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Efficient audits by pooling projects

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Abstract

In a costly state verification model under commitment, the paper shows that jointly financing multiple independent projects reduces the deadweight loss of inefficient audits. This is true for both simultaneous and sequential audit, since each system reveals the same information about the project outcomes at the same cost. Moreover, the audit combination under sequential audit is indeterminate. Audits are decreasing in the reported income and, for sufficiently high projects profitability, deterministic for lower income reports.

Keywords: contracts, auditing, diversification.

JEL Nos: D82, D83, D86.

1 Introduction

In costly state verification models, costly audit is required to implement financial contracts. Borrowers must report their revenue outcomes and, under commitment, random audit is often sufficient to elicit truth-telling and cheaper than universal audit on all debtors who report default. However, even if stochastic, an audit still involves a deadweight loss, as it always just reveals a truthful report. An issue then arises of whether it is possible to reduce this deadweight loss.

In this paper we investigate one possible venue for reducing inefficient audits through multiple project financing. In particular, we consider an entrepreneur who has several projects but no financial resources to carry them out. He can apply for a separate loan for each project, or instead combine the various projects under one roof and apply for a loan from a single lender to finance all projects. The outcome of each project is ex-post private information to the entrepreneur, who, upon its realisation, has to send a report to the lender. The truthfulness of the report can be verified by the lender at a cost, who, in the case of joint project financing, has to decide also whether to audit one or both projects, and, in this latter case, whether to audit them sequentially or simultaneously. We study under what circumstances it is optimal to combine distinct projects under one roof, as opposed to setting them up as stand-alone projects each raising external financing on their own. In doing this, we explore the relative merits of three systems: individual project finance and random audit of individual project reported outcomes; sequential individual random audit of all the projects which are part of the joint loan based on a joint report of the number of defaults; simultaneous audit of all the projects which are part of the joint loan. We provide

a detailed characterization of the optimal contract.

Our principal conclusions are that the joint loan has lower deadweight loss than the individual loans, but that the two forms of the optimal contract, simultaneous or sequential audit of all the projects, are indifferent. This is because putting the projects under one roof allows the expected return to the lender across projects taken together to be set at its fair level, while with single project loans the lender would require the expected fair return on each project separately. In this sense, joint financing allows cross-subsidisation between the two projects. It also allows the policy space for audit to be enlarged, so that policing can be concentrated on states where it is most productive. Thus, the cross subsidisation (diversification) across loans and the richer audit policy lowers the default risk for the lender on multiple relative to individual loans. However, the order with which the audits occur, whether simultaneously or sequentially, is immaterial since, because of projects' independence, there is no gain from conditioning the audit of one project on the result of prior audit of another. Thus, there is an equivalence between simultaneous and sequential audit. Moreover, with sequential audit there is an indeterminacy in the audit combination for steps in the sequence. What matters is the overall net probability of the audit of a reported list of fails. For example with a report of two fails, a sequential audit strategy sets an audit probability for the first fail and then another audit probability (conditional on the audit outcome of the first fail) on the second reported fail. The overall probability of auditing the two fail report is the product of these two audit probabilities and only this matters in the deadweight loss. Hence auditing the first fail intensively and then the second fail with a light touch is equivalent in welfare to auditing the first lightly and the second

intensively.

Audits are decreasing in the reported income and, depending on projects profitability, random or deterministic for lower income reports. In particular, when the project revenues are sufficiently high, then it is possible to cover the lender's investment and observation cost by auditing stochastically only lower income reports and never auditing higher income reports. However, when projects' profitability is not too high, then it is necessary to collect resources also from auditing higher income reports. In this case, lower income reports are audited deterministically, while higher income ones are audited randomly. These results recall some of the features of a standard debt contract and complements some early literature. Townsend (1979) and Gale and Hellwig (1985) establish that the optimal repayment and audit strategy in a costly state verification framework with commitment form a standard debt contract. That has deterministic monitoring of all states below some critical level, but all states above this level are not audited. Below the critical level repayments take the total project return and in all higher states (which are not audited) there is a constant repayment. Here we have pooling in the high states.

In addition, the result reflect also some of the features of costly state verification models. Allowing for stochastic monitoring, Border and Sobel (1985) and Mookherjee and Png (1989) show that generally the audit probabilities are interior and fall with the profitability of the state. In particular the highest revenue state is not audited. In our case the ordering of states is effectively by the number of projects which have failed. We find that states with a lower number of fails are audited less frequently than states with a higher number of fails. With relatively unprofitable projects we find that audit of the worst state must be with

certainty but with more profitable projects overall, audit of the worst state can be random.

In the two results of equivalence of simultaneous and sequential audit and indeterminacy in the audit combination under sequential audit, independence of the project returns matters. If returns on different projects are correlated, then knowing the outcome on one project gives information about the expected outcomes (and hence the need to audit) others. Within a no commitment setting, Phelan (2017) shows if there is correlation, sequential audit may dominate simultaneous audit. Our results imply that a non zero correlation is necessary for this (not just sufficient). If the project returns are uncorrelated, knowing the outcome on one project gives no information about the outcome of the other. Hence, auditing both project outcomes simultaneously is as desirable as auditing them sequentially.

There are various areas of relevant application. There is a literature on how multidivisional firms should finance and control the activities of their units. The firm secures finance for projects in its divisions. It should allow cross-subsidisation between divisions in appropriate circumstance. And if overall the firm is sufficiently profitable, it may be that light touch auditing and regulation of the divisions is socially best.

The remainder of the article is organised as follows. Section 1 discusses the related literature. Section 2 lays out the model assumption. Section 3 develops a standard costly state verification problem in which several individual projects are financed as standalones in the arm's length external capital market. Section 4 considers the case of two independent projects to illustrate the basic role of joint project financing in reducing the deadweight loss of audits, distinguishing between the case in which audits are sequential (Section 4.1) and simultaneous (Section 4.2). Section 4.3 compares the two scenarios. Section 5 extends

the model to consider the effects of relaxing the main assumption of the model. Section 6 compares the individual and joint project financing setting. Section 7 discusses some robustness issues. Section 8 concludes.

2 Related literature

The paper is related to several strands of the literature. The first and most obvious relation is with the literature on costly state verification. Some of the earliest papers in this field are Townsend (1979) and Gale and Hellwig (1985). With deterministic monitoring they find that the optimal contract involves a standard debt contract. There is zero monitoring of the highest state report, but certain monitoring of reports which do not cover the cost of the loan and the lender takes all the borrower's resources. In any state where the debtor can repay the loan there is a common repayment.

Allowing for stochastic monitoring, Border and Sobel (1985) and Mookherjee and Png (1989) show that generally the audit probabilities are interior and fall with the profitability of the state. In particular the highest revenue state is not audited.

Another strand of the literature considers multiple projects and the ways in which a lower expected audit cost can be achieved. Delegated monitoring (Diamond, 1984) is one such approach. Diamond shows that with many projects and many lenders there is a benefit to using an intermediary to perform audit partly because it eliminates the duplication of audit by different lenders, but also partly because the probability of getting multiple fails is smaller than the probability of getting a single fail. As the number of projects tends to infinity in Diamond's world, the chance of all projects failing tends to zero. With only a finite number of projects, Krasa and Villamil (1992) show that some restriction on the

distribution of project returns is necessary for the total audit cost on all projects to become small. In a multiple project world, Phelan (2017) shows that if the project outcomes are correlated, then the best audit policy is sequential, so that selected audits on a few projects can reveal useful information about outcomes on non-audited projects.

