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# The Rehabilitation of Glidepath Investing

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# The Rehabilitation of Glidepath Investing


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**Abstract**

In this paper we examine the proposition of de-risking through life and the guidance offered by TDFs in the decumulation phase following retirement. Using both Monte Carlo methods along with actual historical experience we investigate the withdrawal experience associated with Glidepath Investing in the US since 1925 for conventional bond-equity portfolios. We find two very powerful and practical conclusions. First, that smoothing the returns on individual assets by simple absolute momentum or trend following techniques is a potent tool to enhance withdrawal rates, often by as much as 50% per annum. And, perhaps of even greater social relevance is that it removes the 'left-tail' of unfortunate withdrawal rate experiences, that is, the bad luck of a poor sequence of returns early in decumulation. We show that diversifying assets over time by switching between a risky asset and cash in a systematic way is potentially more important for the retirement income experience than diversifying one's portfolio across a range of risky asset classes. In particular the willingness to tactically de-risk and re-risk allows the investor to stay exposed to equities in selective calendar fashion for far longer with reduced potential for painful drawdowns and raised potential for higher withdrawal rates. Second, and very importantly, we also show that Glidepath Investing is indeed a very sensible strategy within a few years of the target date. This finding provides succour to enthusiasts for target date investing in the face of the growing hostility in the literature


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## 1. Introduction

*"Buy-and-hold doesn't work anymore. The volatility is too significant. Almost any asset can suddenly become much more risky."* Andrew Lo (2012).

*"We have got a model for cross-section diversification. We have to balance that with time-series diversification because when time diversification fails, cross-section diversification fails too …. The greatest reward comes from time diversification."* Myron Scholes, in Cao, Larry, (2016), Managing Multi-Asset Strategies, CFA Institute.

With over 80 million US baby boomers filing for retirement benefits over the next 20 years, with an overwhelming majority relying on defined contributions savings' plans to accumulate and manage their wealth, the necessity for appropriate, personal investment strategies for these individuals is clearly paramount (Social Security Administration, Annual Performance Plan for Fiscal Year 2012). The move away from defined benefit (DB) towards defined contribution (DC) and personal savings for pensions is well underway for a wide variety of reasons that are comprehensively described in the OECD Pensions Outlook, (2016). This, of course, means that both investment and longevity risk rest with the individual. OECD data on assets and members in DB and DC plans from 2000 to 2015 confirm the increasing prominence of DC plans in many OECD countries. Indeed, most new schemes introduced in recent years have been DC schemes, though the exact arrangements differ between countries (OECD, 2016). Assets in occupational DC plans together with those in personal plans exceeded assets in DB plans in most reporting countries.

The issue of the appropriate investment strategy for both accumulation and decumulation has been a relatively neglected area of study was labelled as being a 'known unknown' by Merton (2014). 'Target Date Funds' or TDFs, have emerged as the means by which many individuals change the composition of their portfolio of assets over their remaining lifetime. TDFs are typically structured around a 'glidepath' under which the individual fund is predominantly invested in higher return, higher risk asset classes while the individual is young, where this exposure gradually reduces as the individual ages in favour of ostensibly lower risk assets, primarily government bonds. The economic motivation for this glidepath, or lifecycle approach to investment strategy is that younger investors are better able to withstand equity risk because a large proportion of their total wealth is in the form of low-risk human capital. This argument for the lifecycle pattern of equity holding was originally made by Merton (1971) developing the original Samuelson (1969) and Merton (1969) models which suggested that consumer/investors hold a constant share of the risky asset throughout their lifetime. The approach of Merton is developed in a more realistic form allowing for the uncertain nature of labour income and costs of participation in equity markets by Cocco et al (2005).

In this paper we examine the appropriateness of de-risking through one's lifetime. This lifecycle strategy is the default investment option in many employer-sponsored and individual retirement plans (see Charlson and Lutton, 2012). In Australia, lifecycle funds are increasing rapidly, and are expected to catch up or surpass the U.S. in the next decade (OECD, 2016). In the US, plan sponsors have flocked to the target-date fund (TDF) as a solution for participants to save for retirement. Regulatory direction has anointed the TDF as a Qualified Default Investment Alternative (QDIA), which has prompted substantial and ongoing inflows to these types of savings vehicles across the options in many employer-sponsored DC plans and individual retirement plans (see Charlson and Lutton, 2012). In addition, many commercial asset managers offer versions of TDFs/Glidepath Investing to retail clients (eg see the presentation of the Vanguard scheme in Donaldson et al, 2015).[1][2]

There is, however, a significant literature that presents an alternative argument to the one made by Vanguard and the literature above; namely that investment should become more aggressive over an individual's lifetime. After all, as Shiller (2005) observed, under some alternative assumptions, it is optimal for young people to invest little in equities and to increase that proportion over their lifetime. Blanchett (2007) compares fixed asset allocations to a wide range of investment paths that reduce the allocation to equity during retirement. He finds that fixed asset allocations provide superior results compared to asset allocations which tend to reduce equity investments in retirement. Arnott, Sherrerd, and Wu (2013) argue that a reverse approach to the target-date fund glidepath with an increasing share of equities delivers greater

[1] Callan Associates' DC Index supports the notion of both the increasing prevalence and utilization of TDFs. As of the third quarter of 2015, Callan estimated that 88% of U.S. DC plans tracked included TDFs, with an average allocation to TDFs of 30%. Also in the third quarter of 2015, an estimated 61% of new cash flows to DC plans were allocated to TDFs. Though TDFs have become the most popular DC investing vehicle, we believe that several shortcomings should be addressed to more reliably maximize retirement outcomes.

[2] An example of a widely held TDF is that offered by Vanguard group and presented in detail in Donaldson et al (2015). This shows a 50% investment of funds for retirement in equities upon retirement, reducing to 30% after a further 7 years. Both of these figures are significantly lower than the 90% investment in equities proposed for the first 15 years of working life followed by a glide down over the remaining 25 working years. In their case, Vanguard are following "the theoretical concept that equity allocations should decline with age to help manage risk through time", Donaldson et al (2015, p4), as per the theoretical models.

terminal wealth levels for investors. They claim that this approach yields higher wealth levels than the traditional lifecycle approach even at the left tail of the wealth distribution.

Using nearly a century of data on monthly US equity, corporate and government bond returns, we examine the consequences of alternative popular investment strategies for the decumulation of funds invested for retirement through a defined contribution pension scheme. The findings also apply just as much to endowments, charities, high net wealth individuals, or indeed in any situation where the investment fund is subject to regular withdrawals. We analyse sustainable withdrawals by focussing on the Perfect Withdrawal Rates (PWR), i.e. the maximum annual withdrawal rate possible if one had perfect foresight of returns, and ran one's wealth down to zero at the end of the period, (see Suarez et al, 2015, Clare et al, 2017).

We examine in detail the viability of specific 'safe' withdrawal rates including the '4%-rule' of Bengen (1994) and also Blanchett et al (2016) for international comparisons. We also emphasise the role of Sequence Risk in damaging the possible withdrawal rates and explore ways of enhancing the safe withdrawal experience (see Clare et al, 2017). Sequence Risk is the possibility of bad portfolio returns occurring at the *worst possible* time, e.g. just before or after retirement. Much of the retirement literature to-date, in exploring the relationship between sustainable withdrawal, longevity and portfolio investment returns, makes the simplifying assumption of constant year-by-year returns, for example, see Estrada (2017b, fn 9). This approach is clearly wrong, (and indeed crucially so!), because it omits the role of path dependency.

Basic insight would suggest that diversification across assets might be the route to reducing risk. However, as Lo (2012) suggests in the introductory quote above, diversification across asset classes is no longer enough - quite simply, the world is too volatile. This has been dramatically revealed with the abject failure of Target Date and similar Lifestyle investment strategies in the last decade, (see Dhillon et al, 2016). To our knowledge, Ezra at al. (2009) was the first to draw attention to the practical issues surrounding some important and large TDFs in the recent era. Long-only portfolios, whether passive or active, will inevitably suffer substantial drawdowns at regular (but unpredictable) intervals: even the famous and well diversified portfolios of the Harvard and Yale endowments suffered drawdowns of nearly 30% around 2008/9. The subsequent layoffs of staff and cancellation of capital projects demonstrated the very real impact of sequence risk for these institutions.

We investigate the withdrawal experience associated with Glidepath Investing in TDFs in the US since 1925 for conventional bond-equity portfolios. We find one very powerful conclusion: that smoothing the returns on individual assets by simple absolute momentum or trend following techniques is a potent tool for enhancing withdrawal rates, often by as much as 50% per annum. And, perhaps of even greater social relevance is that it removes the 'left-tail' of unfortunate withdrawal rate experiences, i.e. the bad luck of a poor sequence of returns early in decumulation. We show that diversifying assets over time by switching between an asset and cash in a systematic way is potentially more important for the retirement income experience than diversifying one's portfolio across asset classes. We also show that recent attacks on Glidepath Investing are misplaced and it is indeed a sensible investing strategy within a few years of the target date. This finding provides succour to enthusiasts for target date investing in the face of the growing hostility in the literature identified above.

Our findings here have direct relevance to some of the key issues facing the retirement industry at present:

i) Should you shift your portfolio composition towards bonds as the decumulation phase begins? Possibly applying Glidepath or Target date investing? We consider the distribution of withdrawal rates associated with different portfolio constructions and time horizons.

ii) Even if the answer to (i) above is in general 'no', is there a short planning horizon at which points you should commence 'gliding'? Even if it is only over the last couple of years? The answer to this is unequivocally 'yes'!

iii) And can we do better than using (long only) bonds and equities in the portfolio? Crucially, what is the benefit, if any, of switching to cash at times in an effort to reduce drawdowns and smooth returns? Will such a systematic smoothing rule such as trend following (which is specifically designed to reduce Sequence Risk) offer a superior decumulation journey? The answer is a most persuasive 'yes'.

The paper is set out as follows: in Section 2 we review some of the main issues that a retiree faces, including the choice of decumulation portfolios, the role and nature of Sequence Risk, and the impact of applying trend following techniques to conventional equity and bond portfolios; Section 3 provides empirical results for US retirement portfolios comprising equities and bonds since 1925; Section 4 examines whether savers should de-risk as they approach retirement, and indeed within retirement; while Section 5 presents our de-risking/glidepath empirical results; and Section 6 concludes with a consideration of how divesting (possibly temporarily) to cash compares with market timing, the addition of low volatility assets or derivatives' use in improving the drawdown experience.

## 2. Retirement Portfolios and Perfect Withdrawal Rates

As we suggested in the introduction, the construction of investment portfolios for both accumulation and decumulation phases has been relatively neglected in the study of retirement planning leading Indeed, the study of long-term accumulation and decumulation usually treats the two processes as completely separate phenomena. For the former, the emphasis is on changing the riskiness of portfolios as retirement beckons. This usually involves de-risking in the form of Glidepath or Target Date investing, by raising the proportion in bonds and reducing the percentage in equities, (e.g. see Blanchett et al, 2016, Estrada, 2017b). Ion the decumulation phase the big question is the percentage of wealth that can be safely withdrawn for consumption each year in a world with uncertain life expectancy and stochastic returns (see Bengen, 1994, Blanchett et al, 2016). Sometimes, of course, the glidepath glides through the retirement date and becomes the decumulation portfolio, though most discussions distinguish between the two for investing purposes. In this paper we analyse the decumulation phase following retirement, by measuring periodic, sustainable withdrawals, focussing on Perfect Withdrawal Rates (PWR), i.e. the maximum annual withdrawal rate possible if one had perfect foresight of returns (see Suarez et al, 2015, Clare et al, 2017), and ran one's wealth down to zero at the end of the period. We emphasise the role of Sequence Risk in damaging the possible withdrawal rates and explore ways of enhancing the safe withdrawal experience. Note that this approach is different from simply assessing investment strategies by looking at wealth accumulation since it explicitly considers withdrawals (eg. pensions) and hence the path dependency of returns.

