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Alberto Caruso, Laura Coroneo

Department of Economics and Related Studies
University of York
Heslington
York, YO10 5DD

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Alberto Caruso*
Confindustria
ECARES, Université Libre de Bruxelles

Laura Coroneo†
University of York

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Abstract

We analyse the predictive ability of real-time macroeconomic information for the yield curve of interest rates. We specify a mixed-frequency macro-yields model in real-time that incorporates interest rate surveys and that treats macroeconomic factors as unobservable components. Results indicate that real-time macroeconomic information is helpful to predict interest rates, and that data revisions drive a superior predictive ability of revised macro data over real-time macro data. Moreover, we find that incorporating interest rate surveys in the model can significantly improve its predictive ability.

JEL classification codes: C33, C53, E43, E44, G12.
Keywords: Government Bonds; Dynamic Factor Models; Real-time Forecasting; Mixed-frequencies.

*Contact: Confindustria, Viale dell'Astronomia 30, 00144 Roma, Italy. Email: alberto.caruso01@gmail.com. Web: www.alberto-caruso.com

†Contact: Department of Economics and Related Studies, University of York, Heslington, YO10 5DD York, United Kingdom. Email: laura.coroneo@york.ac.uk. Web: www.sites.google.com/view/lauracoroneo.

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1 Introduction

Macroeconomic variables may incorporate important information for forecasting the evolution of the yield curve. This is due to both the behaviour of market agents, who closely monitor macroeconomic data and react to macroeconomic news, and policy makers, who operate on interest rates to stimulate aggregate demand and control inflation. Indeed, following the seminal work by Ang & Piazzesi (2003), there is a consensus in the literature that macroeconomic indicators are successful at predicting interest rates and excess bond returns, see among others Mönch (2008), Ludvigson & Ng (2009), Favero, Niu & Sala (2012) and Coroneo, Giannone & Modugno (2016). However, Ghysels, Horan & Moench (2017) find limited evidence of predictive ability of real-time macroeconomic variables for excess bond returns: they argue that the result of the previous literature was an artefact coming from the use of revised data, instead of real-time macroeconomic data.[1]

In this paper, we assess the relevance of real-time macroeconomic information to predict the future path of the yield curve of interest rates. Our contribution is to use filtering techniques to exploit the informative content of real-time macroeconomic data, and to properly specify the information set available to agents in each point in time by taking into account all the characteristics of the real-time macroeconomic data flow.[2] First, most macroeconomic data is released in a non-synchronous way and with different publication lags; therefore the available information at each point in time can be described by a dataset that has a ragged edge, and it is not balanced. Second, macroeconomic data is very often subsequently revised: the revisions might be substantial and affect the estimation and the forecast computed using different vintages of the data. Third, in real-time forecasting, soft information provided by surveys can have an important role as it is timely, not subject to revisions and can readily incorporate any information available to survey participants, such as information about the current state of the economy or forward-looking information

[1] A common denominator of this literature, in fact, is the use of revised macroeconomic data to predict interest rates, which involves using an information set that is different from the one available to market participants when the predictions were made.

[2] Adequately specifying the information set available to agents in real-time is particularly important when evaluating models in macroeconomics and finance, especially when the objective is to forecast asset prices using external information, since according to the efficient market hypothesis asset prices should already incorporate all the available information about their future evolution, see Orphanides (2001), Orphanides & Van Norden (2002) and Croushore & Stark (2003).

contained in monetary policy announcements. However, one drawback of using survey expectations is that they are released only infrequently, most often on a quarterly basis.

In order to study interest rate predictability in real-time while also addressing these drawbacks, we specify a mixed-frequency macro-yields model in real-time that incorporates interest rate surveys and treats macroeconomic factors as unobservable components, which we extract simultaneously with the traditional yield curve factors. Similarly to Coroneo et al. (2016), we identify the factors driving the yield curve by constraining the loadings to follow the smooth pattern proposed by Nelson & Siegel (1987). More specifically, our empirical model is a mixed-frequency dynamic factor model for Treasury zero-coupon yields, a representative set of real-time macroeconomic variables and interest rate surveys with restrictions on the factor loadings.

Our model can be estimated by maximum likelihood – see Doz, Giannone & Reichlin (2012) – using an Expectation-Maximization (EM) algorithm adapted to the presence of restrictions on the factor loadings and to missing data. Using U.S. data from 1972 to 2016, we find that real-time macroeconomic information is helpful to predict interest rates, especially short maturities at mid and long horizons, and that data revisions drive an increase in the predictive power of revised macro information with respect to real-time macro information. Moreover, during a period when a forward guidance policy is implemented, we find that incorporating interest rate surveys in the model significantly improves its predictive ability.

Our finding that data revisions drive the increased predictive ability of revised macro data with respect to real-time macro data is in line with Ghysels et al. (2017). However, while they find that real-time macro information has only a marginal (and often statistically non significant) role in predicting excess bond returns, our results show that real-time macroeconomic information is still helpful to predict interest rates, as its predictive power is similar to that of revised macro data. Moreover, in contrast to Ghysels et al. (2017), we find that real-time macro variables have a stronger predictive ability than their first releases, which is in line with the intuition that revisions of first releases enhance the quality of macroeconomic information; it is also in line with the macro forecasting literature that uses the latest available vintage of data (i.e. real-time data) in each point in time, rather than first releases, to nowcast and forecast macro aggregates (Koenig, Dolmas & Piger 2003, Croushore & Stark 2003). The reason for the superior predictive ability of our real-time macro-yield model is our use of filtering techniques that allow us to efficiently extract information from a truly real-time data set with a ragged edge.

Lastly, we find that incorporating interest rate surveys from the Surveys of Professional Forecasters can improve the predictive ability of models that use only information embedded in the yield curve and in macroeconomic variables. Surveys, in fact, incorporate soft information about the future path of interest rates – that comes from policy announcements, for example – that cannot be taken into account by standard macroeconomic variables. With this in mind, we test the predictive ability of the model by incorporating surveys in a period in which the Federal Reserve implemented a forward guidance policy. The resulting improvement in predictive ability is statistically significant. This intuitively appealing result is in line with Altavilla, Giacomini & Ragusa (2017), who use the selected survey forecast value as their forecast for the specific horizon and maturity. However, our results show that in some periods our model produces more accurate forecasts than the surveys themselves. Therefore, we incorporate the surveys into the model itself. In this way, we combine in a single framework the "soft" information embedded in the surveys with the information carried by interest rates and by the real-time macroeconomic data about the state of the economy, fully exploiting all the relevant available information in forecasting the whole yield curve.

The paper is organized as follows. Section 2 outlines the mixed-frequency real-time macro-yields model. Section 3 describes the data and Section 4 outlines the estimation procedure and some preliminary results. Section 5 describes the out-of-sample forecasting exercise, and Section 6 the results. Finally, Section 7 concludes. Appendix A contains details about the state-space representation of the model and the estimation procedure.

2 Model

We model the joint behavior of monthly government bond yields, real-time macroeconomic indicators, and quarterly interest rate surveys using a mixed-frequency dynamic factor model. Bond yields at different maturities are driven by the traditional level, slope and curvature factors, while real-time macroeconomic variables load on the yield curve factors as well as on some additional macro factors that capture the information in macroeconomic variables over and above the yield curve factors. Finally, interest rate surveys load on quarterly averages of the monthly yield and macro factors. In what follows, we describe each point in detail.

