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Fiscal Deficits as a Source of Boom and Bust under a Common Currency

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#### Abstract

We investigate in depth, using predominantly analytical, rather than numerical, methods, the mechanisms which operate following a one-period, debt-financed, fiscal deficit in a small open economy operating under a common currency. The economy incorporates the New Keynesian features of staggered price setting and overlapping generations. Unsurprisingly, these features cause the impact effect to be a boom, in the sense of a positive output gap. However we also find that the boom must later turn into a bust, or negative output gap. Moreover inflation also follows a boom-bust pattern and on average there is deflation rather than inflation.

#### **JEL Classification**

E62, F45, H62

#### Keywords

staggered prices, overlapping generations, small open economy, currency union, fiscal deficits, government debt, output gap

#### 1. Introduction

Although the fiscal stimulus policies associated with the 2008 financial crisis, and the subsequent fiscal retrenchment policies – particularly conspicuous in the Eurozone – have prompted macroeconomic researchers to renew their interest in fiscal policy, most such work has focused heavily on the effects of government spending, and little on the effects of government deficits and debt per se. To put a spotlight on the latter, one needs a theoretical setting in which Ricardian Equivalence is absent. Settings with this feature include models with a fraction of 'hand-to-mouth' households, intertemporal tax-smoothing models and overlapping-generations models. In this paper, we adopt the last approach, for reasons we discuss below. To provide a role for aggregate demand, we appeal to a New Keynesian framework with staggered price setting of a familiar kind.

We are particularly interested in the case where the country is part of a common currency area. By joining a currency union a country has given up its monetary policy as an independent instrument, so that its fiscal policy, and thus its fiscal deficit, becomes potentially the main implement in its macroeconomic policy toolkit. It is hence vital to have a deep understanding of the effects of fiscal deficits on the business cycle. The need for such an understanding can also be appreciated in another way. In a currency union the supra-national authorities typically feel obliged to impose strict deficit limits on member states for the sake of fiscal discipline. To know how costly this is for a country's ability to continue to manage its own economy, it is again essential to have a full picture of the channels through which fiscal deficits affect macroeconomic variables.

Much recent work on fiscal policy has been inspired by the limitations on monetary policy faced by countries experiencing 'liquidity traps'. This is not our motivation here. Nevertheless, the greater insight into the effects of deficits 'in normal times', which we attempt to provide, is a necessary foundation for assessing their usefulness in these more extreme situations.We utilise a sparsely-featured model which will enable us to derive our main conclusions algebraically rather than numerically, since we believe that this more deeply reveals the mechanisms at work. However the model still needs to be sufficiently rich to include the desired features both of staggered price setting and of overlapping generations, and this inevitably introduces a substantial amount of dynamic structure. Within this structure, we will focus on a very basic fiscal experiment, namely a one-period deficit brought about by a tax cut. Such a change corresponds to a once-and-for-all increase in the stock of government debt. The simplicity of this experiment avoids burdening the analysis with additional sources of dynamics which would confuse an already complex picture. Our paper, then, consists of a thorough, and primarily analytical, exploration of the consequences of a simple one-period deficit for all the major macroeconomic variables. In particular we concentrate on characterising and understanding the shapes of their time paths.

Our main findings are that, while in this setting a fiscal deficit does, as might be expected, cause a short-run boom in the form of a positive 'output gap', such a boom has unorthodox features. First, the boom is always followed by a 'bust', or negative output gap. In other words, the business cycle response of the economy is non-monotonic: after impact, instead of the boom simply decaying to zero, it goes into reverse. The output gap disturbance does eventually fade to zero, but from below, not above. Therefore, although a fiscal deficit has a short-run benefit because it generates a cyclical upturn, it also has a medium-run cost through inducing a subsequent downturn. A second finding is that inflation also exhibits a boom-bust cycle, with the deflation phase starting while the output gap boom is still in progress. Moreover, on average the fiscal deficit shock causes deflation, and not inflation. This runs contrary to the common idea that an aggregate demand stimulus must raise prices. We demonstrate that the boom-bust cycle and the deflation are robust phenomena, occurring for all parameter values. We show how they arise from fundamental features of the macroeconomic structure.

From a technical perspective, a feature of the model is that, in its basic linearised form, it is a fourth-order dynamical system. The perfect-foresight solution is characterised by not one, but two, stable eigenvalues. This means that the path has the potential to be non-monotonic, even when it does not exhibit repeated cycles. The boom-bust cycle in the output gap is an instance of precisely such a non-monotonic time path. A fourth-order system is potentially too complicated a mathematical construct to characterise analytically. In the

present case, what makes this feasible is the fact that the system is separable. It turns out that we can solve for the trajectories of two of the four state variables independently of the rest. By exploiting this separability we are able to derive tractable expressions for all variables of interest.

The practical importance of the boom-bust cycle depends on how large is the bust relative to the boom. The goal of our paper is not to conduct a serious quantitative investigation. However, we have conducted some basic numerical experiments. Not all parameter values generate a significant bust, but some do; and these include values in an empirically relevant range.

Only a relatively small amount of literature has so far been devoted to studying the role of government deficits and debt within a New Keynesian dynamic general equilibrium framework. This is even more true as regards open economies. The most commonly used approach in recent work has been that of assuming that a fraction of households have no access to asset markets ('hand-to-mouth' households). In a closed economy, contributions in this line include, for example, Galí et al. (2007) and McManus (2015). In an open economy, Corsetti et al. (2013) consider such households, as do Farhi and Werning (2016). A second approach is to assume that lump-sum taxation is unavailable, so that government debt enables intertemporal shifting of the distorting effects of marginal taxation. For an open economy, this approach is exemplified by Ferrero (2009). A third approach, which is the one we follow, is to appeal to overlapping generations (OLGs) of households. Leith and Wren-Lewis (2000) first used Blanchard's (1985) 'uncertain lifetimes' OLG structure in a modern New Keynesian framework for a closed economy, while Ganelli (2005) applied it to an open economy. The OLG approach, we believe, allows a more fundamental allocative role for government debt, since debt there helps to match agents' different lifetime profiles of income and desired consumption. The present paper is most similar in its set-up to Leith and Wren-Lewis's open-economy work (2006, 2008), except that our application is not to stability and international coordination issues but to a careful dissection of fiscal multiplier effects. Relative to Ganelli, op. cit., it allows for more realistic price dynamics by moving beyond one-period nominal rigidities.<sup>1</sup>

The paper proceeds, in Section 2, by laying out the microeconomic assumptions of the model. In Section 3 we draw out the implied macroeconomic structure. Section 4 then uses this to analyse the effects of a one-off fiscal deficit, and Section 5 concludes.

## 2. The Microeconomic Elements

We consider a 'small' country which takes world output and interest rates as given. It also takes world prices of foreign-produced goods as given. However domestically-produced goods are differentiated from foreign-produced goods, so that prices of the former may be affected by events at home. Our main interest is in short- to medium-run behaviour. Accordingly we abstract from capital accumulation, treating labour as the only productive input. The potentially 'Keynesian' nature of the macroeconomic equilibrium arises from assuming staggered price setting in the style of Calvo (1983). This is embedded in a dynamic general equilibrium framework by combining it with monopolistically competitive firms, in the manner of Woodford (2003), Galí (2008), and many other authors. On the other hand, the absence of Ricardian Equivalence, which generates the scope for government debt and deficits to have real effects, arises from assuming overlapping generations in the style of Blanchard (1985).

#### 2.1 Household behaviour

Domestic households supply labour (*L*) and consume a foreign good ( $C^F$ ) and a composite domestic good ( $C^H$ ). Although the economy contains money, we assume it is 'cashless' in that households' holdings of real balances are sufficiently small that, as a reasonable approximation, their demand for money can be neglected. The key assumption of Blanchard (here adapted for discrete time) is that each household has an exogenous probability of death, 1-*q*, per period of time (where  $0 < q \le 1$ ). The size of the population is normalised to 1, so that in every period 1-*q* existing households die and 1-*q* new households are born. By varying *q* we can vary the expected lifetime (= 1/[1-q]) of a household. An

<sup>&</sup>lt;sup>1</sup> Other recent papers to use OLGs in a *closed*-economy New Keynesian framework for the study of fiscal policy are Devereux (2011), Annichiarico et al. (2012), and Ascari and Rankin (2013).

insurance market is assumed in which the household agrees to cede all its financial wealth to the insurance company in the event of its death, in return for which it receives an 'annuity' at a gross rate 1/q on its financial wealth in every period in which it remains alive. A household is born with zero financial wealth, but over its life it will generally accumulate financial wealth (or, alternatively, accumulate debt). Households of different ages will therefore have different wealth and consumption levels. We use 's' to denote the birth-period of a household. Households may also hold (or issue) bonds, ( $F^N$ , in nominal terms) which pay a nominal interest rate *i*. The bonds held by domestic households are issued either by the government or by foreigners. Since there is no aggregate uncertainty, such bonds are perfect substitutes from households' viewpoint and hence they all pay the same interest rate.

Given the foregoing, the dynamic optimisation problem of a household may be written as:

maximise 
$$\Sigma_{t=n}^{\infty} (\beta q)^{t-n} \left[ \gamma \ln C_{s,t}^{H} + (1-\gamma) \ln C_{s,t}^{F} + \psi \ln(1-L_{s,t}) \right]$$
(1)

subject to 
$$P_t^H C_{s,t}^H + P_t^F C_{s,t}^F + F_{s,t+1}^N = (1/q)(1+i_t)F_{s,t}^N + W_t L_{s,t} + \Pi_t - T_t,$$
  
for  $t = n, \dots, \infty$ . (2)

Here,  $0 < \beta, \gamma < 1$ ;  $\psi > 0$ ;  $s \le n$ . Note that all households face the same lump-sum tax,  $T_t$ , and receive the same share of profits,  $\Pi_t$ , from the monopolistic firms. (1) gives the expected discounted lifetime utility of a household, accounting for the fact that the probability of survival from one period to the next is q. To enable aggregation across households of different ages we need preferences to be homothetic. This is achieved here by adopting a simple logarithmic utility function.<sup>2</sup>

Clearly, we can separate the static problem of optimally allocating consumption spending between home and foreign goods from the dynamic optimisation problem. To do this, define the sub-utility over goods consumption as a whole as:

<sup>&</sup>lt;sup>2</sup> One consequence of (1) is that there will generally be some, sufficiently old, households, whose labour supply,  $L_{s,t}$ , is negative. This phenomenon, the reason for it, and how it might be avoided, are discussed in Ascari and Rankin (2007). It is unappealing on grounds of microeconomic realism. However, since the negativity of some labour supplies is not a factor on which the macroeconomic properties of the model hinge, here we choose to accept it.

$$(C_{s,t}^{H})^{\gamma}(C_{s,t}^{F})^{1-\gamma} \equiv C_{s,t}.$$
(3)

Maximising this subject to a given total nominal expenditure on goods,  $P_t^H C_{s,t}^H + P_t^F C_{s,t}^F = I_{s,t}$ , leads to the following demand functions for home and foreign goods:

$$C_{s,t}^{H} = \gamma I_{s,t} / P_{t}^{H}, \qquad C_{s,t}^{F} = (1 - \gamma) I_{s,t} / P_{t}^{F}.$$
 (4)

The maximised value of  $C_{s,t}$  is then  $I_{s,t} / P_t$ , where  $P_t$  is the domestic consumer price index,

$$P_{t} \equiv \gamma^{-\gamma} (1 - \gamma)^{-(1 - \gamma)} (P_{t}^{H})^{\gamma} (P_{t}^{F})^{1 - \gamma}.$$
(5)

 $C_{s,t}$  is thus also the real value of spending on all goods by the household, or its 'total composite consumption'. In what follows we will present a number of the relationships in terms of it. Note that the intertemporal relative price of  $C_{s,t}$  is the real interest rate:

$$1 + r_t \equiv (1 + i_t) P_{t-1} / P_t.$$
(6)

We may now return to the full dynamic optimisation problem and re-express it in terms of  $C_{s,t}$ . Solving it then yields two first-order conditions which will constitute key equations of the model. Since these are linear in  $(C_{s,t}, L_{s,t})$  and their coefficients are independent of *s*, they can also be expressed in terms of the aggregate counterparts of these generation-specific variables. We then have:

$$L_t = 1 - \psi \frac{P_t}{W_t} C_t \,, \tag{7}$$

$$C_{t+1} = (1+r_{t+1})\beta C_t - \frac{1-q\beta}{1+\psi}(1/q-1)(1+r_{t+1})F_{t+1}.$$
(8)

Note that, for any variable  $X_{s,t}$ , the corresponding aggregate (or average, since the population has been normalised to one) is  $X_t \equiv \sum_{s=-\infty}^t (1-q)q^{t-s}X_{s,t}$ . (7) is the labour supply function, showing that desired labour supply is positively related to the real wage and negatively to consumption. (8) is the 'consumption Euler equation'. As in Blanchard (1985), this differs from its generation-specific counterpart by the presence of  $F_{t+1}$ , where  $F_{t+1}$  is the real value of households' bond (or 'financial') wealth,  $F_{t+1}^N / P_t$ .<sup>3</sup> Observe that if q = 1 (zero probability of death),  $F_{t+1}$  drops out of (8). (8) then reduces to the Euler equation familiar from models with infinitely-lived agents. More generally, however, q < 1, in which case  $F_{t+1}$  has a negative effect (if  $F_{t+1} > 0$ ) on the growth rate of aggregate consumption. This is due to the 'generational turnover effect'. It arises from the fact that, every period, some already-living agents are replaced by newborn agents between t and t+1. The newborn, who have no financial assets, have lower consumption than the already-living, since the latter have had time to accumulate such assets. This effect therefore tends to reduce aggregate consumption growth. Alternatively viewed, (8) says that, for a given expected value of  $C_{t+1}$ , an increase in  $F_{t+1}$  will increase  $C_t$ . This shows that in the presence of overlapping generations current aggregate consumption demand depends not only on expected future aggregate consumption and the real interest rate, but also on the country's aggregate stock of financial assets.