In addition to the simulation exercise used by virtually all academic and commercial researchers which assume constant ('average') returns over a decumulation period, an alternative way to assess the sustainable spending possible from a retirement pot is to take a retirement period of, say, 30 years, along with a period of history, say the 20$^{th}$ century, and ask what constant real level could have been withdrawn each year without the retiree running out of money? This approach was pioneered by Bengen (1994) who found a lower bound of 4% of the initial pot (indexed to maintain real purchasing power) based on a typical US portfolio of 50% equity and 50% government bonds.

Using data on 21 countries over 115 years and with 11 different combinations of bonds and equities, Estrada (2017a) finds widely varying safe withdrawal rates over time and between countries: the 4% rule and a 60/40 equity/bond portfolio would have left substantial bequests much of the time in the US, whereas a 100% bond portfolio would have run out of money 65% of the time. If the percentage in stocks was above 50% then the failure rate was less than 10%. Note that this (and all similar exercises) focus on equities and bonds only, presumably as other asset class returns are not available back to 1900. However, such data restrictions do not rule out making simple switches between equities, bonds and cash if needed. So effectively we have a third potential asset class.

Blanchett et al (2016) provide a relatively comprehensive international overview of safe withdrawal rates for retirees based on both historical returns and forward-looking returns. Overall these findings suggest that financial advisers and retirees in the United Kingdom, for example, should use lower initial safe withdrawal rates of between 2.5% and 3.0%. The generous capital market returns of the prior century that bolstered a comfortable and long-lasting retirement portfolio may give 21st-century retirees a false sense of security.

As pointed out by Estrada (2017a), many of the empirical exercises in this area focus on varying investment returns and individual longevity but assume the returns are *constant* over the decumulation period: this, of course, assumes away sequence risk and all the associated real-world problems. Hence experimenting with variations on longevity and (average) returns to deduce sustainable withdrawal rates misses a key point: the *order* or *sequence* of returns is of equal importance, but because there is no useful measure of such a concept, it has been largely ignored.

What about diversifying across other asset classes? It might seem trivially obvious that diversification of a portfolio across asset classes – the fabled 'only free lunch in finance' – is the best way for investors to manage risk and enhance returns, and indeed the decumulation journey. Mohamed El-Erian remarked: 'Diversification alone is no longer sufficient to temper risk. You need something more to manage risk well' (Investment Week, May, 2019).

We begin our empirical analysis by reviewing conventional US retirement portfolios since 1925 using stocks and government bonds.

## 3. The Retirement Experience from Investing in US Equities and Government Bonds, 1927-2016.

### (i) Overview: Equity and Government Bond Portfolios

The standard approach to forming US retirement portfolios is to allocate wealth between US equities and government bonds only. Our aim here is to examine how this is affected by applying a trend following rule to this allocation which introduces the possibility of temporary shifts into cash.

It is now increasingly recognised that trend following (otherwise known as Absolute Momentum or Time-Series Momentum, see Moskowitz et al, 2012 and Hurst et al, 2017), has delivered 'strong positive returns and realized a low correlation to traditional asset class returns for more than a century', Hurst et al, (2017). The robustness of this performance over returns since 1800 has been recently demonstrated by Baltussen et al (2019). They also show that the statistical significance of this performance is robust to the development of large numbers of potential factors and to data snooping.

In its simplest form, Absolute Momentum may involve buying an asset whose price is above a moving average of past prices, often looking back in time say 10 months or a year, and divesting this part of a portfolio into cash or Treasury Bills if the asset price falls below the average. Thus, this involves divesting a proportion of the portfolio into cash though a more sophisticated version would short the asset rather than into cash itself.[3]

However, we must be careful to distinguish this from the better known traditional cross-sectional momentum factor which considers recent asset performance relative to other assets, (DeBondt and Thaler, 1986 and Jegadish and Titman, 1991). The recent results of Baltussen et al (2019) show that trend following subsumes momentum in spanning tests on global returns data since 1800. In addition to raising average returns above a long-only, holding strategy, an Absolute Momentum strategy also typically improves a portfolio's risk-adjusted returns (Hurst et al, 2017 and Faber, 2007).[4]

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[3] See for a recent example, Haghani and White, https://elmfunds.com/tell-them-about-mo/ Bitcoin and Natural Gas and can be considered a factor in some researchers' eyes, just like value and size.

[4] A range of recent research papers have highlighted the potential for Absolute Momentum as an investment strategy, e.g. Moskowitz, Ooi and Pedersen (2012). In their 2014 study Hurst, Ooi, and Pedersen extend this study to 67 markets involving commodities, equity indices, bond markets and currency pairs for the period 1880-2013. Again the authors find that time-series momentum offers a unique diversification opportunity. Empirical evidence

What underlying behaviour can explain the success of trend following? Hurst et al, (2017) suggest that 'a large body of research has shown that price trends exist in part due to long-standing behavioural biases exhibited by investors, such as anchoring and herding'. The intuition for the success of Absolute Momentum is that bear markets occur gradually over many months allowing this strategy to divest into a safer asset like cash, in time to benefit from the fall in prices. In practice they show that the strategy has a negative correlation with stocks in negative equity market years and associated periods of downside risk, and performs particularly well in strong up and down years, (also see Baltussen et al, 2019).

Table 1 shows the summary statistics for various combinations of US stock and bond portfolios which have been rebalanced at the end of each month (all values are real USD, stock return data from the Shiller database[5], bond return data from updates of Welch and Goyal (2008) in the Goyal database[6][7]). Panel A shows that, as one would expect, that as the stock component increases so does the return. The combination, however, with the best risk-return trade-off, based on the Sharpe ratio, is approximately 50% stocks and 50% bonds (50-50) which produces an annualised return of around 5.0% and volatility of 9.0%. This asset class combination also almost produces the smallest maximum real drawdown. It is worth pointing out that table 1 also shows that while bonds have less volatile returns, they have generated maximum drawdowns of a similar order of magnitude to equities. The maximum (real) drawdown for portfolios with 80%, 90% and 100% bonds are, respectively, 49%, 53% and 67%. This serves to undermine the core argument in favour of TDFs.

Panel B of Table 1 shows the performance of combinations of stocks and bonds which have each had an independent, 10-month trend following rule applied (see Clare et al (2016), and many others). For example, when the stock price level is above its own 10-month moving average it is deemed to be in an uptrend and thus a long position in stocks is adopted. If, however, the stock price level is below its moving average then the allocation to stocks is

supporting time-series momentum for various asset classes is also found in Hutchison and O'Brien (2014), Baltas and Kosowski (2013) and Baltussen et al (2019).

[5] Stock returns data: S&P 500 index returns from 1926 from Center for Research in Security Press (CRSP) month-end values. Stock returns are the continuously compounded returns on the S&P 500 index, including dividends. Data from 1871, from Robert Shiller's website, see http://www.econ.yale.edu/~shiller/data.htm.

[6] Long-term government bond yield data from 1919 to 1925 is the U.S. Yield on Long-Term United States Bonds series in the NBER's Macrohistory data base. Yields from 1926 are from Ibbotson's Stocks, Bonds, Bills and Inflation Yearbook, from the Goyal database, see http://www.hec.unil.ch/agoyal/docs/PredictorData2017.xlsx.

[7] In unreported work we have also considered replacing Treasury bonds with corporate bonds. The results are essentially unchanged. Results are available from the authors.

invested in T-Bills instead. For bonds we adopt the same rule. Applying this trend following to bonds has a relatively small impact on the usual performance measures; returns are very slightly lower than in Panel A of the table but so too is volatility. Stocks, however, have a return of 160bps higher but volatility falls from 15.6% to just under 10.4%. Maximum drawdown is considerably lower for both asset classes. It is reduced by more than half for the equity-only, and by a third for the bond-only portfolio. Such 'improvements' are consistent with the results for the S&P500 since 1872, (Clare et al, 2017). We can also see that the combination of stocks and bonds with the best risk-return trade-off is now approximately 70% of the former and 30% of the latter (70-30 TF). This strategic allocation produces an annualised return of 6.7% and volatility of 7.7%, compared to 5.8% and 11.3% respectively for the equivalent 70-30 portfolio presented in Panel A. But perhaps more interestingly, the trend following portfolio with the closest volatility to the standard 60-40 is the 100-0 portfolio with trend following that produces a return that is around 250bps higher than the 60-40 portfolio without trend following. Thus we find that adding a simple trend following filter to a 100% US equity portfolio produces a superior risk-return outcome than diversifying between conventional asset classes: it also hints at the major finding of this paper that 'time-diversification' (in the form of trend following) is *more* rewarding than diversifying across asset classes.

Trend following, or Absolute Momentum, thus allows investors to take on a ***greater*** weighting in stocks for the same risk or a reduced level of risk for the same asset allocation (a volatility of 7.06% versus 10.08% for the non-trend adjusted 60-40 portfolio). This is potentially a powerful practical result, that is, diversifying between equities and bonds would seem to offer an ***inferior*** investment solution than sticking solely with equities and moving in and out of the asset into cash when the trend signals so indicate. Panel B offers a better all-round investing experience by 'smoothing' both the equity and bond returns by simple trend following: the comparable maximum drawdowns are all much lower than those presented in Panel A.

Of course, as an aside, one may question whether more sophisticated trend following rules could improve performance further? Different versions of trend adjustment are tested and the results are published in detail in Clare et al (2012). Basically, the results are relatively insensitive to the specific method of adjustment, or indeed the specific choice of look-back period and we apply this finding across asset classes (also in Clare et al, 2016).

**(ii) Comparing Decumulation Strategies: Perfect Withdrawal Rates**

A key comparative metric in assessing the relative qualities of different strategies in the decumulation phase is the Perfect Withdrawal Rate (PWR) described in Section 2 above. Table 2 displays statistics for PWRs (see Suarez et al, 2015 and Clare et al, 2017 for more on this) for a variety of portfolios over a range of decumulation periods: 30, 20, 10, 5, 3 and 2 years. These periods are measured in numbers of years before the final point in time when the retirement pot is empty. The reason we choose very short periods as well as the more usual 20 or 30 years is so that we can assess the benefits of relatively short glidepath strategies immediately before the final point. Does there come a point when switching to a high percentage in bonds is a sensible strategy? The answer is indeed yes as we discuss below.

Table 2 shows values of PWRs from investment portfolios comprising only long only holdings of equities and bonds, while Table 3 shows values based on the asset classes having had trend-following applied to them, meaning that 'cash' is effectively introduced as a third asset class. Naturally as the time horizon of the investment pot decreases so the PWR rises although this is not a linear relationship with PWRs increasing rapidly as the number of years of decumulation remaining gets smaller. This suggests that there should still be a role for the introduction of a glidepath, probably once the planning horizon is reduced to 5 years or less.

Using the 30-year decumulation period (see the upper panel of Table 2) as our reference length (see also Bengen, 1994), we see that the median PWR for a 100% long only (i.e. no trend adjustment) equity portfolio is 7.5% p.a. for a US investor for the period 1927-2016, with maximum and minimum values of 12.8% and 3.7% respectively. The equivalent numbers if this 100% equity portfolio has been trend adjusted is 8.2%, 12.3% and 5.1 suggesting a far better left-tail experience of 5.1% v 3.7% for those unfortunate to begin decumulating at the 'wrong' time. The lesson here is that the accident of one's birth has a large effect on the retirement journey if wealth is held predominantly in equity.

But if 100% equity is rather extreme, what about introducing government bonds? Moving to the more familiar 60-40 savings portfolio (also in the upper panel of table 2), the median PWR is just under 6% pa with a minimum of 3.6% with no trend adjustment, but rises to 6.4% and 4.4% respectively with smoothing. Effectively the distribution of the PWRs has shifted to the right by applying a trend adjustment to the asset class holdings.

Our results suggest that the withdrawal experience in general is enhanced by a trend-following portfolio adjustment, both for the representative 'median' experience, and, importantly, the lower bound experiences. The only time when long only really 'beats' the smoothed experience is when it is 100% equity and we are comparing the maximum, far right tail (see Figure 2). Interestingly, the fabled '4%' Sustainable Withdrawal Rate over a 30-year decumulation period of Bengen (1994) and others is not supported by results in Table 2, with a *minimum* withdrawal rate of 3.57% for a representative portfolio; this compares to 4.43% for the smoothed version, comfortably above the 4% threshold.

