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Abstract

The paper attempts to find the socially best loan contract by comparing exante welfare, interest and default rates of individual and group lending. We introduce a general framework which allows auditing policies and interest rates to be simultaneously determined by maximising the social welfare. Both variables vary with the types of risk considered: independently identically distributed and positively correlated risk. An individual project outcome is private information of its owner, but reported outcomes can be audited at a cost which then publicly reveals the true project outcome. We find that incentive compatibility in a group loan context is delicate: the conditions for truth telling vary with the borrowers' perception of the overall solvency of the group. In addition, group loans are often made to local groups who have established local networks. This may mean that the group has cheaper policing of truthtelling, but also that the risks on projects within the group are likely to be correlated. To explore this, we numerically solve for the optimal contracts with varying audit cost differences and correlation, using a betabinomial distribution. We find that with an audit cost advantage, small group loans (typically to two borrowers) dominate individual loans even with correlation. But if audit costs are identical, the individual loan dominates. In the larger the group, the higher the audit probability is required to ensure truthtelling. Our finding provides an argument for why the number of borrowers should be limited to 2-5.

Key words: Group lending, Heterogeneous and Correlated risk, Welfare, Loan Auditing

JEL Codes: D81,G21

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1 Introduction

Group lending has been hailed as an innovation which will assist economic and social development in poorer regions (Ahlin et al., 2011; IMF, 2005). Its advantages are seen as a way of enabling excluded borrowers to access finance. In the best circumstances, a group of poor individuals with no collateral can successfully secure a joint loan with joint liability for repayment. Once the group has a good credit record, the borrowers can enter the formal financial system and secure loans from banks. The two driving forces in reducing default risk and solving costly state verification of the risky project outcomes of individual borrowers are spreading liability across multiple individual risks and the peer pressure for good behaviour exerted by one borrower on another. Thus, group lending can reduce the problems of lack of collateral, asymmetric information (ex-ante adverse selection and ex-post monitoring and audit costs), and high administration costs of small loans. Peer to peer lending which operates and shares information on online platforms with several lenders and borrowers also shows some of these features (Assadi and Ashta, 2014; Everett, 2015).

If individual borrowers finance risky projects by a loan and the project outcome is private information to each borrower, then some incentive compatibility devices must be used to ensure truthtelling of outcomes by individual borrowers. Most commonly, this is by auditing the reported outcomes of borrowers who declare low or defaulting returns. In the context of small sized loans to individuals in small, usually isolated social groups, the literature emphasises that group lending to small local groups can overcome asymmetric information problems and argues that auditing is most efficiently done by fellow borrowers in the group rather than by lenders and that a group loan with joint liability for group failure gives individual borrowers the necessary incentive to undertake this auditing. Similarly, local peer borrowers are better informed about each others expost situation than the lender is (Assadi and Ashta, 2014; Brau and Wooler, 2004), so the investigation costs of auditing by fellow borrowers are lower. In addition, because the borrowers belong to a social group and both pecuniary and non-pecuniary social sanctions can be imposed on cheats (Karlan, 2007), social pressure helps preventing cheating[1] and audit between borrowers is therefore more efficient than audit by the lender. However, borrowers will only have the incentive to audit their fellow borrowers if there is joint liability for loans to different borrowers i.e. if one borrower defaults, then, when possible, the remaining borrowers cover the cost so that the group as a whole can remain solvent.

In this paper, we study individual and group loans with costly state verification under risk neutrality. Borrowers have zero collateral and use the loan to generate risky returns which may be low enough to compel a borrower to default on repayment, but on average each loan is profitable and socially desirable. Borrowers' true realised returns are their private information but a report of a low defaulting return can be randomly audited at a fixed unit cost per audit. With individual loans, the lender performs the audit. In a group loan, since each audit is cheaper if performed by a borrower than the lender, it makes sense for two of the borrowers to have special status. One is appointed as chief auditor and audits the reports of other defaulting borrowers, the second audits the outcome of the chief auditor when the latter declares default. This is a generalization of the setup first proposed by Banerjee et al (1994) and is also related to the idea of delegated audit of Diamond (1984).

We characterise the incentive compatible audit probability and the terms and outcomes of the loans such as the interest payments required, the minimum audit probability, the expected default rate and the surpluses of the individual borrowers in each of individual and group loans. It turns out that, with a group loan, the way incentive compatibility works depends crucially on how each borrower thinks his own report will affect the default rate of the group. We show that different group conditions (e.g. the size of group, the distribution of returns of individual borrowers) will result in different incentive compatible auditing contracts.

With risk neutrality of all parties, we can evaluate loans by social welfare to find the socially best loan contract. While most of the empirical literature evaluates individual or group lending by industrial performance criteria such as the interest rate required or the default rate on past loans, only a handful of studies evaluate the welfare effect of group lending. For example, Baland et al. (2013)

[1] Agarwal and Hauswald (2010) documented that some private credit information is primarily local and that it is more likely that the bank will face nonpayment of borrowers located farther away.

allow for some self-finance and a variable loan size and under their assumptions find that the expected surplus for a risk neutral borrower in a group loan is highest when the default rate on the group loan is lowest. Unlike their framework which has deterministic exogenous audit and punishment large enough to ensure a fair return to lenders, our framework allows auditing policies as well as interest and default rates to be endogenous and vary with the nature of risk and group size. The expected audit cost required to generate truthtelling is a deadweight loss on the loan which reduces the surplus of borrowers and lender. The lender sets the repayment required to generate a fair return whatever the loan form, so we can compute the relative welfare of individual and group lending to detemine their relative social merits by calculating the expected surplus of each (exante identical) borrower.

Most studies assume identical independent risks (iid) of each borrower while some studies analyse lending with independent but heterogenous risks of borrowers (for example, see Stiglitz, 1990). To date, only a handful of studies have looked at the role of correlation of returns between borrowers (Ahlin and Townsend, 2007; Katzur and Lensink, 2012; Kurosaki and Khan, 2012). In fact, most group loans are to borrowers who are similar to one another or live in the same area (Assadi and Ashta, 2014). This implies that local shocks are likely to affect all borrowers, leading to positive correlation of revenues and reducing the possibilities for risk diversification between group members. Goodstein et al. (2013) find that a higher delinquency rate in surrounding zip codes increases the probability of a strategic default and Varian (1990) notes that borrowers' homogeneity can make the lender worse off. How to model correlated risks is still an issue. The few existing studies on group lending take very specific models of correlation (for example, see Sinn, 2013). By contrast, this paper illustrates how incentive compatibility audit policies, default and interest rates as well as borrowers' welfare vary with general degrees of correlation. In particular, we introduce the use of the beta-binomial distribution[2] in the analysis of group lending, as it allows for varying degrees of correlation.

The plan of the paper is outlined as follows. The next section introduces a simple model for individual and group loans which can be applied to both independent and correlated risks facing

[2]Here a local pool of borrowers are subject to systemic risk as well as idiosyncratic risk.

individuals. The section also derives the interest rate, the minimum required number of successes, and auditing strategy that a risk neutral lender would experience on each loan type and each risk distribution. This section also establishes the borrower's welfare for each loan form. Section 3 analyses the relative social merits of individual and group lending based on welfare for different group sizes and risks using simulation when necessary. Finally, Section 4 concludes.

2 A Simple Model

2.1 Assumptions

2.1.1 Distribution of risk

In our framework, there are a risk neutral lender who has access to a safe interest rate (r) and n risk-neutral borrowers. Each borrower i has a project requiring finance of B. Each project yields one of two returns; with some probability p_i, the project succeeds and yields high revenue of H, and with probability $1 - p_i$ the project fails and yields low revenue of L[3]. A set of success probabilities $(p_1 \,..p_n)$ is a sample draw from a distribution $f(p_1 \,..p_n)$ in which each p_i has an identical mean of $\overline{p}$. In general, the random probabilities of successes $(p_1 \,..p_n)$ may be mutually correlated. Define the number of successes as k and the joint distribution of risk as $g(k, p_1..p_n)$. The distribution of the outcomes is common knowledge. Let C_k be any permutation of k integers for $k\varepsilon[0, n]$ e.g. if $n = 5$ and $k = 4$, there are five permutations $\{1, 2, 3, 4\}, \{1, 2, 3, 5\}, \{1, 2, 4, 5\}, \{1, 3, 4, 5\}, \{2, 3, 4, 5\}$. The number of elements $i\varepsilon C_k$ is actually $\begin{pmatrix} n \\ k \end{pmatrix}$. The conditional density of the number of successes being k is $h(k|p_1, ...p_n) = \Sigma_{C_k}\Pi_{i\varepsilon C_k} p_i \Pi_{i\notin C_k}(1 - p_i)$. So

$$\begin{aligned} g(k, p_1..p_n) &= h(k|p_1..p_n)f(p_1..p_n) \\ &= \Sigma_{C_k}\Pi_{i\varepsilon C_k} p_i \Pi_{i\notin C_k}(1 - p_i)f(p_1..p_n) \end{aligned}$$

[3] The setup is equivalent to one in which the project yields 0 if it fails, but the borrowers have to post collateral of L that can then be seized if the project fails and the borrower defaults. Jiménez et al. (2006) found a negative relationship between collateral and default risk.

It follows that[4]

$$
\begin{aligned}
E_k(k|p_1...p_n) &= \Sigma p_i \\
E_p E_k(k|p_1...p_n) &= n\overline{p}
\end{aligned}
$$

Even if individual project risks are identically distributed, success on different projects can be correlated. Allowing (especially positively) correlated risks is empirically important. Given that group loans are typically to geographically close borrowers, the idea of common systemic risk on a group loan is attractive. Due to the localised nature of group lending, group members may face some common risks and thus the success of projects may be correlated (Varian, 1990). The dependence of outcomes violates the independence assumption of the binomial distribution. The tail probabilities (especially the lower tail) are likely to be higher than with independent risks, since downside catastrophic risk is probably more common in the developing economy context in which group lending occurs. Particular events such as extreme weather (droughts, floods), geological events (earthquakes) and economic and political events (commodity price shocks, revolution) are likely to cause common high downside risk in the borrower group. Similarly, good shocks are likely to be correlated across borrowers if they are localised. To model this needs an exante situation in which the risks facing different borrowers are initially random variables. These related risks imply correlation between borrowers in success or fail outcomes on their individual projects (ρ).

[4] As stated above the contract is written on the basis of the individual risks being a draw from the distribution $f(p_1, ...p_n)$. We are particularly interested in the distribution of the number of successes on the n projects. We give some general results: let $(p_1...p_n)$ be a sample from an arbitrary multivariate distribution with mean vector $(\overline{p}..\overline{p})$.