2.1 Yields

We model the cross-section of bond yields using the Dynamic Nelson-Siegel framework of Diebold & Li (2006). Denoting by y_t the $N_y \times 1$ vector of yields with N_y different maturities at time t, we have

$$y_t \;=\; a_y + \; \Gamma_{yy} \; F_t^y + v_t^y, \tag{1}$$

where F_t^y is a 3×1 vector containing the latent yield-curve factors at time t, Γ_{yy} is a $N_y \times 3$ matrix of factor loadings, and v_t^y is an $N_y \times 1$ vector of idiosyncratic components. The yield curve factors F_t^y are identified by constraining the factor loadings to follow the smooth pattern proposed by Nelson & Siegel (1987)

$$a_y = 0; \quad \Gamma_{yy}^{(\tau)} = \left[1 \quad \frac{1-e^{-\lambda\tau}}{\lambda\tau} \quad \frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau} \right] \equiv \Gamma_{NS}^{(\tau)}, \tag{2}$$

where $\Gamma_{yy}^{(\tau)}$ is the row of the matrix of factor loadings corresponding to the yield with maturity τ months and λ is a decay parameter of the factor loadings. Diebold & Li (2006) show that this functional form of the factor loadings implies that the three yield curve factors can be interpreted as the level, slope, and curvature of the yield curve. The specific shape of the loadings depends on the decay parameter λ, which we calibrate to the value that maximizes the loading on the curvature factor for the yields with maturity 30 months, as in Diebold & Li (2006). Due to its flexibility and parsimony, the Nelson & Siegel (1987) model accurately fits the yield curve and performs well in out-of-sample forecasting exercises, see Diebold & Li (2006) and Coroneo, Nyholm & Vidova-Koleva (2011).

2.2 Real-time macro variables

We assume that real-time macroeconomic variables are potentially driven by two sources of co-movement: the yield curve factors F_t^y and some macro specific factors F_t^x. Denoting by x_t the $N_x \times 1$ vector of real-time macroeconomic variables at time t, we have

$$x_t \;=\; a_x + \Gamma_{xy}\, F_t^y + \Gamma_{xx}\, F_t^x + v_t^x, \tag{3}$$

where F_t^x is an $r \times 1$ vector of macroeconomic latent factors, Γ_{xy} is a $N_x \times 3$ matrix of factor loadings of the real-time macro variables on the yield curve factors, Γ_{xx} is a $N_x \times r$ matrix of factor loadings of the real-time macro variables on the macro factors, and v_t^x is an $N_x \times 1$ vector of idiosyncratic components.

To accommodate for the features of the real-time macroeconomic information set, we allow x_t to contain missing values due to publication lags. As for data revisions, these can be easily accommodated in an out-of-sample exercise by using the latest vintage of data available at the date in which the forecasts are made.

Allowing Γ_{xy} to be different from zero is crucial to ensure that the macroeconomic factors F_t^x capture only those source of co-movement in the macroeconomic variables that are not already spanned by the yield curve factors. Also, assuming that macroeconomic factors do not provide any information about the contemporaneous shape of the yield curve ($\Gamma_{yx} = 0$ in (1)) restricts the macroeconomic factors F_t^x to be unspanned by the cross-section of yields. This restriction is expected to be immaterial since the yield factors F_t^y are notoriously effective at fitting the entire yield curve. Coroneo et al. (2016) perform a likelihood ratio test for $\Gamma_{yx} = 0$ and do not reject the restriction. They also show that imposing a block-diagonal structure of the factor loadings ($\Gamma_{xy} = 0$ and $\Gamma_{yx} = 0$) implies a duplication of factors and, as a consequence of the loss of parsimony of the model, a deterioration of the forecasting performance. Accordingly, in the remainder of the paper, we will maintain the restriction $\Gamma_{yx} = 0$ and leave Γ_{xy} unrestricted.

2.3 Interest rate surveys

The information set that forecasters use in real-time to form their expectations about future interest rates includes not only current and past interest rates, and real-time macroeconomic information, but also interest rate surveys that are usually available at a lower frequency than interest rates.

Survey expectations might be good predictors for the yield curve, because they can embed "soft" and forward-looking information which is difficult to incorporate in econometric models. For example, surveys can take into account policy announcements, which are of fundamental importance in periods in which forward guidance is used by central banks, or they can consider the existence of possible non-linearities, for example the presence of a zero lower bound for interest rates.

A successful attempt to incorporate information from surveys in econometric models for forecasting the yield curve is in Altavilla et al. (2017). They anchor the model forecasts to interest rate surveys and find that using survey data on the 3-month Treasury Bill can significantly improve the forecasting performance of the Dynamic Nelson-Siegel model. Accordingly, we exploit the informational content of the Survey of Professional Forecasters (SPF) on the 3-month Treasury Bill. However, while Altavilla et al. (2017) use the selected survey forecast value as their forecast for the specific horizon and maturity, in our case we incorporate survey forecasts into our model such that all forecasts take into account all the available information (yields, real-time macro variables and survey expectations).

Forecasts from the SPF are released the middle of the quarter for the current quarter and the following four quarters. Given that the values reported are quarterly averages, we can denote the SPF forecast for the quarterly yield at time t made at time $t-h$ as $E^s_{t-h}(y^q_{t,\tau})$. This forecast is related to the unobservable monthly forecasts as follows

$$E^s_{t-h}(y^q_{t,\tau}) = \frac{1}{3}\left[E^s_{t-h}(y_{t,\tau}) + E^s_{t-h}(y_{t-1,\tau}) + E^s_{t-h}(y_{t-2,\tau})\right], \; t = 3, 6, 9, \ldots \tag{4}$$

We assume that the unobservable monthly forecast is related to the monthly factors as follows

$$E^s_{t-h}(y_{t,\tau}) = a_s + \Gamma_{h,\tau} F_t + \upsilon_{t,h,\tau}$$

where $F_t = [F_t^y, F_t^x]$. Substituting in (4) we get

$$E^s_{t-h}(y^q_{t,\tau}) = a_s + \Gamma_{h,\tau}\left(\frac{1}{3}F_t + \frac{1}{3}F_{t-1} + \frac{1}{3}F_{t-2}\right) + v^q_{t,h,\tau} = a_s + \Gamma_{h,\tau}F_t^q + v^q_{t,h,\tau},\ t = 3, 6, 9, \dots \tag{5}$$

where F_t^q are the quarterly factors measured as quarterly averages of the monthly factors F_t, F_{t-1} and F_{t-2}, and $v^q_{t,h,\tau}$ follows an AR(1) to allow for persistent divergences between SPF and model based forecasts.

We can write the quarterly factors at a monthly frequency, such that at the end of the quarter they represent the quarterly average, as follows

$$F_t^q = \begin{cases} F_t, & t = 1, 4, 7, 10, \dots \\ \frac{1}{2}F^q_{t-1} + \frac{1}{2}F_t, & t = 2, 5, 8, 11, \dots \\ \frac{2}{3}F^q_{t-1} + \frac{1}{3}F_t, & \text{otherwise.} \end{cases}$$

This can be represented as

$$F_t^q - w_t F_t = \iota_t F^q_{t-1} \tag{6}$$

where w_t is equal to $1, 1/2, 1/3$ respectively the first, second and third month of the quarter, and ι_t is equal to $0, 1/2, 2/3$ respectively the first, second and third month of the quarter.