Above we referred to  $C^H$  as consumption of a 'composite' domestic good. Specifically, we assume that the home economy produces a continuum of varieties of consumption good, each indexed by z, where  $z \in [0,1]$ .  $C_{s,t}^H$  is a CES sub-utility function over these varieties:

$$C_{s,t}^{H} = \left[ \int_{0}^{1} (C_{s,t}(z))^{(\theta-1)/\theta} dz \right]^{\theta/(\theta-1)},$$
(9)

where  $\theta(>1)$  is the constant elasticity of substitution. The household faces the sub-problem of maximising (9) subject to a given nominal expenditure on home goods,  $I_{s,t}^{H}$ , where  $I_{s,t}^{H} = \int_{0}^{1} P_{t}(z)C_{s,t}(z)dz$ . This leads to a constant-elasticity demand function for any variety, z:

$$C_{s,t}(z) = \left(P_t(z) / P_t^H\right)^{-\theta} \left(I_{s,t}^H / P_t^H\right), \tag{10}$$

where  $P_t^H$  is the price index  $[\int_0^1 (P_t(z))^{1-\theta} dz]^{1/(1-\theta)}$ . The maximised value of  $C_{s,t}^H$  is then  $I_{s,t}^H / P_t^H$ .

#### 2.2 Firm behaviour

Domestic goods are produced by an industry which is monopolistically competitive. Each variety, z, is the output of a single firm with a production function  $Y_t(z) = [L_t(z)]^{\sigma}$ ,

<sup>&</sup>lt;sup>3</sup> Unlike (7), (8) is not obtained simply by replacing generation-specific variables by aggregate variables in the corresponding first-order condition of an individual household. Its derivation also makes use of the expression for an individual household's consumption as a function of its total lifetime wealth.

where  $0 < \sigma \le 1$  and  $L_t(z)$  is the labour input. Labour is homogeneous and traded in a perfectly competitive, flexible-wage, market.

The total domestic demand for any variety, z, may be found by summing the demand function (10) across all domestic households. This leads to a function of a similar form to (10) but in which the 'shift parameter',  $I_{s,t}^H / P_t^H$ , is replaced by total domestic consumption of the 'composite' home good,  $C_t^H$ . We furthermore assume that there is an analogous foreign demand function for the domestic good variety, z, also having the price elasticity  $\theta$ . The shift parameter for this foreign demand function will similarly be equal to total foreign consumption of the composite home good, which we denote as  $C_t^{H^*}$ . Moreover, we shall not assume any barriers to international arbitrage in goods, so that the price of good z in the foreign market must also be  $P_t(z)$ . The global demand for good z is hence:

$$Y_t(z) = \left(P_t(z)/P_t^H\right)^{-\theta} \left(C_t^H + C_t^{H^*}\right).$$
(11)

Since firm z is infinitesimal relative to the whole economy, it takes the macroeconomic variables  $(P_t^H, C_t^H, C_t^{H*})$  as given when choosing its price and output.

Staggered price setting is introduced by using Calvo's (1983) assumption that firms may only adjust prices in periods when they receive a random signal permitting this, and that in other periods they must keep prices fixed.  $1-\alpha$  is the exogenous probability of receiving such a signal. Symmetry amongst firms implies that any firm which is permitted to adjust its price in period *t* will choose the same price, and we denote this 'new' price as  $X_t$ . Since the optimisation problem of a firm under these assumptions is by now very familiar from the expositions of Woodford (2003), Galí (2008), and many others, we will simply present the solution which results for the new price, namely:

$$X_{t} = \frac{\theta}{\theta - 1} \frac{\sum_{j=0}^{\infty} \alpha^{j} \Delta_{t,t+j} [\overline{Y}_{t+j}(z) / P_{t+j}] \frac{1}{\sigma} W_{t+j} [\overline{Y}_{t+j}(z)]^{1/\sigma - 1}}{\sum_{j=0}^{\infty} \alpha^{j} \Delta_{t,t+j} [\overline{Y}_{t+j}(z) / P_{t+j}]}.$$
(12)

Here  $\Delta_{t,t+j} \equiv (1+r_{t+1})^{-1}(1+r_{t+2})^{-1}\dots(1+r_{t+j})^{-1}$  is the intertemporal discount factor used by the firm,<sup>4</sup> and  $\overline{Y}_{t+j}(z) = (X_t / P_{t+j}^H)^{-\theta}(C_{t+j}^H + C_{t+j}^{H^*})$  is the demand for its product in period t+j contingent on the firm having last received a price-adjustment opportunity in period *t*. As is standard, this equation says that the new price is given by a mark-up ( $\mathcal{A}[\mathcal{A}1]$ ) over a weighted average of current and expected future marginal costs.

The formula for the index of prices of all domestically-produced goods,  $P_t^H$ , was given above, in conjunction with equation (10). Combining this with the assumption of Calvo-style price staggering, we obtain a well-known relationship between the price index and lagged values of the 'new' price:

$$P_t^H = \left[ (1-\alpha) \Sigma_{j=0}^{\infty} \alpha^j X_{t-j}^{1-\theta} \right]^{1/(1-\theta)}.$$
(13)

Together, (12) and (13) form the key equations of the price-setting sector of the model.

# 2.3 Government behaviour

The government's budget constraint, expressed in real terms using the composite consumption good of domestic households as the numeraire, is:

$$(1+r_t)D_t = \tau_t + D_{t+1}, (14)$$

where  $D_t$  is the real value of government debt at the end of period *t*-1. We treat debt as being 'real' (or 'indexed') debt. Hence we abstract from effects caused by unanticipated price-level changes on the real value of nominally-denominated debt, leaving this as a possible topic for future study.  $\tau_t$  (= $T_t/P_t$ ) is the lump-sum tax levied on households. Government spending could easily be included in the budget constraint, but since this aspect of fiscal policy is not our concern here, we omit it. Another omission from the budget constraint is any seigniorage revenue. In a common currency area, such revenue in principle accrues to the area's central bank, which determines the area's monetary policy. A share of it could be distributed to the home country's government. However, under our assumption of a 'cashless' economy, even if the central bank did engage in monetary expansion (which we shall not consider here, since

<sup>&</sup>lt;sup>4</sup> Since there is no aggregate uncertainty in the model, the appropriate discount factor is simply the inverse of the product of the gross real riskless interest rates between *t* and t+j.

our interest is not in monetary policy), the seigniorage revenue generated would be zero, since holdings of real balances by households are treated as negligible.

#### 2.4 Market equilibrium conditions and some elementary macroeconomic relationships

For each domestically-produced good, z, the output must equal the sum of the total amounts consumed both at home and abroad. Hence  $Y_t(z) = C_t(z) + C_t^*(z)$ , where  $C_t^*(z)$  denotes the foreign consumption. An analogous relationship prevails at the level of the 'composite' home good (defined in (9)):

$$Y_t = C_t^H + C_t^{H^*}.$$
 (15)

This holds because not only have we assumed that the 'law of one price' applies for all individual goods, but also that foreign preferences over the different varieties of home goods, z, are given by a CES sub-utility function like (9), with the same value of  $\theta$ . Hence both domestic and foreign consumers will choose the same composite bundles of domestic goods. Note that  $C_t^{H^*}$  is a measure of the home country's exports.

Thinking of the right-hand side of (15) as the sum of two demands for domesticallyproduced goods, namely home demand and export demand, it is useful to be able to relate these to the terms of trade, i.e. to the relative price of home goods to foreign goods. For  $C_t^H$ , we can use (4), (3), and the fact that  $C_t = I_t/P_t$  to obtain<sup>5</sup>:

$$C_t^H = \gamma(P_t / P_t^H)C_t.$$
<sup>(16)</sup>

Let us now define the terms of trade as  $\rho_t \equiv P_t^F / P_t^H$ . Then, using the expression (5) for the domestic overall consumer price index,

$$P_t^H / P_t = \tilde{\gamma} p_t^{\gamma - 1} \quad \text{where } \tilde{\gamma} \equiv \gamma^{\gamma} (1 - \gamma)^{1 - \gamma}.$$
(17)

This enables us to re-write (16) as:

$$C_t^H = \gamma C_t / (\tilde{\gamma} \rho_t^{\gamma-1}).$$
<sup>(18)</sup>

<sup>&</sup>lt;sup>5</sup> Since these relationships are the same for every generation, s, we can also apply them to the corresponding aggregates across the whole population, as done here.

This confirms that, for a given real value of domestic consumption of *all* goods, a worsening of the terms of trade (a rise in  $\rho_i$ ) will raise the domestic demand for home goods.

For  $C_t^{H^*}$ , the demand will depend on foreign households' preferences. We treat these in a similar way to domestic households' preferences, supposing that they are logarithmic over foreign goods and the composite home good (cf. (1)). This implies that foreign households will devote a constant fraction of their total nominal consumption spending to domestic goods. Analogously to (4) above, we may thus write the demand for  $C_t^{H^*}$  as:

$$C_t^{H^*} = I_t^{H^*} / P_t^H, (19)$$

where  $I_t^{H^*}$  is the nominal budget which foreign households allocate to spending on home goods. Given that the home country is 'small' in the world economy,  $I_t^{H^*}$  can be treated as exogenous. We can also write (19) as:

$$C_t^{H^*} = K \rho_t$$
 where  $K = I_t^{H^*} / P_t^F$ . (20)

*K* is exogenous to the home country since, being small, it also takes  $P_t^F$  as given. Henceforth we shall moreover treat *K* as time-invariant. (20) therefore says that a worsening of the home country's terms of trade (a rise in  $\rho_t$ ) will raise its export demand.

In the labour market, the money wage,  $W_t$ , adjusts flexibly to equate the aggregate supply of labour, given by (7), to firms' aggregate demand for it. Note that the price staggering assumption causes a dispersion of prices across firms. It hence also causes a dispersion in their labour demands. Consequently, when labour demands are summed across firms, aggregate labour demand becomes:

$$L_t = s_t Y_t^{1/\sigma} \quad \text{where } s_t \equiv \sum_{j=0}^{\infty} (1-\alpha) \alpha^j (X_{t-j} / P_t^H)_{t-j}^{-\theta/\sigma}$$
(21)

 $s_t$  is a measure of price dispersion.<sup>6</sup> If there is no dispersion, as happens in a zero-inflation steady state, then  $s_t = 1$  and (21) is just the inverse of an individual firm's production function, but here applied to aggregates. More generally, when there is dispersion,  $s_t > 1$ .

<sup>&</sup>lt;sup>6</sup> It was originally defined in this way, for staggered-price models, by Schmitt-Grohé and Uribe (2007).