For $n = 1$ it is just the binomial case $E(1|p) = p$. For $n = 2$,

$$
\begin{aligned}
E_k(k|p_1 p_2) &= 2\Pr(k=2) + 1 * \Pr(k=1) + 0\Pr(k=0) \\
&= 2p_1 p_2 + [p_1(1-p_2) + p_2(1-p_1)] = p_1 + p_2
\end{aligned}
$$

Suppose that its true for $n-1$: $E_k(k|p_1...p_{n-1}) = \Sigma_1^{n-1} p_i$; then, for n borrowers, we have the recurrence rule

$$
\begin{aligned}
E_k(k|p_1...p_n) &= p_n(1 + E_k(k|p_1...p_{n-1}) + (1-p_n)E_k(k|p_1...p_{n-1}) \\
&= p_n + E_k(k|p_1...p_{n-1})
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
E_k(k|p_1...p_n) &= \Sigma p_i \\
E_p E_k(k|p_1...p_n) &= n\overline{p}
\end{aligned}
$$

2.1.2 Auditing

Only the borrower can costlessly see the outcome of his project. The outcome of a project can be revealed to the lender and all other borrowers only by costly audit which will then reveal publicly the revenue of audited borrowers. With individual loans on multiple projects, there is a separate contract between each borrower and the lender which is set to give the lender a fair return allowing for borrower default and the cost of audit. Each borrower only has an interest in his own contract, so audit of defaulting outcomes must be performed by the lender.

With a single group loan financing the projects of all individual borrowers, the group contract again ensures that with truthful reports the lender receives the fair return on the total group loan. Unlike individual loans, there is joint liability for the group loan amongst borrowers. So long as the total group revenue can cover the fair repayment on the group loan, successful borrowers bail out defaulting borrowers to avoid penalties. Thus, each borrower has an incentive to audit his fellow borrowers to ensure that they truthfully report successful outcomes. There are strong arguments in the literature that the group is better informed about the situation of group members than the lender and is also in a position to impose tougher sanctions for truthtelling on group members than the lender (Assadi and Ashta, 2014; Everett; 2015). In Stiglitz (1990) and Karlan (2007), strong emphasis is put on the role of the group of borrowers in enforcing truthful revelation and repayment through peer pressure. In such cases, it is the group of borrower(s), not the lender, who pay the audit cost. In Banerjee et al (1994), one group member is designated as the auditor, but most group loans are to symmetrically placed borrowers, each with a risky project. To operationalise this in our framework, two group members have an audit role set by the group collectively: the first borrower receives all $n-1$ reports from the other borrowers and audits each of the fail reports, and the second audits only the reported fail outcome of the first auditor.

We assume the lender can audit fail reports at a cost, C per borrower. Thus, as loan size increases, the audit cost per unit of loan is lower[5]. On the other hand, borrowers can audit a failed report at the

[5] Vigenina and Kritikos (2004) find that the size of individual loans tends to be larger than that of group loans. Using 124 institutions in 49 countries, Cull et al. (2007) find that an increase in loan size is associated with lower cost, leading

cost per borrower of c ($\leq C$). In common with the literature, we assume that the audit probability is selected to ensure truthful reporting (to satisfy the revelation principle (Townsend, 1979; Gale and Hellwig, 1985)), and that all revenues of detected false reporters are seized (maximum punishment)[6].

2.1.3 Lending and repayment

In our framework, the timing is as follows:

(i) Initially, each borrower receives finance for a risky project. The individual or group repayments for solvent agents are agreed. The lender does not know p_i for each borrower i but knows its distribution. The amount of repayment is set to give the lender a zero expected excess return above the safe rate interest rate. The rule that, if default occurs with either an individual or group loan, the lender seizes all the discovered assets of the borrowers is agreed. The audit probability on an agent who declares a fail (m) is also set in the initial contract.

(ii) Each borrower executes the project, observes his revenue outcome and reports either a success or fail to the lender and other borrowers. In particular, with individual loan contracts between a borrower and the single lender, all borrowers report their outcome to the lender. With a group loan, each borrower reports their outcome to the group. These individual borrower reports are made simultaneously.

(iii) The auditor(s) carry out random audits on all reported fails and, after audit by the group, the group reports its revenue to the lender[7].With any loan form and any audit arrangements, if there are k reported successes, $n-k$ audits must be undertaken each with probability m. With individual loans, the lender performs the audit. With a group loan, audit by two group members is socially preferred to audit by the lender. The results of these audits become public knowledge to the lender and all borrowers. If a borrower is audited and is found to have cheated, the group seizes all the revenue of that borrower and denies him any share of group surplus.

to a higher rate of return on assets for individual-based lenders. Brick and Palia (2007) find that smaller loans have a higher operation cost and so lenders set a higher interest rate premium. This is reinforced by Gonzalez (2010).

[6] It is well known that applying maximum punishment on false defaulters minimises the risk of false reporting and helps attain incentive compatibility under risk neutrality (see, for example, Border and Sobel (1987), Besley and Coate (1995)).

[7] Since audit results are public to all including the lender, the group itself or its auditors cannot cheat.

(iv) Based on truthful reports, payments are made out of the project(s) revenues. For a group loan, the group either repays and divides up its remaining surplus equally between all borrowers or defaults in which case the lender seizes the group revenue.

2.1.4 Social desirability condition

We assume social desirability conditions of the project: $H > (1+r)B + C > L$ and

$$\overline{p}H + (1-\overline{p})L > (1+r)B + (1-\overline{p})C > (1+r)B + (1-\overline{p})c$$

where $\overline{p}$ is the mean probability of success for each project. Therefore, only successes can repay their own loan in full, and each loan has a positive net expected social return even with the higher audit cost Multiplying the second inequality by n yields

$$n\overline{p}H + (n - n\overline{p})L > (1+r)nB + (n - n\overline{p})c \tag{1}$$

As shown above, the mean number of successes in a population of n borrowers: $E_pE_k(k|p_1..p_n) = n\overline{p}$, so (1) can be rewritten as

$$E_pE_k(k|p_1..p_n)H + (n - E_pE_k(k|p_1..p_n))L > (1+r)nB + (n - E_pE_k(k|p_1..p_n))c \tag{2}$$

That is, on average each project is profitable whatever happens to the other projects, and at the mean number of successes the group revenues are sufficient to repay the loan. There is the usual commitment problem for the lender; he has to commit to pay the cost to discover the borrower assets even though he knows that the borrowers is truthfully reporting. If $L > c$, then the lender can always cover the audit cost.

2.2 Individual loans

The lender directly contracts with each borrower, setting the required repayment from a successful borrower (P_I) and the audit probability on a loan which reports a failure. The borrower reports the outcome to the lender. If fail is reported, he is audited with the agreed probability. A successful borrower makes the repayment to the lender, a truthful failed borrower pays his entire revenue L and a detected cheating successful borrower loses his whole revenue H to the lender. The lender sets the terms of each loan identically on each borrower. With an assumption that the lender knows only the distribution of p_i ($\bar{p}$ is the mean of p_i in the marginal distribution), the repayment per loan P_I is set to give the lender a non-negative expected surplus above the risk-free interest rate on each individual loan[8]:

$$P_I\bar{p} + L(1-\bar{p}) \geq (1+r)B + m_I C(1-\bar{p})$$

where the audit probability is m_I and the audit cost is C. Given our assumptions on returns and loan size, borrowers who fail must default, but successful borrowers should repay. Borrower surplus is

$$\begin{aligned} S_I &= (H - P_I) \text{ with probability } \bar{p} \\ &= 0 \text{ with probability } 1-\bar{p} \end{aligned}$$

Then, the expected surplus of the i^{th} borrower is

$$ES_I = \bar{p}(H - P_I)$$

If the successful borrower reports fail and the loan is audited (with probability m_I), all his revenue is confiscated. As a result, a successful borrower will truthfully report if

$$(1-m_I)(H-L) \leq H - P_I$$

The individual loan contract solves the problem of choosing m_I and P_I to maximise the total welfare between the lender and borrower subject to the lender at least breaking even in expected value terms and a constraint on the borrower which requires truthtelling behaviour. We note that if the lender participation constraint binds, the lender's surplus is zero and the total welfare is equal to the expected individual borrower surplus per loan.

[8] If the lender treats each borrowers i loan in isolation from other loans, the marginal distribution of p_i is used to compute the repayment on each loan, but the mean of the marginal distribution is again $\overline{p}$.
The lender sets the common repayment per loan to just give a zero expected surplus on the n loans in total

$$P_I E_p E_k(k|p_1..p_n) + L(n - E_p E_k(k|p_1..p_n)) = (1+r)nB + m_I C(n - E_p E_k(k|p_1..p_n))$$

but $E_p E_k(k|p_1..p_n) = n\overline{p}$ so the repayment is set identically for n borrowers.

That is, the contract problem is

$$\begin{aligned}
\max_{m_I,P_I} ES_I &= \overline{p}(H-P_I) \\
\text{s.t. lender participation constraint} &: \overline{p}P_I \geq (1+r)B+(m_IC-L)(1-\overline{p}) \\
\text{incentive compatibility} &: (1-m_I)(H-L) \leq H-P_I
\end{aligned}$$

Both the lenders participation constraint and the incentive compatibility constraint must bind. If the participation constraint is slack, P_I can be reduced which slackens the incentive constraint and raises the objective. If the incentive constraint is slack, m_I can be reduced which slackens the participation constraint allowing P_I to also be reduced. Jointly solving the two binding constraints for the variables P_I and m_I yields the solutions

$$m_I = \frac{(1+r)B-L}{\overline{p}(H-L)-C(1-\overline{p})} \tag{3}$$

$$\begin{aligned}
P_I &= \frac{(1+r)B+(m_IC-L)(1-\overline{p})}{\overline{p}} \\
&= \frac{(H-L)((1+r)B-(1-\overline{p})L)-(1-\overline{p})LC}{\overline{p}(H-L)-C(1-\overline{p})} \\
&= (H-L)\frac{(1+r)B-L}{\overline{p}(H-L)-C(1-\overline{p})}+L
\end{aligned} \tag{4}$$

With social desirability $\overline{p}H+(1-\overline{p})L > (1+r)B+(1-\overline{p})C$, the optimal audit probability is strictly between zero and one. The required repayment P_I reflects the feasibility of the project with the first term's denominator reflecting the audit cost transferred from the lender. (4) is consistent with many empirical studies of conventional bank lending which document a positive relationship between loan size and repayment (interest rate) (Godlewski and Weill, 2011). Also P_I is equal to $(1+R_I)B$, so, knowing P_I, we can find the interest rate (R_I) required for an individual loan. The optimal expected surplus of the i^{th} borrower is then

$$\begin{aligned}
ES_I &= \overline{p}(H-P_I) \\
&= \overline{p}(H-L)\left[1-\frac{(1+r)B-L}{\overline{p}(H-L)-C(1-\overline{p})}\right] > 0
\end{aligned}$$

Since individual borrowers fail with probability $1 - \overline{p}$, this is also the default rate of an individual loan.