The difference between the withdrawal experience for standard stock and bond investment is largest when the PWR length is greatest. For instance, the 30-year PWR mean is 7.5% for 100-0 and 4.1% for 0-100 compared to 51.6% and 50.6% respectively for the 2-year PWRs. Given that, in the case of the former, the investment pot size is likely to be close to its largest, this is a very significant difference. Furthermore, the minimum PWR for 0-100 is just 2.4% compared to 3.7% for 100-0 and thus a very conservative asset allocation would have done more harm than good historically over extended periods. The characterisation of the retirement journey is very much in line with Estrada (2017b), Blanchett et al (2016), in reinforcing the benefit of equity exposure in long expected decumulation periods.

As the decumulation length shortens so the possibility of portfolios with large stock allocations underperforming higher bond allocations increases. This is the case for glidepath strategies which we will explore in Sections 4 and 5.

**(iii) Absolute Momentum: the Desirability of a More Concentrated PWR Distribution**

Figure 1 shows how the 20-year PWR values vary over time for 100% stocks (100-0), 100% bonds (0-100) and the traditional equity/bond portfolio (60-40), both with and without the addition of the trend following filter (TF). For a long time, the PWR of stocks was much higher than bonds although this changed somewhat for decumulation periods starting from around 1980. The beginning of a substantial decline in interest rates generated much higher bond returns than previously and hence higher PWRs. Standard and trend following *bond-based* PWRs track each other fairly consistently over the entire period. There is much greater deviation between standard and trend following *stock-based* PWRs. The big differences occur when a period of large, negative stock returns take place in the early years of decumulation, e.g. see starting points in late 1920's, early 1970's and late 1990's. Periods of generally positive returns often see small outperformance for the standard approach, e.g. late 1980's.

Figure 2 shows the frequency distribution of 20,000 Monte Carlo simulated 20-year PWRs of the same investment strategies depicted in Figure 1. We observe that as the portfolio moves away from bonds and towards stocks, so the average PWR increases and the distribution becomes more spread out. It is evident that having a large weighting in bonds when the PWR horizon is long has been a poor strategy. There is a similar chance of achieving a low PWR outcome as most of the other portfolios but without the upside. As one transitions to higher stock weightings, so the risk of a low PWR outcomes is considerably reduced by following the trend following method. The trade-off is a small reduction in the possibility of the very highest PWRs compared to portfolios that are not subject to the trend following rule.

As a comparison, Figure 3 shows another 20,000 Monte Carlo simulations but this time of 3-year PWRs. Following on from our earlier discussion, the outperformance of higher stock weighted portfolios is still present but the difference is less marked now. We note the longer left-hand tail of the 60-40 and 100-0 standard portfolios. There is a clear area which shows an increased risk of PWRs less than 0-100 portfolios. ***It is avoiding these low PWR outcomes that is the premise for glidepath strategies.*** If one adopts trend following, however, the probability of observing the poor PWRs is substantially reduced such that there is little reason to transition to anything more conservative than a 60-40 TF portfolio. Trend Following (or similar smoothing of returns) therefore seems to be the key to reducing Sequence Risk (or see Ge, 2019) and enhancing the decumulation experience.

**4. Glidepath Investing and Perfect Withdrawal Rates**

Should portfolio composition ***change as we go through life?*** Target Date Funds (TDF) and mechanical "glidepath" approaches to de-risking have become popular, accepted practice, both for those approaching retirement and indeed during decumulation. But much of the time the diversification literature tends to ignore an issue which is critical to the discussion here, namely, that investors make periodic contributions to their retirement funds during their working years, or indeed periodic withdrawals during retirement. Hence, the capital accumulated at retirement is a function of both the asset allocation and the size and timing of the contributions. Shiller (2005) was the first to emphasize that investors following a lifecycle strategy choose to have a large exposure to stocks when they are young and their savings are low, and a small exposure to stocks when they are older and their savings are higher. He evaluated the wisdom of this approach through simulations, and found that lifecycle strategies are too conservative, usually underperforming portfolios fully invested in stocks, concluding that these strategies may not be optimal for investors saving for retirement.

Basu and Drew (2009) consider several lifecycle strategies and their mirrors, that is, strategies that remain invested in stocks, bonds, and cash for the same amount of time as lifecycle strategies, but where proportions evolve in the opposite direction, from less aggressive to more aggressive. They find that investors should become *more*, rather than less aggressive over time. The literature that evaluates the plausibility of lifecycle strategies taking into account the critical role that periodic contributions play on wealth accumulation, besides being scarce, is both recent and largely limited to US data. This is the case with the works of Shiller (2005), Basu and Drew (2009), Ayres and Nalebuff (2010), Basu, Byrne, and Drew (2011), and Arnott (2012).

So, should portfolio construction veer towards 'reduced risk' as we approach the point of maximum accumulation or even during decumulation itself?

While there is no consensus about the best investment strategy, the current debate questions the benefits of the popular target-date or lifecycle funds in the *accumulation* phase. In a similar vein, Blanchett (2007) compares fixed asset allocations to a wide range of investment paths that reduce the allocation to equity during retirement. He finds that fixed asset allocations provide superior results compared to asset allocations which tend to reduce equity investments in retirement. Arnott, Sherrerd, and Wu (2013) argue that a reverse approach to the target-date fund glidepath with an increasing share of equities delivers greater terminal wealth levels for investors. They yield higher wealth levels than the traditional lifecycle approach even at the left tail of the wealth distribution.

Estrada (2014) provides a comprehensive study of the lifecycle investor glidepath in 19 countries, finding that contrarian strategies provide higher upside potential, and more limited downside potential, although with higher uncertainty. Arnott (2012) also argues this case. In a provocative article, he argues that investors would be better off by following a strategy opposite to that implemented by target date funds. In fact, he argues that if investors focus on the capital accumulated at retirement, instead of making their portfolios more conservative, they should make their portfolios more aggressive as retirement approaches, along the lines of Shiller (2005). This counterintuitive recommendation follows from a rather obvious but often overlooked fact first highlighted by Shiller (2005): *TDFs expose investors to stocks more in the early years, when the accumulated capital is not large, and less in later years, when the accumulated capital is much larger*. Put differently, these funds are aggressive when the portfolio is small and conservative when the portfolio is large, which could be suboptimal in terms of capital accumulation.

Despite their growing popularity as long-term savings' vehicles, the idea that conventional target date funds are inherently dangerous has been addressed by Ezra et al (2009) and more recently by Capone and Akant (2016) who note that the great financial crisis wreaked havoc on 2010 target date funds with the 3 largest in the US losing around 30% each in 2008: this unfortunate turn of events reflects *sequence risk* as outlined above; namely poor, if not disastrous, returns in the investment portfolio occurring at exactly the worst time for the long-term saver, that is, just before retirement. Capone and Akant (2016) identify "too much equity" and not enough diversification across sources of risk premia as the fundamental problems here and suggest integrating their multi-asset trend following strategy with conventional target date fund strategies. Although trend following appears in the title of their paper, they are not advocating the trend following, 'smoothing' technique, but rather they add their multi-asset trend following investing solution ('fund') to existing target date fund solutions (i.e. an extra diversifying asset which happens to be a conventional trend following Commodity Trade Advisors (CTA) fund). While any diversification is to be welcomed, and meets their first objection to target date funds with limited asset diversification, it still does not address the underlying problem of sequence risk, the potentially large drawdowns at the 'wrong' time. They add up to 15% of portfolio value in the form of their CTA to conventional target date fund allocations, still leaving the bond allocation vulnerable to the next (unforeseeable) secular shift upwards in interest rates.

However, their acknowledgement that maximum loss is much reduced relative to conventional target date funds is a step in the right direction in addressing sequence risk. In anticipation of our proposal later in this paper and elsewhere, we would advocate subjecting all asset classes to smoothing by trend following (see Faber, 2007, Clare et al, 2016) rather than just adding a CTA component to an otherwise long only conventional target date fund allocation. Indeed, as also advocated by Capone and Akant (2016), an additional benefit to smoothing is the long-run extra performance (both risk-adjusted and absolute) due to trend following as evidenced across a wide range of asset classes by Faber (2007) and Clare et al (2016).

In practical terms, Bengen (1994) advises that if future market returns follow behaviour as seen in the past, then a retirement portfolio should hold a 50 to 75% allocation to equities. Milevsky and Huang (2011) and Ameriks, Veres, and Warshawsky (2001) demonstrate through simulation the need for holding a substantial equity allocation in a retirement portfolio. Cooley, Hubbard, and Walz (2011) propose that at least 50% of a retirement portfolio should be invested in equities and their findings show increased sustainability of the fund as it tilts more towards equities. They explain that the presence of bonds is mainly to restrain portfolio volatility and provide liquidity to cover an investor's living expenses.

The lifecycle approach is implemented with the aim of avoiding insufficient diversification as well as to avoid investment choices that may be age-inappropriate. While it undoubtedly achieves these aims, the relevance of having investments following a predetermined glidepath solely dependent on age appears to us simplistic. Important factors such as account balance, gender, marital status, retirement income expectations and life expectancy improvements influence the asset allocation decision. It is clear from this discussion that there is no consensus as yet on de-risking both towards and during retirement.

**5. Perfect Withdrawal Rates for Decumulation Periods of Different Lengths**

We examine the distribution of perfect withdrawal rates using the actual returns data for 1927-2016 from 100% equity,100% bond and 60-40 equity/bond portfolios in more detail in Tables 3 and 4. These involve a range of planning horizons to capture the idea of glidepath investing, from 30 years down to 2 years. Looking at the median results in Table 3, we observe that the 100% Stocks portfolio delivers a higher PWR in every case than the 60-40 portfolio, which in turn delivers a higher value than the 100% Bond portfolio. When one looks at the minimum value columns a different picture emerges, however. The 60-40 has a higher PWR than 100% Stocks in decumulation periods of 23 years and this persists as shorter decumulation periods are examined. We also observe that for periods shorter than 12 years, the 100% Bond portfolio generally has a higher PWR than 60-40.

Table 4 shows the same set of results but this time with trend following applied to each of the asset classes. Compared to the standard results, we note that the average maximum PWR for

100% Stocks TF is around 40 bps lower and fairly similar for 60-40 TF and 100% Bonds TF. The average median value is around 25 bps higher for 100% Stocks TF, however, and this is replicated for the other two portfolios. We observe the most significant differences in the minimum value columns. The 30 Year 100% Stocks PWR is 5.13% compared to just 3.69% for the standard equivalent. Substantial differences are observed across decumulation periods of all lengths. There is much less variation between the standard and trend following in the case of 100% Bonds and thus the 60-40 TF reflects the combination of these two. When we compared the minimum values within Table 4, the 60-40 TF value first surpasses 100% Stocks for the 9 year period, some 14 years shorter than in Table 5, and remains greater in nearly all of the remaining shorter periods. The minimum value for 100% Bonds TF never exceeds that of 60-40 TF and indeed only exceeds 100% Stocks TF for one shorter period. What this means from an investment context is that investors wishing to follow a glidepath strategy can stay in stocks for longer by adopting a trend following approach and hence have the possibility to earn higher returns while also reducing the risk of the very worst outcomes.

**(a) The Performance of Portfolios with the Highest Minimum PWR given a Fixed Decumulation Period.**

Next, rather than sticking with 100% Bonds or Equities, or indeed the 60-40 portfolio, what if we seek the optimal portfolio as we move through time for each remaining period of years, searching combinations from 100% equities through 100% bonds, in increments of 1%, and choosing that portfolio which, with the *benefit of hindsight*, the highest minimum PWR for each number of decumulation years remaining. In other words, focus on the portfolio shares that provide the least-worst possible experience. This is the meaning of ‘optimal’ in our context.

