	6.2	-0.99	-1.86	6.17	-0.84	-1.8	6.53	3.89	4.26	6.38	2.53	2.99	6.28	1.89	2.5
	100	5.53	-0.93	-1.83	5.5	-0.86	-1.67	5.5	-0.91	-1.78	5.76	2.73	3.2	5.6	1.85	2.37	5.56	1.48	2.16
		Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																	
	20	0.12	-0.06	-0.07	0.02	0.09	0	-0.03	-0.08	0.02	4.11	5.8	5.64	2.59	3.62	3.69	2.05	2.73	2.67
	50	-0.05	-0.06	-0.2	0	-0.09	-0.03	-0.02	0.07	0.01	2.36	3.75	3.73	1.54	2.33	2.33	1.14	1.69	1.71
	100	0	-0.03	-0.02	0.02	0.03	0.1	0.08	-0.02	0	1.68	2.57	2.59	1.09	1.64	1.68	0.81	1.17	1.18
		Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	0.1	-0.04	-0.07	0.08	0.08	-0.01	-0.02	-0.08	0.02	5.32	5.78	5.67	3.33	3.62	3.69	2.53	2.73	2.69
$\bar{\boldsymbol{x}}_t$	50	0	-0.07	-0.21	0.07	-0.1	-0.06	-0.06	0.07	0.03	2.9	3.77	3.75	1.87	2.33	2.36	1.39	1.69	1.72
	100	0.01	-0.02	-0.03	0	0.03	0.1	0.05	-0.02	0.01	1.97	2.58	2.63	1.31	1.64	1.71	0.94	1.17	1.2
		Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																	
	20	0.06	-0.05	-0.06	0.04	0.09	0	-0.04	-0.08	0.02	4.12	5.8	5.64	2.61	3.62	3.67	2.08	2.73	2.67
	50	-0.06	-0.05	-0.19	0.03	-0.09	-0.04	-0.04	0.07	0.02	2.39	3.75	3.73	1.56	2.33	2.34	1.14	1.69	1.72
	100	0	-0.02	-0.02	0.01	0.03	0.1	0.06	-0.02	0.01	1.68	2.57	2.58	1.09	1.64	1.68	0.81	1.17	1.18
		Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	0.16	-0.06	-0.07	0.05	0.09	0	-0.02	-0.08	0.02	3.97	5.79	5.63	2.52	3.62	3.68	2.01	2.73	2.67
	50	-0.03	-0.06	-0.2	0.02	-0.09	-0.04	-0.03	0.07	0.02	2.3	3.75	3.72	1.51	2.33	2.33	1.12	1.69	1.71
	100	0.01	-0.03	-0.03	0.02	0.03	0.1	0.07	-0.02	0	1.63	2.57	2.59	1.07	1.64	1.68	0.79	1.17	1.18
		OLS																	
	20	2.62	-0.26	-0.69	2.56	-0.18	-0.53	2.63	-0.37	-0.57	4.56	5.87	5.82	3.56	3.64	3.79	3.27	2.75	2.71
	50	3.74	-0.63	-1.25	3.83	-0.67	-1.15	3.78	-0.5	-1.1	4.34	3.89	4.06	4.11	2.45	2.6	3.93	1.76	2.04
	100	4.19	-0.7	-1.37	4.14	-0.65	-1.23	4.17	-0.7	-1.35	4.47	2.72	3.01	4.26	1.77	2.09	4.24	1.36	1.81
		Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																	
	20	-0.09	-0.04	-0.2	-0.24	0.1	0.03	-0.25	-0.09	0	4.15	5.9	5.78	2.59	3.64	3.76	2.05	2.72	2.64
	50	-0.08	-0.06	-0.16	-0.03	-0.11	-0.02	-0.07	0.06	0.02	2.44	3.84	3.84	1.57	2.36	2.33	1.14	1.68	1.7
	100	0	-0.02	0	0.02	0.02	0.1	0.06	-0.03	0	1.72	2.63	2.65	1.09	1.66	1.68	0.81	1.17	1.18
		Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	-0.26	-0.01	-0.16	-0.4	0.11	0.06	-0.45	-0.07	0.04	5.37	5.9	5.84	3.3	3.64	3.76	2.53	2.73	2.67
$(\bar{y}_t, \bar{\boldsymbol{x}}_t')'$	50	0	-0.08	-0.18	0.02	-0.11	-0.04	-0.12	0.07	0.04	2.96	3.86	3.88	1.88	2.37	2.36	1.38	1.69	1.71
	100	0.01	-0.02	-0.01	-0.01	0.02	0.11	0.03	-0.03	0.01	1.99	2.64	2.69	1.3	1.66	1.72	0.94	1.17	1.19
		Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																	
	20	-0.28	-0.02	-0.17	-0.36	0.11	0.05	-0.4	-0.08	0.03	4.14	5.9	5.79	2.59	3.63	3.74	2.09	2.72	2.64
	50	-0.08	-0.06	-0.15	-0.01	-0.11	-0.03	-0.1	0.07	0.03	2.45	3.84	3.85	1.58	2.37	2.34	1.14	1.68	1.71
	100	0	-0.02	0	0	0.02	0.11	0.04	-0.03	0.01	1.71	2.63	2.65	1.08	1.66	1.69	0.81	1.17	1.18
		Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	-0.1	-0.04	-0.2	-0.26	0.1	0.03	-0.29	-0.09	0.01	3.99	5.89	5.78	2.5	3.64	3.75	2.01	2.72	2.64
	50	-0.05	-0.07	-0.16	-0.01	-0.11	-0.03	-0.08	0.06	0.02	2.37	3.84	3.84	1.54	2.36	2.33	1.12	1.68	1.7
	100	0.01	-0.02	-0.01	0.01	0.02	0.1	0.05	-0.03	0	1.67	2.63	2.66	1.07	1.66	1.69	0.79	1.17	1.18
		OLS																	
	20	3.15	-1.13	-2.31	3.06	-1.01	-2.2	3.12	-1.2	-2.19	4.92	6.17	6.54	3.95	3.79	4.45	3.68	3	3.5
	50	3.89	-0.89	-1.91	4.03	-1	-1.81	3.97	-0.82	-1.76	4.5	4	4.47	4.29	2.58	2.98	4.11	1.88	2.47
	100	4.3	-0.89	-1.72	4.23	-0.8	-1.55	4.26	-0.85	-1.67	4.6	2.86	3.25	4.35	1.83	2.32	4.32	1.45	2.06
		Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																	
	20	0.46	-0.88	-1.77	0.25	-0.7	-1.58	0.24	-0.88	-1.57	4.23	6.16	6.36	2.6	3.71	4.18	2.06	2.88	3.14
	50	0.02	-0.3	-0.77	0.15	-0.42	-0.63	0.09	-0.23	-0.59	2.5	3.92	4.08	1.58	2.42	2.45	1.15	1.7	1.83
	100	0.08	-0.19	-0.32	0.08	-0.12	-0.19	0.12	-0.17	-0.29	1.78	2.73	2.75	1.09	1.66	1.72	0.82	1.19	1.23
		Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	0.54	-0.88	-1.78	0.32	-0.71	-1.59	0.28	-0.89	-1.58	4.09	6.15	6.36	2.53	3.71	4.18	2.02	2.89	3.14
$(\bar{y}_t, \bar{y}_t^{*}, \bar{\boldsymbol{x}}_t')'$	50	0.08	-0.32	-0.79	0.18	-0.42	-0.64	0.1	-0.23	-0.59	2.44	3.92	4.08	1.56	2.42	2.45	1.13	1.7	1.82
	100	0.1	-0.19	-0.33	0.09	-0.12	-0.2	0.13	-0.17	-0.29	1.72	2.73	2.76	1.07	1.66	1.72	0.8	1.19	1.23

Table S7 (Continued)

		Bias (× 100)									RMSE (× 100)								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
	Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																		
	20	0.54	-0.88	-1.79	0.37	-0.71	-1.61	0.33	-0.9	-1.59	4.22	6.15	6.38	2.62	3.71	4.17	2.09	2.89	3.14
	50	0.11	-0.32	-0.79	0.22	-0.43	-0.65	0.12	-0.24	-0.6	2.52	3.92	4.1	1.6	2.43	2.46	1.15	1.7	1.83
	100	0.1	-0.19	-0.33	0.1	-0.12	-0.2	0.13	-0.17	-0.29	1.76	2.73	2.75	1.08	1.66	1.72	0.82	1.19	1.22
	Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																		
	20	0.49	-0.94	-1.77	0.26	-0.73	-1.58	0.24	-0.9	-1.58	4.35	6.24	6.43	2.63	3.72	4.18	2.08	2.91	3.15
	50	0.06	-0.32	-0.78	0.17	-0.42	-0.64	0.09	-0.23	-0.59	2.48	3.93	4.1	1.57	2.42	2.46	1.14	1.7	1.82
	100	0.1	-0.2	-0.32	0.09	-0.11	-0.19	0.13	-0.17	-0.29	1.74	2.73	2.76	1.07	1.66	1.72	0.81	1.19	1.23
	OLS																		
	20	4.13	-0.96	-1.66	4.18	-0.72	-1.59	4.13	-0.94	-1.61	5.13	5.74	5.83	4.69	3.61	3.96	4.45	2.75	3.04
	50	4.24	-0.86	-1.73	4.33	-0.9	-1.63	4.27	-0.76	-1.56	4.71	3.85	4.09	4.53	2.43	2.84	4.38	1.81	2.3
	100	4.4	-0.82	-1.64	4.35	-0.75	-1.47	4.39	-0.8	-1.59	4.64	2.7	3.04	4.47	1.78	2.2	4.45	1.4	2
	Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																		
	20	0.06	-0.21	-0.15	0.03	0.08	0.01	-0.08	-0.14	0.02	3.45	5.7	5.59	2.24	3.53	3.62	1.75	2.58	2.56
	50	0	-0.08	-0.21	0.02	-0.11	-0.04	-0.03	0.03	0.02	2.27	3.75	3.68	1.45	2.27	2.33	1.06	1.64	1.67
	100	-0.01	-0.02	-0.04	0.02	0.04	0.09	0.06	-0.01	-0.01	1.61	2.58	2.54	1.07	1.63	1.64	0.77	1.16	1.19
	Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																		
	20	0.02	-0.21	-0.15	0.06	0.07	0	-0.06	-0.14	0.02	4.01	5.72	5.64	2.6	3.54	3.65	1.95	2.57	2.6
Infeasible	50	-0.02	-0.08	-0.2	0.07	-0.12	-0.05	-0.06	0.03	0.03	2.68	3.77	3.71	1.68	2.28	2.36	1.24	1.65	1.69
	100	0.01	-0.02	-0.04	0	0.04	0.1	0.03	-0.01	0	1.87	2.6	2.58	1.24	1.63	1.68	0.88	1.17	1.2
	Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																		
	20	0.01	-0.21	-0.13	0.04	0.08	0	-0.09	-0.13	0.02	3.39	5.7	5.58	2.22	3.53	3.6	1.75	2.58	2.57
	50	-0.03	-0.07	-0.2	0.03	-0.12	-0.04	-0.05	0.03	0.03	2.29	3.75	3.68	1.46	2.28	2.34	1.06	1.64	1.68
	100	0	-0.02	-0.04	0.01	0.04	0.1	0.04	-0.01	-0.01	1.6	2.59	2.54	1.06	1.63	1.64	0.77	1.16	1.18
	Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																		
	20	0.08	-0.22	-0.16	0.05	0.07	0	-0.07	-0.14	0.02	3.32	5.7	5.58	2.18	3.53	3.62	1.71	2.58	2.56
	50	0.01	-0.08	-0.21	0.03	-0.12	-0.04	-0.04	0.03	0.02	2.22	3.75	3.68	1.42	2.27	2.33	1.04	1.64	1.67
	100	0	-0.02	-0.04	0.02	0.04	0.09	0.05	-0.01	-0.01	1.57	2.59	2.54	1.05	1.63	1.64	0.76	1.16	1.19
	OLS																		
	20	7.96	0.2	-1.84	8.12	0.22	-1.76	8.13	0.01	-1.78	8.57	5.55	5.88	8.42	3.72	4.16	8.35	2.84	3.33
	50	8.34	-0.05	-1.91	8.53	0.02	-1.83	8.51	0.21	-1.81	8.62	3.64	4.23	8.66	2.35	3.03	8.6	1.86	2.59
	100	8.58	0.08	-1.88	8.69	0.15	-1.78	8.69	0.12	-1.85	8.77	2.55	3.23	8.78	1.74	2.48	8.74	1.28	2.28
	Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																		
	20	3.1	1.5	0.59	3.15	1.63	0.86	3.12	1.45	0.86	4.69	5.85	5.59	3.89	4.08	3.87	3.68	3.23	2.94
	50	3.22	1.36	0.64	3.38	1.45	0.79	3.37	1.66	0.82	3.98	3.94	3.77	3.73	2.8	2.54	3.59	2.52	2.03
	100	3.39	1.55	0.74	3.49	1.6	0.84	3.53	1.56	0.78	3.87	3.05	2.7	3.73	2.41	1.93	3.65	2.05	1.54
	Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																		
	20	4.33	1.14	-0.06	4.48	1.23	0.13	4.44	1.06	0.16	5.89	5.74	5.6	5.19	3.93	3.76	4.94	3.05	2.84
Naive	50	4.67	0.94	-0.1	4.85	1.03	0.02	4.81	1.25	0.08	5.39	3.79	3.75	5.2	2.59	2.41	5.02	2.26	1.84
	100	4.89	1.11	-0.04	4.99	1.17	0.08	5.02	1.14	0.02	5.37	2.83	2.61	5.21	2.13	1.74	5.13	1.74	1.31
	Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																		
	20	3.97	1.24	0.12	4.11	1.35	0.33	4.08	1.17	0.34	5.35	5.76	5.54	4.73	3.96	3.73	4.55	3.09	2.82
	50	4.3	1.05	0.08	4.5	1.13	0.2	4.49	1.34	0.24	4.95	3.81	3.71	4.82	2.63	2.4	4.67	2.31	1.86
	100	4.54	1.21	0.14	4.67	1.26	0.24	4.7	1.23	0.18	4.98	2.87	2.58	4.88	2.18	1.73	4.81	1.8	1.32
	Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																		
	20	3.5	1.38	0.37	3.57	1.5	0.62	3.53	1.33	0.64	4.95	5.8	5.56	4.24	4.02	3.8	4.05	3.16	2.88
	50	3.69	1.22	0.4	3.86	1.31	0.54	3.83	1.53	0.58	4.39	3.88	3.73	4.19	2.72	2.46	4.04	2.43	1.93
	100	3.88	1.4	0.48	3.98	1.46	0.59	4.01	1.43	0.53	4.34	2.97	2.63	4.2	2.31	1.83	4.13	1.95	1.42

Table S8: The Finite Sample Performance of Alternative Mean Group Estimators under Experiment 2

		Bias (× 100)									RMSE (× 100)								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
$\bar{\boldsymbol{x}}_t$	OLS																		
	20	7.91	-0.84	-1.54	7.83	-0.69	-1.53	7.82	-0.87	-1.52	8.89	6.27	6.52	8.27	3.89	4.15	8.09	2.91	3.15
	50	6.19	-1.03	-1.92	6.21	-1	-1.85	6.16	-0.85	-1.8	6.64	4.15	4.46	6.4	2.6	3.03	6.27	1.93	2.51
	100	5.58	-0.92	-1.89	5.44	-0.86	-1.68	5.45	-0.89	-1.78	5.83	3.07	3.39	5.54	1.84	2.41	5.51	1.47	2.16
	Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																		
	20	0.08	-0.12	0.01	0.05	0.11	0.04	-0.06	-0.08	0.06	5	6.41	6.5	2.79	3.83	3.83	2.14	2.78	2.72
	50	-0.1	-0.07	-0.09	0.02	-0.07	0.01	-0.04	0.07	0.03	2.79	4.08	4.08	1.59	2.4	2.4	1.17	1.74	1.73
	100	-0.02	-0.01	-0.02	-0.01	0.04	0.09	0.04	-0.01	0	1.88	3.01	2.8	1.11	1.66	1.74	0.82	1.18	1.2
	Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																		
	20	0.71	0.08	-0.2	0.08	0.11	0.07	-0.08	-0.08	0.07	8.34	7.1	7.3	3.85	3.89	3.93	2.72	2.79	2.77
	50	0.11	-0.02	-0.14	0.07	-0.09	-0.02	-0.04	0.07	0.03	4.2	4.25	4.43	2.02	2.47	2.48	1.43	1.75	1.75
	100	0.1	-0.01	-0.01	-0.05	0.03	0.13	0.02	-0.01	0.01	2.65	3.15	3.02	1.39	1.68	1.79	0.97	1.2	1.21
	Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																		
	20	0.06	-0.07	-0.01	0.05	0.1	0.06	-0.07	-0.08	0.07	4.92	6.38	6.62	2.83	3.82	3.8	2.2	2.78	2.72
	50	-0.1	-0.07	-0.13	0.04	-0.06	0	-0.04	0.07	0.03	2.81	4.06	4.06	1.61	2.41	2.41	1.18	1.74	1.73
	100	0	0.01	-0.01	-0.03	0.04	0.1	0.03	-0.01	0	1.94	3.01	2.82	1.12	1.66	1.74	0.82	1.18	1.2
	Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																		
	20	2.05	-0.29	-0.35	0.83	0.03	-0.1	0.37	-0.13	-0.02	4.87	6.29	6.4	2.79	3.81	3.81	2.1	2.77	2.72
	50	1.36	-0.29	-0.53	0.56	-0.15	-0.16	0.24	0.03	-0.05	2.92	4.05	4.04	1.64	2.41	2.4	1.17	1.73	1.72
	100	1.26	-0.2	-0.45	0.42	-0.03	-0.05	0.26	-0.05	-0.07	2.17	2.97	2.83	1.17	1.65	1.74	0.84	1.18	1.2
$(\bar{y}_t, \bar{\boldsymbol{x}}_t')'$	OLS																		
	20	2.91	-0.33	-0.62	2.75	-0.19	-0.53	2.72	-0.38	-0.58	5.14	6.25	6.43	3.79	3.75	3.92	3.35	2.78	2.73
	50	3.81	-0.62	-1.19	3.89	-0.69	-1.16	3.81	-0.51	-1.11	4.52	4.13	4.32	4.17	2.51	2.66	3.96	1.79	2.05
	100	4.25	-0.69	-1.43	4.11	-0.65	-1.24	4.14	-0.69	-1.35	4.58	3.09	3.22	4.24	1.77	2.14	4.21	1.36	1.82
	Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																		
	20	-0.17	-0.08	0.01	-0.2	0.12	0.06	-0.28	-0.07	0.04	4.86	6.41	6.52	2.77	3.77	3.9	2.09	2.76	2.68
	50	-0.12	-0.02	-0.01	0	-0.11	0.01	-0.07	0.06	0.03	2.8	4.13	4.2	1.6	2.41	2.41	1.17	1.72	1.72
	100	-0.03	-0.01	-0.01	0	0.03	0.09	0.03	-0.02	0	1.97	3.08	2.88	1.11	1.68	1.74	0.82	1.18	1.2
	Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																		
	20	-0.03	0.27	-0.09	-0.38	0.15	0.13	-0.5	-0.06	0.09	9.46	7.91	7.26	3.72	3.83	3.97	2.65	2.77	2.71
	50	0.01	0.04	-0.09	0.03	-0.13	-0.02	-0.09	0.07	0.04	3.96	4.35	4.51	2.01	2.48	2.46	1.42	1.74	1.75
	100	0.04	0.01	0.05	-0.06	0.03	0.13	0.01	-0.02	0.01	2.81	3.23	3.14	1.39	1.71	1.8	0.96	1.19	1.2
	Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																		
	20	-0.25	-0.03	0.01	-0.3	0.13	0.09	-0.4	-0.06	0.07	4.83	6.43	6.58	2.76	3.76	3.88	2.13	2.76	2.67
	50	-0.12	-0.01	-0.05	0.01	-0.11	0.01	-0.08	0.06	0.03	2.8	4.16	4.2	1.62	2.42	2.41	1.17	1.72	1.73
	100	-0.03	0.02	0.02	-0.02	0.03	0.1	0.02	-0.02	0.01	1.97	3.09	2.91	1.11	1.68	1.74	0.82	1.18	1.19
	Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																		
	20	0.58	-0.13	-0.15	0	0.09	0.03	-0.2	-0.08	0.02	4.54	6.35	6.44	2.64	3.76	3.89	2.02	2.75	2.67
	50	0.76	-0.16	-0.3	0.3	-0.15	-0.08	0.06	0.04	-0.01	2.73	4.11	4.17	1.58	2.42	2.4	1.14	1.72	1.72
	100	0.97	-0.14	-0.33	0.3	-0.02	-0.01	0.18	-0.05	-0.05	2.07	3.05	2.9	1.13	1.68	1.74	0.82	1.18	1.19
$(\bar{y}_t, \bar{y}_t^{*}, \bar{\boldsymbol{x}}_t')'$	OLS																		
	20	3.43	-1.48	-2.79	3.2	-1.27	-2.69	3.19	-1.45	-2.66	5.56	6.69	7.39	4.15	3.97	4.85	3.75	3.14	3.83
	50	3.96	-0.96	-2.13	4.07	-1.14	-2.03	3.98	-0.93	-1.96	4.71	4.35	4.89	4.34	2.69	3.18	4.13	1.96	2.64
	100	4.35	-0.94	-1.91	4.2	-0.86	-1.67	4.22	-0.9	-1.78	4.7	3.25	3.57	4.32	1.87	2.44	4.29	1.48	2.16
	Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																		
	20	0.35	-1.15	-2.07	0.2	-0.91	-1.99	0.15	-1.09	-1.95	5	6.8	7.3	2.78	3.89	4.49	2.1	3	3.38
	50	-0.02	-0.31	-0.88	0.14	-0.53	-0.79	0.06	-0.32	-0.76	2.9	4.3	4.57	1.61	2.5	2.58	1.17	1.75	1.92
	100	0.05	-0.23	-0.45	0.05	-0.16	-0.31	0.08	-0.22	-0.4	2.06	3.17	3.08	1.12	1.69	1.81	0.83	1.21	1.28
	Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																		
	20	0.95	-1.07	-2.3	0.58	-0.99	-2.13	0.44	-1.17	-2.07	7.93	7.35	8.18	3.67	3.97	4.63	2.65	3.03	3.49
	50	0.31	-0.27	-1.07	0.3	-0.58	-0.89	0.19	-0.35	-0.83	4.27	4.6	5.01	2.05	2.56	2.66	1.44	1.78	1.97
	100	0.21	-0.27	-0.4	0.07	-0.18	-0.31	0.13	-0.23	-0.42	2.8	3.35	3.29	1.37	1.73	1.85	0.96	1.23	1.29

Table S8 (Continued)

		Bias (× 100)									RMSE (× 100)								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																			
	20	0.43	-1.14	-2.14	0.29	-0.94	-2.02	0.2	-1.11	-1.98	4.99	6.81	7.36	2.78	3.88	4.49	2.12	3	3.39
	50	0.05	-0.31	-0.94	0.2	-0.54	-0.83	0.1	-0.33	-0.78	2.92	4.33	4.59	1.64	2.51	2.59	1.18	1.76	1.93
	100	0.08	-0.23	-0.43	0.06	-0.17	-0.31	0.09	-0.22	-0.4	2.13	3.2	3.08	1.11	1.7	1.8	0.82	1.21	1.27
Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																			
	20	1.2	-1.25	-2.28	0.51	-0.96	-2.07	0.32	-1.12	-2	4.79	6.75	7.28	2.72	3.89	4.52	2.06	3	3.4
	50	0.97	-0.47	-1.21	0.48	-0.58	-0.91	0.23	-0.35	-0.82	2.9	4.29	4.61	1.63	2.51	2.61	1.17	1.76	1.94
	100	1.12	-0.4	-0.81	0.38	-0.22	-0.41	0.25	-0.25	-0.45	2.21	3.16	3.13	1.15	1.69	1.82	0.84	1.21	1.29
OLS																			
Infeasible	20	4.36	-0.96	-1.76	4.31	-0.75	-1.65	4.22	-0.95	-1.62	5.45	6.19	6.53	4.83	3.74	4.09	4.53	2.82	3.07
	50	4.24	-0.83	-1.72	4.34	-0.91	-1.61	4.26	-0.78	-1.57	4.76	4.03	4.36	4.55	2.49	2.86	4.38	1.85	2.32
	100	4.45	-0.8	-1.68	4.32	-0.76	-1.47	4.35	-0.78	-1.6	4.73	3	3.23	4.44	1.8	2.23	4.41	1.4	2
Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																			
	20	0.05	-0.17	-0.07	0.04	0.07	0.02	-0.1	-0.1	0.07	3.9	6.22	6.39	2.35	3.66	3.73	1.79	2.66	2.59
	50	-0.11	0.01	-0.13	0.01	-0.1	0.01	-0.05	0.02	0.03	2.48	3.98	3.99	1.48	2.33	2.38	1.08	1.68	1.7
	100	-0.02	0.01	-0.04	0	0.03	0.08	0.03	0	-0.02	1.84	2.96	2.75	1.1	1.66	1.68	0.78	1.17	1.2
Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																			
	20	0.04	-0.1	-0.04	-0.01	0.05	0.07	-0.11	-0.1	0.08	4.79	6.42	6.72	2.7	3.73	3.77	1.99	2.66	2.63
	50	-0.08	0.02	-0.18	0.05	-0.12	-0.02	-0.06	0.02	0.03	3.57	4.15	4.26	1.77	2.36	2.42	1.27	1.7	1.72
	100	0.09	0.02	-0.05	-0.04	0.03	0.12	0	0.01	-0.01	2.42	3.06	2.97	1.32	1.69	1.73	0.9	1.19	1.2
Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																			
	20	-0.03	-0.13	-0.06	0.03	0.06	0.04	-0.11	-0.1	0.08	3.84	6.23	6.39	2.31	3.67	3.7	1.78	2.66	2.59
	50	-0.13	0.01	-0.15	0.02	-0.1	0.01	-0.06	0.02	0.03	2.52	3.98	4	1.49	2.33	2.38	1.08	1.68	1.7
	100	0	0.03	-0.01	-0.02	0.03	0.1	0.01	0	-0.01	1.85	2.95	2.75	1.1	1.67	1.68	0.78	1.17	1.19
Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																			
	20	0.98	-0.34	-0.43	0.35	0	-0.09	0.06	-0.13	0.01	3.75	6.17	6.37	2.27	3.66	3.71	1.74	2.65	2.59
	50	0.81	-0.18	-0.48	0.34	-0.16	-0.11	0.1	-0.01	-0.02	2.47	3.95	3.98	1.48	2.33	2.37	1.06	1.68	1.69
	100	0.96	-0.16	-0.4	0.31	-0.02	-0.02	0.18	-0.03	-0.07	1.97	2.93	2.77	1.11	1.66	1.68	0.79	1.17	1.19
OLS																			
Naive	20	8.06	0.07	-1.9	8.15	0.09	-1.82	8.11	-0.07	-1.86	8.68	5.76	6.42	8.45	3.78	4.17	8.32	2.85	3.37
	50	8.35	-0.1	-1.93	8.45	-0.05	-1.81	8.38	0.12	-1.8	8.64	3.89	4.49	8.58	2.39	2.99	8.47	1.84	2.55
	100	8.54	0	-1.89	8.53	0.11	-1.76	8.51	0.07	-1.84	8.73	2.77	3.39	8.61	1.72	2.48	8.56	1.24	2.27
Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																			
	20	2.95	1.41	0.68	3.11	1.51	0.83	3.01	1.38	0.8	4.87	6.11	6.23	3.88	4.12	3.86	3.58	3.22	2.94
	50	3.11	1.31	0.7	3.27	1.37	0.79	3.21	1.54	0.79	3.98	4.22	4.13	3.65	2.8	2.53	3.44	2.44	1.98
	100	3.28	1.42	0.74	3.3	1.53	0.8	3.33	1.47	0.73	3.8	3.22	2.95	3.54	2.35	1.95	3.45	1.95	1.5
Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																			
	20	4.19	1.18	0.03	4.4	1.16	0.19	4.31	1.02	0.17	6.16	6.18	6.53	5.14	4.01	3.77	4.82	3.04	2.86
	50	4.55	0.96	-0.06	4.72	0.97	0.07	4.65	1.16	0.08	5.49	4.08	4.24	5.1	2.62	2.41	4.86	2.2	1.8
	100	4.84	1.04	0.01	4.78	1.13	0.1	4.79	1.08	0.02	5.39	3.09	2.86	5.01	2.1	1.79	4.91	1.68	1.31
Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																			
	20	3.86	1.22	0.21	4.08	1.23	0.34	3.99	1.1	0.31	5.45	6.03	6.17	4.72	4	3.72	4.47	3.08	2.83
	50	4.22	1.04	0.13	4.4	1.07	0.23	4.34	1.24	0.23	4.94	4.05	4.07	4.73	2.64	2.38	4.53	2.24	1.81
	100	4.45	1.13	0.18	4.49	1.21	0.23	4.49	1.16	0.16	4.91	3.07	2.8	4.69	2.14	1.76	4.6	1.72	1.3
Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																			
	20	4.34	1.06	0.01	3.96	1.27	0.4	3.69	1.19	0.46	5.66	5.95	6.18	4.57	4.01	3.75	4.17	3.11	2.85
	50	4.6	0.91	-0.06	4.17	1.12	0.34	3.94	1.35	0.43	5.19	4.03	4.03	4.49	2.67	2.4	4.13	2.31	1.85
	100	4.79	1.03	0	4.22	1.28	0.36	4.07	1.27	0.37	5.16	3.01	2.8	4.42	2.18	1.79	4.17	1.8	1.35

Table S9: The Finite Sample Performance of Alternative Pooled Estimators under Experiment 3

		Bias (× 100)									RMSE (× 100)								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
								OLS											
	20	8.34	-0.96	-1.51	8.27	-0.89	-1.42	8.22	-0.9	-1.57	9.11	5.99	6.25	8.6	3.66	3.98	8.41	2.73	3.01
	50	7.01	-0.95	-1.99	7.07	-1.03	-2.03	7.11	-1.06	-2.1	7.37	3.72	4.18	7.23	2.43	3.13	7.21	1.87	2.66
	100	6.66	-1.03	-2.12	6.63	-1.11	-2.21	6.6	-1.08	-2.14	6.87	2.73	3.41	6.72	1.91	2.72	6.65	1.54	2.41
								Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments											
	20	0.09	-0.18	-0.02	0.07	-0.07	0.17	0.01	-0.1	0.04	3.84	5.86	5.98	2.46	3.51	3.63	1.69	2.56	2.52
	50	-0.09	0.09	0.04	-0.05	0	0.05	0.01	0	-0.01	2.31	3.59	3.63	1.46	2.18	2.35	1.01	1.54	1.59
	100	-0.03	0.06	0.04	0.01	-0.03	-0.07	0	-0.01	0.01	1.58	2.53	2.63	1	1.54	1.57	0.66	1.1	1.1
								Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	0.05	-0.16	0.01	0.07	-0.08	0.17	0.02	-0.1	0.04	5.16	5.94	5.99	3.26	3.5	3.65	2.31	2.58	2.55
$\bar{\boldsymbol{x}}_t$	50	-0.1	0.1	0.05	-0.08	0	0.05	0.02	-0.01	-0.01	2.78	3.58	3.64	1.78	2.18	2.35	1.27	1.54	1.59
	100	0.01	0.05	0.03	0	-0.03	-0.07	-0.02	-0.01	0.02	1.91	2.53	2.66	1.22	1.54	1.59	0.82	1.11	1.11
								Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments											
	20	0.02	-0.17	0.01	0.1	-0.08	0.17	0.01	-0.1	0.04	3.91	5.88	5.97	2.48	3.5	3.63	1.7	2.57	2.52
	50	-0.1	0.09	0.04	-0.06	0	0.05	0.02	-0.01	-0.01	2.26	3.58	3.64	1.45	2.17	2.34	1.02	1.54	1.58
	100	-0.01	0.05	0.04	0	-0.03	-0.07	-0.01	-0.01	0.02	1.59	2.52	2.63	1	1.54	1.57	0.67	1.1	1.1
								Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	0.12	-0.18	-0.02	0.09	-0.08	0.17	0.02	-0.1	0.04	3.71	5.87	5.97	2.42	3.5	3.63	1.65	2.56	2.52
	50	-0.08	0.09	0.04	-0.06	0	0.05	0.01	-0.01	-0.01	2.24	3.58	3.62	1.42	2.18	2.34	0.99	1.54	1.58
	100	-0.01	0.05	0.04	0.01	-0.03	-0.07	0	-0.01	0.01	1.53	2.52	2.63	0.99	1.54	1.57	0.65	1.1	1.1
								OLS											
	20	2.64	-0.46	-0.53	2.71	-0.34	-0.32	2.69	-0.37	-0.51	4.42	5.93	6.01	3.49	3.54	3.66	3.09	2.56	2.54
	50	3.75	-0.55	-1.05	3.77	-0.52	-1.07	3.83	-0.59	-1.12	4.31	3.63	3.85	4	2.22	2.58	3.94	1.64	1.93
	100	4.15	-0.64	-1.3	4.1	-0.68	-1.41	4.12	-0.68	-1.33	4.39	2.68	2.98	4.2	1.68	2.12	4.16	1.28	1.73
								Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments											
	20	-0.09	-0.22	-0.06	-0.1	-0.07	0.22	-0.2	-0.09	0.06	3.9	5.9	5.99	2.44	3.51	3.64	1.71	2.53	2.49
	50	-0.18	0.02	0.06	-0.08	0.03	0.05	-0.02	-0.02	0.01	2.4	3.6	3.71	1.47	2.16	2.34	0.99	1.54	1.56
	100	-0.04	0.03	0.05	0	-0.01	-0.08	0	-0.02	0.01	1.59	2.61	2.67	0.99	1.54	1.59	0.65	1.09	1.1
								Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
$(\bar{y}_t, \bar{\boldsymbol{x}}_t')'$	20	-0.24	-0.2	-0.02	-0.29	-0.05	0.25	-0.39	-0.08	0.1	5.22	5.98	6.01	3.22	3.51	3.65	2.32	2.55	2.53
	50	-0.23	0.03	0.09	-0.13	0.04	0.06	-0.03	-0.01	0.02	2.88	3.6	3.73	1.78	2.17	2.34	1.25	1.54	1.56
	100	-0.01	0.02	0.04	-0.02	-0.01	-0.08	-0.02	-0.01	0.02	1.94	2.61	2.71	1.21	1.53	1.61	0.8	1.09	1.1
								Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments											
	20	-0.26	-0.2	-0.02	-0.19	-0.06	0.24	-0.33	-0.08	0.09	3.96	5.92	5.98	2.44	3.51	3.64	1.73	2.54	2.49
	50	-0.22	0.03	0.08	-0.1	0.04	0.06	-0.03	-0.01	0.02	2.36	3.59	3.72	1.45	2.16	2.34	1.01	1.54	1.55
	100	-0.02	0.03	0.05	-0.02	-0.01	-0.08	-0.01	-0.01	0.02	1.61	2.6	2.68	0.99	1.53	1.59	0.65	1.09	1.09
								Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	-0.1	-0.21	-0.05	-0.12	-0.07	0.23	-0.24	-0.09	0.07	3.77	5.91	5.98	2.38	3.51	3.63	1.67	2.53	2.49
	50	-0.18	0.02	0.06	-0.09	0.03	0.05	-0.02	-0.02	0.01	2.32	3.59	3.7	1.43	2.16	2.33	0.97	1.54	1.55
	100	-0.03	0.03	0.05	0	-0.01	-0.08	0	-0.02	0.02	1.54	2.61	2.67	0.97	1.53	1.59	0.63	1.09	1.09
								OLS											
	20	3.15	-1.22	-2.11	3.22	-1.16	-1.96	3.18	-1.16	-2.12	4.84	6.26	6.56	3.91	3.74	4.21	3.54	2.82	3.32
	50	3.94	-0.86	-1.68	3.97	-0.85	-1.73	4.01	-0.91	-1.77	4.5	3.77	4.24	4.19	2.35	2.95	4.12	1.78	2.38
	100	4.25	-0.83	-1.61	4.19	-0.85	-1.72	4.21	-0.84	-1.66	4.49	2.79	3.19	4.28	1.76	2.35	4.26	1.38	2
								Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments											
	20	0.41	-0.96	-1.59	0.4	-0.86	-1.35	0.28	-0.85	-1.49	4.11	6.22	6.41	2.51	3.64	3.95	1.75	2.7	2.95
	50	-0.04	-0.26	-0.52	0.07	-0.27	-0.56	0.14	-0.32	-0.59	2.47	3.69	3.94	1.46	2.2	2.44	1	1.57	1.68
	100	0.04	-0.14	-0.23	0.07	-0.16	-0.37	0.07	-0.16	-0.28	1.63	2.67	2.76	1	1.55	1.64	0.66	1.11	1.15
								Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	0.49	-0.96	-1.6	0.46	-0.86	-1.37	0.33	-0.86	-1.51	3.98	6.23	6.41	2.46	3.64	3.95	1.72	2.7	2.96
$(\bar{y}_t, \bar{y}_t^*, \bar{\boldsymbol{x}}_t')'$	50	-0.01	-0.26	-0.53	0.09	-0.27	-0.56	0.16	-0.32	-0.6	2.39	3.69	3.94	1.43	2.2	2.44	0.99	1.57	1.68
	100	0.06	-0.14	-0.24	0.07	-0.16	-0.37	0.08	-0.16	-0.29	1.59	2.66	2.76	0.98	1.55	1.64	0.64	1.11	1.15

Table S9 (Continued)

		Bias (× 100)									RMSE (× 100)								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
		Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																	
	20	0.5	-0.96	-1.6	0.54	-0.87	-1.39	0.39	-0.86	-1.52	4.14	6.24	6.4	2.52	3.64	3.96	1.77	2.7	2.97
	50	-0.01	-0.26	-0.52	0.12	-0.27	-0.57	0.19	-0.32	-0.61	2.42	3.68	3.95	1.46	2.2	2.44	1.03	1.57	1.68
	100	0.08	-0.15	-0.24	0.07	-0.16	-0.37	0.08	-0.16	-0.29	1.67	2.66	2.77	1	1.55	1.64	0.66	1.11	1.15
		Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	0.45	-0.95	-1.61	0.41	-0.88	-1.37	0.27	-0.88	-1.51	4.22	6.33	6.43	2.53	3.69	3.96	1.77	2.73	2.97
	50	-0.02	-0.26	-0.53	0.08	-0.26	-0.56	0.15	-0.32	-0.6	2.44	3.69	3.94	1.45	2.22	2.44	0.99	1.58	1.69
	100	0.05	-0.14	-0.24	0.07	-0.17	-0.37	0.08	-0.16	-0.28	1.6	2.67	2.77	0.99	1.56	1.64	0.65	1.11	1.15
		OLS																	
Infeasible	20	3.79	-0.84	-1.77	3.73	-0.81	-1.47	3.7	-0.77	-1.74	4.71	5.63	5.86	4.1	3.44	3.83	3.89	2.53	2.97
	50	3.8	-0.69	-1.68	3.75	-0.64	-1.64	3.79	-0.71	-1.78	4.23	3.53	4	3.93	2.21	2.77	3.88	1.64	2.33
	100	3.83	-0.64	-1.65	3.79	-0.72	-1.79	3.8	-0.7	-1.72	4.04	2.51	3.05	3.87	1.67	2.37	3.84	1.28	2.03
		Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																	
	20	0.07	-0.16	-0.08	0.06	-0.1	0.28	0.01	-0.06	0.02	3.16	5.56	5.57	1.89	3.34	3.52	1.31	2.4	2.4
	50	-0.09	0.02	0.06	-0.04	0.04	0.1	0	-0.01	-0.02	2.09	3.47	3.64	1.24	2.12	2.21	0.88	1.48	1.5
	100	0.01	0.04	0.09	0.01	-0.04	-0.07	0	-0.02	0.02	1.4	2.43	2.56	0.86	1.5	1.54	0.59	1.07	1.07
		Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	0.04	-0.11	-0.06	0.07	-0.1	0.27	0.03	-0.06	0.01	3.64	5.61	5.59	2.2	3.34	3.56	1.57	2.41	2.45
	50	-0.14	0.03	0.08	-0.05	0.05	0.1	0.01	-0.01	-0.03	2.39	3.47	3.67	1.38	2.13	2.22	1.04	1.48	1.51
	100	0.03	0.04	0.08	-0.01	-0.03	-0.06	-0.02	-0.01	0.03	1.65	2.43	2.61	0.99	1.5	1.56	0.7	1.08	1.08
		Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																	
	20	0.01	-0.13	-0.05	0.08	-0.1	0.27	0.02	-0.06	0.02	3.17	5.57	5.56	1.87	3.33	3.52	1.31	2.4	2.4
	50	-0.11	0.02	0.06	-0.04	0.04	0.1	0	-0.01	-0.03	2.07	3.47	3.65	1.2	2.12	2.21	0.89	1.48	1.49
	100	0.02	0.04	0.08	0	-0.03	-0.06	-0.01	-0.01	0.02	1.42	2.43	2.56	0.85	1.5	1.54	0.6	1.08	1.07
		Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	0.08	-0.15	-0.08	0.07	-0.1	0.27	0.02	-0.06	0.01	3.06	5.56	5.55	1.84	3.33	3.51	1.28	2.4	2.4
	50	-0.09	0.02	0.06	-0.04	0.04	0.1	0	-0.01	-0.03	2.04	3.47	3.63	1.19	2.12	2.2	0.86	1.48	1.49
	100	0.02	0.04	0.08	0	-0.03	-0.07	-0.01	-0.01	0.02	1.36	2.43	2.56	0.84	1.5	1.54	0.58	1.07	1.07
		OLS																	
	20	6.8	0.26	-3.18	6.83	0.49	-3.1	6.81	0.51	-3.26	7.46	5.7	6.54	7.15	3.51	4.84	7	2.67	4.16
	50	6.98	0.58	-3.11	7.02	0.54	-3.17	7.12	0.53	-3.31	7.34	3.62	4.79	7.2	2.31	3.98	7.22	1.7	3.74
	100	7.02	0.53	-3.18	7.16	0.51	-3.32	7.17	0.56	-3.23	7.27	2.49	4.12	7.27	1.73	3.7	7.23	1.32	3.45
		Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																	
	20	1.24	1.8	-0.68	1.28	2.11	-0.47	1.19	2.15	-0.61	3.58	6	5.69	2.52	4.11	3.66	2.02	3.42	2.62
	50	1.22	2.18	-0.56	1.22	2.15	-0.53	1.29	2.15	-0.64	2.64	4.27	3.65	1.97	3.16	2.41	1.71	2.71	1.79
	100	1.2	2.13	-0.57	1.34	2.13	-0.67	1.33	2.19	-0.58	2.24	3.29	2.65	1.77	2.73	1.75	1.59	2.52	1.32
		Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
Naive	20	1.76	1.65	-0.91	1.87	1.91	-0.75	1.77	1.96	-0.89	4.46	6.01	5.76	3.24	3.99	3.74	2.7	3.29	2.74
	50	1.86	1.97	-0.86	1.84	1.96	-0.82	1.96	1.96	-0.95	3.46	4.14	3.73	2.65	3.02	2.48	2.42	2.55	1.92
	100	1.87	1.93	-0.88	2	1.93	-0.98	1.99	1.99	-0.88	3	3.16	2.73	2.52	2.57	1.91	2.3	2.34	1.48
		Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																	
	20	1.57	1.69	-0.83	1.71	1.97	-0.68	1.61	2.01	-0.81	3.95	5.97	5.7	2.88	4.02	3.69	2.41	3.32	2.67
	50	1.71	2.03	-0.78	1.7	2.01	-0.76	1.79	2.01	-0.87	3.1	4.16	3.69	2.4	3.05	2.46	2.2	2.59	1.88
	100	1.7	1.98	-0.8	1.86	1.98	-0.92	1.85	2.04	-0.82	2.73	3.19	2.68	2.31	2.6	1.85	2.12	2.38	1.43
		Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	1.44	1.74	-0.77	1.48	2.04	-0.57	1.38	2.09	-0.7	3.65	5.98	5.69	2.64	4.06	3.67	2.17	3.37	2.64
	50	1.45	2.11	-0.66	1.43	2.09	-0.63	1.51	2.09	-0.74	2.8	4.22	3.66	2.13	3.11	2.42	1.91	2.66	1.83
	100	1.43	2.06	-0.68	1.57	2.07	-0.78	1.55	2.12	-0.68	2.43	3.24	2.66	1.99	2.67	1.79	1.81	2.46	1.36

Table S10: The Finite Sample Performance of Alternative Mean Group Estimators under Experiment 3