2.3 Group loan

The group has a fixed size n with a known distribution of the number of successes k. $E_p E_k(k|p_1..p_n)$ is the mean number of successes from a population of size n (Hereinafter $E(k|n)$). Following from (2), the n individual projects are socially desirable in the sense that $E(k|n)(H - L) + nL - (1 + r)nB - c(n - E(k|n)) > 0$. That is, the mean group revenue covers the safe group loan opportunity cost plus the audit cost of auditing each of the mean number of fails. The realised outcome of any one borrower is private information to that borrower unless the borrower is audited. Each borrower makes a report of his outcome to the group. If audited, the true outcome of the borrower is known by all borrowers and the lender. Audit per borrower costs c.

Recall the time line, loans to individuals are made and the lender only knows the group size and the distribution of the number of successes. The loan contract sets the exogenous individual loan size, the repayment required from the group (P) and the probability with which each reported fail will be audited (m). The audit probability on a fail is set before the project outcomes are realised and thus is independent of the reported number of fails[9]. The group repayment and m are set so that the lender gets a non-negative expected surplus above the safe rate, allowing for joint-liability between borrowers, for group default when many borrowers fail and also for the audit cost on reported fails.

The group undertakes and pays for the audit. So with truthful reporting, group revenue received is

$$kH + (n - k)L - cm(n - k) = k(H - L + cm) + n(L - cm)$$

when there are k successes. If the revenue received by the group is lower than the group loan repayment, the group defaults and the lender seizes all the group revenue; otherwise, the group repays the amount set in the group loan contract. There will be a critical number of successes k^*, the lowest number of

[9] To ensure truthful reports on all projects requires each reported fail to have a positive probability of audit ($m > 0$). This is costly and decreases the group's surplus. The audit probability is optimally set at its lowest level ensuring truthful reporting.

successes which allows the group to repay (See proof in Appendix A). If $k < k^*$, the group defaults and pays all its revenue $k(H-L+cm)+n(L-cm)$ to the lender. So conditional on the group defaulting, the average return to the lender from all the defaulting states is $E^*(k|k < k^*, n)(H-L+cm)+n(L-cm)$ where $E^*(.|k < k^*)$ is the mean of the truncated distribution of the number of successes i.e. $E(k|k < k^*, n)/\Pr(k < k^*, n)$. $\Pr(k < k^*, n)$ is the default probability on the group loan. Hence, the lender receives (i) P with probability $\Pr(k \geq k^*, n)$ and (ii) $E^*(k|k < k^*, n)(H-L+cm)+n(L-cm)$ with probability $\Pr(k < k^*, n)$. The lender has to at least break even on the group loan in expected terms. Thus, the group repayment P and k^* are jointly determined by

$$\Pr(k \geq k^*, n)P + E(k|k < k^*, n)(H-L+cm) + n(L-cm)\Pr(k < k^*, n) \geq (1+r)nB$$

$$k^*(H-L+cm) + n(L-cm) \geq P$$

The surplus of the group (S_G) depends on k :

$$\begin{aligned} S_G &= kH + (n-k)L - cm(n-k) - P \text{ if } k \geq k^* \\ &= 0 \text{ if } k < k^* \end{aligned}$$

Hence,

$$ES_G = E(k|k \geq k^*, n)(H-L+cm) + \Pr(k \geq k^*, n)[n(L-cm) - P]$$

2.3.1 Group Incentive Compatibility

Each borrower has to decide what he should report: a success or a fail, knowing the audit probability m but not at this stage knowing the outcome of other borrowers. If truly a fail, the best the borrower can do is report a fail and pay L to the lender/group, such a borrower still has a chance of a share of the surplus if enough other borrowers have succeeded and the group does not default.

If they are a success, what should they report? Their payoff depends on whether the group defaults after their report. In the case that successes tell the truth, they report H and get (i) a share of the surplus if the group does not default or (ii) nothing if the group does default and has no surplus to distribute. Alternatively, they could cheat, report a fail, keep H for themself and give the group L.

If they are audited, their cheating is discovered for sure and they lose both H and also any right to a share of the group surplus if the group does not default. If they are not audited, then they gain $H-L$ directly and, if the group does not default even after their cheating, they also get a share of the surplus coming from the other $n-1$ borrowers. Hence, the best report for a success to make depends on the reports of other borrowers which sets whether the group defaults or not.

In deciding whether to cheat in his report, a successful borrower has to assess whether the group will default if he either tells the truth or cheats. His decision depends on information/beliefs he has about the outcomes of other borrowers. He knows the distribution of the number of successes and that the audit probability is incentive compatible for all borrowers. Hence, he assumes rationally that all other borrowers report truthfully and thus that expected reported group revenue from the other $n-1$ borrowers is $E(k|n-1)(H-L)+(n-1)L-cm(n-1-E(k|n-1))$. Here, $E(k|n-1)$ is the mean number of successes out of $n-1$ borrowers.

With truthful reporting, this successful borrower expects reported group revenue will be

$$\begin{aligned} G_T^e &= (E(k|n-1)+1)H+(n-1-E(k|n-1))L-cm(n-1-E(k|n-1)) \\ &= (E(k|n-1)+1)(H-L+cm)+n(L-cm) \\ &> E(k|n)(H-L+cm)+n(L-cm) \end{aligned}$$

since, with the class of probability distributions we are using for k, it is true that

$$E(k|n-1) < E(k|n) < E(k|n-1)+1$$

and that there is at most one integer k between any adjacent pair of these expectations. Note that $E(k|n)$ can be any real number between 0 and n, but k^* and k must be integers. If $k^* > E(k|n-1)+1, G_T^e < P$; therefore, a successful borrower thinks that the group will default even if he tells the truth. We call this an unprofitable group. If $k^* \leq E(k|n-1)+1$, then $G_T^e \geq P$; the successful borrower thinks that the group will be solvent if he tells the truth.

Instead, suppose the successful borrower cheats paying just L to the group and retaining $H-L$ for himself. He knows there is a chance he will be audited and the expected cost of this cm has to be added into his view of group revenue if he cheats. If he is audited, he loses $H - L$ (in addition to L that has already paid to the group loan) and the right to a share of the group surplus (if any). But if not audited, he keeps $H - L$ and gets a share of the group surplus if any.

So if he cheats and may be audited with probability m, he thinks group revenue will be

$$\begin{aligned} G_c^e &= E(k|n-1)(H-L) + (n-1)L + L - cm(n - E(k|n-1)) \\ &= E(k|n-1)(H-L+cm) + n(L-cm) \end{aligned}$$

If $k^* \le E(k|n-1)$, a successful borrower thinks that on average the group will be solvent whatever he reports. We call this a non-marginal group. If $E(k|n-1) < k^*$, then $G_C^e < P$, implying that he thinks the number of successes will be $E(k|n-1)$ if he cheats and then the group will be insolvent. Of course $G_c^e < G_T^e$ (in fact $G_c^e = G_T^e - (H - L + cm)$). It shows that a successful borrower expects group revenue to be higher if he truthfully declares a success rather than cheats and, given that a borrower expects the group to be insolvent if he tells the truth, he also expects the group to default if he cheats. We call the group with $E(k|n-1) < k^* \le E(k|n-1)+1$ a marginal group. He believes that, on average, the group will be solvent if he truthfully reports and that if any single success cheats, the group will be insolvent. This gives us three cases:

(i) unprofitable group: $G_c^e < G_T^e < P$

(ii) marginal group: $G_c^e < P < G_T^e$

(iii) nonmarginal group: $P < G_c^e < G_T^e$

To work out his own return from his report, the individual borrower has to judge if the group will be solvent or insolvent after he either cheats or tells the truth since this determines if the group has any surplus to distribute. Thus, incentive compatibility requires the expected gain from unaudited cheating to be lower than the expected gain from telling the truth:

$$(1-m)[H - L + \frac{\max(0, G_c^e - P)}{n}] \le \frac{\max(0, G_T^e - P)}{n}$$

The group contract sets m, P and k^* to maximise the expected surplus per borrower whilst ensuring that the lender participation and the incentive compatibility constraints are satisfied. Thus, the contract problem is

$$\max_{m,P,k^*} E(k \geq k^*)(H - L + cm)/n + \Pr(k \geq k^*)[L - cm - P/n] \tag{5}$$

$$\text{s.t. } \Pr(k \geq k^*, n)P + E(k|k < k^*, n)(H - L + cm) + n(L - cm)\Pr(k < k^*, n) \geq (1+r)nB \tag{6}$$

$$k^*(H - L + cm) + n(L - cm) \geq P \tag{7}$$

$$(1 - m)[H - L + \frac{\max(0, G_c^e - P)}{n}] \leq \frac{\max(0, G_T^e - P)}{n} \tag{8}$$

(6) and (7) are the lender's participation constraints. (8) is the incentive compatibility constraint. Notice that the precise way in which incentive compatibility controls truthtelling is determined endogenously through the contracts choice of m, P, k^*; hence, the form of the group (non-marginal, marginal or unprofitable) is also endogenous. We can combine the conditions (6) and (7) yielding

$$[\Pr(k \geq k^*, n)k^* + E(k|k < k^*, n)](H - L + cm) + n(L - cm) \geq (1+r)nB$$

Optimally, the lenders participation constraint (6) must bind. If it were slack, then P could be reduced which would also slacken the second constraint (7) and either slacken or have no effect on the incentive compatibility constraint depending on whether it is a marginal or nonmarginal group. Similarly, the incentive compatibility constraint must bind optimally since otherwise m could be reduced which would raise the objective. With a binding lenders participation constraint, the group repayment in terms of k^* and m is

$$P = \frac{(1+r)nB - (H - L + cm)E(k|k < k^*, n) - n(L - cm)\Pr(k < k^*, n)}{\Pr(k \geq k^*)}$$

Substituting out P, a non-defaulting group has surplus

$$\begin{aligned} S_G &= k(H - L + mc) + n(L - cm) - P \\ &= k(H - L + mc) + \frac{E(k|k < k^*, n)(H - L + mc)}{\Pr(k \geq k^*)} + \frac{n(L - cm) - (1+r)nB}{\Pr(k \geq k^*)} \end{aligned}$$

Hence,

$$\begin{aligned} ES_G &= E(k|k \geq k^*)(H - L + mc) + E(k|k < k^*, n)(H - L + mc) + n(L - cm) - (1+r)nB \\ &= E(k|n)(H - L) - mc(n - E(k|n)) - (1+r)nB + nL \end{aligned}$$

As mentioned earlier, there are three possible cases: (i) an unprofitable group if $k^* > E(k|n-1)+1$; (ii) a marginal group if $E(k|n-1) < k^* \leq E(k|n-1)+1$ and (iii) a non-marginal group if $k^* \leq E(k|n-1)$. The form of the incentive compatibility (IC) constraint differs between unprofitable, marginal and non-marginal groups:

$$\begin{aligned}
(1-m_u)[H-L] &\leq 0 \text{ for an unprofitable group} \\
(1-m_m)[H-L] &\leq \frac{G_T^e - P}{n} \text{ for a marginal group} \\
(1-m_n)[H-L+\frac{G_c^e - P}{n}] &\leq \frac{G_T^e - P}{n} \text{ for a nonmarginal group}
\end{aligned}$$

and hence the solutions will vary between these groups.