Figure 4 shows the asset allocation to *equities* that would have produced this result based on the data and results in Tables 3 and 4, although the PWRs are not apparent from Figure 4.This looks very much like the glidepath recommended asset allocations for commercial savings’ products and is, in effect, the ideal asset allocation for the most conservative investor, since it would have given the *least* bad outcome in every case. These equity percentage allocations for the long only equity and bond portfolios are given by the blue dots. The first point of note is that investing heavily in bonds when there was a long decumulation period remaining would have been a long way from optimal for both the smoothed (brown dot) and unsmoothed (blue dot). The most conservative standard, unsmoothed portfolio has around 85% in stocks with 30 years of decumulation remaining falling to around 70% when there were 20 years left (brown dots). After this, the switch from stocks to bonds becomes more dramatic and also we note that the transition becomes a lot less smooth in the final 10 years.

In the case of trend following, smoothed, portfolios we find that the optimal portfolio for the worst-case scenario remains at 100% in TF equities all the way down to 10 years apart from 13 years, which sees a small fall in the equity share. Given the higher returns to stocks this looks very favourable. Again, from 10 years the transition becomes rather choppy but a glidepath to somewhere around 65-70% TF Stocks looks a reasonable conclusion. This is much higher than the standard approach where one would have been almost exclusively in bonds.

**(b) Smoother Gliding**

In an attempt to provide glidepaths which are somewhat smoother, and also for investors of varying risk tolerances, we turn to Monte Carlo simulations. We create 101 portfolios, moving from stocks to bonds in 1% increments, and run 100,000 simulations for each calculating the PWR in each case. For each asset allocation we calculate the following PWR percentiles: 1$^{st}$, 5$^{th}$, 10$^{th}$, 25$^{th}$ and 50$^{th}$. We then look to see which allocation gives the highest value for each percentile for each length of decumulation period and this becomes the basis for the glidepath. The 1$^{st}$ percentile is thus in many ways similar to a 1% VaR calculation in that it calculates the best asset allocation that gives a 1% chance of getting a lower PWR. Figure 5 shows the approximate (line of best fit) glidepaths for standard stock-bond allocations expressed as the percentage of the portfolio invested in stocks (left hand axis) for the years left for decumulation ranging from 30 down to 2 years reading right to left.

For example, the stock-bond combination which has the highest 1$^{st}$ percentile value is 35% stocks - 65% bonds. This is a point on the lowest blue line at 30 years on the x-axis and 35% on the y-axis. The 30-yr PWR associated with this asset allocation is 3.01%, which is shown in column 1 of Table 5. Any other combination of stocks and bonds have a greater than 1% chance of delivering a PWR less than 3.01. Less risk-averse investors would look to move up the percentiles and adopt a different glidepath.

We observe from, for example, for the 30-year PWRs in Figure 5 and Table 5, the stock proportion is much lower than that in Figure 4, which is derived from actual historic data. This is due to the returns of bonds being much higher during the last 30 years of the return sequence than at the beginning. As a result, these observations appear disproportionately near the end of a lot of the PWR calculations in Figure 4 when a large amount of decumulation had already occurred and thus had little impact. In the case of the Monte Carlo simulations, shown in Figure 5 and Table 5, these observations are now randomly dispersed and hence have a greater influence on the outcomes, resulting in a lower stock proportion. That aside, the pattern is one of a relatively linear transition from stocks to bonds from 30 to 20 years before the pace starts to accelerate, particularly so in the last 10 years.

We observe that as risk aversion falls (and comfort with a higher failure percentages rises) so the percentage allocation to stocks increases. The 1st percentile starts at around 35% stocks and ends with near 15% compared to over 80% at the beginning for the 25th percentile and declining to just under 40%. The 50th percentile remains at 100% stocks for virtually the entire range of decumulation periods.

Figure 6 displays the same set of glidepaths but this time for stocks and bonds with trend following applied. Consistent with our earlier findings, the proportion of the portfolio held in stocks is much higher. The 1st percentile glidepath starts at over 80% TF equities and finishes around 40%; this is higher than the standard portfolio from the 30-year period downwards. When the line ends at a 2 year decumulation period, the stock allocation is still around 40%.

In Figure 5 the stock allocation (with no TF) for the 1st percentile starts at around 35%, i.e. the TF version of 1st percentile has a higher stock allocation at close to the end of decumulation than the no TF version has for the 30 years decumulation period. All of the other percentiles commence at 100% (TF) Stocks and transition down in a similar pattern with the switch to bonds accelerating for the shortest periods. In the case of the 25th percentile glidepath this only begins at around a 7-year period. Once again, the 50th percentile is resolutely 100% in equity.

Table 5 displays the effects of the different glidepaths in terms of the PWRs garnered with optimal asset allocations. Optimal is defined here as that asset allocation which gives the highest PWR for a given number of years remaining and a given failure rate, that is, a VaR-type concept. In the case of the 1st percentile the PWR using trend following is 4.17% compared to just 3.01% for the standard approach. This effectively equates to a 38% higher retirement income in the case of the former. Substantial differences remain across all percentiles and decumulation periods until the latter gets short and the level of risk tolerance increases. Trend following increases the proportion of the portfolio that can be invested in equities and thus earn higher returns.

## 6. Summary

The application of an absolute momentum or trend following overlay on the assets in a drawdown retirement portfolio reduces maximum drawdowns across asset classes, countries and historical periods (see Clare et al, 2017 and 2019, Faber 2007) reducing sequence risk and potentially improving the retirement journey. But why not use derivatives to achieve similar ends? But as AQR emphasise: "Unfortunately, in the typical case, put options are quite ineffective at reducing drawdowns versus the simple alternative of statically reducing exposure to the underlying asset." (Israelov, 2017, p.1).

Basically, in the case of equities, unless the options' purchases and their maturities are timed precisely around equity drawdowns of uncertain length, then they may result in little downside protection, and even make things worse by increasing rather than decreasing drawdowns and volatility per unit of expected return.

Is buying protective puts combined with the S&P500 equity portfolio an effective tail hedge? Israelov (2017) shows that portfolios that are protected with put options have worse peak-to-trough drawdown characteristics per unit of expected return than portfolios that have instead simply reduced their equity exposure in order to reduce risk.

So changing portfolio allocation between the risky asset and cash (so-called 'divestment') will give a better result than buying put protection, unless the drawdown coincides with the option expiry cycle. In fact the paper suggests that investing 40% in equity and 60% in cash has given similar returns as the protected puts strategy but with under half the volatility and a much improved peak-to-trough drawdown experience.

As Israelov (2017) states: "*For those who are concerned about their equity's downside risk, reducing their equity position is significantly more effective than buying protection. Sized to achieve the same average return, divesting has lower drawdowns, lower volatility, lower equity beta, and a higher Sharpe ratio than does buying put options*."

This approach also echoes the views of Ilmanen (2016). He refers to index put buying as protection for equity portfolios as (looking at historical data) "roughly a minus one Sharpe strategy". Of course, very fast bear markets and crashes can be protected against by using puts, but this is so expensive relative to the slow crash alternative (moving into cash) which is so successful in the long run: "Trend following has a clear positive Sharpe ratio, and it has done well in most of the historical bear markets over the past hundred years". Direct hedging is costly and only delivers value when combined with the ability to predict short-term market crashes and unwind the positions quickly after a crash.

How about more sophisticated timing mechanisms for buying protection? Strub (2013) introduces an algorithm for tail risk hedging and compares it with using Extreme Value Theory (EVT) to estimate Conditional Value at Risk (CVaR) and applies it to the S&P 500 and MSCI Emerging Markets equity indexes between 2000 and 2012. Performance is compared to cash and options-based tail hedging strategies. The cash-based methods are shown to significantly increase risk-adjusted returns and reduce drawdowns, while the options-based strategy suffers a decrease in performance from 2003 onwards due to the increase in the cost of puts with respect to calls.

It would seem then that divesting (albeit temporarily) offers a much better solution to reducing drawdowns (i.e. managing tail risk) than either buying puts systematically or trying to time their purchases using conditioning information as in Strub (2013), though of course there are an infinity of such alternatives. Unless one knows when a 'fast crash' is about to occur and can time the option purchase cycle to good effect, then switching to cash via a trend following rule appears to be the best solution.

However, there is an industry-wide aversion by advisers to recommending holding large proportions in cash even when the evidence is stacked up in its favour: they feel that they cannot charge fees for advocating cash holdings, whereas in fact this form of timely diversification may well be the only sensible game in town.

Our results have shown the impact on retirees of smoothing investment returns through trend adjustment (i.e. divesting at times into cash) over a long period of US equity and bond returns. We find that the withdrawal experience from conventional equity/bond portfolios can be substantially enhanced by applying these smoothing techniques, effectively creating a much higher lower bound for withdrawal rates in the 20th century (4.4% pa versus 3.6% p.a. for the popular 60-40 allocation with withdrawals lasting 30 years), and also improving withdrawal rates for the 'average' experience (6% v 6.4%, see Table 2). In all cases strategies targeting a set volatility (as is popular with retirement consultants) allows far higher equity proportions and consequent withdrawal rates for a given volatility level (or 'bucket').

The failure of Target Date Funds and Glidepath Investing, with the associated disastrous experience for such strategies targeting 2010 among the largest commercial providers (see Dhillon et al, 2016 and Ezra, 2009) prompted research on alternative approaches including maintaining constant proportions of equities and bonds, or indeed reversing the glide proportions, have been explored by, for example, Estrada (2014). Our findings for varying decumulation period lengths suggests that unless one is very risk averse it would only appear sensible to apply conventional glidepath strategies *towards the last few years* of decumulation, and then only move towards a 50-50 equity-bond allocation.

Ge (2019) suggests adding low-volatility assets to the portfolio of equities and bonds to reduce Sequence Risk, while Israelov (2017) emphasises the possible advantages of devoting a percentage of the portfolio to trend following assets which are robust to large equity drawdowns and are largely uncorrelated with conventional assets. They have in mind a CTA 'hedge' fund, for example, and possibly allocating to it up to 15% of wealth. This certainly improves the experience in reducing drawdowns (and hence Sequence Risk) but begs the question as to why stop at 15% - why not consider the advantages of trend adjustment for the whole portfolio?

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**Table 1**

**Summary Statistics for Stock and Bond Portfolios 1927-2016**

This table presents summary statistical properties for stock and government bond portfolios constructed from: Stock returns data: S&P 500 end of month index returns from Jan 1926 to Dec 2016 from Center for Research in Security Press (CRSP) month-end values. Stock returns are the continuously compounded returns on the S&P 500 index, including dividends. Data is available from 1871, from Robert Shiller's website, see http://www.econ.yale.edu/~shiller/data.htm. Long-term government bond yield data from 1919 to 1925 is the U.S. Yield on Long-Term United States Bonds series in the NBER's Macrohistory data base. Yields from 1926 are from Ibbotson's *Stocks, Bonds, Bills and Inflation Yearbook*, from the Goyal database, see http://www.hec.unil.ch/agoyal/docs/PredictorData2017.xlsx .

| *Stocks (%)* | 100 | 90 | 80 | 70 | 60 | 50 | 40 | 30 | 20 | 10 | 0 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| *Bonds (%)* | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |
| *A: Standard* | | | | | | | | | | | |
| Real Annual Return (%) | 6.81 | 6.52 | 6.20 | 5.84 | 5.46 | 5.04 | 4.59 | 4.10 | 3.59 | 3.05 | 2.48 |
| Real Annualised Volatility (%) | 15.66 | 14.14 | 12.67 | 11.31 | 10.08 | 9.03 | 8.25 | 7.81 | 7.76 | 8.11 | 8.83 |
| Sharpe Ratio | 0.40 | 0.42 | 0.45 | 0.47 | 0.49 | 0.50 | 0.49 | 0.46 | 0.40 | 0.31 | 0.22 |
| Maximum Real Drawdown (%) | 76.80 | 71.57 | 65.34 | 57.95 | 49.21 | 40.49 | 42.65 | 45.30 | 49.10 | 52.87 | 67.24 |
| *B: Trend Following* | | | | | | | | | | | |
| Real Annual Return (%) | 8.39 | 7.85 | 7.31 | 6.74 | 6.17 | 5.58 | 4.98 | 4.36 | 3.73 | 3.09 | 2.44 |
| Real Annualised Volatility (%) | 10.44 | 9.46 | 8.55 | 7.74 | 7.06 | 6.54 | 6.24 | 6.18 | 6.38 | 6.80 | 7.40 |
| Sharpe Ratio | 0.75 | 0.78 | 0.79 | 0.80 | 0.80 | 0.77 | 0.71 | 0.62 | 0.50 | 0.38 | 0.26 |
| Maximum Real Drawdown (%) | 34.88 | 32.02 | 29.51 | 27.57 | 27.33 | 27.34 | 27.54 | 28.98 | 32.12 | 35.18 | 44.34 |