However, when the model is log-linearised around a zero-inflation steady state,  $s_t$  drops out, so in fact it will not play a role in our analysis below.

In the bond market, the equilibrium condition is that the aggregate demand for bonds by domestic households,  $F_t$ , should equal the supply of bonds by the home government,  $D_t$ , plus the supply of bonds by foreign residents. We denote the latter as  $V_t$ . They constitute the 'net foreign assets' of the home country as a whole, i.e. the combined private and public sectors' net holdings of bonds.<sup>7</sup> Hence:

$$F_t = D_t + V_t. \tag{22}$$

As earlier, all bond stocks are measured in real terms using the domestic household's composite consumption good as the numeraire.<sup>8</sup>

The accumulation or decumulation of net foreign assets arises from surpluses or deficits in the country's balance of payments. The trade surplus, expressed in units of domestic households' composite consumption, is:

$$B_t = (P_t^H / P_t) C_t^{H^*} - (P_t^F / P_t) C_t^F.$$
(23)

Replacing  $(C_t^{H^*}, C_t^F)$  by their respective demand functions, and replacing the relative prices by writing them in terms of  $\rho_t$ , we have:

$$B_t = K \tilde{\gamma} \rho_t^{\gamma} - (1 - \gamma) C_t.$$
(24)

Here we have used (17) and (20), and similar expressions which can be obtained for  $P_t^F / P_t$ and  $C_t^F$ . (24) tells us that a worsening of the terms of trade (increase in  $\rho_t$ ) improves the trade balance by raising export demand. Meanwhile, an increase in domestic consumption worsens the trade balance by expanding import demand. Given the trade balance, net foreign assets then evolve according to the standard identity:

$$V_{t+1} = (1+r_t)V_t + B_t.$$
(25)

<sup>&</sup>lt;sup>7</sup> We here speak as if  $F_t$ ,  $D_t$  and  $V_t$  are all positive, but they may also be negative, as indeed occurs for  $V_t$  in the policy experiment studied below.

<sup>&</sup>lt;sup>8</sup> As with government debt, we treat the debt of foreigners held by domestic residents as 'real', or 'indexed', debt.

 $B_t + r_t V_t$  is the current account surplus on the balance of payments, so this says that net foreign assets increase or decrease over time according as there is a current account surplus or deficit (respectively).

Under a single currency, the nominal interest rate is the same at home and abroad. However, the same does not necessarily apply to the *real* interest rate, which is the intertemporal relative price of goods. In the case of the foreign-produced good, its intertemporal relative price is exogenous to the home country, since the latter is 'small' in world markets. Hence, defining the foreign real interest rate as  $1+r_t^F \equiv (1+i_t)P_{t-1}^F/P_t^F, r_t^F$ must be treated as given. In the case of home-produced goods, on the other hand, their intertemporal relative price will generally be affected by events at home, since these goods are differentiated from foreign goods and so their prices are not completely tied down by world prices. Now, we previously defined the domestic real interest rate as the intertemporal relative price of domestic households' composite consumption (see (6)). Since homeproduced goods make up a significant part of this, it follows that the domestic real interest rate is also not exogenous. Its relationship to the foreign real interest rate is given by:

$$1 + r_{t+1} = (1 + r_{t+1}^F)(\rho_{t+1} / \rho_t)^{\gamma},$$
(26)

as can be derived from the foregoing definitions. This is an 'uncovered interest parity' condition for the interest rate, but in real terms. It says that the domestic real interest rate can deviate from the foreign one to the extent that the country's terms of trade are expected to deteriorate or improve between this period and the next.

# 3. The Macroeconomic Structure

In this section we assemble the elements introduced above into an apparatus suitable for studying the full macroeconomic effects of changing government debt. We begin with an analysis of the economy's steady state equilibrium. To understand the steady state is important for two reasons. First, a permanent change in government debt permanently alters the steady state and, under forward-looking expectations, this also plays a role in determining

the short-run effects. Second, the steady state is the point around which we approximate the model, so to understand it helps to explain some features of this approximation. We then present the log-linear approximation which will be used for the study of the dynamics. We proceed to highlight a key property of this linearised model: namely, the separability of the sector consisting of net foreign assets and the trade balance. This is what makes it feasible to characterise it without reliance on numerical simulations. Finally, we rearrange the equations in a more helpful form by employing the concept of the 'output gap'.

## 3.1 The zero-inflation steady state equilibrium

We assume the central bank of the common currency area conducts monetary policy to ensure that foreign inflation is zero. In the domestic economy, the steady state then has to be one in which domestic inflation is also zero. Otherwise, the terms of trade would be permanently changing, which is not consistent with domestic real variables taking timeinvariant values. Note that in a zero-inflation steady state, the staggered pricing has no real effects. This is because in the long run all prices have had time to adjust. Moreover, symmetry amongst domestic firms implies that the prices of all domestic goods varieties will be the same, i.e. there will be no price dispersion. Otherwise expressed,  $P(z) = P^H = X$  for all z (where absence of time subscripts denotes a steady state).

To determine steady-state output, we start in the labour market. Under the conditions explained, the price-setting condition, (12), reduces to  $W/P^H = (1-1/\theta)\sigma Y^{1-1/\sigma}$ . In essence, this is the economy's labour demand function, since output and employment move together (recalling the discussion of (21)). Combining it with the labour supply function, (7) (and also using (17)), we obtain:

$$Y^{1/\sigma} = 1 - \frac{\psi}{\tilde{\gamma}\rho^{\gamma-1}} \frac{\theta}{\theta-1} \frac{1}{\sigma} Y^{1/\sigma-1} C.$$
<sup>(27)</sup>

It may readily be observed that this equation implies that Y is negatively related to C and  $\rho$ . Intuitively, C represents the 'income effect' on labour supply. If domestic households are wealthier (as measured by C), they will supply less labour, so that output will fall. An increase in  $\rho$ , on the other hand, representing a higher relative price of foreign goods, means that domestic households need to work more hours to buy a unit of foreign goods, and thus also to buy a unit of their composite consumption good. This fall in their real consumption wage again lowers labour supply and thereby domestic output.

A second relationship of Y to C and  $\rho$  comes from the aggregate demand for homeproduced goods. This is obtained by substituting (18) and (20) into (15):

$$Y = \gamma C / (\tilde{\gamma} \rho^{\gamma - 1}) + K \rho.$$
<sup>(28)</sup>

(28) in fact holds in every period, not just in the steady state. It indicates that higher total domestic consumption (*C*) and a higher relative price of foreign goods ( $\rho$ ) raise aggregate demand for home-produced goods.

Now notice that (27) and (28) can be used to jointly determine *Y* and  $\rho$  as a function only of *C*. It is straightforward to show that *Y* and  $\rho$  are decreasing in *C*. We can represent these relationships as:

$$Y = Y(C), \quad \rho = \rho(C). \tag{29}$$

These negative linkages arise because greater wealth of domestic households, as reflected in greater *C*, simultaneously increases the demand for the home-produced good and reduces the supply of it. It is clear that such forces must raise its relative price, i.e. must reduce  $\rho$ . It is less immediately clear how they affect the quantity produced, but in fact formal analysis shows that the supply effect dominates.

For the economy to be in a steady state, its terms of trade,  $\rho_t$ , must be constant over time. The 'real UIP' condition, (26), then implies that the domestic real interest rate,  $r_t$ , must be equal to the foreign real interest rate,  $r_t^F$ . As discussed earlier, the latter is exogenous. Thus, even though  $r_t$  is endogenous when the domestic economy is not in the steady state, in the long run  $r_t$  must take the exogenous value  $r_t^F$ . Henceforth we furthermore take  $r_t^F$  to be time-invariant, denoting it as  $r^F$ .

In a steady state it must also be true that the country's net foreign assets are unchanging. This requires the current account of the balance of payments to be zero, or  $B + r^F V = 0$ . Otherwise expressed, in a steady state (and taking the foreign real interest rate to be positive), if the country is a net creditor (V > 0), then it must run a trade deficit. In this case, it is using the interest receipts on its net foreign assets to finance a level of consumption permanently greater than the value of its output. Conversely, if the country is a net debtor (V < 0), then it must run a trade surplus. This means it is financing the interest on its net foreign debt by consuming permanently less that the value of its output. Using (24) in this condition yields:

$$r^{F}V = (1 - \gamma)C - K\tilde{\gamma}[\rho(C)]^{\gamma}.$$
(30)

Here we have also substituted out  $\rho$  as a function of *C* using (29). (30) provides a steady-state relationship between *V* and *C*. It is unambiguously positive. Intuitively, a country which is a large net creditor can afford to consume more. The residents of such a country can also afford to work less. The output of the country will then be lower, and its comparative scarcity on the world market will cause its relative price to be higher.

To complete the determination of the steady state equilibrium, we now derive a second relationship between *V* and *C*. This is obtained from the consumption Euler equation, (8). We also here use the bond market equilibrium condition, (22), and the fact that  $r_t = r^F$  in the steady state. Imposing time-invariance on (8) and rearranging then gives:<sup>9</sup>

$$V + D = \frac{1 + \psi}{(1 - q\beta)(1/q - 1)} \left[\beta - \frac{1}{1 + r^F}\right] C$$
(31)

It is clear that this provides a second equation linking V and C.Hence from (30) and (31) we can in principle solve jointly for (V,C). Thence we can also obtain the final solutions for  $(Y,\rho)$ . (31) can be interpreted as being domestic households' aggregate steady-state demand for financial assets. It arises from the life-cycle pattern of asset accumulation which occurs under overlapping generations. To see this, note that if  $r (= r^F) > 1/\beta - 1$ , then each household will choose an ever-rising path for its consumption, since this is implied by the individual household's Euler equation. To achieve such a path, given the 'flat' path of the household's after-tax labour income, it has to continually accumulate financial assets over its lifetime. In a steady state the typical household's financial assets thus grow at a constant rate, starting from

<sup>&</sup>lt;sup>9</sup> Note that (31) is not valid when q = 1 since in deriving it we would be dividing by zero.

zero; but upon death they drop abruptly back to zero. Therefore if  $r > 1/\beta - 1$ , the average demand for financial assets in the country is positive, and the greater is r, the greater is the demand. Conversely, if  $r < 1/\beta - 1$ , the average demand for financial assets is negative, so that households accumulate debt over their lifetimes. The smaller is r, the greater is the average debt. This is the relationship captured by (31), treating C as given. C itself acts as a scale factor for the aggregate demand for assets, with higher C implying higher asset demand.