2.4 Joint model

The yield curve and the macroeconomic factors are extracted by estimating (1), (3) and (5) simultaneously

$$\begin{pmatrix} y_t \\ x_t \\ E^s(y_t^q) \end{pmatrix} = \begin{pmatrix} 0 \\ a_x \\ a_s \end{pmatrix} + \begin{bmatrix} \Gamma_{yy} & \Gamma_{yx} & 0 \\ \Gamma_{xy} & \Gamma_{xx} & 0 \\ 0 & 0 & \Gamma_q \end{bmatrix} \begin{pmatrix} F_t^y \\ F_t^x \\ F_t^q \end{pmatrix} + \begin{pmatrix} v_t^y \\ v_t^x \\ v_t^q \end{pmatrix},\ \Gamma_{yy} = \Gamma_{NS},\ \Gamma_{yx} = 0, \tag{7}$$

where $F_t^q = [F_t^{yq}, F_t^{xq}]$. The joint dynamics of the yield curve and the macroeconomic factors follow

$$\begin{pmatrix} F_t \\ F_t^q \end{pmatrix} = \begin{pmatrix} \mu \\ w_t\mu \end{pmatrix} + \begin{bmatrix} A & 0 \\ w_tA & \iota_t I_r \end{bmatrix} \begin{pmatrix} F_{t-1} \\ F_{t-1}^q \end{pmatrix} + \begin{pmatrix} u_t \\ w_tu_t \end{pmatrix}, u_t \sim N(0, Q), \tag{8}$$

where $F_t = [F_t^y, F_t^x]$. This is a VAR(1) with time-varying coefficients, where w_t is equal to $1, 1/2, 1/3$ respectively the first, second and third month of the quarter, and ι_t is equal to $0, 1/2, 2/3$ respectively the first, second and third month of the quarter, as in (6).

The idiosyncratic components collected in $v_t = [v_t^y \quad v_t^x \quad v_t^q]'$ are modelled to follow independent autoregressive processes

$$v_t = Bv_{t-1} + \xi_t, \quad \xi_t \sim N(0, R) \tag{9}$$

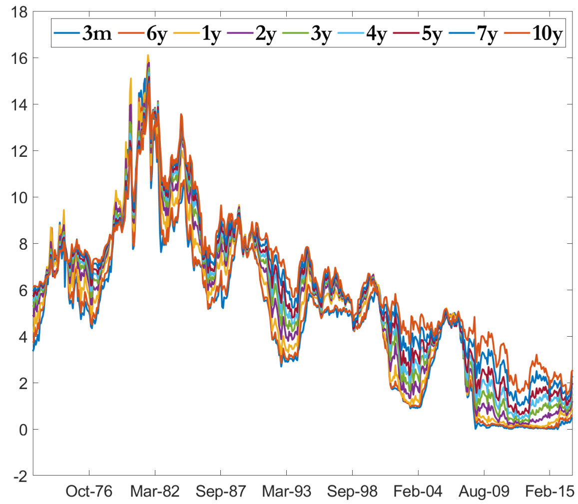
where B and R are diagonal matrices, implying that the common factors fully account for the joint correlation of the observations. The residuals to the idiosyncratic components of the individual variables, ξ_t, and the innovations driving the common factors, u_t, are assumed to be normally distributed and mutually independent. This assumption implies that the common factors are not allowed to react to variable specific shocks.

3 Data

Our dataset for interest rates and macroeconomic variables consists of U.S. observations from January 1972 to December 2016. For interest rates, we use end-of-month zero-coupon yields on 3-month and 6-month Treasury Bills from the FRED dataset, and on 1, 2, 3, 4, 5, 7 and 10-year bonds from the Federal Reserve Board dataset. In Figure 1 we plot the time series of interest rates in our sample. The figure shows a strong comovement among interest rates, and that, in the last period, short term interest rates are close to the zero lower bound.

As for macro variables, we use a monthly real-time data set using the vintages available in the Archival Federal Reserve Economic Database (ALFRED) of the Federal Reserve Bank of St. Louis and the accurate publication pattern. Macroeconomic data and the

Figure 1: Interest rates data



The chart shows the interest rates data used in our analysis.

publication delay of the variables are described in Table 1. We use 16 macroeconomic variables, including real activity indicators, inflation measures, surveys, one money aggregate and the Federal Funds rate.[3] We use annual growth rates for all variables, except for capacity utilization, the federal funds rate, the unemployment and the surveys, that we keep in levels. With the exception of the Conference Board Consumer Confidence survey and the GBA Philadelphia Fed Outlook survey, this is the same macro data set considered in Coroneo et al. (2016). We add these two surveys because of their timeliness and therefore the possibility to include early information in the forecasts: they are released before the start of the reference period (3 and 15 days before), so being amongst the first macroeconomic signals about economic activity taken into account by a forecaster. All the other macroeconomic indicators, with the exception of the Federal Funds rate, are released only after the end of the reference period, which means that in real-time their value for the current month is not available when forming expectations about future interest rates.

To illustrate the relevance of revisions in macroeconomic series, in Figure 2 we look at an example. The chart refers to the data for US Industrial Production as released in

[3]We use a medium-size data set as it has been proven that such dimension provides the best results in forecasting macroeconomic variables using dynamic factor models (see Boivin & Ng 2006, Banbura, Giannone, Modugno & Reichlin 2013, Bańbura & Modugno 2014).

Table 1: Real-time macroeconomic data

Series N.	Mnemonic	Description	Transf.	Delay (days)
1	AHE	Average Hourly Earnings: Total Private	1	4
2	CPI	Consumer Price Index: All Items	1	15
3	INC	Real Disposable Personal Income	1	28
4	FFR	Effective Federal Funds Rate	0	0
5	HSal	New One Family Houses Sold	1	24
6	IP	Industrial Production Index	1	16
7	M1	M1 Money Stock	1	3
8	Manf	PMI Composite Index (NAPM)	0	1
9	Paym	All Employees: Total nonfarm	1	4
10	PCE	Personal Consumption Expenditures	1	28
11	PPIc	Producer Price Index: Crude Materials	1	16
12	PPIf	Producer Price Index: Finished Goods	1	16
13	CU	Capacity Utilization: Total Industry	0	16
14	Unem	Civilian Unemployment Rate	0	14
15	CC	Conf. Board Consumer Confidence	0	-3
16	GBA	Philadelphia Fed Outlook survey	0	-15

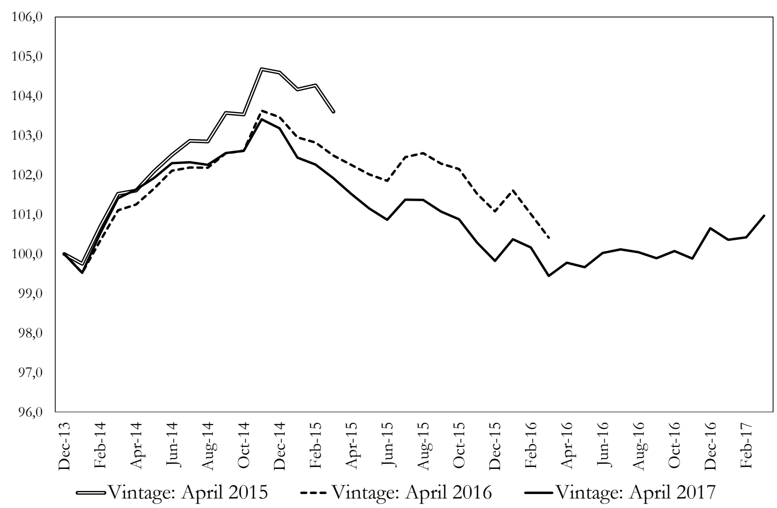
Note: real-time macroeconomic data descriptions, transformations and publication delays (number of days from the end of the reference month). Transformation codes: 0 = no transformation, 1 = annual growth rate. Source: Archival Federal Reserve Economic Database (ALFRED).

three different vintages, in April 2015, 2016 and 2017. As shown in the chart, the series is subject to substantial revisions: the information in real-time can be substantially different from the one that we can get using revised data. It is, therefore, important to use the information available in real-time when evaluating the forecasting performance.[4]

In Table 2, we give an example of the information set relative to Industrial Production in different points in time, in what is called a “revision triangle”. In the top panel, the columns represent the publication date of a vintage of data, and correspond to the information set that a forecaster has until the following release. The rows represent the reference period. If a forecaster needs the data relative to April, she must wait until the 15th May, date in which the April data gets released. However, that data point, the “first release” (104.1), is subject to revisions: on the 15th June, the data is revised to 104.0; then, after other revisions, she reads the final revised data (last column), 102.9. The series of “Revised data” for Industrial Production, therefore, corresponds to the last column. “First Releases” corresponds to the

[4]We recall that, however, if the revisions are weakly cross-correlated, factor extraction is robust to data revisions (Giannone, Reichlin & Small 2008).