**Table 2**

**Perfect Withdrawal Rates for Stock and Bond Combinations: 1927-2016**

This table shows Perfect Withdrawal Rates (PWRs) for a range of portfolios of stocks and government bonds for both unadjusted, standard returns and where the returns have been generated by applying a trend following rule to both standard stock and bond returns, as described in the text. Each panel shows the summary statistical properties of the PWRs for decumulation periods of lengths which vary from 30 to 2 years in length.

| | Standard | | | | | | Trend Following | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Stock(%) | 100 | 80 | 60 | 40 | 20 | 0 | 100 | 80 | 60 | 40 | 20 | 0 |
| Bond(%) | 0 | 20 | 40 | 60 | 80 | 100 | 0 | 20 | 40 | 60 | 80 | 100 |
| *30-year PWRs* | | | | | | | | | | | | |
| Mean | 7.51 | 6.86 | 6.18 | 5.48 | 4.77 | 4.09 | 8.19 | 7.41 | 6.63 | 5.87 | 5.14 | 4.45 |
| Median | 7.50 | 6.66 | 5.97 | 5.23 | 4.05 | 3.17 | 7.88 | 7.09 | 6.37 | 5.24 | 4.58 | 3.83 |
| Max | 12.81 | 10.96 | 10.85 | 10.64 | 10.33 | 9.92 | 12.32 | 11.09 | 10.88 | 10.61 | 10.27 | 9.87 |
| Min | 3.69 | 3.71 | 3.57 | 3.38 | 3.14 | 2.42 | 5.13 | 4.88 | 4.43 | 3.91 | 3.32 | 2.57 |
| StDev | 2.55 | 2.06 | 1.77 | 1.70 | 1.75 | 1.83 | 2.01 | 1.71 | 1.57 | 1.57 | 1.66 | 1.78 |
| *20-year PWRs* | | | | | | | | | | | | |
| Mean | 9.01 | 8.47 | 7.90 | 7.30 | 6.68 | 6.04 | 9.55 | 8.90 | 8.24 | 7.59 | 6.94 | 6.30 |
| Median | 8.78 | 8.68 | 7.86 | 7.08 | 6.08 | 5.10 | 9.75 | 8.87 | 8.01 | 7.12 | 6.40 | 5.59 |
| Max | 15.06 | 12.68 | 12.48 | 12.31 | 12.05 | 11.70 | 14.27 | 12.81 | 12.47 | 12.30 | 12.08 | 11.81 |
| Min | 4.28 | 4.42 | 4.39 | 4.22 | 4.01 | 3.75 | 5.81 | 5.59 | 5.33 | 5.01 | 4.55 | 3.79 |
| StDev | 2.84 | 2.36 | 2.05 | 1.94 | 1.97 | 2.06 | 2.20 | 1.93 | 1.80 | 1.79 | 1.87 | 1.99 |
| *10-year PWRs* | | | | | | | | | | | | |
| Mean | 13.13 | 12.81 | 12.45 | 12.04 | 11.60 | 11.12 | 13.85 | 13.34 | 12.82 | 12.27 | 11.72 | 11.16 |
| Median | 13.60 | 12.94 | 12.34 | 11.86 | 11.57 | 10.76 | 13.46 | 13.15 | 12.60 | 11.99 | 11.25 | 10.46 |
| Max | 20.21 | 18.06 | 17.23 | 16.98 | 17.00 | 16.95 | 20.27 | 18.32 | 17.25 | 17.11 | 16.91 | 16.68 |
| Min | 7.00 | 7.27 | 7.50 | 7.68 | 7.81 | 7.83 | 9.15 | 9.03 | 8.89 | 8.73 | 8.40 | 7.82 |
| StDev | 3.31 | 2.73 | 2.30 | 2.06 | 2.03 | 2.19 | 2.57 | 2.29 | 2.10 | 2.01 | 2.03 | 2.11 |
| *5-year PWRs* | | | | | | | | | | | | |
| Mean | 22.66 | 22.41 | 22.12 | 21.79 | 21.42 | 21.01 | 23.28 | 22.86 | 22.43 | 21.97 | 21.49 | 20.99 |
| Median | 22.86 | 22.62 | 22.31 | 21.92 | 21.54 | 20.96 | 22.84 | 22.71 | 22.30 | 21.82 | 21.36 | 20.74 |
| Max | 31.56 | 29.92 | 28.82 | 27.68 | 26.65 | 26.79 | 31.76 | 29.42 | 28.30 | 27.34 | 27.05 | 26.72 |
| Min | 13.53 | 14.85 | 15.72 | 16.57 | 16.63 | 15.54 | 16.27 | 16.90 | 16.85 | 16.72 | 16.58 | 16.36 |
| StDev | 4.19 | 3.43 | 2.78 | 2.34 | 2.18 | 2.34 | 3.04 | 2.64 | 2.32 | 2.12 | 2.08 | 2.17 |
| *3-year PWRs* | | | | | | | | | | | | |
| Mean | 35.58 | 35.38 | 35.13 | 34.85 | 34.53 | 34.18 | 36.06 | 35.72 | 35.36 | 34.98 | 34.58 | 34.16 |
| Median | 36.02 | 35.90 | 35.50 | 35.01 | 34.57 | 34.22 | 35.79 | 35.54 | 35.31 | 34.94 | 34.59 | 34.01 |
| Max | 46.10 | 43.88 | 41.64 | 41.14 | 41.11 | 41.05 | 46.10 | 43.88 | 41.63 | 41.14 | 41.11 | 41.05 |
| Min | 23.28 | 24.73 | 26.18 | 27.62 | 28.37 | 28.54 | 29.24 | 29.10 | 28.96 | 28.80 | 28.64 | 28.47 |
| StDev | 4.87 | 3.98 | 3.20 | 2.63 | 2.40 | 2.57 | 3.34 | 2.84 | 2.44 | 2.19 | 2.14 | 2.29 |
| *2-year PWRs* | | | | | | | | | | | | |
| Mean | 51.64 | 51.50 | 51.32 | 51.12 | 50.89 | 50.62 | 52.00 | 51.76 | 51.49 | 51.21 | 50.92 | 50.60 |
| Median | 52.70 | 52.12 | 51.74 | 51.21 | 50.91 | 50.52 | 51.98 | 51.88 | 51.35 | 50.92 | 50.65 | 50.31 |
| Max | 60.51 | 58.64 | 56.97 | 56.73 | 56.72 | 57.48 | 59.87 | 58.33 | 56.76 | 56.54 | 56.94 | 57.48 |
| Min | 37.78 | 41.44 | 43.16 | 44.86 | 45.42 | 45.31 | 45.87 | 46.33 | 46.43 | 46.12 | 45.80 | 44.08 |
| StDev | 4.75 | 3.87 | 3.10 | 2.55 | 2.37 | 2.61 | 3.21 | 2.71 | 2.31 | 2.07 | 2.06 | 2.27 |

**Table 3**

**Summary Statistics for Stock, Bond & 60-40 Perfect Withdrawal Rates**

This table shows Perfect Withdrawal Rates (PWRs) for three different portfolios of stocks and government bonds for unadjusted, standard returns for 1926 – 2016. Each panel summarises the distributions of the PWRs for decumulation periods of lengths which vary from 30 to 2 years in length.

| | 100% Stocks | | | 60-40 | | | 100% Bonds | | |
|---|---|---|---|---|---|---|---|---|---|
| **PWR Years** | Max | Median | Min | Max | Median | Min | Max | Median | Min |
| **30** | 12.81 | 7.50 | 3.69 | 10.85 | 5.97 | 3.57 | 9.92 | 3.17 | 2.42 |
| **29** | 13.03 | 7.61 | 3.72 | 10.97 | 6.07 | 3.62 | 10.05 | 3.34 | 2.53 |
| **28** | 13.23 | 7.67 | 3.75 | 11.12 | 6.15 | 3.66 | 10.20 | 3.48 | 2.63 |
| **27** | 13.46 | 7.76 | 3.79 | 11.28 | 6.24 | 3.72 | 10.32 | 3.62 | 2.74 |
| **26** | 13.78 | 7.88 | 3.84 | 11.41 | 6.36 | 3.78 | 10.48 | 3.79 | 2.85 |
| **25** | 14.00 | 8.21 | 3.89 | 11.55 | 6.61 | 3.86 | 10.66 | 3.96 | 2.97 |
| **24** | 14.17 | 8.36 | 3.95 | 11.71 | 6.85 | 3.93 | 10.83 | 4.14 | 3.10 |
| **23** | 14.38 | 8.60 | 4.02 | 11.88 | 6.99 | 4.03 | 11.02 | 4.30 | 3.23 |
| **22** | 14.62 | 9.03 | 4.10 | 12.06 | 7.14 | 4.13 | 11.23 | 4.58 | 3.38 |
| **21** | 14.85 | 8.88 | 4.18 | 12.28 | 7.52 | 4.24 | 11.45 | 4.83 | 3.56 |
| **20** | 15.06 | 8.78 | 4.28 | 12.48 | 7.86 | 4.39 | 11.70 | 5.10 | 3.75 |
| **19** | 15.29 | 8.97 | 4.41 | 12.68 | 7.99 | 4.58 | 11.98 | 5.33 | 3.91 |
| **18** | 15.57 | 9.26 | 4.53 | 12.88 | 8.20 | 4.80 | 12.32 | 5.64 | 4.13 |
| **17** | 15.82 | 9.59 | 4.70 | 13.10 | 8.41 | 4.99 | 12.64 | 6.08 | 4.40 |
| **16** | 16.12 | 9.87 | 4.92 | 13.38 | 8.73 | 5.25 | 13.01 | 6.44 | 4.70 |
| **15** | 16.47 | 10.30 | 5.17 | 13.73 | 9.14 | 5.49 | 13.46 | 6.85 | 5.05 |
| **14** | 16.97 | 10.84 | 5.44 | 14.14 | 9.64 | 5.70 | 13.93 | 7.41 | 5.47 |
| **13** | 17.68 | 11.44 | 5.69 | 14.73 | 10.19 | 5.98 | 14.57 | 7.98 | 5.94 |
| **12** | 18.36 | 12.14 | 6.05 | 15.33 | 10.78 | 6.39 | 15.21 | 8.67 | 6.46 |
| **11** | 19.18 | 12.86 | 6.45 | 16.05 | 11.51 | 6.87 | 16.01 | 9.54 | 7.11 |
| **10** | 20.21 | 13.60 | 7.00 | 17.23 | 12.34 | 7.50 | 16.95 | 10.76 | 7.83 |
| **9** | 21.61 | 14.44 | 7.76 | 18.72 | 13.26 | 8.45 | 18.18 | 12.12 | 8.59 |
| **8** | 23.10 | 15.61 | 8.60 | 20.72 | 14.72 | 9.55 | 19.60 | 13.51 | 9.55 |
| **7** | 25.57 | 17.33 | 9.83 | 22.98 | 16.63 | 11.04 | 21.51 | 15.34 | 10.90 |
| **6** | 28.15 | 19.65 | 11.47 | 25.55 | 18.77 | 13.02 | 23.95 | 17.64 | 12.63 |
| **5** | 31.56 | 22.86 | 13.53 | 28.82 | 22.31 | 15.72 | 26.79 | 20.96 | 15.54 |
| **4** | 37.71 | 27.98 | 16.43 | 33.32 | 27.45 | 19.31 | 31.98 | 25.93 | 20.29 |
| **3** | 46.10 | 36.02 | 23.28 | 41.64 | 35.50 | 26.18 | 41.05 | 34.22 | 28.54 |
| **2** | 60.51 | 52.70 | 37.78 | 56.97 | 51.74 | 43.16 | 57.48 | 50.52 | 45.31 |