(i) Unprofitable group: $G_T^e, G_c^e < P$

Any successful borrower, who thinks the group will default regardless of his decision, should always cheat since he gets 0 from telling truth but has a chance of $H-L$ from cheating. Each of the successful borrowers thinks the same (they all use the expected return to form beliefs) and cheats, resulting in default of the group loan. In order to stop everyone cheating, the group has to set the audit probability $m_u = 1$.

(ii) Marginal group $G_c^e < P < G_T^e$

For a marginal group, we know $E(k|n-1) < k^* < E(k|n-1)+1$ and IC requires

$$(1-m_m)[H-L] \leq \frac{G_T^e - P}{n}$$

We must have the audit probability $m_m > 0$. A detailed proof is in Appendix B. Basically, in a marginal group if $m_m = 0$, the payoff from truthtelling (his share of the expected group surplus) is less than the gain from cheating, violating the above IC constraint. Therefore, to ensure truthtelling requires $m_m > 0$.

For a marginal group which requires audit, m_m solves

$$(1-m_m)[H-L] = \frac{(G_T^e - P)_{m=m_m}}{n} \tag{9}$$

We must also have $m_m < 1$ optimally. As the minimum value of the left-hand side (LHS) of (9) is 0 and $G^e_T - P > 0$ for a marginal group, it would be possible to reduce m_m, raise borrower surplus, and still satisfy (9). So, in fact the group must have $0 < m_m < 1$.

Solving (9) for m_m (see Appendix C)

$$m_m = \frac{n((1+r)B - L) + [\Pr(k \geq k^*)(n-1-E(k|n-1)) - E(k|k<k^*,n)](H-L)}{c[\Pr(k \geq k^*)(E(k|n-1)+1) + E(k|k<k^*,n) - n] + n[\Pr(k \geq k^*)(H-L))}$$

The relative audit probabilities of an individual loan and a group loan for a marginal group play a large role in their relative efficiency. One comparison of interest is in the optimal audit probabilities of individual and group loans. We can show (the proof is in Appendix C):

Proposition 1 $m_m > m_I$ *iff*

$$\frac{\bar{p}H + (1-\bar{p})L - (1+r)B - C(1-\bar{p})}{(1+r)B - L} > \frac{(G^e_T - P)_{m=1}/n}{(H-L) - (G^e_T - P)_{m=0}/n} \tag{10}$$

The LHS of (10) is the profit rate on an individual loan with truthtelling and $m_I = 1$. In the LHS of (10), the numerator is the expected social revenue net of the audit cost and the denominator is the social cost of the project, so the ratio measures the social profit rate on an individual loan. While $m = 1$ ensures truthtelling, borrowers are tempted to cheat if $m = 0$. Thus, the ratio on the right-hand side is the borrower's gain with $m = 1$ relative to the net gain from cheating with $m = 0$. If the net gain from cheating is large enough to make condition (10) hold, m_m will need to be sufficiently high to deter cheating.

(iii) Nonmarginal group: $G^e_T, G^e_c > P$

IC requires

$$(1-m_n)[H - L + \frac{(G^e_c - P)_{m=m_n}}{n}] = \frac{(G^e_T - P)_{m=m_n}}{n}$$

Since optimally IC binds, using the expressions for $G^e_c - P$ and $G^e_T - P$ generates a convex quadratic function of m (see Appendix D):

$$\begin{aligned} F(m_n) \quad = \quad & \frac{n - E(k|n-1)\Pr(k \geq k^*) - E(k|k < k^*)}{n \Pr(k \geq k^*)} c m_n^2 \\ & + \left[\frac{(1+r)B - L}{\Pr(k \geq k^*)} - \frac{(H-L)}{n}\left(n + E(k|n-1) + \frac{E(k|k < k^*)}{\Pr(k \geq k^*)}\right) - \frac{c}{n}\right] m_n \\ & + \frac{(H-L)(n-1)}{n} = 0 \end{aligned}$$

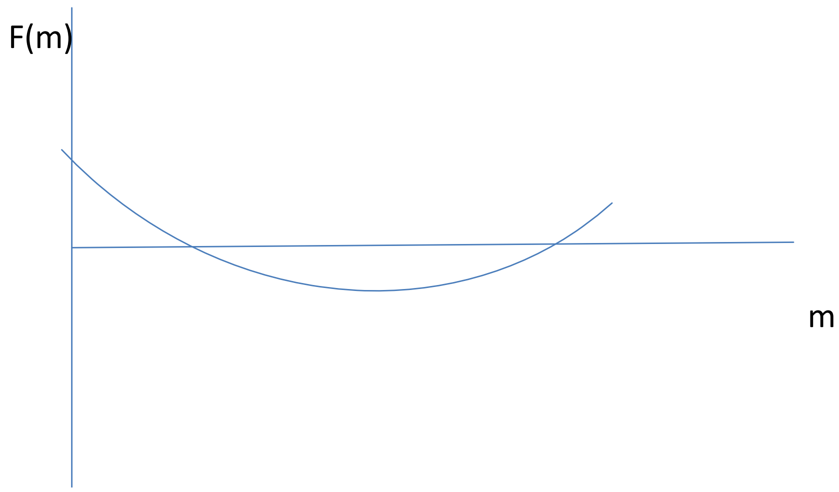

Figure 1: Audit probability for nonmarginal group (m_n)

The intercept is positive. The first term is positive because $(n - E(k|n-1))\Pr(k \geq k^*) + E(n-k|k < k^*) > 0$. The slope at $m_n = 0$ is negative because the second term is negative (see proof in Appendix D). Thus, $F(m_n)$ is convex. So there are two positive roots (λ_1, λ_2) as shown in Figure 1. Incentive compatibility with efficient audit requires the lowest m ensuring that $F(m_n)$ is non-positive; hence, the lower root of the quadratic gives the required audit probability.

The root must be less than unity. If $m_n = 1, G_T^e - P \geq 0$. So the IC constaint is slack and reducing m_n marginally still satisfies IC but raises group surplus with truth telling

$$\frac{\partial ES_G}{\partial m} = -c[n - E(k|n)] < 0$$

Hence, any m_n for a nonmarginal group must be in $(0, 1)$.

The efficiency of individual and group loans depends on the relative expected surpluses per borrower of the two systems. In turn, this depends on the difference in the audit cost per loan and in the audit probabilities. We can establish a result under which the audit probability for a nonmarginal group is higher than that with an individual loan for any given individual loan audit probability as follows (see the detail in Appendix D).

Proposition 2 $m_n > m_I$ *iff*

$$c < \frac{(n - 1 - nm_I)(H - L)}{m_I} - (G_c^e - P)_{m=m_I},$$

equivalent to

$$(1-m_I)\left[(H-L)+\frac{(G_c^e-P)_{m=m_I}}{n}\right]>\frac{(G_T^e-P)_{m=m_I}}{n}$$

where at the individual loan optimum

$$m_I=\frac{(1+r)B-L}{p(H-L)-C(1-p)}$$

Proposition 2 shows that if the group applies $m=m_I$ and the gain from truthtelling is lower than that from non-audited cheating, more cheats are encouraged until all successes cheat as they know that the group will not default no matter whether they tell the truth or lie. This indicates that the audit probability applied is too low. To avoid this, the group should raise the audit probability, resulting in $m_n>m_I$.

2.3.2 Group lending vs individual lending

The difference in interest rates between individual and group loans is

$$R_G-R_I=\frac{(1+r)nB-G(k|k<k^*,n)}{\Pr(k\geq k^*,n)nB}-\frac{m_I(H-L)+cm}{B}$$

See Appendix E for derivation of this equation. When $k<k^*$, the group revenue is less than the repayment required which is set to break even with the lender's cost of funding the projects; therefore, the first term is positive while the second term is negative. Therefore, whether R_G is less or greater than R_I depends on the distribution of p_i and values of $n, H-L, B, c, r$ and optimal audit probabilities for both individual and group lending.

The ranking of the welfare (expected surplus per borrower) of individual and group loans is identical to that by the expected audit cost:

$$\begin{aligned}ES_G-nES_I &= E(k|n)(H-L)-mc(n-E(k|n))+nL-n\bar{p}H-n(L-m_IC)(1-\bar{p})\\ &= (n-n\bar{p})(m_IC-mc)\end{aligned}$$

where m is the audit probability varying with whether the group is unprofitable, marginal or non-marginal. Because $n>n\bar{p}$, whether the group loan's expected surplus per borrower is greater than the individual loan's depends on the difference between the audit cost per borrower of both loan forms.

3 Simulation

The above comparisons of the features of optimal individual and group loans are implicit. In order to identify the loan form with the highest welfare or best industrial criteria (lowest interest rate and expected default rate), we choose a parametric form for the distribution of p_i which allows for a wide range of the correlation of risks between borrowers, its mean and its variance. In Section 2, the distribution of k was in a general form $g(k, p_1..p_n) = h(k|p_1..p_n)f(p_1..p_n)$. $h(k|p_1..p_n)$ is a binomial distribution, but the restriction we add is that $f(p_1..p_n)$ is the product of beta distributions. That is, the probability of $(p_1..p_n)$ is the probability of a sample of size n from a beta distribution with positive parameters α and β. This is called the betabinomial[10] which can cover a variety of skewness situations and degrees of correlation between project outcomes. Each borrower receives a draw from the beta distribution of a chance of success p_i; given this, the actual number of successes follows a binomial distribution[11]. Here, the number of successes coming from the sample drawn for n borrowers has density

$$h(k \mid p_i; \alpha, \beta) = \frac{n!}{k!(n-k)!} p_i^k (1-p_i)^{n-k} \frac{p_i^{\alpha-1}(1-p_i)^{\beta-1}}{B(\alpha, \beta)} \tag{11}$$

where

$$B(\alpha, \beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)} = \frac{(\alpha-1)!(\beta-1)!}{(\alpha+\beta-1)!}$$

As the parameters α and β vary, the values of mean, variance and correlation between the chance of success for different borrowers differ. With positive α and β, the correlation must always be positive. Keeping $\bar{p}$ constant, the lower are both α and β, the higher are the correlation and the variance of risks (see Figure 2).

For the simulations, we take a range of values of α and β yielding combinations of the probability of success $\bar{p}$ and correlations ρ (e.g. $\alpha = 0.132, \beta = 0.198$ yields $\bar{p} = 0.4$ and $\rho = 0.75$ and $\alpha = 0.5, \beta = 0.5$

[10] The correlated binomial distribution can be viewed as a special case of heterogeneous distribution where risks are heterogeneous but correlated.