Figure 2: Example: revisions in Industrial Production



The chart shows an example of data revisions in macroeconomic series, showing data for US Industrial Production as released in April 2015, April 2016, April 2017. Data are normalized (avoiding rebasing issues) constructing indexes, putting 100=December 2013. Source: authors' calculations on ALFRED data.

first available data for each reference period (the bold diagonal). A series in real-time corresponds to any of the first four columns of the table. Keeping this revision process in mind, we consider the following definitions of our macro dataset:

- Revised data: we consider the data as available on 31 March 2017, incorporating all data revisions, in a balanced dataset.

- Pseudo Real-Time: we still consider the revised data as available on 31 March 2017, but using the correct calendar of macroeconomic releases and publication lags, in a "ragged edge" dataset with missing data at the end.

- Real-Time: this is the proper real-time dataset that uses both real-time vintages and the correct publication lag structure, as such it takes into account the exact information set at the vintage date (ragged edge dataset). The last value of a series is the first release of that data point, while the previous data points are reported as revised on that specific vintage date.

- First Releases: we consider only the first release for each data point, taking into account the correct publication lag structure (ragged edge dataset).

Table 2: Industrial production - revision triangle and construction of the dataset

	15-Apr-16	15-May-16	15-Jun-16	15-Jul-16	...	31-Mar-17
Mar-16	**103.4**	103.5	103.4	103.4	...	**102.5**
Apr-16		**104.1**	104.0	103.8	...	**102.9**
May-16			**103.6**	103.5	...	**102.8**
Jun-16				**104.1**	...	**103.1**

	Revised	*Pseudo RT*	*Real-Time*	*First release*
Mar-16	102.5	102.5	103.4	103.4
Apr-16	102.9	102.9	104.0	104.1
May-16	102.8	102.8	103.6	103.6
Jun-16	103.1	-	-	-

The top panel reports five vintages of data relative to Industrial Production. The names of the columns represent the release dates of the vintages, the rows represent the reference period of each data point. The bottom panel reports the different possibilities in constructing the macro series for a forecast conducted on the 30 of June 2016, following the definitions reported in the main text.

The bottom panel of Table 2 reports an example of these definitions for the case of a forecast made on the 30th of June 2016. The table shows that using the Revised dataset, we have one extra data point (Jun-16) that in reality was not available to forecasters at the end of June 2016; taking this point away, we have the Pseudo Real-Time dataset. Both these datasets use finally revised values that can be different from the data available at the end of June 2016. The Real-Time dataset has values for the Industrial Production that are as released on the 15th of June 2016. The last value of this series is the first release for May 2016, while the previous data points are reported as revised on the 15th of June 2016. Finally, the First Release data collects all the first releases: the data point for March 2016 (released the 15th of April 2016), the data point for April 2016 (released the 15th of May 2016) and the data point for May 2016 (released on the 15th of June 2016).

Surveys of Professional Forecasters data on the 3-month Treasury Bill are provided by the Federal Reserve Bank of Philadelphia at quarterly frequency. Data are quarterly averages of the daily levels of interest rates, available since 1981:Q3, and we use the median forecast of the 3-month Treasury Bill collected for three and four quarters ahead.[5]. The surveys were conducted by the American Statistical Association (ASA) and the National Bureau of Economic Research (NBER) until 1990:Q2, and then by the Philadelphia Fed. The deadlines for the answers are known since 1990:Q3 and are in the middle of the second

[5]The horizon up to one year is the same as in Kim & Orphanides (2012).

month of the quarter. Since the deadlines for the respondents define their information set, we fix the release dates in correspondence to those deadlines on the 15th of the second month of the quarter.

4 Estimation and preliminary results

The mixed-frequency real-time macro-yields model in equations (7)-(9) can be cast in a state-space form by augmenting the state variables to include the intercept and the idiosyncratic components, for details see Appendix A.1. Following Doz et al. (2012), we estimate the model by quasi-maximum likelihood using an Expectation-Maximization (EM) algorithm initialized by Principal Components. The complication of having a ragged edge data set, which involves missing data also at the end of the sample, can be solved by adapting the EM algorithm to the presence of missing data, as in Bańbura & Modugno (2014). Also, the factor loading restrictions that identify the yield curve factors can be imposed by performing a constrained maximization in the EM algorithm, for more details see Appendix A.2.

For comparison, we also estimate an only-yields model, which uses only the information contained in yields. This is a restricted version of the macro-yields model in Equations (7)-(9) with $\Gamma_{xy} = 0$, $A_{yx} = 0$ and $Q_{yx} = 0$, and can hence be estimated using the same procedure.

To select the number of factors, we use the information criterion (IC) of Coroneo et al. (2016), which is a modification of the Bai & Ng (2002) criterion to account for the fact that the estimation is performed by quasi-maximum likelihood. We report in Table 3 the IC and the average variance of the idiosyncratic components when different numbers of factors are estimated. Results refer to both the subsample up to the Great Recession (from 1972 to 2008) and to the full sample (from 1972 to 2016). In the sample up to 2008, we find that the IC is minimized for the model with 5 factors, as in Coroneo et al. (2016). In the full sample, however, the information criterion does not deliver clear-cut results as the IC is minimised in correspondence of both 4 and 5 factors. This is due to the fact that the decrease in the variance of the idiosyncratic component achieved by adding the fifth factor is lower in the full sample than in the sample up to 2008, indicating a more marginal role

Table 3: Model selection

	1972-2008		1972-2016	
N. of factors	**IC**	**V**	**IC**	**V**
3	-0.05	0.43	-0.06	0.43
4	-0.16	0.30	-0.15	0.30
5	-0.22	0.21	-0.15	0.23
6	-0.19	0.17	-0.10	0.19
7	-0.07	0.15	0.01	0.16
8	0.06	0.13	0.09	0.13

Note: the table reports the IC criterion relative to models with different numbers of factors, following the modified version of the Bai & Ng (2002) criterion described in Coroneo et al. (2016). Columns IC report the information criteria, columns V report the average variance of the idiosyncratic components.

of the fifth factor in the last part of the sample. Therefore, after 2008, we select the more parsimonious model, with four factors.

5 Out-of-sample forecast

We design a forecasting exercise in a truly real-time out-of-sample fashion. We perform a recursive estimation using data starting in January 1972 and use the out-of-sample evaluation period from January 1995 to December 2016. We reconstruct the information set available to forecasters at each point in time in which the forecast is computed, that is at the end of each month of the out-of-sample period, using the information available at that time. This entails using the real-time vintages for all the variables in the dataset, and also reconstructing the exact calendar of the releases. Since the macroeconomic data releases are not synchronous, we have to deal with the ragged edge of the dataset: as stated above, the estimation performed within an Expectation-Maximization algorithm conveniently helps us in this respect.