		Bias (× 100)									RMSE (× 100)								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
								OLS											
	20	8.71	-0.82	-1.82	8.63	-0.98	-1.46	8.56	-0.93	-1.67	9.59	6.7	6.8	8.99	3.81	4.09	8.76	2.76	3.12
	50	7.15	-1.01	-2.02	7.17	-1.06	-2.07	7.19	-1.06	-2.13	7.59	4.12	4.48	7.34	2.55	3.2	7.28	1.91	2.7
	100	6.76	-1.01	-2.15	6.67	-1.11	-2.22	6.63	-1.08	-2.16	6.99	3.02	3.61	6.77	1.97	2.78	6.68	1.55	2.44
								Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments											
	20	0.02	0.09	-0.04	0.08	-0.15	0.23	0.01	-0.07	0.03	4.99	6.77	6.74	2.72	3.66	3.83	1.82	2.58	2.63
	50	-0.11	0.06	0.09	-0.05	0	0.06	0.02	0	-0.01	2.78	4.11	4.08	1.56	2.31	2.42	1.05	1.59	1.63
	100	-0.05	0.12	0.08	0.01	-0.01	-0.06	0.01	0	0.01	1.84	2.87	2.94	1.07	1.63	1.66	0.69	1.12	1.13
								Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	0.59	0.04	-0.11	0.16	-0.15	0.19	-0.01	-0.06	0.05	8.72	7.55	7.53	3.86	3.78	3.92	2.53	2.62	2.65
$\bar{\boldsymbol{x}}_t$	50	-0.07	0.05	0.12	-0.08	-0.02	0.07	0.02	-0.01	-0.01	5.04	5.08	4.52	1.97	2.36	2.47	1.35	1.6	1.64
	100	0.21	0.03	0	-0.03	-0.01	-0.04	-0.01	0	0.02	2.52	3.03	3.15	1.32	1.65	1.7	0.85	1.13	1.14
								Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments											
	20	-0.11	0.02	-0.01	0.11	-0.14	0.21	-0.01	-0.07	0.04	5.61	6.82	6.72	2.76	3.64	3.8	1.86	2.58	2.63
	50	-0.15	0.03	0.11	-0.07	-0.01	0.06	0.02	-0.01	-0.01	2.86	4.08	4.06	1.57	2.32	2.42	1.06	1.59	1.62
	100	0.01	0.09	0.06	-0.01	-0.02	-0.05	-0.01	0	0.01	1.84	2.86	2.95	1.05	1.63	1.66	0.7	1.12	1.13
								Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	2.24	-0.13	-0.48	0.91	-0.22	0.07	0.44	-0.11	-0.05	4.84	6.68	6.63	2.74	3.66	3.8	1.82	2.58	2.62
	50	1.57	-0.2	-0.38	0.55	-0.09	-0.12	0.34	-0.05	-0.11	3.04	4.05	4.04	1.61	2.31	2.42	1.07	1.58	1.63
	100	1.51	-0.13	-0.42	0.55	-0.11	-0.24	0.27	-0.04	-0.08	2.27	2.86	2.95	1.16	1.63	1.67	0.72	1.12	1.13
								OLS											
	20	2.85	-0.22	-0.71	2.87	-0.42	-0.33	2.81	-0.39	-0.56	5.03	6.61	6.54	3.72	3.64	3.77	3.23	2.58	2.62
	50	3.89	-0.61	-1.07	3.88	-0.55	-1.09	3.9	-0.59	-1.15	4.57	4.03	4.22	4.13	2.36	2.66	4.01	1.68	1.97
	100	4.26	-0.58	-1.32	4.15	-0.68	-1.42	4.16	-0.68	-1.36	4.55	2.92	3.22	4.25	1.74	2.19	4.2	1.3	1.76
								Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments											
	20	-0.15	0.06	-0.05	-0.09	-0.12	0.28	-0.2	-0.08	0.05	4.75	6.71	6.62	2.62	3.61	3.79	1.8	2.55	2.57
	50	-0.18	0.02	0.16	-0.07	0.03	0.07	-0.02	-0.01	0.02	2.78	4.05	4.12	1.55	2.3	2.42	1.03	1.58	1.6
	100	-0.05	0.13	0.1	0	0.01	-0.06	0	-0.01	0	1.83	2.93	2.97	1.04	1.62	1.67	0.67	1.11	1.12
								Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	-0.17	0.12	-0.09	-0.26	-0.08	0.29	-0.39	-0.03	0.11	7.76	7.26	7.04	3.49	3.7	3.84	2.45	2.58	2.59
$(\bar{y}_t, \bar{\boldsymbol{x}}_t')'$	50	-0.15	-0.13	0.08	-0.12	0.02	0.09	-0.03	-0.01	0.02	3.96	4.36	4.51	1.94	2.35	2.47	1.32	1.59	1.61
	100	0.16	0.12	0.1	-0.04	0.01	-0.05	-0.02	-0.01	0.01	2.53	3.09	3.16	1.3	1.64	1.71	0.83	1.12	1.14
								Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments											
	20	-0.31	0.06	0	-0.17	-0.1	0.28	-0.32	-0.06	0.08	4.7	6.71	6.63	2.6	3.6	3.76	1.81	2.55	2.56
	50	-0.23	-0.01	0.17	-0.09	0.02	0.07	-0.02	-0.01	0.02	2.79	4.05	4.13	1.55	2.31	2.42	1.04	1.58	1.6
	100	0	0.12	0.09	-0.02	0.01	-0.06	-0.01	-0.01	0.01	1.86	2.92	2.99	1.03	1.62	1.68	0.67	1.11	1.12
								Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	0.54	0.02	-0.21	0.13	-0.14	0.22	-0.11	-0.08	0.04	4.39	6.65	6.56	2.53	3.61	3.76	1.74	2.55	2.56
	50	0.69	-0.15	-0.12	0.21	-0.01	-0.02	0.13	-0.04	-0.03	2.69	4.01	4.09	1.52	2.3	2.42	1.01	1.58	1.6
	100	0.92	-0.03	-0.22	0.31	-0.05	-0.16	0.15	-0.03	-0.04	1.93	2.9	2.97	1.04	1.62	1.67	0.67	1.11	1.12
								OLS											
	20	3.31	-1.31	-2.7	3.35	-1.49	-2.49	3.26	-1.42	-2.68	5.41	7.05	7.22	4.11	3.95	4.58	3.65	2.94	3.77
	50	4.08	-1.04	-1.86	4.05	-0.99	-1.97	4.06	-1.02	-2	4.78	4.28	4.66	4.29	2.5	3.17	4.17	1.87	2.58
	100	4.35	-0.84	-1.73	4.23	-0.9	-1.84	4.24	-0.89	-1.79	4.66	3.06	3.5	4.33	1.86	2.49	4.29	1.42	2.13
								Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments											
	20	0.27	-0.96	-1.94	0.34	-1.14	-1.79	0.21	-1.07	-1.97	5.01	7.12	7.12	2.67	3.83	4.26	1.84	2.78	3.31
	50	-0.02	-0.37	-0.59	0.06	-0.37	-0.74	0.11	-0.41	-0.78	2.88	4.23	4.36	1.55	2.34	2.58	1.03	1.63	1.8
	100	0.04	-0.09	-0.3	0.06	-0.2	-0.45	0.06	-0.2	-0.4	1.97	3.04	3.13	1.04	1.65	1.74	0.69	1.13	1.21
								Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	0.93	-0.96	-2.08	0.7	-1.2	-1.93	0.55	-1.11	-2.1	7.79	7.65	7.66	3.63	3.92	4.38	2.5	2.83	3.42
$(\bar{y}_t, \bar{y}_t^{*}, \bar{\boldsymbol{x}}_t')'$	50	0.19	-0.59	-0.77	0.15	-0.42	-0.8	0.24	-0.44	-0.85	4.42	4.69	4.92	1.97	2.4	2.66	1.34	1.65	1.84
	100	0.28	-0.16	-0.32	0.08	-0.22	-0.48	0.11	-0.22	-0.42	2.79	3.22	3.42	1.3	1.68	1.79	0.84	1.14	1.23

Table S10 (Continued)

		Bias (× 100)									RMSE (× 100)								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
								Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments											
	20	0.29	-0.97	-1.91	0.43	-1.15	-1.84	0.27	-1.07	-1.99	4.86	7.1	7.1	2.67	3.81	4.26	1.83	2.78	3.32
	50	-0.02	-0.41	-0.59	0.09	-0.39	-0.77	0.16	-0.42	-0.8	2.87	4.22	4.37	1.56	2.35	2.59	1.06	1.63	1.8
	100	0.11	-0.13	-0.31	0.05	-0.21	-0.45	0.07	-0.2	-0.4	1.95	3.02	3.11	1.04	1.65	1.75	0.68	1.14	1.21
								Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	1.07	-1.04	-2.14	0.66	-1.18	-1.88	0.4	-1.09	-2.02	4.65	7.06	7.09	2.65	3.84	4.28	1.82	2.79	3.33
	50	0.94	-0.56	-0.91	0.38	-0.43	-0.85	0.29	-0.44	-0.84	2.84	4.21	4.38	1.56	2.35	2.61	1.04	1.64	1.82
	100	1.08	-0.28	-0.64	0.38	-0.26	-0.56	0.23	-0.23	-0.45	2.06	3.01	3.14	1.07	1.65	1.77	0.7	1.14	1.23
								OLS											
Infeasible	20	3.97	-0.87	-1.96	3.87	-0.86	-1.51	3.84	-0.81	-1.79	5.11	6.23	6.46	4.27	3.55	3.89	4.03	2.58	3.06
	50	3.94	-0.74	-1.7	3.87	-0.68	-1.68	3.87	-0.73	-1.8	4.45	3.97	4.23	4.05	2.34	2.86	3.97	1.68	2.37
	100	3.98	-0.57	-1.67	3.87	-0.73	-1.79	3.87	-0.71	-1.73	4.23	2.72	3.26	3.95	1.73	2.43	3.92	1.3	2.05
								Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments											
	20	0.01	-0.09	-0.1	0.05	-0.09	0.35	0.02	-0.06	0.03	3.82	6.26	6.22	2.02	3.45	3.61	1.36	2.46	2.49
	50	-0.08	0.03	0.1	-0.04	0.04	0.1	-0.01	-0.01	-0.02	2.39	3.97	3.98	1.32	2.24	2.31	0.92	1.52	1.55
	100	0.02	0.15	0.11	0	-0.02	-0.04	0	-0.01	0.01	1.61	2.71	2.84	0.9	1.58	1.65	0.62	1.1	1.1
								Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	0.03	-0.04	-0.1	0.08	-0.08	0.31	0.01	-0.05	0.04	4.7	6.59	6.58	2.29	3.47	3.67	1.64	2.46	2.51
	50	-0.1	0	0.14	-0.08	0.03	0.13	0	-0.01	-0.02	3.02	4.12	4.12	1.52	2.29	2.35	1.1	1.52	1.55
	100	0.13	0.13	0.13	-0.03	-0.02	-0.04	-0.01	-0.01	0.03	2.22	2.85	3.05	1.05	1.6	1.68	0.73	1.11	1.12
								Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments											
	20	-0.07	-0.07	-0.02	0.07	-0.09	0.33	0.01	-0.05	0.04	3.81	6.29	6.24	1.96	3.42	3.6	1.35	2.45	2.48
	50	-0.1	0.04	0.14	-0.05	0.03	0.11	0	-0.01	-0.02	2.35	3.98	3.94	1.31	2.25	2.31	0.93	1.51	1.54
	100	0.05	0.13	0.11	-0.02	-0.02	-0.03	-0.01	-0.01	0.02	1.6	2.71	2.85	0.89	1.58	1.64	0.62	1.1	1.1
								Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	0.86	-0.24	-0.47	0.34	-0.14	0.2	0.16	-0.08	-0.03	3.61	6.23	6.19	1.96	3.44	3.59	1.34	2.45	2.48
	50	0.74	-0.14	-0.25	0.23	-0.01	-0.02	0.13	-0.04	-0.08	2.34	3.96	3.92	1.29	2.24	2.3	0.91	1.51	1.54
	100	0.84	0.01	-0.25	0.28	-0.07	-0.17	0.13	-0.03	-0.04	1.73	2.69	2.83	0.9	1.58	1.65	0.61	1.1	1.1
								OLS											
	20	7.02	0.21	-3.45	6.98	0.34	-3.09	6.93	0.37	-3.3	7.77	6.16	7.15	7.31	3.64	4.87	7.12	2.64	4.23
	50	7.13	0.42	-3.12	7.09	0.39	-3.16	7.15	0.42	-3.28	7.52	3.98	4.98	7.26	2.37	3.99	7.25	1.69	3.71
	100	7.15	0.49	-3.17	7.19	0.42	-3.26	7.17	0.45	-3.18	7.4	2.72	4.23	7.29	1.74	3.69	7.22	1.27	3.41
								Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments											
	20	1.17	1.88	-0.66	1.24	2	-0.35	1.16	2.02	-0.57	4.03	6.57	6.41	2.61	4.17	3.75	2.04	3.34	2.7
	50	1.16	2.07	-0.45	1.16	2	-0.47	1.23	2.02	-0.59	2.84	4.62	3.98	1.97	3.13	2.45	1.68	2.63	1.8
	100	1.17	2.1	-0.47	1.27	2.02	-0.6	1.26	2.05	-0.53	2.3	3.49	2.88	1.71	2.68	1.81	1.52	2.39	1.3
								Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
Naive	20	1.76	1.73	-0.97	1.83	1.83	-0.65	1.71	1.86	-0.81	5.12	6.76	6.67	3.33	4.1	3.87	2.7	3.24	2.77
	50	1.79	1.91	-0.69	1.78	1.83	-0.73	1.88	1.84	-0.88	3.72	4.66	4.11	2.65	3.04	2.54	2.38	2.49	1.9
	100	1.92	1.93	-0.79	1.91	1.84	-0.89	1.91	1.87	-0.82	3.19	3.45	3.02	2.44	2.55	1.94	2.22	2.23	1.46
								Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments											
	20	1.56	1.78	-0.82	1.68	1.88	-0.56	1.57	1.91	-0.75	4.32	6.53	6.38	2.96	4.08	3.77	2.41	3.25	2.72
	50	1.64	1.94	-0.64	1.65	1.87	-0.68	1.74	1.89	-0.81	3.21	4.51	3.95	2.4	3.03	2.48	2.16	2.52	1.86
	100	1.68	1.96	-0.71	1.78	1.88	-0.83	1.77	1.91	-0.76	2.76	3.39	2.91	2.21	2.56	1.88	2.05	2.26	1.41
								Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	2.58	1.5	-1.34	1.91	1.81	-0.67	1.61	1.89	-0.78	4.44	6.41	6.43	2.97	4.05	3.79	2.35	3.25	2.74
	50	2.59	1.68	-1.06	1.85	1.82	-0.78	1.72	1.89	-0.81	3.6	4.38	4.03	2.45	3	2.52	2.08	2.52	1.87
	100	2.59	1.72	-1.11	1.97	1.83	-0.91	1.74	1.92	-0.74	3.25	3.23	3.02	2.3	2.52	1.93	1.96	2.27	1.4

Table S11: The Size and Power for Alternative Pooled Estimators under Experiment 1

		Size									Power								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
										OLS									
	20	0.644	0.068	0.097	0.935	0.071	0.079	0.994	0.066	0.12	0.19	0.42	0.37	0.41	0.74	0.76	0.5	0.97	0.95
	50	0.848	0.077	0.086	0.998	0.077	0.145	1	0.105	0.223	0.56	0.73	0.8	0.83	0.99	0.99	0.97	1	1
	100	0.954	0.075	0.117	1	0.096	0.228	1	0.121	0.347	0.82	0.98	0.98	0.99	1	1	1	1	1
										Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments									
	20	0.093	0.06	0.078	0.072	0.063	0.058	0.059	0.054	0.062	0.2	0.41	0.38	0.48	0.73	0.76	0.81	0.96	0.93
	50	0.048	0.073	0.045	0.049	0.061	0.05	0.055	0.049	0.047	0.63	0.73	0.81	0.94	0.99	1	1	1	1
	100	0.056	0.054	0.046	0.059	0.054	0.057	0.055	0.036	0.056	0.82	0.97	0.98	1	1	1	1	1	1
										Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments									
	20	0.072	0.062	0.073	0.07	0.065	0.054	0.063	0.055	0.063	0.18	0.37	0.36	0.31	0.73	0.76	0.51	0.96	0.94
$\bar{\boldsymbol{x}}_t$	50	0.067	0.069	0.044	0.05	0.06	0.056	0.056	0.045	0.044	0.35	0.74	0.79	0.83	0.99	0.99	0.96	1	1
	100	0.046	0.049	0.05	0.061	0.052	0.051	0.053	0.041	0.055	0.71	0.98	0.98	0.97	1	1	1	1	1
										Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments									
	20	0.076	0.06	0.076	0.062	0.062	0.054	0.062	0.055	0.061	0.19	0.4	0.36	0.5	0.73	0.77	0.79	0.96	0.93
	50	0.055	0.069	0.043	0.056	0.062	0.055	0.041	0.047	0.046	0.57	0.73	0.8	0.94	0.99	0.99	1	1	1
	100	0.061	0.05	0.047	0.054	0.056	0.056	0.056	0.04	0.053	0.81	0.97	0.98	1	1	1	1	1	1
										Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments									
	20	0.088	0.061	0.077	0.071	0.062	0.056	0.061	0.056	0.061	0.22	0.41	0.37	0.49	0.74	0.76	0.83	0.96	0.94
	50	0.053	0.07	0.043	0.05	0.061	0.054	0.041	0.049	0.048	0.6	0.73	0.8	0.95	0.99	0.99	1	1	1
	100	0.058	0.054	0.044	0.061	0.054	0.058	0.061	0.038	0.058	0.85	0.97	0.98	1	1	1	1	1	1
										OLS									
	20	0.164	0.088	0.096	0.272	0.087	0.079	0.438	0.077	0.088	0.26	0.37	0.39	0.54	0.73	0.76	0.77	0.97	0.96
	50	0.438	0.075	0.065	0.84	0.074	0.103	0.986	0.079	0.14	0.55	0.71	0.8	0.93	0.99	1	1	1	1
	100	0.789	0.065	0.092	0.994	0.09	0.152	1	0.088	0.237	0.85	0.98	0.97	1	1	1	1	1	1
										Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments									
	20	0.102	0.087	0.089	0.073	0.085	0.082	0.09	0.07	0.089	0.23	0.36	0.37	0.54	0.73	0.76	0.81	0.97	0.95
	50	0.063	0.07	0.053	0.057	0.062	0.058	0.057	0.057	0.065	0.58	0.73	0.79	0.94	0.99	0.99	1	1	1
	100	0.063	0.059	0.059	0.061	0.056	0.061	0.056	0.044	0.065	0.83	0.97	0.97	1	1	1	1	1	1
										Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments									
	20	0.076	0.085	0.084	0.078	0.086	0.087	0.073	0.074	0.089	0.18	0.35	0.36	0.32	0.74	0.76	0.6	0.97	0.95
$(\bar{y}_t, \bar{\boldsymbol{x}}_t')'$	50	0.082	0.072	0.053	0.056	0.065	0.064	0.066	0.055	0.061	0.37	0.71	0.77	0.82	0.99	0.99	0.96	1	1
	100	0.054	0.06	0.058	0.066	0.058	0.065	0.063	0.043	0.059	0.68	0.97	0.96	0.97	1	1	1	1	1
										Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments									
	20	0.085	0.085	0.085	0.067	0.083	0.083	0.093	0.07	0.086	0.21	0.35	0.36	0.54	0.72	0.76	0.76	0.97	0.95
	50	0.067	0.073	0.052	0.058	0.061	0.06	0.045	0.057	0.066	0.54	0.72	0.78	0.94	0.99	0.99	1	1	1
	100	0.066	0.057	0.058	0.061	0.057	0.065	0.056	0.043	0.063	0.82	0.97	0.96	1	1	1	1	1	1
										Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments									
	20	0.095	0.085	0.087	0.064	0.085	0.082	0.087	0.07	0.087	0.21	0.36	0.37	0.55	0.73	0.76	0.84	0.97	0.95
	50	0.067	0.069	0.053	0.057	0.061	0.058	0.051	0.056	0.066	0.6	0.72	0.79	0.95	0.99	0.99	1	1	1
	100	0.066	0.059	0.061	0.062	0.056	0.066	0.062	0.045	0.063	0.84	0.97	0.96	1	1	1	1	1	1
										OLS									
	20	0.191	0.099	0.113	0.351	0.101	0.106	0.552	0.096	0.149	0.26	0.35	0.35	0.51	0.76	0.78	0.78	0.97	0.96
	50	0.452	0.073	0.078	0.859	0.085	0.142	0.99	0.103	0.216	0.55	0.7	0.76	0.93	0.99	0.99	1	1	1
	100	0.783	0.071	0.103	0.996	0.097	0.202	1	0.107	0.32	0.81	0.97	0.96	1	1	1	1	1	1
										Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments									
	20	0.089	0.09	0.108	0.087	0.092	0.091	0.085	0.087	0.111	0.2	0.36	0.35	0.48	0.74	0.79	0.82	0.97	0.95
	50	0.062	0.07	0.054	0.057	0.064	0.077	0.058	0.065	0.079	0.54	0.71	0.73	0.93	0.99	0.99	1	1	1
	100	0.054	0.064	0.053	0.063	0.05	0.063	0.063	0.053	0.07	0.82	0.96	0.96	1	1	1	1	1	1
										Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments									
	20	0.093	0.092	0.104	0.085	0.094	0.09	0.088	0.086	0.113	0.2	0.35	0.36	0.47	0.74	0.78	0.82	0.97	0.95
$(\bar{y}_t, \bar{y}_t^*, \bar{\boldsymbol{x}}_t')'$	50	0.075	0.07	0.054	0.056	0.065	0.075	0.058	0.063	0.076	0.56	0.71	0.73	0.94	0.99	0.99	1	1	1
	100	0.058	0.063	0.053	0.069	0.05	0.068	0.065	0.053	0.067	0.82	0.96	0.96	1	1	1	1	1	1

Table S11 (Continued)

	(N,T)	Size 20 ρ	Size 20 β_1	Size 20 β_2	Size 50 ρ	Size 50 β_1	Size 50 β_2	Size 100 ρ	Size 100 β_1	Size 100 β_2	Power 20 ρ	Power 20 β_1	Power 20 β_2	Power 50 ρ	Power 50 β_1	Power 50 β_2	Power 100 ρ	Power 100 β_1	Power 100 β_2
								Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments											
	20	0.092	0.097	0.103	0.074	0.093	0.092	0.073	0.085	0.11	0.2	0.35	0.36	0.53	0.75	0.76	0.75	0.97	0.96
	50	0.074	0.07	0.056	0.054	0.064	0.074	0.057	0.064	0.077	0.46	0.71	0.75	0.94	0.99	0.99	1	1	1
	100	0.067	0.064	0.056	0.069	0.051	0.068	0.06	0.052	0.068	0.8	0.96	0.96	1	1	1	1	1	1
								Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	0.098	0.07	0.079	0.099	0.08	0.076	0.09	0.087	0.081	0.17)	(0.43)	(0.38)	(0.33)	(0.76)	(0.73)	(0.62)	(0.94)	(0.96)
	50	0.068	0.065	0.048	0.056	0.053	0.055	0.066	0.065	0.064	0.38)	(0.77)	(0.81)	(0.75)	(1)	(0.99)	(0.97)	(1)	(1)
	100	0.063	0.065	0.058	0.065	0.063	0.067	0.066	0.057	0.057	0.65)	(0.96)	(0.96)	(0.97)	(1)	(1)	(1)	(1)	(1)
								OLS											
Infeasible	20	0.328	0.085	0.101	0.638	0.08	0.103	0.898	0.09	0.141	0.32	0.48	0.41	0.65	0.8	0.8	0.86	0.98	0.97
	50	0.59	0.074	0.077	0.959	0.077	0.129	1	0.105	0.201	0.6	0.73	0.82	0.95	0.99	0.99	1	1	1
	100	0.862	0.075	0.106	1	0.092	0.207	1	0.104	0.294	0.87	0.98	0.97	1	1	1	1	1	1
								Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments											
	20	0.089	0.073	0.087	0.067	0.08	0.066	0.077	0.07	0.082	0.3	0.47	0.34	0.68	0.79	0.79	0.9	0.98	0.97
	50	0.065	0.072	0.05	0.048	0.064	0.059	0.047	0.053	0.066	0.59	0.73	0.82	0.97	0.99	0.99	1	1	1
	100	0.055	0.054	0.057	0.056	0.061	0.062	0.058	0.045	0.06	0.87	0.98	0.97	1	1	1	1	1	1
								Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	0.077	0.077	0.081	0.076	0.076	0.065	0.064	0.067	0.086	0.28	0.47	0.41	0.52	0.79	0.8	0.8	0.99	0.96
	50	0.067	0.07	0.051	0.052	0.065	0.061	0.061	0.044	0.062	0.46	0.74	0.81	0.89	0.99	0.99	0.99	1	1
	100	0.055	0.049	0.063	0.058	0.06	0.063	0.057	0.045	0.062	0.73	0.99	0.97	0.99	1	1	1	1	1
								Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments											
	20	0.072	0.075	0.08	0.068	0.08	0.063	0.078	0.067	0.089	0.34	0.47	0.37	0.68	0.79	0.8	0.91	0.99	0.97
	50	0.055	0.072	0.044	0.053	0.064	0.057	0.043	0.049	0.063	0.59	0.74	0.81	0.95	0.99	0.99	1	1	1
	100	0.061	0.052	0.065	0.058	0.061	0.059	0.049	0.046	0.062	0.85	0.98	0.96	1	1	1	1	1	1
								Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	0.081	0.074	0.086	0.068	0.081	0.065	0.078	0.066	0.086	0.35	0.47	0.38	0.69	0.8	0.8	0.92	0.99	0.97
	50	0.06	0.071	0.048	0.061	0.062	0.058	0.048	0.051	0.062	0.61	0.74	0.81	0.96	0.99	0.99	1	1	1
	100	0.057	0.055	0.063	0.056	0.061	0.059	0.058	0.046	0.064	0.87	0.98	0.97	1	1	1	1	1	1
								OLS											
Naive	20	0.817	0.066	0.098	0.983	0.07	0.104	1	0.07	0.132	0.3	0.39	0.43	0.4	0.75	0.76	0.52	0.95	0.94
	50	0.987	0.073	0.098	1	0.062	0.149	1	0.056	0.211	0.46	0.72	0.78	0.81	0.99	1	0.89	1	1
	100	1	0.067	0.132	1	0.051	0.235	1	0.055	0.331	0.6	0.97	0.97	0.95	1	1	0.99	1	1
								Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments											
	20	0.282	0.067	0.087	0.451	0.09	0.072	0.624	0.124	0.094	0.26	0.4	0.42	0.51	0.7	0.75	0.71	0.93	0.92
	50	0.447	0.091	0.057	0.73	0.137	0.056	0.903	0.175	0.083	0.47	0.7	0.78	0.85	0.98	0.99	0.97	1	1
	100	0.654	0.111	0.071	0.92	0.158	0.094	0.992	0.258	0.114	0.66	0.96	0.97	0.96	1	1	1	1	1
								Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	0.372	0.061	0.074	0.549	0.084	0.064	0.74	0.1	0.08	0.17	0.4	0.43	0.34	0.72	0.75	0.5	0.93	0.92
	50	0.583	0.077	0.06	0.865	0.1	0.051	0.974	0.118	0.056	0.37	0.73	0.77	0.68	0.98	0.99	0.85	1	1
	100	0.799	0.087	0.06	0.976	0.106	0.05	0.998	0.165	0.069	10.45	0.96	0.97	0.86	1	1	0.99	1	1
								Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments											
	20	0.385	0.064	0.08	0.604	0.085	0.074	0.795	0.106	0.078	0.24	0.38	0.42	0.39	0.73	0.77	0.67	0.93	0.93
	50	0.613	0.085	0.056	0.899	0.109	0.047	0.99	0.127	0.062	0.45	0.72	0.79	0.78	0.99	0.99	0.94	1	1
	100	0.835	0.092	0.064	0.987	0.118	0.052	1	0.185	0.065	0.57	0.96	0.97	0.93	1	1	1	1	1
								Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	0.341	0.063	0.085	0.518	0.088	0.073	0.705	0.119	0.087	0.23	0.41	0.43	0.46	0.72	0.76	0.74	0.94	0.92
	50	0.542	0.092	0.059	0.835	0.125	0.047	0.958	0.15	0.071	0.48	0.7	0.78	0.82	0.98	0.99	0.96	1	1
	100	0.762	0.104	0.07	0.958	0.146	0.072	0.998	0.231	0.085	0.61	0.96	0.97	0.95	1	1	1	1	1

Table S12: The Size and Power for Alternative Mean Group Estimators under Experiment 1

		Size									Power								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
	OLS																		
	20	0.559	0.054	0.086	0.922	0.074	0.082	0.997	0.068	0.104	0.21	0.42	0.35	0.39	0.69	0.73	0.53	0.96	0.94
	50	0.792	0.066	0.076	0.994	0.077	0.129	1	0.09	0.225	0.48	0.69	0.68	0.84	0.98	0.99	0.98	1	1
	100	0.921	0.069	0.114	1	0.095	0.199	1	0.113	0.347	0.78	0.95	0.94	0.99	1	1	1	1	1
	Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																		
	20	0.064	0.053	0.062	0.067	0.072	0.05	0.056	0.055	0.057	0.18	0.36	0.36	0.42	0.68	0.74	0.73	0.96	0.94
	50	0.059	0.064	0.05	0.045	0.064	0.048	0.048	0.048	0.059	0.46	0.65	0.69	0.9	0.98	0.99	1	1	1
	100	0.048	0.051	0.05	0.054	0.049	0.058	0.051	0.037	0.057	0.74	0.95	0.94	1	1	1	1	1	1
	Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																		
	20	0.058	0.048	0.059	0.064	0.057	0.055	0.061	0.054	0.057	0.12	0.35	0.32	0.3	0.71	0.69	0.48	0.95	0.94
$\bar{\boldsymbol{x}}_t$	50	0.053	0.061	0.048	0.044	0.059	0.051	0.05	0.045	0.049	0.3	0.58	0.61	0.75	0.98	0.98	0.94	1	1
	100	0.048	0.055	0.05	0.068	0.053	0.057	0.05	0.046	0.057	.49	0.91	0.92	0.95	1	1	1	1	1
	Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																		
	20	0.073	0.051	0.067	0.068	0.067	0.054	0.064	0.051	0.061	0.17	0.35	0.32	0.42	0.69	0.73	0.74	0.96	0.93
	50	0.059	0.057	0.042	0.048	0.058	0.044	0.039	0.046	0.05	0.43	0.64	0.71	0.89	0.98	0.99	1	1	1
	100	0.057	0.053	0.052	0.057	0.047	0.061	0.047	0.038	0.059	0.7	0.95	0.93	0.99	1	1	1	1	1
	Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																		
	20	0.111	0.048	0.064	0.081	0.069	0.052	0.072	0.053	0.06	0.21	0.37	0.35	0.42	0.69	0.75	0.75	0.96	0.94
	50	0.112	0.054	0.051	0.054	0.063	0.045	0.055	0.046	0.06	0.49	0.66	0.69	0.92	0.98	0.99	1	1	1
	100	0.115	0.049	0.056	0.071	0.051	0.061	0.071	0.042	0.057	0.8	0.95	0.94	1	1	1	1	1	1
	OLS																		
	20	0.138	0.064	0.069	0.267	0.078	0.072	0.409	0.082	0.082	0.22	0.35	0.35	0.53	0.71	0.73	0.76	0.96	0.95
	50	0.406	0.069	0.061	0.829	0.071	0.092	0.986	0.072	0.134	0.45	0.62	0.69	0.92	0.99	0.99	1	1	1
	100	0.735	0.061	0.089	0.992	0.083	0.132	1	0.089	0.228	0.78	0.94	0.93	1	1	1	1	1	1
	Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																		
	20	0.061	0.071	0.064	0.062	0.082	0.073	0.088	0.07	0.08	0.2	0.34	0.34	0.46	0.71	0.75	0.72	0.96	0.95
	50	0.062	0.066	0.046	0.043	0.059	0.052	0.05	0.053	0.073	0.38	0.58	0.68	0.93	0.99	0.99	1	1	1
	100	0.048	0.05	0.066	0.054	0.052	0.059	0.055	0.049	0.063	0.73	0.93	0.92	1	1	1	1	1	1
	Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																		
	20	0.058	0.057	0.058	0.071	0.074	0.078	0.061	0.08	0.072	0.16	0.35	0.3	0.29	0.72	0.68	0.57	0.95	0.95
$(\bar{y}_t, \bar{\boldsymbol{x}}_t')'$	50	0.049	0.067	0.045	0.05	0.06	0.059	0.052	0.056	0.064	0.3	0.55	0.62	0.77	0.98	0.98	0.96	1	1
	100	0.055	0.048	0.06	0.061	0.053	0.056	0.057	0.045	0.062	0.49	0.92	0.87	0.94	1	1	1	1	1
	Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																		
	20	0.061	0.065	0.07	0.057	0.079	0.074	0.078	0.074	0.078	0.2	0.33	0.32	0.51	0.71	0.72	0.73	0.96	0.95
	50	0.062	0.067	0.051	0.045	0.061	0.056	0.042	0.056	0.07	0.4	0.61	0.66	0.92	0.99	0.99	1	1	1
	100	0.056	0.051	0.067	0.057	0.054	0.063	0.051	0.048	0.063	0.68	0.93	0.91	0.99	1	1	1	1	1
	Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																		
	20	0.071	0.068	0.064	0.057	0.081	0.073	0.077	0.072	0.078	0.21	0.34	0.33	0.49	0.71	0.74	0.78	0.96	0.95
	50	0.077	0.066	0.045	0.045	0.063	0.052	0.048	0.052	0.075	0.45	0.59	0.69	0.95	0.99	0.99	1	1	1
	100	0.085	0.049	0.068	0.068	0.058	0.06	0.06	0.047	0.064	0.75	0.94	0.92	1	1	1	1	1	1
	OLS																		
	20	0.165	0.082	0.093	0.324	0.087	0.122	0.514	0.103	0.154	0.19	0.35	0.28	0.5	0.72	0.73	0.76	0.96	0.94
	50	0.401	0.077	0.072	0.835	0.085	0.141	0.992	0.101	0.237	0.47	0.65	0.69	0.91	0.99	0.98	1	1	1
	100	0.724	0.067	0.111	0.991	0.088	0.185	1	0.121	0.331	0.72	0.92	0.92	1	1	1	1	1	1
	Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																		
	20	0.072	0.077	0.083	0.064	0.082	0.088	0.076	0.093	0.107	0.18	0.29	0.28	0.42	0.7	0.7	0.71	0.96	0.94
	50	0.049	0.059	0.05	0.046	0.059	0.068	0.05	0.058	0.086	0.39	0.64	0.65	0.92	0.98	0.98	1	1	1
	100	0.06	0.054	0.057	0.06	0.049	0.067	0.059	0.05	0.065	0.68	0.92	0.9	1	1	1	1	1	1
	Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																		
	20	0.061	0.069	0.072	0.064	0.073	0.1	0.076	0.091	0.112	0.15	0.32	0.27	0.31	0.71	0.7	0.51	0.95	0.94
$(\bar{y}_t, \bar{y}_t^{*}, \bar{\boldsymbol{x}}_t')'$	50	0.052	0.069	0.062	0.054	0.059	0.07	0.059	0.059	0.082	0.26	0.57	0.56	0.76	0.99	0.98	0.96	1	1
	100	0.064	0.065	0.047	0.064	0.056	0.069	0.053	0.048	0.07	0.5	0.86	0.87	0.94	1	1	1	1	1

Table S12 (Continued)

		Size									Power								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
		Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																	
	20	0.065	0.076	0.08	0.059	0.083	0.093	0.068	0.089	0.112	0.19	0.32	0.29	0.48	0.72	0.7	0.7	0.96	0.95
	50	0.055	0.064	0.054	0.043	0.05	0.073	0.044	0.062	0.086	0.38	0.64	0.65	0.92	0.99	0.99	1	1	1
	100	0.069	0.051	0.06	0.061	0.054	0.065	0.053	0.049	0.065	0.65	0.92	0.89	0.99	1	1	1	1	1
		Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	0.071	0.075	0.087	0.075	0.082	0.092	0.073	0.092	0.11	0.21	0.31	0.29	0.44	0.7	0.72	0.77	0.96	0.95
	50	0.076	0.067	0.051	0.052	0.051	0.069	0.061	0.061	0.092	0.46	0.63	0.68	0.94	0.99	0.99	1	1	1
	100	0.101	0.057	0.073	0.071	0.056	0.073	0.066	0.054	0.07	0.7	0.92	0.91	1	1	1	1	1	1
Infeasible		OLS																	
	20	0.292	0.057	0.085	0.595	0.077	0.09	0.895	0.078	0.129	0.3	0.39	0.37	0.6	0.77	0.79	0.84	0.97	0.96
	50	0.554	0.055	0.066	0.95	0.079	0.129	1	0.092	0.204	0.61	0.7	0.72	0.96	0.99	0.99	1	1	1
	100	0.803	0.061	0.102	0.998	0.087	0.182	1	0.104	0.286	0.84	0.96	0.94	1	1	1	1	1	1
		Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																	
	20	0.066	0.054	0.068	0.063	0.076	0.053	0.072	0.073	0.079	0.26	0.38	0.36	0.61	0.74	0.78	0.89	0.97	0.96
	50	0.057	0.057	0.045	0.047	0.062	0.053	0.048	0.045	0.07	0.53	0.7	0.72	0.95	0.99	0.99	1	1	1
	100	0.055	0.051	0.058	0.054	0.056	0.057	0.053	0.045	0.062	0.77	0.95	0.93	1	1	1	1	1	1
		Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	0.062	0.046	0.067	0.074	0.067	0.059	0.061	0.069	0.072	0.23	0.4	0.32	0.41	0.73	0.77	0.77	0.96	0.95
	50	0.06	0.06	0.044	0.053	0.066	0.051	0.057	0.045	0.062	0.35	0.64	0.71	0.83	0.98	0.98	0.99	1	1
	100	0.052	0.049	0.046	0.059	0.058	0.049	0.048	0.046	0.062	0.55	0.94	0.92	0.97	1	1	1	1	1
		Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																	
	20	0.07	0.054	0.071	0.07	0.067	0.053	0.061	0.072	0.073	0.29	0.38	0.37	0.61	0.74	0.78	0.89	0.97	0.96
	50	0.058	0.057	0.05	0.046	0.066	0.058	0.048	0.044	0.064	0.51	0.67	0.7	0.94	0.99	0.99	1	1	1
	100	0.053	0.05	0.056	0.063	0.058	0.052	0.05	0.042	0.058	0.74	0.95	0.93	1	1	1	1	1	1
		Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	0.079	0.056	0.07	0.07	0.071	0.051	0.067	0.073	0.075	0.28	0.38	0.37	0.6	0.74	0.77	0.91	0.97	0.96
	50	0.073	0.052	0.042	0.053	0.066	0.057	0.05	0.047	0.069	0.57	0.71	0.72	0.96	0.99	0.99	1	1	1
	100	0.092	0.051	0.062	0.07	0.057	0.055	0.062	0.041	0.063	0.8	0.96	0.94	1	1	1	1	1	1
Naive		OLS																	
	20	0.79	0.043	0.084	0.971	0.063	0.107	1	0.064	0.127	0.28	0.43	0.4	0.37	0.73	0.74	0.53	0.95	0.93
	50	0.98	0.057	0.075	1	0.06	0.134	1	0.052	0.204	0.48	0.69	0.73	0.79	0.99	0.99	0.92	1	1
	100	1	0.05	0.13	1	0.053	0.219	1	0.047	0.321	0.61	0.96	0.96	0.96	1	1	1	1	1
		Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																	
	20	0.257	0.051	0.06	0.383	0.092	0.061	0.562	0.119	0.088	0.25	0.4	0.38	0.43	0.69	0.76	0.73	0.93	0.92
	50	0.376	0.078	0.044	0.676	0.125	0.053	0.87	0.16	0.074	0.44	0.68	0.74	0.84	0.98	0.98	0.96	1	1
	100	0.563	0.095	0.051	0.892	0.142	0.084	0.984	0.233	0.097	0.61	0.94	0.94	0.96	1	1	1	1	1
		Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	0.326	0.044	0.069	0.494	0.079	0.054	0.699	0.103	0.075	0.23	0.41	0.34	0.32	0.71	0.75	0.51	0.94	0.92
	50	0.491	0.069	0.053	0.824	0.089	0.04	0.958	0.103	0.057	0.32	0.64	0.7	0.69	0.98	0.99	0.88	1	1
	100	0.704	0.066	0.052	0.965	0.099	0.042	0.996	0.15	0.058	0.41	0.93	0.93	0.88	1	1	0.98	1	1
		Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																	
	20	0.353	0.044	0.069	0.549	0.081	0.056	0.758	0.111	0.078	0.23	0.43	0.36	0.43	0.69	0.75	0.66	0.94	0.92
	50	0.559	0.071	0.04	0.884	0.098	0.046	0.979	0.113	0.055	0.41	0.66	0.76	0.8	0.98	0.99	0.93	1	1
	100	0.763	0.08	0.046	0.98	0.113	0.053	0.999	0.16	0.07	0.54	0.94	0.95	0.91	1	1	0.99	1	1
		Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	0.401	0.042	0.063	0.548	0.087	0.055	0.713	0.109	0.076	0.24	0.42	0.37	0.44	0.69	0.76	0.69	0.94	0.91
	50	0.642	0.07	0.046	0.87	0.1	0.046	0.965	0.126	0.062	0.45	0.69	0.76	0.84	0.98	0.99	0.96	1	1
	100	0.84	0.071	0.046	0.978	0.115	0.059	0.999	0.178	0.075	0.55	0.94	0.95	0.94	1	1	1	1	1

Table S13: The Size and Power for Alternative Pooled Estimators under Experiment 2

		Size									Power								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
		OLS																	
	20	0.625	0.075	0.067	0.874	0.076	0.092	0.967	0.081	0.108	0.19	0.41	0.37	0.31	0.7	0.75	0.44	0.94	0.94
	50	0.81	0.073	0.105	0.993	0.078	0.134	1	0.089	0.193	0.5	0.75	0.71	0.74	0.98	0.98	0.91	1	1
	100	0.955	0.079	0.111	0.999	0.091	0.194	1	0.127	0.332	0.81	0.98	0.97	0.98	1	1	1	1	1
		Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																	
	20	0.075	0.065	0.062	0.068	0.064	0.054	0.074	0.053	0.062	0.21	0.4	0.36	0.44	0.7	0.74	0.64	0.94	0.93
	50	0.043	0.058	0.066	0.06	0.055	0.055	0.043	0.053	0.056	0.61	0.75	0.72	0.85	0.98	0.98	0.99	1	1
	100	0.06	0.048	0.042	0.056	0.054	0.056	0.052	0.049	0.057	0.87	0.98	0.98	0.99	1	1	1	1	1
		Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	0.059	0.071	0.06	0.067	0.062	0.058	0.073	0.055	0.059	0.2	0.41	0.37	0.32	0.71	0.74	0.51	0.95	0.92
$\bar{\boldsymbol{x}}_t$	50	0.052	0.059	0.065	0.06	0.057	0.058	0.055	0.054	0.061	0.41	0.76	0.72	0.71	0.98	0.98	0.94	1	1
	100	0.054	0.047	0.047	0.059	0.05	0.056	0.055	0.049	0.052	0.73	0.98	0.97	0.95	1	1	1	1	1
		Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																	
	20	0.078	0.069	0.061	0.064	0.067	0.056	0.066	0.056	0.063	0.26	0.39	0.35	0.45	0.71	0.75	0.63	0.94	0.92
	50	0.052	0.058	0.068	0.07	0.055	0.055	0.045	0.054	0.057	0.56	0.76	0.72	0.82	0.98	0.98	0.98	1	1
	100	0.067	0.045	0.043	0.049	0.051	0.055	0.056	0.046	0.055	0.82	0.98	0.97	0.99	1	1	1	1	1
		Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	0.066	0.066	0.064	0.065	0.063	0.056	0.07	0.055	0.06	0.26	0.4	0.36	0.51	0.7	0.74	0.66	0.95	0.92
	50	0.046	0.06	0.069	0.067	0.056	0.057	0.043	0.053	0.058	0.58	0.75	0.71	0.86	0.98	0.98	0.99	1	1
	100	0.06	0.048	0.043	0.058	0.053	0.054	0.053	0.047	0.057	0.85	0.98	0.97	0.99	1	1	1	1	1
		OLS																	
	20	0.186	0.081	0.075	0.259	0.075	0.093	0.384	0.086	0.086	0.26	0.4	0.37	0.42	0.73	0.71	0.6	0.93	0.95
	50	0.418	0.07	0.09	0.77	0.079	0.097	0.938	0.069	0.112	0.6	0.74	0.72	0.82	0.98	0.98	0.96	1	1
	100	0.783	0.064	0.088	0.984	0.079	0.13	1	0.111	0.22	0.84	0.97	0.97	0.99	1	1	1	1	1
		Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																	
	20	0.089	0.079	0.073	0.081	0.07	0.087	0.094	0.081	0.075	0.23	0.4	0.36	0.46	0.7	0.7	0.64	0.93	0.94
	50	0.06	0.069	0.075	0.072	0.068	0.063	0.05	0.056	0.064	0.57	0.72	0.72	0.84	0.98	0.98	0.99	1	1
	100	0.064	0.054	0.049	0.052	0.06	0.058	0.053	0.053	0.054	0.84	0.97	0.97	0.99	1	1	1	1	1
		Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	0.067	0.078	0.071	0.067	0.073	0.087	0.087	0.082	0.079	0.19	0.39	0.36	0.35	0.72	0.72	0.52	0.93	0.92
$(\bar{y}_t, \bar{\boldsymbol{x}}_t')'$	50	0.058	0.069	0.072	0.07	0.071	0.062	0.062	0.059	0.065	0.4	0.75	0.75	0.68	0.98	0.98	0.94	1	1
	100	0.051	0.053	0.052	0.067	0.063	0.062	0.059	0.055	0.052	0.74	0.97	0.97	0.96	1	1	1	1	1
		Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																	
	20	0.08	0.081	0.075	0.072	0.071	0.088	0.089	0.08	0.077	0.26	0.4	0.37	0.5	0.72	0.7	0.66	0.93	0.94
	50	0.065	0.069	0.076	0.074	0.067	0.062	0.053	0.058	0.067	0.54	0.72	0.73	0.8	0.98	0.98	0.99	1	1
	100	0.065	0.053	0.046	0.059	0.059	0.061	0.061	0.057	0.052	0.84	0.96	0.97	0.99	1	1	1	1	1
		Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	0.082	0.082	0.073	0.077	0.071	0.084	0.088	0.081	0.077	0.25	0.4	0.37	0.48	0.71	0.7	0.69	0.93	0.94
	50	0.06	0.07	0.077	0.075	0.068	0.062	0.047	0.056	0.064	0.59	0.73	0.73	0.86	0.98	0.98	0.99	1	1
	100	0.063	0.055	0.046	0.063	0.058	0.06	0.052	0.055	0.056	0.86	0.97	0.97	0.99	1	1	1	1	1
		OLS																	
	20	0.21	0.09	0.093	0.318	0.076	0.127	0.478	0.105	0.16	0.22	0.37	0.35	0.41	0.75	0.66	0.62	0.93	0.95
	50	0.44	0.072	0.113	0.801	0.089	0.126	0.951	0.093	0.191	0.61	0.7	0.66	0.83	0.98	0.99	0.96	1	1
	100	0.782	0.077	0.105	0.99	0.084	0.16	1	0.135	0.289	0.82	0.95	0.95	0.99	1	1	1	1	1
		Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																	
	20	0.092	0.089	0.083	0.077	0.066	0.107	0.095	0.101	0.108	0.21	0.38	0.34	0.48	0.76	0.65	0.65	0.93	0.95
	50	0.065	0.069	0.081	0.071	0.082	0.072	0.051	0.056	0.069	0.54	0.72	0.67	0.82	0.98	0.98	0.99	1	1
	100	0.066	0.059	0.052	0.057	0.051	0.058	0.061	0.057	0.064	0.8	0.96	0.95	0.99	1	1	1	1	1
		Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	0.088	0.09	0.085	0.083	0.069	0.103	0.093	0.1	0.104	0.25	0.38	0.34	0.49	0.76	0.66	0.66	0.92	0.94
$(\bar{y}_t, \bar{y}_t^*, \bar{\boldsymbol{x}}_t')'$	50	0.064	0.073	0.08	0.075	0.081	0.071	0.052	0.057	0.067	0.59	0.72	0.68	0.84	0.98	0.98	0.99	1	1
	100	0.064	0.058	0.053	0.054	0.055	0.061	0.063	0.056	0.062	0.82	0.96	0.95	0.99	1	1	1	1	1