An alternative to reduce deadweight loss of costly audit is Menichini and Simmons (2014), who show that acquiring a costly public signal correlated with the project outcome is a valuable tool.

3 The Model Assumptions

An entrepreneur/borrower has two investment projects with uncorrelated returns, each costing I, which can be funded from a risk neutral investor. Each project gives a random return s, $s \in \{H, L\}$, with $H > I > L > 0$, with probabilities p,and $1 - p$ respectively, that is freely observable only to the entrepreneur and not to the investor. Once the return is realised, the borrower makes a report to the investor. Because of output unobservability, the entrepreneur has an incentive to report $s = L$. But since $I > L$, the only way for the investor to recoup the investment cost is to carry out an audit. Audit is observable and the result of it verifiable, and involves a cost $c > 0$.

When the two projects are brought under a single roof, four outcomes are possible: two successes, with probability p^2, two failures, with probability $(1-p)^2$, one success and one failure, with probability $2p(1-p)$. Thus, $s \in \{LL, HL, LH, HH\}$ with $2H > L+H > 2I > 2L$.The reports can be $i \in \{0, 1, 2\}$, where $i = 0$ denotes a report of zero successes, $i = 1$ denotes a report of one success (and one failure), and $i = 2$ denotes a report of 2 successes. If a single success is reported, the borrower also reports which project has failed.

In the following we consider two possible levels of ex-ante profitability of the project. In the first, we assume that the expected project returns using the revenues of two failures and those of one-success-one-failure with certain audit of every report in which there may be cheating covers the cost of the two loans fully, i.e., $[p^2 + 2p(1-p)](H+L) + (1-p)^2 2L > 2I + 2(1-p)^2 c$. This is equivalent to assuming:

Assumption 1

$$p(2-p)(H-L) - 2(1-p)^2 c > 2(I-L). \tag{1}$$

In the second case, there is sufficient expected profitability from the revenues of the three states for a contract to give a fair return to the lender even with audit for sure of the bottom two states, and for sure monitoring a single and a zero success report, i.e., $2p^2 H + 2p(1-p)(H+L-c) + 2(1-p)^2 L > 2I + 2(1-p)^2 c$. This is equivalent to assuming:

Assumption 2 *The NPV of each project net of its expected audit cost is strictly positive, i.e.,*

$$pH + (1-p)L - I - (1-p)c > 0. \tag{2}$$

This assumption ensures that the project is ex-ante profitable even if enforcement involves auditing every low state report.

Last, we assume that the left hand side of condition **(2)** is larger than the left hand side of condition (1), i.e.,

$$2[pH + (1-p)L - I - (1-p)c] > \left[p(2-p)(H-L) - 2(1-p)^2 c - 2(I-L)\right].$$

This is equivalent to assuming:

Assumption 3

$$p\left[p\left(H-L\right)-2\left(1-p\right)c\right]>0 \tag{3}$$

This means that there are pairs of projects which satisfy condition (2), but not condition (1). Throughout the paper we assume that condition (2) holds.

We first derive the optimal contract under Assumption (1) holding (Sections 5.1 and 6). We then relax Assumption (1) and derive the optimal contract (Section 7). For each of these scenarios we consider the case in which the borrower makes a report on the joint outcome of the two projects and, conditional on the report, the investor audits, sequentially or simultaneously. We take as a benchmark the standard case of single project financing, and, by comparing the expected values of these contracts, we determine whether projects should be financed as standalones or jointly, and, in the latter case, the optimal timing of audit.

4 Single project financing

Under single project financing, each project is funded as a stand-alone. A contract specifies repayments and the probability with which an audit will occur. Let $R_{\sigma|s}$ be the repayment due following a report $\sigma \in \{H, L\}$, and an audit which reveals that the state is $s \in \{H, L\}$, and $R_{\sigma|\cdot}$ be the repayment with report σ, but with no audit. Let m_σ be the probability of auditing a report of σ. Assume, without loss of generality that $m_H = 0$, so we can write $m_L = m$ without confusion.[1]

The contract has commitment so that in the play of the game the random monitoring

[1] Since $L < I$, $R_H, R_{L|H} > L > R_{L|\cdot}$, i.e., repayments are non-decreasing with the state. Thus, any report of the high state must be truthful and it is a dominant strategy never to audit the H report.

must actually occur even though the lender knows that a low report must be truthful. This monitoring has a deadweight loss which must be paid. All repayments are non-negative and the agent has limited liability.

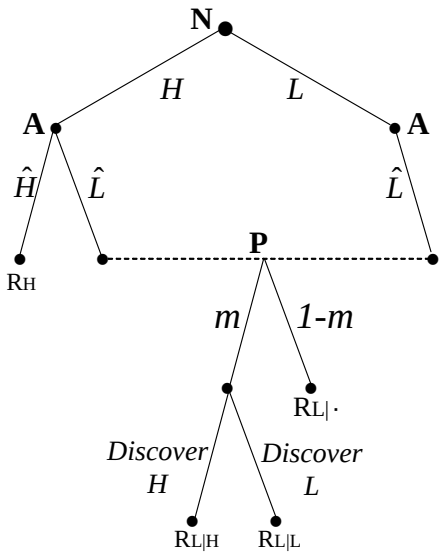
The sequence of events is as follows.

1. A financing contract is offered and, if accepted, the borrower is committed to the investment.

2. Nature chooses the project outcome, $s = \{H, L\}$. This is only observed by the borrower, who makes a report σ to the investor.

3. If $\sigma = L$ is reported, the investor can audit with probability m to discover the true project outcome, or not audit, with probability $1 - m$. If he does not audit, he gets a repayment $R_{L|}$. If he does audit, instead, he gets $R_{L|L}$ if the report is found to be true, and $R_{L|H}$ if the report is found to be false.

4. Payoffs are distributed. A game tree is sketched in Fig. 1.



The contract is set to solve

$$\max E\Pi_S = p\left(H - R_H\right) + (1-p)\left[m\left(L - R_{L|L}\right) + (1-m)\left(L - R_{L|\cdot}\right)\right] \tag{4}$$

$$pR_H + (1-p)\left[(1-m)R_L + m\left(R_{LL} - c\right)\right] \geq I \tag{5}$$

$$R_H \leq mR_{LH} + (1-m)R_L \tag{6}$$

$$0 \leq R_H, R_{LH} \leq H \text{ and } 0 \leq R_L, R_{LL} \leq L \tag{7}$$

where (4) is the objective function, constraint (5) is the participation constraint, ensuring that the lender breaks even in expected terms on each project, (6) is the truth-telling constraint, ensuring that, upon a high state occurring, the borrower prefers to report truthfully, rather than cheating and be audited with probability m, and (7) are the limited liability conditions.