**Table 4**

**Summary Statistics for Stock, Bond & 60-40 with Trend Following Perfect Withdrawal Rates**

This table shows Perfect Withdrawal Rates (PWRs) for three different portfolios of stocks and government bonds where the returns have been generated by applying a trend following rule to both standard stock and bond returns, as described in the text. Each panel shows the summary statistical properties of the PWRs for decumulation periods of lengths which vary from 30 to 2 years in length

| | 100% Stocks TF | | | 60-40 TF | | | 100% Bonds TF | | |
|---|---|---|---|---|---|---|---|---|---|
| **PWR Years** | Max | Median | Min | Max | Median | Min | Max | Median | Min |
| **30** | 12.32 | 7.88 | 5.13 | 10.88 | 6.37 | 4.43 | 9.87 | 3.83 | 2.57 |
| **29** | 12.46 | 7.97 | 5.19 | 11.00 | 6.48 | 4.49 | 10.06 | 3.94 | 2.66 |
| **28** | 12.59 | 8.06 | 5.25 | 11.13 | 6.59 | 4.54 | 10.25 | 4.07 | 2.75 |
| **27** | 12.75 | 8.09 | 5.31 | 11.27 | 6.71 | 4.61 | 10.41 | 4.23 | 2.84 |
| **26** | 12.92 | 8.20 | 5.39 | 11.41 | 6.82 | 4.69 | 10.59 | 4.42 | 2.94 |
| **25** | 13.10 | 8.41 | 5.46 | 11.57 | 6.98 | 4.76 | 10.78 | 4.57 | 3.05 |
| **24** | 13.25 | 8.60 | 5.52 | 11.73 | 7.10 | 4.86 | 10.97 | 4.70 | 3.17 |
| **23** | 13.43 | 8.89 | 5.58 | 11.89 | 7.26 | 4.96 | 11.17 | 4.94 | 3.30 |
| **22** | 13.63 | 9.14 | 5.64 | 12.07 | 7.50 | 5.07 | 11.38 | 5.16 | 3.44 |
| **21** | 13.91 | 9.51 | 5.73 | 12.26 | 7.77 | 5.19 | 11.58 | 5.38 | 3.60 |
| **20** | 14.27 | 9.75 | 5.81 | 12.47 | 8.01 | 5.33 | 11.81 | 5.59 | 3.79 |
| **19** | 14.59 | 9.85 | 5.90 | 12.69 | 8.17 | 5.47 | 12.05 | 5.87 | 4.01 |
| **18** | 15.00 | 9.96 | 6.02 | 12.91 | 8.29 | 5.64 | 12.34 | 6.17 | 4.25 |
| **17** | 15.46 | 10.30 | 6.22 | 13.17 | 8.65 | 5.89 | 12.63 | 6.48 | 4.53 |
| **16** | 15.98 | 10.62 | 6.42 | 13.46 | 8.96 | 6.17 | 12.98 | 6.84 | 4.84 |
| **15** | 16.49 | 10.86 | 6.66 | 13.91 | 9.34 | 6.49 | 13.38 | 7.25 | 5.15 |
| **14** | 17.05 | 11.29 | 6.98 | 14.49 | 9.78 | 6.85 | 13.80 | 7.73 | 5.54 |
| **13** | 17.74 | 11.65 | 7.40 | 15.11 | 10.33 | 7.20 | 14.37 | 8.17 | 5.98 |
| **12** | 18.43 | 12.08 | 7.93 | 15.72 | 10.98 | 7.66 | 14.99 | 8.82 | 6.50 |
| **11** | 19.24 | 12.68 | 8.47 | 16.45 | 11.75 | 8.21 | 15.77 | 9.53 | 7.11 |
| **10** | 20.27 | 13.46 | 9.15 | 17.25 | 12.60 | 8.89 | 16.68 | 10.46 | 7.82 |
| **9** | 21.66 | 14.52 | 9.91 | 18.50 | 13.59 | 9.94 | 17.88 | 11.60 | 8.67 |
| **8** | 23.19 | 15.92 | 10.98 | 20.20 | 14.82 | 11.25 | 19.25 | 13.03 | 9.83 |
| **7** | 25.53 | 17.41 | 12.11 | 22.28 | 16.66 | 12.53 | 21.09 | 14.84 | 11.40 |
| **6** | 28.19 | 19.67 | 13.79 | 24.89 | 19.24 | 14.29 | 23.44 | 17.35 | 13.22 |
| **5** | 31.76 | 22.84 | 16.27 | 28.30 | 22.30 | 16.85 | 26.72 | 20.74 | 16.36 |
| **4** | 37.67 | 27.79 | 20.80 | 33.06 | 27.18 | 21.00 | 32.20 | 25.72 | 20.60 |
| **3** | 46.10 | 35.79 | 29.24 | 41.63 | 35.31 | 28.96 | 41.05 | 34.01 | 28.47 |
| **2** | 59.87 | 51.98 | 45.87 | 56.76 | 51.35 | 46.43 | 57.48 | 50.31 | 44.08 |

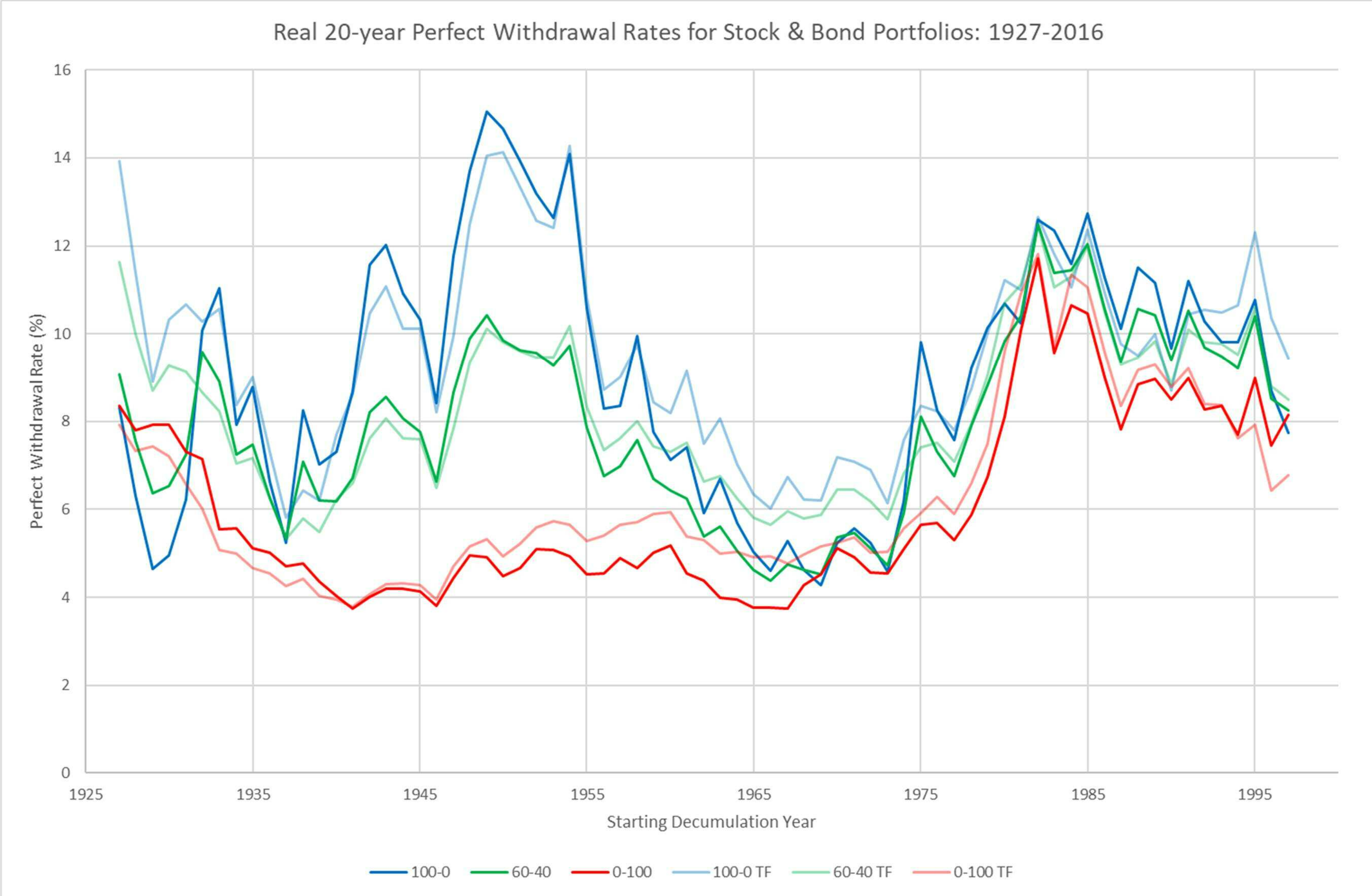
**Table 5**

**Perfect Withdrawal Rates Based on Optimal Asset Allocation using Percentiles from Monte Carlo Simulations With and Without Trend Following**

This table summarises the properties of 101 portfolios for decumulation periods of lengths between 30 and 2 years, moving from stocks to bonds in 1% increments. Where 100,000 simulations are run for each portfolio calculating the PWR in each case. For each asset allocation we summarise the distributions of the PWRs by giving their values at four percentiles. In the text we describe that we then look to see which allocation gives the highest value for each percentile for each length of decumulation period and this becomes the basis for the glidepath and show these in Figure 5.