Table S13 (Continued)

		Size									Power								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
		Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																	
	20	0.098	0.087	0.086	0.079	0.067	0.101	0.095	0.1	0.107	0.25	0.39	0.35	0.45	0.76	0.65	0.65	0.92	0.94
	50	0.06	0.069	0.083	0.078	0.081	0.07	0.056	0.059	0.069	0.55	0.71	0.66	0.8	0.98	0.98	0.98	1	1
	100	0.061	0.058	0.053	0.058	0.055	0.058	0.062	0.058	0.064	0.83	0.95	0.95	0.99	1	1	1	1	1
		Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	0.109	0.089	0.086	0.097	0.086	0.106	0.096	0.095	0.109	0.18)	(0.29)	(0.4)	(0.36)	(0.74)	(0.71)	(0.57)	(0.96)	(0.94)
	50	0.057	0.063	0.073	0.072	0.058	0.068	0.064	0.055	0.059	0.4)	(0.78)	(0.7)	(0.72)	(0.99)	(0.98)	(0.96)	(1)	(1)
	100	0.059	0.048	0.069	0.062	0.058	0.064	0.061	0.056	0.051	0.63)	(0.97)	(0.96)	(0.97)	(1)	(1)	(1)	(1)	(1)
		OLS																	
Infeasible	20	0.336	0.083	0.084	0.586	0.072	0.096	0.744	0.087	0.126	0.29	0.39	0.42	0.5	0.78	0.72	0.7	0.97	0.96
	50	0.566	0.086	0.111	0.904	0.079	0.125	0.983	0.092	0.165	0.61	0.76	0.72	0.89	0.99	0.99	0.97	1	1
	100	0.851	0.077	0.094	0.994	0.094	0.145	1	0.118	0.283	0.89	0.98	0.98	0.99	1	1	1	1	1
		Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																	
	20	0.082	0.083	0.066	0.07	0.068	0.065	0.081	0.064	0.071	0.24	0.38	0.41	0.59	0.77	0.73	0.73	0.96	0.95
	50	0.056	0.069	0.06	0.055	0.059	0.072	0.047	0.06	0.064	0.57	0.76	0.73	0.93	0.99	0.98	0.99	1	1
	100	0.066	0.054	0.046	0.062	0.054	0.052	0.049	0.055	0.059	0.86	0.98	0.98	0.99	1	1	1	1	1
		Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	0.069	0.078	0.059	0.08	0.067	0.071	0.08	0.064	0.074	0.27	0.4	0.42	0.52	0.78	0.75	0.62	0.96	0.94
	50	0.064	0.069	0.065	0.058	0.058	0.066	0.057	0.063	0.057	0.46	0.75	0.73	0.83	0.99	0.98	0.97	1	1
	100	0.062	0.054	0.049	0.055	0.05	0.057	0.057	0.059	0.05	0.76	0.97	0.97	0.98	1	1	1	1	1
		Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																	
	20	0.072	0.08	0.063	0.074	0.068	0.067	0.083	0.064	0.074	0.33	0.38	0.42	0.64	0.79	0.73	0.76	0.96	0.96
	50	0.064	0.07	0.065	0.059	0.061	0.067	0.038	0.062	0.062	0.61	0.75	0.73	0.89	0.99	0.98	0.99	1	1
	100	0.066	0.054	0.048	0.054	0.053	0.058	0.055	0.057	0.054	0.86	0.98	0.98	1	1	1	1	1	1
		Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	0.078	0.08	0.064	0.07	0.067	0.069	0.087	0.063	0.07	0.28	0.39	0.42	0.64	0.78	0.75	0.77	0.96	0.96
	50	0.064	0.069	0.064	0.053	0.06	0.07	0.04	0.06	0.062	0.63	0.75	0.72	0.93	0.99	0.98	1	1	1
	100	0.059	0.056	0.048	0.061	0.054	0.053	0.054	0.058	0.056	0.87	0.98	0.98	1	1	1	1	1	1
		OLS																	
Naive	20	0.781	0.076	0.089	0.96	0.077	0.108	0.992	0.067	0.124	0.22	0.44	0.39	0.39	0.7	0.72	0.39	0.91	0.93
	50	0.986	0.064	0.099	1	0.062	0.12	1	0.076	0.195	0.45	0.78	0.78	0.7	0.98	0.98	0.86	1	1
	100	0.998	0.065	0.114	1	0.055	0.192	1	0.057	0.311	0.7	0.98	0.97	0.91	1	1	0.99	1	1
		Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																	
	20	0.274	0.087	0.059	0.359	0.088	0.075	0.474	0.104	0.065	0.26	0.46	0.39	0.41	0.65	0.72	0.52	0.88	0.89
	50	0.432	0.084	0.06	0.669	0.106	0.061	0.83	0.164	0.087	0.49	0.77	0.77	0.77	0.97	0.97	0.95	1	1
	100	0.64	0.109	0.067	0.876	0.175	0.081	0.98	0.227	0.093	0.71	0.96	0.97	0.91	1	1	1	1	1
		Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	0.347	0.078	0.061	0.515	0.085	0.062	0.642	0.092	0.062	0.19	0.45	0.39	0.31	0.69	0.72	0.42	0.88	0.9
	50	0.577	0.076	0.057	0.825	0.08	0.056	0.944	0.123	0.057	0.35	0.78	0.77	0.66	0.97	0.97	0.84	1	1
	100	0.792	0.081	0.057	0.966	0.125	0.063	0.997	0.141	0.06	0.55	0.96	0.97	0.85	1	1	0.97	1	1
		Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																	
	20	0.385	0.08	0.062	0.527	0.084	0.064	0.653	0.097	0.059	0.24	0.47	0.43	0.39	0.68	0.72	0.46	0.9	0.9
	50	0.59	0.078	0.053	0.855	0.081	0.056	0.948	0.129	0.059	0.42	0.78	0.77	0.72	0.97	0.97	0.92	1	1
	100	0.806	0.083	0.052	0.976	0.135	0.059	0.998	0.15	0.064	0.64	0.96	0.97	0.89	1	1	0.99	1	1
		Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	0.336	0.083	0.06	0.448	0.085	0.069	0.563	0.102	0.061	0.22	0.47	0.4	0.4	0.68	0.72	0.48	0.89	0.91
	50	0.524	0.086	0.053	0.769	0.091	0.06	0.9	0.143	0.074	0.44	0.78	0.77	0.75	0.98	0.97	0.94	1	1
	100	0.732	0.097	0.06	0.932	0.161	0.072	0.99	0.193	0.074	0.68	0.96	0.97	0.91	1	1	1	1	1

Table S14: The Size and Power for Alternative Mean Group Estimators under Experiment 2

		Size									Power								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
$\bar{\boldsymbol{x}}_t$	OLS																		
	20	0.569	0.055	0.07	0.861	0.069	0.084	0.976	0.069	0.099	0.19	0.35	0.33	0.33	0.69	0.76	0.42	0.93	0.93
	50	0.772	0.061	0.086	0.985	0.083	0.126	1	0.098	0.201	0.43	0.66	0.71	0.74	0.98	0.98	0.93	1	1
	100	0.922	0.077	0.117	0.999	0.077	0.183	1	0.121	0.311	0.79	0.94	0.94	0.99	1	1	1	1	1
	Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																		
	20	0.063	0.048	0.051	0.062	0.06	0.059	0.061	0.058	0.052	0.2	0.35	0.35	0.4	0.69	0.7	0.58	0.94	0.92
	50	0.057	0.055	0.05	0.049	0.057	0.051	0.042	0.052	0.05	0.45	0.65	0.65	0.82	0.98	0.98	0.99	1	1
	100	0.047	0.06	0.052	0.049	0.051	0.051	0.056	0.046	0.06	0.75	0.93	0.92	0.99	1	1	1	1	1
	Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																		
	20	0.045	0.055	0.056	0.062	0.064	0.056	0.057	0.05	0.054	0.15	0.34	0.29	0.24	0.69	0.68	0.49	0.94	0.92
	50	0.047	0.05	0.051	0.059	0.059	0.061	0.058	0.054	0.051	0.33	0.62	0.66	0.62	0.97	0.97	0.91	1	1
	100	0.057	0.059	0.044	0.058	0.048	0.057	0.055	0.047	0.047	0.52	0.91	0.9	0.93	1	1	1	1	1
	Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																		
	20	0.061	0.05	0.059	0.073	0.067	0.061	0.07	0.055	0.049	0.19	0.37	0.34	0.39	0.69	0.69	0.59	0.94	0.93
	50	0.061	0.059	0.045	0.061	0.056	0.054	0.047	0.053	0.052	0.46	0.64	0.69	0.83	0.98	0.98	0.98	1	1
	100	0.06	0.06	0.046	0.051	0.047	0.058	0.054	0.046	0.053	0.72	0.93	0.94	0.99	1	1	1	1	1
	Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																		
	20	0.098	0.054	0.055	0.08	0.066	0.063	0.071	0.056	0.056	0.23	0.34	0.34	0.4	0.69	0.69	0.63	0.94	0.93
	50	0.105	0.056	0.051	0.078	0.06	0.054	0.043	0.049	0.055	0.46	0.66	0.7	0.84	0.98	0.98	0.99	1	1
	100	0.118	0.06	0.051	0.081	0.05	0.052	0.063	0.047	0.05	0.72	0.93	0.94	0.99	1	1	1	1	1
$(\bar{y}_t, \bar{\boldsymbol{x}}_t')'$	OLS																		
	20	0.155	0.058	0.066	0.241	0.057	0.087	0.341	0.079	0.074	0.22	0.32	0.31	0.43	0.75	0.67	0.59	0.93	0.95
	50	0.355	0.052	0.06	0.77	0.08	0.088	0.932	0.081	0.112	0.52	0.67	0.68	0.85	0.98	0.98	0.97	1	1
	100	0.722	0.076	0.093	0.979	0.064	0.128	1	0.105	0.211	.81	0.92	0.94	0.99	1	1	1	1	1
	Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																		
	20	0.068	0.066	0.057	0.067	0.061	0.077	0.076	0.069	0.059	0.22	0.3	0.33	0.46	0.74	0.64	0.64	0.92	0.95
	50	0.065	0.057	0.05	0.057	0.068	0.062	0.045	0.053	0.055	0.43	0.65	0.65	0.81	0.98	0.98	0.99	1	1
	100	0.055	0.063	0.046	0.054	0.055	0.058	0.057	0.056	0.052	0.68	0.92	0.93	0.99	1	1	1	1	1
	Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																		
	20	0.053	0.068	0.079	0.067	0.068	0.077	0.072	0.072	0.061	0.17	0.3	0.28	0.26	0.72	0.65	0.51	0.92	0.94
	50	0.053	0.059	0.051	0.064	0.067	0.062	0.067	0.057	0.063	0.33	0.63	0.64	0.65	0.97	0.97	0.91	1	1
	100	0.055	0.064	0.052	0.068	0.049	0.062	0.06	0.055	0.047	0.52	0.89	0.91	0.93	1	1	1	1	1
	Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																		
	20	0.063	0.062	0.065	0.075	0.063	0.08	0.072	0.071	0.058	0.23	0.29	0.33	0.44	0.74	0.64	0.66	0.93	0.95
	50	0.068	0.052	0.047	0.065	0.068	0.065	0.051	0.056	0.06	0.45	0.66	0.68	0.81	0.98	0.98	0.99	1	1
	100	0.06	0.064	0.047	0.055	0.052	0.062	0.058	0.057	0.051	0.74	0.92	0.93	0.99	1	1	1	1	1
	Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																		
	20	0.076	0.066	0.061	0.07	0.06	0.081	0.075	0.071	0.056	0.22	0.29	0.33	0.47	0.74	0.64	0.63	0.93	0.95
	50	0.074	0.059	0.051	0.076	0.07	0.063	0.046	0.053	0.056	0.46	0.65	0.66	0.85	0.98	0.98	0.99	1	1
	100	0.09	0.065	0.051	0.067	0.051	0.059	0.061	0.053	0.051	0.71	0.92	0.94	0.99	1	1	1	1	1
$(\bar{y}_t, \bar{y}_t^*, \bar{\boldsymbol{x}}_t')'$	OLS																		
	20	0.174	0.069	0.096	0.298	0.072	0.113	0.438	0.103	0.153	0.21	0.29	0.3	0.43	0.75	0.63	0.59	0.92	0.94
	50	0.368	0.053	0.096	0.785	0.088	0.137	0.949	0.1	0.209	0.49	0.6	0.62	0.83	0.99	0.98	0.96	1	1
	100	0.704	0.086	0.11	0.983	0.082	0.17	1	0.127	0.303	0.8	0.91	0.92	0.99	1	1	1	1	1
	Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																		
	20	0.073	0.074	0.084	0.07	0.062	0.096	0.078	0.088	0.108	0.19	0.27	0.3	0.43	0.75	0.62	0.59	0.92	0.93
	50	0.061	0.059	0.071	0.062	0.073	0.064	0.048	0.066	0.076	0.4	0.59	0.54	0.82	0.99	0.98	0.99	1	1
	100	0.063	0.059	0.051	0.057	0.048	0.058	0.064	0.057	0.064	0.64	0.91	0.92	0.99	1	1	1	1	1
	Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																		
	20	0.053	0.062	0.081	0.067	0.059	0.101	0.072	0.088	0.114	0.14	0.3	0.29	0.28	0.73	0.59	0.5	0.92	0.92
	50	0.058	0.057	0.062	0.066	0.075	0.07	0.064	0.066	0.079	0.3	0.6	0.55	0.63	0.98	0.96	0.92	1	1
	100	0.055	0.058	0.049	0.061	0.054	0.059	0.064	0.066	0.063	0.51	0.87	0.87	0.94	1	1	1	1	1

Table S14 (Continued)

		Size									Power								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
		Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																	
	20	0.067	0.065	0.086	0.069	0.066	0.101	0.082	0.09	0.11	0.22	0.28	0.3	0.46	0.74	0.63	0.64	0.92	0.93
	50	0.058	0.054	0.065	0.063	0.072	0.067	0.048	0.065	0.081	0.41	0.61	0.59	0.8	0.98	0.97	0.98	1	1
	100	0.062	0.065	0.054	0.053	0.053	0.057	0.058	0.06	0.06	0.68	0.9	0.91	0.99	1	1	1	1	1
		Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	0.101	0.066	0.085	0.076	0.062	0.101	0.074	0.092	0.117	0.19	0.3	0.3	0.45	0.74	0.65	0.61	0.92	0.93
	50	0.086	0.054	0.072	0.081	0.075	0.067	0.048	0.069	0.081	0.48	0.63	0.57	0.84	0.99	0.98	0.99	1	1
	100	0.114	0.064	0.053	0.072	0.052	0.065	0.068	0.061	0.065	0.69	0.91	0.92	0.99	1	1	1	1	1
Infeasible		OLS																	
	20	0.275	0.067	0.072	0.555	0.064	0.101	0.722	0.08	0.117	0.3	0.33	0.36	0.53	0.75	0.7	0.73	0.95	0.96
	50	0.498	0.062	0.079	0.887	0.084	0.114	0.981	0.093	0.174	0.57	0.72	0.68	0.89	0.99	0.99	0.98	1	1
	100	0.794	0.071	0.092	0.989	0.088	0.152	1	0.109	0.277	0.84	0.94	0.94	0.99	1	1	1	1	1
		Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																	
	20	0.06	0.063	0.058	0.068	0.069	0.064	0.067	0.059	0.062	0.25	0.34	0.37	0.54	0.73	0.65	0.74	0.95	0.95
	50	0.044	0.057	0.047	0.055	0.057	0.059	0.048	0.057	0.064	0.53	0.71	0.67	0.89	0.98	0.98	0.99	1	1
	100	0.058	0.062	0.051	0.055	0.047	0.055	0.054	0.053	0.055	0.74	0.93	0.94	0.99	1	1	1	1	1
		Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	0.05	0.062	0.062	0.067	0.06	0.075	0.061	0.065	0.057	0.24	0.31	0.32	0.48	0.75	0.69	0.67	0.95	0.94
	50	0.054	0.052	0.05	0.05	0.058	0.06	0.062	0.056	0.057	0.39	0.7	0.66	0.78	0.98	0.97	0.96	1	1
	100	0.066	0.06	0.047	0.055	0.055	0.053	0.056	0.051	0.054	0.58	0.92	0.93	0.96	1	1	1	1	1
		Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																	
	20	0.056	0.064	0.057	0.067	0.062	0.061	0.067	0.062	0.056	0.31	0.33	0.35	0.59	0.73	0.65	0.77	0.94	0.96
	50	0.05	0.053	0.053	0.046	0.055	0.061	0.047	0.056	0.057	0.53	0.74	0.67	0.9	0.99	0.98	0.99	1	1
	100	0.066	0.065	0.044	0.056	0.052	0.054	0.05	0.054	0.055	0.76	0.94	0.95	0.99	1	1	1	1	1
		Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	0.081	0.065	0.056	0.075	0.067	0.066	0.059	0.06	0.06	0.29	0.33	0.38	0.59	0.73	0.68	0.79	0.95	0.95
	50	0.074	0.052	0.054	0.057	0.059	0.06	0.044	0.057	0.061	0.58	0.72	0.69	0.91	0.99	0.98	1	1	1
	100	0.1	0.056	0.05	0.078	0.052	0.055	0.058	0.053	0.052	0.81	0.94	0.95	0.99	1	1	1	1	1
Naive		OLS																	
	20	0.751	0.061	0.069	0.957	0.073	0.091	0.992	0.064	0.115	0.25	0.41	0.37	0.32	0.67	0.74	0.4	0.94	0.94
	50	0.975	0.053	0.106	1	0.056	0.124	1	0.078	0.191	0.45	0.71	0.72	0.73	0.98	0.99	0.87	1	1
	100	0.999	0.065	0.116	1	0.06	0.187	1	0.056	0.308	0.63	0.94	0.92	0.93	1	1	0.99	1	1
		Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																	
	20	0.232	0.055	0.06	0.313	0.085	0.062	0.431	0.096	0.063	0.22	0.39	0.36	0.44	0.65	0.7	0.55	0.9	0.92
	50	0.356	0.072	0.057	0.624	0.105	0.058	0.796	0.159	0.074	0.42	0.68	0.7	0.77	0.97	0.99	0.93	1	1
	100	0.549	0.092	0.06	0.832	0.146	0.074	0.966	0.208	0.09	0.65	0.92	0.92	0.93	1	1	1	1	1
		Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	0.283	0.064	0.062	0.477	0.086	0.049	0.608	0.095	0.062	0.2	0.37	0.31	0.33	0.69	0.73	0.41	0.9	0.91
	50	0.475	0.057	0.055	0.791	0.076	0.052	0.925	0.116	0.05	0.32	0.69	0.67	0.63	0.98	0.98	0.85	1	1
	100	0.711	0.072	0.056	0.932	0.113	0.054	0.996	0.138	0.066	0.48	0.92	0.9	0.82	1	1	0.98	1	1
		Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																	
	20	0.316	0.056	0.064	0.488	0.086	0.049	0.624	0.089	0.066	0.25	0.37	0.37	0.37	0.68	0.74	0.54	0.91	0.92
	50	0.525	0.067	0.054	0.825	0.083	0.05	0.939	0.12	0.057	0.42	0.69	0.71	0.73	0.97	0.99	0.93	1	1
	100	0.759	0.076	0.053	0.953	0.118	0.048	0.998	0.151	0.07	0.56	0.94	0.91	0.89	1	1	0.99	1	1
		Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	0.361	0.06	0.064	0.469	0.087	0.056	0.573	0.093	0.06	0.24	0.4	0.37	0.38	0.68	0.71	0.55	0.91	0.92
	50	0.603	0.066	0.061	0.808	0.09	0.052	0.907	0.135	0.057	0.42	0.69	0.72	0.75	0.98	0.99	0.94	1	1
	100	0.822	0.078	0.059	0.951	0.127	0.058	0.994	0.165	0.074	0.62	0.93	0.93	0.92	1	1	1	1	1

Table S15: The Size and Power for Alternative Pooled Estimators under Experiment 3

		Size									Power								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
								OLS											
	20	0.708	0.086	0.099	0.957	0.074	0.098	0.999	0.074	0.121	0.28	0.36	0.36	0.38	0.8	0.72	0.45	0.95	0.97
	50	0.908	0.066	0.099	0.999	0.079	0.155	1	0.11	0.258	0.5	0.79	0.79	0.81	0.99	0.99	0.91	1	1
	100	0.997	0.072	0.142	1	0.105	0.281	1	0.149	0.474	0.79	0.97	0.97	0.98	1	1	1	1	1
								Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments											
	20	0.054	0.073	0.074	0.069	0.061	0.067	0.062	0.063	0.067	0.31	0.37	0.42	0.5	0.8	0.74	0.75	0.94	0.95
	50	0.049	0.055	0.049	0.055	0.049	0.059	0.049	0.04	0.052	0.57	0.78	0.79	0.91	0.99	0.99	1	1	1
	100	0.047	0.049	0.049	0.049	0.049	0.047	0.034	0.054	0.046	0.85	0.97	0.97	1	1	1	1	1	1
								Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	0.053	0.077	0.073	0.071	0.058	0.071	0.073	0.064	0.072	0.18	0.36	0.39	0.31	0.81	0.76	0.49	0.93	0.94
$\bar{\boldsymbol{x}}_t$	50	0.049	0.052	0.05	0.054	0.047	0.057	0.048	0.037	0.048	0.43	0.78	0.79	0.77	0.99	0.99	0.97	1	1
	100	0.045	0.054	0.049	0.047	0.047	0.048	0.038	0.055	0.045	0.72	0.97	0.97	0.98	1	1	1	1	1
								Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments											
	20	0.061	0.072	0.078	0.071	0.059	0.067	0.065	0.063	0.071	0.3	0.37	0.4	0.47	0.81	0.76	0.77	0.94	0.94
	50	0.055	0.055	0.052	0.054	0.049	0.06	0.048	0.041	0.05	0.56	0.78	0.79	0.91	0.99	0.99	1	1	1
	100	0.044	0.051	0.049	0.039	0.048	0.045	0.041	0.054	0.045	0.85	0.97	0.97	1	1	1	1	1	1
								Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	0.05	0.072	0.077	0.066	0.058	0.067	0.063	0.063	0.07	0.33	0.37	0.41	0.51	0.81	0.73	0.79	0.93	0.94
	50	0.054	0.056	0.048	0.064	0.048	0.06	0.055	0.039	0.051	0.58	0.78	0.8	0.92	0.99	0.99	1	1	1
	100	0.045	0.052	0.048	0.044	0.048	0.046	0.037	0.053	0.046	0.87	0.97	0.97	1	1	1	1	1	1
								OLS											
	20	0.161	0.075	0.084	0.304	0.079	0.088	0.473	0.087	0.085	0.3	0.42	0.4	0.51	0.8	0.77	0.75	0.94	0.97
	50	0.46	0.061	0.081	0.83	0.068	0.098	0.982	0.079	0.121	0.66	0.79	0.73	0.9	0.99	0.99	0.99	1	1
	100	0.81	0.075	0.085	0.997	0.073	0.161	1	0.102	0.233	0.86	0.96	0.96	1	1	1	1	1	1
								Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments											
	20	0.069	0.069	0.084	0.083	0.077	0.09	0.077	0.086	0.077	0.29	0.43	0.4	0.52	0.79	0.76	0.81	0.95	0.96
	50	0.065	0.059	0.065	0.074	0.055	0.065	0.06	0.058	0.068	0.55	0.79	0.73	0.89	0.99	0.99	0.99	1	1
	100	0.051	0.062	0.065	0.053	0.049	0.054	0.041	0.059	0.048	0.84	0.96	0.96	1	1	1	1	1	1
								Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
$(\bar{y}_t, \bar{\boldsymbol{x}}_t')'$	20	0.074	0.081	0.083	0.079	0.081	0.088	0.087	0.087	0.084	0.19	0.42	0.41	0.35	0.79	0.78	0.58	0.94	0.96
	50	0.058	0.055	0.063	0.05	0.052	0.065	0.054	0.06	0.06	0.43	0.8	0.74	0.81	0.99	0.99	0.98	1	1
	100	0.052	0.065	0.062	0.056	0.045	0.055	0.049	0.06	0.045	0.72	0.96	0.96	0.98	1	1	1	1	1
								Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments											
	20	0.077	0.074	0.087	0.083	0.075	0.09	0.079	0.086	0.076	0.29	0.42	0.41	0.53	0.79	0.77	0.81	0.95	0.96
	50	0.058	0.056	0.065	0.063	0.054	0.066	0.059	0.059	0.066	0.53	0.8	0.74	0.91	0.99	0.99	1	1	1
	100	0.047	0.06	0.061	0.046	0.048	0.055	0.04	0.06	0.048	0.86	0.97	0.96	1	1	1	1	1	1
								Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	0.065	0.073	0.085	0.084	0.077	0.092	0.081	0.086	0.077	0.29	0.43	0.41	0.55	0.79	0.76	0.8	0.95	0.96
	50	0.059	0.057	0.063	0.064	0.054	0.065	0.061	0.058	0.068	0.59	0.79	0.74	0.92	0.99	0.99	1	1	1
	100	0.047	0.064	0.063	0.056	0.048	0.055	0.035	0.059	0.046	0.89	0.96	0.96	1	1	1	1	1	1
								OLS											
	20	0.207	0.084	0.092	0.377	0.081	0.117	0.589	0.099	0.144	0.26	0.39	0.36	0.5	0.81	0.75	0.72	0.96	0.97
	50	0.493	0.064	0.099	0.849	0.079	0.14	0.984	0.106	0.205	0.58	0.77	0.71	0.91	0.99	0.99	0.99	1	1
	100	0.808	0.08	0.1	0.998	0.085	0.199	1	0.124	0.329	0.87	0.96	0.95	1	1	1	1	1	1
								Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments											
	20	0.084	0.083	0.087	0.088	0.073	0.097	0.078	0.089	0.098	0.24	0.37	0.36	0.5	0.8	0.74	0.78	0.95	0.97
	50	0.063	0.062	0.068	0.063	0.063	0.068	0.063	0.063	0.07	0.49	0.75	0.72	0.9	0.99	0.99	1	1	1
	100	0.056	0.068	0.063	0.052	0.044	0.062	0.039	0.063	0.062	0.81	0.97	0.95	1	1	1	1	1	1
								Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	0.077	0.081	0.086	0.093	0.074	0.095	0.083	0.089	0.097	0.25	0.38	0.35	0.51	0.8	0.74	0.78	0.95	0.97
$(\bar{y}_t, \bar{y}_t^*, \bar{\boldsymbol{x}}_t')'$	50	0.063	0.058	0.066	0.058	0.067	0.068	0.059	0.062	0.072	0.52	0.76	0.72	0.91	0.99	0.99	1	1	1
	100	0.056	0.066	0.065	0.052	0.045	0.06	0.034	0.062	0.063	0.83	0.96	0.95	1	1	1	1	1	1

Table S15 (Continued)

		Size									Power								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
		Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																	
	20	0.088	0.083	0.086	0.1	0.074	0.1	0.081	0.091	0.099	0.26	0.38	0.35	0.5	0.81	0.74	0.79	0.95	0.97
	50	0.061	0.055	0.072	0.056	0.066	0.069	0.063	0.059	0.071	0.49	0.77	0.71	0.91	0.99	0.99	1	1	1
	100	0.056	0.067	0.063	0.049	0.046	0.056	0.039	0.061	0.063	0.82	0.97	0.95	1	1	1	1	1	1
		Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	0.084	0.073	0.066	0.088	0.091	0.082	0.093	0.08	0.078	0.16	0.75	0.73	0.18	0.63	0.73	0.22	0.95	0.93
	50	0.064	0.071	0.077	0.063	0.062	0.066	0.051	0.058	0.064	0.19	0.77	0.74	0.44	1	0.99	0.71	1	1
	100	0.063	0.062	0.058	0.067	0.047	0.065	0.054	0.054	0.063	0.39	0.96	0.97	0.7	1	1	0.97	1	1
Infeasible		OLS																	
	20	0.325	0.084	0.09	0.622	0.079	0.115	0.86	0.078	0.141	0.37	0.38	0.48	0.67	0.82	0.79	0.87	0.97	0.98
	50	0.58	0.058	0.096	0.911	0.07	0.142	0.996	0.085	0.218	0.74	0.81	0.74	0.98	0.99	1	1	1	1
	100	0.831	0.062	0.108	0.998	0.086	0.236	1	0.109	0.382	0.94	0.98	0.98	1	1	1	1	1	1
		Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																	
	20	0.077	0.081	0.072	0.071	0.076	0.076	0.064	0.068	0.067	0.39	0.38	0.43	0.72	0.83	0.77	0.94	0.98	0.98
	50	0.071	0.06	0.073	0.062	0.061	0.066	0.064	0.047	0.053	0.69	0.81	0.74	0.97	0.99	1	1	1	1
	100	0.056	0.061	0.062	0.048	0.052	0.051	0.046	0.065	0.047	0.92	0.98	0.98	1	1	1	1	1	1
		Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	0.065	0.081	0.07	0.076	0.07	0.079	0.082	0.07	0.076	0.31	0.37	0.41	0.58	0.84	0.76	0.84	0.97	0.98
	50	0.063	0.064	0.069	0.045	0.058	0.061	0.058	0.05	0.055	0.59	0.81	0.73	0.93	1	1	0.99	1	1
	100	0.046	0.052	0.065	0.041	0.053	0.047	0.039	0.067	0.045	0.83	0.99	0.98	1	1	1	1	1	1
		Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																	
	20	0.08	0.079	0.07	0.072	0.071	0.077	0.062	0.073	0.074	0.38	0.37	0.42	0.71	0.84	0.76	0.96	0.98	0.98
	50	0.076	0.06	0.065	0.052	0.061	0.064	0.067	0.049	0.054	0.68	0.81	0.73	0.97	0.99	1	1	1	1
	100	0.058	0.055	0.058	0.048	0.055	0.048	0.038	0.061	0.05	0.92	0.99	0.98	1	1	1	1	1	1
		Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	0.076	0.08	0.069	0.066	0.073	0.077	0.076	0.07	0.07	0.38	0.38	0.42	0.75	0.83	0.78	0.96	0.98	0.97
	50	0.072	0.062	0.07	0.06	0.061	0.063	0.071	0.049	0.055	0.72	0.82	0.74	0.98	0.99	1	1	1	1
	100	0.052	0.057	0.063	0.051	0.053	0.05	0.042	0.065	0.044	0.93	0.99	0.98	1	1	1	1	1	1
Naive		OLS																	
	20	0.716	0.073	0.129	0.94	0.066	0.167	0.997	0.074	0.284	0.29	0.41	0.43	0.43	0.79	0.76	0.54	0.94	0.98
	50	0.94	0.067	0.168	0.999	0.069	0.322	1	0.06	0.522	0.49	0.8	0.79	0.84	0.99	0.99	0.92	1	1
	100	0.995	0.053	0.257	1	0.074	0.536	1	0.082	0.782	0.65	0.98	0.97	0.96	1	1	1	1	1
		Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																	
	20	0.128	0.081	0.08	0.168	0.11	0.083	0.205	0.142	0.072	0.33	0.41	0.42	0.55	0.75	0.75	0.75	0.93	0.95
	50	0.17	0.115	0.062	0.225	0.171	0.064	0.338	0.271	0.07	0.53	0.77	0.8	0.83	0.98	0.99	0.97	1	1
	100	0.233	0.145	0.073	0.37	0.278	0.065	0.519	0.476	0.079	0.69	0.98	0.97	0.98	1	1	1	1	1
		Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	0.17	0.082	0.084	0.224	0.101	0.084	0.277	0.146	0.084	0.25	0.41	0.4	0.42	0.75	0.78	0.53	0.94	0.96
	50	0.235	0.115	0.066	0.322	0.144	0.068	0.474	0.229	0.089	0.37	0.79	0.81	0.65	0.98	1	0.89	1	1
	100	0.331	0.127	0.073	0.484	0.232	0.094	0.667	0.411	0.118	0.52	0.98	0.97	0.83	1	1	0.97	1	1
		Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																	
	20	0.195	0.081	0.083	0.245	0.105	0.085	0.294	0.141	0.077	0.32	0.4	0.43	0.54	0.75	0.76	0.66	0.93	0.96
	50	0.261	0.114	0.068	0.337	0.149	0.074	0.517	0.244	0.089	0.48	0.8	0.8	0.75	0.98	0.99	0.94	1	1
	100	0.345	0.132	0.071	0.539	0.241	0.091	0.715	0.429	0.103	0.61	0.98	0.98	0.9	1	1	0.99	1	1
		Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	0.16	0.082	0.081	0.221	0.109	0.086	0.251	0.142	0.075	0.34	0.41	0.43	0.5	0.74	0.75	0.7	0.93	0.95
	50	0.204	0.114	0.067	0.285	0.16	0.07	0.418	0.257	0.079	0.5	0.77	0.81	0.78	0.98	1	0.97	1	1
	100	0.286	0.139	0.07	0.448	0.261	0.079	0.621	0.462	0.095	0.68	0.98	0.97	0.96	1	1	1	1	1

Table S16: The Size and Power for Alternative Mean Group Estimators under Experiment 3

		Size									Power								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
OLS																			
	20	0.615	0.068	0.084	0.948	0.062	0.088	0.998	0.07	0.108	0.19	0.33	0.36	0.38	0.78	0.69	0.47	0.94	0.96
	50	0.852	0.059	0.081	1	0.076	0.146	1	0.107	0.245	0.45	0.71	0.74	0.78	0.99	0.98	0.92	1	1
	100	0.984	0.07	0.121	1	0.101	0.251	1	0.159	0.479	0.78	0.93	0.94	0.98	1	1	1	1	1
Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																			
	20	0.059	0.06	0.061	0.063	0.047	0.062	0.055	0.054	0.048	0.17	0.34	0.35	0.43	0.77	0.65	0.69	0.95	0.95
	50	0.058	0.047	0.041	0.046	0.054	0.061	0.056	0.036	0.051	0.4	0.69	0.72	0.86	0.99	0.98	1	1	1
	100	0.046	0.05	0.05	0.055	0.044	0.052	0.035	0.051	0.049	0.74	0.93	0.93	1	1	1	1	1	1
Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																			
	20	0.056	0.055	0.065	0.072	0.06	0.068	0.078	0.052	0.054	0.13	0.3	0.28	0.27	0.73	0.67	0.45	0.93	0.93
$\bar{\boldsymbol{x}}_t$	50	0.059	0.051	0.037	0.049	0.05	0.057	0.055	0.039	0.053	0.27	0.63	0.68	0.7	0.99	0.98	0.94	1	1
	100	0.034	0.048	0.038	0.054	0.038	0.048	0.038	0.056	0.046	0.59	0.91	0.9	0.94	1	1	1	1	1
Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																			
	20	0.058	0.061	0.066	0.065	0.055	0.067	0.062	0.05	0.054	0.19	0.34	0.32	0.43	0.79	0.67	0.72	0.95	0.94
	50	0.048	0.048	0.046	0.055	0.049	0.065	0.039	0.041	0.054	0.43	0.68	0.72	0.87	0.99	0.99	0.99	1	1
	100	0.035	0.053	0.047	0.04	0.047	0.045	0.042	0.053	0.048	0.75	0.93	0.93	1	1	1	1	1	1
Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																			
	20	0.099	0.066	0.067	0.074	0.051	0.07	0.071	0.053	0.052	0.2	0.33	0.35	0.42	0.77	0.65	0.72	0.95	0.95
	50	0.122	0.044	0.047	0.09	0.053	0.061	0.07	0.038	0.054	0.47	0.71	0.73	0.88	0.99	0.98	1	1	1
	100	0.14	0.049	0.054	0.085	0.046	0.057	0.053	0.052	0.049	0.8	0.94	0.94	1	1	1	1	1	1
OLS																			
	20	0.143	0.062	0.071	0.286	0.077	0.079	0.462	0.078	0.076	0.19	0.35	0.38	0.46	0.77	0.75	0.74	0.94	0.97
	50	0.41	0.053	0.068	0.785	0.068	0.096	0.983	0.071	0.114	0.54	0.69	0.65	0.85	0.99	0.98	0.99	1	1
	100	0.754	0.063	0.089	0.996	0.075	0.149	1	0.106	0.229	0.84	0.94	0.92	1	1	1	1	1	1
Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																			
	20	0.068	0.065	0.067	0.071	0.068	0.083	0.075	0.077	0.067	0.19	0.34	0.33	0.49	0.76	0.72	0.78	0.93	0.97
	50	0.055	0.058	0.062	0.066	0.055	0.063	0.059	0.049	0.059	0.46	0.68	0.67	0.87	0.99	0.98	1	1	1
	100	0.056	0.05	0.051	0.057	0.048	0.054	0.037	0.055	0.056	0.74	0.93	0.91	1	1	1	1	1	1
Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																			
	20	0.051	0.067	0.058	0.059	0.07	0.085	0.073	0.076	0.063	0.17	0.3	0.31	0.33	0.74	0.7	0.56	0.94	0.95
$(\bar{y}_t, \bar{\boldsymbol{x}}_t')'$	50	0.052	0.054	0.05	0.058	0.05	0.073	0.065	0.048	0.054	0.31	0.6	0.63	0.71	0.99	0.97	0.95	1	1
	100	0.047	0.053	0.054	0.057	0.052	0.055	0.043	0.053	0.049	0.56	0.91	0.89	0.95	1	1	1	1	1
Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																			
	20	0.063	0.064	0.068	0.066	0.072	0.082	0.072	0.076	0.07	0.22	0.33	0.34	0.49	0.79	0.73	0.77	0.94	0.96
	50	0.053	0.046	0.061	0.051	0.053	0.062	0.06	0.048	0.056	0.44	0.7	0.67	0.87	0.99	0.98	0.99	1	1
	100	0.048	0.051	0.055	0.046	0.047	0.053	0.039	0.056	0.056	0.72	0.93	0.91	1	1	1	1	1	1
Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																			
	20	0.063	0.066	0.069	0.077	0.069	0.081	0.08	0.076	0.063	0.22	0.34	0.36	0.49	0.77	0.73	0.76	0.94	0.97
	50	0.072	0.051	0.053	0.075	0.054	0.071	0.061	0.048	0.056	0.49	0.69	0.69	0.88	0.99	0.98	1	1	1
	100	0.073	0.055	0.058	0.067	0.048	0.059	0.039	0.057	0.052	0.81	0.94	0.92	1	1	1	1	1	1
OLS																			
	20	0.16	0.075	0.084	0.339	0.084	0.118	0.559	0.101	0.172	0.17	0.31	0.34	0.46	0.77	0.69	0.69	0.95	0.96
	50	0.408	0.068	0.085	0.82	0.077	0.142	0.988	0.108	0.208	0.53	0.66	0.67	0.84	0.99	0.98	0.99	1	1
	100	0.735	0.068	0.103	0.997	0.091	0.205	1	0.13	0.352	0.8	0.93	0.91	1	1	1	1	1	1
Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																			
	20	0.061	0.084	0.067	0.073	0.08	0.095	0.076	0.087	0.111	0.18	0.29	0.33	0.47	0.76	0.67	0.76	0.95	0.96
	50	0.06	0.053	0.065	0.06	0.061	0.076	0.062	0.06	0.077	0.41	0.67	0.64	0.87	0.99	0.98	0.99	1	1
	100	0.045	0.06	0.058	0.052	0.055	0.059	0.038	0.06	0.058	0.71	0.92	0.9	1	1	1	1	1	1
Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																			
	20	0.05	0.07	0.064	0.076	0.08	0.087	0.077	0.092	0.121	0.16	0.28	0.29	0.32	0.75	0.66	0.48	0.94	0.94
$(\bar{y}_t, \bar{y}_t^*, \bar{\boldsymbol{x}}_t')'$	50	0.054	0.054	0.062	0.061	0.053	0.072	0.066	0.061	0.084	0.25	0.61	0.6	0.69	0.99	0.98	0.95	1	1
	100	0.049	0.051	0.054	0.053	0.046	0.057	0.049	0.061	0.061	0.5	0.89	0.84	0.96	1	1	1	1	1

Table S16 (Continued)

		Size									Power								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
	Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																		
	20	0.069	0.073	0.067	0.073	0.082	0.096	0.084	0.086	0.114	0.2	0.31	0.35	0.48	0.76	0.69	0.72	0.95	0.95
	50	0.066	0.057	0.06	0.048	0.056	0.071	0.06	0.06	0.085	0.42	0.64	0.65	0.86	0.99	0.98	0.99	1	1
	100	0.059	0.059	0.054	0.049	0.05	0.058	0.036	0.065	0.056	0.68	0.91	0.9	1	1	1	1	1	1
	Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																		
	20	0.072	0.078	0.068	0.087	0.079	0.098	0.082	0.085	0.117	0.19	0.29	0.33	0.47	0.76	0.7	0.77	0.95	0.96
	50	0.088	0.059	0.065	0.079	0.059	0.078	0.072	0.058	0.08	0.47	0.65	0.65	0.89	0.99	0.99	1	1	1
	100	0.084	0.061	0.056	0.062	0.052	0.058	0.046	0.064	0.064	0.78	0.92	0.89	1	1	1	1	1	1
Infeasible	OLS																		
	20	0.278	0.061	0.078	0.566	0.069	0.095	0.852	0.075	0.134	0.28	0.39	0.39	0.62	0.81	0.81	0.87	0.97	0.97
	50	0.52	0.057	0.074	0.891	0.072	0.134	0.995	0.069	0.199	0.65	0.71	0.7	0.95	0.99	0.99	1	1	1
	100	0.797	0.058	0.096	0.997	0.079	0.215	1	0.105	0.372	0.9	0.98	0.95	1	1	1	1	1	1
	Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																		
	20	0.077	0.057	0.064	0.06	0.056	0.054	0.061	0.065	0.066	0.3	0.38	0.38	0.65	0.79	0.77	0.92	0.97	0.97
	50	0.065	0.055	0.059	0.063	0.052	0.065	0.072	0.049	0.055	0.56	0.69	0.72	0.95	0.99	0.99	1	1	1
	100	0.046	0.036	0.049	0.04	0.059	0.057	0.043	0.059	0.053	0.84	0.97	0.95	1	1	1	1	1	1
	Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																		
	20	0.065	0.058	0.064	0.068	0.057	0.056	0.075	0.069	0.072	0.24	0.38	0.34	0.51	0.8	0.8	0.78	0.96	0.96
	50	0.056	0.057	0.046	0.051	0.048	0.064	0.063	0.05	0.053	0.41	0.62	0.72	0.89	0.99	0.98	0.99	1	1
	100	0.042	0.037	0.052	0.041	0.054	0.052	0.048	0.061	0.044	0.71	0.95	0.92	1	1	1	1	1	1
	Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																		
	20	0.08	0.058	0.061	0.059	0.055	0.048	0.058	0.064	0.067	0.29	0.39	0.4	0.69	0.81	0.8	0.92	0.97	0.96
	50	0.061	0.056	0.049	0.056	0.049	0.063	0.07	0.045	0.054	0.56	0.7	0.72	0.95	0.99	0.99	1	1	1
	100	0.041	0.035	0.051	0.036	0.063	0.052	0.039	0.06	0.054	0.86	0.97	0.94	1	1	1	1	1	1
	Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																		
	20	0.09	0.059	0.066	0.063	0.053	0.053	0.067	0.067	0.066	0.3	0.39	0.38	0.66	0.81	0.8	0.92	0.97	0.96
	50	0.081	0.058	0.05	0.06	0.049	0.068	0.07	0.049	0.055	0.62	0.68	0.72	0.97	0.99	0.99	1	1	1
	100	0.079	0.039	0.049	0.058	0.061	0.056	0.047	0.06	0.052	0.88	0.98	0.95	1	1	1	1	1	1
Naive	OLS																		
	20	0.651	0.058	0.106	0.944	0.061	0.156	0.996	0.058	0.26	0.27	0.38	0.37	0.43	0.77	0.78	0.55	0.94	0.96
	50	0.913	0.065	0.142	1	0.064	0.273	1	0.054	0.496	0.47	0.7	0.73	0.81	0.98	0.99	0.94	1	1
	100	0.997	0.042	0.229	1	0.068	0.488	1	0.071	0.769	0.67	0.97	0.95	0.98	1	1	1	1	1
	Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																		
	20	0.115	0.067	0.062	0.158	0.09	0.066	0.186	0.122	0.066	0.27	0.35	0.36	0.51	0.74	0.74	0.68	0.92	0.95
	50	0.151	0.091	0.05	0.198	0.133	0.061	0.298	0.243	0.068	0.46	0.66	0.73	0.83	0.98	0.99	0.97	1	1
	100	0.184	0.114	0.05	0.313	0.232	0.065	0.469	0.405	0.066	0.62	0.94	0.95	0.98	1	1	1	1	1
	Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																		
	20	0.13	0.067	0.069	0.208	0.08	0.069	0.242	0.113	0.066	0.23	0.33	0.32	0.36	0.75	0.77	0.5	0.93	0.95
	50	0.194	0.091	0.048	0.29	0.12	0.066	0.438	0.205	0.081	0.34	0.64	0.74	0.66	0.98	0.98	0.88	1	1
	100	0.261	0.105	0.055	0.436	0.204	0.081	0.631	0.351	0.094	0.45	0.95	0.94	0.85	1	1	0.98	1	1
	Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																		
	20	0.15	0.07	0.061	0.214	0.087	0.066	0.265	0.112	0.065	0.26	0.36	0.36	0.46	0.76	0.76	0.66	0.93	0.95
	50	0.237	0.092	0.049	0.302	0.129	0.071	0.473	0.218	0.074	0.43	0.66	0.72	0.72	0.98	0.99	0.93	1	1
	100	0.287	0.108	0.056	0.497	0.207	0.074	0.664	0.368	0.091	0.52	0.94	0.94	0.94	1	1	0.99	1	1
	Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																		
	20	0.191	0.066	0.072	0.238	0.085	0.075	0.259	0.114	0.068	0.25	0.37	0.35	0.47	0.75	0.76	0.68	0.93	0.95
	50	0.306	0.083	0.058	0.351	0.12	0.071	0.472	0.22	0.076	0.47	0.65	0.74	0.78	0.98	0.98	0.96	1	1
	100	0.436	0.089	0.066	0.561	0.2	0.086	0.686	0.369	0.089	0.63	0.95	0.95	0.97	1	1	1	1	1

S2.3 Supplementary Results for $\rho = 0.2$

This subsection provides Monte Carlo simulation results for $\rho = 0.2$ while all the other settings remain the same as in the main text. And to save space, we only report the results for the most general DGP, that is Experiment 4. [2] The influence of the decrease in the spatial coefficient on the simulation results is very limited and indeed almost negligible compared to those reported in the main text. Therefore, the same findings apply here.