[11] Here, X_i is the outcome for the i^{th} borrower ($X_i = 0, 1$ with 1 being success), the conditional distribution $X_i|p_i$ is Bernouilli(p_i) and the marginal distribution of p_i is Beta(α, β). Thus the joint probability of (X_i, p_i) =Bernouilli(p_i)Beta(α, β). Recall that the Bernouilli(p) density is $p^k(1-p)^{1-k}, k = 0, 1$; Bernouilli(p) has mean p and variance $p(1-p)$. The mean of the beta-distributed probability $\bar{p}$ is $\alpha/(\alpha+\beta)$ and its variance is $\rho\bar{p}(1-\bar{p}) = \alpha\beta/[(\alpha+\beta)^2(1+\alpha+\beta)]$ (Moraux, 2010). The correlation between the binary outcomes across any two individual projects is $\rho = 1/(1+\alpha+\beta)$.

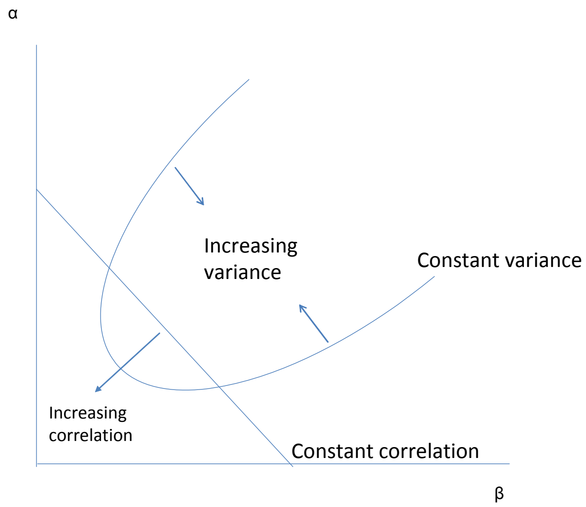

Figure 2: Correlation and Variance of p_i with varying α and β

yields $\bar{p} = 0.5$ and $\rho = 0.5$). We also provide the simulation result for the case of zero correlation which is equivalent to the binomial case with identical and independent risks (iid). For the individual loan, the mean probability of success from the binomial and betabinomial are identical. For each case, we take $H = 10, B = 4, r = 0.1$ and vary the combinations of group size (n), "collateral" value ($L = 2, 3$), audit cost for individual loans ($C = 0.1, 0.2$) and audit cost for group loans ($c = 0.1, 0.2$) to simulate the impact of the variation of $H - L$ (gain from cheating), audit costs (c, C) and group size on the interest rate, default rate, welfare and thus the optimal lending form (individual loans correspond to $n = 1$).

The optimal m, P and k^* are obtained for each group. These variables depend on each other as well as the parameters set above; k^* determines whether the group is unprofitable, marginal or nonmarginal. The simulation results in Figure (3-6) shows four selected cases where $L = 2, c = 0.1, C = 0.2$: (i) $\rho = 0, \bar{p} = 0.5$;(ii) $\rho = 0.5, \bar{p} = 0.5$; (iii)$\rho = 0, \bar{p} = 0.4$; (iv) $\rho = 0.75, \bar{p} = 0.4$ and two additional cases where (v) $L = 2, c = C = 0.2; \rho = 0.75, \bar{p} = 0.4$ and (vi) $L = 3, c = 0.1, C = 0.2, \rho = 0, \bar{p} = 0.4$.

3.1 Optimal contract variables

For each combination of parameters, we contrast the optimal outcomes for the group loan with those of an individual loan. From P, we derive the interest rate on each loan form. Figure 3 illustrates that the individual loan's interest rate is higher than the group loan's interest rates for all group sizes - that is, effectively repayment per borrower is lower with a group loan. The exceptions to this generalisation are cases (iv) and (v) where the mean probability of success for individual projects is low (e.g. $\bar{p} = 0.4$) but the correlation between the chance of success for different borrowers is high (e.g. $\rho = 0.75$); the interest rate for the group with 2 borrowers (115.37%) is higher than that for the individual loan (105.84%). It also shows that the interest rate generally declines as n increases, especially when the outcomes of projects are iid. Consequently, the ratio k^*/n shown in Figure 4 declines in a scissor pattern as the group becomes larger, consistent with Baland et al. (2013). In the groups with correlated risk, although the decrease in interest rates becomes insignificant as n becomes larger (e.g. $n > 3$), the default rate does not decline with n, unlike the iid cases.

For a given mean, higher correlation raises the risk for group loans and hence the interest rate and repayment required. This requires a higher number of successes for the group to be solvent. As a result, the expected default rate is higher (comparing cases (i) and (ii) with $\bar{p} = 0.5$ in Figure 4). For cases (iv) and (v) where $\rho = 0.75$, k^*/n in Figure 4 is equal to 1 in small groups ($n = 1, 2$, and 3); the groups are classified as unprofitable group ($k^* > E(k|n)$) and the individual loan's default rate is lower than that of the group loan[12]. In these cases, the interest rate for a 2-borrower group loan is higher than that for an individual loan, causing the default rate in the group loan to be higher than the individual loan. Although the interest rate declines for a 3-borrower group, the default rate does not fall; the high correlation raises the chance of a large number of fails requiring high k^*/n for the group to be solvent. For the groups with more than 3 borrowers, neither the interest rate nor the default rate drops as the group becomes larger.

Figure 4 also highlights that not only does k^*/n fall with an increase in n in iid cases, but it also falls with an increase in L and $\bar{p}$ (compare (iii) with (vi) for L and compare (i) and (iii) for $\bar{p}$). This finding is consistent with the evidence of a negative relationship between collateral and the interest rate in the literature (see for example, Agarwal and Hauswald (2010) and Menkhoff et al. (2012)).

[12] Baland et al. (2013) also documented that with bank and social sanctions, MFIs' lending may shift toward individual loan when there is a risk of strategic default by risk-neutral borrowers.

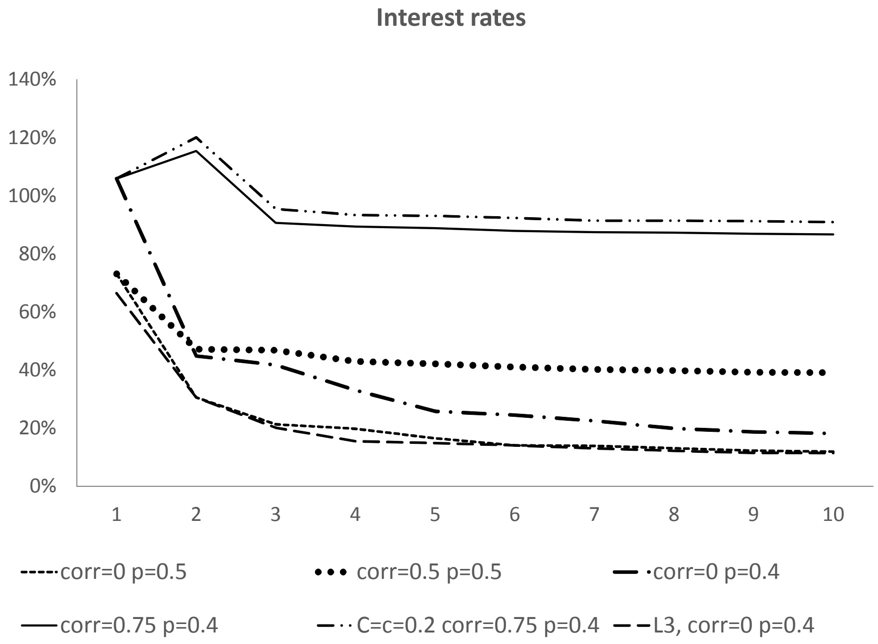

Figure 3: Interest rates for individual and group lending

Moreover, Figure 3 shows that the interest rate rises with an increase in the audit cost. When the group's audit cost (c) increases from 0.1 to 0.2, the interest rate shifts up (comparing cases (iv) and (v)).

3.2 The relative social merits of individual and group lending

As shown in Figure 4, the groups can be classified into three types: (i) an unprofitable group if $k^* > E(k|n-1)+1$ e.g. $n = 2$ and 3 with $\rho = 0.75, \bar{p} = 0.4$; (ii) a marginal group if $E(k|n-1) < k^* \leq E(k|n-1)+1$ e.g. any n with $\rho = 0.5, \bar{p} = 0.5$ and (iii) a non-marginal group if $k^* \leq E(k|n-1)$ e.g. $n > 4$ with $\rho = 0, \bar{p} = 0.5$. In accordance with the type of group, Figure 5 plots the values of m for $1 \leq n \leq 10$. The group should apply $m = 1$ if the mean and correlation of risk is high (the group is unprofitable). In other cases optimally, the value of m is lower than 1. The higher the value of $\bar{p}$, the lower the value of m. In addition, the lender or borrowers can apply a lower m as the collateral value (L) increases. Given a group size n, non-marginal groups can apply a lower m compared to their

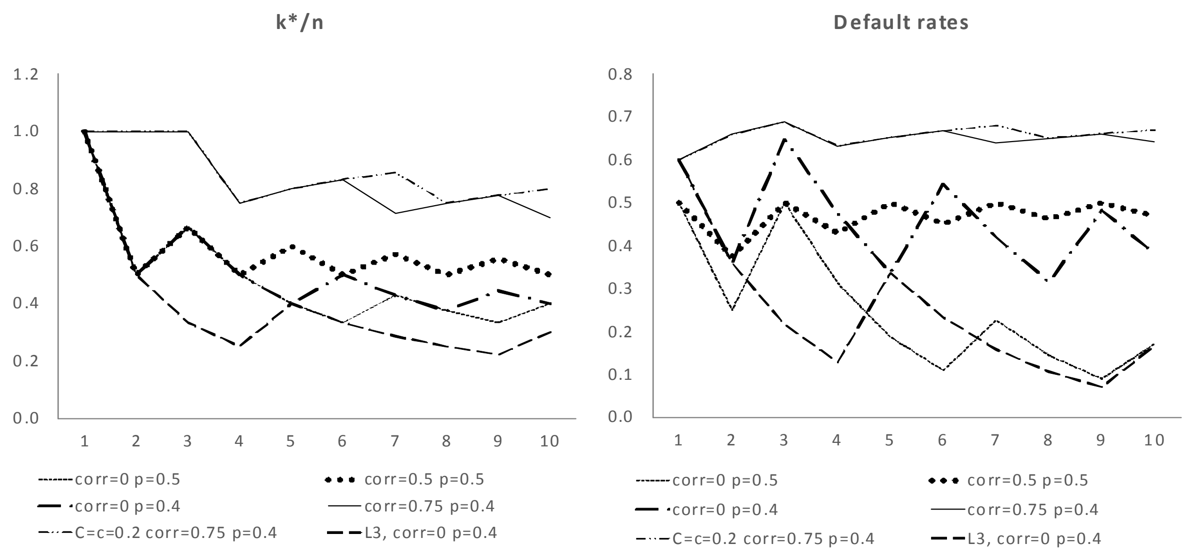

Figure 4: The success ratios and default rates

marginal counterparts. With the same value of c but lower m, the expected audit cost is lower and the expected surplus is higher in nonmarginal groups. So the borrower would prefer to be in a group whose optimal policy makes it nonmarginal (e.g. groups with no correlation between project outcomes or high mean probability of success) to marginal groups (e.g. groups with high correlation between project outcomes and low mean probability of success). In contrast, the audit probability should be higher in larger groups. A reason for this can be that the interest rate and k^*/n for group loans are typically lower than individual loans and thus the excess number of successes $(n - k^*)$ is higher. For a given case as n increases, the gain from undetected cheating tends to increase due to the fall in k^*/n and default rate which allows the undetected cheat a share of the group surplus (e.g. the group tends to become nonmarginal). Thus, group loans require higher audit probability than individual loans as shown in Figure 5. In a larger group, this problem increases and A higher audit probability is required.