Of the two relationships which link V and C, namely (30) and (31), the former is positive while the sign of the latter depends on the sign of  $\beta$ - 1/(1 +  $r^F$ ) (or equivalently of  $r^F$ - (1/ $\beta$ - 1)). Since both  $r^F$  and  $\beta$  are exogenous parameters, it is a matter of assumption which case applies. For present purposes, it is convenient to focus on the case where  $r^F = 1/\beta - 1$ . Although this is special, our later results have no special sensitivity to this precise assumption, and it simplifies the algebra of what follows. When  $r^F = 1/\beta - 1$ , (31) implies that:

$$V + D = 0. \tag{32}$$

In other words, in this case domestic households' steady-state demand for financial assets, F, is zero. It follows that V = -D, i.e. that the country's steady-state net foreign assets(which are an endogenous variable) are simply given by minus the level of government debt (which is an exogenous policy variable). Hence an increase in government debt 'crowds out' net foreign assets one-for-one in the steady state. Intuitively, this is because when the government increases the supply of an asset which can be used by households for optimising the 'tilt' of their lifetime consumption paths, they need less of the other asset which could be used for this, namely net foreign assets.<sup>10</sup>

Let us summarise the steady-state response of the economy to an increase in government debt. An increase in D, by crowding out net foreign assets, V, reduces aggregate consumption, C. This linkage to C occurs through (30). The mechanism is that, with a fall in V, interest receipts from abroad decline, and this obliges domestic households to cut back

<sup>&</sup>lt;sup>10</sup> The same result arises in Blanchard (1985) in the small open economy case. More generally, both here and there, if the foreign real interest rate is above (below) the domestic time preference rate, then an increase in government debt crowds out steady-state net foreign assets by less (more) than 100%.

their consumption. At the same time, being poorer, they also cut back on leisure and so increase labour supply. This boosts domestic output. The greater availability of domestic goods on world markets then lowers their price, so that the country's terms of trade worsen. These linkages to *Y* and  $\rho$  occur through (29).<sup>11</sup>

#### 3.2 Log-linearisation of the model

The macroeconomic structural equations contain significant non-linearities. Although the full non-linear version of the model can be studied numerically, if we wish to directly inspect the linkages between macroeconomic variables using algebra then it is essential, for tractability, to take a log-linear approximation of the model.<sup>12</sup> The non-linearity also means that the log-linearised equations are sensitive to the steady state around which the approximation is taken. Here we choose the steady state with zero inflation and zero government debt (designated henceforth as the 'reference' steady state). Note that D = 0implies V = 0 in the reference steady state (see (32)). In turn, V = 0 means that, in the reference steady-state, the trade balance is zero (since, as noted,  $B + r^FV = 0$  must hold).<sup>13</sup>

In (33)-(39) below, we present a set of log-linearised equations which summarises the 'raw' version of the model. Lower-case letters here generally denote log-deviations: i.e.  $z_t \equiv \ln Z_t - \ln Z_R$  for any variable  $Z_t$ , where  $Z_R$  is its value in the reference steady state. However, where the variable is already denoted in the lower case, ^ is used. In the case of interest rates, we use log-deviations of 'gross' values:  $\hat{r}_t \equiv \ln(1+r_t) - \ln(1+r_R)$ , etc. Moreover, where the reference steady state value is zero, so its log is not defined, the 'deviation' form of the variable is specified by scaling it by composite consumption evaluated in the reference steady state: thus  $b_t \equiv B_t/C_R$ ,  $v_t \equiv V_t/C_R$ ,  $d_t \equiv D_t/C_R$ . We define the inflation rates for the consumer price index,  $P_t$ , and the producer price index,  $P_t^H$ , directly as  $\pi_t \equiv p_t - p_{t-1}$  and  $\pi_t^H \equiv p_t^H - p_{t-1}^{H}$ .<sup>14</sup>

<sup>&</sup>lt;sup>11</sup> These results are qualitatively unaffected by assuming  $r^F$  is in some region either side of  $1/\beta - 1$ .

<sup>&</sup>lt;sup>12</sup> Log-linearisation of models with Calvo-style price staggering also eliminates some potentially interesting features of the dynamics: see Ascari (2004). Nevertheless it is a natural first step towards obtaining a deeper understanding of the model's characteristics.

<sup>&</sup>lt;sup>13</sup> To ensure consistency, it is also helpful to define reference steady state values for the nominal 'scale' variables  $(P_t, P_t^H, P_t^F)$ . We therefore let  $P_R \equiv 1$ . The CPI formula, (5), then implies  $P_R^H = \tilde{\gamma} \rho_R^{\gamma-1}$  and  $P_R^F = \tilde{\gamma} \rho_R^{\gamma}$ , where  $\rho_R$  is tied down by the steady state equations.

$$y_t = \gamma c_t + (1 - \gamma^2) \hat{\rho}_t$$
 (33)

$$b_t = (1 - \gamma)(\gamma \hat{p}_t - c_t)$$
(34)

$$c_t = c_{t+1} - \hat{r}_{t+1} + \zeta(v_{t+1} + d_{t+1})$$
(35)

$$\hat{r}_{t+1} = \gamma(\hat{\rho}_{t+1} - \hat{\rho}_t)$$
(36)

$$v_{t+1} = (1+r^F)v_t + b_t$$
(37)

$$\pi_t^H = (1+r^F)^{-1}\pi_{t+1}^H + \kappa_a \left\{ \frac{[1/\sigma - 1 + (\theta - 1)/\theta \psi]y_t}{y_t + c_t} + (1-\gamma)\hat{\rho}_t \right\}$$
(38)

$$\hat{\rho}_t = -p_t^H \tag{39}$$

The first of these equations, (33), is just the log-linearised version of (28), giving aggregate demand for home-produced goods. (34) is the deviation form of the expression for the trade balance, (24). (35) is the consumption Euler equation in log-linearised form, its original version being (8). Here  $\zeta$  is the composite parameter  $(1/q-1)(1+r^F-q)(1+\psi)^{-1}$ .  $\zeta$ is zero when 1 - q, the probability of death, is zero. Observe that (35) can be interpreted as the economy's 'IS' equation. The presence in it of households' financial assets,  $v_{t+1} + d_{t+1}$ , when q < 1, is due to the OLG structure, as pointed out earlier. Log-linearisation of the 'real UIP' condition gives rise to (36), the counterpart of (26); while the balance of payments identity in deviation form is (37), the counterpart of (25). Equation (38) can be recognised as a form of the New Keynesian Phillips Curve (NKPC) equation. It arises from log-linearising the expression for the 'new' price, (12), and the price index formula, (13), while also imposing that the wage continuously clears the labour market (so using the labour supply and demand functions and (21)).The composite coefficient (7) $K_a$ is given by  $[1/\alpha - 1][1 - \alpha(1 + r^F)^{-1}]\sigma[\sigma + (1 - \sigma)\theta]^{-1}$ . The presence of  $(c_t, \hat{\rho}_t)$  in this version of the NKPC equation is due to the economy being open. In a closed economy we would have  $c_t =$  $y_t$ , but in an open economy international trade permits consumption and output to differ; and in a closed economy there are no terms of trade. As explained earlier, increases in  $c_t$  and  $\hat{\rho}_t$ 

<sup>&</sup>lt;sup>14</sup> See also footnote 13, which implies that  $p_t \equiv \ln P_t$  and  $p_t^H \equiv \ln P_t^H - \ln P_R^H$ .

act as disincentives to labour supply, so pushing up the wage and inflation.<sup>15</sup> Lastly, (39) follows directly from the definition of the terms of trade,  $\rho_t \equiv P_t^F / P_t^H$ , plus the exogeneity of the foreign goods price.

#### 3.3 A separability property

Overall, (33)-(39) is a fourth-order dynamical system, as may be seen from the fact that it contains four first-order difference equations. In general, a fourth-order system is too unwieldy to study purely algebraically. Fortunately, however, it proves possible to separate out a second-order sub-system from the larger one, such that the sub-system can be solved independently of the other components. This is extremely helpful, since the solution to the sub-system is easy to derive. It can then be re-united with the residual equations, thus enabling us to obtain a tractable algebraic solution to the overall model.

The separable sub-system is expressed in terms of  $(b_t, v_t)$  as the state variables. To obtain it, we use (36) to eliminate  $\hat{r}_{t+1}$  from (35), obtaining:

$$c_t - \gamma \hat{p}_t = c_{t+1} - \gamma \hat{p}_{t+1} + \zeta (v_{t+1} + d_{t+1}).$$
(40)

Notice that this is a first-order difference equation in the composite variable  $c_t - \hat{p_t}$ . This composite variable is directly and negatively related to the trade balance,  $b_t$ , as can be seen from (34). Hence we can re-express (40) as:

$$b_t = b_{t+1} - \zeta(1-\gamma)(v_{t+1}+d_{t+1}).$$
(41)

(41) implies that the current trade balance is determined only by the next period's expected trade balance and by the financial assets of domestic households. In the special case where households are infinitely-lived (q = 1), we know that  $\zeta = 0$ . Therefore in this case the trade balance is time-invariant along the perfect-foresight path. More generally, when q < 1, the trade balance is time-varying, but it is driven only by the evolution of domestic households' financial assets.

 $<sup>^{15}</sup>$  Having said this, when the equation is re-written in terms of the 'output gap' – as is done below – the separate influence of these variables will disappear.

The other component of the sub-system is given by the balance of payments identity, (37). This determines the evolution of net foreign assets,  $v_t$ , as a function of  $b_t$ . It is then clear that, together, (37) and (41) constitute a self-contained second-order difference equation system just in  $(b_t, v_t)$ . The only other variable in this system is  $d_{t+1}$ , the stock of government debt, which here is an exogenous policy variable. We summarise this finding as:

**Proposition 1** The fourth-order difference equation system which describes the model can be separated into an independent second-order system in the trade balance and net foreign assets,  $(b_t, v_t)$ , and a residual system in which  $(b_t, v_t)$  are forcing variables.

# 3.4 The output gap and the residual equation system

As well as being interested in the effects of deficits and debt on output itself, we may also wish to consider their effects on the deviation of output from the value which it would take if prices were perfectly flexible, i.e. on the 'output gap'. The sign of the output gap is the best indicator of whether the economy is in a boom or a recession, since it is the distortion caused by the Keynesian feature of price stickiness. To calculate the output gap we first need to calculate the flexible-price, or 'natural', level of output. If this were just a constant, then little would be added by studying the output gap. However in fact in our model the natural level of output has its own dynamics, since it is affected by government debt and by net foreign assets as they evolve over time. Intuitively this is because these variables affect households' desired labour supply, which then generates a supply-side effect on output. We denote the natural level of output (in log-deviation form) as  $y_t^N$ , and the output gap as  $\tilde{y}_t \equiv y_t - y_t^N$ .

In considering the determinants of  $y_t^N$ , notice that the behaviour of the  $(b_t, v_t)$  subsystem is independent of whether prices are flexible or sticky. This is because neither of the equations which govern it – (37) or (41) – are influenced by the staggered-pricing assumption. Hence this sub-system must be part of the set of determinants of  $y_t^N$ . Other determinants of  $y_t^N$  are the relationships (29). Under flexible prices, these will hold in every period and not just in the steady state. If we combine log-linearised versions of them with the balance of trade equation, (34), we can obtain the natural level of output as a function of the trade balance:

$$y_t^N = \delta b_t$$
 where  $\delta \equiv [(\theta - 1)/\theta \psi + 1/\sigma]^{-1}$ . (42)

Although it may not seem intuitive that the natural level of output should be a function of the trade balance, note that when we solve the  $(b_t, v_t)$  sub-system,  $b_t$  will become a function of  $v_t$  along the economy's perfect foresight path. Thus  $y_t^N$  can also be viewed as a function of  $v_t$ , i.e. of net foreign assets. As net foreign assets evolve over time, they also alter the natural level of output in the economy. The details of this will be discussed below.

We now return to the 'residual' equations in the system (33)-(39), i.e. those other than the equations governing  $(b_t, v_t)$ . The NKPC equation, (38), can be re-expressed in terms of the output gap. To do this, we first use (33)-(34) to solve for  $(c_t, \hat{\rho}_t)$  as functions of  $(y_t, b_t)$ , thereby allowing us to eliminate  $(c_t, \hat{\rho}_t)$  from (38).  $y_t$  can then be replaced by  $y_t^N + \tilde{y}_t$ , and  $y_t^N$  by the function (42). This yields:

$$\pi_t^H = (1 + r^F)^{-1} \pi_{t+1}^H + \kappa \tilde{y}_t, \qquad (43)$$

where  $\kappa \equiv \delta^{-1} \kappa_a$ . (43) has the canonical form of the NKPC equation, with  $\tilde{y}_t$  having replaced all of  $(y_t, c_t, \hat{\rho}_t)$ . As elsewhere in the literature, this equation says that current inflation depends on two variables: expected future inflation, and a measure of economic activity which is fully summarised by the output gap.

While the NKPC captures the 'aggregate supply' side of the economy, the 'aggregate demand' side is given in part by (33). It is helpful to eliminate  $c_t$  from this by using the rearranged trade balance equation, (34), obtaining:

$$y_t = \hat{\rho}_t - \gamma (1 - \gamma)^{-1} b_t.$$
(44)

This directly links output to  $b_t$ , a forcing variable of the residual system. Although it may seem puzzling that the trade balance negatively affects output, this is because (44) is a semireduced form: for given terms of trade, a rise in the trade surplus has to be generated by lower domestic consumption, and thus (via (33)) lower output, in order to reduce the demand for imports. Replacing output by the output gap in the same way as above, (44) can be converted to a similar relationship:

$$\tilde{y}_{t} = \hat{\rho}_{t} - [\gamma(1-\gamma)^{-1} + \delta]b_{t}.$$
(45)

The positive effect of  $\hat{\rho}_t$  on  $y_t$  or  $\tilde{y}_t$ , on the other hand, as exhibited in (44) or (45), mainly reflects the standard 'expenditure-switching' effect: as home goods become cheaper, consumers switch their expenditure into domestic goods and away from foreign goods.