Generally the participation constraint (5) must bind since otherwise the objective could be increased by reducing R_H. Substituting R_H from the participation constraint gives the objective function as the expected return net of the expected audit cost:

$$E\Pi_S = pH + (1-p)L - I - (1-p)mc \tag{8}$$

Proposition 1 *When each project is funded as a stand-alone, the optimal contract has:*

$$\begin{aligned} R_{L|H} &= H \\ m^{\sin} &= \tfrac{I-L}{p(H-L)-(1-p)c} < 1 \\ R_{L|L} &= R_L = L \\ R_H &= \tfrac{(H-L)I-(1-p)L(H-L+c)}{p(H-L)-(1-p)c} \end{aligned} \tag{9}$$

and expected return to the borrower

$$pH + (1-p)\,L - I - \underbrace{\frac{(I-L)}{p\,(H-L) - (1-p)\,c}}_{m^{sim}}(1-p)\,c. \tag{10}$$

The intuition behind these results is the following. The deadweight loss of monitoring is minimised by raising R_{LH} to its maximum level of H and reducing m until (6) holds with equality. In addition $0 < m < 1$ (since if $m = 0$, $R_H = R_{L|\cdot} \leq L$ and there is then insufficient revenue to meet the investment cost). Moreover, low state repayments whether monitored or not are set to give zero surplus to the borrower: $R_{L|\cdot}, R_{L|L} = L$. However, since $R_H < H$ ($H - R_H = (H-L)[pH + (1-p)L - (1-p)c]$, she gets a reward in the high state. These results are analogous to those obtained by Khalil and Parigi (1998), except here investment is exogenous.

5 Joint project financing

We now introduce the possibility of jointly financing two projects with independent ex-post private returns costlessly observed by the borrower. The two projects may give four possible outcomes: $s \in \{LL, HL, LH, HH\}$. We assume that the borrower makes a single report to the lender stating the number of successes, and which project has succeeded, in case only one has. To simplify the notation, we write the report as $\sigma \in \{0, 1, 2\}$.

Any report the borrower makes must be feasible in that she has to have funds to make the appropriate repayment. Conditional on the report, the lender can audit. The contract has to list an audit strategy that overcomes the temptations to cheat in the report. In particular, in case of a report of zero successes, the lender can audit sequentially, i.e., first audit one project and then, possibly, conditional on the outcome of the first audit, the other,

or simultaneously, i.e., audit both projects reported to have failed. In both cases we assume there is commitment in the contract so the lender has to carry through the audit policy although he knows that he will never catch a cheat.

5.1 Sequential audits

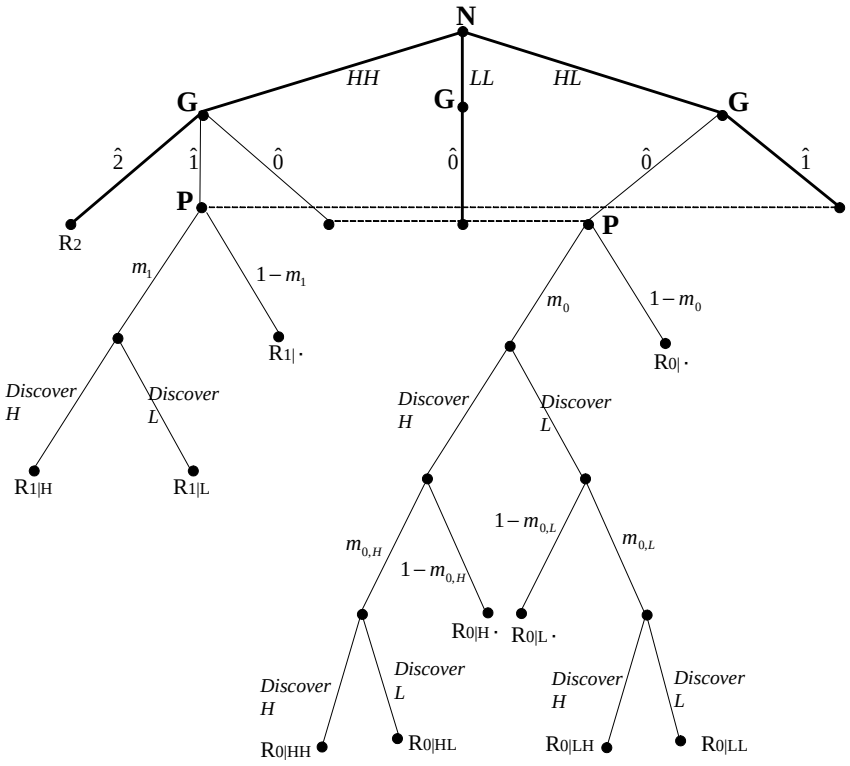
We first consider the case of sequential audits. In this case the borrower can report two successes, one success, or no success, $\sigma \in \{0, 1, 2\}$.

In case of a report 2, because the reports must be feasible, the lender knows that both projects have been successful. So, there is no audit: $m_2 = 0$.

In case of a report 1, the lender knows that at least one project has been successful, and which one has, while the other may have succeeded or failed. So, to minimise audit cost, the lender can target monitoring on the failed project only, with probability m_1, or not audit at all, with probability $1 - m_1$. In the case in which he does not audit, he demands a repayment $R_{1|\cdot}$ in total on the two projects, where the first subscript denotes the report $\sigma \in \{0, 1, 2\}$, while the second denotes whether there has been an audit, and the outcome of it, if any (so, a dot stands for no audit). So, upon an audit of a report 1 with probability m_1, if the lender finds that the project has truly failed, he demands repayment $R_{1|L}$, where the second subscript denotes that the outcome of the audit has been L. If he discovers a success on the reportedly failed project, he demands repayment $R_{1|H}$.

In case of a report 0, knowing that the borrower claims that both projects have failed, the lender can randomly choose which one to audit, if any, with probability 1/2 on each (by the principle of insufficient reason). Denote with m_0 the probability to audit one of the two projects, and with $1 - m_0$ the probability *of no* audit. In case in which he does

not audit, he demands a repayment $R_{0|\cdot}$ and the game ends. In case in which the lender does audit and discovers the outcome for the selected project, then he decides whether to go further and audit the second project, with probability $m_{0,i}$, $i = H, L$, where the second subscript denotes the outcome of the first audit. Denote moreover with $R_{0|ij}$, $i, j = H, L$, the repayment when the lender first discovers the state of one project is i and then goes on to audit the second project discovering its state to be j.That is, with zero reported successes, learning the outcome of one project gives no immediate information about the outcome of the other. In case of no second stage audit, with prob. $1 - m_{0,i}$, $i = H, L$, the repayments are $R_{0|i\cdot}$, $i = H, L$.



The borrowers joint payoff function is

$$E\Pi_J = p^2 (2H - R_2) + 2p(1-p)\left\{H + L - m_1 R_{1|L} - (1 - m_1) R_{1|\cdot}\right\} \tag{11}$$

$$+ (1-p)^2 \left\{2L - m_0 \left[m_{0,L} R_{0|LL} + (1 - m_{0,L}) R_{0|L\cdot}\right] - (1 - m_0) R_{0|\cdot}\right\}.$$

The lender's payoff function when financing two joint projects is:

$$E\Pi_L = p^2 R_2 + 2p(1-p)\left\{m_1\left(R_{1|L} - c\right) + (1-m_1)R_{1|\cdot}\right\} \tag{12}$$

$$+ (1-p)^2\left\{m_0\left[m_{0,L}\left(R_{0|LL} - c\right) + (1-m_{0,L})R_{0|L\cdot} - c\right] + (1-m_0)R_{0|\cdot}\right\} - 2I$$

With two true successes, there are two ways of cheating. To declare one success, or to declare none. Thus, the incentive constraints that ensure that a borrower with two successes reports truthfully are:

$$R_2 \leq (1-m_0)R_{0|\cdot} + m_0\left[m_{0,H}R_{0|HH} + (1-m_{0,H})R_{0|H\cdot}\right] \text{ if report 0} \tag{13}$$

$$R_2 \leq (1-m_1)R_{1|\cdot} + m_1 R_{1|H} \text{ if report 1} \tag{14}$$

With one true success, the only way of cheating is to declare zero successes. The incentive constraint is:

$$(1-m_1)R_{1|\cdot} + m_1 R_{1|L} \leq (1-m_0)R_{0|\cdot} + \tag{15}$$

$$m_0\left[\frac{1}{2}\left(m_{0,H}R_{0|HL} + (1-m_{0,H})R_{0|H\cdot}\right) + \frac{1}{2}\left(m_{0,L}R_{0|LH} + (1-m_{0,L})R_{0|L\cdot}\right)\right] \text{ if report 0}$$

The first two incentive constraints ensure that an HH type prefers to make a report 2 rather than 0 or 1. Similarly, the third incentive constraint ensures that an HL type prefers to make a report 1 rather than 0.