The expected audit cost shows two main features of interest. Firstly, it is generally highest for individual loans largely due the audit cost advantage of the group. Secondly, for group loans, the 2-member group has the lowest expected audit cost. The audit cost in a group loan rises with group

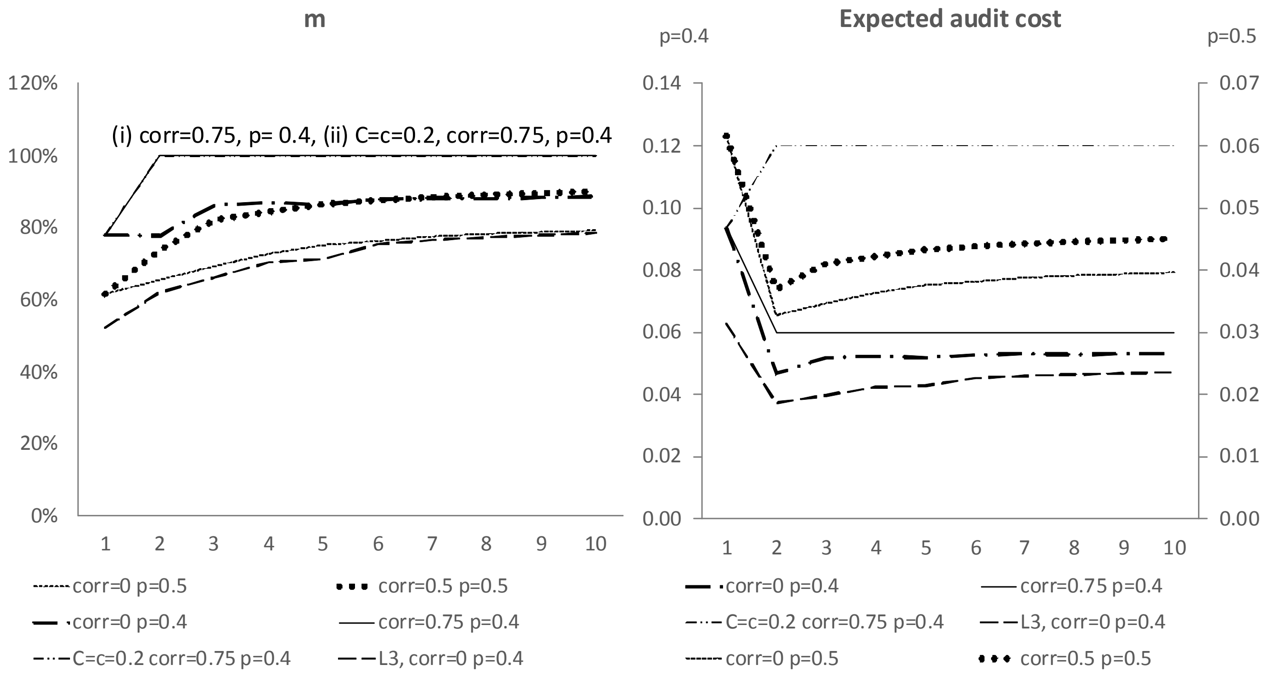

Figure 5: Audit probability and cost per borrower

size both because the number of reported fails rises and because m rises although the rise is small. Overall on balance when $c < C$, a group with 2 members has the lowest expected audit cost and hence the highest expected surplus per borrower (see Figure 6). When the cost of audit by borrowers is as large as the cost of audit by the lender, the expected audit cost of an individual loan is below the per capita cost of a group loan (e.g. comparing cases (iv) $\rho = 0.75, \bar{p} = 0.4$ and (v) $L = 2, c = C = 0.2; \rho = 0.75, \bar{p} = 0.4$). In other words, in per capita terms the advantage of group lending over individual lending disappears if the audit costs per failed borrower are identical in the two loan forms. The results indicate that, choosing m optimally, a two-person group loan dominates individual loans exactly because of the lower policing cost of the group loan. Other indications are that the expected surplus per borrower rises with $\bar{p}$ and L but falls slightly as the group becomes larger and the borrowers' outcomes have higher correlation. The overall picture is then that small group loans ($n = 2$) can dominate individual loans when the group has a cost advantage in audit. But if the audit costs are identical, then individual loans dominate group loans in welfare terms.

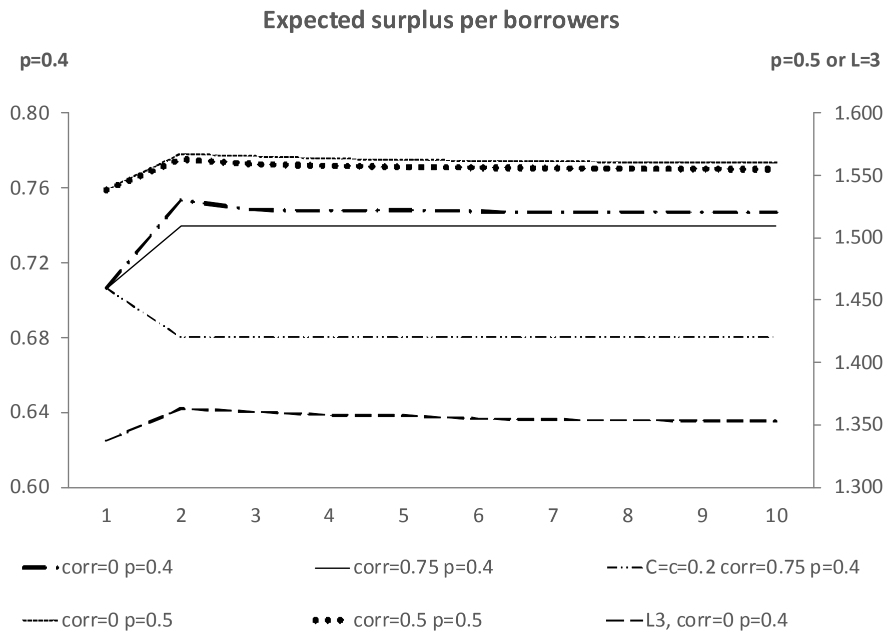

Figure 6: Expected surplus per borrowers for individual and group lending.

To sum up, we know that with risk neutrality and an audit cost advantage for the group, group loans dominate individual loans in terms of interest and default rates and indeed welfare (expected surplus per borrower). On interest and default rates, the only exception is when the group has high individual risk which is also highly correlated among borrowers. With iid risk, the group loan with joint liability is often seen as having better risk diversification possibilities than individual loans (because it has possibilities of cross subsidisation within the group), and largely because the chance of a high number of simultaneous fails and group default is reduced. This diversification gain should increase with the size of the group. However, with asymmetric information and the need for costly audit, the advantage can be dissipated since all fail reports within the group must be audited.

We find that the benefit of group loans from decreasing interest rates becomes insignificant as the group has more than 5 members. But in terms of expected surplus, the advantage of the group loan rests strongly on the group having a cost advantage in audit. Amongst different size groups, small groups ($n = 2$) are welfare preferable but the difference between $n = 2$ and $n = 10$ in welfare is small.

With a slight fall in expected surplus per borrower as n increases, the group' s lower interest rate and default rate outweighs the smaller gain in surplus, so the group could choose $2 \leq n \leq 5$ (except for the group which has low mean probability of success and outcomes are highly correlated). Similarly, Devereux and Fishe (1993) find that a small group size is important in determining group loan success and a common group size is 3-5 members. The heuristic argument of Abbink et al. (2006) is that 3 is sufficient to get reasonable risk diversification and 5 is an upper bound set by the requirement for high solidarity in the borrower group to police repayment by individuals. Therefore, on all these counts group loans to small groups tend to be preferred to individual loans unless there is high correlation and low $\overline{p}$. If group lending does not have a cost advantage in auditing ($C = c$), group lending still has lower interest and default rates, but individual loans dominate group loans on welfare grounds. This may explain why individual loans are more common in urban areas where there is less information asymmetry (e.g. commercial banks have better information system about borrowers living nearby. In rural areas, transaction and information costs are higher for urban banks, so group loans through cooperatives or self-help groups are more common).

4 Conclusions

In this paper, we try to compare the structure of individual and group lending under asymmetric information about individual project outcomes between a borrower and all other borrowers and the single lender. We characterise the optimal contract forms in terms of welfare.

There are some conceptual innovations. With asymmetric information and costly audit possibilities, incentive compatibility comes into play. The nature of the incentive compatibility restrictions are different between individual and group loans. For a group loan, there are alternative ways to achieve truthtelling; the optimal way is determined as part of the contract problem which depends on the expected profitability of the group and the risk distribution. Hence, the optimal audit probability of reported fails varies between groups of different average profitability and between individual and group loans. The key magnitudes are the interest rate, the probability of default and the audit probability.

We derive some theoretical comparisons of these in different loan situations.

Our framework deals with one important feature of most group loan settings which are geographically concentrated creating correlation between different borrower risks. Another issue is the idea that since group loans are usually to quite small groups in a local area, self enforcement of good behaviour by group members is cheaper than enforcement by an external lender. To identify the best form of loan and the best size of group, we conduct some numerical simulations using a betabinomial distribution of individual project risks. This allows for varying degrees of positive correlation between individual borrower risks. We also allow for a cost advantage of group members in auditing reports of the project outcome of different borrowers. Here, we find that the audit cost advantage, the degree of correlation and the chance of success on individual projects all play important parts in determining the incentive compatible contract, the best type of loan and the optimal group size. Usually small group loans (ie 2 members in the group) are the best form by most criteria so long as the group has an audit cost advantage over the external lender. With a slight fall in expected surplus per borrower as n increases, the group' s lower interest rate and default rate outweighs the smaller gain in surplus when $2 \leq n \leq 5$. Our results support the empirical evidence that group lending institutions reporting low defaults provide loans to small groups (Assadi and Ashta, 2014).