Being aware of the presence of the zero lower bound for interest rates, a serious issue since 2008, we impose non-negativity of the predicted interest rates as follows

$$E_t(y_{t+h}^{(\tau)}) \equiv \hat{y}_{t+h|t} = \max(\hat{\Gamma}_{|t}^{y*} \hat{F}_{t+h|t}^{*}, 0)$$

where $\hat{\Gamma}^{y*}_{|t}$ contains the factor loadings for yields and is estimated using information up to time t and $\hat{F}^*_{t+h|t} \equiv E_t(F^*_{t+h})$ is the out-of-sample iterative forecast of the factors.[6] We take as benchmark the forecast at horizon h for the maturity τ produced by a random walk at time t

$$E_t(y^{(\tau)}_{t+h}) \equiv \hat{y}^{(\tau)}_{t+h|t} = y^{(\tau)}_t.$$

6 Results

Our empirical results are organised into two parts. First, we assess the predictive content of real-time macroeconomic information for interest rates by comparing the out-of-sample performance of a macro-yield model in real-time with one that uses revised macro data. We then add the interest rate surveys into the model, and analyse their role over and above real-time macroeconomic information.

6.1 Real-time macro data: is it useful?

In order to assess the role of the real-time macroeconomic data-flow for interest rate predictions, in this section, we report the out-of-sample evaluation of the macro-yields model using real-time data.

In Table 4, we report the MSFE (relative to the random walk) of the only-yields model, the macro-yields model using revised macro data and the macro-yields model using real-time macro data. We test for the significance of their outperformance with respect to the random walk using the Diebold & Mariano (2002) test statistic with fixed-b asymptotics to avoid size distortions due to small sample size and autocorrelation in the loss differentials, see Coroneo & Iacone (2015). Results indicate that macroeconomic data has a strong predictive ability for interest rates especially at long forecasting horizons and short-mid maturities, while the only-yields model never outperforms the random walk. The forecasting ability is stronger using revised data, but robust to the use of real-time macro data: the real-time macro-yields model forecasts significantly better than the random walk at short maturities for mid-long forecasting horizons.

[6] See Appendix A.2 for the definitions of Γ^* and F^*_t.

Table 4: Relative MSFE, Evaluation: 1995-2016

Only-yields model

	3m	6m	1y	2y	3y	4y	5y	7y	10y
1	1.02	1.04	1.02	1.05	1.05	1.04	1.04	1.04	1.02
3	1.06	1.10	1.07	1.12	1.12	1.11	1.10	1.10	1.05
6	1.06	1.13	1.13	1.21	1.21	1.20	1.19	1.18	1.12
12	1.05	1.10	1.12	1.28	1.35	1.38	1.38	1.37	1.23
24	1.13	1.15	1.13	1.33	1.51	1.64	1.75	1.91	1.80

Macro-yields model

	3m	6m	1y	2y	3y	4y	5y	7y	10y
1	0.84	0.89	0.98	1.03	1.04	1.04	1.03	1.03	1.02
3	0.72*	0.78*	0.93	1.03	1.05	1.05	1.04	1.03	1.00
6	0.64**	0.70*	0.81*	0.95	1.00	1.01	1.02	1.01	0.97
12	0.60**	0.64**	0.70**	0.82*	0.94	1.00	1.04	1.05	0.98
24	0.62**	0.64**	0.65**	0.76*	0.91	1.03	1.15	1.29	1.27

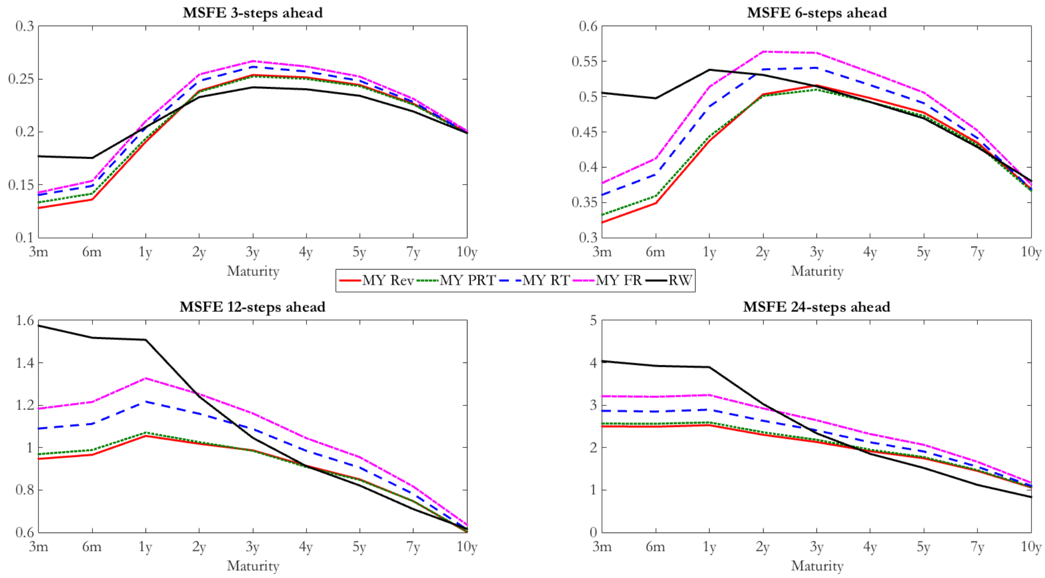
Real-time macro-yields model

	3m	6m	1y	2y	3y	4y	5y	7y	10y
1	0.90	0.93	0.98	1.06	1.06	1.05	1.04	1.03	1.02
3	0.79	0.85	1.00	1.07	1.08	1.07	1.06	1.04	1.00
6	0.71**	0.78*	0.90	0.95	1.05	1.05	1.05	1.03	0.97
12	0.69**	0.73*	0.81*	0.93	1.04	1.08	1.10	1.10	1.00
24	0.71*	0.73*	0.74*	0.87	1.03	1.15	1.26	1.38	1.32

Note: The table reports the relative Mean Squared Forecast Error relative to the random walk of the only-yields model (top panel), the macro-yields model (middle panel) and the Real-time macro-yields model (bottom panel), for the evaluation period 1995-2016. A number smaller than one indicates that the model performs better than the random walk. (*) and (**) indicate one-side significance at the 10% and 5%, respectively, using the Diebold & Mariano (2002) test statistic with fixed-b asymptotics, as in Coroneo & Iacone (2015).

In order to understand the drivers of the difference in the forecasting performance between revised and real-time macroeconomic information, in Figure 3 we plot the Mean Squared Forecast Error of the macro-yields model using the four different definitions of the macroeconomic dataset described in Section 3: the revised, the pseudo real-time, the real-time and the first releases datasets. Results indicate that the macro-yields model consistently outperforms the random walk at short maturities for all horizons. The real-time macro-yields model is slightly worse than the macro-yields using revised data and pseudo real-time data, but it still outperforms the random walk at short maturities for all horizons: this indicates that macroeconomic information is useful in predicting interest rates, even when using real-time data. Taking into account only the publication lags plays a