Last, the limited liability conditions are

$$R_2, R_{0|HH}, R_{1|H} \leq 2H, \tag{16}$$

$$R_{1|\cdot}, R_{1|L}, R_{0|H\cdot}, R_{0|HL}, R_{0|LH} \leq H + L,$$

$$R_{0|\cdot}, R_{0|L\cdot}, R_{0|LL} \leq 2L.$$

The contract problem is to choose R_2, $R_{1|\cdot}$, $R_{1|L}$, $R_{0|H\cdot}$, $R_{0|HL}$, $R_{0|LH}$, $R_{0|\cdot}$, $R_{0|L\cdot}$, $R_{0|LL}$, $R_{0|HH}$, $R_{1|H}$, m_0, $m_{0,H}$, $m_{0,L}$, m_1 to maximise (11), subject to the participation constraint (12) being non-negative, to the incentive constraints (13), (14), and (15), and to the limited liability conditions (16).

The participation constraint must bind since otherwise R_2 could be reduced. Using the participation constraint to eliminate R_2 gives the objective function as the expected return net of the expected audit cost:

$$2p^2H + 2p(1-p)(H+L) + 2(1-p)^2 L - 2p(1-p)m_1 c - (1-p)^2 m_0 (1+m_{0,L})c - (17)$$

$$2[pH + (1-p)L - I] - 2p(1-p)m_1 c - (1-p)^2 m_0 (1+m_{0,L})c$$

By setting all the m's equal to one, the expected group payoff reduces to :

$$2[pH + (1-p)L - I] - 2(1-p)c. \tag{18}$$

which we know to be positive from Assumption 2. Hence this condition ensures that, even with full monitoring of all state reports, the joint projects have positive NPV.

The solution to the above problem is summarised in Proposition 2:

Proposition 2 *With joint contracting and sequential monitoring the optimal contract has:*

1. *maximum punishment for false reporting:*

$$R_{0|HH} = R_{1|H} = 2H;$$

$$R_{0|H\cdot} = R_{0|HL} = R_{0|LH} = H + L;$$

2. *zero rent to the borrower in the lowest true state (both projects fail):*

$$R_{0|L\cdot} = R_{0|\cdot} = R_{0|LL} = 2L;$$

3. *probability of monitoring at the second stage having discovered a cheat by first stage audit set at its highest value,* $m_{0,H} = 1$;

4. *the repayment after a report of a single success giving a truth-telling reward since* $R_{1|L} = 0$ *and* $R_{1|\cdot} < H + L$;

5. $m_0, m_{0,L}$ *not uniquely defined but* $0 < m_0 (1 + m_{0,L}) < 1$, *with* $m_0 > 0$, *and* $m_{0,L} \in [0, 1]$. *In particular, any combination* $m_0, m_{0,L}$ *along the curve satisfying*

$$m_0 (1 + m_{0,L}) = \frac{4 (I - L)}{p (2 - p) (H - L) - 2 (1 - p)^2 c} \tag{19}$$

is optimal;

6. *probability of auditing reports of one success set at its lowest value,* $m_1 = 0$. *Because of this,* $R_{1|L}$ *is never paid and it can be set to any value between zero and* $H + L$.

7. *repayments pooled in the top two states, so that there is a common repayment after a report of two successes or a single success. This gives a reward to truthfully reporting one or two successes:*

$$R_{1|\cdot} = R_2 = \frac{2 (I - L) (H - L)}{p (2 - p) (H - L) - 2 (1 - p)^2 c} + 2L < H + L;$$

8. *borrowers indifferent between truthfully reporting one or two successes, getting a zero return only from reporting no success but a positive return from reporting a single or two successes that is equal to*

$$2 [pH + (1 - p) L - I] - \frac{4 (1 - p)^2 (I - L)}{p (2 - p) (H - L) - 2 (1 - p)^2 c} c. \tag{20}$$

The intuition behind these results is the following. Maximum punishment and zero rent to the borrowers in the lowest truthfully reported states maximises the incentives for truth-telling whilst also keeping the observation cost of low reports as small as possible.

There must always be some audit of a report of 0 successes, since, if not, the borrower could always report 0 and get away with cheating. But this would lead to repayments which do not cover the investment cost.

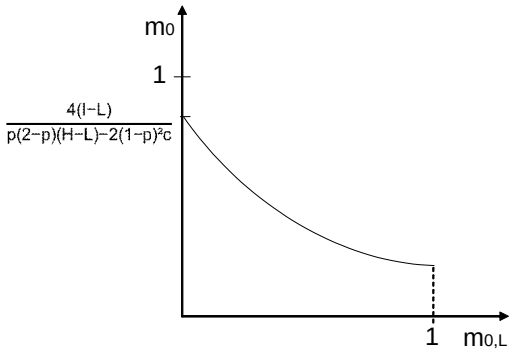
The incentive to cheat between a report of one success or zero successes is controlled by a binding incentive constraint in which $R_{1|\cdot}$ is set at the lowest possible level compatible with the lender's participation constraint. Thus, repayments and audit probabilities are set so as ensure that the borrower is indifferent between declaring one success and zero successes.

The incentive to cheat between a report of one or two successes is controlled by a binding incentive constraint again, which involves pooling the repayments due after reporting one or two successes, $R_{1|\cdot} = R_2$. These repayments must both be above $2L$, since otherwise there would be insufficient revenue to the lender to recoup the loans cost.

The reasons for setting $m_1 = 0$ is that condition (1) allows pooling the repayments in the top two states, thus removing any cheating incentives. So audit costs can be minimised by setting $m_1 = 0$.

Conversely, there are many combinations of efficient monitoring of a zero success report and they lie along the curve satisfying Eq. (19). One extreme possibility has zero monitoring at the second stage, $m_{0,L} = 0$, and $m_0 = \dfrac{4(I-L)}{p(2-p)(H-L) - 2(1-p)^2 c}$. The other extreme possibility has $m_{0,L} = 1$, and $m_0 = \dfrac{2(I-L)}{p(2-p)(H-L) - 2(1-p)^2 c}$, so that $m_0(1+m_{0L}) = \dfrac{4(I-L)}{p(2-p)(H-L) - 2(1-p)^2 c}$. At the extremes, monitoring only the first

stage but with twice the intensity is equivalent in deadweight loss to monitoring both stages, the second stage with probability one. The intuition for the irrelevance of the timing of monitoring a zero report is that the repayments are both equal to $2L$ after the first or second stage of monitoring.