[2] The results for the other three experiments are available upon request.

Table S17: The Finite Sample Performance of Alternative Pooled Estimators under Experiment 4 with $\rho = 0.2$

		Bias (× 100)									RMSE (×) 100								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
OLS																			
	20	5.84	0	-0.19	5.96	0.14	0.04	5.86	-0.04	-0.06	6.76	3.68	3.64	6.8	3.62	3.8	6.42	2.82	2.67
	50	5.21	-0.16	-0.58	5.22	-0.15	-0.47	5.22	-0.25	-0.41	6.09	3.71	3.77	5.61	2.36	2.38	5.46	1.68	1.81
	100	4.91	-0.3	-0.49	4.83	-0.2	-0.45	4.96	-0.27	-0.42	5.39	2.59	2.7	5.06	1.66	1.68	5.08	1.23	1.23
Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																			
	20	-0.05	-0.04	-0.26	0.01	0.11	-0.02	-0.03	-0.05	-0.11	3.23	3.66	3.62	3.12	3.59	3.75	2.4	2.8	2.65
	50	0.04	-0.01	-0.26	-0.03	0.01	-0.13	-0.02	-0.08	-0.06	3.02	3.68	3.71	1.94	2.35	2.32	1.41	1.66	1.75
	100	-0.01	-0.1	-0.08	-0.01	0.01	-0.03	0.07	-0.05	0.01	2.07	2.56	2.64	1.35	1.64	1.61	1	1.2	1.15
Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																			
	20	-0.12	-0.03	-0.27	-0.2	0.11	-0.04	-0.05	-0.05	-0.1	5.05	3.67	3.63	4.99	3.6	3.76	3.58	2.79	2.65
$\bar{\boldsymbol{x}}_t$	50	0.07	-0.02	-0.26	0.1	0	-0.15	0.01	-0.08	-0.06	4.22	3.68	3.71	2.78	2.34	2.33	1.89	1.66	1.75
	100	0.06	-0.1	-0.08	-0.01	0.01	-0.03	0.04	-0.05	0.02	2.82	2.56	2.65	1.84	1.64	1.62	1.29	1.2	1.15
Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																			
	20	-0.05	-0.04	-0.26	-0.03	0.11	-0.03	-0.06	-0.05	-0.11	3.43	3.67	3.62	3.39	3.59	3.75	2.57	2.8	2.65
	50	0.05	-0.02	-0.26	0.01	0	-0.14	-0.02	-0.08	-0.06	3.18	3.68	3.71	2.1	2.34	2.32	1.48	1.66	1.75
	100	0.01	-0.1	-0.08	-0.02	0.01	-0.03	0.06	-0.05	0.02	2.19	2.56	2.65	1.42	1.64	1.61	1.03	1.2	1.15
Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																			
	20	-0.03	-0.04	-0.26	0.02	0.11	-0.02	-0.02	-0.05	-0.11	3.21	3.66	3.62	3.11	3.59	3.75	2.38	2.8	2.65
	50	0.07	-0.01	-0.26	-0.01	0.01	-0.14	-0.01	-0.08	-0.06	3	3.68	3.71	1.94	2.35	2.32	1.4	1.66	1.75
	100	0	-0.1	-0.08	-0.01	0.01	-0.03	0.07	-0.05	0.01	2.05	2.56	2.64	1.34	1.64	1.61	1	1.2	1.15
OLS																			
	20	-0.5	-0.05	-0.3	-0.39	0.12	-0.02	-0.37	-0.06	-0.14	3.33	3.67	3.65	3.12	3.61	3.76	2.43	2.79	2.63
	50	1.45	-0.12	-0.36	1.44	-0.03	-0.23	1.39	-0.14	-0.14	3.31	3.77	3.79	2.39	2.34	2.28	1.97	1.66	1.73
	100	1.98	-0.15	-0.25	1.88	-0.06	-0.22	2.03	-0.15	-0.16	2.81	2.61	2.67	2.31	1.63	1.62	2.26	1.21	1.15
Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																			
	20	-0.14	-0.05	-0.29	-0.12	0.11	-0.02	-0.14	-0.06	-0.14	3.22	3.66	3.64	3.09	3.61	3.75	2.37	2.79	2.63
	50	0.06	-0.08	-0.29	-0.04	0.01	-0.14	-0.04	-0.09	-0.04	3.09	3.77	3.79	1.93	2.33	2.27	1.4	1.66	1.72
	100	0	-0.07	-0.09	-0.02	0.03	-0.06	0.06	-0.06	0.01	2.07	2.61	2.66	1.34	1.63	1.6	1	1.2	1.14
Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																			
	20	-0.32	-0.04	-0.31	-0.45	0.11	-0.04	-0.3	-0.06	-0.14	4.87	3.67	3.65	4.95	3.62	3.76	3.49	2.78	2.63
$(\bar{y}_t, \bar{\boldsymbol{x}}_t')'$	50	0.05	-0.09	-0.28	0.07	0.01	-0.15	-0.02	-0.09	-0.04	4.22	3.76	3.78	2.75	2.33	2.28	1.88	1.65	1.72
	100	0.09	-0.07	-0.09	-0.02	0.03	-0.05	0.03	-0.06	0.02	2.83	2.6	2.67	1.81	1.63	1.61	1.28	1.2	1.14
Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																			
	20	-0.2	-0.04	-0.3	-0.24	0.11	-0.03	-0.24	-0.06	-0.14	3.35	3.67	3.65	3.32	3.61	3.75	2.5	2.79	2.63
	50	0.05	-0.08	-0.28	-0.02	0.01	-0.14	-0.04	-0.09	-0.04	3.21	3.77	3.79	2.09	2.33	2.27	1.48	1.66	1.72
	100	0.03	-0.07	-0.09	-0.03	0.03	-0.05	0.05	-0.06	0.01	2.18	2.6	2.66	1.41	1.63	1.6	1.02	1.2	1.14
Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																			
	20	-0.14	-0.05	-0.29	-0.12	0.11	-0.02	-0.14	-0.06	-0.14	3.19	3.66	3.64	3.08	3.61	3.75	2.35	2.79	2.63
	50	0.08	-0.08	-0.29	-0.02	0.01	-0.14	-0.03	-0.09	-0.04	3.06	3.77	3.78	1.93	2.33	2.27	1.39	1.66	1.72
	100	0.02	-0.07	-0.09	-0.02	0.03	-0.06	0.06	-0.06	0.01	2.06	2.61	2.66	1.33	1.63	1.6	0.99	1.2	1.14
OLS																			
	20	0.19	-0.78	-1.8	0.3	-0.62	-1.51	0.35	-0.79	-1.65	3.36	3.79	4.17	3.15	3.7	4.12	2.45	2.91	3.16
	50	1.75	-0.39	-1	1.72	-0.35	-0.83	1.68	-0.45	-0.74	3.55	3.83	4.07	2.59	2.36	2.45	2.19	1.72	1.88
	100	2.14	-0.31	-0.56	2.03	-0.2	-0.51	2.17	-0.29	-0.46	2.98	2.68	2.77	2.44	1.64	1.7	2.39	1.23	1.24
Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																			
	20	0.58	-0.79	-1.81	0.58	-0.63	-1.52	0.6	-0.8	-1.65	3.36	3.79	4.17	3.21	3.7	4.12	2.46	2.91	3.17
	50	0.35	-0.35	-0.91	0.23	-0.3	-0.72	0.24	-0.39	-0.62	3.19	3.82	4.05	1.97	2.35	2.42	1.43	1.71	1.83
	100	0.13	-0.22	-0.38	0.11	-0.11	-0.34	0.19	-0.2	-0.27	2.17	2.67	2.74	1.35	1.63	1.66	1.01	1.21	1.18
Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																			
	20	0.6	-0.79	-1.81	0.59	-0.63	-1.52	0.62	-0.8	-1.65	3.33	3.79	4.17	3.2	3.7	4.12	2.46	2.91	3.17
$(\bar{y}_t, \bar{y}_t^{*}, \bar{\boldsymbol{x}}_t')'$	50	0.37	-0.35	-0.91	0.25	-0.3	-0.72	0.24	-0.39	-0.62	3.16	3.82	4.05	1.97	2.35	2.42	1.42	1.71	1.83
	100	0.15	-0.22	-0.38	0.11	-0.11	-0.34	0.19	-0.2	-0.27	2.16	2.67	2.74	1.34	1.63	1.66	1	1.21	1.18

Table S17 (Continued)

		Bias (× 100)									RMSE (×) 100								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
								Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments											
	20	0.65	-0.79	-1.81	0.6	-0.63	-1.52	0.63	-0.8	-1.66	3.48	3.79	4.17	3.44	3.7	4.12	2.61	2.91	3.17
	50	0.38	-0.35	-0.9	0.29	-0.3	-0.72	0.26	-0.4	-0.62	3.32	3.82	4.05	2.13	2.35	2.42	1.51	1.71	1.83
	100	0.17	-0.22	-0.38	0.12	-0.11	-0.34	0.19	-0.2	-0.27	2.28	2.67	2.74	1.43	1.63	1.66	1.04	1.21	1.18
								Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	0.6	-0.79	-1.79	0.63	-0.62	-1.49	0.6	-0.79	-1.66	3.48	3.8	4.2	3.29	3.72	4.15	2.52	2.94	3.18
	50	0.39	-0.31	-0.92	0.24	-0.31	-0.71	0.24	-0.39	-0.62	3.24	3.84	4.07	1.99	2.36	2.44	1.44	1.71	1.84
	100	0.15	-0.22	-0.38	0.11	-0.11	-0.34	0.19	-0.2	-0.27	2.19	2.68	2.75	1.36	1.63	1.66	1.02	1.22	1.18
								OLS											
	20	1.98	-0.13	-0.59	2.14	0	-0.36	2.02	-0.18	-0.51	3.37	3.51	3.37	3.38	3.41	3.52	2.87	2.62	2.52
	50	2.17	-0.11	-0.56	2.12	-0.07	-0.52	2.09	-0.21	-0.41	3.4	3.54	3.75	2.73	2.27	2.27	2.44	1.63	1.71
	100	2.23	-0.2	-0.44	2.1	-0.1	-0.4	2.2	-0.18	-0.35	2.89	2.48	2.5	2.43	1.57	1.63	2.39	1.18	1.15
								Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments											
	20	0	-0.02	-0.24	0.09	0.12	0	-0.04	-0.05	-0.14	2.76	3.5	3.32	2.64	3.41	3.5	2.02	2.61	2.47
	50	0.07	0	-0.2	-0.03	0.05	-0.14	-0.02	-0.09	-0.03	2.75	3.53	3.72	1.73	2.27	2.22	1.27	1.62	1.66
	100	0.05	-0.09	-0.07	-0.01	0.02	-0.03	0.05	-0.07	0.02	1.91	2.46	2.46	1.21	1.57	1.58	0.91	1.17	1.09
								Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
Infeasible	20	0	-0.02	-0.24	0.01	0.13	0	-0.06	-0.05	-0.14	3.5	3.51	3.33	3.5	3.42	3.54	2.55	2.61	2.48
	50	0.13	0	-0.21	0.05	0.05	-0.16	0	-0.1	-0.03	3.41	3.54	3.73	2.23	2.27	2.23	1.58	1.61	1.67
	100	0.13	-0.09	-0.08	0.01	0.02	-0.03	0.01	-0.06	0.03	2.48	2.46	2.49	1.55	1.57	1.59	1.12	1.17	1.1
								Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments											
	20	0.01	-0.02	-0.24	0.09	0.12	-0.01	-0.05	-0.05	-0.14	2.87	3.51	3.32	2.83	3.42	3.51	2.13	2.61	2.47
	50	0.08	0	-0.2	0	0.05	-0.15	-0.02	-0.09	-0.03	2.84	3.54	3.72	1.82	2.26	2.22	1.32	1.62	1.66
	100	0.08	-0.09	-0.07	0	0.02	-0.03	0.03	-0.06	0.02	2	2.46	2.47	1.26	1.57	1.58	0.93	1.17	1.09
								Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	0.01	-0.02	-0.24	0.1	0.12	-0.01	-0.03	-0.05	-0.14	2.74	3.51	3.31	2.64	3.41	3.5	2.01	2.61	2.47
	50	0.09	0	-0.21	-0.02	0.05	-0.15	-0.02	-0.09	-0.03	2.72	3.53	3.72	1.72	2.27	2.22	1.26	1.62	1.66
	100	0.07	-0.09	-0.07	-0.01	0.02	-0.03	0.05	-0.06	0.02	1.9	2.46	2.46	1.2	1.57	1.58	0.9	1.17	1.09
								OLS											
	20	5.82	1.74	-1.29	6.03	1.9	-1.05	5.83	1.77	-1.22	6.64	4.15	3.86	6.79	4.07	3.9	6.32	3.35	3.04
	50	6.17	1.73	-1.37	6.23	1.78	-1.23	6.37	1.71	-1.15	6.98	4.05	3.99	6.62	3.01	2.7	6.62	2.47	2.19
	100	6.35	1.6	-1.16	6.39	1.85	-1.15	6.51	1.73	-1.09	6.9	3.11	2.86	6.66	2.54	2.08	6.64	2.17	1.63
								Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments											
	20	1.46	2.23	-0.56	1.66	2.42	-0.29	1.43	2.31	-0.45	3.52	4.4	3.66	3.48	4.37	3.76	2.75	3.68	2.79
	50	1.63	2.24	-0.61	1.64	2.31	-0.44	1.72	2.26	-0.33	3.57	4.33	3.78	2.69	3.38	2.43	2.39	2.9	1.88
	100	1.67	2.14	-0.39	1.71	2.4	-0.35	1.8	2.3	-0.28	3.09	3.46	2.63	2.4	3	1.76	2.18	2.65	1.24
								Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
Naive	20	2.51	2.08	-0.74	2.69	2.26	-0.49	2.43	2.17	-0.63	5	4.31	3.69	5.04	4.26	3.8	4.04	3.58	2.81
	50	2.82	2.08	-0.81	2.94	2.14	-0.67	2.97	2.09	-0.55	5.23	4.22	3.8	4.13	3.25	2.47	3.76	2.76	1.91
	100	2.95	1.96	-0.61	2.99	2.23	-0.57	3.04	2.14	-0.49	4.76	3.31	2.66	3.88	2.85	1.8	3.48	2.51	1.3
								Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments											
	20	2.13	2.14	-0.68	2.34	2.32	-0.42	2.06	2.22	-0.57	4.16	4.34	3.67	4.24	4.3	3.77	3.37	3.61	2.8
	50	2.4	2.14	-0.75	2.45	2.21	-0.59	2.54	2.15	-0.48	4.39	4.26	3.78	3.48	3.29	2.45	3.22	2.81	1.89
	100	2.5	2.03	-0.54	2.57	2.29	-0.5	2.64	2.19	-0.42	4.04	3.36	2.64	3.3	2.9	1.78	3.01	2.56	1.27
								Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	1.64	2.21	-0.59	1.85	2.4	-0.32	1.6	2.29	-0.48	3.63	4.38	3.67	3.63	4.35	3.76	2.87	3.66	2.79
	50	1.85	2.21	-0.65	1.85	2.29	-0.48	1.92	2.23	-0.37	3.74	4.31	3.78	2.86	3.35	2.44	2.57	2.88	1.88
	100	1.9	2.11	-0.43	1.92	2.37	-0.39	2	2.27	-0.31	3.31	3.43	2.64	2.61	2.97	1.76	2.36	2.63	1.25

Table S18: The Finite Sample Performance of Alternative Mean Group Estimators under Experiment 4 with $\rho = 0.2$

		Bias (× 100)									RMSE (×) 100								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
									OLS										
	20	6.11	-0.01	-0.22	6.22	0.11	0.11	6.09	-0.02	-0.06	7.06	3.82	3.73	7.09	3.67	3.91	6.66	2.84	2.7
	50	5.17	-0.19	-0.44	5.28	-0.16	-0.49	5.26	-0.25	-0.41	6.22	4.11	4.26	5.69	2.45	2.45	5.5	1.71	1.82
	100	4.95	-0.34	-0.45	4.83	-0.21	-0.46	4.94	-0.27	-0.41	5.46	2.85	2.91	5.07	1.68	1.79	5.07	1.25	1.24
									Use only $\tilde{\boldsymbol{X}}^*$ as Instruments										
	20	0.01	-0.02	-0.25	-0.01	0.11	0.07	-0.03	-0.02	-0.08	3.42	3.82	3.75	3.36	3.68	3.86	2.55	2.83	2.68
	50	-0.14	0	-0.05	-0.02	0	-0.14	-0.01	-0.07	-0.05	3.57	4.16	4.27	2.05	2.45	2.4	1.47	1.69	1.77
	100	-0.01	-0.1	0.02	-0.05	0.01	-0.03	0.06	-0.05	0.02	2.35	2.87	2.9	1.43	1.67	1.75	1.03	1.21	1.17
									Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments										
	20	0.08	-0.04	-0.3	-0.13	0.17	0.06	-0.08	0	-0.1	6.08	3.95	3.9	6.43	3.95	4.04	3.94	2.87	2.71
$\bar{\boldsymbol{x}}_t$	50	0	0.06	-0.17	0.03	0.01	-0.14	0.05	-0.07	-0.05	6.98	4.8	4.62	3.23	2.51	2.48	2.01	1.7	1.78
	100	0.43	-0.1	-0.07	-0.03	0	-0.01	0.03	-0.05	0.02	5.52	3.29	3.4	2	1.72	1.76	1.33	1.22	1.17
									Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments										
	20	0.06	-0.03	-0.26	-0.03	0.12	0.06	-0.06	-0.01	-0.09	3.67	3.81	3.76	3.71	3.69	3.88	2.74	2.84	2.67
	50	-0.17	0.04	-0.06	0	0.01	-0.15	-0.01	-0.08	-0.04	3.94	4.23	4.27	2.26	2.45	2.42	1.56	1.69	1.77
	100	0.07	-0.1	0.02	-0.05	0.01	-0.03	0.05	-0.05	0.02	2.57	2.9	2.94	1.52	1.67	1.74	1.06	1.21	1.17
									Use $\tilde{\boldsymbol{X}}^*$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments										
	20	0.61	-0.03	-0.24	0.58	0.11	0.07	0.27	-0.02	-0.08	3.39	3.81	3.75	3.35	3.67	3.86	2.54	2.83	2.68
	50	1.05	-0.04	-0.15	0.43	-0.01	-0.17	0.22	-0.08	-0.06	3.55	4.14	4.26	2.08	2.44	2.4	1.47	1.69	1.77
	100	1.1	-0.17	-0.09	0.35	-0.01	-0.07	0.25	-0.06	0	2.52	2.86	2.9	1.46	1.67	1.75	1.05	1.21	1.17
									OLS										
	20	-0.44	-0.05	-0.28	-0.36	0.08	0.06	-0.37	-0.04	-0.13	3.44	3.78	3.79	3.24	3.65	3.85	2.48	2.8	2.66
	50	1.42	-0.11	-0.23	1.5	-0.04	-0.23	1.42	-0.14	-0.13	3.61	4.2	4.23	2.48	2.42	2.36	2.03	1.69	1.75
	100	2.04	-0.22	-0.19	1.89	-0.06	-0.23	2.02	-0.15	-0.16	2.97	2.86	2.91	2.34	1.64	1.73	2.25	1.22	1.16
									Use only $\tilde{\boldsymbol{X}}^*$ as Instruments										
	20	-0.11	-0.03	-0.26	-0.16	0.08	0.06	-0.13	-0.04	-0.12	3.39	3.77	3.78	3.29	3.66	3.85	2.47	2.81	2.66
	50	-0.11	-0.05	-0.12	-0.01	0.01	-0.13	-0.03	-0.08	-0.03	3.51	4.24	4.26	1.99	2.43	2.36	1.45	1.69	1.74
	100	-0.01	-0.1	0.01	-0.05	0.03	-0.06	0.06	-0.06	0.02	2.33	2.87	2.92	1.41	1.64	1.73	1.01	1.21	1.15
									Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments										
	20	-0.29	-0.08	-0.29	-0.34	0.11	0.06	-0.29	-0.01	-0.13	5.98	3.9	3.93	5.72	3.78	3.95	3.67	2.84	2.69
$(\bar{y}_t, \bar{\boldsymbol{x}}_t')'$	50	-0.54	0.02	-0.17	0.07	0.01	-0.14	0.02	-0.09	-0.03	7.32	5.18	4.83	2.99	2.47	2.44	1.98	1.7	1.74
	100	0.29	-0.17	-0.07	-0.09	0.02	-0.04	0.02	-0.05	0.02	4.33	3.18	3.32	1.96	1.69	1.75	1.29	1.22	1.15
									Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments										
	20	-0.14	-0.04	-0.27	-0.23	0.09	0.06	-0.21	-0.03	-0.13	3.53	3.77	3.8	3.5	3.67	3.86	2.56	2.81	2.65
	50	-0.13	-0.01	-0.13	0	0.01	-0.14	-0.03	-0.09	-0.03	3.75	4.28	4.27	2.18	2.43	2.38	1.54	1.69	1.74
	100	0.05	-0.1	0.02	-0.05	0.02	-0.06	0.05	-0.06	0.02	2.53	2.9	2.96	1.49	1.64	1.72	1.04	1.21	1.15
									Use $\tilde{\boldsymbol{X}}^*$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments										
	20	-0.14	-0.04	-0.26	-0.17	0.09	0.06	-0.14	-0.04	-0.12	3.34	3.77	3.79	3.26	3.65	3.85	2.45	2.81	2.66
	50	0.2	-0.06	-0.14	0.11	0.01	-0.14	0.03	-0.09	-0.03	3.43	4.22	4.26	1.99	2.42	2.36	1.43	1.69	1.74
	100	0.45	-0.13	-0.04	0.1	0.02	-0.07	0.12	-0.06	0.01	2.28	2.86	2.92	1.4	1.64	1.73	1	1.21	1.15
									OLS										
	20	0.15	-1.02	-2.3	0.24	-0.91	-1.96	0.26	-1.03	-2.13	3.47	3.94	4.57	3.26	3.83	4.43	2.49	3.01	3.49
	50	1.68	-0.47	-1.11	1.77	-0.47	-1.04	1.67	-0.55	-0.95	3.84	4.28	4.53	2.66	2.48	2.62	2.22	1.78	1.99
	100	2.19	-0.44	-0.63	2.02	-0.26	-0.65	2.15	-0.35	-0.56	3.13	2.99	3.05	2.46	1.66	1.85	2.37	1.26	1.29
									Use only $\tilde{\boldsymbol{X}}^*$ as Instruments										
	20	0.49	-1	-2.28	0.43	-0.9	-1.95	0.49	-1.02	-2.11	3.5	3.93	4.55	3.38	3.83	4.42	2.54	3.01	3.47
	50	0.12	-0.41	-0.96	0.24	-0.41	-0.92	0.21	-0.49	-0.83	3.66	4.31	4.55	2.04	2.47	2.58	1.48	1.77	1.93
	100	0.12	-0.32	-0.4	0.07	-0.16	-0.46	0.17	-0.26	-0.37	2.44	3	3.06	1.42	1.65	1.8	1.03	1.24	1.23
									Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments										
	20	0.76	-1.1	-2.38	0.72	-0.91	-2.03	0.87	-1.05	-2.22	5.85	4.06	4.76	5.97	3.99	4.61	3.82	3.05	3.6
$(\bar{y}_t, \bar{y}_t^*, \bar{\boldsymbol{x}}_t')'$	50	0.16	-0.4	-1.11	0.5	-0.43	-0.98	0.41	-0.52	-0.88	7.07	4.93	5.56	3.07	2.52	2.68	2.03	1.79	1.97
	100	0.4	-0.36	-0.41	0.12	-0.18	-0.47	0.21	-0.26	-0.39	4.93	3.33	3.79	1.96	1.71	1.84	1.32	1.25	1.24

Table S18 (Continued)

		Bias (× 100)									RMSE (×) 100								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
		Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																	
	20	0.57	-1.02	-2.31	0.46	-0.9	-1.96	0.51	-1.02	-2.13	3.64	3.93	4.57	3.58	3.85	4.45	2.65	3.01	3.49
	50	0.16	-0.4	-0.98	0.28	-0.41	-0.94	0.23	-0.5	-0.84	3.91	4.36	4.57	2.22	2.47	2.6	1.57	1.78	1.94
	100	0.18	-0.34	-0.4	0.08	-0.17	-0.46	0.17	-0.26	-0.38	2.66	3.02	3.09	1.51	1.66	1.8	1.05	1.24	1.23
		Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	0.48	-1.01	-2.28	0.45	-0.9	-1.95	0.5	-1.02	-2.11	3.45	3.93	4.56	3.35	3.83	4.43	2.53	3.01	3.48
	50	0.46	-0.43	-1	0.37	-0.41	-0.93	0.27	-0.49	-0.83	3.61	4.3	4.54	2.05	2.47	2.58	1.47	1.77	1.94
	100	0.63	-0.35	-0.47	0.22	-0.18	-0.47	0.24	-0.26	-0.38	2.41	2.99	3.05	1.42	1.65	1.81	1.03	1.24	1.23
Infeasible		OLS																	
	20	2.12	-0.17	-0.59	2.25	-0.06	-0.33	2.15	-0.15	-0.53	3.56	3.63	3.53	3.53	3.5	3.64	3	2.64	2.59
	50	2.17	-0.11	-0.47	2.16	-0.1	-0.51	2.12	-0.22	-0.4	3.66	3.96	4.1	2.81	2.34	2.36	2.47	1.64	1.75
	100	2.31	-0.26	-0.4	2.13	-0.11	-0.4	2.21	-0.18	-0.34	3.06	2.75	2.71	2.47	1.6	1.7	2.39	1.19	1.16
		Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																	
	20	0.03	-0.03	-0.2	0.05	0.08	0.06	-0.01	-0.01	-0.14	2.89	3.63	3.48	2.78	3.53	3.61	2.12	2.63	2.53
	50	-0.03	0.03	-0.07	-0.04	0.03	-0.12	-0.03	-0.1	-0.03	3.18	4	4.13	1.82	2.35	2.32	1.31	1.64	1.7
	100	0.06	-0.11	0	-0.02	0.01	-0.04	0.04	-0.06	0.03	2.13	2.76	2.69	1.28	1.6	1.66	0.92	1.18	1.11
		Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	0.05	-0.05	-0.23	-0.04	0.1	0.06	-0.02	0.01	-0.15	3.71	3.7	3.53	3.84	3.6	3.75	2.67	2.65	2.56
	50	-0.06	0.08	-0.13	0.01	0.05	-0.13	0.01	-0.1	-0.03	4.85	4.34	4.48	2.4	2.38	2.39	1.65	1.64	1.72
	100	0.23	-0.15	-0.09	-0.01	0.01	-0.04	0	-0.05	0.04	3.74	2.97	3.15	1.66	1.64	1.67	1.16	1.18	1.11
		Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																	
	20	0.03	-0.03	-0.22	0.05	0.08	0.04	-0.01	0	-0.14	2.99	3.63	3.49	2.99	3.55	3.65	2.25	2.64	2.53
	50	-0.05	0.01	-0.08	-0.03	0.03	-0.13	-0.02	-0.1	-0.03	3.39	4.05	4.17	1.93	2.35	2.34	1.37	1.64	1.7
	100	0.08	-0.12	-0.01	-0.01	0.01	-0.04	0.02	-0.05	0.03	2.3	2.79	2.75	1.32	1.61	1.65	0.95	1.18	1.11
		Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	0.19	-0.04	-0.23	0.21	0.08	0.02	0.07	-0.01	-0.16	2.86	3.63	3.48	2.77	3.53	3.61	2.12	2.63	2.53
	50	0.41	-0.01	-0.16	0.13	0.02	-0.15	0.05	-0.1	-0.04	3.15	3.99	4.11	1.82	2.35	2.32	1.29	1.64	1.7
	100	0.55	-0.15	-0.09	0.13	0	-0.07	0.11	-0.06	0.02	2.14	2.75	2.69	1.28	1.6	1.66	0.92	1.18	1.11
Naive		OLS																	
	20	6	1.59	-1.31	6.22	1.7	-0.96	5.95	1.66	-1.21	6.83	4.18	3.93	6.98	4	3.97	6.45	3.29	3.04
	50	6.14	1.63	-1.17	6.2	1.65	-1.2	6.32	1.56	-1.11	7.01	4.32	4.3	6.59	3	2.72	6.56	2.36	2.16
	100	6.41	1.45	-1.08	6.34	1.71	-1.13	6.4	1.58	-1.02	6.94	3.17	3.01	6.59	2.45	2.11	6.53	2.04	1.58
		Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																	
	20	1.49	2.13	-0.53	1.65	2.25	-0.16	1.4	2.21	-0.4	3.57	4.44	3.74	3.58	4.31	3.83	2.82	3.61	2.78
	50	1.39	2.15	-0.33	1.52	2.16	-0.39	1.63	2.1	-0.29	3.77	4.62	4.18	2.65	3.34	2.46	2.33	2.77	1.87
	100	1.59	2.02	-0.27	1.59	2.25	-0.34	1.68	2.12	-0.22	3.08	3.54	2.83	2.32	2.88	1.82	2.06	2.49	1.23
		Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	2.55	1.98	-0.71	2.65	2.11	-0.34	2.36	2.11	-0.56	5.15	4.44	3.8	5.23	4.3	3.93	4.1	3.54	2.82
	50	2.71	2.06	-0.6	2.74	2.06	-0.58	2.85	1.96	-0.5	6.07	4.77	4.35	4.1	3.28	2.55	3.66	2.65	1.91
	100	2.96	1.82	-0.5	2.86	2.11	-0.53	2.86	1.99	-0.42	5.15	3.53	3.16	3.77	2.79	1.86	3.31	2.38	1.27
		Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																	
	20	2.15	2.05	-0.65	2.36	2.16	-0.3	2.02	2.15	-0.51	4.24	4.4	3.75	4.35	4.26	3.86	3.44	3.57	2.78
	50	2.19	2.07	-0.47	2.31	2.09	-0.52	2.43	2	-0.43	4.59	4.57	4.17	3.41	3.28	2.49	3.14	2.69	1.88
	100	2.4	1.91	-0.39	2.45	2.15	-0.47	2.49	2.03	-0.36	3.94	3.46	2.87	3.18	2.8	1.82	2.86	2.41	1.25
		Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	2.04	2.06	-0.62	2.19	2.19	-0.27	1.77	2.17	-0.47	3.84	4.4	3.74	3.9	4.26	3.84	3.05	3.58	2.79
	50	2.53	2.02	-0.53	2.09	2.11	-0.49	2.03	2.05	-0.36	4.31	4.52	4.18	3.05	3.3	2.48	2.66	2.73	1.88
	100	2.76	1.87	-0.47	2.18	2.18	-0.43	2.08	2.07	-0.29	3.85	3.42	2.86	2.79	2.83	1.83	2.42	2.45	1.24

Table S19: The Size and Power for Alternative Pooled Estimators under Experiment 4 with $\rho = 0.2$

		Size									Power								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
OLS																			
$\bar{\boldsymbol{x}}_t$	20	0.53	0.059	0.06	0.545	0.056	0.072	0.739	0.066	0.067	0.2	0.74	0.75	0.23	0.75	0.73	0.34	0.93	0.93
	50	0.488	0.052	0.067	0.799	0.047	0.06	0.946	0.055	0.075	0.33	0.78	0.74	0.59	0.99	0.99	0.77	1	1
	100	0.673	0.055	0.073	0.923	0.052	0.06	0.998	0.063	0.064	0.6	0.97	0.95	0.88	1	1	0.99	1	1
Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																			
	20	0.086	0.057	0.057	0.06	0.049	0.07	0.069	0.066	0.063	0.28	0.75	0.74	0.35	0.77	0.73	0.55	0.93	0.94
	50	0.058	0.055	0.059	0.06	0.045	0.052	0.058	0.054	0.067	0.39	0.76	0.72	0.66	0.99	0.99	0.91	1	1
	100	0.039	0.055	0.06	0.052	0.049	0.048	0.054	0.052	0.04	0.7	0.96	0.95	0.95	1	1	1	1	1
Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																			
	20	0.077	0.059	0.059	0.076	0.05	0.073	0.061	0.068	0.063	0.18	0.76	0.74	0.18	0.76	0.72	0.34	0.92	0.93
	50	0.042	0.054	0.057	0.058	0.047	0.055	0.049	0.054	0.066	0.25	0.76	0.71	0.4	0.99	0.99	0.73	1	1
	100	0.044	0.054	0.057	0.05	0.048	0.051	0.039	0.054	0.04	0.44	0.97	0.96	0.77	1	1	0.97	1	1
Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																			
	20	0.081	0.058	0.057	0.073	0.05	0.069	0.072	0.068	0.062	0.29	0.76	0.75	0.29	0.76	0.73	0.47	0.93	0.94
	50	0.05	0.053	0.058	0.07	0.045	0.053	0.05	0.055	0.066	0.37	0.77	0.73	0.63	0.99	0.99	0.89	1	1
	100	0.036	0.057	0.059	0.051	0.048	0.048	0.049	0.053	0.039	0.67	0.96	0.96	0.92	1	1	1	1	1
Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																			
	20	0.081	0.057	0.057	0.063	0.049	0.071	0.065	0.066	0.063	0.3	0.75	0.74	0.34	0.77	0.73	0.54	0.93	0.94
	50	0.056	0.054	0.059	0.064	0.045	0.052	0.055	0.054	0.066	0.38	0.77	0.72	0.64	0.99	0.99	0.91	1	1
	100	0.039	0.055	0.06	0.055	0.049	0.048	0.053	0.052	0.04	0.69	0.96	0.96	0.95	1	1	1	1	1
OLS																			
$(\bar{y}_t, \bar{\boldsymbol{x}}_t')'$	20	0.128	0.074	0.074	0.094	0.07	0.078	0.117	0.083	0.085	0.29	0.76	0.72	0.35	0.75	0.75	0.51	0.92	0.94
	50	0.099	0.058	0.066	0.161	0.067	0.057	0.204	0.067	0.079	0.36	0.75	0.76	0.71	0.99	0.99	0.9	1	1
	100	0.174	0.066	0.074	0.346	0.059	0.056	0.582	0.061	0.053	0.69	0.96	0.95	0.94	1	1	1	1	1
Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																			
	20	0.099	0.07	0.074	0.073	0.07	0.077	0.082	0.085	0.083	0.3	0.77	0.71	0.33	0.75	0.76	0.55	0.93	0.94
	50	0.062	0.059	0.066	0.064	0.066	0.053	0.063	0.068	0.077	0.42	0.75	0.75	0.7	0.99	0.99	0.91	1	1
	100	0.048	0.066	0.071	0.057	0.058	0.053	0.066	0.058	0.044	0.68	0.96	0.95	0.95	1	1	1	1	1
Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																			
	20	0.084	0.077	0.074	0.081	0.071	0.077	0.077	0.085	0.083	0.18	0.77	0.72	0.2	0.74	0.74	0.33	0.93	0.93
	50	0.049	0.054	0.062	0.067	0.063	0.055	0.058	0.068	0.078	0.23	0.75	0.75	0.41	0.99	0.99	0.71	1	1
	100	0.049	0.06	0.067	0.057	0.059	0.055	0.049	0.06	0.045	0.46	0.96	0.95	0.76	1	1	0.96	1	1
Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																			
	20	0.095	0.073	0.073	0.08	0.07	0.076	0.082	0.085	0.081	0.32	0.77	0.71	0.31	0.75	0.75	0.5	0.93	0.94
	50	0.06	0.056	0.065	0.076	0.064	0.056	0.052	0.068	0.077	0.39	0.76	0.75	0.65	0.99	0.99	0.9	1	1
	100	0.047	0.065	0.07	0.055	0.061	0.053	0.047	0.057	0.045	0.67	0.96	0.95	0.93	1	1	1	1	1
Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																			
	20	0.101	0.071	0.075	0.078	0.07	0.077	0.079	0.085	0.083	0.31	0.77	0.71	0.36	0.75	0.75	0.57	0.93	0.94
	50	0.062	0.058	0.065	0.067	0.066	0.054	0.059	0.068	0.078	0.41	0.75	0.75	0.68	0.99	0.99	0.91	1	1
	100	0.048	0.066	0.07	0.051	0.059	0.054	0.062	0.058	0.044	0.68	0.96	0.95	0.95	1	1	1	1	1
OLS																			
$(\bar{y}_t, \bar{y}_t^{*}, \bar{\boldsymbol{x}}_t')'$	20	0.124	0.08	0.107	0.096	0.07	0.097	0.11	0.095	0.11	0.29	0.75	0.7	0.37	0.78	0.72	0.51	0.93	0.94
	50	0.115	0.055	0.076	0.195	0.065	0.071	0.255	0.069	0.096	0.33	0.75	0.74	0.67	0.99	0.99	0.91	1	1
	100	0.199	0.064	0.084	0.383	0.059	0.066	0.632	0.071	0.069	0.63	0.95	0.93	0.94	1	1	1	1	1
Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																			
	20	0.108	0.083	0.104	0.076	0.07	0.094	0.093	0.095	0.112	0.32	0.75	0.7	0.35	0.78	0.71	0.55	0.93	0.94
	50	0.059	0.054	0.074	0.074	0.064	0.066	0.06	0.068	0.087	0.38	0.75	0.74	0.65	0.99	0.99	0.92	1	1
	100	0.047	0.062	0.076	0.052	0.056	0.057	0.063	0.064	0.055	0.65	0.95	0.93	0.96	1	1	1	1	1
Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																			
	20	0.104	0.081	0.103	0.075	0.069	0.094	0.091	0.095	0.111	0.31	0.75	0.7	0.36	0.78	0.71	0.54	0.93	0.94
	50	0.06	0.055	0.075	0.077	0.063	0.066	0.057	0.068	0.087	0.35	0.74	0.74	0.65	0.99	0.99	0.92	1	1
	100	0.043	0.062	0.078	0.051	0.056	0.057	0.063	0.064	0.054	0.68	0.95	0.93	0.96	1	1	1	1	1

Table S19 (Continued)

		Size										Power										
		20			50			100				20				50				100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2		ρ	β_1	β_2		ρ	β_1	β_2		ρ	β_1	β_2
								Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments														
	20	0.093	0.08	0.105	0.083	0.069	0.096	0.095	0.094	0.111		0.3	0.74	0.7		0.34	0.77	0.72		0.49	0.93	0.94
	50	0.048	0.054	0.076	0.085	0.061	0.065	0.062	0.066	0.088		0.41	0.74	0.74		0.61	0.99	0.99		0.9	1	1
	100	0.052	0.062	0.078	0.056	0.057	0.06	0.054	0.065	0.053		0.62	0.95	0.93		0.93	1	1		1	1	1
								Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments														
	20	0.098	0.098	0.093	0.09	0.083	0.11	0.085	0.087	0.092		0.28	0.73	0.71		0.34	0.73	0.73		0.48	0.91	0.94
	50	0.079	0.054	0.065	0.051	0.052	0.074	0.063	0.062	0.073		0.34	0.74	0.74		0.62	0.98	0.99		0.89	1	1
	100	0.062	0.064	0.066	0.06	0.059	0.054	0.055	0.048	0.054		0.65	0.95	0.93		0.95	1	1		1	1	1
								OLS														
Infeasible	20	0.196	0.074	0.062	0.196	0.062	0.078	0.243	0.085	0.077		0.38	0.78	0.85		0.41	0.81	0.79		0.57	0.94	0.97
	50	0.166	0.057	0.077	0.286	0.057	0.066	0.423	0.065	0.085		0.47	0.81	0.77		0.83	0.99	1		0.95	1	1
	100	0.254	0.062	0.063	0.451	0.05	0.063	0.709	0.058	0.062		0.79	0.97	0.99		0.97	1	1		1	1	1
								Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments														
	20	0.095	0.07	0.056	0.075	0.065	0.08	0.075	0.086	0.065		0.42	0.78	0.85		0.44	0.81	0.79		0.61	0.94	0.97
	50	0.068	0.057	0.08	0.062	0.053	0.065	0.056	0.064	0.067		0.45	0.8	0.77		0.83	0.99	1		0.96	1	1
	100	0.051	0.061	0.046	0.046	0.049	0.051	0.051	0.059	0.044		0.73	0.98	0.99		0.98	1	1		1	1	1
								Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments														
	20	0.074	0.07	0.057	0.074	0.065	0.078	0.068	0.091	0.07		0.26	0.78	0.84		0.29	0.8	0.75		0.55	0.94	0.97
	50	0.06	0.053	0.075	0.057	0.054	0.064	0.053	0.064	0.071		0.3	0.8	0.77		0.6	0.99	0.99		0.88	1	1
	100	0.047	0.063	0.051	0.047	0.05	0.057	0.05	0.059	0.056		0.57	0.97	0.98		0.88	1	1		0.99	1	1
								Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments														
	20	0.09	0.072	0.057	0.077	0.065	0.078	0.077	0.086	0.063		0.42	0.78	0.84		0.43	0.81	0.78		0.61	0.94	0.97
	50	0.062	0.057	0.075	0.06	0.054	0.062	0.055	0.065	0.065		0.43	0.8	0.76		0.79	0.99	1		0.95	1	1
	100	0.051	0.06	0.047	0.05	0.049	0.051	0.054	0.059	0.05		0.7	0.98	0.99		0.97	1	1		1	1	1
								Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments														
	20	0.091	0.07	0.056	0.077	0.065	0.081	0.074	0.086	0.064		0.4	0.78	0.85		0.44	0.81	0.79		0.6	0.94	0.97
	50	0.061	0.057	0.079	0.062	0.053	0.065	0.057	0.064	0.067		0.46	0.8	0.77		0.8	0.99	1		0.96	1	1
	100	0.054	0.061	0.045	0.048	0.048	0.051	0.052	0.059	0.045		0.74	0.98	0.99		0.98	1	1		1	1	1
								OLS														
	20	0.622	0.105	0.084	0.64	0.111	0.091	0.794	0.133	0.096		0.24	0.71	0.78		0.32	0.73	0.72		0.4	0.88	0.92
	50	0.661	0.095	0.086	0.928	0.143	0.106	0.979	0.168	0.138		0.31	0.73	0.75		0.49	0.98	0.98		0.69	1	1
	100	0.853	0.121	0.086	0.987	0.226	0.124	1	0.296	0.142		0.41	0.95	0.97		0.71	1	1		0.96	1	1
								Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments														
	20	0.169	0.122	0.072	0.166	0.132	0.077	0.167	0.168	0.079		0.33	0.71	0.75		0.35	0.72	0.73		0.51	0.86	0.92
	50	0.154	0.117	0.076	0.213	0.183	0.068	0.307	0.271	0.074		0.32	0.73	0.75		0.57	0.98	0.98		0.82	1	1
	100	0.239	0.15	0.065	0.333	0.316	0.078	0.466	0.447	0.049		0.46	0.94	0.97		0.81	1	1		0.98	1	1
								Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments														
Naive	20	0.215	0.119	0.071	0.227	0.126	0.085	0.254	0.15	0.08		0.2	0.69	0.75		0.21	0.73	0.71		0.38	0.86	0.92
	50	0.225	0.108	0.072	0.339	0.174	0.078	0.451	0.237	0.081		0.19	0.73	0.76		0.31	0.98	0.98		0.48	1	1
	100	0.328	0.135	0.064	0.479	0.287	0.072	0.662	0.408	0.24	0.95	0.97		0.51	1	1		0.8	1	1		
								Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments														
	20	0.22	0.12	0.074	0.24	0.128	0.08	0.254	0.158	0.079		0.27	0.71	0.76		0.27	0.72	0.74		0.47	0.87	0.92
	50	0.242	0.107	0.074	0.34	0.175	0.074	0.46	0.256	0.079		0.25	0.74	0.77		0.46	0.98	0.98		0.64	1	1
	100	0.356	0.138	0.059	0.494	0.294	0.078	0.676	0.421	0.055		0.28	0.94	0.97		0.64	1	1		0.91	1	1
								Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments														
	20	0.181	0.12	0.071	0.187	0.13	0.077	0.196	0.165	0.079		0.32	0.71	0.75		0.32	0.72	0.73		0.51	0.87	0.92
	50	0.186	0.116	0.076	0.249	0.183	0.068	0.347	0.268	0.077		0.31	0.73	0.76		0.54	0.98	0.98		0.79	1	1
	100	0.278	0.147	0.063	0.377	0.315	0.078	0.536	0.441	0.053		0.42	0.94	0.97		0.77	1	1		0.96	1	1

Table S20: The Size and Power for Alternative Mean Group Estimators under Experiment 4 with $\rho = 0.2$