The aggregate supply and aggregate demand sides of the model, (43) and (45), are linked by recalling that  $\pi_t^H \equiv p_t^H - p_{t-1}^H$  and  $\hat{\rho}_t = -p_t^H$  (see (39)). If we substitute these into (43) and (45), and then use (45) to eliminate  $\tilde{y}_t$  from (43), we end up with:

$$(1+r^{F})^{-1}p_{t+1}^{H} - [(1+r^{F})^{-1}+1+\kappa]p_{t}^{H} + p_{t-1}^{H} = \kappa[\gamma(1-\gamma)^{-1}+\delta]b_{t}.$$
(46)

The 'residual' system of our model has thus been resolved into a single second-order difference equation in  $p_t^H$ , with  $b_t$  as a forcing variable.

#### 4. Effects of a One-Period Budget Deficit

As stated earlier, for our fiscal policy experiment we consider a one-period budget deficit, implemented through a cut in the level of lump-sum taxation,  $\tau_i$ , and financed by issuing government debt. After the impact period, taxation is raised again in order to balance the budget, so that government debt ( $d_{t+1}$ ) is thereafter constant at the new, higher, level. Although more 'realistic' policy changes could be examined, we choose this one in order to reveal as clearly as possible the dynamic forces which are unleashed. Under this experiment, government debt does not continuously evolve, so the subsequent time path of the economy is driven only by the responses emanating from the domestic and foreign private sectors, and not by the dynamics of the policy instrument itself. We start by examining the effects on the trade balance and net foreign assets, since, as noted, these are independent of the rest. This also enables us to see how the natural level of output responds. We then study the implications for the output gap and inflation. Here we explore, first, the impact effects; and,

second, the transition path to the new steady state. We conclude with a numerical example illustrating the possible magnitudes involved.

# 4.1 Effects on the trade balance, net foreign assets and the natural level of output

As just seen, the time paths of  $(b_t, v_t)$  are determined only by (37) and (41). In this system,  $v_t$  is predetermined as of period t, since it is the real stock of net foreign assets held in the domestic economy at the start of period t, and is the outcome of accumulated current account deficits or surpluses in previous periods.  $b_t$ , on the other hand, is non-predetermined, since (via (34)) it is directly linked to consumption and the terms of trade, neither of which are naturally predetermined. Therefore, for a unique bounded perfect foresight equilibrium to exist, we need the second-order system comprised of (37) and (41) to have one eigenvalue inside, and one outside, the unit circle. Appendix A demonstrates that this is in fact satisfied.

We depict the equilibrium graphically in Figure 1. The stationary loci in the diagram are given by:

$$\Delta b_{t+1} = 0; \quad b_t = -(1+r^F)v_t - d, \tag{47}$$

$$\Delta v_{t+1} = 0: \quad b_t = -r^F v_t. \tag{48}$$

We assume that the economy is initially in a steady state at point O, with d = 0. When d is permanently increased to a positive value, the  $\Delta b_{t+1} = 0$  locus shifts down, as (47) shows. The new steady state is at point S, to which there is a unique convergent path – the saddlepath – given by the dashed line. On impact, the economy jumps down from O and onto this path at point A. The trade balance thus goes into deficit in the short run. Over time, the economy then converges along the saddlepath to point S. Hence, during the transition, domestic households steadily accumulate foreign debt. The trade balance, meanwhile, gradually turns from deficit to surplus. In the new long-run equilibrium, the surplus must be just enough to offset the continuous outflow of interest payments needed to service the permanently higher foreign debt. Overall, therefore, the trade balance exhibits a 'J-curve' response: an initial deficit, which over time changes to a surplus and remains there.

Algebraically, the solution for  $v_t$ , following a tax cut in t = 0, can be expressed as:

$$v_t - v = \lambda_1^t [v_0 - v], (49)$$

where *v* is the new steady-state value of  $v_t$  (= -*d*),  $v_0$  is the pre-shock steady-state value (= 0), and  $\lambda_1$  is the stable eigenvalue of the system. We can show (see Appendix A) that  $\lambda_1$  in fact lies in the interval (0,1). The associated saddlepath solution for  $b_t$  is (again see Appendix A):

$$b_t - b = \eta_b [v_t - v], \tag{50}$$

where *b* is the new steady-state value of  $b_t (= r^F d)$  and  $\eta_b \equiv \lambda_1 - 1 - r^F (< 0)$ .<sup>16</sup>

It follows from the behaviour just described for  $b_t$ , and from (42), that the natural level of output  $(y_t^N)$  also falls on impact. Along the transition path, however,  $y_t^N$  rises, ending up higher than at the point from which it started. This is illustrated in the first panel of Figure 2. We can understand these effects as resulting from labour supply changes. The budget deficit and increase in government debt initially raise domestic households' perceived total lifetime wealth. This is due to q < 1, which causes the increased current bond holdings not to be fully offset by the expected higher present value of the taxes during the lifetime of currently alive households. Such, of course, is the standard mechanism causing absence of Ricardian Equivalence under OLGs. Feeling wealthier, households demand more leisure and supply less labour. Under fully flexible prices, this has a negative supply-side effect on output. Over time, however, households' wealth declines due to the accumulation of net foreign debt. In the new steady state the country is poorer than when it started, since it has to make permanently higher interest payments to its foreign creditors. This has the opposite effect to the initial shock, with domestic households now demanding less leisure and supplying more labour, implying an expansionary long-run effect on the natural level of output.

# 4.2 Impact effects on the domestic price level and the output gap

As seen above, the system of 'residual' equations of the model can be reduced to a second-order difference equation in  $p_t^H$ , given by (46).  $b_t$  is a forcing variable in this system.

<sup>&</sup>lt;sup>16</sup> Note that the foregoing only applies when q < 1 (and thus  $\zeta > 0$ ), i.e. when Ricardian Equivalence is absent. When q = 1, the system exhibits path-dependence. In this case,  $b_t = -r^F v_0$ , where  $v_0$  is the predetermined initial stock of net foreign assets.  $b_t$  and  $v_t$  are then time-invariant along the perfect-foresight path and the system is independent of d.

As also seen, along the perfect-foresight path,  $b_t$  is negatively related to  $v_t$  via (50), while  $v_t$  obeys (49). We may re-express (49) as a first-order difference equation:

$$v_{t+1} = \lambda_1 v_t - (1 - \lambda_1) d.$$
(51)

It is then clear that (46) and (51), together with the static relationship (50), constitute a thirdorder dynamical system.

We now write this system as a set of three simultaneous first-order difference equations by defining the new state variable  $p_t^{HU}$ .  $p_t^{HU} (\equiv p_{t-1}^H)$  is the lagged value of the domestic price index.<sup>17</sup> Equivalently, it is the index of those prices which are 'unchanged' since period *t*-1. It is obviously predetermined in period *t*. Combining (46), (50) and (51), the model's dynamics can then be fully described by the matrix equation:

$$\begin{bmatrix} p_{t+1}^{H} \\ p_{t+1}^{HU} \\ v_{t+1} \end{bmatrix} = \begin{bmatrix} 1 + (1+r^{F})(1+\kappa) & -(1+r^{F}) & (1+r^{F})\kappa[\gamma(1-\gamma)^{-1}+\delta](\lambda_{1}-1-r^{F}) \\ 1 & 0 & 0 \\ 0 & 0 & \lambda_{1} \end{bmatrix} \begin{bmatrix} p_{t}^{H} \\ p_{t}^{HU} \\ v_{t} \end{bmatrix} + \begin{bmatrix} -(1+r^{F})\kappa[\gamma(1-\gamma)^{-1}+\delta](1-\lambda_{1})d \\ 0 \\ -(1-\lambda_{1})d \end{bmatrix}.$$
 (52)

Of the three state variables,  $(p_t^H, p_t^{HU}, v_t)$ , in this system,  $(p_t^{HU}, v_t)$  are predetermined while  $p_t^H$  is non-predetermined. For a unique bounded perfect-foresight solution to exist, we hence require that the coefficient matrix in (52) should possess two eigenvalues inside the unit circle and one outside. In Appendix B we show that this condition is indeed satisfied. Unsurprisingly, in view of the independence of  $v_t$  within this system, one of the stable eigenvalues is  $\lambda_1$ , discussed above. As regards the other two, Appendix B proves that the stable eigenvalue (which we label as  $\lambda_2$ ), lies in the interval (0,1), while the unstable eigenvalue (which we label as  $\lambda_3$ ), lies in the interval (1, $\infty$ ). The appendix moreover demonstrates that  $\lambda_2$  is decreasing in  $\kappa$ ; the slope of the NKPC. This is a standard property of

<sup>&</sup>lt;sup>17</sup> By 'price index', we henceforth refer to the *producer* price index,  $p_t^H$ , and not to the consumer price index,  $p_t$ , unless otherwise stated.

the canonical model with Calvo-style staggered pricing. It means, for example, that a higher value of  $\alpha$  (the probability of an individual price remaining unchanged), by lowering  $\kappa$ ; causes a higher value of  $\lambda_2$ .

We now seek to discover, first, the effect which the budget deficit has on the price level in the period in which the deficit occurs,  $p_0^H$ . This can be found from the saddlepath solution of the system (52), relating the initial value of the non-predetermined variable,  $p_0^H$ , to the initial values of the predetermined variables,  $(p_0^{HU}, v_0)$ . Written in a general form, the saddlepath solution is given by:

$$p_0^H - p^H = \left[ (b_{32}^N - b_{31}^N)(p_0^{HU} - p^{HU}) + (b_{21}^N - b_{22}^N)(v_0 - v) \right] \left[ b_{21}^N b_{32}^N - b_{31}^N b_{22}^N \right]^{-1}, \quad (53)$$

where the  $b_{ij}^N$ 's are elements of the matrix of normalised eigenvectors associated with the coefficient matrix in (52).<sup>18</sup>

We have assumed that prior to the shock the economy is in a steady state with zero government debt, zero net foreign assets and a zero trade balance. It is thus in the 'reference' steady state as defined in Section 3.2, implying that  $v_0 = p_0^{HU} = 0$ . Of the new, post-shock, steady state values which appear in (53), we have already seen that v = -d. The new steady state value of the price level,  $p^H (= p^{HU})$ , on the other hand, must obey (39), and hence is given by the new steady state value of the terms of trade. As discussed in Section 3.1, in the long run the terms of trade worsen, which means that the domestic price level  $p^H$  must fall. Specifically,

$$p^{H} = -[\gamma(1-\gamma)^{-1} + \delta]r^{F}d.$$
(54)

It is therefore already clear that, along the transition path to the new steady state, on average the economy must experience deflation rather than inflation. The fact that the country shares a common currency is key to this. If it had its own currency, the necessary long-run deterioration of the terms of trade could be achieved through exchange rate depreciation.

<sup>&</sup>lt;sup>18</sup> More precisely, the *j*th column of the eigenvector matrix is the eigenvector corresponding to the *j*th eigenvalue, with the elements of the first row normalised to unity. (53) is an application and slight adaptation of the formula in Blanchard and Kahn (1980).

Without such a possibility, however, a fall in the domestic price level is the only way to achieve it.

Now substituting the above values into (53), after evaluating the  $b_{ij}^N$  parameters and some rearrangement (see Appendix B), we obtain:

$$p_0^H = \frac{(1+r^F)(1-\lambda_1)(1-\lambda_2)^2}{1+r^F-\lambda_1\lambda_2} [\gamma(1-\gamma)^{-1}+\delta]d.$$
(55)

(55) demonstrates that the impact effect of the fiscal deficit on the price level (or, equivalently, on the current inflation rate,  $\pi_0^H$ ) is unambiguously positive. Intuitively, this is what we would expect as the short-run effect of a fiscal stimulus to aggregate demand. Only if q = 1, implying  $\lambda_1 = 1$  (see Appendix A), does (55) show that the price level would be unaffected. This is the special case in which Ricardian Equivalence holds, and the fiscal deficit has no effect on demand. In the general, q < 1, case, however, the higher government debt adds to domestic households' perceived net wealth, boosting consumption demand and thus prices.

The positive effect on  $p_0^H$  contrasts with the negative effect on  $p^H$ . In other words, the short-run and long-run price levels move in opposite directions. This is part of the key to understanding what happens along the transition path, something which we examine directly in the next sub-section.