We have used a simple setting of two state outcomes for each individual. This could obviously be extended to a continuum of states. With iid risks, a single continuous distribution could be used for each project. The mixture distribution idea underlying the beta-binomial distribution could still be used, leading to a compound distribution of revenues. A more important limit is the static nature of the analysis. The dynamics are primarily very important in analysing the compliance mechanisms in individual and group lending (Sinn,2013). However, abstracting from these highlights the determinants of default risk, interest rate and welfare for different group sizes and probability distributions of project returns.

Appendix A: Group lending is feasible

In general, k^* may not exist - that is, even with n successes there may not be enough group revenue. However, under our assumptions about project returns and cost, there is always a unique smallest k^*. With zero successes the group cannot afford to repay and hence must default. If all group members succeed, then group revenue is $G(n) = nH > (1+r)nB$ and the group can certainly afford to repay. Group revenue is increasing in the number of successes; hence, there must be a smallest critical number of successes k^* above which the group can repay and below which the group defaults. This just requires $H > (1+r)B + C > L$. To see this formally

$$\begin{aligned}(1+r)nB &\leq \Pr(k \geq k^*)G(k^*) + E(G(k)|k < k^*) \qquad (12)\\ &\leq \Pr(k \geq k^*)[(H - L + cm)k^* + n(L - cm)] + E((H - L + cm)k + n(L - cm)|k < k^*)\\ &\leq n(L - cm) + (H - L + cm)[\Pr(k \geq k^*)k^* + E(k|k < k^*)]\\ &\leq n(L - cm) + (H - L + cm)Ek\end{aligned}$$

Applying the maximum audit cost ($m = 1$ and $c = C$) yields $(1+r)B \leq L - C + (H - L + C)\frac{Ek}{n}$. Thus, there is a lowest k^* so long as $[(1+r)B - L + C]/(H - L + C)$ is not greater than the mean ratio of successes (Ek/n). The minimal k^* is unique since the group revenue is increasing in k.

Appendix B: Audit probability in a marginal group must be positive

If $m_m = 0$, the successful borrower could cheat; if he does, he expects the group to default and his expected total gain is $H - L$. If he tells the truth, he gets an equal share of the group surplus with $1 + E(k|n-1)$ successes:

$$\frac{(1 + E(k|n-1))}{n}(H - L) + \frac{E(k|k < k^*, n)](H - L)}{n\Pr(k \geq k^*)} + \frac{L - (1+r)B}{\Pr(k \geq k^*)}$$

since

$$\begin{aligned}G_T^e - P &= \frac{[\Pr(k \geq k^*)(E(k|n-1) + 1) + E(k|k < k^*, n)](H - L + cm_m)}{\Pr(k \geq k^*)}\\ &\quad + \frac{n(L - cm_m - (1+r)B)}{\Pr(k \geq k^*)}\\ (G_T^e - P)_{m=0} &= \frac{[\Pr(k \geq k^*)(E(k|n-1) + 1) + E(k|k < k^*, n)](H - L) + n(L - (1+r)B)}{\Pr(k \geq k^*)}\end{aligned}$$

His gain from truthtelling as compared with cheating at $m_m = 0$ is

$$\begin{aligned}
\frac{(G_T^e - P)_{m=0}}{n} - (H-L) &= \frac{[\Pr(k \geq k^*)(E(k|n-1)+1-n) + E(k|k<k^*,n)](H-L)}{n\Pr(k \geq k^*)} \\
&\quad + \frac{L-(1+r)B}{\Pr(k \geq k^*)} \\
&= \frac{[\Pr(k \geq k^*)E(k|n-1) + E(k|k<k^*,n)](H-L) + n(L-(1+r)B)}{n\Pr(k \geq k^*)} \\
&\quad + \frac{1-n}{n}(H-L) \\
&= \frac{(G_c^e - P)_{m=0}}{n} - \frac{n-1}{n}(H-L) < 0
\end{aligned}$$

as $G_c^e < P$ for the marginal group. To prevent this, we need $m_m > 0$.

Appendix C: Optimal audit probabilities for individual and marginal group loan

For a marginal group,

$$(1-m_m)[H-L] \leq \frac{(G_T - P)_{m=m_m}}{n}$$

We can then derive an explicit solution for m_m:

$$n(1-m_m)[H-L] = G_T - P \tag{13}$$

$$\begin{aligned}
n(1-m_m)[H-L] &= \frac{[\Pr(k \geq k^*)(E(k|n-1)+1) + E(k|k<k^*,n)](H-L+cm_m)}{\Pr(k \geq k^*)} \\
&\quad + \frac{n(L-cm_m)-(1+r)nB}{\Pr(k \geq k^*)}
\end{aligned}$$

$$\begin{aligned}
(1+r)nB - [\Pr(k &\geq k^*)(E(k|n-1)+1-n) + E(k|k<k^*,n)](H-L) - nL \\
&= m_m[\Pr(k \geq k^*)n(H-L) + c\,(\Pr(k \geq k^*)(E(k|n-1)+1) + E(k|k<k^*,n) - n)]
\end{aligned}$$

$$m_m = \frac{(1+r)nB - [\Pr(k \geq k^*)(E(k|n-1)+1-n) + E(k|k<k^*,n)](H-L) - nL}{\Pr(k \geq k^*)n(H-L) + c\,(\Pr(k \geq k^*)(E(k|n-1)+1) + E(k|k<k^*,n) - n)}$$

Compare m_m with m_I :

$$m_I = \frac{(1+r)B - L}{\bar{p}(H-L) - C(1-\bar{p})},$$

$$\begin{aligned}
m_m &= \frac{(1+r)nB - [\Pr(k \geq k^*)(E(k|n-1)+1-n) + E(k|k<k^*,n)](H-L) - nL}{\Pr(k \geq k^*)n(H-L) + c\,(\Pr(k \geq k^*)(E(k|n-1)+1) + E(k|k<k^*,n) - n)} \\
&= \frac{(1+r)nB - nL + [n\Pr(k \geq k^*) - X](H-L)}{n\Pr(k \geq k^*)(H-L) - c[n-X]}
\end{aligned}$$

where

$$X = E(k|k < k^*, n) + \Pr(k \geq k^*)(E(k|n-1) + 1)$$

At $m = 0$,

$$(G_T^e - P)_{m=0} = \frac{X(H-L)}{\Pr(k \geq k^*, n)} - \frac{(1+r)nB - nL}{\Pr(k \geq k^*, n)}$$

and at $m = 1$,

$$\begin{aligned}
(G_T^e - P)_{m=1} &= \frac{X(H-L)}{\Pr(k \geq k^*, n)} - \frac{(1+r)nB - nL}{\Pr(k \geq k^*, n)} - \frac{c(n-X)}{\Pr(k \geq k^*, n)} \\
(G_T^e - P)_{m=1} &= (G_T^e - P)_{m=0} - \frac{c(n-X)}{\Pr(k \geq k^*, n)}
\end{aligned}$$

So

$$\begin{aligned}
m_m &= \frac{(1+r)nB - nL + n\Pr(k \geq k^*)(H-L) - X(H-L)}{n\Pr(k \geq k^*)(H-L) - c[n-X]} \\
&= \frac{\Pr(k \geq k^*)\left[n(H-L) - G_T^e - P)_{m=0}\right]}{\Pr(k \geq k^*)\left[n(H-L) + (G_T^e - P)_{m=1} - (G_T^e - P)_{m=0}\right]} \\
&= \frac{(G_T^e - P)_{m=0} - n(H-L)}{(G_T^e - P)_{m=0} - n(H-L) - (G_T^e - P)_{m=1}} \\
&= \frac{1}{1 - \frac{(G_T^e - P)_{m=1}}{(G_T^e - P)_{m=0} - n(H-L)}}
\end{aligned}$$

m_m would be greater than m_I if

$$\begin{aligned}
\frac{1}{1 - \frac{(G_T^e - P)_{m=1}}{(G_T^e - P)_{m=0} - n(H-L)}} &> \frac{(1+r)B - L}{\bar{p}(H-L) - C(1-\bar{p})} \\
\bar{p}(H-L) - C(1-\bar{p}) &> ((1+r)B - L)\left(1 - \frac{(G_T^e - P)_{m=1}}{(G_T^e - P)_{m=0} - n(H-L)}\right) \qquad (14) \\
\bar{p}H + (1-\bar{p})L - C(1-\bar{p}) &> (1+r)B\left(1 - \frac{(G_T^e - P)_{m=1}}{(G_T^e - P)_{m=0} - n(H-L)}\right) \\
&\quad + L\frac{(G_T^e - P)_{m=1}}{(G_T^e - P)_{m=0} - n(H-L)} \\
\bar{p}H + (1-\bar{p})L - C(1-\bar{p}) &> (1+r)B - \frac{(G_T^e - P)_{m=1}}{(G_T^e - P)_{m=0} - n(H-L)}((1+r)B - L) \\
\frac{\bar{p}H + (1-\bar{p})L - (1+r)B - C(1-\bar{p})}{(1+r)B - L} &> \frac{(G_T^e - P)_{m=1}/n}{(H-L) - (G_T^e - P)_{m=0}/n} \qquad (15)
\end{aligned}$$

where

$$(G_T^e - P)_{m=1} = (G_c^e - P)_{m=0} + (H-L) - \frac{c(n - \Pr(k \geq k^*)(E(k|n-1) + 1) - E(k|k < k^*, n))}{\Pr(k \geq k^*, n)}$$

Note that, from social desirability, $0 > L-(1+r)B > C(1-\bar{p})-\bar{p}(H-L)$ and $\bar{p}(H-L)-C(1-\bar{p}) > 0$.

The sign of LHS of (10) depends on the social desirability condition. The numerator of the right-hand side is the borrower's share of surplus and the denominator is the difference between the borrower's gain from cheating and his share of surplus if telling the truth.