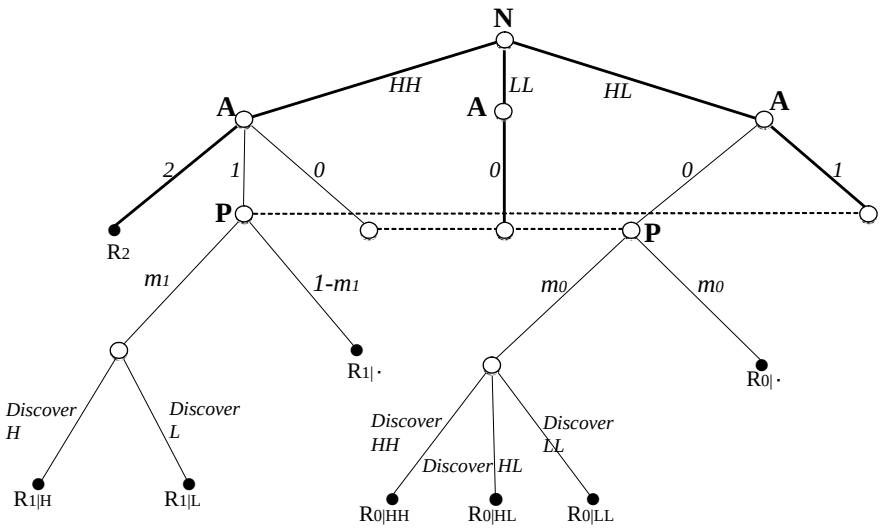
Note that the borrower is indifferent between a point on the curve and an alternative contract which has a lower m_0 but a more than proportionately higher $m_{0,L}$. m_0 is a more powerful instrument than $m_{0,L}$ in controlling the incentive to cheat because it affects incentives directly, at the first stage monitoring following a zero report, and indirectly, through the effect that it has at the second stage monitoring, after first stage monitoring has occurred upon a zero report. The combinations have the same expected audit cost.

Thus, some of the basic properties of the solution mirror those of the fundamental literature: maximum punishment on detected cheats; zero rent for the borrower in the lowest state. But there are many novelties. If a cheat is found at stage 1 then optimally the lender should seriously look for another cheat ($m_{0,H} = 1$). But if in the first round there is a report of one success then it is best not to monitor the second project at all ($m_1 = 0$). If the projects taken together are sufficiently profitable then optimally there is a common repayment from reports of one or two successes. And finally only the net repayment from a

report of zero successes matters; it can be built up from different combinations of first and second stage audit.

6 Simultaneous audits

We now consider the case of simultaneous audits. The reporting and auditing structure are similar to the those seen in the previous section. In case of a report 2, or 1, the audit and repayment structure is analogous to the sequential one. In particular there is no audit following a report of two successes, while it is only necessary to audit the project reported to have failed in case of report of just one success. Different is the case in which no success is reported. In this case the lender will audit both projects simultaneously with probability m_0, and neither of them with probability $1 - m_0$. As for the sequential case, if he does not audit, he demands a repayment $R_{0|\cdot}$ and the game ends. When the lender does audit, instead, he can discover that both projects have failed, that only one has failed, or that none has, getting respectively $R_{0|LL}, R_{0|LH}, R_{0|HH}$.



As for the sequential case, we have the following incentive compatibility conditions:

1. declare zero rather than two successes

$$R_2 \leq (1 - m_0) R_{0|\cdot} + m_0 R_{0|HH} \tag{21}$$

2. declare one rather than two successes

$$R_2 \leq (1 - m_1) R_{1|\cdot} + m_1 R_{1|H} \tag{22}$$

3. declare zero rather than one success

$$m_1 R_{1|L} + (1 - m_1) R_{1|\cdot} \leq (1 - m_0) R_{0|\cdot} + m_0 R_{0|HL}. \tag{23}$$

Thus, the first two IC's ensure that a borrower with two successful projects (HH) prefers to report 2 successes rather than 0 or 1. Similarly, the third incentive constraint ensures that a borrower with a successful and a failed project (HL) prefers to report 1 success rather than 0.

The objective function is

$$p^2 (2H - R_2) + 2p (1 - p) \left[H + L - m_1 R_{1|L} - (1 - m_1) R_{1|\cdot}\right] + (1 - p)^2 \left[2L - m_0 R_{0|LL} - (1 - m_0) R_{0|\cdot}\right]. \tag{24}$$

The participation constraint requires the expected return to the lender from both projects to cover the two loan costs and the expected audit costs:

$$p^2 R_2 + 2p (1 - p) \left[m_1 \left(R_{1|L} - c\right) + (1 - m_1) R_1|.\right] + (1 - p)^2 \left[m_0 \left(R_{0|LL} - 2c\right) + (1 - m_0) R_{0|\cdot}\right] \geq 2I. \tag{25}$$

Last, the limited liability conditions are

$$\begin{aligned} R_{0|HH}, R_{1|H}, R_2 &\leq 2H, \\ R_{1|\cdot}, R_{1|L}, R_{0|HL} &\leq H + L, \\ R_{0|\cdot}, R_{0|LL} &\leq 2L. \end{aligned} \tag{26}$$

The contract problem is to choose R_2, $R_{1|\cdot}$, $R_{1|L}$, $R_{1|H}$, $R_{0|\cdot}$, $R_{0|LL}$, $R_{0|HL}$, $R_{0|HH}$, m_0, m_1 to maximise (24), subject to the participation constraint (25), to the incentive constraints (21), (22), and (23), and to the limited liability conditions (26).

In the following we show that the optimal simultaneous contract is closely related to the optimal sequential one, as stated in Proposition 3.

Proposition 3 *(Irrelevance of the timing of monitoring) If we set* $m_0^{\text{seq}}\left(1 + m_{0,L}^{\text{seq}}\right) = 2m_0^{sim}$, *the two contract problems are identical.*

This follows just from inspecting the two contract problems. Since the contract problems are identical (and we know that only the combined value $m_0^{\text{seq}}\left(1 + m_{0,L}^{\text{seq}}\right)$ matters in the sequential problem), the optimal audit strategies in both problems must coincide with $m_0^{\text{seq}}\left(1 + m_{0,L}^{\text{seq}}\right) = 2m_0^{sim}$.

Corollary 1 *Optimal sequential and simultaneous audits have identical ex-ante welfare.*[2]

Choosing $m_0^{sim} = \dfrac{2(I-L)}{p(2-p)(H-L) - 2(1-p)^2 c} = \dfrac{1}{2} m_0^{\text{seq}}\left(1 + m_{0,L}\right)$ (their optimal values), the optimal ex-ante welfare coincide in the two contracts.

The equivalence arises because the returns on the two projects are independent. So, knowing the outcome on one project does not change the information about the distribution

[2] With n project we would still expect to get an equivalence between simultaneous and sequential contracts.

of returns on the other project. There is no gain from auditing one project first to learn its outcome. By contrast, if the returns were correlated, there would be a potential information gain in sequential audit (Phelan, 2017).