		Size									Power								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
		OLS																	
	20	0.504	0.058	0.053	0.528	0.044	0.053	0.721	0.063	0.053	0.22	0.7	0.73	0.25	0.73	0.72	0.4	0.93	0.94
	50	0.398	0.052	0.068	0.779	0.052	0.058	0.946	0.055	0.076	0.3	0.69	0.67	0.57	0.99	0.99	0.77	1	1
	100	0.624	0.05	0.056	0.919	0.041	0.065	0.998	0.065	0.062	0.55	0.95	0.93	0.89	1	1	0.99	1	1
		Use only $\tilde{\boldsymbol{X}}^*$ as Instruments																	
	20	0.058	0.056	0.052	0.05	0.04	0.045	0.064	0.063	0.049	0.32	0.74	0.72	0.31	0.75	0.72	0.47	0.93	0.94
	50	0.059	0.048	0.058	0.056	0.05	0.052	0.05	0.05	0.064	0.27	0.68	0.67	0.67	0.99	0.98	0.91	1	1
	100	0.045	0.045	0.057	0.054	0.042	0.06	0.054	0.048	0.041	0.55	0.94	0.94	0.91	1	1	1	1	1
		Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	0.055	0.049	0.051	0.071	0.045	0.05	0.051	0.058	0.049	0.19	0.72	0.68	0.15	0.69	0.7	0.3	0.93	0.94
$\bar{\boldsymbol{x}}_t$	50	0.059	0.052	0.057	0.053	0.052	0.046	0.046	0.046	0.059	0.15	0.58	0.62	0.32	0.98	0.98	0.69	1	1
	100	0.044	0.045	0.056	0.042	0.042	0.055	0.043	0.05	0.042	0.25	0.88	0.87	0.7	1	1	0.95	1	1
		Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																	
	20	0.063	0.054	0.052	0.077	0.044	0.043	0.058	0.065	0.049	0.28	0.73	0.71	0.25	0.73	0.73	0.43	0.93	0.94
	50	0.052	0.054	0.055	0.068	0.051	0.048	0.053	0.051	0.064	0.25	0.64	0.68	0.58	0.99	0.98	0.87	1	1
	100	0.041	0.049	0.056	0.048	0.043	0.053	0.043	0.051	0.043	0.51	0.92	0.93	0.89	1	1	1	1	1
		Use $\tilde{\boldsymbol{X}}^*$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	0.067	0.059	0.051	0.057	0.041	0.044	0.068	0.062	0.049	0.32	0.75	0.73	0.32	0.75	0.74	0.46	0.93	0.94
	50	0.07	0.05	0.064	0.069	0.05	0.051	0.055	0.05	0.064	0.27	0.68	0.66	0.68	0.99	0.98	0.92	1	1
	100	0.079	0.044	0.055	0.06	0.042	0.058	0.058	0.049	0.04	0.59	0.94	0.94	0.91	1	1	1	1	1
		OLS																	
	20	0.109	0.073	0.061	0.069	0.061	0.06	0.096	0.082	0.069	0.3	0.72	0.69	0.36	0.75	0.72	0.48	0.93	0.92
	50	0.09	0.064	0.067	0.161	0.057	0.052	0.204	0.067	0.072	0.31	0.64	0.65	0.71	0.99	0.99	0.88	1	1
	100	0.162	0.051	0.062	0.334	0.045	0.068	0.576	0.063	0.052	0.57	0.95	0.93	0.94	1	1	1	1	1
		Use only $\tilde{\boldsymbol{X}}^*$ as Instruments																	
	20	0.081	0.071	0.065	0.062	0.056	0.06	0.073	0.082	0.066	0.29	0.71	0.7	0.3	0.76	0.72	0.47	0.93	0.93
	50	0.058	0.063	0.067	0.056	0.06	0.052	0.059	0.064	0.072	0.28	0.65	0.64	0.68	0.99	0.99	0.92	1	1
	100	0.041	0.05	0.058	0.054	0.039	0.067	0.058	0.057	0.048	0.58	0.94	0.93	0.93	1	1	1	1	1
		Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	0.063	0.06	0.064	0.069	0.058	0.055	0.063	0.078	0.062	0.2	0.72	0.65	0.2	0.7	0.71	0.33	0.91	0.92
$(\bar{y}_t, \bar{\boldsymbol{x}}_t')'$	50	0.051	0.057	0.055	0.06	0.058	0.056	0.047	0.063	0.068	0.15	0.54	0.62	0.34	0.98	0.98	0.71	1	1
	100	0.048	0.047	0.06	0.044	0.046	0.069	0.041	0.059	0.046	0.3	0.91	0.88	0.72	1	1	0.96	1	1
		Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																	
	20	0.055	0.07	0.062	0.069	0.059	0.06	0.071	0.08	0.067	0.3	0.72	0.66	0.29	0.75	0.72	0.47	0.92	0.93
	50	0.058	0.063	0.065	0.063	0.06	0.052	0.056	0.067	0.069	0.23	0.63	0.65	0.6	0.99	0.99	0.89	1	1
	100	0.045	0.051	0.051	0.056	0.041	0.072	0.045	0.057	0.052	0.5	0.94	0.92	0.91	1	1	1	1	1
		Use $\tilde{\boldsymbol{X}}^*$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	0.081	0.071	0.062	0.065	0.058	0.058	0.069	0.082	0.068	0.29	0.72	0.7	0.32	0.75	0.72	0.49	0.93	0.93
	50	0.066	0.061	0.066	0.057	0.06	0.052	0.058	0.065	0.072	0.27	0.66	0.63	0.7	0.99	0.99	0.93	1	1
	100	0.048	0.052	0.058	0.056	0.038	0.068	0.055	0.057	0.048	0.57	0.94	0.93	0.92	1	1	1	1	1
		OLS																	
	20	0.094	0.08	0.109	0.082	0.062	0.095	0.098	0.098	0.11	0.29	0.74	0.65	0.3	0.74	0.68	0.51	0.93	0.93
	50	0.105	0.057	0.065	0.191	0.072	0.083	0.25	0.07	0.102	0.27	0.64	0.64	0.7	0.99	0.98	0.87	1	1
	100	0.169	0.064	0.06	0.36	0.051	0.079	0.614	0.07	0.069	0.54	0.94	0.91	0.95	1	1	1	1	1
		Use only $\tilde{\boldsymbol{X}}^*$ as Instruments																	
	20	0.08	0.075	0.104	0.065	0.062	0.089	0.077	0.096	0.115	0.33	0.75	0.65	0.27	0.75	0.69	0.45	0.93	0.93
	50	0.064	0.055	0.06	0.064	0.072	0.076	0.058	0.069	0.099	0.24	0.63	0.63	0.67	0.98	0.98	0.92	1	1
	100	0.046	0.06	0.062	0.049	0.048	0.075	0.056	0.066	0.057	0.53	0.92	0.9	0.94	1	1	1	1	1
		Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	0.071	0.071	0.107	0.068	0.058	0.09	0.076	0.1	0.115	0.18	0.7	0.62	0.17	0.71	0.65	0.34	0.91	0.92
$(\bar{y}_t, \bar{y}_t^*, \bar{\boldsymbol{x}}_t')'$	50	0.046	0.057	0.066	0.07	0.067	0.077	0.065	0.074	0.093	0.17	0.59	0.58	0.33	0.98	0.97	0.69	1	1
	100	0.053	0.058	0.062	0.043	0.052	0.072	0.045	0.067	0.056	0.25	0.87	0.84	0.71	1	1	0.96	1	1

Table S20 (Continued)

		Size									Power								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																			
	20	0.067	0.073	0.108	0.062	0.061	0.093	0.073	0.091	0.116	0.34	0.73	0.65	0.28	0.77	0.67	0.43	0.92	0.93
	50	0.057	0.06	0.064	0.07	0.071	0.07	0.06	0.071	0.093	0.22	0.64	0.64	0.59	0.99	0.99	0.88	1	1
	100	0.036	0.051	0.061	0.055	0.045	0.077	0.053	0.066	0.056	0.49	0.93	0.89	0.89	1	1	1	1	1
Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																			
	20	0.078	0.077	0.109	0.062	0.062	0.089	0.077	0.097	0.116	0.31	0.75	0.65	0.31	0.75	0.69	0.47	0.92	0.93
	50	0.067	0.054	0.061	0.066	0.073	0.076	0.061	0.07	0.099	0.24	0.64	0.65	0.68	0.98	0.98	0.92	1	1
	100	0.055	0.059	0.062	0.054	0.05	0.075	0.06	0.066	0.056	0.55	0.92	0.91	0.93	1	1	1	1	1
OLS																			
	20	0.177	0.074	0.057	0.186	0.049	0.062	0.247	0.07	0.071	0.34	0.76	0.82	0.39	0.78	0.77	0.58	0.95	0.97
	50	0.152	0.064	0.081	0.266	0.053	0.065	0.429	0.063	0.083	0.41	0.71	0.64	0.76	0.99	0.99	0.93	1	1
	100	0.227	0.049	0.045	0.439	0.048	0.07	0.702	0.055	0.057	0.69	0.96	0.96	0.96	1	1	1	1	1
Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																			
	20	0.079	0.075	0.047	0.062	0.05	0.06	0.076	0.072	0.066	0.39	0.75	0.81	0.47	0.78	0.78	0.62	0.95	0.96
	50	0.062	0.063	0.078	0.06	0.052	0.053	0.055	0.063	0.077	0.37	0.69	0.62	0.79	0.99	0.99	0.96	1	1
	100	0.05	0.051	0.05	0.057	0.049	0.062	0.054	0.051	0.048	0.66	0.96	0.96	0.96	1	1	1	1	1
Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																			
Infeasible	20	0.074	0.067	0.053	0.071	0.051	0.067	0.07	0.073	0.063	0.27	0.76	0.78	0.28	0.77	0.73	0.51	0.94	0.96
	50	0.051	0.068	0.073	0.049	0.057	0.058	0.053	0.06	0.069	0.25	0.61	0.61	0.52	0.98	0.99	0.82	1	1
	100	0.059	0.051	0.055	0.045	0.042	0.056	0.048	0.054	0.049	0.37	0.92	0.91	0.85	1	1	0.99	1	1
Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																			
	20	0.076	0.076	0.054	0.068	0.048	0.065	0.075	0.074	0.06	0.39	0.75	0.81	0.41	0.77	0.77	0.58	0.95	0.97
	50	0.061	0.056	0.077	0.058	0.056	0.052	0.052	0.063	0.078	0.32	0.69	0.65	0.74	0.99	0.99	0.94	1	1
	100	0.05	0.051	0.05	0.053	0.045	0.055	0.052	0.052	0.053	0.55	0.95	0.95	0.94	1	1	1	1	1
Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																			
	20	0.08	0.073	0.052	0.062	0.046	0.062	0.084	0.074	0.064	0.39	0.75	0.81	0.47	0.79	0.76	0.62	0.95	0.96
	50	0.072	0.06	0.079	0.057	0.054	0.055	0.056	0.061	0.077	0.38	0.69	0.66	0.78	0.99	0.99	0.96	1	1
	100	0.065	0.046	0.046	0.056	0.049	0.06	0.048	0.051	0.049	0.68	0.96	0.96	0.96	1	1	1	1	1
OLS																			
	20	0.588	0.092	0.083	0.611	0.08	0.079	0.792	0.114	0.085	0.23	0.68	0.79	0.25	0.77	0.72	0.42	0.9	0.94
	50	0.592	0.09	0.087	0.902	0.129	0.093	0.98	0.142	0.117	0.3	0.68	0.68	0.53	0.98	0.98	0.72	1	1
	100	0.837	0.092	0.071	0.986	0.185	0.124	1	0.253	0.14	0.46	0.92	0.95	0.75	1	1	0.95	1	1
Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																			
	20	0.146	0.108	0.061	0.154	0.096	0.064	0.153	0.146	0.059	0.35	0.66	0.75	0.32	0.75	0.71	0.5	0.88	0.93
	50	0.128	0.104	0.069	0.185	0.16	0.054	0.27	0.219	0.069	0.29	0.63	0.64	0.52	0.96	0.98	0.81	1	1
	100	0.187	0.128	0.056	0.29	0.279	0.058	0.432	0.393	0.047	0.44	0.92	0.94	0.8	1	1	0.98	1	1
Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																			
Naive	20	0.175	0.099	0.068	0.193	0.097	0.062	0.219	0.139	0.063	0.19	0.66	0.71	0.15	0.74	0.71	0.35	0.89	0.93
	50	0.193	0.086	0.065	0.272	0.149	0.061	0.412	0.198	0.072	0.18	0.61	0.62	0.31	0.97	0.98	0.49	1	1
	100	0.246	0.107	0.057	0.42	0.251	0.073	0.596	0.356	0.064	0.19	0.9	0.92	0.48	1	1	0.81	1	1
Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																			
	20	0.189	0.102	0.066	0.2	0.093	0.068	0.23	0.142	0.06	0.3	0.66	0.74	0.28	0.74	0.71	0.41	0.88	0.94
	50	0.208	0.093	0.065	0.288	0.158	0.058	0.431	0.207	0.069	0.23	0.64	0.68	0.42	0.97	0.98	0.61	1	1
	100	0.273	0.124	0.057	0.443	0.264	0.066	0.624	0.363	0.059	0.3	0.92	0.94	0.64	1	1	0.93	1	1
Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																			
	20	0.186	0.105	0.062	0.188	0.097	0.064	0.191	0.145	0.058	0.34	0.67	0.74	0.33	0.74	0.72	0.5	0.88	0.94
	50	0.216	0.096	0.07	0.267	0.157	0.055	0.355	0.209	0.074	0.3	0.64	0.68	0.5	0.97	0.98	0.74	1	1
	100	0.323	0.128	0.06	0.414	0.269	0.062	0.554	0.378	0.052	0.43	0.92	0.95	0.77	1	1	0.97	1	1

S2.4 Supplementary Results for $\rho = 0.8$

This subsection provides Monte Carlo simulation results for $\rho = 0.8$ while all the other settings remain the same as in the main text. To save space, we only report the results for Experiment 4.[3] The results here are also qualitatively similar to those reported in Section 4.3 in the main text. One thing worthy of mentioning is that while the increase of spatial coefficient has limited (and almost negligible) impact on the performance of our proposed estimator, it generally makes the standard CCE estimator that utilises $(\bar{y}_t, \bar{\boldsymbol{x}}_t')'$ as factor proxies worse, especially when sample size is small.

[3] The results for the other three experiments are available upon request.

Table S21: The Finite Sample Performance of Alternative Pooled Estimators under Experiment 4 with $\rho = 0.8$

		Bias ($\times$ 100)									RMSE ($\times$) 100								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
OLS																			
	20	6.48	-2.19	-4.14	6.44	-2.43	-4.12	6.45	-2.16	-4.44	6.85	6.48	7.7	6.61	4.52	5.68	6.58	3.49	5.25
	50	5.3	-2.15	-4.45	5.34	-2.44	-4.65	5.23	-2.3	-4.68	5.45	4.31	6.02	5.42	3.47	5.26	5.29	2.92	5.05
	100	4.92	-2.31	-4.58	4.88	-2.32	-4.66	4.87	-2.27	-4.54	5.01	3.58	5.36	4.92	2.84	4.97	4.91	2.57	4.72
Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																			
	20	0.03	-0.05	0.11	0.08	0.05	0.19	0.06	0.02	-0.05	2.62	6.1	6.47	1.74	3.81	3.87	1.4	2.77	2.77
	50	0	0.15	0.17	0.04	-0.09	0	0	0.01	-0.05	1.45	3.74	3.98	0.99	2.44	2.43	0.85	1.8	1.85
	100	0.05	-0.01	0.02	0.04	-0.02	-0.07	0.05	0.01	0.04	1.05	2.71	2.76	0.7	1.64	1.69	0.59	1.19	1.26
Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																			
	20	-0.04	-0.05	0.12	0.08	0.05	0.2	0.1	0.01	-0.08	2.9	6.11	6.44	1.88	3.81	3.87	1.47	2.77	2.76
$\bar{\boldsymbol{x}}_t$	50	-0.04	0.16	0.2	0.03	-0.1	-0.02	-0.01	0.01	-0.06	1.52	3.72	4	1.01	2.45	2.46	0.88	1.8	1.86
	100	0.04	-0.01	0.03	0.04	-0.02	-0.07	0.04	0.02	0.06	1.06	2.72	2.76	0.7	1.64	1.71	0.6	1.19	1.26
Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																			
	20	0	-0.05	0.12	0.09	0.05	0.19	0.07	0.02	-0.05	2.46	6.09	6.42	1.64	3.81	3.84	1.35	2.77	2.74
	50	-0.02	0.15	0.18	0.03	-0.1	-0.01	-0.01	0.01	-0.05	1.31	3.72	3.94	0.93	2.44	2.41	0.82	1.8	1.84
	100	0.05	-0.02	0.02	0.04	-0.02	-0.07	0.04	0.02	0.05	0.96	2.71	2.72	0.65	1.63	1.68	0.56	1.19	1.24
Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																			
	20	0.03	-0.05	0.1	0.09	0.05	0.19	0.08	0.02	-0.06	2.45	6.09	6.4	1.66	3.8	3.84	1.34	2.77	2.74
	50	-0.02	0.15	0.18	0.04	-0.1	-0.01	-0.01	0.01	-0.06	1.35	3.72	3.95	0.93	2.44	2.42	0.82	1.8	1.84
	100	0.05	-0.01	0.02	0.04	-0.02	-0.07	0.05	0.02	0.05	0.98	2.71	2.73	0.66	1.63	1.69	0.57	1.19	1.25
OLS																			
	20	2.99	-1.11	-1.95	3.04	-1.28	-1.93	3.04	-1.04	-2.2	3.7	6.2	6.7	3.38	4.02	4.24	3.29	2.92	3.5
	50	3.28	-1.25	-2.72	3.32	-1.55	-2.88	3.28	-1.45	-2.95	3.5	4.03	4.81	3.43	2.89	3.76	3.36	2.29	3.47
	100	3.42	-1.66	-3.18	3.37	-1.61	-3.22	3.36	-1.56	-3.11	3.52	3.18	4.21	3.42	2.28	3.64	3.4	1.96	3.34
Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																			
	20	-0.22	-0.06	0.13	-0.15	0.05	0.21	-0.16	0.05	-0.01	2.68	6.14	6.47	1.75	3.83	3.79	1.4	2.75	2.74
	50	-0.03	0.18	0.16	0.08	-0.09	0.02	0.04	0.01	-0.03	1.5	3.89	4	0.99	2.44	2.43	0.84	1.79	1.82
	100	0.03	-0.07	0.01	0.07	-0.02	-0.05	0.07	0.01	0.05	1.07	2.73	2.79	0.69	1.64	1.69	0.58	1.18	1.25
Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																			
	20	-0.41	-0.02	0.22	-0.27	0.09	0.3	-0.25	0.07	0.04	2.98	6.16	6.44	1.9	3.84	3.8	1.5	2.75	2.74
$(\bar{y}_t, \bar{\boldsymbol{x}}_t')'$	50	-0.08	0.21	0.2	0.11	-0.09	0.02	0.05	0.01	-0.02	1.56	3.86	4	1.01	2.45	2.44	0.86	1.79	1.84
	100	0.03	-0.07	0.02	0.07	-0.02	-0.05	0.06	0.02	0.07	1.08	2.73	2.79	0.69	1.64	1.71	0.6	1.19	1.25
Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																			
	20	-0.36	-0.03	0.2	-0.23	0.08	0.27	-0.25	0.08	0.05	2.53	6.13	6.41	1.66	3.83	3.77	1.37	2.75	2.71
	50	-0.06	0.2	0.19	0.1	-0.09	0.02	0.04	0.01	-0.02	1.37	3.87	3.95	0.93	2.44	2.41	0.81	1.78	1.81
	100	0.03	-0.07	0.01	0.07	-0.02	-0.05	0.06	0.02	0.06	0.97	2.72	2.76	0.64	1.63	1.68	0.56	1.18	1.23
Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																			
	20	-0.28	-0.05	0.16	-0.19	0.06	0.24	-0.19	0.06	0.01	2.49	6.13	6.4	1.66	3.82	3.77	1.35	2.75	2.71
	50	-0.05	0.19	0.18	0.09	-0.09	0.02	0.05	0.01	-0.02	1.39	3.87	3.96	0.93	2.44	2.41	0.81	1.78	1.81
	100	0.03	-0.07	0.01	0.07	-0.02	-0.05	0.07	0.02	0.06	0.99	2.72	2.76	0.65	1.63	1.69	0.56	1.18	1.24
OLS																			
	20	3.16	-1.9	-3.56	3.2	-2.11	-3.6	3.19	-1.83	-3.79	3.87	6.55	7.68	3.53	4.41	5.3	3.42	3.3	4.73
	50	3.35	-1.57	-3.41	3.38	-1.87	-3.53	3.35	-1.8	-3.62	3.57	4.24	5.3	3.49	3.09	4.27	3.43	2.54	4.06
	100	3.47	-1.84	-3.51	3.4	-1.78	-3.56	3.4	-1.73	-3.44	3.57	3.33	4.52	3.45	2.42	3.95	3.44	2.1	3.66
Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																			
	20	-0.07	-0.84	-1.44	0.01	-1.02	-1.41	-0.01	-0.72	-1.56	2.71	6.35	7.01	1.74	4.01	4.14	1.37	2.86	3.23
	50	-0.02	-0.1	-0.44	0.06	-0.38	-0.57	0.03	-0.31	-0.63	1.55	3.99	4.11	0.99	2.49	2.49	0.84	1.82	1.95
	100	0.05	-0.23	-0.29	0.05	-0.17	-0.34	0.06	-0.14	-0.24	1.09	2.8	2.88	0.69	1.67	1.74	0.59	1.2	1.27
Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																			
	20	0.03	-0.88	-1.53	0.12	-1.05	-1.48	0.1	-0.76	-1.64	2.54	6.34	6.97	1.65	4.01	4.13	1.32	2.86	3.26
$(\bar{y}_t, \bar{y}_t^{*}, \bar{\boldsymbol{x}}_t')'$	50	0.01	-0.11	-0.47	0.1	-0.39	-0.6	0.06	-0.32	-0.66	1.44	3.97	4.08	0.93	2.49	2.48	0.81	1.82	1.95
	100	0.07	-0.23	-0.3	0.06	-0.18	-0.36	0.07	-0.14	-0.25	1.01	2.8	2.85	0.65	1.67	1.74	0.57	1.2	1.26

Table S21 (Continued)

		Bias (× 100)									RMSE (×) 100								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
								Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments											
	20	0.03	-0.9	-1.54	0.17	-1.07	-1.51	0.13	-0.77	-1.66	2.55	6.34	6.98	1.63	4.02	4.15	1.32	2.87	3.27
	50	0.01	-0.11	-0.47	0.12	-0.4	-0.62	0.08	-0.33	-0.68	1.42	3.97	4.07	0.94	2.49	2.48	0.81	1.82	1.95
	100	0.08	-0.24	-0.31	0.07	-0.18	-0.36	0.08	-0.14	-0.25	0.99	2.8	2.85	0.64	1.67	1.73	0.56	1.2	1.26
								Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	0.02	-0.87	-1.5	0.08	-1.06	-1.45	0.05	-0.76	-1.63	2.63	6.44	7.01	1.69	4.04	4.14	1.33	2.87	3.26
	50	0	-0.11	-0.45	0.09	-0.39	-0.6	0.06	-0.32	-0.66	1.46	3.98	4.1	0.94	2.51	2.48	0.81	1.83	1.95
	100	0.07	-0.24	-0.3	0.06	-0.18	-0.36	0.07	-0.14	-0.25	1.02	2.8	2.86	0.65	1.66	1.74	0.57	1.21	1.26
								OLS											
Infeasible	20	2.7	-1.77	-3.65	2.73	-1.68	-3.42	2.77	-1.57	-3.77	3.12	6.15	7.2	2.97	4.04	5.07	2.96	3.06	4.63
	50	2.83	-1.39	-3.45	2.82	-1.59	-3.54	2.79	-1.47	-3.66	3.02	3.99	5.18	2.92	2.86	4.24	2.87	2.27	4.08
	100	2.86	-1.45	-3.49	2.85	-1.45	-3.58	2.82	-1.43	-3.48	2.95	2.98	4.36	2.9	2.14	3.94	2.87	1.83	3.68
								Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments											
	20	-0.02	-0.25	-0.07	0.04	-0.19	0.21	0.07	-0.03	-0.05	1.91	5.93	6.26	1.36	3.69	3.79	1.14	2.65	2.74
	50	0.02	0.08	0.11	0.07	-0.11	0.03	0.03	0.01	-0.07	1.24	3.78	3.93	0.87	2.39	2.36	0.76	1.74	1.81
	100	0.05	0	0.05	0.07	0.01	-0.04	0.07	0.01	0.05	0.9	2.6	2.64	0.63	1.6	1.67	0.54	1.13	1.23
								Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	-0.03	-0.24	-0.09	0.03	-0.18	0.22	0.09	-0.04	-0.09	1.95	5.92	6.27	1.36	3.67	3.77	1.13	2.65	2.72
	50	-0.01	0.09	0.15	0.07	-0.11	0.02	0.03	0.01	-0.07	1.25	3.76	3.9	0.89	2.39	2.36	0.77	1.74	1.82
	100	0.05	0	0.06	0.08	0	-0.05	0.06	0.02	0.06	0.92	2.61	2.64	0.62	1.59	1.68	0.55	1.13	1.23
								Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments											
	20	-0.02	-0.26	-0.08	0.04	-0.19	0.21	0.08	-0.03	-0.06	1.78	5.91	6.18	1.29	3.68	3.73	1.1	2.64	2.7
	50	0.01	0.08	0.13	0.07	-0.11	0.02	0.03	0.01	-0.07	1.15	3.77	3.87	0.83	2.38	2.33	0.74	1.74	1.8
	100	0.05	-0.01	0.05	0.07	0.01	-0.05	0.06	0.01	0.05	0.83	2.6	2.6	0.59	1.59	1.65	0.52	1.13	1.21
								Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	-0.02	-0.25	-0.09	0.04	-0.19	0.21	0.08	-0.04	-0.07	1.8	5.91	6.19	1.3	3.67	3.74	1.1	2.64	2.7
	50	0.01	0.08	0.13	0.07	-0.11	0.02	0.03	0.01	-0.07	1.16	3.76	3.87	0.83	2.38	2.33	0.74	1.74	1.8
	100	0.05	0	0.06	0.07	0.01	-0.05	0.06	0.01	0.05	0.85	2.6	2.61	0.6	1.59	1.66	0.53	1.13	1.22
								OLS											
	20	4.27	-1.15	-5.61	4.29	-1.15	-5.52	4.31	-0.9	-5.9	4.57	6.02	8.49	4.48	4.02	6.85	4.46	2.96	6.56
	50	4.48	-0.77	-5.63	4.52	-1.03	-5.78	4.46	-0.9	-5.84	4.65	3.77	6.9	4.61	2.7	6.31	4.53	2.09	6.18
	100	4.54	-0.98	-5.65	4.6	-0.95	-5.79	4.59	-0.84	-5.71	4.65	2.83	6.29	4.65	1.92	6.06	4.63	1.51	5.88
								Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments											
	20	0.62	1.7	-0.87	0.69	1.71	-0.67	0.69	2.01	-0.95	2.08	6.27	6.41	1.64	4.25	4.07	1.4	3.54	3.01
	50	0.66	2.11	-0.82	0.74	1.89	-0.91	0.72	2.02	-1	1.55	4.34	4.02	1.22	3.2	2.65	1.1	2.81	2.19
	100	0.74	1.86	-0.9	0.81	1.94	-0.97	0.79	2.07	-0.88	1.31	3.31	2.86	1.11	2.61	2.02	1	2.45	1.63
								Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
Naive	20	0.76	1.55	-1.1	0.84	1.56	-0.89	0.87	1.85	-1.2	2.23	6.23	6.43	1.77	4.18	4.1	1.53	3.43	3.07
	50	0.83	1.95	-1.04	0.93	1.73	-1.16	0.92	1.86	-1.26	1.74	4.21	4.09	1.4	3.1	2.76	1.26	2.69	2.32
	100	0.93	1.7	-1.14	1.03	1.76	-1.25	0.99	1.91	-1.14	1.5	3.21	2.95	1.31	2.46	2.19	1.19	2.31	1.78
								Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments											
	20	0.75	1.56	-1.07	0.83	1.58	-0.88	0.84	1.89	-1.15	2.08	6.21	6.37	1.7	4.18	4.07	1.47	3.45	3.04
	50	0.82	1.97	-1.03	0.91	1.75	-1.14	0.9	1.88	-1.23	1.64	4.23	4.03	1.35	3.1	2.71	1.22	2.7	2.28
	100	0.92	1.71	-1.13	1	1.78	-1.22	0.97	1.93	-1.12	1.44	3.21	2.91	1.27	2.48	2.14	1.16	2.31	1.76
								Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	0.71	1.61	-1.01	0.78	1.63	-0.8	0.79	1.93	-1.09	2.05	6.22	6.37	1.66	4.2	4.06	1.44	3.47	3.02
	50	0.77	2.01	-0.95	0.85	1.8	-1.05	0.84	1.93	-1.15	1.6	4.25	4.03	1.29	3.14	2.69	1.17	2.74	2.25
	100	0.85	1.77	-1.04	0.93	1.84	-1.13	0.9	1.98	-1.03	1.39	3.24	2.89	1.21	2.52	2.1	1.1	2.36	1.7

Table S22: The Finite Sample Performance of Alternative Mean Group Estimators under Experiment 4 with $\rho = 0.8$

		Bias (× 100)									RMSE (×) 100								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
		OLS																	
	20	6.67	-2.47	-4.47	6.62	-2.36	-4.26	6.65	-2.27	-4.57	7.05	7.09	8.28	6.79	4.56	5.81	6.77	3.57	5.37
	50	5.3	-2.2	-4.58	5.26	-2.51	-4.64	5.15	-2.3	-4.65	5.47	4.62	6.31	5.33	3.52	5.27	5.21	2.9	5.02
	100	4.82	-2.28	-4.56	4.76	-2.3	-4.58	4.74	-2.23	-4.45	4.91	3.71	5.47	4.8	2.84	4.9	4.77	2.53	4.62
		Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																	
	20	0.02	-0.21	0.19	-0.03	-0.1	0.28	-0.02	0	-0.02	3.65	7.26	7.76	1.86	3.98	4.08	1.38	2.81	2.83
	50	0.07	0.06	0.05	-0.02	-0.16	0.05	-0.04	-0.01	-0.03	1.76	4.3	4.68	1.01	2.5	2.49	0.85	1.8	1.87
	100	-0.03	0.1	0.05	-0.02	-0.03	-0.03	0	0.02	0.06	1.25	3.03	3.2	0.73	1.67	1.74	0.58	1.2	1.25
		Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	0.14	-0.03	-0.07	-0.02	-0.08	0.3	0.01	-0.03	-0.04	3.99	7.54	7.8	2.09	3.96	4.17	1.52	2.83	2.85
$\bar{\boldsymbol{x}}_t$	50	0	0.18	0.11	0.02	-0.16	0	-0.03	0	-0.04	2.26	4.37	4.59	1.05	2.54	2.5	0.87	1.79	1.87
	100	-0.01	0.02	0.02	-0.01	-0.03	-0.04	-0.02	0.02	0.08	1.28	3.13	3.2	0.74	1.68	1.78	0.6	1.2	1.26
		Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																	
	20	-0.02	-0.08	0	-0.01	-0.08	0.28	-0.02	0	-0.02	3.45	6.9	7.53	1.76	3.94	4.04	1.34	2.79	2.81
	50	-0.05	0.13	0.16	0	-0.16	0.03	-0.03	0	-0.04	1.67	4.18	4.59	0.93	2.49	2.44	0.81	1.79	1.85
	100	-0.05	0.08	0.06	-0.02	-0.03	-0.03	-0.01	0.02	0.07	1.07	2.98	3.06	0.68	1.67	1.72	0.56	1.2	1.24
		Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	1.86	-0.76	-1.18	0.71	-0.33	-0.2	0.37	-0.14	-0.28	3.22	6.79	7.31	1.82	3.92	4.02	1.35	2.8	2.81
	50	1.33	-0.48	-1.08	0.51	-0.38	-0.42	0.21	-0.11	-0.26	1.99	4.15	4.51	1.04	2.51	2.48	0.83	1.79	1.86
	100	1.17	-0.5	-1.09	0.41	-0.23	-0.44	0.2	-0.08	-0.13	1.55	2.99	3.25	0.79	1.68	1.79	0.6	1.2	1.24
		OLS																	
	20	3.11	-1.38	-2.15	3.11	-1.21	-1.98	3.12	-1.13	-2.25	3.92	6.87	7.29	3.45	4.09	4.35	3.35	2.97	3.57
	50	3.31	-1.28	-2.87	3.27	-1.63	-2.9	3.22	-1.46	-2.94	3.55	4.31	5.17	3.37	2.97	3.77	3.3	2.28	3.45
	100	3.34	-1.62	-3.12	3.26	-1.59	-3.14	3.26	-1.54	-3.04	3.45	3.35	4.32	3.32	2.28	3.58	3.29	1.94	3.28
		Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																	
	20	-0.37	-0.28	0.23	-0.24	-0.07	0.32	-0.26	0.02	0.05	3.09	6.95	7.3	1.81	3.95	3.95	1.38	2.77	2.78
	50	-0.06	0.16	0.13	-0.04	-0.17	0.04	-0.07	-0.01	0	1.75	4.33	4.59	1.01	2.5	2.46	0.84	1.78	1.82
	100	-0.05	0.04	0.11	-0.03	-0.01	0	-0.01	0.02	0.07	1.22	3.01	3.16	0.72	1.67	1.74	0.58	1.19	1.24
		Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	-0.42	-0.13	0.35	-0.36	-0.02	0.42	-0.34	0.04	0.11	3.45	7.16	7.43	2.01	3.98	4.04	1.49	2.79	2.82
$(\bar{y}_t, \bar{\boldsymbol{x}}_t')'$	50	-0.07	0.21	0.12	-0.02	-0.17	0.03	-0.07	0	0	1.86	4.31	4.5	1.03	2.52	2.46	0.86	1.76	1.82
	100	-0.03	-0.01	0.09	-0.02	-0.02	-0.01	-0.03	0.02	0.1	1.22	3.1	3.17	0.72	1.67	1.78	0.6	1.19	1.24
		Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																	
	20	-0.43	-0.19	0.27	-0.31	-0.03	0.38	-0.34	0.04	0.11	2.76	6.84	7.14	1.71	3.94	3.92	1.33	2.76	2.77
	50	-0.07	0.2	0.14	-0.03	-0.16	0.04	-0.07	0	0	1.49	4.23	4.38	0.93	2.49	2.41	0.8	1.76	1.8
	100	-0.04	0	0.11	-0.03	-0.01	0	-0.02	0.02	0.09	1.06	2.98	3.04	0.66	1.66	1.73	0.56	1.19	1.22
		Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	0.41	-0.46	-0.3	0	-0.15	0.15	-0.15	-0.03	-0.02	2.64	6.84	7.12	1.67	3.93	3.91	1.29	2.76	2.77
	50	0.71	-0.14	-0.58	0.24	-0.28	-0.2	0.06	-0.06	-0.12	1.63	4.19	4.39	0.95	2.49	2.43	0.8	1.76	1.81
	100	0.75	-0.35	-0.66	0.24	-0.14	-0.26	0.11	-0.04	-0.03	1.26	2.98	3.1	0.69	1.66	1.76	0.57	1.19	1.22
		OLS																	
	20	3.34	-2.33	-4.18	3.33	-2.28	-4.15	3.32	-2.16	-4.33	4.14	7.3	8.63	3.65	4.57	5.75	3.53	3.5	5.2
	50	3.35	-1.68	-3.71	3.32	-2.03	-3.73	3.28	-1.91	-3.79	3.61	4.62	5.81	3.43	3.21	4.46	3.36	2.59	4.22
	100	3.37	-1.81	-3.51	3.29	-1.81	-3.57	3.29	-1.75	-3.47	3.49	3.56	4.68	3.34	2.46	3.98	3.33	2.12	3.68
		Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																	
	20	-0.19	-1.14	-1.68	-0.03	-1.09	-1.74	-0.07	-0.96	-1.92	3.26	7.22	8.09	1.8	4.15	4.4	1.36	2.93	3.46
	50	-0.06	-0.23	-0.61	-0.01	-0.52	-0.71	-0.04	-0.41	-0.78	1.91	4.51	4.85	1.02	2.57	2.58	0.84	1.83	2.01
	100	-0.06	-0.11	-0.19	-0.03	-0.2	-0.38	0	-0.17	-0.3	1.3	3.17	3.34	0.71	1.71	1.8	0.59	1.21	1.27
		Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	0.15	-1.27	-1.92	0.21	-1.2	-1.95	0.19	-1.09	-2.16	3.6	7.5	8.49	1.97	4.18	4.54	1.47	2.99	3.63
$(\bar{y}_t, \bar{y}_t^{*}, \bar{\boldsymbol{x}}_t')'$	50	0.02	-0.21	-0.72	0.08	-0.58	-0.82	0.05	-0.45	-0.89	1.96	4.53	4.76	1.04	2.6	2.61	0.86	1.83	2.05
	100	0.03	-0.21	-0.32	0.03	-0.24	-0.44	0.03	-0.2	-0.33	1.27	3.25	3.3	0.71	1.71	1.86	0.6	1.22	1.29

Table S22 (Continued)

		Bias (× 100)									RMSE (×) 100								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																			
	20	-0.07	-1.17	-1.8	0.06	-1.12	-1.82	0.02	-1	-2	2.81	7.11	7.93	1.67	4.15	4.4	1.29	2.94	3.49
	50	-0.04	-0.19	-0.66	0.03	-0.54	-0.75	0	-0.42	-0.82	1.59	4.42	4.58	0.94	2.56	2.55	0.8	1.82	2.01
	100	-0.01	-0.16	-0.25	-0.01	-0.21	-0.4	0.01	-0.18	-0.31	1.09	3.12	3.18	0.66	1.7	1.79	0.56	1.21	1.26
Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																			
	20	0.78	-1.44	-2.37	0.34	-1.23	-2.02	0.17	-1.06	-2.11	2.8	7.14	8.04	1.69	4.15	4.48	1.3	2.96	3.57
	50	0.79	-0.56	-1.42	0.3	-0.67	-1	0.13	-0.49	-0.95	1.74	4.39	4.76	0.98	2.59	2.64	0.81	1.84	2.06
	100	0.84	-0.57	-1.08	0.26	-0.35	-0.66	0.14	-0.25	-0.44	1.33	3.14	3.33	0.7	1.72	1.88	0.58	1.22	1.3
OLS																			
Infeasible	20	2.69	-1.85	-3.76	2.65	-1.67	-3.43	2.7	-1.62	-3.78	3.14	6.55	7.67	2.9	4.11	5.09	2.88	3.09	4.63
	50	2.78	-1.41	-3.46	2.71	-1.62	-3.5	2.68	-1.49	-3.57	2.99	4.17	5.47	2.81	2.88	4.17	2.76	2.27	4
	100	2.78	-1.39	-3.42	2.75	-1.45	-3.49	2.72	-1.43	-3.37	2.89	3.13	4.4	2.8	2.15	3.87	2.76	1.83	3.57
Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																			
	20	-0.12	-0.22	0.11	-0.03	-0.12	0.27	0.01	-0.05	-0.05	2.1	6.57	7.05	1.37	3.8	3.85	1.11	2.65	2.73
	50	-0.04	0.06	0.16	-0.03	-0.13	0.04	-0.05	-0.01	-0.05	1.39	4.1	4.38	0.86	2.42	2.34	0.76	1.73	1.81
	100	-0.06	0.08	0.17	-0.01	-0.01	-0.03	0	0.01	0.07	1.12	3.05	2.98	0.65	1.62	1.71	0.54	1.14	1.21
Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																			
	20	-0.09	-0.08	0.06	-0.04	-0.11	0.3	0.03	-0.08	-0.07	2.12	6.55	7.03	1.38	3.79	3.89	1.11	2.64	2.71
	50	-0.07	0.09	0.15	-0.02	-0.14	0.02	-0.05	-0.01	-0.04	1.39	3.99	4.4	0.87	2.43	2.32	0.77	1.72	1.8
	100	0	0.05	0.08	0	0	-0.04	-0.01	0.01	0.08	1.01	2.89	2.9	0.64	1.61	1.73	0.55	1.14	1.22
Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																			
	20	-0.08	-0.17	0.04	-0.04	-0.1	0.28	0.02	-0.06	-0.05	1.9	6.44	6.83	1.28	3.78	3.79	1.07	2.63	2.68
	50	-0.05	0.09	0.15	-0.02	-0.13	0.03	-0.05	-0.01	-0.04	1.22	3.98	4.29	0.81	2.4	2.29	0.73	1.72	1.78
	100	-0.02	0.06	0.13	-0.01	0	-0.03	-0.01	0.01	0.07	0.89	2.84	2.79	0.6	1.61	1.68	0.52	1.14	1.2
Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																			
	20	0.51	-0.55	-0.77	0.17	-0.23	0.01	0.12	-0.13	-0.2	1.88	6.41	6.87	1.3	3.77	3.8	1.07	2.64	2.7
	50	0.55	-0.23	-0.61	0.19	-0.25	-0.24	0.05	-0.06	-0.18	1.32	3.95	4.31	0.83	2.42	2.3	0.73	1.72	1.79
	100	0.58	-0.23	-0.65	0.21	-0.12	-0.3	0.09	-0.04	-0.05	1.04	2.82	2.88	0.64	1.61	1.72	0.53	1.14	1.2
OLS																			
	20	4.27	-1.29	-5.65	4.22	-1.15	-5.5	4.26	-1.04	-5.9	4.59	6.41	8.69	4.42	4.03	6.83	4.4	2.94	6.56
	50	4.45	-0.86	-5.63	4.43	-1.12	-5.7	4.35	-0.99	-5.73	4.62	3.97	7.04	4.51	2.7	6.21	4.41	2.08	6.06
	100	4.47	-0.95	-5.58	4.49	-1	-5.64	4.46	-0.89	-5.54	4.57	2.93	6.26	4.55	1.94	5.9	4.5	1.52	5.7
Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																			
	20	0.48	1.88	-0.56	0.59	1.75	-0.5	0.62	1.87	-0.91	2.4	7.06	7.29	1.6	4.36	4.15	1.34	3.41	3.02
	50	0.6	1.99	-0.75	0.62	1.79	-0.81	0.62	1.87	-0.93	1.72	4.54	4.52	1.15	3.15	2.61	1.02	2.69	2.15
	100	0.63	1.9	-0.76	0.71	1.81	-0.88	0.68	1.95	-0.79	1.35	3.57	3.07	1.03	2.51	1.97	0.91	2.32	1.53
Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																			
Naive	20	0.66	1.79	-0.76	0.75	1.62	-0.69	0.8	1.72	-1.15	2.41	6.98	6.91	1.74	4.26	4.18	1.47	3.3	3.06
	50	0.75	1.91	-0.92	0.83	1.63	-1.06	0.82	1.73	-1.17	1.78	4.41	4.5	1.33	3.05	2.68	1.18	2.57	2.25
	100	0.86	1.74	-1.06	0.92	1.66	-1.13	0.88	1.79	-1.04	1.49	3.39	3.11	1.22	2.39	2.11	1.09	2.2	1.68
Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																			
	20	0.67	1.71	-0.78	0.74	1.63	-0.69	0.77	1.76	-1.1	2.17	6.8	6.77	1.65	4.26	4.1	1.42	3.31	3.03
	50	0.76	1.9	-0.94	0.81	1.65	-1.04	0.8	1.74	-1.15	1.67	4.36	4.34	1.27	3.03	2.64	1.14	2.58	2.23
	100	0.82	1.76	-1	0.9	1.68	-1.11	0.87	1.8	-1.03	1.39	3.35	3.01	1.18	2.4	2.07	1.06	2.2	1.65
Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																			
	20	1.41	1.11	-1.79	1.02	1.4	-1.07	0.9	1.64	-1.29	2.38	6.56	6.93	1.76	4.15	4.19	1.48	3.25	3.1
	50	1.53	1.3	-1.92	1.09	1.44	-1.39	0.91	1.65	-1.3	2.07	4.09	4.67	1.43	2.92	2.79	1.22	2.52	2.32
	100	1.59	1.22	-1.96	1.16	1.48	-1.45	0.98	1.72	-1.17	1.9	3.06	3.46	1.38	2.25	2.27	1.15	2.13	1.75

Table S23: The Size and Power for Alternative Pooled Estimators under Experiment 4 with $\rho = 0.8$