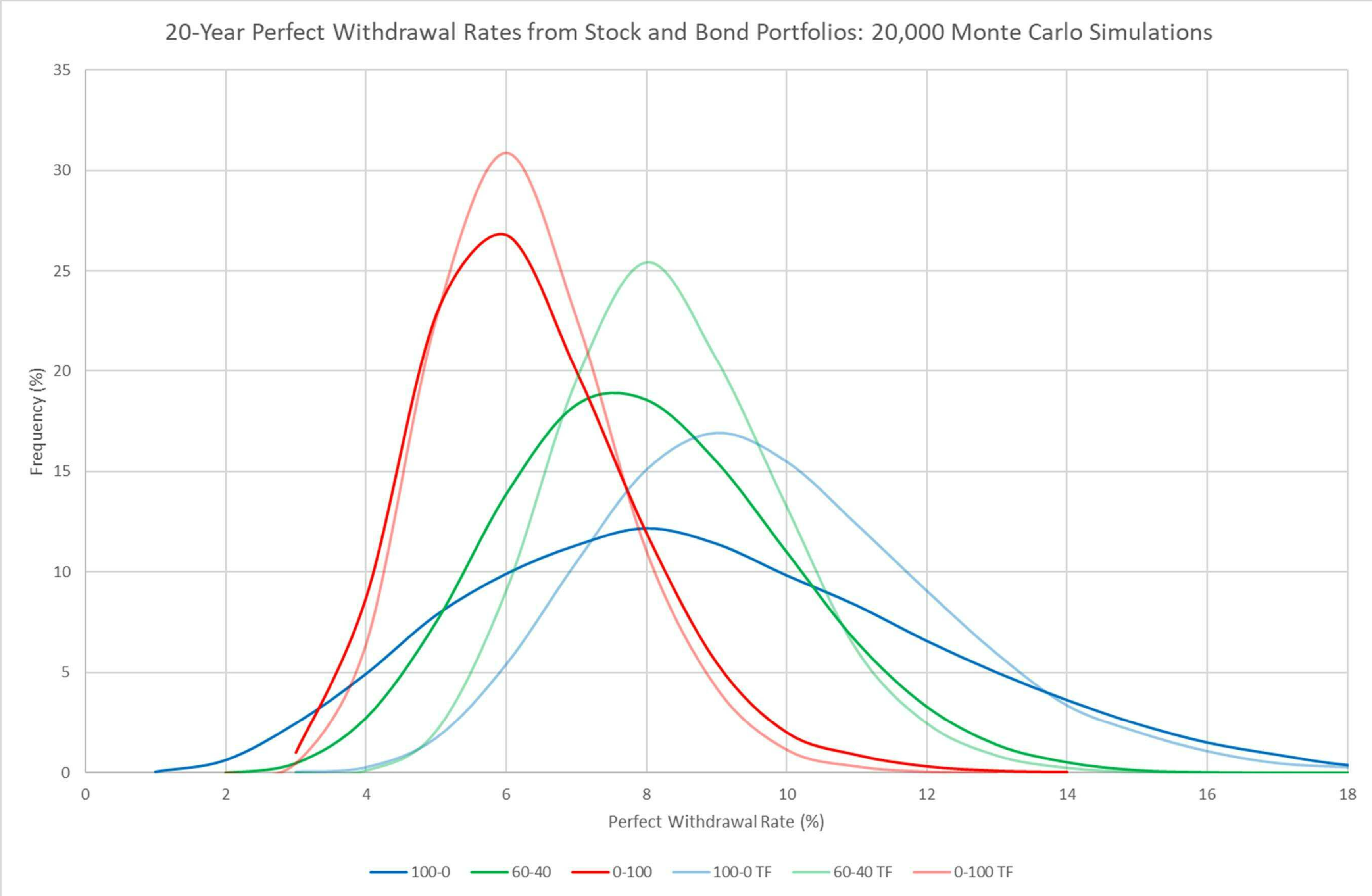
| | Standard | | | | | Trend Following | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 1st | 5th | 10th | 25th | 50th | 1st | 5th | 10th | 25th | 50th |
| **30** | 3.01 | 3.72 | 4.18 | 5.25 | 7.20 | 4.17 | 5.08 | 5.68 | 6.83 | 8.31 |
| **29** | 3.08 | 3.80 | 4.28 | 5.34 | 7.29 | 4.22 | 5.16 | 5.76 | 6.89 | 8.37 |
| **28** | 3.15 | 3.90 | 4.36 | 5.45 | 7.39 | 4.29 | 5.22 | 5.83 | 6.97 | 8.44 |
| **27** | 3.26 | 3.99 | 4.46 | 5.53 | 7.51 | 4.38 | 5.32 | 5.91 | 7.04 | 8.53 |
| **26** | 3.36 | 4.09 | 4.57 | 5.64 | 7.60 | 4.46 | 5.41 | 6.01 | 7.16 | 8.64 |
| **25** | 3.46 | 4.20 | 4.68 | 5.75 | 7.73 | 4.56 | 5.51 | 6.11 | 7.25 | 8.74 |
| **24** | 3.59 | 4.33 | 4.82 | 5.89 | 7.85 | 4.67 | 5.59 | 6.21 | 7.37 | 8.85 |
| **23** | 3.72 | 4.46 | 4.95 | 6.04 | 8.02 | 4.82 | 5.73 | 6.33 | 7.49 | 9.00 |
| **22** | 3.86 | 4.64 | 5.12 | 6.19 | 8.20 | 4.94 | 5.86 | 6.47 | 7.64 | 9.14 |
| **21** | 4.03 | 4.79 | 5.28 | 6.38 | 8.38 | 5.08 | 6.01 | 6.62 | 7.81 | 9.30 |
| **20** | 4.20 | 4.99 | 5.48 | 6.57 | 8.59 | 5.27 | 6.18 | 6.80 | 7.98 | 9.49 |
| **19** | 4.39 | 5.19 | 5.69 | 6.80 | 8.82 | 5.45 | 6.36 | 6.98 | 8.17 | 9.71 |
| **18** | 4.61 | 5.42 | 5.94 | 7.04 | 9.09 | 5.64 | 6.60 | 7.22 | 8.40 | 9.94 |
| **17** | 4.86 | 5.70 | 6.20 | 7.32 | 9.38 | 5.91 | 6.83 | 7.45 | 8.66 | 10.23 |
| **16** | 5.16 | 6.00 | 6.52 | 7.63 | 9.71 | 6.18 | 7.11 | 7.73 | 8.96 | 10.53 |
| **15** | 5.48 | 6.33 | 6.86 | 7.99 | 10.09 | 6.52 | 7.44 | 8.06 | 9.30 | 10.87 |
| **14** | 5.85 | 6.72 | 7.26 | 8.40 | 10.54 | 6.87 | 7.81 | 8.44 | 9.68 | 11.27 |
| **13** | 6.30 | 7.20 | 7.75 | 8.89 | 11.07 | 7.32 | 8.26 | 8.89 | 10.15 | 11.77 |
| **12** | 6.82 | 7.73 | 8.29 | 9.47 | 11.67 | 7.85 | 8.79 | 9.41 | 10.68 | 12.32 |
| **11** | 7.45 | 8.39 | 8.96 | 10.16 | 12.39 | 8.45 | 9.41 | 10.05 | 11.34 | 13.00 |
| **10** | 8.19 | 9.17 | 9.77 | 10.98 | 13.26 | 9.22 | 10.19 | 10.82 | 12.14 | 13.84 |
| **9** | 9.16 | 10.15 | 10.77 | 12.00 | 14.34 | 10.17 | 11.15 | 11.78 | 13.12 | 14.84 |
| **8** | 10.34 | 11.38 | 12.00 | 13.29 | 15.66 | 11.36 | 12.37 | 13.00 | 14.34 | 16.14 |
| **7** | 11.88 | 12.98 | 13.65 | 14.95 | 17.42 | 12.92 | 13.97 | 14.61 | 15.97 | 17.82 |
| **6** | 13.99 | 15.14 | 15.82 | 17.20 | 19.74 | 15.03 | 16.12 | 16.76 | 18.12 | 20.04 |
| **5** | 16.98 | 18.20 | 18.92 | 20.36 | 23.04 | 18.02 | 19.17 | 19.85 | 21.19 | 23.19 |
| **4** | 21.63 | 22.86 | 23.63 | 25.16 | 27.94 | 22.63 | 23.83 | 24.54 | 25.85 | 27.92 |
| **3** | 29.64 | 30.77 | 31.62 | 33.23 | 36.05 | 30.50 | 31.70 | 32.49 | 33.81 | 35.90 |
| **2** | 45.82 | 47.34 | 48.00 | 49.74 | 52.63 | 46.57 | 47.81 | 48.98 | 50.17 | 51.96 |

**Figure 1.**



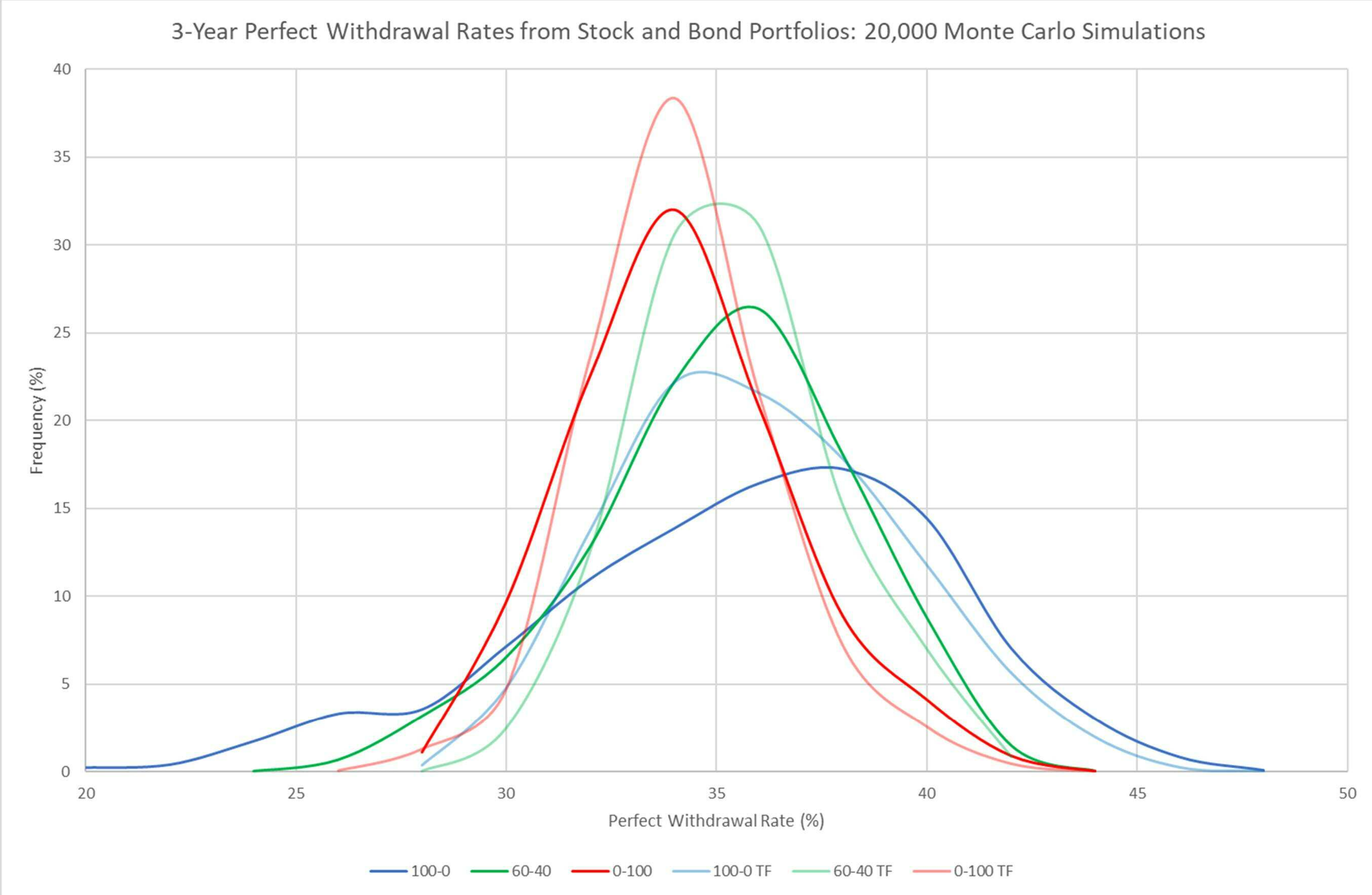
Real 20-year Perfect Withdrawal Rates for Stock & Bond Portfolios: 1927-2016
Perfect Withdrawal Rate (%)
16
14
12
10
8
6
4
2
0
1925
1935
1945
1955
1965
1975
1985
1995
Starting Decumulation Year
100-0
60-40
0-100
100-0 TF
60-40 TF
0-100 TF