We may now insert the results for the impact effects on the trade balance and the price level into equation (45), to obtain the impact effect on the output gap. After some simplification, the formula obtained is:

$$\tilde{y}_{0} = \frac{\lambda_{2}(1-\lambda_{1})[1+r^{F}-\lambda_{1}+(1-\lambda_{2})(1+r^{F})]}{1+r^{F}-\lambda_{1}\lambda_{2}}[\gamma(1-\gamma)^{-1}+\delta]d.$$
(56)

It is clear that this is positive. Thus it is unambiguous that the fiscal deficit causes a boom, in the sense of a positive output gap. This is what we would intuitively expect: the tax cut makes domestic households feel wealthier, stimulating consumption demand, and in the presence of sticky prices this pushes output above its natural level. Unsurprisingly, then, given the Keynesian elements of staggered prices and OLGs, the economy's very short-run behaviour is typically Keynesian. If prices were fully flexible (as would occur if  $\alpha = 0$ ), then  $\lambda_2$  would go to zero: see Appendix B. (56) tells us that the output gap would then also be zero. Alternatively, if agents were infinitely-lived (as would occur if q = 1), then  $\lambda_1$  would go to unity: see Appendix A. (56) tells us that the output gap would then again be zero. The strength of the impact effect on the output gap hence increases as  $\alpha$  and 1-q move away from zero.

One might also be interested in the impact effect on the absolute level of output, and not just on its value relative to the natural level. We saw earlier that the natural level of output falls on impact. Since  $y_t = \tilde{y}_t + y_t^N$ , it follows from what has just been seen that, for sufficiently flexible prices, the rise in the output gap will be too small to outweigh the fall in the natural level of output, and thus output itself will fall. On the other hand, for sufficiently sticky prices, we can show that the rise in  $\tilde{y}_t$  will be large enough to outweigh the fall in  $y_t^N$ , and thus  $y_t$  will rise. An expansionary impact effect on output itself is hence not guaranteed. This is because the fiscal deficit has a contractionary supply-side effect, via labour supply, which counteracts the expansionary demand-side effect, via consumption demand. For the expansionary effect to dominate, a sufficient degree of price stickiness is required.

These findings regarding the impact effects we encapsulate as:

**Proposition 2**. The impact effect on the output gap of a one-period debt-financed tax cut is positive, except when either the probability of being forbidden to change price ( $\alpha$ ), or the probability of dying (1-q), is zero. It is increasing in  $\alpha$  and 1-q. The impact effect on output itself is positive if  $\alpha$  is sufficiently close to one, but negative if  $\alpha$  is sufficiently close to zero.

## 4.3 Behaviour of the output gap and inflation along the transition path

How do the output gap and inflation evolve during the transition to the new steady state? We know they must eventually tend to zero. However, since the dynamics along the perfect-foresight path are governed here by two stable eigenvalues, rather than just one, it is possible that this adjustment is non-monotonic, even though both eigenvalues are real and lie in the (0,1) interval.

To examine this, we derive the 'final form' solutions for the variables of interest, expressing them as explicit functions of time. In the case of the output gap, the equation obtained is (for the derivation, see Appendix C):

$$\tilde{y}_{t} = [w_{y1}'\lambda_{1}^{t} + w_{y2}'\lambda_{2}^{t}][\gamma(1-\gamma)^{-1} + \delta]d, \qquad (57)$$

where

$$w'_{y1} = \frac{\lambda_2 (1 - \lambda_1)(1 + r^F - \lambda_1)^2}{(\lambda_2 - \lambda_1)(1 + r^F - \lambda_1 \lambda_2)},$$
  
$$w'_{y2} = \frac{\lambda_2 (1 - \lambda_1)[r^F (1 + r^F - \lambda_1 \lambda_2) + (1 + r^F)(1 - \lambda_2)^2]}{(\lambda_1 - \lambda_2)(1 + r^F - \lambda_1 \lambda_2)}.$$

Now consider the signs of the coefficients  $(w'_{y1}, w'_{y2})$ . Both have positive numerators, but their denominators depend on the sign of  $\lambda_2 - \lambda_1$ . If  $\lambda_2 > \lambda_1$ , then  $w'_{y1}$  is positive and  $w'_{y2}$  is negative; while if  $\lambda_2 < \lambda_1$ , the reverse is true. Note also that both  $\lambda_2 > \lambda_1$  and  $\lambda_2 < \lambda_1$  are possible. For example,  $\lambda_2 > \lambda_1$  would occur if  $\alpha$  is close to 1 and q is close to 0; while  $\lambda_2 < \lambda_1$ would occur if  $\alpha$  is close to 0 and q is close to 1.

Suppose first that  $\lambda_2 > \lambda_1$ , implying  $w'_{y1} > 0$ ,  $w'_{y2} < 0$ . Slightly rearranging (57) gives:

$$\tilde{y}_{t} = \lambda_{2}^{t} \{ w_{y1}^{\prime} (\lambda_{1} / \lambda_{2})^{t} + w_{y2}^{\prime} \} [\gamma (1 - \gamma)^{-1} + \delta] d .$$
(58)

The sign of the term {.} at first appears ambiguous. However, we already know that  $\tilde{y}_t$  is positive when t = 0, from which it follows that  $w'_{y1} + w'_{y2} > 0$ . Equivalently, the positive term involving  $w'_{y1}$  inside {.} must dominate the negative term  $w'_{y2}$ . Since, by assumption,  $\lambda_1/\lambda_2 < 1$ , it is then clear that, over time, the size of the positive term inside {.} shrinks towards zero, so that the sign of {.} must eventually switch from positive to negative. Thus the output gap, having started out positive, must become negative at some point along the transition path. The initial 'boom' inevitably turns into a 'bust'. It is also clear that this change of sign can only occur once. The boom-bust cycle is thus not repeated: output will subsequently tend to zero, but always from below.

Next suppose that  $\lambda_2 < \lambda_1$ . Since the signs of  $(w'_{y1}, w'_{y2})$  are then reversed, an exactly parallel argument – this time factorising (57) by  $\lambda_1^t$  – shows that the output gap must again exhibit a boom-bust cycle.

Thirdly, consider the case  $\lambda_1 = \lambda_2$ . This could only arise for a chance combination of the underlying parameter values, but since it might appear to be a pivotal case and different from the other two, it is worthy of attention. It should be noted that when  $\lambda_1 = \lambda_2$ , (57) is no longer correct. The system (52) now has 'repeated' eigenvalues, implying that the final form solutions for  $p_t^H$ ,  $\tilde{y}_t$ , etc., contain not only  $\lambda_1^t$  but also  $t\lambda_1^t$ . This means that (57) should be replaced by:<sup>19</sup>

$$\tilde{y}_{t} = (1 - \lambda_{1}) \frac{\lambda_{1}(1 - \lambda_{1})(1 + r^{F}) + \lambda_{1}(1 + r^{F} - \lambda_{1}) - (1 + r^{F} - \lambda_{1})^{2}t}{1 + r^{F} - \lambda_{1}^{2}} \lambda_{1}^{t} [\gamma(1 - \gamma)^{-1} + \delta]d.$$
(59)

It is clear from inspection of this expression that it is positive for t = 0, but that it must turn negative for sufficiently high *t*. Hence, even in the case  $\lambda_1 = \lambda_2$ , the output gap still exhibits a boom-bust cycle.<sup>20</sup>

Thus we have established our main result:

**Proposition 3**. Although the output gap is positive on impact, it must later become negative. It then tends to zero asymptotically from below. This holds for all parameter values (other than the special cases associated with a zero impact effect).

A similar analysis to that just described for  $\tilde{y}_t$  can be conducted for  $\Delta \tilde{y}_t$ . This shows that  $\Delta \tilde{y}_t$  must start out negative (in t = 1) and then turn positive in some period. Its sign can change only once. Together with the foregoing, we may therefore deduce that the general shape of the time path for the output gap must be asdepicted in the second panel of Figure 2.

Next, consider the transition path of inflation. An expression analogous to (57) can be derived for  $\pi_t^H$ . Similar arguments to those used for  $\tilde{y}_t$  can be employed to characterise the

<sup>&</sup>lt;sup>19</sup> The calculations for this case are available on request.

<sup>&</sup>lt;sup>20</sup> Strictly speaking, the saddlepath solution (53), from which we derived expressions for the impact effects, is also not valid when  $\lambda_1 = \lambda_2$ , since its numerator and denominator are then both zero. However, we can confirm that, in the limit as  $\lambda_1 \rightarrow \lambda_2$ , our earlier formula (56) for  $\tilde{y}_0$  yields the same as is obtained by setting t = 0 in (59).

transition path of  $\pi_t^H$ . For brevity we omit the details here. We have already noted that, on average, inflation must be negative along the adjustment path, since in the long run the price level has to fallrelative to its pre-shock level. A detailed analysis of this path shows that, while inflation is initially positive, it must change to being negative at some point and cannot change sign again thereafter. Thus the economy experiences deflation in the later phase of the adjustment, and the total amount of deflation in fact outweighs the total amount of inflation in the earlier phase. This is illustrated in the third panel of Figure 2. The main features of inflation's behaviour can be summed up as:

**Proposition 4**. Inflation becomes positive on impact but later turns negative. It then tends asymptotically to zero from below. On average, over the whole transition path, inflation is negative. This holds for all parameter values (other than the special cases associated with a zero impact effect).

Both the output gap and the inflation rate therefore go through their own boom-bust cycles. To see their correlation, consider again the NKPC equation, (43). We can arrange this as:

$$\tilde{y}_{t} = \kappa^{-1} \Big[ (\pi_{t}^{H} - \pi_{t+1}^{H}) + r^{F} (1 + r^{F})^{-1} \pi_{t+1}^{H} \Big].$$
(60)

(60) makes clear that, if the term in  $r^F$  is neglected, the end of the boom in the output gap must occur where  $\pi_t^H - \pi_{t+1}^H$  changes sign from positive to negative, i.e. where the rate of deflation peaks. With a positive and non-negligible  $r^F$  (and noting that  $\pi_{t+1}^H < 0$  in this vicinity), the boom in fact ends a bit earlier than this. Nevertheless, the output gap boom is still likely to end after the inflation gap boom, rather than before it. Such a correlation is also illustrated in Figure 2. The later part of the boom in the output gap is hence accompanied not by inflation, but by deflation.

What is the reason why the boom inevitably turns into a bust, as opposed to just fading away? We can explain this using equation (45), which provides an expression for the output gap in terms of the factors affecting aggregate demand for domestic goods. It is helpful first to rewrite it slightly. We observed earlier that the composite variable  $c_t - \hat{p}_t$  was uniquely and negatively related to the trade balance,  $b_t$ . It is also noteworthy that  $c_t - \hat{p}_t$  is equal to domestic composite consumption, but measured in the equivalent number of units of foreign goods. In other words, it is  $P_t C_t / P_t^F$  (in log-deviation terms), since, using our earlier definitions, this can also be expressed as  $C_t / (\tilde{p}_t^{\gamma})$ . Let us then define this variable as  $c_t^W \equiv c_t - \hat{p}_t$  (domestic composite consumption measured in foreign, or 'world', goods). In view of (34),

$$c_t^W = -(1-\gamma)^{-1}b_t.$$
(61)

It is clear that  $c_t^W$  is part of the same separable sub-system as  $b_t$  and  $v_t$ , so that its time path is likewise not affected by price stickiness. Now using  $c_t^W$  in place of  $b_t$  in (45), and defining  $[\gamma + (1-\gamma)\delta]c_t^W \equiv c_t^A$  ('adjusted' domestic composite consumption), we have:

$$\tilde{y}_t = c_t^A - p_t^H.$$
(62)

According to this view of the aggregate demand function, then, the sign of the output gap depends on the balance between  $c_t^A$  and  $p_t^H$ . The former is unaffected by price stickiness, but the latter is very directly affected by it.