Figure 3: Real-time macroeconomic information



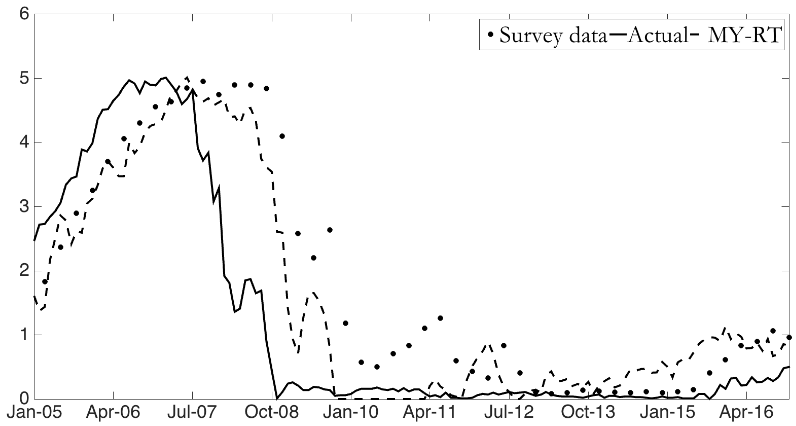
Mean Squared Forecast Error for the macro-yields model with revised data (MY Rev), the macro-yields model with pseudo real-time data (MY PRT), the macro-yields model with truly real-time data (MY RT), the macro-yields model with first releases (MY FR), and the random walk. Evaluation period 1995-2016.

lesser role, since the model in pseudo real-time has a forecasting performance very similar to the one with revised data. The model that uses the first releases, instead, performs worse than the others: this is consistent with the intuition that revisions improve the quality of macroeconomic data and therefore the signal they convey about the future path of interest rates. Therefore, we can conclude that the main drivers of the difference in forecasting performance between the model that uses revised macro data and the one that uses real-time macro data are the data revisions.

Our results are different from the general message of Ghysels et al. (2017)[7]. In addition to the finding that a real-time dataset is significantly less powerful in such a forecasting exercise (our results are milder in this respect), they also find that it also performs worse than a dataset with first releases. However, their estimation method cannot take into account missing variables, so the information set cannot exactly represent the information set faced by a forecaster in real-time. For this reason, their definition of a "real-time" dataset corresponds to our definition of "first releases", lagged by one period (for macro

[7]Note that their analysis refers to excess returns, and is based on a different sample.

Figure 4: Forecasts: macro-yield model in real-time vs. interest rate surveys



The charts report the 12-month ahead forecast for the 3-month interest rate obtained from the macro-yield model in real-time (MY-RT, dashed line), the four-quarters ahead survey for the quarterly average of the 3-month Treasury Bill (Survey Data), and the realised value (Actual, solid line).

variables with a "standard" publication lag, like Industrial Production). Our method, instead, which relies on filtering techniques widely used in the nowcasting literature (see Banbura et al. (2013) for details), can efficiently incorporate missing variables and properly treat a ragged edge dataset, maintaining the contemporaneous relationships between macro variables and interest rates.

6.2 Interest rate surveys: do they help?

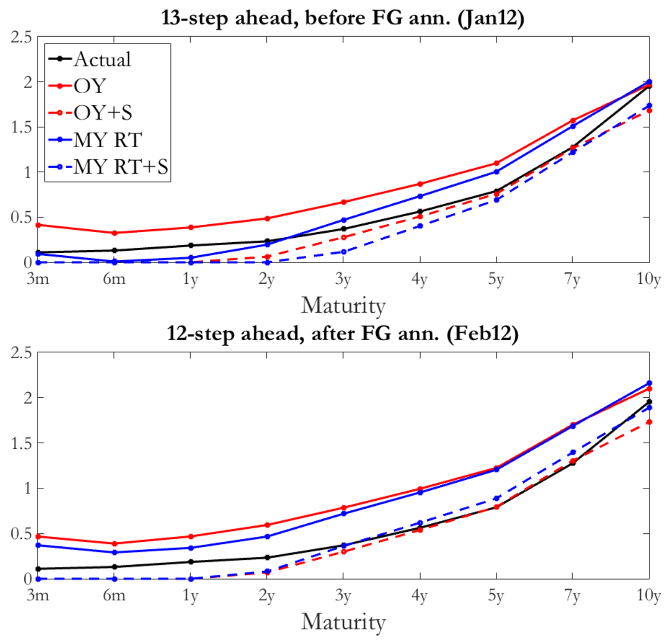
We now add the Surveys of Professional Forecasters data on the 3-month Treasury Bill to our real-time dataset, in order to evaluate if they contain additional information to predict the yield curve. We recall that we use the median forecast of the 3-month Treasury Bill collected for three and four quarters ahead. In fact, we assume that at this horizon soft information about monetary policy can play a strong role, especially during periods in which the FOMC uses forward guidance.

In Figure 4, we report the 12-month ahead forecast for the 3-month interest rate obtained from the macro-yield model in real-time, along with the four-quarters ahead survey for the quarterly average of the 3-month Treasury Bill, and the realised value. The figure shows how the macro-yield model is able to provide predictions that are closer to the realised value than the survey forecasts from 2007Q3 up to 2012Q2. After this date, the surveys consistently predict very low values for the 3-month rate, and these predictions are correct. Only at the very end of the sample, the macro-yield model in real-time provides more accurate predictions than the surveys again. This is due to the fact that on August 9, 2011 the FOMC announced that it would likely keep the federal funds rate at exceptionally low levels "at least through mid-2013". In the figure, we can see the effect of the announcement on the decline of the survey forecast for 2012Q3, which was formed one year ahead, i.e. just after the forward guidance announcement. The figure also shows that forward guidance announcements have been effective at stabilizing expectations up to mid-2014 when the one-year ahead predictions for mid-2015 indicated a rise in interest rates. On the other hand, the macro-yield model in real-time could not incorporate this type of announcement, and for all this period predicted low interest rates but higher than expected from the survey. Notice also that the time varying relative importance of model-based and survey-based forecasts signals the advantage of efficiently combining the different sources of information.

In Figure 5 we give an intuitive example of the mechanism for which the use of surveys helps to improve the forecasts of the model. In the top panel, we report the 13-month ahead forecasts from the only-yields and the real-time macro-yields model (both with and without the interest rate surveys) made in January 2012, along with the actual realization of the yield curve in February 2013. In the bottom panel, we report the forecasts for February 2013 made one month later, i.e. in February 2012. In between, there have been some macroeconomic releases and revisions, which induced the revisions of the forecasts made using the macro-yields model, but more importantly, there has been an FOMC release with a "forward guidance type" announcement on the 25th of January 2012.[8] In February, the macroeconomic news releases (and the revisions) brought up the forecasts produced by the macro-yields model (solid blue line), but the forecasts obtained using the information in the surveys are lower and closer to the realised values. Notice how, despite including

[8]The statement reads as follows "(...) the Committee decided today to keep the target range for the federal funds rate at 0 to 1/4 percent and currently anticipates that economic conditions–including low rates of resource utilization and a subdued outlook for inflation over the medium run–are likely to warrant exceptionally low levels for the federal funds rate at least through late 2014." Source: `https://www.federalreserve.gov/newsevents/pressreleases/monetary20120125a.htm`.