7 Violation of the strong feasibility condition (1)

In the previous section we have seen that if the strong feasibility condition (1) holds, then only reports of two fails are audited randomly, while reports of only one fail are never audited ($m_1 = 0$). The intuition is that there are enough revenues from bottom and intermediate states to cover the investment and audit cost, so that there is no need to collect resources from the top state. The repayment from either of the top two states is identical. In the present section we consider what happens if the strong feasibility condition (1) is violated, i.e., if

$$p(2-p)(H-L) - 2(I-L) - 2(1-p)^2 c \leq 0.$$

To this aim, consider the reduced form contract problem with variables $R_2, R_{1|\cdot}, R_{1|L}, m_0, m_{0,L}, m_1$:[3]

$$E\Pi_J = p^2(2H - R_2) + 2p(1-p)\left[H + L - m_1 R_{1|L} - (1-m_1) R_{1|\cdot}\right]$$

$$\text{st } E\Pi_L = p^2 R_2 + 2p(1-p)\left[m_1\left(R_{1|L} - c\right) + (1-m_1) R_{1|\cdot}\right] + (1-p)^2\left[2L - m_0(1+m_{0,L})c\right] = 2I$$

$$\begin{aligned} R_2 &\leq 2m_0(H-L) + 2L \\ R_2 &\leq m_1 2H + (1-m_1) R_{1|\cdot} \end{aligned}$$

$$m_1 R_{1|L} + (1-m_1) R_{1|\cdot} = \frac{1}{2} m_0 (1+m_{0,L})(H-L) + 2L$$

[3]The remaining variables, whose value is set by maximum punishment and zero rent in the low state, are independent of the precise feasibility condition being used.

In the first part of the paper, to prove that $m_1 = 0$, we have used a variational argument. Starting from any feasible position in the variables, we have locally varied them in ways which kept each constraint unchanged, seeing which directions of change improved the objective function. We have seen that this required reducing m_1 as far as possible. However, the implied optimal values of m_0^{sim} (and $m_0^{\text{seq}}(1 + m_{0,L})$) and $R_{1|\cdot}$ required that condition (1) was satisfied.

If the parameter values are such that condition (1) is violated, then the optimal values of m_0^{sim} (and $m_0^{\text{seq}}(1 + m_{0,L})$) and $R_{1|\cdot}$ implied by $m_1 = 0$ are infeasible, namely $R_{1|\cdot} \geq H + L$ and $m_0^{sim} \geq 1$ ($m_0^{\text{seq}}(1 + m_{0,L}) \geq 2$). Thus, we could at most set m_0^{sim} ($m_0^{\text{seq}}(1 + m_{0,L})$ in the sequential case) and $R_{1|\cdot}$ at their corners and "collect" the missing resources from monitoring reports of one success ($m_1 > 0$) and from not pooling the top two states ($R_2 > H + L$).

Using the corner values of $m_0(1 + m_{0,L}) = 2$ and $R_{1|\cdot} = H + L$ in the reduced form optimisation problem above, the contract problem becomes ($\mathcal{P}_{weak}$):

$$E\Pi_J = p^2(2H - R_2) + 2p(1-p)m_1\left(H + L - R_{1|L}\right)$$

$$\text{st } E\Pi_L = p^2 R_2 + 2p(1-p)\left[H + L - m_1\left(H + L - R_{1|L} + c\right)\right] + (1-p)^2 2(L - c) = 2I$$

$$R_2 \leq 2H \tag{27}$$

$$R_2 \leq m_1 2H + (1 - m_1)(H + L) \tag{28}$$

$$m_1\left(R_{1|L} - H - L\right) = 0 \tag{29}$$

Notice that if constraint (28) is satisfied, then certainly constraint (27) is. So we can ignore (27). Moreover, from the binding incentive constraint (29), we see that there are two mutually

exclusive cases, $m_1 = 0$ or $R_{1|L} = H + L$. We analyse these two cases and find the results in Proposition 4.

Proposition 4 *If condition (1) is violated, with joint contracting and simultaneous or sequential audits the optimal contract has:*

1. *maximum punishment for false reporting:*

$$\begin{aligned} R_{0|HH} &= R_{1|H} = 2H; \\ R_{0|HL} &= H + L; \end{aligned}$$

2. *zero rent to the borrower in the lowest true state (both projects fail):*

$$R_{0|L} = R_{0|\cdot} = R_{0|LL} = 2L;$$

3. $m_0 = m_{0,L} = 1$;

4. $R_{1|L} = R_{1|\cdot} = H + L$;

5. $$m_1 = \frac{2(I-L) - p(2-p)(H-L) + 2(1-p)^2 c}{p^2(H-L) - 2p(1-p)c} \geq 0;$$

6. $$H + L \leq R_2 = 2H - 2(H-L)\frac{[pH + (1-p)L - I - (1-p)c]}{p^2(H-L) - 2p(1-p)c} < 2H;$$

7. *Expected return to the borrower*

$$2[pH + (1-p)L - I] - 2p(1-p)\frac{2(I-L) - p(2-p)(H-L) + 2(1-p)^2 c}{p^2(H-L) - 2p(1-p)c}c - 2(1-p)^2 c \tag{30}$$

The intuition behind these results is that, since the revenues from just one success or no successes do not cover the investment plus audit cost of the two projects, additional revenues in excess of $H+L$ must be raised from the report of two successes. But in this case, to ensure truthful reports of two successes and prevent cheating by declaring one success, reports of only one success must sometimes be audited ($m_1 \geq 0$).

Notice that the optimal audit probabilities are decreasing in the profitability of the state. In particular, while a report of zero successes is always audited deterministically ($m_0 = m_{0,L} = 1$), a report of one success is audited with a probability strictly less than one.[4] The intuition behind this result is that setting the audit probabilities far apart improves the incentive to truthfully declare one success instead of no successes.

8 Comparisons

In the present section we compare the relative efficiency of the various contract problems.

Under single project financing, the expected profits obtainable from two stand-alone projects are

$$2\left[pH + (1-p)L - I\right] - 2(1-p)\, m^{\sin} c, \tag{31}$$

where $m^{\sin} = \dfrac{I-L}{p(H-L) - (1-p)c}$ is the probability of monitoring a low state report under single project financing, as defined in (9).

When all the projects are financed by a unique lender, under sequential audit, from (17), expected profits are

$$2\left[pH + (1-p)L - I\right] - 2p(1-p)\, m_1 c - (1-p)^2\, m_0 (1 + m_{0,L})\, c. \tag{32}$$

[4] Having $m_1 = 1$ would violate condition (2).

If condition (1) is satisfied, $m_1 = 0$, $m_0(1+m_{0,L}) = \dfrac{4(I-L)}{p(2-p)(H-L)-2(1-p)^2 c}$ from (19).

By comparing the expected audit cost under single (31) and joint project finance (32), we see that joint project financing dominates single project financing iff:

$$2(1-p)m^{sin}c > (1-p)^2 m_0(1+m_{0,L})c$$

$$2(1-p)c\underbrace{\frac{(I-L)}{p(H-L)-(1-p)c}}_{m^{sin}} - (1-p)^2 c\underbrace{\frac{4(I-L)}{p(2-p)(H-L)-2(1-p)^2 c}}_{m_0(1+m_{0,L})} > 0$$

The difference reduces to

$$\frac{2p^2(H-L)(1-p)(I-L)}{[p(H-L)-(1-p)c]\left[p(2-p)(H-L)-2(1-p)^2 c\right]}$$

which is always positive so long as condition (1) holds. Thus, under our assumptions, joint project financing always has lower audit cost and so dominates single project financing.