Following the increase in government debt, adjusted consumption jumps up and then gradually falls back, ending at a level below its pre-shock level. This is simply a reinterpretation of the time path of  $b_t$ , discussed above. We can understand its shape in a different way by rewriting (50), the solution for  $b_t$ , as instead an expression for  $c_t^A$ :

$$c_t^A = [\gamma(1-\gamma)^{-1} + \delta][(1-\lambda_1)(v_t + d) + r^F v_t].$$
(63)

The term  $v_t+d$  was also noted to be domestic households' financial assets,  $f_t$ . We saw that, after increasing on impact with the increase in d, they decay back to zero over time. This would bring  $c_t^A$  back exactly to its pre-shock, zero, level, were it not for the extra effect of the term in  $r^F$ , representing increased interest payments to foreigners. The latter effect means that  $c_t^A$  ends up being negative. This is illustrated in the fourth panel of Figure 2. From that panel we see that the stimulus to demand which is generated by the higher level of government debt is not a permanent one, even though the government debt increase itself is

permanent. Indeed, what starts out as a stimulus to demand eventually turns into a permanent drag on demand.

The time path of the home goods price index,  $p_t^H$ , on the other hand, has already been studied through consideration of the inflation rate,  $\pi_t^H$ . If we convert the information in panel 3 of Figure 2 into a trajectory for  $p_t^H$ , we obtain the picture shown in panel 4.

As (62) makes clear, in order to keep the output gap at all times equal to zero, the price level would need to track  $c_t^A$  exactly. However, the sluggishness in  $p_t^H$  due to price staggering prevents this.  $p_t^H$  lags behind  $c_t^A$  in the first few periods after the policy shock. Hence a positive output gap, i.e. a boom, emerges. But in later periods, as  $c_t^A$  declines,  $p_t^H$  is now too high to keep  $\tilde{y}_t$  at zero, and a bust emerges. Again the culprit is inertia in price setting, which this time causes the price level to overshoot the level required to negate the demand stimulus. The price level eventually follows the source of the stimulus downwards, but only with a lag. Overall, prices are simply not nimble enough to instantaneously offset the demand disturbance. Despite this, if the demand stimulus from the higher government debt were permanent, the bust phase would not emerge, since the price level would not need to fall as well as rise. It is the combination of price stickiness with the gradual 'crowding out' effect of government debt that causes the 'bust'.

# 4.4 A numerical example

Our model is designed to facilitate a qualitative understanding of the mechanisms at work, rather than to produce realistic quantitative estimates. Nevertheless one might be interested to know the sort of magnitudes that even such a stripped-down apparatus can yield, for plausible parameter values. Here we present a numerical example which provides an idea of this.

We will take one time period to be a quarter. In the uncertain lifetimes model, it is well known that if an agent's probability of death, 1-q, is chosen to equate the length of the expected lifetime (1/[1-q]) to a typical human lifetime, this implies a trivial departure from full Ricardian Equivalence. When choosing parameter values it is hence better to interpret 'death' as representing a broader set of events than just physical death. The interesting study

of Bayoumi and Sgherri (2006) econometrically estimates q based on US data on consumption, income and taxation, and finds that an expected 'economic lifetime' of about 5 years provides a good fit. In line with this, we therefore use q = 0.95 here. For the world real interest rate,  $r^{F}$  (and the pure time preference rate of home households,  $1/\beta - 1$ , which is assumed to be the same) we adopt a value of 0.02.  $\chi$  the weight on home goods in consumers' utility function, is fixed at 0.7;  $\theta$ , the elasticity of substitution between varieties of home goods, is set at 15; and  $\psi$ , the weight on utility of leisure, is taken to be 0.9333, resulting in one third of the household's time endowment being devoted to work in the reference steady state. As regards firms,  $\sigma$ , the elasticity of output with respect to labour input, is set at  $\frac{1}{2}$ .  $\alpha$ , the probability of being forbidden to change price in any period, is pegged at 0.75, implying the expected duration of a 'price spell'  $(1/[1-\alpha])$  is one year. The policy experiment is the one considered throughout, namely a tax cut and thus fiscal deficit which lasts for just one period. The deficit is chosen as 1% of pre-shock GDP. Although we illustrate only one case, we also discuss how varying some of the parameter values alters the outcomes. The time paths for the main variables of interest are easily computed using the explicit algebraic solutions obtained earlier, and the results are presented in Figure 3.

Regarding the relative magnitudes of the impact effects, it is notable that the increase in the output gap dominates the decrease in natural output, such that output itself (which is the simple unweighted sum of the two) clearly increases.<sup>21</sup> This is what we would expect: in the short run, the demand-driven, expansionary effect strongly outweighs the supply-driven contractionary effect. Nevertheless the latter is not completely trivial. A lower value of  $\psi$  would reduce its importance, by weakening the income effect on labour supply.

The dynamic effects are strongly persistent. This reflects the fact that both eigenvalues turn out to be high, with  $\lambda_1 = 0.9843$  and  $\lambda_2 = 0.8872$ . As already seen, q and  $\alpha$  (respectively) are the main determinants of these. For plausible values of q, a near-unit value of  $\lambda_1$  is always likely, because a 'biologically realistic' q would raise  $\lambda_1$  still further. If such a value were used instead, its main consequence would be to scale down the absolute magnitudes of all the effects: the shapes of the time paths would not change markedly. As regards  $\lambda_2$ , this would be

<sup>&</sup>lt;sup>21</sup> For ease of comparison the first two panels are drawn to the same vertical scale.

lower if  $\alpha$  were lower; or if  $\theta$  were lower; or if  $\sigma$  were higher. However  $\lambda_2$  is unlikely to fall below about 0.75, for plausible parameter choices.

The point of greatest interest is to observe the 'bust' relative to the 'boom'. The plot of the output gap in Figure 3 shows that the bust is shallow relative to the boom. Measuring each at its maximum extent, the former is 3.8% of the latter. On the other hand, we can also see that the bust is prolonged. If we consider the *cumulative* loss of output as measured by the output gap, in the bust phase we find that it is 33.1% of the cumulative gain of output in the boom phase.<sup>22</sup> Viewed this way, the bust is of significant magnitude relative to the boom. Other numerical experiments confirm that, for parameter values in an empirically relevant range, the bust is likely to be shallow but long. The parameter to which it is most sensitive is  $r^{F}$ , the foreign real interest rate. With a lower value of  $r^{F}$ , such as 0.01, the bust becomes relatively insignificant, unless other parameters such as  $\alpha$  are pushed to fairly extreme values. Both 0.01 and 0.02, however, lie in the range of international borrowing and asset return rates which are broadly realistic. This therefore suggests that while, on the one hand, a fiscal deficit is not guaranteed to be the source of a sizeable delayed recession, on the other hand, we cannot safely ignore this possibility.

# 5. Conclusions

We have studied in depth the mechanisms which operate following a one-period, debtfinanced, fiscal deficit in a small open economy. The economy incorporates New Keynesian features which capture effects which are present in simple ad hoc Keynesian models, in particular price stickiness and non-neutrality of public debt, but in a way consistent with general equilibrium principles. To do this it assumes monopolistic competition combined with Calvo-style staggered price setting, and overlapping generations based on uncertain lifetimes. Since our objective is not the quantitative one of accurately replicating a real-world data set, but the analytical one of being able to clearly observe the inner workings of the economy, we have eliminated many inessential, but undeniably 'realistic', features which

<sup>&</sup>lt;sup>22</sup> This calculation is done by truncating the bust at 120 periods, as in Figure 3.

would otherwise confuse the picture. Despite this parsimony in our modelling approach, the resulting dynamical system is still non-trivial, exhibiting fourth-order dynamics.

Notwithstanding the order of the dynamics, we have been able to fully characterise the dynamic responses of variables to the deficit algebraically, by exploiting a separability property of the system. This consists in the fact that the time paths of net foreign assets and the trade balance can be solved for independently of other variables. Taking advantage of this, we were able to show that- as might be expected from the Keynesian structural characteristics of the economy - the fiscal deficit reliably causes a boom, i.e. a positive output gap, on impact and in general for some periods thereafter. However we also found that the boom at some point inevitably turns into a bust, i.e. a negative output gap. This second finding is not a well-known consequence of a fiscal deficit, and yet it follows robustly from the same New Keynesian structural features which give rise to the initial boom. The wider implications of our analysis are that, while fiscal deficits can indeed be used as a tool for business cycle stabilisation, they should not be used in a naive way. A simple one-period deficit sets off a dynamic reaction which can itself be destabilising. A careful plan for the future path of the fiscal deficit and public debt levels therefore needs to be mapped out which can mitigate the disturbance exhibited in the dynamic reaction. Future work with more sophisticated fiscal policy experiments would permit an analysis of this.



Figure 1



Determinants of the output gap

Figure 2





# Appendix A

We may rewrite the difference equation system comprised of (37) and (41) in matrix form as:

$$\begin{bmatrix} b_{t+1} \\ v_{t+1} \end{bmatrix} = \begin{bmatrix} 1+\zeta(1-\gamma) & \zeta(1-\gamma)(1+r^F) \\ 1 & 1+r^F \end{bmatrix} \begin{bmatrix} b_t \\ v_t \end{bmatrix} + \begin{bmatrix} \zeta(1-\gamma)d \\ 0 \end{bmatrix}.$$
 (A1)

Let an eigenvalue of the coefficient matrix in (A1) be denoted by  $\lambda$ .  $\lambda$  is determined by the characteristic equation of the matrix, namely:

$$\lambda^{2} - [1 + \zeta(1 - \gamma) + (1 + r^{F})]\lambda + (1 + r^{F}) = 0.$$
(A2)

This has the structure:

$$\lambda^2 - (a+b)\lambda + b = 0, \tag{A3}$$

where  $a \equiv 1 + \zeta(1-\gamma) \ge 1, b \equiv 1 + r^{F} > 1$ . The properties of its roots, i.e. of the eigenvalues, can most easily be examined by graphing it. To do this, rearrange (A3) as:

$$\lambda^2 = (a+b)\lambda - b. \tag{A4}$$

The left-hand side (LHS) and right-hand side (RHS) expressions are both simple functions of  $\lambda$ . These can readily be sketched as in Figure A1.



Figure A1

It is easy to see from (A4) that the LHS function is a parabola which passes through the points (0,0) and (1,1). On the other hand, the RHS function is a straight line which passes through the points (1,*a*) (labelled A), and (0,-*b*) (labelled B). Given that points A and B lie on opposite sides of the parabola, it follows that the two loci must intersect. Hence the eigenvalues are real, rather than complex, numbers. Given the location of points A and B, it is also clear that one intersection of the two loci must occur for  $\lambda$  in the range (0,1), while there will be another intersection (not shown) for  $\lambda$  in the range (1, $\infty$ ). Hence, as asserted in the main text, we have one eigenvalue inside, and one outside, the unit circle.

Next consider how changes in the parameters (a,b) affect the size of the smaller eigenvalue,  $\lambda_1$ . Note that both *a* and *b* can vary between 1 and  $\infty$ . In particular, q = 1 implies  $\zeta = 0$  and hence a = 1; while as *q* falls towards zero,  $\zeta$ , and hence *a*, increase towards infinity. Meanwhile,  $r^F = 0$  implies b = 1; while as  $r^F$  rises towards infinity, so does *b*. (A rise in  $r^F$  in addition increases  $\zeta$ , and thus also increases *a*.) From Figure A1, it is easy to see that as *a* increases from 1 to  $\infty$ , the line 'RHS' pivots anti-clockwise about point B (assuming unchanged *b*), and hence  $\lambda_1$  falls from 1 to 0.  $\lambda_1$  is therefore an increasing function of *q*, tending to 0 as *q* tends to 0, and tending to 1 as *q* tends to 1. On the other hand, as *b* increases from 1 to  $\infty$ , the RHS line pivots anti-clockwise about point A (assuming unchanged *a*). Hence  $\lambda_1$  rises towards 1.

In the main text, the relationship of  $b_t$  to  $v_t$  in the perfect foresight solution is given by eqn. (50). The coefficient  $\eta_b$  which appears in this comes from the normalised stable eigenvector of the matrix in (A1): i.e. the eigenvector associated with  $\lambda_1$ .  $\eta_b$  therefore satisfies:

$$\begin{bmatrix} 1+\zeta(1-\gamma)-\lambda_{1} & \zeta(1-\gamma)(1+r^{F})\\ 1 & 1+r^{F}-\lambda_{1} \end{bmatrix} \begin{bmatrix} \eta_{b}\\ 1 \end{bmatrix} = \begin{bmatrix} 0\\ 0 \end{bmatrix}.$$
 (A5)

The expression for  $\eta_b$  in the main text is taken from the second equation in (A5).