Figure 5: Example: forecasts made before and after a FG announcement



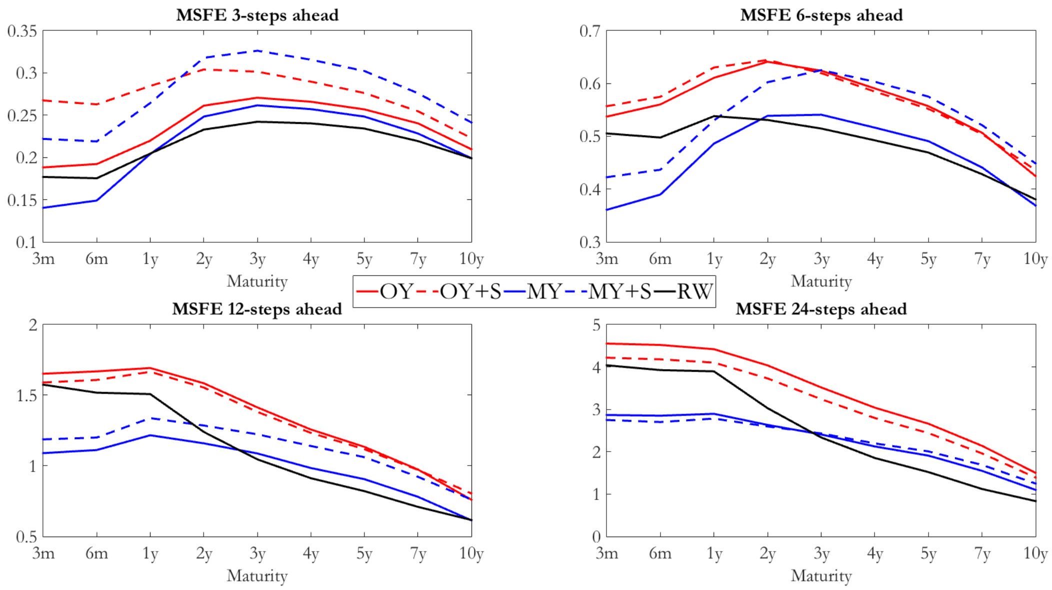
The charts report the forecasts relative to the yield curve in February 2013, made in January 2012 (top panel) and in February 2012 (bottom panel), produced by the only-yields model (OY), the only-yields model plus surveys (OY+S), the real-time macro-yields model (MY) and the real-time macro-yields model plus surveys (MY+S), along with the realized yield curve (Actual) in February 2013.

information only on survey forecasts for the 3-month Treasury Bill, the forecasts for all interest rates incorporate this information and, as a consequence, are much lower than in the models that do not use interest rate survey forecasts.

To assess whether interest rate surveys indeed help in predicting interest rates, in Figure 6 we report MSFEs for the macro-yields model in real-time and the only-yields model, both with and without surveys. Results show that in general surveys worsen the results almost on all occasions, and when they bring useful information the impact is marginal (as in the case of the 24-month forecasting horizon). This indicates that, overall, all the relevant information to predict interest rates in real-time can be extracted only from yields and macroeconomic variables. However, as stated above, we expect that there are circumstances in which soft information from surveys can bring additional value to the model, as in the case of forward guidance announcements.

To this aim, we test the relevance of the 3 and 4 quarters ahead survey forecasts for

Figure 6: Information from surveys



Mean Squared Forecast Error for the only-yields model without (OY) and with surveys (OY+S), of the macro-yields model without (MY) and with surveys (MY+S), and the random walk (RW). Evaluation period: 1995-2016.

the 3-month Treasury Bill rate from the Survey of Professional Forecasters in a period in which the Federal Reserve adopted an "Odyssean" forward guidance, i.e. since when in FOMC statements we can find an explicit reference to future dates.[9]. In Table 5, we report relative Mean Squared Forecast Error of the only-yields model with surveys relative to the only-yields model (top panel) and of the real-time macro-yields model with surveys relative to the real-time macro-yields (bottom panel), for the evaluation period going from August 2011 to June 2015. Results indicate that the use of Survey of Professional Forecasters, in a mixed-frequency model, improves the predictive power of both the only-yields model and the real-time macro-yields model. In particular, the improvement is statistically significant for long horizons and short maturities, which are the cases in which a forward guidance announcement is hoped to be effective. Note that the results show that adding the Surveys of Professional Forecasters is more beneficial to the only-yields model than to the macro-yields one, as surveys may carry some information about the state of the economy that is already embedded in "standard" real-time macroeconomic variables.

[9]On this topic, see Campbell, Evans, Fisher, Justiniano, Calomiris & Woodford (2012).

Table 5: The usefulness of SPF (RMSFE, August 2011 to June 2015)

(OnlyYields with SPF) vs OnlyYields

	3m	6m	1y	2y	3y	4y	5y	7y	10y
1	1.13	1.41	1.89	1.32	1.1	1.06	1.04	1.03	1.05
3	0.48*	0.72	0.87	1.04	0.98	0.99	1.00	0.98	1.05
6	0.11**	0.3**	0.52	0.73	0.92	1.04	1.08	1.03	1.15
12	0.05**	0.17**	0.45*	0.62	0.91	1.04	1.08	1.07	1.19
24	0.25**	0.19**	0.23**	0.51**	0.81	1.00	1.07	1.05	1.18

(RT MacroYields with SPF) vs RT MacroYields

	3m	6m	1y	2y	3y	4y	5y	7y	10y
1	1.23	2.41	2.13	1.23	1.06	1.02	1.01	1.00	1.00
3	0.74**	1.14	1.25	1.21	1.03	1.04	1.02	1.00	1.03
6	0.49**	0.61*	0.68	0.99	1.00	1.10	1.09	1.04	1.08
12	0.34**	0.33**	0.44*	0.65	0.75	0.83	0.86	0.88	0.94
24	0.87	0.87	0.87	0.91	0.93	0.95	0.96	0.94	0.98

Note: The table reports the relative Mean Squared Forecast Error of the only-yields model with SPF relative to the only-yields model (top panel) and of the macro-yields model with SPF relative to the macro-yields (bottom panel), for the evaluation period Aug2011-Jun2015. A number smaller than one indicates that the model with SPF performs better. (*) and (**) indicate one-side significance at 10% and 5%, respectively, Diebold and Mariano (1995) test statistic with fixed-b asymptotics, as in Coroneo and Iacone (2018).

7 Conclusions

In this paper, we assess the predictive ability of real-time macroeconomic information and interest rates surveys for the yield curve of interest rates. We propose a mixed-frequency dynamic factor model with restrictions on the factor loadings which includes Treasury yields, a set of real-time macroeconomic variables and interest rate survey expectations. Through the lens of a real-time out-of-sample exercise, we document the following findings.

First, we show the importance of macroeconomic information in predicting interest rates in a fully real-time out-of-sample exercise in which, in order to reconstruct the information set available to market participants at each point in time, we use the real-time vintages and the exact calendar of data releases.

Second, we document that survey expectations can play an important role in improving interest rate forecasts at long horizons for short maturities. An interpretation of this finding is that surveys incorporate soft information which might be neglected in "standard" data: for example, they can consider forward-looking information coming from policy announcements (e.g. forward guidance). In fact, we prove that properly adding surveys to our model in a forward guidance period significantly enhances its predictive power, especially for short maturities.

In future research, we plan to extend our empirical specification to explicitly incorporate long-run trends, to account for the recent decline in interest rates. The macro-yields model presented in this paper cannot identify trends as it is estimated on real-time macroeconomic variables transformed to achieve stationarity; however, our model can be easily extended to deal with trends along the lines of Del Negro, Giannone, Giannoni & Tambalotti (2017).

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A APPENDIX - Estimation procedure

A.1 State-space representation

The mixed-frequency macro-yields model with real-time macro information in equations (7)-(9) can be cast in a state-space form by augmenting the state variables to include the intercept and the idiosyncratic components. In particular, the measurement equation can be written as

$$\begin{pmatrix} y_t \\ x_t \\ E^s(y_t^q) \end{pmatrix} = \begin{bmatrix} \Gamma_{yy}^{NS} & 0 & 0 & 0 & I_n & 0 & 0 \\ \Gamma_{xy} & \Gamma_{yy} & 0 & a_x & 0 & I_m & 0 \\ 0 & 0 & \Gamma_q & a_s & 0 & 0 & I_s \end{bmatrix} \begin{pmatrix} F_t^y \\ F_t^x \\ F_t^q \\ c_t \\ v_t^y \\ v_t^x \\ v_t^s \end{pmatrix} + \begin{pmatrix} \eta_t^y \\ \eta_t^x \\ \eta_t^s \end{pmatrix} \tag{10}$$

where $(\eta_t^y, \eta_t^x, \eta_t^s)' \sim N(0, \epsilon I_{n+m+s})$ with ε a very small fixed coefficient. Γ_{yy}^{NS} is the matrix whose rows correspond to the smooth patterns proposed by Nelson & Siegel (1987) and shown in equation (2). Also notice that, since we are using real-time macro data, x_t contains missing values.