**Figure 2.**



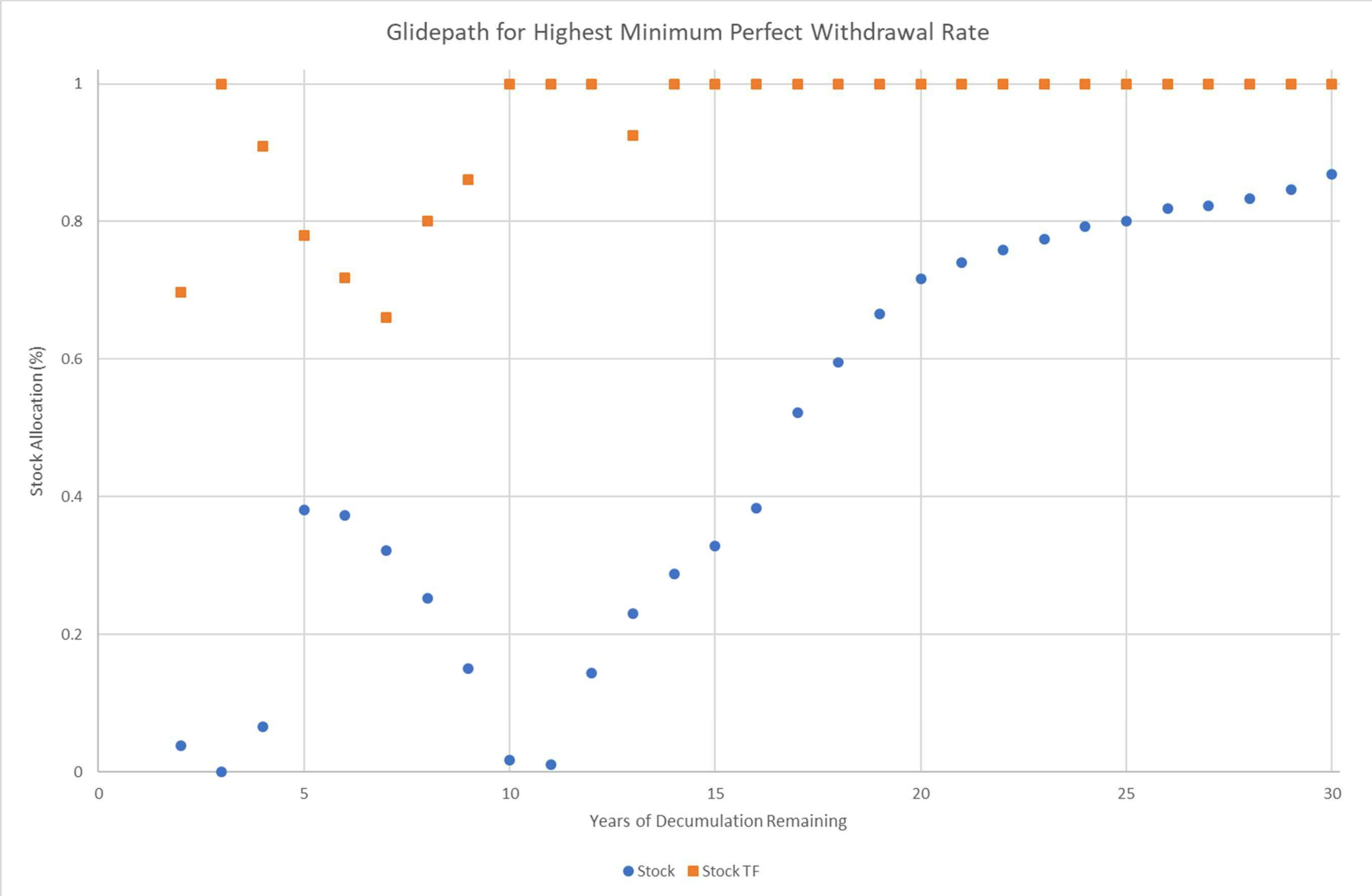
20-Year Perfect Withdrawal Rates from Stock and Bond Portfolios: 20,000 Monte Carlo Simulations
35
30
25
20
15
10
5
0
Frequency (%)
0
2
4
6
8
10
12
14
16
18
Perfect Withdrawal Rate (%)
100-0
60-40
0-100
100-0 TF
60-40 TF
0-100 TF

**Figure 3.**



3-Year Perfect Withdrawal Rates from Stock and Bond Portfolios: 20,000 Monte Carlo Simulations
40
35
30
25
20
15
10
5
0
Frequency (%)
20
25
30
35
40
45
50
Perfect Withdrawal Rate (%)
100-0
60-40
0-100
100-0 TF
60-40 TF
0-100 TF

**Figure 4.**



Glidepath for Highest Minimum Perfect Withdrawal Rate
Stock Allocation (%)
1
0.8
0.6
0.4
0.2
0
0
5
10
15
20
25
30
Years of Decumulation Remaining
Stock
Stock TF

**Figure 5.**

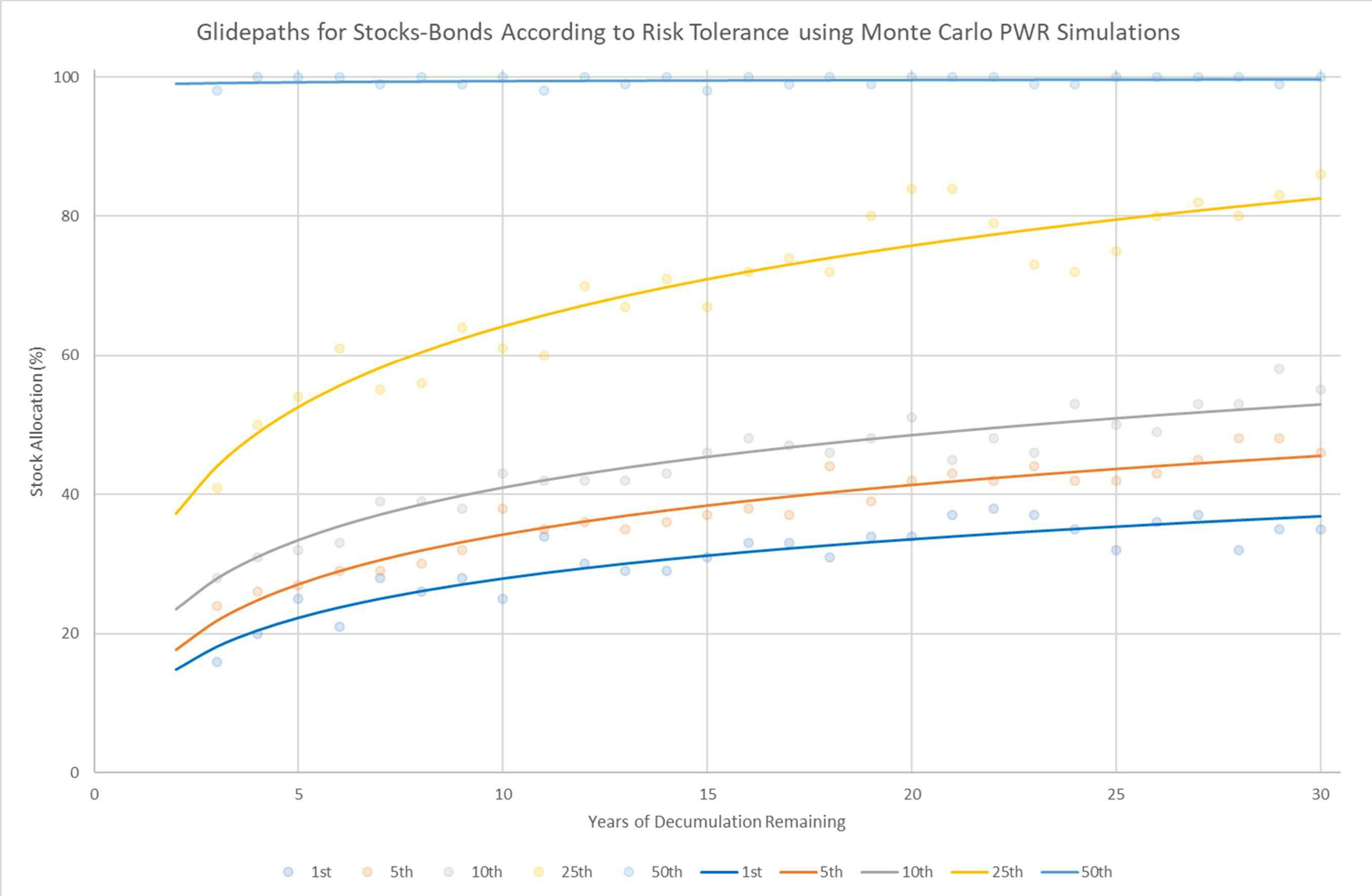
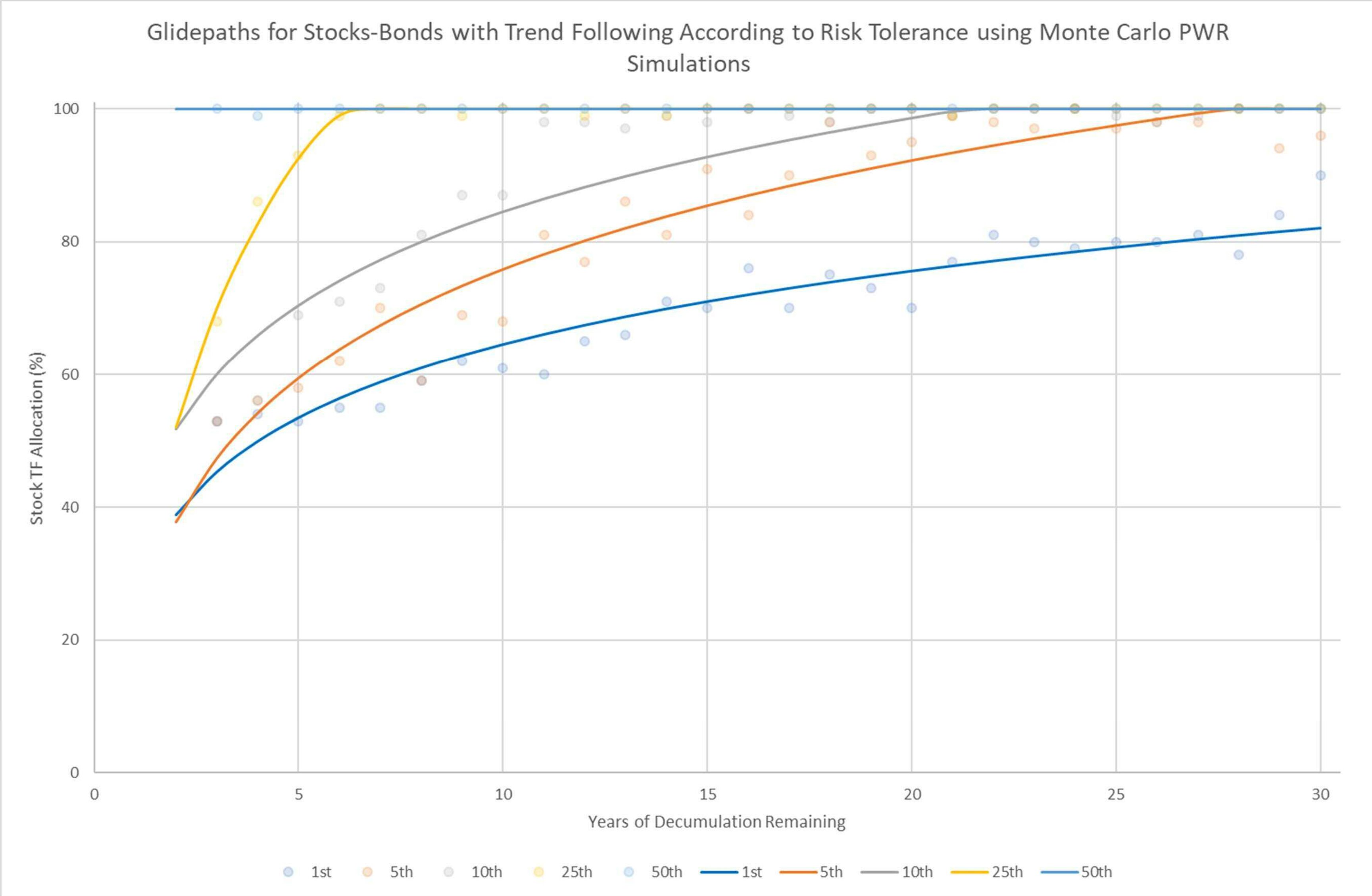

Glidepaths for Stocks-Bonds According to Risk Tolerance using Monte Carlo PWR Simulations
100
80
60
40
20
0
Stock Allocation (%)
0
5
10
15
20
25
30
Years of Decumulation Remaining
1st
5th
10th
25th
50th
1st
5th
10th
25th
50th

**Figure 6.**



Glidepaths for Stocks-Bonds with Trend Following According to Risk Tolerance using Monte Carlo PWR Simulations
Stock TF Allocation (%)
100
80
60
40
20
0
0
5
10
15
20
25
30
Years of Decumulation Remaining
1st
5th
10th
25th
50th
1st
5th
10th
25th
50th