# **Appendix B**

The characteristic equation of the matrix in the equation system (52) is:

$$(\lambda - \lambda_1) \left\{ \lambda^2 - [1 + (1 + r^F)\kappa + (1 + r^F)]\lambda + (1 + r^F) \right\} = 0$$
(B1)

where  $\lambda$  is again the general symbol for an eigenvalue, and  $\lambda_1$  is the stable eigenvalue from the  $(b_t, v_t)$  sub-system.  $\lambda = \lambda_1$  is clearly also a solution of (B1), and hence  $\lambda_1$  is one of the three eigenvalues of the system (52). The other two eigenvalues – which we have denoted as  $(\lambda_2, \lambda_3)$  – are solutions of the equation obtained by setting  $\{.\} = 0$  in (B1), namely:

$$\lambda^{2} - [1 + (1 + r^{F})\kappa + (1 + r^{F})]\lambda + (1 + r^{F}) = 0.$$
(B2)

Now note that (B2) is a quadratic equation which has a similar parameter structure to (A2) in Appendix A. Specifically, (B2) can also be written in the form (A3), where *b* is defined as before, while *a* is now defined as  $a \equiv 1+(1+r^F)\kappa$ . Despite this re-definition of '*a*', it remains true that the parameters (*a*,*b*) can take any values in the range  $(1,\infty)$ . It then follows that the characterisation performed in Appendix A applies to the eigenvalues ( $\lambda_2, \lambda_3$ ), too. Thus ( $\lambda_2, \lambda_3$ ) are real rather than complex, and the smaller of them ( $\lambda_2$ ) lies in the range (0,1), while the larger ( $\lambda_3$ ) lies in the range (1, $\infty$ ). Hence, as asserted in the main text, two eigenvalues of the system (52) lie inside the unit circle, while one lies outside.

Consider next how  $\lambda_2$  is affected by changes in the underlying parameter values. From the definition of  $\kappa$ ; for  $\alpha$  arbitrarily close to 1,  $\kappa$  is arbitrarily close to zero, and hence a is arbitrarily close to 1. On the other hand, as  $\alpha$  falls towards zero,  $\kappa$  increases towards infinity, and hence a also increases towards infinity. The diagrammatic analysis in Appendix A then implies that as  $\alpha$  goes from 0 to 1,  $\lambda_2$  goes from 0 to 1.  $\lambda_2$  as a function of  $\alpha$  is thus very similar to  $\lambda_1$  as a function of q.

Thirdly we turn to the derivation of eqn. (55) in the main text. The coefficient matrix in (52) has a normalised eigenvector associated with the eigenvalue  $\lambda_1$  which is given by the matrix equation:

$$\begin{bmatrix} 1+(1+r^{F})(1+\kappa)-\lambda_{1} & -(1+r^{F}) & (1+r^{F})\kappa[\gamma(1-\gamma)^{-1}+\delta](\lambda_{1}-1-r^{F})\\ 1 & -\lambda_{1} & 0\\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1\\b_{21}^{N}\\b_{31}^{N} \end{bmatrix} = \begin{bmatrix} 0\\0\\0 \end{bmatrix}.$$

From this we may solve for  $(b_{21}^N, b_{31}^N)$  as:

$$b_{21}^{N} = 1/\lambda_{1},$$
  

$$b_{31}^{N} = \frac{\lambda_{1} + (1+r^{F})/\lambda_{1} - [1+(1+r^{F})(1+\kappa)]}{(1+r^{F})\kappa[\gamma(1-\gamma)^{-1}+\delta](\lambda_{1}-1-r^{F})}.$$

Similarly, the coefficient matrix in (52) has a normalised eigenvector associated with the eigenvalue  $\lambda_2$  which is given by the matrix equation:

$$\begin{bmatrix} 1 + (1 + r^{F})(1 + \kappa) - \lambda_{2} & -(1 + r^{F}) & (1 + r^{F})\kappa[\gamma(1 - \gamma)^{-1} + \delta](\lambda_{1} - 1 - r^{F}) \\ 1 & -\lambda_{2} & 0 \\ 0 & 0 & \lambda_{1} - \lambda_{2} \end{bmatrix} \begin{bmatrix} 1 \\ b_{22}^{N} \\ b_{32}^{N} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

From this we may solve for  $(b_{22}^N, b_{32}^N)$  as:

$$b_{22}^N = 1/\lambda_2,$$
  
 $b_{32}^N = 0.$ 

These terms may now be substituted into the formula (53). Using the expressions for  $(b_{21}^N, b_{22}^N, b_{32}^N)$ , but not yet  $b_{31}^N$ , we initially obtain:

$$p_0^H - p^H = \lambda_2 (p_0^{HU} - p^{HU}) + \frac{\lambda_1 - \lambda_2}{\lambda_1 b_{31}^N} (v_0 - v)$$

To assist with interpreting the somewhat complex expression for  $b_{31}^N$ , notice that the characteristic equation (B2), evaluated at  $\lambda = \lambda_{2}$ , can be rearranged to yield:

$$\kappa = (1 + r^{F})^{-1} \lambda_{2}^{-1} (1 - \lambda_{2}) (1 + r^{F} - \lambda_{2}).$$
(B3)

Inserting this into the formula for  $b_{31}^N$  to eliminate  $\kappa$  then gives, after simplification:

$$b_{31}^{N} = \frac{(\lambda_{1} - \lambda_{2})(1 + r^{F} - \lambda_{1}\lambda_{2})}{\lambda_{1}(1 - \lambda_{2})(1 + r^{F} - \lambda_{1})(1 + r^{F} - \lambda_{2})[\gamma(1 - \gamma)^{-1} + \delta]}.$$
 (B4)

We may now use this in the above equation for  $p_0^H$ . By design of the policy experiment,  $v_0 = p_0^{HU} = 0$ , as noted in the main text. The main text also gives expressions for

the new steady state values  $p^{H}(=p^{HU})$  and v. After making these substitutions and some further simplification we then arrive at (55) in the main text.

# Appendix C

In order to obtain the final-form solution for  $\tilde{y}_t$  ((57) in the main text), we first need the final-form solutions for the state variables  $(p_t^H, p_t^{HU}, v_t)$ . These can then be used in the 'aggregate demand' equation, (45) (in which we recall that  $\hat{\rho}_t = -p_t^H$ , while  $b_t$  is uniquely linked to  $v_t$  via (50)). In fact the final-form solution for  $v_t$ , being part of the separable subsystem in  $(b_t, v_t)$ , was already given in (49).

The final-form solutions for  $(p_t^H, p_t^{HU}, v_t)$ , which are the solutions of the matrix difference equation (52), are in general given by:

$$p_t^H - p^H = w_1 \lambda_1^t + w_2 \lambda_2^t, (C1)$$

$$p_t^{HU} - p^{HU} = w_1 b_{21}^N \lambda_1^t + w_2 b_{22}^N \lambda_2^t,$$
(C2)

$$v_t - v = w_1 b_{31}^N \lambda_1^t + w_2 b_{32}^N \lambda_2^t.$$
(C3)

Values for the  $b_{ij}^N$  coefficients were presented in Appendix B. The remaining unknowns are the weights  $(w_1, w_2)$ . These are determined by the initial conditions for the two predetermined state variables,  $(p_t^{HU}, v_t)$ . As explained in the main text, under our assumed policy experiment these initial conditions are  $v_0 = p_0^{HU} = 0$ . If we apply these to (C2) and (C3) we may now solve for  $(w_1, w_2)$  as:

$$w_{1} = (1/b_{31}^{N})d,$$
  

$$w_{2} = \lambda_{2} \left\{ \gamma (1-\gamma)^{-1} + \delta \right] r^{F} - 1/(\lambda_{1}b_{31}^{N}) d$$

(Here  $b_{31}^N$  has not yet been substituted out, but this can be done using (B4) in Appendix B.)

As just noted, from (45) and (50) we may write:

$$\tilde{y}_t = -p_t^H - [\gamma(1-\gamma)^{-1} + \delta][(\lambda_1 - 1 - r^F)v_t - (1-\lambda_1)d].$$

Using the solutions (C1) and (C3) in this, together with the expressions already provided for their constituent terms, and simplifying the resulting formula, we then arrive at (57) in the main text.

#### References

- Annicchiarico, B., Giammarioli, N. and Piergallini, A. (2012) 'Budgetary Policies in a DSGE Model with Finite Horizons', *Research in Economics* 66(2), 111-130
- Ascari, G. (2004) 'Staggered Prices and Trend Inflation: Some Nuisances', *Review of Economic Dynamics*, 7(3), 642-667
- Ascari, G. and Rankin, N. (2007) 'Perpetual Youth and Endogenous Labour Supply: A Problem and a Possible Solution', *Journal of Macroeconomics* 29(4), 708-723
- Ascari, G. and Rankin, N. (2013) 'The Effectiveness of Government Debt for Demand Management: Sensitivity to Monetary Policy Rules', *Journal of Economic Dynamics and Control* 37, 1544-1566
- Bayoumi, T. and Sgherri, S. (2006) 'Mr Ricardo's Great Adventure: Estimating Fiscal Multipliers in a Truly Intertemporal Model', *IMF Working Paper*, WP/06/168
- Blanchard, O.J. (1985) 'Debt, Deficits and Finite Horizons', *Journal of Political Economy* 93(2), 223-247
- Blanchard, O.J. and Kahn, C.M. (1980) 'The Solution of Linear Difference Models under Rational Expectations', *Econometrica* 48(5), 1305-1311
- Calvo, G. (1983) 'Staggered Prices in a Utility-Maximising Framework', Journal of Monetary Economics 12(3), 383-398
- Corsetti, G., Kuester, K. and Muller, G.J. (2013) 'Floats, Pegs and the Transmission of Fiscal Policy', in Cespedes, L.F. and Gali, J. (eds.), *Fiscal Policy and Macroeconomic Performance*, Santiago: Central Bank of Chile
- Devereux, M.B. (2011) 'Fiscal Deficits, Debt and Monetary Policy in a Liquidity Trap', in Cespedes, L.F., Chang, R. and Saravia, D. (eds.), *Monetary Policy under Financial Turbulence*, Santiago: Central Bank of Chile
- Farhi, E. and Werning, I. (2016) 'Fiscal Multipliers: Liquidity Traps and Currency Unions', in Taylor, J. and Uhlig, H. (eds.). *Handbook of Macroeconomics*, vol. 2B, Amsterdam: Elsevier
- Ferrero, A. (2009) 'Fiscal and Monetary Rules for a Currency Union', Journal of International Economics, 77(1), 1-10
- Galí, J. (2008) Monetary Policy, Inflation and the Business Cycle: An Introduction to the New Keynesian Framework, Princeton NJ: Princeton University Press
- Galí, J., Lopez-Salido, D., and Valles, J. (2007) 'Understanding the Effects of Government Spending on Consumption', *Journal of the European Economic Association* 5(1), 227-270
- Ganelli, G. (2005) 'The New Open Economy Macroeconomics of Government Debt', Journal of International Economics 65(1), 167-184

- Leith, C. and Wren-Lewis, S. (2000) 'Interactions between Monetary and Fiscal Policy Rules', *Economic Journal* 110(462), 93-108
- Leith, C. and Wren-Lewis, S. (2006) 'Compatibility between Monetary and Fiscal Policy under EMU', *European Economic Review* 50(6), 1529-1556
- Leith, C. and Wren-Lewis, S. (2008). 'Interactions between Monetary and Fiscal Policy under Flexible Exchange Rates', *Journal of Economic Dynamics and Control*, 32(9), 2854-2882
- McManus, R. (2015) 'Austerity versus Stimulus: The Polarizing Effect of Fiscal Policy', *Oxford Economic Papers*, 67(3), 581-597
- Obstfeld, M. and Rogoff, K. (1995), 'Exchange Rate Dynamics Redux', *Journal of Political Economy*, 103(3), 624-660
- Obstfeld, M. and Rogoff, K. (1996), *Foundations of International Macroeconomics*, Cambridge MA, MIT Press
- Schmitt-Grohé, S. and Uribe, M. (2007) 'Optimal Simple and Implementable Monetary and Fiscal Rules', *Journal of Monetary Economics*, 54(6), 1702-1725.

Woodford, M. (2003) Interest and Prices, Princeton, NJ: Princeton University Press