If we denote by $F_t = [F_t^y, F_t^x]$ and $v_t = [v_t^y, v_t^x]$, then we can write the state equation as

$$\begin{pmatrix} F_t \\ F_t^q \\ c_t \\ v_t \\ v_t^s \end{pmatrix} = \begin{bmatrix} A & 0 & \mu & 0 & 0 \\ w_t A & \iota_t I_r & w_t \mu & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & B & 0 \\ 0 & 0 & 0 & 0 & B_s \end{bmatrix} \begin{pmatrix} F_{t-1} \\ F_{t-1}^q \\ c_{t-1} \\ v_{t-1} \\ v_{t-1}^s \end{pmatrix} + \begin{pmatrix} u_t \\ u_t^s \\ \nu_t \\ \xi_t \\ \xi_t^s \end{pmatrix} \tag{11}$$

with $(u_t, u_t^s, \nu_t, \xi_t, \xi_t^s)' \sim N\left(0, blkdiag(Q, w_t' Q w_t, \epsilon, R, R_s)\right)$ and where the coefficients w_t and ι_t are known (w_t is equal to $1, 1/2, 1/3$ and ι_t is equal to $0, 1/2, 2/3$ respectively the first, second and third month of the quarter). In this state-space form, c_t an additional state variable restricted to one at every time t.

A.2 Estimation

The state-space model in (10)-(11) can be written compactly as

$$\begin{aligned} z_t &= \Gamma^* F_t^* + v_t^*, \quad v_t^* \sim N(0, R^*) \\ F_t^* &= A_t^* F_{t-1}^* + u_t^*, \quad u_t^* \sim N(0, Q_t^*) \end{aligned}$$

where $z_t = \begin{bmatrix} y_t \\ x_t \\ E^s(y_t^q) \end{bmatrix}$, $F_t^* = \begin{bmatrix} F_t \\ F_t^q \\ c_t \\ v_t \\ v_t^s \end{bmatrix}$, $v_t^* = \begin{bmatrix} \eta_t^y \\ \eta_t^x \\ \eta_t^s \end{bmatrix}$ and $u_t^* = \begin{bmatrix} u_t \\ u_t^s \\ \epsilon_t \\ \xi_t \\ \xi_t^s \end{bmatrix}$.

The restrictions on the factor loadings Γ^* and on the transition matrix A_t^* can be written as

$$H_1 \ \mathrm{vec}(\Gamma^*) = q_1, \qquad H_2 \ \mathrm{vec}(A_t^*) = q_{2t},$$

where H_1 and H_2 are selection matrices, and q_1 and q_{2t} contain the restrictions.

We assume that $F_1^* \sim N(\pi_1, V_1)$, and define $z = [z_1, \ldots, z_T]$ and $F^* = [F_1^*, \ldots, F_T^*]$. Then denoting the parameters by $\theta_t = \{\Gamma^*, A_t^*, Q_t^*, \pi_1, V_1\}$, we can write the joint loglikelihood of z_t and F_t, for $t = 1, \ldots, T$, as

$$\begin{aligned} L(z, F^*; \theta) = & -\sum_{t=1}^{T} \left(\frac{1}{2} \left[z_t - \Gamma^* F_t^* \right]' (R^*)^{-1} \left[z_t - \Gamma^* F_t^* \right] \right) + \\ & -\frac{T}{2} \log |R^*| - \sum_{t=2}^{T} \left(\frac{1}{2} [F_t^* - A_t^* F_{t-1}^*]' (Q_t^*)^{-1} [F_t^* - A_t^* F_{t-1}^*] \right) + \\ & -\frac{T-1}{2} \log |Q_t^*| + \frac{1}{2} [F_1^* - \pi_1]' V_1^{-1} [F_1^* - \pi_1] + \\ & -\frac{1}{2} \log |V_1| - \frac{T(p+k)}{2} \log 2\pi + \lambda_1' \left(H_1 \,\mathrm{vec}(\Gamma^*) - q_1 \right) + \lambda_2' \left(H_2 \,\mathrm{vec}(A_t^*) - q_2 \right) \end{aligned}$$

where λ_1 contains the lagrangian multipliers associate with the constraints on the factor loadings Γ^* and λ_2 contains the lagrangian multipliers associated with the constraints on the transition matrix A_t^*.

The computation of the Maximum Likelihood estimates is performed using the EM algorithm. Broadly speaking, the algorithm consists in a sequence of simple steps, each of which uses the time-varying parameter Kalman smoother to extract the common factors for a given set of parameters and closed form solutions to estimate the parameters given the factors. In practice, we use the restricted version of the EM algorithm, the Expectation Restricted Maximization, since we need to impose the smooth pattern on the factor loadings of the yields on the Nelson-Siegel factors. The ERM algorithm alternates Kalman filter extraction of the factors to the restricted maximization of the likelihood. At the j-th iteration the ERM algorithm performs two steps:

1. In the Expectation-step, we compute the expected log-likelihood conditional on the data and the estimates from the previous iteration, i.e.

$$\mathcal{L}(\theta) = E[L(z, F^*; \theta^{(j-1)})|z]$$

   which depends on three expectations

$$\begin{aligned} \hat{F}_t^* &\equiv E[F_t^*; \theta^{(j-1)}|z] \\ P_t &\equiv E[F_t^*(F_t^*)'; \theta^{(j-1)}|z] \\ P_{t,t-1} &\equiv E[F_t^*(F_{t-1}^*)'; \theta^{(j-1)}|z] \end{aligned}$$

   Given that our observables contain missing values, these expectations can be computed, for given parameters of the model, using the time-varying parameters Kalman smoother. This entails pre-multiplying the measurement equation by a selection matrix S_t of dimension $(n - \#missing) \times n$, as follows

$$S_t z_t = S_t \Gamma^* F_t^* + S_t v_t^*, \quad S_t v_t^* \sim N(0, S_t R^* S_t)$$

   and apply the Kalman filter to a time-varying measurement equation with parameters $S_t\Gamma^*$ and $S_t R^* S_t$, and observables $S_t z_t$.

2. In the Restricted Maximization-step, we update the parameters maximizing the ex-

pected the expected lagrangian with missing values with respect to θ:

$$\theta^{(j)} = \arg\max_{\theta} \mathcal{L}(\theta)$$

This can be implemented taking the corresponding partial derivative of the expected log likelihood, setting to zero, and solving. In particular, the measurement equation parameters are estimated by using a selection matrix W_t with diagonal element equal to 1 if non-missing, and 0 otherwise, so that only the available data are used in the calculations.

Following Coroneo et al. (2016), we initialize the yield curve factors with the Nelson-Siegel factors using the two-steps ordinary least squares (OLS) procedure introduced by Diebold & Li (2006). We then project the balanced panel of macroeconomic variables on the Nelson-Siegel factors and use the principal components of the residuals of this regression to initialize the unspanned macroeconomic factors. The quarterly factors are then computed by time aggregating the monthly yield curve and macro factors. All the parameters are initialized with the OLS estimates obtained using the initial guesses of yield and macro factors described above. The initial values for the factor loadings of surveys are obtained by projecting the linearly interpolated quarterly surveys on the quarterly factors observed at a monthly frequency.