Appendix D: Optimal audit probabilities for individual and non-marginal group loan

For a nonmarginal group,

$$(1-m_n)[H-L+\frac{(G_c-P)_{m=m_n}}{n}] \leq \frac{(G_T-P)_{m=m_n}}{n}$$

m_n solves

$$\begin{aligned}
(1-m_n)[H-L+\frac{(G_c-P)_{m=m_n}}{n}] &= \frac{(G_T-P)_{m=m_n}}{n} \\
(1-m_n)(H-L)-\frac{(G_T-G_c)_{m=m_n}}{n}-m_n\frac{(G_c-P)_{m=m_n}}{n} &= 0
\end{aligned}$$

$G_T - G_c = H - L + cm$ for all m, so

$$\begin{aligned}
(1-m_n)(H-L)-\frac{H-L+cm_n}{n}-m_n\frac{G_c-P}{n} &= 0 \\
(1-m_n-\frac{1}{n})(H-L)-\frac{cm_n}{n}-m_n\frac{G_c-P}{n} &= 0 \qquad (16)
\end{aligned}$$

Substituting

$$G_c-P=[E(k|n-1)+\frac{E(k|k<k^*,n)}{\Pr(k\geq k^*,n)}](H-L+cm)+\frac{n(L-cm)}{\Pr(k\geq k^*)}-\frac{(1+r)nB}{\Pr(k\geq k^*,n)}$$

into the LHS of (16):

$$\begin{aligned}
& (1-m_n-\frac{1}{n})(H-L)-\frac{cm_n}{n}-m_n\frac{G_c-P}{n} \\
= \; & (1-m_n-\frac{1}{n})(H-L)-\frac{cm_n}{n} \\
& -\frac{m_n}{n}\{[E(k|n-1)+\frac{E(k|k<k^*,n)}{\Pr(k\geq k^*,n)}](H-L+cm_n)+\frac{n(L-cm)}{\Pr(k\geq k^*)}-\frac{(1+r)nB}{\Pr(k\geq k^*,n)}\} \\
= \; & \left(1-m_n-\frac{1}{n}-\frac{m_n}{n}[E(k|n-1)+\frac{E(k|k<k^*,n)}{\Pr(k\geq k^*,n)}]\right)(H-L)-\frac{m_nL}{\Pr(k\geq k^*)} \\
& -\left(\frac{1}{n}+\frac{m}{n}[E(k|n-1)+\frac{E(k|k<k^*,n)}{\Pr(k\geq k^*,n)}-\frac{n}{\Pr(k\geq k^*)}]\right)cm+\frac{m_n(1+r)B}{\Pr(k\geq k^*,n)}
\end{aligned}$$

Therefore, IC for a nonmarginal group requires

$$\begin{aligned}
F(m_n) &= \frac{(n - E(k|n-1)\Pr(k \geq k^*) - E(k|k < k^*))cm_n^2}{n\Pr(k \geq k^*)} \\
&+ \frac{(1+r)nB - nL - (H-L)((n + E(k|n-1))\Pr(k \geq k^*) + E(k|k < k^*)) - c\Pr(k \geq k^*)}{n\Pr(k \geq k^*)} m_n \\
&+ \frac{(H-L)(n-1)}{n} \\
&= \frac{c(n - X + \Pr(k \geq k^*))}{n\Pr(k \geq k^*)} m_n^2 - ((H-L) + \frac{(G_c^e - P)_{m=0}}{n} + \frac{c}{n})m_n + \frac{(H-L)(n-1)}{n} = 0
\end{aligned}$$

as for any m

$$n - X + \Pr(k \geq k^*) = n - E(k|n-1)\Pr(k \geq k^*) - E(k|k < k^*)$$

$$\begin{aligned}
G_c^e - P &= \frac{[\Pr(k \geq k^*)E(k|n-1) + E(k|k < k^*, n)](H - L + mc)}{\Pr(k \geq k^*, n)} \\
&+ \frac{n(L - mc) - (1+r)nB}{\Pr(k \geq k^*, n)} \\
&= \frac{(X - \Pr(k \geq k^*, n))(H - L + mc)}{\Pr(k \geq k^*, n)} + \frac{n(L - mc) - (1+r)nB}{\Pr(k \geq k^*, n)}
\end{aligned}$$

If $m = 1$,

$$\begin{aligned}
(G_c^e - P)_{m=1} &= \frac{(X - \Pr(k \geq k^*, n))(H-L)}{\Pr(k \geq k^*, n)} + \frac{nL - (1+r)nB}{\Pr(k \geq k^*, n)} - \frac{c(n - X + \Pr(k \geq k^*, n))}{\Pr(k \geq k^*, n)} \\
&= (G_c^e - P)_{m=0} - \frac{c(n - X + \Pr(k \geq k^*, n))}{\Pr(k \geq k^*, n)}
\end{aligned}$$

Substituting this into $F(m_n)$ yields

$$F(m_n) = \frac{[(G_c^e - P)_{m=0} - (G_c^e - P)_{m=1}]}{n} m_n^2 - [\frac{(G_c^e - P)_{m=0} + n(H-L) + c}{n}]m_n + \frac{(H-L)(n-1)}{n}$$

The intercept is positive while the first and second terms are negative. Thus,

$$\begin{aligned}
m_n &= \frac{(G_c^e - P)_{m=0} + n(H-L) + c}{2\left[(G_c^e - P)_{m=0} - (G_c^e - P)_{m=1}\right]} \\
&- \frac{\sqrt{[(G_c^e - P)_{m=0} + n(H-L) + c]^2 - 4(H-L)(n-1)\left[(G_c^e - P)_{m=0} - (G_c^e - P)_{m=1}\right]}}{2\left[(G_c^e - P)_{m=0} - (G_c^e - P)_{m=1}\right]}
\end{aligned}$$

If $m_N > m_I$,

$$\begin{gathered}
\frac{(G_c^e - P)_{m=0} + n(H-L) + c}{2\left[(G_c^e - P)_{m=0} - (G_c^e - P)_{m=1}\right]} \\
- \frac{\sqrt{[(G_c^e - P)_{m=0} + n(H-L) + c]^2 - 4(H-L)(n-1)\left[(G_c^e - P)_{m=0} - (G_c^e - P)_{m=1}\right]}}{2\left[(G_c^e - P)_{m=0} - (G_c^e - P)_{m=1}\right]} > m_I \qquad (17)
\end{gathered}$$

Substituting

$$m_I = \frac{(1+r)B - L}{\bar{p}(H-L) - C(1-\bar{p})},$$

into (17) yields

$$\begin{aligned} & (G_c^e - P)_{m=0} + n(H-L) + c - 2m_I \left[(G_c^e - P)_{m=0} - (G_c^e - P)_{m=1} \right] \\ > \quad & \sqrt{\left[(G_c^e - P)_{m=0} + n(H-L) + c \right]^2 - 4(H-L)(n-1)\left[(G_c^e - P)_{m=0} - (G_c^e - P)_{m=1} \right]} \end{aligned}$$

That is,

$$\begin{aligned} & m_I^2 \left[(G_c^e - P)_{m=0} - (G_c^e - P)_{m=1} \right]^2 \\ & -m_I \left[(G_c^e - P)_{m=0} + n(H-L) + c \right] \left[(G_c^e - P)_{m=0} - (G_c^e - P)_{m=1} \right] \\ & +(H-L)(n-1) \left[(G_c^e - P)_{m=0} - (G_c^e - P)_{m=1} \right] \\ > \quad & 0 \end{aligned}$$

$$\begin{aligned} (H-L)(n-1) \quad &> \quad m_I((G_c^e - P)_{m=0} + n(H-L) + c) - m_I^2 \left[(G_c^e - P)_{m=0} - (G_c^e - P)_{m=1} \right] \\ (H-L)(n-1) \quad &> \quad m_I((G_c^e - P)_{m=0} + n(H-L)) - m_I^2 \left[(G_c^e - P)_{m=0} - (G_c^e - P)_{m=1} \right] + c m_I \end{aligned}$$

$$\begin{aligned} c \quad &< \quad \frac{(H-L)(n-1-nm_I)}{m_I} - (G_c^e - P)_{m=0} + m_I \left[(G_c^e - P)_{m=0} - (G_c^e - P)_{m=1} \right] \\ &< \quad \frac{(n-1-nm_I)(H-L)}{m_I} - \{ [m_I (G_c^e - P)_{m=1} + (1-m_I)(G_c^e - P)_{m=0}] \} \end{aligned} \tag{18}$$

So if c is smaller than $\frac{(n(1-m_I)-1)(H-L)}{m_I} - \{[m_I(G_c^e - P)_{m=1} + (1-m_I)(G_c^e - P)_{m=0}]\}$, $m_N > m_I$.

$$\begin{aligned} & (G_c^e - P)_{m=0} - m_I \left[(G_c^e - P)_{m=0} - (G_c^e - P)_{m=1} \right] \\ = \quad & (G_c^e - P)_{m=0} - m_I \frac{c(n - X + \Pr(k \geq k^*, n))}{\Pr(k \geq k^*, n)} \\ = \quad & \frac{(X - \Pr(k \geq k^*, n))(H-L)}{\Pr(k \geq k^*, n)} + \frac{nL - (1+r)nB}{\Pr(k \geq k^*, n)} - m_I \frac{c(n - X + \Pr(k \geq k^*, n))}{\Pr(k \geq k^*, n)} \\ = \quad & (G_c^e - P)_{m=m_I} \end{aligned}$$

Consequently, (18) can be rewritten as

$$\begin{aligned}
c &< \frac{(n-1-nm_I)(H-L)}{m_I} - (G_c^e - P)_{m=m_I} \\
n(H-L)+c &< \frac{(n-1)(H-L)}{m_I} - (G_c^e - P)_{m=m_I} \\
(n-1)(H-L) &> m_I\left[n(H-L)+(G_c^e-P)_{m=m_I}\right] + m_I c \\
(1-m_I)\left[n(H-L)+(G_c^e-P)_{m=m_I}\right] - m_I c &> -(n-1)(H-L) + \left[n(H-L)+(G_c^e-P)_{m=m_I}\right] \\
(1-m_I)\left[n(H-L)+(G_c^e-P)_{m=m_I}\right] &> (H-L) + m_I c + (G_c^e-P)_{m=m_I} \\
(1-m_I)\left[(H-L)+\frac{(G_c^e-P)_{m=m_I}}{n}\right] &> \frac{(G_T^e-P)_{m=m_I}}{n}
\end{aligned}$$

Appendix E Comparing Individual and Group Interest rates

From P_I,

$$1+R_I = \frac{m_I(H-L)+L}{B}$$

From P,

$$1+R_G = \frac{(1+r)B-(H-L+cm)E(\frac{k}{n}|k<k^*,n)-(L-cm)\Pr(k<k^*,n)}{\Pr(k\geq k^*)B}$$

$$\begin{aligned}
R_G - R_I &= \frac{(1+r)B-(H-L+cm)E(\frac{k}{n}|k<k^*,n)-(L-cm)\Pr(k<k^*,n)}{\Pr(k\geq k^*)B} - \frac{m_I(H-L)+L}{B} \\
&= \frac{(1+r)B-L+cm}{\Pr(k\geq k^*)B} - (H-L+cm)\frac{E(\frac{k}{n}|k<k^*,n)}{\Pr(k\geq k^*)B} - \frac{m_I(H-L)+cm}{B} \\
&= \frac{(1+r)nB-n(L-cm)}{\Pr(k\geq k^*,n)nB} - \frac{E(k|k<k^*,n)(H-L+cm)}{\Pr(k\geq k^*,n)nB} - \frac{m_I(H-L)+cm}{B} \\
&= \frac{(1+r)nB-G(k|k<k^*,n)}{\Pr(k\geq k^*,n)nB} - \frac{m_I(H-L)+cm}{B}
\end{aligned}$$

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