		Size									Power								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
$\bar{\boldsymbol{x}}_t$		OLS																	
	20	0.859	0.104	0.154	0.988	0.13	0.221	0.998	0.151	0.395	0.38	0.39	0.31	0.54	0.72	0.73	0.59	0.94	0.95
	50	0.995	0.102	0.248	0.999	0.201	0.503	1	0.292	0.755	0.88	0.74	0.68	0.97	0.98	0.97	0.99	1	1
	100	1	0.153	0.399	1	0.274	0.791	1	0.479	0.95	0.99	0.95	0.96	1	1	1	1	1	1
		Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																	
	20	0.083	0.078	0.084	0.071	0.067	0.067	0.069	0.068	0.054	0.41	0.38	0.25	0.73	0.72	0.69	0.93	0.94	0.94
	50	0.048	0.053	0.063	0.049	0.064	0.054	0.061	0.063	0.055	0.88	0.76	0.67	1	0.97	0.98	1	1	1
	100	0.056	0.054	0.057	0.05	0.038	0.041	0.06	0.047	0.058	0.99	0.95	0.95	1	1	1	1	1	1
		Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	0.08	0.075	0.088	0.077	0.071	0.069	0.07	0.07	0.05	0.39	0.37	0.26	0.68	0.7	0.66	0.87	0.94	0.93
	50	0.048	0.043	0.055	0.037	0.067	0.06	0.065	0.061	0.063	0.87	0.76	0.67	1	0.98	0.98	1	1	1
	100	0.05	0.056	0.053	0.044	0.039	0.047	0.063	0.051	0.049	0.99	0.94	0.95	1	1	1	1	1	1
		Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																	
	20	0.089	0.077	0.083	0.066	0.068	0.068	0.07	0.071	0.054	0.47	0.39	0.25	0.77	0.7	0.66	0.93	0.94	0.93
	50	0.039	0.048	0.056	0.047	0.068	0.056	0.059	0.062	0.058	0.94	0.77	0.69	1	0.97	0.98	1	1	1
	100	0.051	0.055	0.051	0.051	0.04	0.044	0.061	0.045	0.054	1	0.95	0.95	1	1	1	1	1	1
		Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	0.081	0.077	0.085	0.071	0.073	0.068	0.073	0.071	0.055	0.44	0.37	0.25	0.78	0.71	0.68	0.93	0.93	0.93
	50	0.05	0.049	0.056	0.043	0.068	0.059	0.062	0.062	0.061	0.93	0.77	0.69	1	0.98	0.98	1	1	1
	100	0.058	0.056	0.054	0.054	0.038	0.044	0.065	0.048	0.052	0.99	0.95	0.95	1	1	1	1	1	1
$(\bar{y}_t, \bar{\boldsymbol{x}}_t')'$		OLS																	
	20	0.353	0.094	0.131	0.555	0.096	0.103	0.69	0.097	0.159	0.42	0.34	0.28	0.73	0.72	0.73	0.78	0.94	0.95
	50	0.756	0.095	0.129	0.965	0.131	0.26	0.992	0.168	0.425	0.92	0.74	0.72	0.99	0.98	0.98	1	1	1
	100	0.981	0.112	0.233	1	0.173	0.486	1	0.271	0.739	1	0.95	0.94	1	1	1	1	1	1
		Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																	
	20	0.097	0.082	0.089	0.087	0.086	0.072	0.088	0.084	0.057	0.43	0.35	0.28	0.72	0.72	0.68	0.92	0.94	0.94
	50	0.06	0.076	0.067	0.051	0.075	0.066	0.067	0.068	0.064	0.85	0.71	0.68	1	0.98	0.98	1	1	1
	100	0.061	0.073	0.059	0.054	0.045	0.049	0.067	0.06	0.055	0.99	0.95	0.94	1	1	1	1	1	1
		Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	0.086	0.081	0.093	0.089	0.087	0.064	0.094	0.083	0.065	0.4	0.34	0.27	0.68	0.7	0.72	0.88	0.94	0.92
	50	0.064	0.076	0.069	0.052	0.073	0.063	0.073	0.073	0.069	0.86	0.7	0.68	1	0.98	0.97	1	1	1
	100	0.061	0.071	0.062	0.05	0.044	0.05	0.068	0.057	0.053	0.99	0.94	0.94	1	1	1	1	1	1
		Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																	
	20	0.09	0.084	0.091	0.083	0.086	0.071	0.094	0.083	0.067	0.46	0.35	0.27	0.79	0.71	0.71	0.92	0.94	0.94
	50	0.056	0.072	0.065	0.053	0.075	0.066	0.065	0.074	0.068	0.91	0.71	0.67	1	0.98	0.98	1	1	1
	100	0.059	0.071	0.06	0.059	0.046	0.043	0.059	0.059	0.053	1	0.95	0.95	1	1	1	1	1	1
		Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	0.09	0.084	0.092	0.075	0.088	0.073	0.08	0.086	0.065	0.46	0.35	0.27	0.79	0.71	0.71	0.93	0.94	0.94
	50	0.068	0.073	0.069	0.048	0.074	0.066	0.066	0.076	0.068	0.9	0.71	0.68	1	0.98	0.98	1	1	1
	100	0.065	0.072	0.059	0.055	0.047	0.048	0.071	0.058	0.055	1	0.95	0.95	1	1	1	1	1	1
$(\bar{y}_t, \bar{y}_t^*, \bar{\boldsymbol{x}}_t')'$		OLS																	
	20	0.37	0.099	0.144	0.6	0.132	0.171	0.742	0.137	0.3	0.45	0.38	0.25	0.69	0.71	0.67	0.79	0.94	0.95
	50	0.759	0.101	0.16	0.965	0.162	0.314	0.995	0.212	0.548	0.92	0.72	0.67	1	0.97	0.99	1	1	1
	100	0.981	0.122	0.263	1	0.194	0.547	1	0.323	0.808	1	0.93	0.93	1	1	1	1	1	1
		Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																	
	20	0.086	0.082	0.112	0.078	0.091	0.081	0.082	0.086	0.103	0.46	0.34	0.25	0.72	0.71	0.68	0.93	0.94	0.94
	50	0.062	0.076	0.067	0.052	0.081	0.068	0.069	0.074	0.081	0.86	0.7	0.64	1	0.98	0.99	1	1	1
	100	0.06	0.066	0.056	0.054	0.051	0.056	0.069	0.06	0.061	0.99	0.93	0.94	1	1	1	1	1	1
		Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	0.086	0.083	0.115	0.081	0.095	0.078	0.079	0.088	0.104	0.5	0.34	0.23	0.79	0.71	0.68	0.94	0.95	0.93
	50	0.063	0.074	0.069	0.043	0.084	0.066	0.068	0.072	0.085	0.89	0.69	0.65	1	0.98	0.98	1	1	1
	100	0.064	0.067	0.059	0.054	0.05	0.054	0.072	0.06	0.067	1	0.93	0.95	1	1	1	1	1	1

Table S23 (Continued)

		Size									Power								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
								Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments											
	20	0.091	0.083	0.119	0.082	0.097	0.076	0.076	0.088	0.101	0.53	0.36	0.25	0.79	0.71	0.68	0.94	0.95	0.93
	50	0.069	0.076	0.07	0.05	0.077	0.071	0.071	0.073	0.081	0.89	0.7	0.64	1	0.98	0.99	1	1	1
	100	0.058	0.07	0.059	0.054	0.051	0.052	0.066	0.055	0.0691	0.93 0.94		1	1 1		1	1 1		
								Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	0.082	0.091	0.098	0.082	0.084	0.104	0.081	0.077	0.12	0.43	0.33	0.23	0.73	0.69	0.67	0.92	0.92	0.93
	50	0.072	0.076	0.064	0.063	0.072	0.077	0.052	0.066	0.076	0.88	0.69	0.65	1	0.98	0.98	1	1	1
	100	0.056	0.055	0.052	0.062	0.051	0.065	0.05	0.061	0.064	0.99	0.93	0.94	1	1	1	1	1	1
								OLS											
	20	0.422	0.095	0.149	0.631	0.109	0.173	0.728	0.119	0.318	0.77	0.36	0.31	0.86	0.75	0.75	0.93	0.94	0.96
	50	0.747	0.093	0.176	0.941	0.137	0.351	0.976	0.159	0.578	0.98	0.75	0.73	1	0.99	0.99	1	1	1
	100	0.96	0.104	0.257	0.999	0.142	0.592	1	0.242	0.835	1	0.97	0.97	1	1	1	1	1	1
								Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments											
	20	0.091	0.086	0.098	0.077	0.081	0.07	0.076	0.075	0.066	0.62	0.33	0.34	0.88	0.74	0.73	0.99	0.94	0.94
	50	0.051	0.076	0.07	0.057	0.072	0.059	0.066	0.072	0.064	0.96	0.73	0.67	1	0.98	0.99	1	1	1
	100	0.054	0.054	0.057	0.058	0.038	0.05	0.062	0.04	0.057	1	0.96	0.96	1	1	1	1	1	1
								Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	0.076	0.083	0.087	0.073	0.085	0.066	0.069	0.076	0.077	0.68	0.34	0.31	0.91	0.72	0.73	0.99	0.94	0.93
Infeasible	50	0.057	0.077	0.065	0.051	0.071	0.059	0.067	0.076	0.063	0.96	0.75	0.71	1	0.99	0.99	1	1	1
	100	0.06	0.056	0.052	0.045	0.048	0.049	0.067	0.039	0.056	1	0.97	0.96	1	1	1	1	1	1
								Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments											
	20	0.085	0.084	0.095	0.069	0.079	0.067	0.073	0.076	0.07	0.71	0.33	0.33	0.93	0.72	0.76	0.99	0.94	0.94
	50	0.049	0.076	0.069	0.061	0.077	0.056	0.067	0.072	0.07	0.98	0.74	0.7	1	0.98	0.99	1	1	1
	100	0.051	0.056	0.05	0.058	0.045	0.05	0.064	0.041	0.06	1	0.97	0.96	1	1	1	1	1	1
								Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	0.073	0.086	0.091	0.066	0.082	0.068	0.074	0.077	0.07	0.68	0.33	0.33	0.92	0.72	0.75	0.99	0.94	0.93
	50	0.055	0.077	0.068	0.051	0.074	0.058	0.064	0.069	0.067	0.98	0.75	0.71	1	0.98	0.99	1	1	1
	100	0.059	0.055	0.053	0.058	0.044	0.049	0.068	0.04	0.056	1	0.97	0.96	1	1	1	1	1	1
								OLS											
	20	0.764	0.086	0.225	0.917	0.099	0.331	0.972	0.094	0.565	0.58	0.37	0.36	0.73	0.69	0.69	0.8	0.93	0.93
	50	0.981	0.084	0.337	1	0.089	0.658	1	0.091	0.875	0.9	0.76	0.74	0.99	0.97	0.97	1	1	1
	100	0.999	0.086	0.585	1	0.094	0.913	1	0.116	0.987	0.99	0.96	0.95	1	1	1	1	1	1
								Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments											
	20	0.121	0.088	0.088	0.127	0.098	0.084	0.142	0.136	0.082	0.63	0.33	0.34	0.83	0.64	0.64	0.95	0.91	0.89
	50	0.149	0.101	0.066	0.18	0.136	0.069	0.2	0.231	0.105	0.86	0.76	0.75	1	0.95	0.98	1	1	1
	100	0.212	0.117	0.069	0.296	0.214	0.091	0.361	0.38	0.112	0.98	0.95	0.95	1	1	1	1	1	1
								Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	0.146	0.091	0.091	0.166	0.096	0.08	0.177	0.126	0.078	0.57	0.34	0.3	0.8	0.62	0.64	0.91	0.92	0.91
Naive	50	0.221	0.086	0.075	0.246	0.122	0.086	0.289	0.213	0.12	0.84	0.78	0.77	0.99	0.95	0.97	1	1	1
	100	0.291	0.11	0.074	0.39	0.187	0.128	0.478	0.327	0.147	0.97	0.95	0.96	1	1	1	1	1	1
								Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments											
	20	0.149	0.087	0.09	0.167	0.093	0.078	0.177	0.125	0.082	0.65	0.34	0.31	0.84	0.63	0.65	0.94	0.91	0.92
	50	0.218	0.096	0.074	0.256	0.125	0.085	0.282	0.21	0.126	0.87	0.77	0.77	1	0.95	0.98	1	1	1
	100	0.302	0.102	0.07	0.394	0.187	0.122	0.486	0.329	0.15	0.98	0.95	0.95	1	1	1	1	1	1
								Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	0.128	0.092	0.096	0.151	0.099	0.081	0.166	0.129	0.08	0.62	0.35	0.32	0.83	0.62	0.65	0.95	0.91	0.91
	50	0.202	0.094	0.072	0.216	0.124	0.077	0.258	0.216	0.114	0.88	0.78	0.76	1	0.95	0.98	1	1	1
	100	0.276	0.109	0.068	0.364	0.199	0.111	0.444	0.349	0.134	0.99	0.95	0.95	1	1	1	1	1	1

Table S24: The Size and Power for Alternative Mean Group Estimators under Experiment 4 with $\rho = 0.8$

		Size									Power								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
								OLS											
	20	0.833	0.096	0.138	0.987	0.108	0.218	1	0.154	0.386	0.36	0.31	0.3	0.6	0.71	0.68	0.73	0.95	0.95
	50	0.98	0.083	0.219	0.999	0.192	0.479	1	0.286	0.744	0.86	0.72	0.64	0.98	0.98	0.98	1	1	1
	100	0.999	0.13	0.353	1	0.275	0.754	1	0.449	0.947	1	0.91	0.92	1	1	1	1	1	1
								Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments											
	20	0.062	0.064	0.062	0.063	0.056	0.067	0.05	0.064	0.054	0.32	0.3	0.31	0.67	0.68	0.65	0.91	0.94	0.92
	50	0.043	0.047	0.055	0.039	0.056	0.049	0.061	0.063	0.054	0.77	0.71	0.6	0.99	0.98	0.98	1	1	1
	100	0.044	0.053	0.05	0.055	0.048	0.048	0.057	0.049	0.056	0.96	0.92	0.89	1	1	1	1	1	1
								Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	0.063	0.061	0.058	0.06	0.057	0.065	0.058	0.062	0.055	0.32	0.31	0.25	0.59	0.7	0.66	0.86	0.93	0.91
$\bar{\boldsymbol{x}}_t$	50	0.052	0.045	0.052	0.035	0.058	0.048	0.063	0.056	0.067	0.72	0.66	0.6	0.99	0.97	0.98	1	1	1
	100	0.051	0.055	0.045	0.047	0.043	0.044	0.067	0.051	0.054	0.95	0.88	0.89	1	1	1	1	1	1
								Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments											
	20	0.072	0.063	0.066	0.057	0.056	0.068	0.055	0.064	0.054	0.39	0.3	0.31	0.67	0.7	0.65	0.93	0.94	0.93
	50	0.046	0.048	0.051	0.033	0.057	0.042	0.061	0.058	0.059	0.86	0.71	0.61	1	0.98	0.99	1	1	1
	100	0.056	0.059	0.051	0.049	0.052	0.047	0.058	0.048	0.056	0.98	0.9	0.92	1	1	1	1	1	1
								Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	0.139	0.063	0.077	0.082	0.054	0.066	0.065	0.065	0.059	0.43	0.33	0.3	0.74	0.71	0.67	0.94	0.93	0.93
	50	0.171	0.045	0.064	0.077	0.061	0.052	0.077	0.06	0.061	0.88	0.71	0.61	1	0.97	0.98	1	1	1
	100	0.22	0.058	0.061	0.104	0.047	0.06	0.075	0.046	0.057	0.99	0.91	0.92	1	1	1	1	1	1
								OLS											
	20	0.284	0.078	0.1	0.538	0.085	0.1	0.709	0.093	0.158	0.46	0.35	0.28	0.75	0.7	0.71	0.87	0.95	0.94
	50	0.685	0.068	0.121	0.957	0.134	0.221	0.986	0.164	0.421	0.9	0.71	0.64	1	0.98	0.98	1	1	1
	100	0.956	0.097	0.187	1	0.157	0.449	1	0.249	0.726	1	0.91	0.91	1	1	1	1	1	1
								Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments											
	20	0.078	0.078	0.082	0.076	0.074	0.071	0.07	0.075	0.058	0.38	0.3	0.28	0.7	0.68	0.68	0.95	0.94	0.92
	50	0.057	0.061	0.063	0.048	0.06	0.063	0.067	0.07	0.063	0.74	0.69	0.58	1	0.98	0.97	1	1	1
	100	0.054	0.06	0.054	0.059	0.051	0.053	0.064	0.051	0.055	0.96	0.9	0.91	1	1	1	1	1	1
								Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	0.049	0.08	0.083	0.069	0.076	0.066	0.071	0.07	0.06	0.39	0.3	0.27	0.66	0.67	0.68	0.91	0.94	0.91
$(\bar{y}_t, \bar{\boldsymbol{x}}_t')'$	50	0.063	0.059	0.063	0.035	0.068	0.058	0.063	0.07	0.067	0.77	0.68	0.56	1	0.97	0.98	1	1	1
	100	0.051	0.067	0.05	0.048	0.047	0.054	0.07	0.053	0.058	0.96	0.89	0.91	1	1	1	1	1	1
								Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments											
	20	0.056	0.08	0.081	0.069	0.076	0.068	0.07	0.074	0.061	0.45	0.3	0.3	0.78	0.69	0.7	0.96	0.95	0.93
	50	0.049	0.063	0.057	0.037	0.064	0.059	0.059	0.065	0.064	0.88	0.68	0.59	1	0.98	0.98	1	1	1
	100	0.053	0.067	0.053	0.052	0.048	0.055	0.067	0.052	0.058	0.99	0.9	0.91	1	1	1	1	1	1
								Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	0.065	0.081	0.084	0.066	0.071	0.068	0.061	0.076	0.06	0.48	0.32	0.3	0.78	0.69	0.7	0.95	0.95	0.93
	50	0.091	0.061	0.071	0.051	0.066	0.06	0.065	0.067	0.063	0.87	0.68	0.59	1	0.98	0.98	1	1	1
	100	0.123	0.064	0.061	0.075	0.052	0.06	0.074	0.051	0.058	0.99	0.9	0.91	1	1	1	1	1	1
								OLS											
	20	0.307	0.092	0.139	0.587	0.119	0.186	0.768	0.144	0.31	0.44	0.33	0.2	0.75	0.7	0.71	0.87	0.95	0.94
	50	0.671	0.082	0.154	0.959	0.15	0.328	0.992	0.219	0.567	0.88	0.65	0.61	1	0.98	0.98	1	1	1
	100	0.952	0.104	0.212	1	0.184	0.526	1	0.317	0.817	1	0.9	0.89	1	1	1	1	1	1
								Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments											
	20	0.072	0.079	0.093	0.067	0.07	0.074	0.062	0.084	0.112	0.36	0.3	0.21	0.7	0.69	0.7	0.95	0.94	0.93
	50	0.056	0.061	0.06	0.047	0.075	0.061	0.061	0.084	0.083	0.76	0.64	0.52	0.99	0.97	0.98	1	1	1
	100	0.055	0.061	0.058	0.052	0.055	0.054	0.064	0.05	0.058	0.95	0.89	0.86	1	1	1	1	1	1
								Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	0.057	0.074	0.093	0.06	0.086	0.08	0.067	0.085	0.122	0.36	0.3	0.22	0.62	0.68	0.65	0.89	0.94	0.93
$(\bar{y}_t, \bar{y}_t^{*}, \bar{\boldsymbol{x}}_t')'$	50	0.076	0.061	0.064	0.037	0.083	0.063	0.069	0.074	0.089	0.68	0.62	0.53	1	0.97	0.98	1	1	1
	100	0.047	0.06	0.057	0.051	0.057	0.068	0.075	0.054	0.06	0.96	0.88	0.87	1	1	1	1	1	1

Table S24 (Continued)

		Size									Power								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
		Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																	
	20	0.062	0.086	0.095	0.061	0.075	0.072	0.056	0.089	0.111	0.45	0.29	0.2	0.79	0.68	0.67	0.95	0.94	0.93
	50	0.061	0.066	0.058	0.039	0.073	0.059	0.07	0.084	0.082	0.84	0.65	0.56	1	0.97	0.98	1	1	1
	100	0.049	0.06	0.052	0.051	0.057	0.062	0.062	0.053	0.06	0.99	0.89	0.9	1	1	1	1	1	1
		Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	0.08	0.086	0.099	0.067	0.081	0.078	0.066	0.09	0.125	0.44	0.32	0.23	0.76	0.7	0.66	0.95	0.95	0.93
	50	0.105	0.064	0.073	0.056	0.079	0.073	0.076	0.085	0.095	0.86	0.63	0.56	1	0.98	0.98	1	1	1
	100	0.145	0.065	0.081	0.077	0.059	0.075	0.079	0.055	0.064	0.99	0.9	0.9	1	1	1	1	1	1
Infeasible		OLS																	
	20	0.367	0.073	0.123	0.56	0.098	0.162	0.713	0.118	0.307	0.7	0.34	0.32	0.91	0.71	0.73	0.98	0.96	0.97
	50	0.678	0.078	0.155	0.927	0.127	0.312	0.973	0.169	0.57	0.98	0.73	0.64	1	0.98	0.99	1	1	1
	100	0.935	0.091	0.234	0.998	0.143	0.535	1	0.236	0.816	1	0.94	0.97	1	1	1	1	1	1
		Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																	
	20	0.059	0.063	0.086	0.069	0.064	0.061	0.062	0.059	0.062	0.62	0.34	0.28	0.87	0.74	0.75	0.99	0.95	0.93
	50	0.052	0.059	0.067	0.038	0.067	0.047	0.061	0.068	0.065	0.9	0.71	0.59	1	0.98	0.99	1	1	1
	100	0.052	0.056	0.04	0.061	0.048	0.06	0.061	0.045	0.058	0.99	0.93	0.94	1	1	1	1	1	1
		Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	0.066	0.072	0.077	0.068	0.063	0.063	0.057	0.055	0.071	0.58	0.34	0.31	0.89	0.7	0.7	0.99	0.96	0.92
	50	0.046	0.061	0.072	0.046	0.069	0.05	0.059	0.07	0.065	0.91	0.72	0.63	1	0.97	0.98	1	1	1
	100	0.053	0.056	0.041	0.054	0.049	0.053	0.064	0.045	0.059	0.99	0.94	0.94	1	1	1	1	1	1
		Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																	
	20	0.062	0.063	0.082	0.063	0.068	0.063	0.058	0.064	0.068	0.65	0.36	0.29	0.91	0.74	0.74	1	0.96	0.93
	50	0.046	0.064	0.064	0.042	0.067	0.055	0.058	0.068	0.062	0.95	0.71	0.64	1	0.98	0.99	1	1	1
	100	0.048	0.063	0.043	0.053	0.047	0.056	0.054	0.048	0.061	1	0.94	0.96	1	1	1	1	1	1
		Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	0.072	0.061	0.084	0.064	0.065	0.065	0.06	0.058	0.073	0.7	0.35	0.33	0.93	0.72	0.74	1	0.96	0.93
	50	0.081	0.062	0.068	0.046	0.07	0.053	0.055	0.072	0.062	0.96	0.7	0.63	1	0.97	0.99	1	1	1
	100	0.113	0.055	0.044	0.06	0.049	0.054	0.069	0.048	0.062	1	0.95	0.96	1	1	1	1	1	1
Naive		OLS																	
	20	0.728	0.073	0.172	0.9	0.082	0.312	0.97	0.086	0.538	0.61	0.36	0.34	0.78	0.66	0.67	0.87	0.94	0.94
	50	0.969	0.059	0.304	0.999	0.088	0.634	1	0.105	0.873	0.92	0.75	0.68	0.99	0.98	0.99	1	1	1
	100	0.998	0.073	0.521	1	0.093	0.902	1	0.128	0.994	1	0.95	0.95	1	1	1	1	1	1
		Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																	
	20	0.087	0.087	0.077	0.119	0.092	0.077	0.105	0.113	0.077	0.58	0.3	0.3	0.85	0.64	0.66	0.97	0.91	0.91
	50	0.134	0.072	0.076	0.144	0.12	0.06	0.163	0.203	0.092	0.8	0.71	0.63	0.99	0.97	0.98	1	1	1
	100	0.162	0.104	0.049	0.239	0.182	0.078	0.291	0.344	0.098	0.97	0.93	0.93	1	1	1	1	1	1
		Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	0.127	0.073	0.075	0.154	0.083	0.085	0.156	0.095	0.081	0.52	0.33	0.31	0.79	0.63	0.64	0.94	0.92	0.93
	50	0.173	0.061	0.08	0.213	0.121	0.067	0.246	0.18	0.118	0.76	0.74	0.62	0.99	0.97	0.97	1	1	1
	100	0.238	0.099	0.056	0.35	0.162	0.099	0.41	0.296	0.123	0.93	0.91	0.93	1	1	1	1	1	1
		Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																	
	20	0.128	0.084	0.081	0.159	0.083	0.081	0.155	0.104	0.078	0.59	0.34	0.31	0.84	0.64	0.68	0.97	0.92	0.93
	50	0.19	0.07	0.074	0.212	0.116	0.067	0.24	0.186	0.12	0.84	0.75	0.64	1	0.97	0.98	1	1	1
	100	0.257	0.102	0.059	0.357	0.165	0.094	0.44	0.307	0.125	0.98	0.93	0.93	1	1	1	1	1	1
		Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	0.183	0.073	0.085	0.178	0.086	0.089	0.171	0.097	0.086	0.61	0.34	0.32	0.82	0.64	0.65	0.97	0.92	0.91
	50	0.318	0.055	0.1	0.281	0.105	0.076	0.284	0.177	0.132	0.88	0.76	0.63	1	0.97	0.98	1	1	1
	100	0.461	0.083	0.101	0.47	0.142	0.121	0.493	0.278	0.145	0.98	0.95	0.94	1	1	1	1	1	1

S2.5 Supplementary Results for $h = 4$

This subsection provides Monte Carlo simulation results for $h = 4$ while all the other settings remain the same as in the main text. To save space, we again only report the results for Experiment 4. [4] It is noteworthy that when the density of the spatial weight matrix increases, all the other estimators' performance becomes worse, especially when sample size is small. However, the performance of our proposed estimator and the infeasible estimator remains almost unchanged. This confirms the robustness of our proposed estimator and the previous conclusion that the additional use of $\bar{y}_t$ and $\bar{y}_t^*$ as factor approximations could bias the estimation, especially when the spatial correlation is dense.

[4] The results for the other three experiments are available upon request.

Table S25: The Finite Sample Performance of Alternative Pooled Estimators under Experiment 4 with $h = 4$

		Bias (× 100)									RMSE (×) 100								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
		OLS																	
	20	11.45	-0.1	-0.22	11.46	-0.15	-0.37	11.51	-0.21	-0.23	12.74	6.03	5.93	12.04	3.86	3.86	11.88	2.6	2.78
	50	8.75	-0.53	-0.98	8.9	-0.55	-1.13	8.85	-0.54	-1.15	9.35	3.65	3.76	9.17	2.43	2.7	9.02	1.76	2
	100	7.91	-0.76	-1.2	8	-0.6	-1.28	8	-0.66	-1.33	8.28	2.7	2.95	8.15	1.74	2.08	8.09	1.38	1.81
		Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																	
	20	0.03	-0.07	-0.12	-0.09	0.01	-0.08	0.02	-0.06	0.08	5.85	5.96	5.87	3.73	3.82	3.77	2.74	2.57	2.74
	50	0.06	0	0.04	0.03	0.01	-0.05	-0.04	0.03	-0.02	3.19	3.58	3.63	2.09	2.37	2.42	1.53	1.67	1.63
	100	-0.05	-0.13	0.05	0.03	0.04	0	-0.01	-0.01	-0.04	2.19	2.6	2.66	1.35	1.63	1.62	0.98	1.21	1.2
		Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	-0.08	-0.07	-0.11	0	0.02	-0.08	0.04	-0.05	0.07	6.8	5.95	5.91	4.19	3.82	3.78	3.09	2.57	2.74
$\bar{\boldsymbol{x}}_t$	50	-0.07	0.02	0.06	-0.02	0.01	-0.05	0.01	0.03	-0.03	3.31	3.59	3.62	2.25	2.37	2.42	1.63	1.67	1.63
	100	-0.06	-0.12	0.07	0.04	0.04	0	-0.05	-0.01	-0.04	2.41	2.6	2.66	1.44	1.62	1.62	1.06	1.21	1.2
		Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																	
	20	-0.05	-0.07	-0.11	-0.08	0.01	-0.08	0.06	-0.05	0.08	6.18	5.95	5.89	3.94	3.82	3.77	2.89	2.57	2.74
	50	-0.02	0.01	0.05	-0.02	0.01	-0.05	-0.01	0.03	-0.03	3.16	3.58	3.63	2.14	2.37	2.42	1.58	1.67	1.63
	100	-0.08	-0.12	0.06	0.02	0.04	0	-0.04	-0.01	-0.04	2.31	2.6	2.65	1.38	1.63	1.62	1.02	1.21	1.2
		Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	0.17	-0.07	-0.12	-0.01	0.01	-0.08	0.06	-0.06	0.07	5.66	5.96	5.88	3.64	3.82	3.77	2.69	2.57	2.74
	50	0.05	0.01	0.04	0.03	0.01	-0.05	-0.02	0.03	-0.03	3.07	3.58	3.62	2.05	2.37	2.42	1.5	1.67	1.63
	100	-0.06	-0.13	0.06	0.04	0.04	0	-0.02	-0.01	-0.04	2.17	2.6	2.65	1.33	1.62	1.62	0.97	1.21	1.2
		OLS																	
	20	-0.24	-0.1	-0.23	0.23	-0.05	-0.16	0.41	-0.06	0.04	5.95	6.12	5.88	3.63	3.78	3.8	2.72	2.57	2.73
	50	2.93	-0.19	-0.27	3.05	-0.19	-0.41	3.02	-0.15	-0.42	4.13	3.64	3.72	3.58	2.35	2.44	3.33	1.68	1.66
	100	3.54	-0.4	-0.5	3.68	-0.26	-0.57	3.64	-0.3	-0.63	4.12	2.61	2.78	3.88	1.65	1.71	3.75	1.26	1.33
		Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																	
	20	-0.5	-0.11	-0.24	-0.62	-0.04	-0.15	-0.48	-0.05	0.06	5.94	6.14	5.88	3.76	3.78	3.8	2.73	2.57	2.74
	50	-0.07	-0.01	0.08	-0.04	0	-0.03	-0.12	0.05	-0.03	3.18	3.63	3.72	2.06	2.35	2.4	1.51	1.68	1.61
	100	-0.1	-0.11	0.08	0.04	0.03	0.01	-0.03	-0.01	-0.05	2.26	2.6	2.73	1.36	1.63	1.61	0.96	1.23	1.17
		Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	-1.07	-0.11	-0.23	-0.95	-0.03	-0.15	-0.91	-0.04	0.07	6.94	6.13	5.92	4.18	3.78	3.81	3.13	2.57	2.73
$(\bar{y}_t, \bar{\boldsymbol{x}}_t')'$	50	-0.23	0.01	0.1	-0.14	0.01	-0.02	-0.12	0.04	-0.03	3.37	3.64	3.72	2.2	2.36	2.41	1.59	1.68	1.61
	100	-0.2	-0.11	0.09	0.06	0.03	0.01	-0.06	-0.01	-0.04	2.47	2.59	2.74	1.43	1.62	1.6	1.03	1.23	1.18
		Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																	
	20	-1.09	-0.11	-0.22	-1.12	-0.03	-0.14	-0.97	-0.04	0.07	6.21	6.13	5.9	3.91	3.78	3.8	2.9	2.57	2.73
	50	-0.21	0	0.1	-0.14	0.01	-0.02	-0.14	0.05	-0.02	3.19	3.64	3.72	2.07	2.36	2.4	1.54	1.68	1.61
	100	-0.18	-0.11	0.09	0.05	0.03	0.01	-0.05	-0.01	-0.04	2.37	2.59	2.73	1.37	1.62	1.61	0.99	1.23	1.18
		Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	-0.56	-0.11	-0.24	-0.68	-0.04	-0.15	-0.58	-0.05	0.06	5.75	6.13	5.89	3.64	3.78	3.8	2.68	2.57	2.73
	50	-0.09	-0.01	0.08	-0.06	0.01	-0.03	-0.11	0.05	-0.03	3.08	3.63	3.72	2.01	2.35	2.4	1.48	1.68	1.61
	100	-0.12	-0.11	0.08	0.06	0.03	0.01	-0.04	-0.01	-0.05	2.23	2.59	2.73	1.33	1.62	1.61	0.94	1.23	1.17
		OLS																	
	20	-0.36	-0.76	-1.65	0.14	-0.82	-1.57	0.38	-0.78	-1.39	6.22	6.28	6.21	3.69	3.9	4.22	2.79	2.72	3.14
	50	3	-0.49	-0.85	3.08	-0.45	-0.98	3.04	-0.44	-0.98	4.23	3.71	3.92	3.62	2.43	2.62	3.36	1.73	1.9
	100	3.55	-0.54	-0.77	3.71	-0.41	-0.85	3.65	-0.46	-0.9	4.16	2.72	2.9	3.92	1.7	1.83	3.77	1.31	1.48
		Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																	
	20	-0.53	-0.77	-1.66	-0.74	-0.82	-1.57	-0.55	-0.77	-1.38	6.27	6.3	6.21	3.92	3.9	4.22	2.83	2.72	3.13
	50	-0.06	-0.31	-0.51	-0.08	-0.25	-0.6	-0.16	-0.24	-0.58	3.26	3.68	3.87	2.09	2.41	2.5	1.51	1.69	1.72
	100	-0.14	-0.24	-0.19	0.02	-0.12	-0.26	-0.05	-0.16	-0.31	2.34	2.68	2.79	1.37	1.65	1.64	0.96	1.24	1.22
		Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	-0.38	-0.77	-1.66	-0.55	-0.82	-1.58	-0.39	-0.77	-1.38	6.1	6.3	6.22	3.77	3.9	4.23	2.76	2.72	3.13
$(\bar{y}_t, \bar{y}_t^{*}, \bar{\boldsymbol{x}}_t')'$	50	-0.02	-0.31	-0.51	-0.03	-0.25	-0.61	-0.08	-0.24	-0.59	3.15	3.68	3.87	2.04	2.41	2.5	1.47	1.69	1.73
	100	-0.12	-0.25	-0.2	0.05	-0.12	-0.27	-0.03	-0.16	-0.31	2.3	2.68	2.79	1.34	1.65	1.64	0.95	1.24	1.22

Table S25 (Continued)

		Bias (× 100)									RMSE (×) 100								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																			
	20	-0.17	-0.78	-1.66	-0.2	-0.82	-1.58	-0.02	-0.77	-1.39	6.48	6.29	6.23	3.87	3.9	4.23	2.84	2.72	3.13
	50	0.03	-0.31	-0.52	0.06	-0.26	-0.62	0.06	-0.25	-0.61	3.23	3.69	3.87	2.09	2.41	2.5	1.53	1.69	1.73
	100	-0.1	-0.25	-0.2	0.11	-0.12	-0.28	0.02	-0.17	-0.32	2.42	2.68	2.78	1.39	1.65	1.64	0.99	1.24	1.22
Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																			
	20	-0.5	-0.75	-1.63	-0.93	-0.83	-1.64	-0.72	-0.8	-1.43	7.1	6.4	6.24	4.47	3.94	4.26	3.16	2.74	3.17
	50	-0.02	-0.32	-0.48	-0.07	-0.26	-0.6	-0.17	-0.25	-0.6	3.39	3.72	3.87	2.12	2.42	2.51	1.54	1.7	1.73
	100	-0.11	-0.24	-0.18	0.03	-0.12	-0.27	-0.04	-0.16	-0.31	2.34	2.69	2.78	1.38	1.65	1.63	0.96	1.24	1.22
OLS																			
	20	3.18	-0.45	-0.96	3.23	-0.34	-1.12	3.21	-0.41	-0.92	4.79	5.85	5.72	3.92	3.57	3.81	3.69	2.54	2.74
	50	3.25	-0.38	-0.88	3.31	-0.37	-1.03	3.25	-0.29	-1.02	4	3.47	3.66	3.64	2.24	2.53	3.46	1.66	1.89
	100	3.27	-0.37	-0.96	3.32	-0.31	-0.97	3.29	-0.34	-1.01	3.71	2.5	2.79	3.5	1.58	1.83	3.39	1.22	1.53
Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																			
	20	0.07	-0.15	-0.09	0.01	0.02	-0.11	-0.02	-0.05	0.1	3.84	5.82	5.64	2.39	3.57	3.62	1.88	2.5	2.58
	50	-0.03	-0.05	0.06	0.03	-0.04	-0.07	-0.01	0.04	-0.04	2.49	3.44	3.55	1.64	2.21	2.29	1.22	1.63	1.59
	100	-0.03	-0.04	0.01	0.03	0.02	0.01	-0.02	-0.01	-0.02	1.9	2.48	2.61	1.18	1.55	1.55	0.84	1.17	1.15
Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																			
	20	-0.01	-0.14	-0.07	0.07	0.01	-0.13	-0.04	-0.05	0.1	4.01	5.8	5.62	2.43	3.57	3.63	1.96	2.5	2.57
Infeasible	50	-0.11	-0.05	0.08	-0.01	-0.03	-0.06	0.02	0.04	-0.05	2.61	3.44	3.56	1.7	2.22	2.31	1.24	1.63	1.59
	100	-0.08	-0.04	0.01	0.04	0.02	0	-0.05	-0.01	-0.01	1.96	2.48	2.61	1.22	1.54	1.55	0.86	1.17	1.16
Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																			
	20	-0.01	-0.15	-0.08	0.05	0.02	-0.13	-0.02	-0.05	0.1	3.9	5.81	5.64	2.37	3.57	3.63	1.92	2.5	2.57
	50	-0.09	-0.05	0.08	0	-0.04	-0.06	0.01	0.04	-0.05	2.54	3.44	3.57	1.64	2.22	2.3	1.23	1.63	1.59
	100	-0.06	-0.04	0.01	0.03	0.02	0.01	-0.04	-0.01	-0.02	1.92	2.48	2.6	1.18	1.54	1.55	0.85	1.17	1.16
Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																			
	20	0.08	-0.15	-0.1	0.05	0.02	-0.12	-0.01	-0.05	0.1	3.75	5.81	5.63	2.32	3.57	3.62	1.86	2.5	2.57
	50	-0.04	-0.05	0.06	0.02	-0.04	-0.07	0	0.04	-0.04	2.44	3.44	3.55	1.61	2.21	2.29	1.2	1.63	1.58
	100	-0.04	-0.04	0.01	0.04	0.02	0	-0.03	-0.01	-0.02	1.86	2.48	2.61	1.16	1.54	1.55	0.82	1.17	1.15
OLS																			
	20	7.42	0.81	-2.27	7.55	0.96	-2.44	7.52	0.85	-2.35	8.41	5.79	6.22	8.05	3.8	4.53	7.88	2.77	3.66
	50	7.95	0.88	-2.26	8.15	0.94	-2.41	8.12	0.93	-2.48	8.53	3.58	4.22	8.42	2.6	3.47	8.29	1.99	3.03
	100	8.12	0.74	-2.35	8.31	0.94	-2.44	8.37	0.91	-2.52	8.52	2.65	3.6	8.48	1.94	2.98	8.47	1.6	2.84
Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																			
	20	1.95	1.84	-0.73	1.95	2.12	-0.7	1.86	2	-0.58	4.79	6.05	5.79	3.56	4.28	3.85	3.03	3.34	2.8
	50	2.19	2.02	-0.63	2.22	2.12	-0.69	2.17	2.13	-0.7	3.94	4.04	3.58	3.11	3.26	2.55	2.74	2.78	1.86
	100	2.13	1.91	-0.64	2.25	2.13	-0.66	2.29	2.12	-0.73	3.39	3.26	2.75	2.85	2.75	1.82	2.62	2.52	1.47
Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																			
	20	2.37	1.72	-0.86	2.48	1.99	-0.87	2.33	1.89	-0.73	5.31	5.99	5.77	4.08	4.21	3.88	3.47	3.27	2.83
Naive	50	2.72	1.88	-0.79	2.81	1.99	-0.87	2.8	1.99	-0.89	4.57	3.95	3.6	3.72	3.16	2.62	3.36	2.67	1.95
	100	2.74	1.77	-0.82	2.91	1.99	-0.86	2.91	1.99	-0.92	4.04	3.15	2.79	3.51	2.63	1.88	3.25	2.4	1.58
Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																			
	20	2.43	1.71	-0.88	2.53	1.98	-0.89	2.41	1.88	-0.76	5.29	5.99	5.8	4.08	4.2	3.88	3.52	3.26	2.84
	50	2.84	1.86	-0.82	2.92	1.97	-0.9	2.9	1.97	-0.92	4.64	3.94	3.61	3.8	3.14	2.63	3.45	2.65	1.96
	100	2.86	1.75	-0.85	3.03	1.97	-0.89	3.05	1.97	-0.96	4.15	3.13	2.79	3.62	2.61	1.9	3.37	2.38	1.6
Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																			
	20	2.23	1.77	-0.82	2.23	2.05	-0.79	2.11	1.94	-0.66	4.94	6.02	5.78	3.76	4.24	3.86	3.21	3.3	2.82
	50	2.51	1.94	-0.72	2.53	2.05	-0.79	2.49	2.06	-0.8	4.21	3.99	3.6	3.39	3.2	2.58	3.03	2.72	1.9
	100	2.47	1.83	-0.73	2.6	2.06	-0.76	2.61	2.06	-0.83	3.69	3.19	2.77	3.17	2.69	1.85	2.94	2.46	1.52

Table S26: The Finite Sample Performance of Alternative Mean Group Estimators under Experiment 4 with $h = 4$

		Bias (× 100)									RMSE (×) 100								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
	OLS																		
	20	12.41	-0.07	-0.22	12.32	-0.09	-0.35	12.34	-0.23	-0.22	13.85	6.5	6.46	12.94	3.93	3.96	12.71	2.61	2.8
	50	8.96	-0.51	-1.07	9.06	-0.55	-1.19	8.98	-0.56	-1.16	9.59	3.95	4.17	9.34	2.47	2.76	9.15	1.79	2.03
	100	8.05	-0.81	-1.24	8.02	-0.61	-1.25	7.99	-0.66	-1.33	8.45	2.97	3.21	8.18	1.81	2.1	8.08	1.39	1.81
	Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																		
	20	0.1	0.07	0.18	-0.15	0.1	0	0	-0.04	0.09	8.09	6.88	6.73	4.34	3.98	3.91	3.08	2.62	2.77
	50	0.01	0.12	0.05	-0.01	0.02	-0.06	-0.04	0.01	-0.02	3.81	4.01	4.18	2.24	2.44	2.46	1.6	1.7	1.67
	100	0.04	-0.13	0.08	0	0.04	0.04	-0.06	-0.01	-0.04	2.63	2.92	2.99	1.44	1.72	1.7	1.04	1.23	1.22
	Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																		
	20	1	0.17	0.03	-0.05	0.13	-0.01	0.05	-0.04	0.08	10	7.25	7.25	5.23	4.07	3.98	3.71	2.62	2.79
$\bar{\boldsymbol{x}}_t$	50	-0.06	0.15	0.07	-0.04	0.03	-0.06	-0.01	0.02	-0.02	4.29	4.11	4.25	2.46	2.44	2.48	1.73	1.7	1.67
	100	-0.04	-0.14	0.08	0.03	0.04	0.04	-0.1	-0.01	-0.02	2.98	2.97	3	1.56	1.71	1.69	1.13	1.23	1.23
	Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																		
	20	0.16	0.17	0.18	-0.11	0.11	0.01	0.05	-0.04	0.09	8.6	6.82	6.75	4.67	4.01	3.93	3.33	2.62	2.78
	50	-0.08	0.14	0.07	-0.03	0.03	-0.05	-0.02	0.02	-0.02	3.99	4.05	4.18	2.32	2.43	2.46	1.67	1.7	1.67
	100	-0.02	-0.13	0.09	0	0.05	0.04	-0.09	-0.01	-0.03	2.79	2.94	2.98	1.48	1.72	1.69	1.1	1.23	1.23
	Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																		
	20	3.58	0.06	0.04	1.32	0.09	-0.04	0.77	-0.05	0.07	7.73	6.71	6.56	4.33	3.97	3.9	3.06	2.61	2.77
	50	2.23	-0.03	-0.26	0.82	-0.03	-0.17	0.4	-0.01	-0.07	4.07	3.97	4.13	2.3	2.43	2.46	1.62	1.7	1.67
	100	1.92	-0.29	-0.24	0.71	-0.02	-0.08	0.28	-0.04	-0.09	3.11	2.91	2.96	1.55	1.72	1.69	1.05	1.23	1.22
	OLS																		
	20	-0.12	-0.01	-0.12	-0.16	0	-0.18	-0.11	-0.06	0.01	6.7	6.63	6.34	3.87	3.89	3.94	2.85	2.59	2.75
	50	3.11	-0.13	-0.34	3.09	-0.18	-0.47	3.08	-0.17	-0.42	4.51	3.98	4.12	3.65	2.42	2.49	3.4	1.7	1.69
	100	3.69	-0.43	-0.52	3.7	-0.27	-0.55	3.63	-0.3	-0.63	4.32	2.87	3.05	3.92	1.72	1.75	3.74	1.27	1.34
	Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																		
	20	-0.72	0	-0.03	-0.89	0.02	-0.13	-0.78	-0.04	0.06	7.07	6.76	6.44	4.14	3.91	3.96	3	2.59	2.75
	50	-0.13	0.09	0.08	-0.11	0.02	-0.06	-0.12	0.03	-0.01	3.53	4.03	4.18	2.13	2.43	2.44	1.54	1.69	1.64
	100	-0.06	-0.1	0.13	0.01	0.03	0.05	-0.06	-0.01	-0.04	2.55	2.87	3.02	1.43	1.71	1.67	1	1.24	1.19
	Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																		
	20	-1.03	0.04	-0.05	-1.53	0.06	-0.12	-1.47	-0.03	0.07	10.95	7.19	6.73	4.84	3.95	4	3.55	2.6	2.76
$(\bar{y}_t, \bar{\boldsymbol{x}}_t')'$	50	-0.29	0.13	0.12	-0.2	0.03	-0.05	-0.12	0.03	-0.01	3.99	4.09	4.21	2.29	2.43	2.46	1.64	1.69	1.64
	100	-0.14	-0.09	0.13	0.03	0.03	0.05	-0.09	0	-0.03	2.8	2.92	3.05	1.52	1.71	1.66	1.07	1.24	1.19
	Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																		
	20	-1.31	0.08	-0.03	-1.51	0.04	-0.11	-1.43	-0.03	0.07	7.19	6.71	6.46	4.28	3.93	3.96	3.19	2.59	2.75
	50	-0.29	0.11	0.12	-0.18	0.03	-0.05	-0.14	0.03	-0.01	3.69	4.04	4.17	2.13	2.43	2.44	1.57	1.69	1.64
	100	-0.11	-0.09	0.13	0	0.03	0.05	-0.08	0	-0.03	2.64	2.88	3.03	1.45	1.71	1.66	1.03	1.24	1.19
	Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																		
	20	-0.73	0.02	-0.05	-0.93	0.02	-0.13	-0.88	-0.04	0.06	6.74	6.71	6.39	3.98	3.91	3.95	2.95	2.59	2.75
	50	0.55	0.04	-0.01	0.11	0.01	-0.09	0	0.02	-0.03	3.37	4	4.15	2.06	2.43	2.44	1.51	1.69	1.64
	100	0.75	-0.17	-0.02	0.29	0.01	0	0.07	-0.02	-0.06	2.52	2.86	3	1.41	1.7	1.66	0.98	1.24	1.19
	OLS																		
	20	-0.07	-0.8	-1.76	-0.04	-0.86	-1.82	0.06	-0.88	-1.63	7.32	6.89	6.86	4.04	4	4.47	3	2.78	3.27
	50	3.21	-0.48	-0.96	3.13	-0.49	-1.14	3.12	-0.51	-1.07	4.67	4.12	4.32	3.71	2.49	2.73	3.43	1.77	1.97
	100	3.72	-0.59	-0.87	3.74	-0.44	-0.88	3.64	-0.48	-0.95	4.38	2.98	3.21	3.96	1.77	1.89	3.76	1.33	1.52
	Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																		
	20	-0.71	-0.8	-1.67	-0.79	-0.84	-1.77	-0.67	-0.85	-1.58	7.87	7.01	6.9	4.37	4.01	4.47	3.16	2.78	3.24
	50	-0.13	-0.27	-0.55	-0.14	-0.27	-0.72	-0.15	-0.3	-0.66	3.73	4.19	4.34	2.17	2.48	2.58	1.55	1.72	1.79
	100	-0.08	-0.25	-0.21	0	-0.14	-0.27	-0.08	-0.18	-0.35	2.67	2.98	3.09	1.45	1.73	1.7	1.02	1.26	1.24
	Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																		
	20	-0.66	-0.69	-1.66	-0.7	-0.82	-1.78	-0.59	-0.85	-1.59	11.55	7.2	7.74	5.11	4.04	4.52	3.75	2.78	3.26
$(\bar{y}_t, \bar{y}_t^{*}, \bar{\boldsymbol{x}}_t')'$	50	-0.08	-0.27	-0.51	-0.02	-0.29	-0.74	0.06	-0.31	-0.69	4.2	4.23	4.38	2.28	2.48	2.61	1.64	1.73	1.8
	100	-0.05	-0.25	-0.23	0.1	-0.14	-0.29	-0.02	-0.18	-0.36	2.89	3.02	3.13	1.55	1.73	1.69	1.08	1.26	1.25