To see where this result comes from, we compare the audit probabilities, and see that $m_0(1+m_{0L}) > 2m^{\sin}$ (their difference reduces to $p^2(H-L) - 2p(1-p)c$, positive by Assumption 3). Thus, despite the lower audit probability of the single project finance, the joint random audit is applied with a sufficiently lower frequency, $(1-p)^2$ rather than $(1-p)$, for joint finance to dominate stand-alone finance.

If condition (1) is violated, $m_0 = m_{0,L} = 1$ and $m_1 = \frac{2(I-L)-p(2-p)(H-L)+2(1-p)^2 c}{p^2(H-L)-2p(1-p)c} \geq 0$. By comparing the expected audit cost under single (31) and joint project financing (32), we get:

$$\underbrace{2(1-p)m^{\sin}c}_{\text{single}} - \underbrace{\left[2p(1-p)m_1 c + (1-p)^2 m_0(1+m_{0,L})c\right]}_{\text{joint}}$$

i.e.,

$$\frac{p(H-L)[pH+(1-p)L-I-(1-p)c]}{[p(H-L)-2(1-p)c][p(H-L)-(1-p)c]} > 0.$$

which is strictly positive under Assumptions (2) and (3). Similar considerations of balancing differences in the m's and the probabilities with which they are applied (the p's) arise here as in the case above. The lower monitoring probability of single project finance is dominated by the lower frequency with which they are applied in the joint finance case. Thus, even in the case in which condition (1) is violated and a pooling contract is infeasible, joint-project financing dominates single-project financing, as it involves lower audit costs.

The intuition behind this result is that with joint project finance only the total revenue of the two projects is relevant, so that one project can cross-subsidise the other. The value of this depends on the chance that the project outcomes differ and the size of the difference (the amount of possible cross-subsidisation). In the literature this is also called the diversification gain of joint project finance. In addition, if condition (1) holds, there is also a saving of audit cost since an audit is only necessary following a report of zero successes. Conversely, when projects are financed as standalones, each single report of fail *has* to be audited stochastically.

Summarising these results, we can state Proposition 5.

Proposition 5 *Under Assumptions (2) and (3), joint project financing dominates single project financing.*

The proofs follow from the arguments set out above.

9 Robustness

In this section we consider what happens if, upon a report of one success, the lender does not know which one it is. She has then to choose at random which one to audit, and chooses an arbitrary one to monitor with probability 1/2 again by the principle of insufficient reason.

This makes the monitoring chance m_1 above conditional on the selection of a project to monitor replacing m_1 by m_1^A and m_1^B for the two projects A and B respectively. If the lender discovers the monitored project i ($i = A$ or B) is a fail, it also reveals that the other project must be a success, since the borrower reported one success. Conversely, if the monitored project i is a success, there is a chance that the remaining project could be a success as well, in which case it could be monitored with probability $m_{1|H}^i$. The obvious way to control the cheating incentive in this case is to set $m_{1|H}^i = 1$. This is by analogy with $m_{0|H} = 1$ above.

We have assumed commitment. That is the lender carries out his audit strategy as announced in the contract even though he knows that there is always truthtelling. If the lender is an intermediary in turn financed by shareholders, then they can hold the lender to account to ensure audits are fulfilled. On a repeated contract setting, the lender could get away once with not carrying out his announced audit strategy. But in the next round the borrower should start anticipating that maybe if he cheats the lender will not monitor as stated in the contract. Phelan (2017) finds that with sequential audit of several projects whose outcomes are correlated, the correlation can provide an incentive to audit at least some projects to gain information about the chances of good outcomes on unaudited projects.

10 Conclusion

The costly state verification literature established some general principles in the context of a single risky borrower and lender where only the borrower knows his ex-post outcome. Since then many studies have examined whether these principles hold up under different settings. One part of this has looked at some issues arising with many borrowers and lenders. In this paper we add to these contributions by seeing how the principles extend to a single

risky borrower seeking finance from a single lender for several risky projects. Should such a borrower seek a single loan for all the projects collectively or instead distinct loans for each project? And how should the borrowers incentive to cheat be controlled, namely, what is the cheapest audit strategy?

This problem relates to the ideas of joint project finance and internal capital markets in multidivisional firms. We find that some of the detailed principles of the optimal contract stand up, e.g., monotonicity of the audit probabilities with the profitability of states of the world (defined as the combination of possible outcomes across the projects), zero monitoring of the top state and others. Under some circumstances we find results analogous to a standard debt contract. But there are also some new forces. The joint finance is preferable to multiple single loans because it allows both parties to the loan to cross-subsidise over the different projects. This lowers the fair return to the lender and allows a lower expected audit cost to ensure truthtelling. Partly this works in a way similar to Diamonds theory of delegated monitoring (1984). With joint finance for many projects the tails of the probability distribution of the number of successful or failed projects are less likely to occur.

As far as the audit strategy is concerned, with joint finance there are richer possibilities than with multiple single project loans. With the latter each single bad project outcome must be audited to ensure truthtelling on all projects. But with joint project finance the projects can be audited sequentially one by one. This allows later audits to be conditioned on the results of earlier ones. Phelan shows that if returns are correlated across projects then sequential audit allows updating of probability distribution of returns on unaudited projects allowing a cheaper overall audit strategy. So sequential audit will socially dominate

simultaneous audit. A further possible advantage of sequential audit with multiple projects could be that only as many projects will need to be audited as will ex-post cover the lenders costs across all projects. Hence ex-post the lender can stop auditing as soon as his costs are covered. With uncorrelated projects and commitment to audit we find that these advantages disappear and sequential and simultaneous audit incur the same expected deadweight loss of audit. But joint financing is still preferable to individual project finance.

There are various future research directions. First there are areas of application such as to multidivisional firm finance: firms with multiple semi-independent projects. But this raises the idea immediately of correlated projects, e.g. if such a firm is operating in a broad single sector (transport, say) then aggregate transport shocks are likely to affect all divisions.

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A Appendix

Proof of Proposition 1. Using maximum punishment in the optimisation problem ($R_{LH} = H$) and forming a Lagrangian with multiplier λ and μ, the FOC's wrt R_H, R_L, R_{LL} and m are

$$\begin{aligned}
\tfrac{\partial \mathcal{L}}{\partial R_H} &: (\lambda - 1)\, p - \mu \geq 0, R_H \leq H \\
\tfrac{\partial \mathcal{L}}{\partial m} &: (1-p)\,(R_L - R_{LL})\,(1-\lambda) - \lambda\,(1-p)\,c + \mu\,(H - R_L) \geq 0, m \leq 1 \\
\tfrac{\partial \mathcal{L}}{\partial R_{LL}} &: (\lambda - 1)\, m\,(1-p) \geq 0, R_{LL} \leq L \\
\tfrac{\partial \mathcal{L}}{\partial R_L} &: (1-m)\,[(\lambda - 1)\,(1-p) + \mu] \geq 0, R_L \leq L
\end{aligned}$$