Table S26 (Continued)

		Bias (× 100)									RMSE (×) 100								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
	Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																		
	20	-0.67	-0.75	-1.68	-0.74	-0.84	-1.77	-0.64	-0.85	-1.59	8.04	7	7.01	4.38	4.02	4.48	3.2	2.78	3.25
	50	-0.06	-0.27	-0.52	-0.02	-0.28	-0.74	0.03	-0.31	-0.68	3.84	4.17	4.33	2.14	2.48	2.59	1.57	1.72	1.79
	100	-0.07	-0.25	-0.22	0.07	-0.14	-0.29	-0.02	-0.19	-0.36	2.76	2.98	3.11	1.47	1.73	1.69	1.04	1.26	1.25
	Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																		
	20	-0.54	-0.77	-1.7	-0.65	-0.84	-1.77	-0.57	-0.85	-1.58	7.49	6.95	6.9	4.17	4.01	4.47	3.07	2.77	3.24
	50	0.69	-0.33	-0.65	0.15	-0.29	-0.76	0.04	-0.31	-0.68	3.57	4.16	4.32	2.1	2.48	2.59	1.51	1.72	1.79
	100	0.81	-0.33	-0.38	0.32	-0.16	-0.33	0.08	-0.2	-0.37	2.63	2.97	3.1	1.44	1.73	1.7	0.99	1.26	1.25
Infeasible	OLS																		
	20	3.48	-0.5	-1.21	3.34	-0.33	-1.1	3.31	-0.41	-0.98	5.18	6.4	6.11	4.06	3.7	3.95	3.8	2.54	2.79
	50	3.4	-0.39	-0.85	3.39	-0.36	-1.05	3.31	-0.32	-1.01	4.27	3.75	4.05	3.74	2.34	2.57	3.52	1.67	1.91
	100	3.4	-0.48	-0.96	3.35	-0.33	-0.94	3.32	-0.35	-0.99	3.91	2.73	3.03	3.54	1.63	1.86	3.42	1.25	1.53
	Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																		
	20	0.01	-0.12	-0.11	-0.01	0.08	-0.01	-0.02	-0.02	0.08	4.25	6.46	6.08	2.49	3.71	3.8	1.95	2.51	2.62
	50	-0.03	-0.01	0.18	0	-0.01	-0.07	-0.03	0.02	-0.03	2.87	3.78	4.03	1.72	2.33	2.34	1.25	1.64	1.62
	100	0.01	-0.11	0.05	0	0	0.04	-0.06	-0.02	-0.02	2.14	2.72	2.88	1.25	1.61	1.62	0.87	1.19	1.17
	Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																		
	20	-0.02	-0.11	-0.13	0.04	0.06	-0.01	-0.04	-0.02	0.08	4.56	6.46	6.12	2.51	3.73	3.8	2.02	2.51	2.62
	50	-0.13	0.02	0.23	-0.02	-0.01	-0.07	0	0.02	-0.03	3.07	3.79	4.03	1.78	2.34	2.37	1.28	1.64	1.62
	100	-0.05	-0.1	0.07	0	0.01	0.04	-0.08	-0.01	0	2.21	2.72	2.9	1.29	1.61	1.61	0.9	1.2	1.17
	Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																		
	20	-0.04	-0.11	-0.15	0.01	0.06	-0.01	-0.03	-0.02	0.08	4.38	6.44	6.12	2.46	3.72	3.79	1.97	2.51	2.62
	50	-0.09	0.01	0.21	-0.01	-0.01	-0.06	-0.02	0.02	-0.03	2.94	3.77	4.03	1.7	2.34	2.35	1.25	1.64	1.62
	100	-0.03	-0.11	0.07	-0.01	0.01	0.04	-0.08	-0.01	0	2.15	2.71	2.89	1.25	1.61	1.62	0.88	1.2	1.17
	Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																		
	20	0.71	-0.18	-0.35	0.26	0.04	-0.1	0.09	-0.04	0.04	4.13	6.4	6.04	2.41	3.71	3.78	1.92	2.51	2.61
	50	0.65	-0.09	-0.03	0.24	-0.03	-0.14	0.1	0.01	-0.06	2.82	3.75	4.01	1.69	2.33	2.35	1.23	1.64	1.62
	100	0.69	-0.19	-0.15	0.25	-0.02	-0.03	0.05	-0.03	-0.05	2.14	2.71	2.87	1.24	1.61	1.61	0.85	1.19	1.17
Naive	OLS																		
	20	7.74	0.85	-2.49	7.71	0.94	-2.41	7.65	0.74	-2.36	8.78	6.16	6.73	8.21	3.91	4.58	8.01	2.73	3.66
	50	8.13	0.81	-2.26	8.24	0.83	-2.39	8.14	0.82	-2.44	8.72	3.83	4.45	8.51	2.6	3.45	8.31	1.93	3
	100	8.29	0.54	-2.33	8.32	0.82	-2.35	8.35	0.81	-2.44	8.69	2.78	3.74	8.49	1.9	2.92	8.45	1.53	2.76
	Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																		
	20	1.83	2.06	-0.66	1.91	2.16	-0.52	1.82	1.92	-0.51	5.16	6.54	6.35	3.61	4.43	3.9	3.04	3.31	2.82
	50	2.14	2.04	-0.48	2.14	2.01	-0.63	2.08	1.99	-0.64	4.11	4.34	3.91	3.1	3.23	2.54	2.67	2.68	1.86
	100	2.09	1.74	-0.52	2.15	2	-0.55	2.15	2	-0.65	3.43	3.34	2.96	2.75	2.67	1.81	2.49	2.4	1.42
	Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																		
	20	2.27	1.98	-0.84	2.43	2.04	-0.69	2.28	1.83	-0.66	5.56	6.51	6.37	4.1	4.38	3.93	3.46	3.24	2.85
	50	2.72	1.94	-0.65	2.74	1.9	-0.8	2.7	1.88	-0.82	4.78	4.3	3.92	3.72	3.15	2.6	3.27	2.58	1.92
	100	2.68	1.64	-0.69	2.8	1.89	-0.74	2.78	1.89	-0.82	4.04	3.28	2.99	3.4	2.59	1.87	3.12	2.3	1.52
	Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																		
	20	2.33	1.94	-0.87	2.48	2.03	-0.7	2.35	1.82	-0.68	5.54	6.47	6.35	4.09	4.37	3.92	3.5	3.23	2.85
	50	2.84	1.91	-0.68	2.87	1.87	-0.83	2.81	1.86	-0.85	4.83	4.26	3.9	3.8	3.13	2.6	3.36	2.57	1.94
	100	2.83	1.61	-0.73	2.93	1.87	-0.78	2.92	1.86	-0.86	4.15	3.24	2.98	3.51	2.56	1.89	3.25	2.28	1.53
	Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																		
	20	3.24	1.78	-1.12	2.67	2	-0.78	2.31	1.82	-0.67	5.53	6.41	6.34	4.05	4.34	3.92	3.37	3.24	2.84
	50	3.68	1.73	-0.95	2.95	1.86	-0.86	2.67	1.88	-0.82	5.03	4.17	3.96	3.74	3.12	2.61	3.17	2.59	1.92
	100	3.66	1.44	-0.98	2.99	1.84	-0.8	2.75	1.88	-0.82	4.55	3.13	3.05	3.48	2.55	1.9	3.05	2.3	1.51

Table S27: The Size and Power for Alternative Pooled Estimators under Experiment 4 with $h = 4$

		Size									Power								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
OLS																			
	20	0.678	0.084	0.069	0.911	0.077	0.078	0.988	0.055	0.088	0.12	0.31	0.38	0.18	0.71	0.69	0.21	0.97	0.92
	50	0.849	0.055	0.068	0.99	0.069	0.1	1	0.063	0.098	0.23	0.77	0.78	0.47	0.99	0.98	0.67	1	1
	100	0.959	0.047	0.084	1	0.069	0.117	1	0.082	0.192	0.47	0.97	0.96	0.81	1	1	0.96	1	1
Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																			
	20	0.062	0.077	0.061	0.066	0.069	0.073	0.077	0.052	0.075	0.16	0.32	0.37	0.24	0.74	0.72	0.44	0.97	0.92
	50	0.055	0.051	0.053	0.055	0.053	0.059	0.061	0.054	0.038	0.28	0.77	0.8	0.65	0.99	0.98	0.86	1	1
	100	0.054	0.047	0.065	0.045	0.041	0.041	0.042	0.056	0.054	0.57	0.97	0.96	0.95	1	1	0.99	1	1
Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																			
	20	0.068	0.074	0.06	0.067	0.067	0.071	0.073	0.053	0.074	0.12	0.33	0.37	0.24	0.74	0.71	0.38	0.96	0.92
$\bar{\boldsymbol{x}}_t$	50	0.039	0.049	0.056	0.06	0.053	0.058	0.062	0.052	0.041	0.3	0.76	0.79	0.58	0.99	0.98	0.82	1	1
	100	0.058	0.045	0.068	0.047	0.044	0.043	0.042	0.056	0.05	0.53	0.97	0.96	0.91	1	1	0.99	1	1
Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																			
	20	0.062	0.073	0.06	0.073	0.069	0.073	0.072	0.052	0.074	0.16	0.32	0.37	0.27	0.74	0.72	0.41	0.96	0.92
	50	0.046	0.05	0.053	0.057	0.053	0.061	0.076	0.052	0.039	0.26	0.77	0.8	0.64	0.99	0.98	0.83	1	1
	100	0.059	0.046	0.064	0.046	0.045	0.039	0.04	0.056	0.052	0.53	0.97	0.96	0.94	1	1	1	1	1
Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																			
	20	0.057	0.075	0.059	0.068	0.069	0.073	0.08	0.052	0.075	0.16	0.31	0.37	0.26	0.74	0.72	0.43	0.97	0.92
	50	0.052	0.051	0.052	0.058	0.053	0.06	0.06	0.052	0.04	0.29	0.77	0.8	0.7	0.99	0.98	0.88	1	1
	100	0.054	0.046	0.068	0.041	0.042	0.039	0.046	0.054	0.054	0.6	0.97	0.96	0.96	1	1	1	1	1
OLS																			
	20	0.096	0.096	0.078	0.106	0.086	0.087	0.098	0.066	0.097	0.13	0.28	0.39	0.3	0.76	0.72	0.38	0.96	0.93
	50	0.188	0.058	0.068	0.4	0.063	0.081	0.597	0.058	0.062	0.34	0.77	0.78	0.69	0.99	0.98	0.88	1	1
	100	0.437	0.046	0.078	0.828	0.062	0.059	0.976	0.068	0.076	0.62	0.98	0.94	0.97	1	1	1	1	1
Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																			
	20	0.076	0.098	0.078	0.079	0.086	0.092	0.083	0.066	0.091	0.14	0.28	0.38	0.25	0.76	0.72	0.46	0.96	0.93
	50	0.07	0.059	0.068	0.066	0.065	0.063	0.058	0.057	0.045	0.25	0.76	0.78	0.66	0.99	0.99	0.88	1	1
	100	0.054	0.045	0.071	0.049	0.053	0.046	0.044	0.072	0.058	0.58	0.98	0.94	0.95	1	1	1	1	1
Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																			
	20	0.067	0.092	0.076	0.077	0.085	0.086	0.082	0.067	0.094	0.1	0.28	0.39	0.27	0.76	0.72	0.37	0.96	0.93
$(\bar{y}_t, \bar{\boldsymbol{x}}_t')'$	50	0.046	0.059	0.067	0.07	0.064	0.062	0.067	0.058	0.05	0.32	0.76	0.77	0.62	0.99	0.99	0.83	1	1
	100	0.061	0.047	0.067	0.046	0.053	0.043	0.042	0.072	0.056	0.55	0.98	0.95	0.93	1	1	1	1	1
Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																			
	20	0.071	0.095	0.077	0.078	0.087	0.087	0.088	0.068	0.092	0.13	0.28	0.38	0.31	0.76	0.72	0.42	0.96	0.92
	50	0.05	0.059	0.066	0.063	0.065	0.061	0.065	0.059	0.048	0.33	0.76	0.77	0.64	0.99	0.99	0.86	1	1
	100	0.062	0.048	0.071	0.051	0.054	0.04	0.048	0.07	0.058	0.56	0.98	0.94	0.94	1	1	1	1	1
Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																			
	20	0.072	0.097	0.077	0.07	0.085	0.088	0.086	0.066	0.094	0.13	0.28	0.38	0.3	0.76	0.72	0.47	0.96	0.92
	50	0.059	0.059	0.068	0.064	0.067	0.06	0.065	0.058	0.047	0.31	0.76	0.78	0.67	0.99	0.99	0.9	1	1
	100	0.056	0.046	0.07	0.043	0.055	0.043	0.04	0.07	0.06	0.61	0.97	0.94	0.97	1	1	1	1	1
OLS																			
	20	0.102	0.097	0.078	0.109	0.085	0.116	0.098	0.081	0.124	0.15	0.34	0.38	0.29	0.74	0.65	0.4	0.96	0.93
	50	0.173	0.051	0.073	0.404	0.065	0.096	0.594	0.062	0.096	0.3	0.78	0.75	0.64	0.99	0.98	0.88	1	1
	100	0.421	0.048	0.08	0.829	0.06	0.081	0.976	0.066	0.112	0.61	0.97	0.94	0.96	1	1	1	1	1
Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																			
	20	0.072	0.096	0.079	0.078	0.087	0.119	0.088	0.081	0.122	0.16	0.33	0.38	0.25	0.74	0.65	0.45	0.96	0.93
	50	0.064	0.048	0.073	0.067	0.063	0.076	0.065	0.055	0.06	0.24	0.77	0.75	0.64	0.99	0.98	0.87	1	1
	100	0.06	0.046	0.069	0.045	0.062	0.048	0.042	0.07	0.054	0.53	0.97	0.94	0.95	1	1	1	1	1
Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																			
	20	0.076	0.096	0.079	0.076	0.087	0.12	0.084	0.081	0.124	0.14	0.33	0.37	0.28	0.74	0.65	0.46	0.96	0.93
$(\bar{y}_t, \bar{y}_t^{*}, \bar{\boldsymbol{x}}_t')'$	50	0.061	0.05	0.073	0.073	0.063	0.075	0.066	0.054	0.059	0.26	0.78	0.76	0.63	0.99	0.98	0.89	1	1
	100	0.059	0.047	0.067	0.043	0.062	0.048	0.046	0.07	0.056	0.57	0.97	0.94	0.96	1	1	1	1	1

Table S27 (Continued)

		Size									Power								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
								Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments											
	20	0.071	0.096	0.076	0.076	0.084	0.121	0.087	0.08	0.125	0.15	0.34	0.36	0.29	0.74	0.65	0.43	0.96	0.93
	50	0.054	0.051	0.074	0.059	0.061	0.076	0.068	0.056	0.06	0.28	0.78	0.76	0.66	0.99	0.98	0.85	1	1
	100	0.06	0.047	0.068	0.053	0.062	0.047	0.052	0.066	0.058	0.55	0.97	0.94	0.93	1	1	1	1	1
								Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	0.078	0.054	0.066	0.072	0.075	0.064	0.074	0.07	0.064	0.15	0.32	0.34	0.2	0.7	0.69	0.4	0.95	0.93
	50	0.059	0.055	0.056	0.066	0.054	0.052	0.059	0.053	0.049	0.24	0.75	0.72	0.6	0.99	0.98	0.86	1	1
	100	0.052	0.06	0.051	0.051	0.045	0.064	0.045	0.054	0.051	0.52	0.96	0.94	0.93	1	1	1	1	1
								OLS											
Infeasible	20	0.184	0.091	0.077	0.33	0.083	0.101	0.494	0.074	0.1	0.22	0.37	0.43	0.51	0.75	0.75	0.65	0.98	0.95
	50	0.31	0.049	0.075	0.597	0.056	0.098	0.807	0.074	0.112	0.51	0.8	0.78	0.85	0.99	0.99	0.96	1	1
	100	0.488	0.047	0.082	0.853	0.053	0.095	0.974	0.068	0.136	0.77	0.98	0.97	0.99	1	1	1	1	1
								Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments											
	20	0.083	0.092	0.075	0.059	0.082	0.074	0.079	0.066	0.088	0.25	0.35	0.43	0.52	0.74	0.76	0.72	0.96	0.95
	50	0.057	0.049	0.064	0.053	0.055	0.067	0.056	0.061	0.048	0.48	0.8	0.77	0.83	0.99	0.99	0.98	1	1
	100	0.059	0.046	0.06	0.05	0.05	0.049	0.038	0.06	0.056	0.73	0.98	0.97	0.99	1	1	1	1	1
								Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	0.079	0.092	0.07	0.054	0.085	0.078	0.088	0.068	0.086	0.2	0.35	0.45	0.49	0.73	0.77	0.67	0.97	0.95
	50	0.049	0.047	0.065	0.061	0.058	0.07	0.063	0.062	0.048	0.52	0.81	0.78	0.82	0.99	0.99	0.97	1	1
	100	0.065	0.042	0.063	0.045	0.046	0.048	0.04	0.058	0.06	0.74	0.98	0.97	0.98	1	1	1	1	1
								Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments											
	20	0.079	0.091	0.072	0.065	0.085	0.081	0.084	0.067	0.086	0.22	0.35	0.43	0.55	0.73	0.76	0.69	0.97	0.95
	50	0.051	0.047	0.065	0.057	0.058	0.07	0.064	0.062	0.048	0.52	0.81	0.78	0.82	0.99	0.99	0.97	1	1
	100	0.062	0.041	0.063	0.047	0.048	0.045	0.042	0.058	0.052	0.76	0.98	0.97	0.99	1	1	1	1	1
								Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	0.087	0.093	0.075	0.06	0.085	0.076	0.088	0.066	0.089	0.24	0.36	0.44	0.53	0.73	0.77	0.73	0.97	0.95
	50	0.053	0.049	0.064	0.054	0.056	0.067	0.065	0.061	0.0470.51 0.8	0.78		0.85 0.99	0.99		0.97 1	1		
	100	0.063	0.043	0.06	0.05	0.049	0.048	0.042	0.058	0.054	0.78	0.98	0.97	0.99	1	1	1	1	1
								OLS											
Naive	20	0.613	0.086	0.091	0.87	0.085	0.138	0.953	0.071	0.174	0.14	0.38	0.48	0.24	0.65	0.7	0.43	0.95	0.95
	50	0.881	0.059	0.118	0.99	0.091	0.189	1	0.095	0.32	0.27	0.79	0.81	0.5	0.98	0.98	0.74	1	1
	100	0.976	0.062	0.177	1	0.102	0.326	1	0.13	0.56	0.38	0.97	0.96	0.78	1	1	0.94	1	1
								Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments											
	20	0.173	0.092	0.067	0.224	0.125	0.078	0.264	0.131	0.079	0.15	0.37	0.45	0.3	0.69	0.73	0.51	0.94	0.95
	50	0.253	0.094	0.056	0.353	0.191	0.088	0.448	0.227	0.063	0.27	0.79	0.81	0.58	0.98	0.98	0.74	1	1
	100	0.343	0.124	0.061	0.526	0.26	0.072	0.676	0.406	0.088	0.4	0.96	0.97	0.78	1	1	0.96	1	1
								Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	0.202	0.095	0.065	0.27	0.118	0.084	0.324	0.116	0.086	0.16	0.4	0.45	0.29	0.68	0.74	0.42	0.94	0.95
	50	0.335	0.084	0.064	0.458	0.171	0.095	0.568	0.207	0.081	0.23	0.81	0.81	0.48	0.98	0.98	0.73	1	1
	100	0.442	0.113	0.072	0.647	0.234	0.077	0.772	0.362	0.112	0.34	0.96	0.97	0.67	1	1	0.9	1	1
								Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments											
	20	0.22	0.094	0.067	0.286	0.119	0.085	0.362	0.117	0.088	0.16	0.39	0.45	0.25	0.68	0.73	0.45	0.94	0.95
	50	0.364	0.082	0.061	0.482	0.175	0.099	0.597	0.204	0.084	0.22	0.81	0.8	0.43	0.98	0.98	0.71	1	1
	100	0.467	0.114	0.068	0.672	0.23	0.087	0.812	0.354	0.112	0.34	0.96	0.97	0.68	1	1	0.91	1	1
								Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	0.198	0.089	0.069	0.26	0.119	0.081	0.298	0.125	0.08	0.15	0.39	0.45	0.31	0.69	0.73	0.46	0.94	0.96
	50	0.317	0.089	0.058	0.426	0.181	0.095	0.516	0.216	0.068	0.23	0.8	0.81	0.54	0.98	0.98	0.73	1	1
	100	0.404	0.119	0.067	0.598	0.243	0.08	0.752	0.386	0.096	0.38	0.96	0.97	0.72	1	1	0.95	1	1

Table S28: The Size and Power for Alternative Mean Group Estimators under Experiment 4 with $h = 4$

		Size									Power								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
OLS																			
	20	0.643	0.058	0.066	0.928	0.059	0.068	0.987	0.046	0.074	0.14	0.31	0.34	0.18	0.73	0.67	0.28	0.97	0.92
	50	0.81	0.047	0.06	0.988	0.055	0.086	1	0.063	0.103	0.23	0.7	0.7	0.47	0.98	0.98	0.64	1	1
	100	0.943	0.059	0.075	1	0.057	0.105	1	0.088	0.184	0.45	0.93	0.9	0.84	1	1	0.96	1	1
Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																			
	20	0.066	0.056	0.063	0.066	0.053	0.066	0.073	0.044	0.064	0.12	0.28	0.32	0.22	0.73	0.65	0.37	0.96	0.92
	50	0.049	0.039	0.052	0.061	0.058	0.051	0.06	0.052	0.036	0.24	0.7	0.7	0.58	0.98	0.98	0.82	1	1
	100	0.053	0.044	0.055	0.046	0.049	0.042	0.05	0.052	0.058	0.48	0.92	0.91	0.93	1	1	1	1	1
Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																			
	20	0.07	0.052	0.052	0.078	0.059	0.062	0.093	0.042	0.06	0.09	0.32	0.32	0.19	0.71	0.66	0.28	0.96	0.91
$\bar{\boldsymbol{x}}_t$	50	0.057	0.042	0.053	0.063	0.057	0.052	0.066	0.057	0.037	0.22	0.68	0.64	0.51	0.98	0.98	0.78	1	1
	100	0.061	0.052	0.052	0.046	0.046	0.042	0.05	0.052	0.054	0.4	0.9	0.89	0.87	1	1	0.99	1	1
Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																			
	20	0.075	0.05	0.051	0.081	0.056	0.064	0.096	0.04	0.062	0.1	0.3	0.34	0.22	0.73	0.65	0.29	0.96	0.93
	50	0.058	0.044	0.054	0.056	0.057	0.053	0.065	0.055	0.038	0.24	0.68	0.69	0.59	0.98	0.98	0.8	1	1
	100	0.056	0.047	0.058	0.044	0.048	0.046	0.058	0.054	0.052	0.42	0.92	0.89	0.9	1	1	0.99	1	1
Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																			
	20	0.116	0.056	0.056	0.078	0.056	0.066	0.079	0.042	0.063	0.13	0.29	0.33	0.23	0.73	0.67	0.33	0.96	0.92
	50	0.102	0.043	0.056	0.075	0.056	0.05	0.069	0.055	0.035	0.3	0.7	0.71	0.58	0.98	0.98	0.82	1	1
	100	0.142	0.04	0.055	0.069	0.05	0.044	0.06	0.05	0.062	0.47	0.93	0.9	0.91	1	1	0.99	1	1
OLS																			
	20	0.079	0.067	0.062	0.081	0.069	0.08	0.081	0.063	0.079	0.15	0.33	0.35	0.23	0.74	0.69	0.42	0.95	0.93
	50	0.164	0.045	0.059	0.38	0.064	0.063	0.585	0.062	0.064	0.34	0.72	0.7	0.62	0.98	0.99	0.85	1	1
	100	0.392	0.044	0.06	0.815	0.05	0.059	0.982	0.062	0.074	0.56	0.94	0.9	0.96	1	1	1	1	1
Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																			
	20	0.064	0.072	0.061	0.074	0.071	0.082	0.083	0.062	0.082	0.13	0.29	0.36	0.22	0.73	0.67	0.46	0.95	0.93
	50	0.04	0.042	0.055	0.053	0.07	0.057	0.059	0.058	0.044	0.3	0.71	0.71	0.65	0.98	0.99	0.86	1	1
	100	0.059	0.043	0.06	0.049	0.056	0.041	0.044	0.056	0.052	0.48	0.94	0.91	0.92	1	1	1	1	1
Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																			
	20	0.058	0.072	0.064	0.065	0.068	0.087	0.083	0.062	0.082	0.11	0.31	0.33	0.24	0.71	0.66	0.32	0.95	0.92
$(\bar{y}_t, \bar{\boldsymbol{x}}_t')'$	50	0.046	0.046	0.052	0.055	0.067	0.058	0.064	0.064	0.047	0.24	0.67	0.71	0.58	0.98	0.98	0.82	1	1
	100	0.063	0.048	0.058	0.042	0.052	0.046	0.048	0.058	0.06	0.42	0.92	0.88	0.89	1	1	0.99	1	1
Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																			
	20	0.05	0.068	0.065	0.078	0.07	0.085	0.083	0.063	0.08	0.14	0.3	0.32	0.24	0.71	0.68	0.43	0.95	0.92
	50	0.048	0.048	0.054	0.044	0.069	0.056	0.059	0.062	0.047	0.26	0.68	0.69	0.63	0.98	0.98	0.85	1	1
	100	0.053	0.046	0.063	0.035	0.054	0.045	0.054	0.058	0.058	0.47	0.93	0.88	0.92	1	1	1	1	1
Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																			
	20	0.071	0.065	0.061	0.07	0.07	0.085	0.084	0.063	0.082	0.12	0.3	0.36	0.25	0.73	0.69	0.48	0.95	0.92
	50	0.051	0.042	0.058	0.058	0.072	0.056	0.053	0.059	0.047	0.29	0.71	0.7	0.66	0.98	0.99	0.88	1	1
	100	0.071	0.039	0.059	0.048	0.053	0.044	0.044	0.056	0.054	0.52	0.94	0.91	0.94	1	1	1	1	1
OLS																			
	20	0.086	0.071	0.069	0.073	0.071	0.113	0.089	0.073	0.117	0.12	0.29	0.34	0.21	0.7	0.63	0.39	0.96	0.93
	50	0.176	0.053	0.067	0.379	0.064	0.088	0.583	0.057	0.104	0.29	0.63	0.7	0.64	0.98	0.98	0.86	1	1
	100	0.387	0.05	0.063	0.812	0.061	0.078	0.98	0.07	0.112	0.57	0.92	0.86	0.96	1	1	1	1	1
Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																			
	20	0.07	0.072	0.074	0.074	0.078	0.106	0.083	0.068	0.11	0.09	0.29	0.35	0.21	0.7	0.64	0.38	0.96	0.94
	50	0.043	0.054	0.066	0.054	0.061	0.069	0.063	0.055	0.07	0.26	0.6	0.64	0.64	0.98	0.98	0.87	1	1
	100	0.061	0.051	0.053	0.049	0.055	0.046	0.046	0.06	0.06	0.45	0.92	0.88	0.92	1	1	1	1	1
Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																			
	20	0.063	0.071	0.071	0.062	0.069	0.116	0.064	0.071	0.118	0.1	0.25	0.31	0.18	0.71	0.59	0.3	0.96	0.93
$(\bar{y}_t, \bar{y}_t^{*}, \bar{\boldsymbol{x}}_t')'$	50	0.048	0.048	0.064	0.047	0.06	0.074	0.057	0.055	0.069	0.22	0.62	0.68	0.59	0.98	0.97	0.79	1	1
	100	0.055	0.05	0.053	0.043	0.056	0.045	0.042	0.054	0.058	0.43	0.91	0.88	0.89	1	1	0.99	1	1

Table S28 (Continued)

		Size									Power								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																			
	20	0.058	0.074	0.069	0.059	0.075	0.112	0.076	0.068	0.113	0.11	0.28	0.34	0.23	0.71	0.61	0.38	0.96	0.93
	50	0.054	0.055	0.063	0.047	0.061	0.07	0.059	0.054	0.07	0.26	0.61	0.66	0.61	0.98	0.98	0.84	1	1
	100	0.063	0.048	0.052	0.041	0.057	0.044	0.046	0.058	0.058	0.46	0.92	0.88	0.92	1	1	1	1	1
Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																			
	20	0.072	0.071	0.07	0.062	0.077	0.109	0.084	0.068	0.113	0.12	0.3	0.35	0.24	0.7	0.61	0.4	0.96	0.93
	50	0.051	0.058	0.067	0.055	0.062	0.073	0.059	0.055	0.071	0.28	0.61	0.65	0.65	0.98	0.98	0.87	1	1
	100	0.074	0.05	0.053	0.053	0.055	0.045	0.05	0.058	0.058	0.51	0.91	0.88	0.94	1	1	1	1	1
OLS																			
Infeasible	20	0.168	0.08	0.058	0.325	0.065	0.096	0.477	0.058	0.09	0.22	0.34	0.39	0.51	0.72	0.71	0.69	0.97	0.96
	50	0.278	0.042	0.066	0.587	0.062	0.081	0.797	0.063	0.103	0.42	0.75	0.71	0.79	0.99	0.99	0.96	1	1
	100	0.455	0.05	0.076	0.847	0.052	0.096	0.98	0.07	0.13	0.69	0.96	0.9	0.99	1	1	1	1	1
Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																			
	20	0.057	0.076	0.058	0.059	0.068	0.073	0.071	0.053	0.073	0.2	0.32	0.4	0.49	0.71	0.71	0.7	0.97	0.95
	50	0.05	0.04	0.052	0.056	0.057	0.058	0.052	0.063	0.048	0.4	0.73	0.74	0.8	0.98	0.99	0.97	1	1
	100	0.059	0.042	0.06	0.054	0.049	0.041	0.044	0.058	0.052	0.66	0.96	0.92	0.97	1	1	1	1	1
Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																			
	20	0.069	0.077	0.054	0.054	0.069	0.075	0.077	0.058	0.073	0.18	0.35	0.38	0.45	0.72	0.72	0.68	0.98	0.95
	50	0.061	0.04	0.054	0.056	0.063	0.059	0.06	0.067	0.05	0.36	0.74	0.72	0.81	0.99	0.99	0.95	1	1
	100	0.053	0.041	0.06	0.052	0.045	0.045	0.046	0.062	0.052	0.65	0.95	0.91	0.97	1	1	1	1	1
Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																			
	20	0.074	0.075	0.058	0.058	0.069	0.072	0.075	0.059	0.073	0.19	0.34	0.37	0.49	0.71	0.71	0.69	0.97	0.95
	50	0.065	0.04	0.054	0.048	0.058	0.058	0.058	0.064	0.051	0.37	0.72	0.72	0.8	0.99	0.98	0.96	1	1
	100	0.054	0.04	0.062	0.051	0.047	0.046	0.046	0.058	0.05	0.67	0.96	0.9	0.97	1	1	1	1	1
Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																			
	20	0.065	0.082	0.054	0.055	0.069	0.075	0.07	0.056	0.074	0.22	0.33	0.4	0.56	0.7	0.71	0.72	0.97	0.95
	50	0.069	0.04	0.054	0.055	0.059	0.055	0.059	0.063	0.052	0.42	0.75	0.73	0.81	0.99	0.99	0.97	1	1
	100	0.082	0.043	0.06	0.066	0.05	0.04	0.044	0.056	0.056	0.68	0.96	0.92	0.98	1	1	1	1	1
OLS																			
Naive	20	0.569	0.062	0.082	0.85	0.067	0.126	0.959	0.057	0.155	0.18	0.4	0.38	0.29	0.7	0.69	0.37	0.96	0.93
	50	0.872	0.055	0.093	0.992	0.083	0.183	1	0.08	0.287	0.26	0.73	0.75	0.55	0.98	0.98	0.72	1	1
	100	0.968	0.054	0.145	1	0.088	0.285	1	0.108	0.52	0.38	0.95	0.94	0.81	1	1	0.97	1	1
Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																			
	20	0.146	0.078	0.07	0.185	0.103	0.071	0.254	0.115	0.079	0.14	0.37	0.36	0.29	0.71	0.69	0.46	0.94	0.93
	50	0.231	0.074	0.053	0.322	0.163	0.058	0.41	0.204	0.06	0.27	0.71	0.76	0.53	0.97	0.98	0.77	1	1
	100	0.281	0.1	0.054	0.472	0.228	0.053	0.646	0.372	0.074	0.38	0.93	0.94	0.78	1	1	0.96	1	1
Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																			
	20	0.177	0.076	0.067	0.237	0.105	0.068	0.317	0.1	0.076	0.17	0.37	0.37	0.28	0.69	0.69	0.41	0.94	0.92
	50	0.29	0.076	0.055	0.405	0.152	0.068	0.537	0.189	0.071	0.21	0.71	0.73	0.45	0.98	0.99	0.7	1	1
	100	0.371	0.092	0.064	0.594	0.214	0.068	0.742	0.35	0.094	0.29	0.92	0.92	0.69	1	1	0.89	1	1
Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																			
	20	0.191	0.078	0.076	0.255	0.103	0.07	0.331	0.101	0.077	0.17	0.38	0.37	0.27	0.71	0.69	0.43	0.94	0.93
	50	0.316	0.075	0.052	0.432	0.149	0.068	0.571	0.189	0.079	0.25	0.71	0.73	0.44	0.97	0.99	0.69	1	1
	100	0.413	0.091	0.062	0.635	0.206	0.07	0.776	0.34	0.096	0.29	0.93	0.92	0.69	1	1	0.91	1	1
Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																			
	20	0.215	0.075	0.066	0.265	0.098	0.073	0.322	0.104	0.079	0.17	0.37	0.37	0.28	0.71	0.7	0.46	0.95	0.93
	50	0.366	0.07	0.06	0.448	0.153	0.07	0.546	0.193	0.076	0.24	0.74	0.76	0.51	0.97	0.99	0.76	1	1
	100	0.526	0.079	0.07	0.668	0.209	0.073	0.77	0.342	0.096	0.31	0.94	0.93	0.74	1	1	0.93	1	1

S2.6 Supplementary Results for $h = 6$

This subsection provides Monte Carlo simulation results for the case of $h = 6$ while all the other settings remain the same as in the main text, which increases the density of spatial correlation even further. Again we only report the results for Experiment 4. [5] With the exception of our proposed estimator and the infeasible estimator, the performance of all the other estimators becomes much worse, especially when sample size is small.

[5] The results for the other three experiments are available upon request.

Table S29: The Finite Sample Performance of Alternative Pooled Estimators under Experiment 4 with $h = 6$

		Bias (× 100)									RMSE (×) 100								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
		OLS																	
	20	15.73	0.59	0.76	16.06	0.39	0.65	16.05	0.24	0.54	17.22	6.14	6.23	16.69	3.77	3.81	16.48	2.74	2.77
	50	10.62	-0.32	-0.89	10.83	-0.43	-0.68	10.98	-0.4	-0.8	11.42	3.8	3.71	11.17	2.39	2.38	11.18	1.75	1.85
	100	9.37	-0.51	-0.95	9.49	-0.53	-0.88	9.46	-0.44	-0.91	9.87	2.74	2.79	9.7	1.68	1.87	9.59	1.31	1.53
		Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																	
	20	0.25	0.13	0.04	0.35	0.08	0.04	0.11	-0.03	-0.03	8.47	5.98	6.03	5.22	3.7	3.71	3.8	2.68	2.67
	50	-0.09	0.01	-0.23	0.06	-0.07	0.03	0.01	-0.04	-0.08	4.04	3.76	3.57	2.53	2.34	2.27	1.81	1.69	1.65
	100	-0.05	-0.04	-0.05	0.05	-0.06	0.05	0.04	0.03	0.02	2.67	2.68	2.6	1.69	1.59	1.64	1.2	1.22	1.21
		Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	0.28	0.13	0.06	0.28	0.07	0.04	0.13	-0.03	-0.03	10.29	5.99	6.04	6.57	3.7	3.71	4.35	2.69	2.67
$\bar{\boldsymbol{x}}_t$	50	0.04	0.01	-0.23	-0.01	-0.07	0.03	-0.05	-0.04	-0.07	4.18	3.76	3.57	2.67	2.34	2.27	1.86	1.7	1.65
	100	-0.07	-0.04	-0.05	0.04	-0.06	0.05	0.04	0.03	0.02	2.81	2.68	2.6	1.7	1.58	1.65	1.25	1.22	1.22
		Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																	
	20	0.22	0.13	0.05	0.26	0.07	0.04	0.12	-0.03	-0.03	9.95	5.98	6.04	6.35	3.7	3.71	4.36	2.69	2.67
	50	0.02	0.01	-0.23	-0.02	-0.06	0.03	-0.03	-0.04	-0.07	4.1	3.76	3.57	2.65	2.34	2.27	1.83	1.7	1.65
	100	-0.05	-0.04	-0.05	0.05	-0.06	0.05	0.04	0.03	0.02	2.75	2.68	2.6	1.67	1.58	1.64	1.22	1.22	1.21
		Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	0.81	0.15	0.07	0.57	0.08	0.05	0.24	-0.03	-0.03	8.21	5.98	6.03	5.19	3.7	3.71	3.75	2.69	2.67
	50	0.02	0.01	-0.23	0.07	-0.07	0.03	0	-0.04	-0.07	3.9	3.76	3.57	2.47	2.34	2.26	1.75	1.69	1.65
	100	-0.03	-0.04	-0.05	0.06	-0.06	0.05	0.05	0.03	0.02	2.61	2.68	2.6	1.64	1.58	1.64	1.18	1.22	1.21
		OLS																	
	20	-4.38	0.07	-0.24	-3.32	0.02	-0.18	-3.25	-0.12	-0.19	9.95	5.98	6.12	6.32	3.68	3.71	5.1	2.65	2.7
	50	2.18	-0.02	-0.33	2.38	-0.15	-0.14	2.35	-0.12	-0.24	4.32	3.85	3.57	3.32	2.36	2.25	2.88	1.69	1.68
	100	3.23	-0.21	-0.37	3.34	-0.22	-0.28	3.33	-0.12	-0.3	4.01	2.73	2.67	3.67	1.6	1.65	3.52	1.23	1.24
		Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																	
	20	-0.96	0.15	-0.08	-0.89	0.07	-0.09	-1.07	-0.08	-0.11	8.67	6	6.1	5.22	3.68	3.7	3.93	2.66	2.69
	50	-0.25	0.05	-0.19	-0.06	-0.09	0.02	-0.15	-0.04	-0.09	4.01	3.85	3.57	2.52	2.36	2.25	1.8	1.68	1.67
	100	-0.09	-0.05	-0.06	0.01	-0.05	0.05	0	0.05	0.03	2.71	2.72	2.63	1.68	1.58	1.63	1.18	1.22	1.2
		Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	-2.64	0.1	-0.16	-2.27	0.04	-0.14	-2.44	-0.1	-0.16	10.09	6	6.1	6.41	3.69	3.71	4.81	2.66	2.69
$(\bar{y}_t, \bar{\boldsymbol{x}}_t')'$	50	-0.21	0.05	-0.19	-0.19	-0.09	0.03	-0.3	-0.03	-0.09	4.2	3.85	3.57	2.64	2.36	2.25	1.85	1.69	1.66
	100	-0.14	-0.04	-0.05	0	-0.05	0.05	-0.02	0.05	0.03	2.77	2.72	2.64	1.69	1.58	1.63	1.24	1.22	1.2
		Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																	
	20	-3.27	0.08	-0.19	-2.89	0.03	-0.16	-3.04	-0.11	-0.18	9.46	5.99	6.1	6.13	3.68	3.7	4.91	2.66	2.69
	50	-0.24	0.05	-0.19	-0.23	-0.09	0.03	-0.31	-0.03	-0.09	4.08	3.85	3.57	2.6	2.36	2.25	1.82	1.69	1.66
	100	-0.13	-0.05	-0.06	0	-0.05	0.05	-0.02	0.05	0.03	2.72	2.72	2.64	1.66	1.58	1.63	1.21	1.22	1.2
		Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	-1.28	0.14	-0.1	-1.17	0.07	-0.09	-1.36	-0.08	-0.12	8.36	6	6.09	5.14	3.68	3.7	3.93	2.66	2.69
	50	-0.18	0.05	-0.19	-0.08	-0.09	0.02	-0.19	-0.03	-0.09	3.88	3.85	3.57	2.46	2.36	2.25	1.74	1.68	1.66
	100	-0.08	-0.05	-0.06	0.02	-0.05	0.05	0	0.05	0.03	2.62	2.72	2.63	1.64	1.58	1.63	1.17	1.22	1.2
		OLS																	
	20	-5.92	-0.82	-1.99	-4.25	-0.76	-1.73	-4.08	-0.88	-1.72	11.5	6.12	6.58	7.21	3.8	4.19	5.82	2.82	3.26
	50	2.15	-0.24	-0.88	2.29	-0.39	-0.68	2.25	-0.38	-0.76	4.4	3.94	3.76	3.27	2.4	2.41	2.81	1.72	1.84
	100	3.17	-0.34	-0.66	3.3	-0.36	-0.55	3.3	-0.25	-0.56	3.99	2.83	2.77	3.64	1.63	1.74	3.49	1.25	1.34
		Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																	
	20	-2.57	-0.69	-1.76	-1.97	-0.69	-1.58	-2.11	-0.82	-1.6	9.75	6.12	6.5	5.78	3.78	4.12	4.51	2.8	3.19
	50	-0.39	-0.17	-0.74	-0.25	-0.32	-0.53	-0.33	-0.3	-0.61	4.15	3.95	3.73	2.57	2.39	2.38	1.83	1.71	1.78
	100	-0.26	-0.18	-0.35	-0.1	-0.19	-0.23	-0.1	-0.09	-0.23	2.8	2.81	2.7	1.71	1.6	1.67	1.2	1.23	1.23
		Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	-2.13	-0.68	-1.73	-1.5	-0.67	-1.56	-1.64	-0.81	-1.58	9.43	6.13	6.49	5.6	3.78	4.11	4.27	2.8	3.17
$(\bar{y}_t, \bar{y}_t^*, \bar{\boldsymbol{x}}_t')'$	50	-0.19	-0.18	-0.76	-0.15	-0.32	-0.53	-0.26	-0.31	-0.61	4.03	3.95	3.74	2.5	2.39	2.38	1.77	1.71	1.78
	100	-0.2	-0.18	-0.36	-0.04	-0.2	-0.23	-0.05	-0.09	-0.24	2.72	2.81	2.7	1.66	1.6	1.67	1.18	1.23	1.23

Table S29 (Continued)

		Bias (× 100)									RMSE (×) 100								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
		Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																	
	20	-0.04	-0.6	-1.58	0.52	-0.61	-1.43	0.35	-0.75	-1.46	9.43	6.14	6.45	5.54	3.77	4.05	3.93	2.79	3.11
	50	0.09	-0.19	-0.77	0.03	-0.33	-0.54	-0.06	-0.31	-0.62	4.21	3.95	3.74	2.6	2.4	2.38	1.8	1.71	1.79
	100	-0.11	-0.19	-0.37	0.06	-0.2	-0.24	0.05	-0.09	-0.25	2.81	2.81	2.71	1.68	1.6	1.67	1.23	1.23	1.24
		Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	-3.55	-0.78	-1.92	-3.19	-0.8	-1.81	-3.35	-0.95	-1.84	11.88	6.14	6.68	7.14	3.83	4.29	5.77	2.87	3.35
	50	-0.42	-0.19	-0.76	-0.27	-0.33	-0.54	-0.37	-0.31	-0.62	4.45	3.98	3.74	2.74	2.41	2.4	1.95	1.72	1.79
	100	-0.23	-0.18	-0.35	-0.1	-0.2	-0.23	-0.11	-0.09	-0.24	2.9	2.82	2.71	1.77	1.6	1.68	1.22	1.23	1.24
Infeasible		OLS																	
	20	2.74	0.08	-0.58	3.07	-0.18	-0.78	2.98	-0.33	-0.84	4.96	5.65	5.79	4.01	3.48	3.58	3.57	2.5	2.73
	50	2.85	-0.23	-0.83	2.97	-0.26	-0.68	3	-0.27	-0.78	3.87	3.73	3.51	3.4	2.31	2.29	3.25	1.62	1.78
	100	2.96	-0.22	-0.76	3.02	-0.29	-0.64	2.98	-0.18	-0.69	3.51	2.55	2.63	3.27	1.57	1.72	3.12	1.18	1.35
		Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																	
	20	-0.04	0.26	0.06	0.17	0.05	-0.06	0.03	-0.09	-0.08	4.53	5.65	5.76	2.83	3.5	3.49	2.08	2.48	2.59
	50	-0.18	-0.01	-0.14	0.02	-0.05	0.03	0.02	-0.05	-0.07	2.8	3.71	3.41	1.8	2.3	2.17	1.34	1.6	1.6
	100	-0.08	-0.02	-0.07	0.07	-0.08	0.06	0.02	0.02	0.02	2.05	2.53	2.51	1.31	1.54	1.59	0.96	1.17	1.16
		Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	-0.04	0.27	0.05	0.14	0.05	-0.05	0.04	-0.09	-0.08	4.58	5.64	5.78	2.89	3.49	3.51	2.09	2.48	2.6
	50	-0.09	-0.02	-0.16	0.01	-0.05	0.04	-0.02	-0.05	-0.07	2.85	3.72	3.41	1.8	2.3	2.18	1.34	1.6	1.6
	100	-0.1	-0.01	-0.07	0.07	-0.08	0.06	0.02	0.02	0.02	2.1	2.53	2.52	1.31	1.54	1.6	0.99	1.17	1.17
		Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																	
	20	-0.09	0.27	0.06	0.14	0.05	-0.05	0.04	-0.09	-0.08	4.56	5.64	5.78	2.86	3.49	3.5	2.08	2.48	2.6
	50	-0.08	-0.02	-0.17	0.01	-0.05	0.04	0	-0.05	-0.07	2.82	3.72	3.41	1.79	2.3	2.17	1.34	1.6	1.61
	100	-0.09	-0.02	-0.07	0.08	-0.08	0.06	0.02	0.02	0.02	2.1	2.53	2.52	1.3	1.54	1.6	0.97	1.17	1.16
		Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	0.03	0.26	0.04	0.18	0.05	-0.06	0.05	-0.09	-0.08	4.42	5.65	5.76	2.79	3.49	3.5	2.05	2.48	2.59
	50	-0.11	-0.01	-0.16	0.03	-0.05	0.03	0.01	-0.05	-0.07	2.74	3.71	3.41	1.75	2.3	2.17	1.31	1.6	1.6
	100	-0.07	-0.02	-0.07	0.08	-0.08	0.06	0.02	0.02	0.02	2.01	2.53	2.51	1.28	1.54	1.59	0.95	1.17	1.16
Naive		OLS																	
	20	8.02	1.19	-1.85	8.33	1.07	-2.03	8.39	0.97	-2.24	9.37	5.81	6.09	8.92	3.79	4.2	8.78	2.86	3.54
	50	8.72	0.96	-2.26	8.84	0.96	-2.11	9.04	0.92	-2.21	9.38	3.83	4.24	9.13	2.59	3.1	9.24	1.97	2.8
	100	8.92	0.89	-2.11	9.21	0.9	-2.08	9.17	0.99	-2.17	9.42	2.72	3.37	9.42	1.87	2.7	9.3	1.64	2.53
		Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																	
	20	2.51	2.06	-0.61	2.69	2.02	-0.64	2.66	2	-0.78	6.18	6.03	5.84	4.35	4.21	3.71	3.74	3.38	2.82
	50	2.78	1.93	-0.91	2.88	1.96	-0.69	2.92	1.95	-0.77	4.72	4.19	3.65	3.82	3.14	2.33	3.53	2.63	1.87
	100	2.84	1.88	-0.75	3.1	1.93	-0.65	3.02	2.03	-0.72	4.2	3.21	2.71	3.69	2.55	1.82	3.43	2.42	1.47
		Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	2.86	1.96	-0.72	3.11	1.94	-0.74	3.13	1.9	-0.9	6.67	5.99	5.86	4.82	4.16	3.74	4.19	3.31	2.86
	50	3.43	1.8	-1.06	3.46	1.85	-0.83	3.51	1.84	-0.91	5.31	4.12	3.69	4.37	3.06	2.38	4.1	2.55	1.94
	100	3.44	1.77	-0.89	3.76	1.81	-0.8	3.67	1.91	-0.87	4.88	3.13	2.75	4.36	2.45	1.89	4.08	2.32	1.55
		Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																	
	20	2.95	1.94	-0.74	3.23	1.91	-0.78	3.25	1.88	-0.93	6.79	5.98	5.85	4.91	4.15	3.75	4.3	3.29	2.87
	50	3.59	1.77	-1.11	3.64	1.82	-0.87	3.7	1.81	-0.96	5.48	4.11	3.71	4.54	3.04	2.4	4.28	2.52	1.96
	100	3.64	1.73	-0.94	3.97	1.77	-0.85	3.88	1.88	-0.92	5.06	3.1	2.77	4.56	2.42	1.91	4.28	2.29	1.58
		Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	2.84	1.98	-0.7	2.98	1.97	-0.71	2.95	1.94	-0.85	6.33	6	5.85	4.59	4.18	3.73	3.98	3.33	2.84
	50	3.21	1.85	-1.01	3.24	1.89	-0.78	3.28	1.88	-0.86	5.04	4.15	3.68	4.13	3.09	2.36	3.86	2.58	1.91
	100	3.24	1.81	-0.85	3.5	1.86	-0.74	3.42	1.96	-0.81	4.57	3.15	2.74	4.08	2.49	1.86	3.81	2.36	1.52

Table S30: The Finite Sample Performance of Alternative Mean Group Estimators under Experiment 4 with $h = 6$

		Bias (× 100)									RMSE (×) 100								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
		OLS																	
	20	17.49	0.49	0.63	17.7	0.35	0.6	17.59	0.3	0.64	19.02	6.59	6.74	18.3	3.83	3.89	17.98	2.78	2.86
	50	11.02	-0.41	-0.86	11.15	-0.44	-0.68	11.27	-0.41	-0.78	11.9	4.17	4.08	11.5	2.44	2.44	11.48	1.76	1.88
	100	9.54	-0.39	-0.92	9.6	-0.51	-0.93	9.53	-0.43	-0.91	10.08	2.97	2.97	9.81	1.76	1.94	9.65	1.32	1.54
		Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																	
	20	1	0.17	0.04	0.16	0.02	0.03	0	-0.01	0	13.19	7.17	7.28	6.72	3.89	3.92	4.58	2.76	2.77
	50	-0.19	-0.04	-0.12	0.03	-0.08	0.07	0	-0.05	-0.06	5.16	4.29	4.14	2.83	2.43	2.37	1.87	1.71	1.7
	100	-0.06	0.08	0	0.05	-0.04	0.03	0.01	0.04	0.03	3.3	3.06	2.89	1.85	1.7	1.72	1.24	1.25	1.25
		Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	3.2	0.1	0.14	0.43	0.01	0.02	-0.09	-0.01	0	18.58	8.02	8.08	9.71	3.96	4.15	5.6	2.79	2.78
$\bar{\boldsymbol{x}}_t$	50	0.2	-0.04	-0.11	-0.02	-0.08	0.07	-0.05	-0.04	-0.06	5.58	4.29	4.18	3.18	2.43	2.37	2.03	1.71	1.71
	100	-0.17	0.05	0.02	0.03	-0.04	0.03	-0.01	0.03	0.03	3.76	3.12	2.93	1.94	1.7	1.73	1.3	1.25	1.24
		Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																	
	20	1.06	0.32	0.13	0.07	0.02	-0.01	-0.05	-0.01	0.01	18.18	7.08	7.59	7.92	3.9	3.96	5.14	2.78	2.76
	50	0.09	-0.02	-0.12	-0.03	-0.08	0.07	-0.03	-0.04	-0.06	5.5	4.31	4.17	3.14	2.42	2.37	2	1.71	1.7
	100	-0.09	0.07	0.03	0.03	-0.04	0.03	0	0.04	0.03	3.52	3.1	2.91	1.91	1.7	1.73	1.27	1.25	1.24
		Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	6.43	0.27	0.22	2.85	0.07	0.1	1.42	0.01	0.06	11.64	6.76	6.93	6.77	3.86	3.9	4.55	2.76	2.76
	50	2.89	-0.15	-0.32	1.14	-0.12	-0.01	0.58	-0.06	-0.1	5.54	4.22	4.09	2.94	2.42	2.35	1.91	1.71	1.7
	100	2.3	-0.04	-0.22	0.92	-0.09	-0.06	0.44	0.02	-0.01	3.75	3.03	2.86	2	1.7	1.71	1.28	1.25	1.24
		OLS																	
	20	-7	-0.07	-0.44	-6.89	-0.17	-0.49	-6.92	-0.23	-0.45	12.6	6.55	6.83	9.32	3.77	3.88	8.19	2.69	2.8
	50	2.31	-0.06	-0.23	2.46	-0.16	-0.13	2.37	-0.12	-0.23	4.81	4.25	4.01	3.46	2.42	2.34	2.9	1.69	1.71
	100	3.44	-0.13	-0.34	3.38	-0.21	-0.32	3.35	-0.11	-0.3	4.37	2.99	2.89	3.74	1.69	1.71	3.54	1.24	1.25
		Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																	
	20	-2.48	0.08	-0.17	-2.61	-0.08	-0.24	-2.7	-0.12	-0.21	11.03	6.68	6.93	6.62	3.77	3.85	5.04	2.69	2.75
	50	-0.41	0.05	-0.06	-0.13	-0.07	0.05	-0.2	-0.04	-0.05	4.69	4.32	4.05	2.68	2.43	2.34	1.8	1.69	1.69
	100	-0.04	0.04	0.01	0	-0.04	0.02	-0.02	0.06	0.04	3.09	3.04	2.87	1.79	1.69	1.69	1.2	1.24	1.22
		Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	-3.83	0.13	-0.29	-4.4	-0.1	-0.32	-4.64	-0.15	-0.29	15.41	6.87	7.3	9.03	3.84	3.93	6.75	2.71	2.78
$(\bar{y}_t, \bar{\boldsymbol{x}}_t')'$	50	-0.26	-0.02	-0.09	-0.23	-0.06	0.06	-0.34	-0.03	-0.05	4.99	4.36	4.11	2.86	2.43	2.35	1.9	1.69	1.69
	100	-0.1	0.03	0.04	-0.03	-0.04	0.03	-0.04	0.06	0.04	3.26	3.05	2.9	1.83	1.69	1.69	1.27	1.24	1.22
		Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																	
	20	-4.28	0.13	-0.24	-4.56	-0.11	-0.31	-4.7	-0.14	-0.27	11.83	6.64	7.01	7.66	3.79	3.88	6.21	2.7	2.76
	50	-0.29	0	-0.07	-0.26	-0.07	0.06	-0.34	-0.03	-0.05	4.83	4.34	4.09	2.78	2.43	2.34	1.84	1.69	1.69
	100	-0.07	0.04	0.05	-0.02	-0.04	0.02	-0.04	0.06	0.04	3.16	3.04	2.89	1.79	1.69	1.69	1.24	1.24	1.22
		Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	-3.73	0.05	-0.26	-3.27	-0.09	-0.27	-3.18	-0.13	-0.23	10.98	6.63	6.89	6.77	3.77	3.85	5.21	2.69	2.75
	50	0.23	0.01	-0.1	0.04	-0.07	0.04	-0.14	-0.04	-0.06	4.46	4.29	4.02	2.6	2.43	2.34	1.75	1.69	1.69
	100	0.74	0	-0.06	0.24	-0.05	0	0.09	0.05	0.03	3	3.02	2.87	1.74	1.69	1.68	1.19	1.24	1.22
		OLS																	
	20	-19.55	-1.76	-3.86	-18.96	-1.78	-3.72	-18.42	-1.81	-3.63	25.09	6.91	8.11	21.02	4.17	5.52	19.52	3.27	4.66
	50	2.42	-0.31	-0.84	2.46	-0.43	-0.72	2.36	-0.42	-0.81	4.98	4.32	4.26	3.48	2.46	2.49	2.9	1.73	1.89
	100	3.39	-0.29	-0.66	3.38	-0.36	-0.62	3.34	-0.26	-0.59	4.4	3.07	3.04	3.75	1.72	1.82	3.53	1.27	1.37
		Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																	
	20	-10.71	-1.23	-2.99	-9.4	-1.27	-2.65	-9.04	-1.29	-2.57	24.3	7.12	8.06	13.66	4	4.89	11.45	3.02	3.88
	50	-0.41	-0.19	-0.67	-0.24	-0.34	-0.55	-0.3	-0.34	-0.65	4.87	4.41	4.29	2.73	2.46	2.46	1.83	1.72	1.82
	100	-0.17	-0.12	-0.32	-0.07	-0.19	-0.28	-0.09	-0.09	-0.25	3.25	3.12	2.99	1.81	1.71	1.73	1.22	1.25	1.26
		Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																	
	20	-20.2	-2.02	-3.83	-15.22	-1.63	-3.41	-19.51	-1.93	-3.97	69.93	9.45	12.12	58.97	5.66	8.72	63.94	5.14	9.93
$(\bar{y}_t, \bar{y}_t^{*}, \bar{\boldsymbol{x}}_t')'$	50	0.02	-0.27	-0.72	-0.04	-0.35	-0.55	-0.16	-0.34	-0.65	5.17	4.42	4.35	2.88	2.46	2.47	1.89	1.72	1.83
	100	-0.15	-0.14	-0.29	0.02	-0.2	-0.29	-0.01	-0.09	-0.26	3.49	3.14	3.01	1.87	1.71	1.74	1.28	1.25	1.27

Table S30 (Continued)

		Bias (× 100)									RMSE (×) 100								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
								Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments											
	20	-17.15	-1.69	-3.82	-18.77	-1.91	-3.91	-18.96	-1.96	-3.92	42.3	7.83	9.58	28.94	4.43	6.16	22.98	3.46	5.11
	50	0.03	-0.26	-0.7	-0.01	-0.35	-0.56	-0.1	-0.34	-0.66	5.01	4.39	4.32	2.79	2.46	2.46	1.82	1.71	1.83
	100	-0.09	-0.14	-0.28	0.05	-0.2	-0.29	0.01	-0.09	-0.26	3.36	3.12	2.99	1.82	1.71	1.74	1.26	1.25	1.26
								Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	-13.18	-1.41	-3.23	-10.48	-1.33	-2.79	-9.5	-1.33	-2.64	21.81	7	8	14.24	4.02	4.97	11.73	3.03	3.93
	50	0.35	-0.24	-0.71	0.04	-0.35	-0.56	-0.15	-0.34	-0.66	4.63	4.35	4.26	2.64	2.46	2.46	1.77	1.71	1.82
	100	0.71	-0.17	-0.39	0.22	-0.2	-0.31	0.07	-0.1	-0.27	3.13	3.11	3	1.77	1.71	1.74	1.2	1.25	1.27
								OLS											
Infeasible	20	3.03	0	-0.84	3.24	-0.21	-0.87	3.08	-0.28	-0.81	5.3	5.99	6.19	4.15	3.57	3.74	3.67	2.55	2.76
	50	3.08	-0.16	-0.9	3.11	-0.29	-0.67	3.09	-0.27	-0.77	4.27	4.09	4.03	3.55	2.36	2.35	3.34	1.66	1.8
	100	3.1	-0.15	-0.78	3.11	-0.28	-0.69	3.04	-0.16	-0.67	3.72	2.86	2.82	3.36	1.66	1.78	3.17	1.19	1.36
								Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments											
	20	-0.05	0.27	-0.03	0.14	0.04	-0.07	0.01	-0.02	-0.01	4.83	5.99	6.17	2.91	3.58	3.64	2.13	2.54	2.63
	50	-0.06	0.11	-0.14	0.03	-0.05	0.07	0.03	-0.05	-0.05	3.17	4.13	4.01	1.89	2.35	2.23	1.33	1.64	1.63
	100	-0.04	0.07	-0.04	0.09	-0.07	0.02	0.01	0.05	0.03	2.33	2.88	2.73	1.35	1.64	1.65	0.97	1.18	1.19
								Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	-0.08	0.25	0.01	0.12	0.05	-0.07	0.02	-0.02	-0.01	4.82	6.03	6.27	2.97	3.6	3.67	2.15	2.54	2.64
	50	0	0.1	-0.15	0.02	-0.05	0.07	0	-0.05	-0.05	3.28	4.12	3.99	1.92	2.35	2.25	1.36	1.65	1.64
	100	-0.11	0.07	-0.02	0.07	-0.08	0.02	-0.01	0.05	0.04	2.4	2.89	2.74	1.37	1.64	1.66	1.01	1.18	1.19
								Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments											
	20	-0.13	0.25	0.01	0.13	0.05	-0.08	0.01	-0.02	-0.01	4.81	6	6.23	2.92	3.59	3.66	2.15	2.54	2.64
	50	0.02	0.1	-0.15	0.03	-0.05	0.06	0.01	-0.05	-0.05	3.23	4.11	3.98	1.89	2.35	2.25	1.36	1.64	1.64
	100	-0.1	0.07	-0.03	0.08	-0.07	0.02	0	0.05	0.04	2.38	2.89	2.75	1.36	1.64	1.66	0.99	1.18	1.19
								Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	0.57	0.19	-0.18	0.36	0.03	-0.13	0.12	-0.03	-0.04	4.58	5.99	6.18	2.84	3.58	3.65	2.1	2.54	2.63
	50	0.6	0.05	-0.3	0.25	-0.07	0.01	0.13	-0.05	-0.08	3.13	4.1	3.98	1.85	2.34	2.24	1.32	1.64	1.63
	100	0.59	0.02	-0.19	0.29	-0.09	-0.03	0.11	0.04	0.01	2.28	2.88	2.73	1.35	1.64	1.65	0.97	1.18	1.19
								OLS											
	20	8.37	1.06	-2.16	8.52	0.89	-2.14	8.5	0.92	-2.2	9.69	6.11	6.53	9.08	3.8	4.39	8.89	2.87	3.54
	50	8.97	0.93	-2.22	8.99	0.85	-2.07	9.12	0.83	-2.14	9.64	4.08	4.53	9.29	2.57	3.1	9.3	1.93	2.74
	100	9.18	0.92	-2.1	9.31	0.85	-2.11	9.2	0.91	-2.1	9.65	2.96	3.51	9.51	1.91	2.73	9.33	1.58	2.48
								Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments											
	20	2.55	2.14	-0.64	2.63	1.92	-0.6	2.6	1.99	-0.67	6.33	6.4	6.28	4.35	4.24	3.86	3.71	3.39	2.84
	50	2.78	1.98	-0.75	2.82	1.89	-0.58	2.85	1.84	-0.68	4.89	4.53	4.1	3.83	3.12	2.35	3.45	2.56	1.85
	100	2.83	1.95	-0.64	3.03	1.88	-0.65	2.9	1.93	-0.64	4.24	3.53	2.88	3.63	2.56	1.85	3.3	2.35	1.44
								Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
Naive	20	2.87	2.05	-0.75	3.05	1.85	-0.72	3.05	1.9	-0.78	6.7	6.33	6.33	4.78	4.22	3.89	4.14	3.33	2.86
	50	3.43	1.88	-0.91	3.43	1.79	-0.73	3.45	1.75	-0.81	5.49	4.46	4.11	4.4	3.06	2.4	4.04	2.49	1.9
	100	3.45	1.84	-0.78	3.7	1.77	-0.8	3.56	1.83	-0.79	4.92	3.45	2.93	4.3	2.47	1.92	3.96	2.26	1.51
								Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments											
	20	2.95	2.02	-0.78	3.16	1.83	-0.75	3.15	1.89	-0.81	6.8	6.31	6.31	4.86	4.2	3.88	4.24	3.32	2.87
	50	3.62	1.86	-0.95	3.62	1.76	-0.77	3.64	1.72	-0.86	5.64	4.43	4.11	4.57	3.04	2.42	4.22	2.46	1.92
	100	3.68	1.81	-0.83	3.91	1.73	-0.85	3.78	1.8	-0.83	5.12	3.42	2.95	4.52	2.44	1.94	4.17	2.23	1.53
								Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	3.87	1.87	-1.01	3.36	1.8	-0.79	3.13	1.89	-0.8	6.69	6.28	6.27	4.81	4.17	3.89	4.12	3.33	2.87
	50	4.37	1.71	-1.13	3.69	1.74	-0.79	3.5	1.74	-0.83	5.87	4.36	4.14	4.51	3.02	2.42	4.02	2.48	1.9
	100	4.49	1.68	-1.02	3.93	1.73	-0.86	3.57	1.83	-0.79	5.47	3.33	2.99	4.44	2.44	1.93	3.94	2.25	1.51

Table S31: The Size and Power for Alternative Pooled Estimators under Experiment 4 with $h = 6$

		Size									Power								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
									OLS										
	20	0.723	0.095	0.095	0.949	0.081	0.061	0.998	0.075	0.076	0.1	0.42	0.36	0.12	0.68	0.73	0.12	0.93	0.95
	50	0.835	0.069	0.052	0.994	0.066	0.06	1	0.059	0.092	0.18	0.75	0.79	0.36	0.99	0.99	0.43	1	1
	100	0.948	0.055	0.066	1	0.065	0.083	1	0.074	0.127	0.3	0.96	0.96	0.63	1	1	0.85	1	1
									Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments										
	20	0.076	0.073	0.075	0.079	0.073	0.056	0.079	0.061	0.057	0.13	0.41	0.34	0.17	0.74	0.78	0.2	0.93	0.95
	50	0.055	0.062	0.041	0.055	0.059	0.048	0.049	0.048	0.047	0.24	0.75	0.81	0.5	0.99	0.99	0.73	1	1
	100	0.053	0.051	0.051	0.048	0.048	0.052	0.053	0.056	0.047	0.46	0.97	0.96	0.83	1	1	0.97	1	1
									Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments										
	20	0.101	0.073	0.069	0.093	0.07	0.06	0.094	0.063	0.058	0.1	0.41	0.34	0.14	0.74	0.78	0.24	0.93	0.95
$\bar{\boldsymbol{x}}_t$	50	0.06	0.061	0.042	0.049	0.059	0.051	0.047	0.048	0.048	0.21	0.74	0.8	0.46	0.99	0.99	0.73	1	1
	100	0.052	0.051	0.051	0.055	0.046	0.057	0.051	0.056	0.047	0.44	0.97	0.96	0.79	1	1	0.97	1	1
									Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments										
	20	0.102	0.075	0.072	0.109	0.071	0.057	0.107	0.064	0.058	0.1	0.41	0.35	0.15	0.73	0.77	0.19	0.93	0.95
	50	0.055	0.061	0.042	0.057	0.059	0.052	0.051	0.048	0.049	0.23	0.74	0.81	0.47	0.99	0.99	0.73	1	1
	100	0.055	0.051	0.05	0.05	0.046	0.056	0.058	0.057	0.05	0.46	0.97	0.96	0.8	1	1	0.97	1	1
									Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments										
	20	0.082	0.072	0.074	0.081	0.072	0.056	0.09	0.062	0.056	0.12	0.42	0.36	0.16	0.73	0.78	0.23	0.93	0.95
	50	0.061	0.061	0.041	0.057	0.058	0.048	0.049	0.048	0.047	0.24	0.75	0.81	0.52	0.99	0.99	0.75	1	1
	100	0.047	0.051	0.052	0.058	0.046	0.054	0.055	0.055	0.048	0.47	0.97	0.96	0.82	1	1	0.98	1	1
									OLS										
	20	0.148	0.09	0.103	0.171	0.082	0.076	0.221	0.079	0.082	0.1	0.38	0.34	0.15	0.74	0.73	0.27	0.95	0.95
	50	0.117	0.071	0.056	0.217	0.067	0.058	0.318	0.066	0.068	0.29	0.71	0.79	0.5	0.99	0.99	0.76	1	1
	100	0.262	0.054	0.06	0.62	0.058	0.056	0.853	0.066	0.063	0.5	0.96	0.96	0.87	1	1	0.98	1	1
									Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments										
	20	0.079	0.089	0.103	0.085	0.082	0.076	0.098	0.077	0.08	0.09	0.38	0.34	0.14	0.74	0.74	0.25	0.95	0.94
	50	0.053	0.07	0.053	0.058	0.065	0.057	0.056	0.066	0.066	0.26	0.71	0.79	0.51	0.99	0.99	0.76	1	1
	100	0.051	0.056	0.06	0.059	0.057	0.054	0.058	0.065	0.051	0.45	0.96	0.96	0.83	1	1	0.98	1	1
									Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments										
	20	0.072	0.087	0.098	0.088	0.082	0.078	0.126	0.078	0.081	0.09	0.38	0.34	0.15	0.74	0.76	0.21	0.95	0.95
$(\bar{y}_t, \bar{\boldsymbol{x}}_t')'$	50	0.056	0.071	0.056	0.056	0.068	0.063	0.054	0.066	0.067	0.23	0.71	0.79	0.46	0.99	0.99	0.76	1	1
	100	0.054	0.059	0.057	0.056	0.056	0.055	0.052	0.065	0.051	0.49	0.96	0.96	0.8	1	1	0.97	1	1
									Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments										
	20	0.064	0.091	0.103	0.089	0.083	0.075	0.146	0.079	0.081	0.11	0.38	0.34	0.16	0.74	0.75	0.27	0.95	0.95
	50	0.055	0.071	0.056	0.058	0.067	0.064	0.056	0.066	0.068	0.23	0.71	0.79	0.48	0.99	0.99	0.76	1	1
	100	0.057	0.057	0.058	0.053	0.056	0.056	0.056	0.065	0.052	0.48	0.96	0.96	0.82	1	1	0.97	1	1
									Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments										
	20	0.078	0.09	0.102	0.08	0.081	0.077	0.105	0.078	0.081	0.09	0.38	0.35	0.16	0.74	0.74	0.26	0.95	0.94
	50	0.057	0.07	0.055	0.053	0.068	0.058	0.054	0.066	0.067	0.26	0.7	0.8	0.52	0.99	0.99	0.75	1	1
	100	0.048	0.056	0.06	0.062	0.057	0.054	0.058	0.065	0.052	0.5	0.96	0.96	0.84	1	1	0.98	1	1
									OLS										
	20	0.191	0.089	0.107	0.204	0.079	0.108	0.277	0.095	0.129	0.1	0.39	0.31	0.15	0.74	0.75	0.23	0.94	0.94
	50	0.126	0.074	0.06	0.201	0.067	0.07	0.306	0.069	0.084	0.22	0.65	0.75	0.5	0.98	0.99	0.79	1	1
	100	0.25	0.061	0.066	0.594	0.057	0.07	0.841	0.068	0.084	0.52	0.94	0.96	0.86	1	1	0.98	1	1
									Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments										
	20	0.095	0.086	0.104	0.083	0.077	0.1	0.137	0.095	0.126	0.09	0.38	0.28	0.17	0.74	0.75	0.23	0.95	0.95
	50	0.064	0.075	0.059	0.061	0.068	0.066	0.065	0.067	0.075	0.22	0.65	0.75	0.49	0.98	0.99	0.74	1	1
	100	0.056	0.059	0.056	0.054	0.049	0.056	0.056	0.065	0.048	0.45	0.94	0.96	0.84	1	1	0.98	1	1
									Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments										
	20	0.09	0.088	0.102	0.079	0.078	0.1	0.13	0.095	0.126	0.1	0.38	0.28	0.18	0.74	0.76	0.24	0.95	0.95
$(\bar{y}_t, \bar{y}_t^{*}, \bar{\boldsymbol{x}}_t')'$	50	0.065	0.075	0.06	0.055	0.068	0.069	0.06	0.067	0.074	0.24	0.65	0.75	0.52	0.98	0.99	0.76	1	1
	100	0.059	0.06	0.057	0.055	0.05	0.06	0.059	0.066	0.048	0.45	0.94	0.96	0.84	1	1	0.98	1	1

Table S31 (Continued)

		Size									Power								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																			
	20	0.081	0.088	0.099	0.078	0.081	0.097	0.076	0.093	0.119	0.11	0.38	0.29	0.17	0.74	0.77	0.22	0.95	0.95
	50	0.065	0.077	0.06	0.06	0.068	0.069	0.056	0.067	0.074	0.21	0.64	0.75	0.49	0.98	0.99	0.76	1	1
	100	0.053	0.06	0.057	0.051	0.051	0.061	0.059	0.066	0.052	0.47	0.94	0.96	0.78	1	1	0.98	1	1
Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																			
	20	0.093	0.087	0.094	0.084	0.081	0.101	0.106	0.108	0.133	0.08	0.34	0.27	0.11	0.7	0.71	0.16	0.94	0.94
	50	0.061	0.073	0.069	0.073	0.069	0.045	0.065	0.077	0.072	0.19	0.67	0.74	0.41	0.98	0.98	0.68	1	1
	100	0.059	0.063	0.054	0.065	0.05	0.075	0.053	0.058	0.057	0.4	0.93	0.95	0.77	1	1	0.96	1	1
OLS																			
Infeasible	20	0.168	0.083	0.081	0.272	0.077	0.081	0.398	0.069	0.085	0.22	0.4	0.41	0.35	0.77	0.81	0.61	0.97	0.97
	50	0.212	0.076	0.056	0.441	0.071	0.06	0.648	0.064	0.09	0.44	0.77	0.8	0.84	0.99	0.99	0.92	1	1
	100	0.337	0.059	0.065	0.691	0.058	0.081	0.904	0.06	0.101	0.66	0.98	0.97	0.95	1	1	1	1	1
Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																			
	20	0.093	0.081	0.086	0.087	0.084	0.073	0.084	0.06	0.07	0.2	0.4	0.39	0.37	0.76	0.78	0.58	0.96	0.97
	50	0.051	0.075	0.054	0.052	0.07	0.046	0.065	0.061	0.055	0.43	0.76	0.81	0.77	0.99	0.99	0.93	1	1
	100	0.054	0.06	0.064	0.057	0.054	0.057	0.043	0.056	0.057	0.65	0.97	0.97	0.95	1	1	1	1	1
Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																			
	20	0.102	0.083	0.083	0.089	0.084	0.068	0.081	0.062	0.065	0.2	0.42	0.37	0.36	0.76	0.78	0.62	0.96	0.97
	50	0.051	0.073	0.056	0.054	0.069	0.05	0.058	0.06	0.056	0.42	0.76	0.8	0.8	0.99	0.99	0.92	1	1
	100	0.053	0.059	0.061	0.045	0.053	0.058	0.052	0.058	0.057	0.66	0.97	0.97	0.96	1	1	1	1	1
Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																			
	20	0.105	0.083	0.083	0.092	0.082	0.067	0.085	0.062	0.061	0.21	0.42	0.37	0.39	0.76	0.78	0.61	0.96	0.97
	50	0.05	0.074	0.054	0.054	0.069	0.049	0.058	0.061	0.056	0.42	0.76	0.81	0.77	0.99	0.99	0.92	1	1
	100	0.061	0.059	0.059	0.05	0.053	0.058	0.05	0.058	0.059	0.65	0.97	0.97	0.95	1	1	1	1	1
Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																			
	20	0.095	0.082	0.084	0.093	0.084	0.07	0.083	0.061	0.067	0.2	0.41	0.38	0.37	0.76	0.78	0.63	0.96	0.97
	50	0.052	0.073	0.054	0.047	0.07	0.046	0.065	0.061	0.054	0.42	0.76	0.81	0.8	0.99	0.99	0.94	1	1
	100	0.054	0.059	0.061	0.052	0.053	0.058	0.051	0.057	0.058	0.68	0.97	0.97	0.97	1	1	1	1	1
OLS																			
	20	0.631	0.089	0.101	0.863	0.088	0.117	0.965	0.089	0.179	0.16	0.38	0.34	0.3	0.71	0.75	0.36	0.94	0.94
	50	0.877	0.083	0.109	0.989	0.09	0.15	1	0.089	0.252	0.3	0.77	0.81	0.49	0.98	0.99	0.64	1	1
	100	0.965	0.077	0.145	1	0.096	0.264	1	0.148	0.42	0.32	0.97	0.97	0.7	1	1	0.84	1	1
Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																			
	20	0.251	0.101	0.088	0.291	0.111	0.08	0.372	0.145	0.074	0.17	0.35	0.33	0.31	0.69	0.78	0.43	0.93	0.94
	50	0.296	0.111	0.059	0.422	0.155	0.062	0.571	0.207	0.078	0.23	0.73	0.81	0.43	0.97	0.99	0.66	1	1
	100	0.409	0.131	0.069	0.628	0.214	0.083	0.744	0.375	0.1	0.33	0.96	0.97	0.68	1	1	0.84	1	1
Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																			
Naive	20	0.312	0.097	0.089	0.349	0.105	0.078	0.434	0.137	0.083	0.15	0.36	0.35	0.28	0.69	0.78	0.44	0.92	0.94
	50	0.373	0.103	0.066	0.502	0.141	0.063	0.672	0.19	0.086	0.21	0.73	0.8	0.41	0.98	0.99	0.63	1	1
	100	0.504	0.123	0.072	0.722	0.196	0.101	0.826	0.349	0.119	0.23	0.96	0.97	0.6	1	1	0.77	1	1
Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																			
	20	0.322	0.1	0.089	0.378	0.105	0.08	0.459	0.135	0.084	0.13	0.38	0.34	0.27	0.69	0.77	0.42	0.92	0.94
	50	0.4	0.1	0.068	0.539	0.14	0.063	0.709	0.185	0.086	0.2	0.74	0.82	0.39	0.98	0.99	0.61	1	1
	100	0.536	0.12	0.075	0.755	0.192	0.101	0.862	0.337	0.123	0.23	0.96	0.97	0.58	1	1	0.78	1	1
Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																			
	20	0.295	0.1	0.086	0.337	0.109	0.081	0.426	0.141	0.08	0.15	0.36	0.34	0.28	0.69	0.78	0.44	0.92	0.94
	50	0.357	0.105	0.064	0.477	0.148	0.065	0.635	0.192	0.082	0.22	0.73	0.82	0.45	0.97	0.99	0.65	1	1
	100	0.49	0.126	0.07	0.697	0.199	0.094	0.803	0.359	0.112	0.27	0.96	0.97	0.65	1	1	0.81	1	1

Table S32: The Size and Power for Alternative Mean Group Estimators under Experiment 4 with $h = 6$

		Size									Power								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
								OLS											
	20	0.705	0.068	0.074	0.966	0.064	0.054	0.998	0.07	0.076	0.09	0.35	0.26	0.12	0.73	0.71	0.19	0.93	0.93
	50	0.8	0.067	0.049	0.992	0.06	0.051	1	0.055	0.082	0.19	0.65	0.7	0.33	0.98	0.99	0.46	1	1
	100	0.91	0.06	0.061	1	0.065	0.095	1	0.074	0.129	0.25	0.91	0.94	0.59	1	1	0.85	1	1
								Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments											
	20	0.064	0.064	0.063	0.074	0.052	0.047	0.078	0.053	0.056	0.08	0.3	0.24	0.12	0.71	0.72	0.18	0.93	0.94
	50	0.063	0.06	0.05	0.055	0.055	0.053	0.042	0.051	0.048	0.18	0.65	0.67	0.43	0.98	0.99	0.74	1	1
	100	0.056	0.061	0.051	0.067	0.05	0.049	0.047	0.053	0.052	0.33	0.9	0.92	0.66	1	1	0.97	1	1
								Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	0.082	0.064	0.063	0.088	0.057	0.05	0.084	0.058	0.055	0.06	0.29	0.26	0.12	0.7	0.69	0.18	0.89	0.94
$\bar{\boldsymbol{x}}_t$	50	0.069	0.053	0.052	0.078	0.056	0.05	0.055	0.05	0.055	0.17	0.6	0.65	0.36	0.98	0.98	0.67	1	1
	100	0.06	0.068	0.047	0.059	0.046	0.052	0.051	0.051	0.056	0.31	0.88	0.92	0.68	1	1	0.96	1	1
								Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments											
	20	0.085	0.059	0.06	0.102	0.053	0.049	0.098	0.054	0.054	0.09	0.32	0.23	0.14	0.72	0.73	0.17	0.92	0.93
	50	0.072	0.058	0.044	0.075	0.057	0.053	0.063	0.048	0.052	0.18	0.62	0.66	0.38	0.98	0.98	0.68	1	1
	100	0.059	0.064	0.043	0.068	0.046	0.047	0.05	0.051	0.054	0.3	0.87	0.93	0.72	1	1	0.96	1	1
								Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	0.148	0.063	0.07	0.122	0.051	0.049	0.101	0.056	0.057	0.09	0.32	0.27	0.15	0.71	0.7	0.17	0.92	0.94
	50	0.135	0.058	0.044	0.1	0.055	0.052	0.071	0.049	0.051	0.17	0.65	0.69	0.45	0.98	0.98	0.73	1	1
	100	0.142	0.065	0.049	0.099	0.048	0.051	0.072	0.053	0.056	0.35	0.9	0.94	0.72	1	1	0.97	1	1
								OLS											
	20	0.12	0.06	0.077	0.231	0.065	0.067	0.353	0.074	0.076	0.07	0.35	0.3	0.13	0.76	0.74	0.26	0.94	0.95
	50	0.123	0.066	0.052	0.195	0.06	0.06	0.301	0.061	0.057	0.24	0.67	0.68	0.47	0.98	0.98	0.77	1	1
	100	0.252	0.062	0.058	0.601	0.055	0.052	0.843	0.058	0.059	0.43	0.89	0.93	0.83	1	1	0.97	1	1
								Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments											
	20	0.052	0.063	0.08	0.072	0.06	0.063	0.102	0.07	0.068	0.07	0.34	0.27	0.15	0.75	0.71	0.25	0.94	0.95
	50	0.061	0.064	0.052	0.054	0.062	0.059	0.044	0.061	0.061	0.19	0.66	0.69	0.46	0.98	0.99	0.79	1	1
	100	0.058	0.061	0.058	0.066	0.051	0.051	0.047	0.064	0.057	0.33	0.9	0.93	0.74	1	1	0.98	1	1
								Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	0.044	0.068	0.078	0.091	0.064	0.064	0.129	0.073	0.071	0.09	0.34	0.28	0.13	0.73	0.71	0.2	0.95	0.94
$(\bar{y}_t, \bar{\boldsymbol{x}}_t')'$	50	0.051	0.064	0.05	0.065	0.061	0.062	0.056	0.063	0.059	0.17	0.63	0.68	0.35	0.98	0.98	0.71	1	1
	100	0.051	0.062	0.052	0.054	0.049	0.053	0.051	0.063	0.054	0.37	0.88	0.93	0.75	1	1	0.97	1	1
								Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments											
	20	0.042	0.063	0.074	0.096	0.063	0.067	0.149	0.072	0.071	0.08	0.36	0.28	0.14	0.75	0.71	0.29	0.95	0.94
	50	0.048	0.062	0.052	0.061	0.062	0.062	0.052	0.062	0.06	0.18	0.62	0.67	0.42	0.98	0.98	0.73	1	1
	100	0.054	0.062	0.057	0.057	0.051	0.051	0.055	0.064	0.056	0.35	0.89	0.93	0.78	1	1	0.97	1	1
								Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	0.06	0.059	0.076	0.085	0.059	0.065	0.116	0.07	0.071	0.08	0.36	0.3	0.16	0.76	0.72	0.28	0.94	0.94
	50	0.069	0.062	0.05	0.057	0.064	0.059	0.049	0.063	0.062	0.17	0.68	0.69	0.47	0.98	0.99	0.76	1	1
	100	0.063	0.065	0.058	0.063	0.054	0.051	0.051	0.065	0.057	0.42	0.89	0.93	0.78	1	1	0.98	1	1
								OLS											
	20	0.189	0.07	0.115	0.372	0.084	0.181	0.529	0.115	0.242	0.07	0.34	0.29	0.1	0.75	0.67	0.15	0.94	0.94
	50	0.109	0.06	0.06	0.188	0.065	0.062	0.297	0.063	0.083	0.23	0.64	0.64	0.46	0.98	0.98	0.77	1	1
	100	0.246	0.068	0.054	0.589	0.056	0.072	0.834	0.068	0.089	0.43	0.87	0.92	0.83	1	1	0.98	1	1
								Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments											
	20	0.074	0.076	0.091	0.12	0.075	0.132	0.171	0.095	0.164	0.06	0.3	0.25	0.1	0.74	0.68	0.11	0.94	0.94
	50	0.053	0.067	0.061	0.054	0.065	0.059	0.048	0.061	0.072	0.21	0.61	0.63	0.44	0.98	0.98	0.77	1	1
	100	0.061	0.066	0.05	0.064	0.054	0.06	0.052	0.06	0.054	0.33	0.88	0.92	0.75	1	1	0.97	1	1
								Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments											
	20	0.051	0.064	0.087	0.046	0.072	0.105	0.038	0.094	0.126	0.05	0.25	0.23	0.06	0.63	0.47	0.06	0.8	0.62
$(\bar{y}_t, \bar{y}_t^{*}, \bar{\boldsymbol{x}}_t')'$	50	0.058	0.056	0.066	0.064	0.062	0.061	0.043	0.058	0.068	0.17	0.64	0.63	0.36	0.98	0.98	0.72	1	1
	100	0.055	0.069	0.044	0.063	0.059	0.058	0.048	0.063	0.053	0.37	0.86	0.92	0.71	1	1	0.97	1	1

Table S32 (Continued)

		Size									Power								
		20			50			100			20			50			100		
	(N,T)	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2	ρ	β_1	β_2
Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																			
	20	0.054	0.07	0.1	0.07	0.087	0.154	0.088	0.117	0.212	0.06	0.28	0.19	0.06	0.68	0.61	0.06	0.91	0.85
	50	0.065	0.059	0.065	0.065	0.063	0.058	0.048	0.059	0.068	0.16	0.64	0.64	0.39	0.98	0.98	0.74	1	1
	100	0.052	0.067	0.049	0.058	0.057	0.06	0.052	0.059	0.051	0.37	0.87	0.91	0.74	1	1	0.97	1	1
Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																			
	20	0.095	0.075	0.095	0.139	0.074	0.14	0.174	0.097	0.172	0.07	0.3	0.27	0.11	0.75	0.66	0.12	0.94	0.93
	50	0.061	0.064	0.06	0.057	0.062	0.06	0.046	0.06	0.068	0.18	0.62	0.65	0.47	0.98	0.98	0.77	1	1
	100	0.061	0.066	0.054	0.064	0.058	0.061	0.05	0.059	0.053	0.4	0.86	0.92	0.76	1	1	0.98	1	1
OLS																			
Infeasible	20	0.124	0.064	0.063	0.241	0.07	0.071	0.39	0.062	0.069	0.16	0.38	0.35	0.35	0.77	0.78	0.61	0.97	0.96
	50	0.202	0.072	0.07	0.429	0.072	0.051	0.667	0.065	0.09	0.36	0.67	0.72	0.79	0.98	0.99	0.93	1	1
	100	0.302	0.061	0.054	0.663	0.057	0.085	0.912	0.059	0.094	0.65	0.93	0.95	0.96	1	1	1	1	1
Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																			
	20	0.068	0.062	0.066	0.074	0.067	0.06	0.079	0.056	0.061	0.21	0.39	0.34	0.35	0.75	0.76	0.58	0.97	0.96
	50	0.054	0.075	0.067	0.051	0.071	0.046	0.047	0.064	0.054	0.34	0.67	0.7	0.74	0.98	0.99	0.95	1	1
	100	0.049	0.066	0.051	0.047	0.052	0.044	0.047	0.055	0.058	0.57	0.92	0.95	0.94	1	1	1	1	1
Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																			
	20	0.054	0.062	0.062	0.072	0.068	0.061	0.075	0.055	0.06	0.2	0.38	0.34	0.35	0.75	0.75	0.63	0.96	0.96
	50	0.056	0.075	0.068	0.05	0.069	0.052	0.049	0.062	0.058	0.33	0.68	0.72	0.72	0.98	0.99	0.94	1	1
	100	0.049	0.06	0.049	0.047	0.055	0.041	0.064	0.056	0.062	0.54	0.92	0.95	0.94	1	1	1	1	1
Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																			
	20	0.061	0.058	0.065	0.069	0.069	0.062	0.077	0.055	0.063	0.21	0.39	0.34	0.35	0.75	0.75	0.6	0.96	0.96
	50	0.06	0.07	0.066	0.047	0.069	0.051	0.054	0.062	0.06	0.31	0.68	0.74	0.75	0.98	0.99	0.93	1	1
	100	0.05	0.06	0.051	0.052	0.057	0.041	0.048	0.054	0.064	0.55	0.92	0.95	0.94	1	1	1	1	1
Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																			
	20	0.065	0.063	0.064	0.077	0.068	0.06	0.079	0.055	0.063	0.19	0.38	0.35	0.36	0.75	0.75	0.63	0.97	0.96
	50	0.068	0.072	0.072	0.052	0.071	0.051	0.051	0.062	0.055	0.34	0.66	0.71	0.76	0.98	0.99	0.95	1	1
	100	0.054	0.063	0.051	0.048	0.053	0.041	0.053	0.054	0.058	0.57	0.92	0.95	0.96	1	1	1	1	1
OLS																			
Naive	20	0.595	0.06	0.072	0.863	0.07	0.118	0.962	0.077	0.149	0.16	0.38	0.33	0.29	0.74	0.73	0.35	0.91	0.94
	50	0.852	0.079	0.095	0.988	0.071	0.12	1	0.077	0.231	0.26	0.69	0.71	0.5	0.98	0.99	0.61	1	1
	100	0.961	0.069	0.128	1	0.09	0.259	1	0.144	0.411	0.35	0.94	0.94	0.69	1	1	0.87	1	1
Use only $\tilde{\boldsymbol{X}}^{*}$ as Instruments																			
	20	0.206	0.067	0.06	0.266	0.095	0.067	0.331	0.127	0.069	0.15	0.35	0.32	0.3	0.68	0.72	0.43	0.89	0.93
	50	0.258	0.107	0.069	0.392	0.123	0.05	0.541	0.175	0.066	0.18	0.67	0.69	0.4	0.97	0.99	0.66	1	1
	100	0.364	0.113	0.056	0.597	0.216	0.073	0.715	0.337	0.088	0.36	0.93	0.94	0.64	1	1	0.82	1	1
Use only $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																			
	20	0.23	0.07	0.06	0.311	0.091	0.067	0.388	0.121	0.067	0.17	0.35	0.32	0.24	0.69	0.73	0.39	0.9	0.93
	50	0.313	0.106	0.066	0.475	0.121	0.051	0.632	0.167	0.078	0.17	0.67	0.69	0.4	0.96	0.99	0.62	1	1
	100	0.446	0.115	0.065	0.702	0.203	0.082	0.801	0.311	0.102	0.27	0.93	0.94	0.58	1	1	0.78	1	1
Use only $\tilde{\boldsymbol{X}}^{3*}$ as Instruments																			
	20	0.249	0.066	0.059	0.33	0.086	0.067	0.409	0.12	0.068	0.15	0.37	0.31	0.25	0.71	0.73	0.4	0.89	0.93
	50	0.348	0.105	0.072	0.512	0.112	0.05	0.668	0.164	0.085	0.18	0.66	0.7	0.41	0.96	0.99	0.64	1	1
	100	0.474	0.11	0.069	0.728	0.193	0.087	0.838	0.305	0.107	0.28	0.93	0.93	0.58	1	1	0.76	1	1
Use $\tilde{\boldsymbol{X}}^{*}$ and $\tilde{\boldsymbol{X}}^{2*}$ as Instruments																			
	20	0.278	0.061	0.056	0.336	0.089	0.068	0.408	0.119	0.07	0.13	0.36	0.33	0.28	0.71	0.73	0.41	0.88	0.93
	50	0.395	0.095	0.076	0.525	0.115	0.054	0.665	0.161	0.078	0.21	0.68	0.7	0.42	0.97	0.99	0.64	1	1
	100	0.559	0.105	0.078	0.753	0.203	0.089	0.836	0.312	0.103	0.32	0.94	0.95	0.64	1	1	0.82	1	1