$\lambda > 1$. Suppose $\lambda = 1$. Then by $\frac{\partial \mathcal{L}}{\partial R_H}$, $\mu = 0$. By $\frac{\partial \mathcal{L}}{\partial m}$, this implies $-\lambda\,(1-p)\,c \leq 0$, a contradiction, since $\frac{\partial \mathcal{L}}{\partial m} \geq 0$. By $\lambda > 1$, $\frac{\partial \mathcal{L}}{\partial R_{LL}}, \frac{\partial \mathcal{L}}{\partial R_L} > 0$ and $R_{LL} = R_L = f_L$. Using $R_{LL} = R_L = L$, $R_{LH} = H$ and $m = \frac{R_H - L}{H - L}$ from the incentive constraint, the contract problem becomes

$$\begin{aligned}
&\max_{R_H} p\,(H - R_H) \\
\text{s.t.} \quad &pR_H + (1-p)\left(L - \tfrac{R_H - f_L}{f_H - f_L} c\right) = I
\end{aligned}$$

The objective function is decreasing in R_H, while the participation constraint is increasing in it ($\frac{\partial PC}{\partial R_H} = \frac{1}{H-L}\left[p\,(f_H - f_L) - (1-p)\,c\right]$). R_H is then obtained by solving the participation constraint, giving $R_H = \frac{(H-L)I - (1-p)L(H-L+c)}{p(H-L) - (1-p)c}$. Substituting out in m, gives $m = \frac{I-L}{p(H-L)-(1-p)c}$.
For $m < 1$, $pH + (1-p)\,L - I - (1-p)\,c > 0$, which certainly holds under Assumption 2. This in turn implies from (6) that $R_H < H$.

To work out the expected return to the borrower, take the objective function

$$E\Pi_S = p\,(H - R_H) + (1-p)\,[m\,(L - R_{LL}) + (1-m)\,(L - R_L)]$$

and use the solutions to the programme set out above: zero revenue in the low state ($R_{LL} = R_L = L$):

$$E\Pi_S = pH - pR_H$$

From IC:

$$\begin{aligned} R_H &= mR_{LH} + (1-m) R_L \\ &= m(H-L) + L \end{aligned}$$

Replace in objective function

$$\begin{aligned} E\Pi_S &= pH - p(m(H-L) + L) \\ & \quad p(H-L) - pm(H-L) \end{aligned}$$

Add and subtract $(I-L)$

$$\begin{aligned} & p(H-L) - pm(H-L) + (I-L) - (I-L) \\ & pH + (1-p)L - I - pm(H-L) - (L-I) \end{aligned}$$

Using $m = \frac{I-L}{p(H-L)-(1-p)c}$ from (9)

$$pH + (1-p)L - I + (I-L) - p\frac{I-L}{p(H-L)-(1-p)c}(H-L)$$

and collecting $(I-L)$

$$pH + (1-p)L - I - \underbrace{\frac{(I-L)}{p(H-L)-(1-p)c}}_{m^{sim}}(1-p)c$$

as (8). ■

Proof of Proposition 2. The proof proceeds as follows. First, with the binding participation constraint and the binding incentive constraint giving indifference for the group between truth-telling and cheating when the group has one success, we solve out for values of $1 \geq m_0 > 0$ and $2H \geq R_2 > 2L$. That leaves a reduced problem of choosing $m_1, m_{0,L}, R_{0|\cdot}, R_{1|L}$ to maximise the group return within the two incentive constraints controlling truth-telling when the group actually has two successes. Of course we have to check that any solution we reach for $m_1, m_{0,L}$, $R_{0|\cdot}, R_{1|L}$ leads to values of m_0, R_2 satisfying $1 \geq m_0 > 0$ and $2H \geq R_2 > 2L$. Using a local variational argument we show that from any initial feasible point $m_1, m_{0,L}, R_{0|\cdot}, R_{1|L}$ there are small changes which will continue to satisfy the incentive constraints and will increase the objective function if $m_{0,L}$ and $m_1$are both reduced. The obvious solution is then $m_1 = m_{0,L} = 0$. If $m_1 = 0$ then the value of $R_{1|L}$ is immaterial (it is never paid) and so can be set equal to zero. Since $m_1 = 0$ there is a solution in which the top two state repayments are pooled: $R_2 = R_{1|\cdot}$.The final step then checks that $R_{1|\cdot}$ is feasible and that the resulting m_0, R_2 are feasible.

The contract problem is to choose R_2, $R_{1|\cdot}$, $R_{1|L}$, $R_{0|H\cdot}$, $R_{0|HL}$, $R_{0|LH}$, $R_{0|\cdot}$, $R_{0|L\cdot}$, $R_{0|LL}$, $R_{0|HH}$, $R_{1|H}$, m_0, $m_{0,H}$, $m_{0,L}$, m_1 to maximise

$$\begin{aligned} & p^2(2H - R_2) + 2p(1-p)\left\{H + L - m_1 R_{1|L} - (1-m_1) R_{1|\cdot}\right\} \\ & + (1-p)^2\left\{2L - m_0\left[m_{0,L} R_{0|LL} + (1-m_{0,L}) R_{0|L\cdot}\right] - (1-m_0) R_{0|\cdot}\right\}. \end{aligned}$$

subject to the participation constraint

$$p^2 R_2 + 2p(1-p)\left\{m_1\left(R_{1|L} - c\right) + (1-m_1)R_{1|\cdot}\right\}$$
$$+ (1-p)^2\left\{m_0\left[m_{0,L}\left(R_{0|LL} - c\right) + (1-m_{0,L})R_{0|L\cdot} - c\right] + (1-m_0)R_{0|\cdot}\right\} = 2I$$

the incentive constraints

$$\begin{aligned} R_2 &\leq (1-m_0)R_{0|\cdot} + m_0\left[m_{0,H}R_{0|HH} + (1-m_{0,H})R_{0|H\cdot}\right] \\ R_2 &\leq (1-m_1)R_{1|\cdot} + m_1 R_{1|H} \end{aligned}$$

$$(1-m_1)R_{1|\cdot} + m_1 R_{1|L} \leq (1-m_0)R_{0|\cdot} +$$
$$m_0\left[\frac{1}{2}\left(m_{0,H}R_{0|HL} + (1-m_{0,H})R_{0|H\cdot}\right) + \frac{1}{2}\left(m_{0,L}R_{0|LH} + (1-m_{0,L})R_{0|L\cdot}\right)\right]$$

and the limited liability conditions:

$$\begin{aligned} R_2, R_{0|HH}, R_{1|H} &\leq 2H, \\ R_{1|\cdot}, R_{1|L}, R_{0|H\cdot}, R_{0|HL}, R_{0|LH} &\leq H + L, \\ R_{0|\cdot}, R_{0|L\cdot}, R_{0|LL} &\leq 2L. \end{aligned}$$

Punishment repayments $R_{0|HH}, R_{1|H}, R_{0|H\cdot}, R_{0|HL}, R_{0|LH}$ only enter the incentive constraints. So by setting maximum punishment can raise RHS of these and so lower either m_0 or m_1 or both. For example if $R_{0|HH} < 2H$ then we can increase $R_{0|HH}$ and reduce m_0 keeping $m_0 m_{0H} R_{0|HH}$ constant. This raises the RHS of IC1 because it raises $(1-m_0)R_{0|\cdot}$ and slackens PC due to the decreased frequency of the audit cost $m_0 c$. In turn this allows a reduction in R_2. Similar arguments apply to variations keeping $(1-m_{0,H})R_{0|H\cdot}$ and variations keeping $m_1 R_{1|H}$ constant (which has the added benefit of *coeteris paribus* reducing the left hand side and so slackening IC3. And finally variations keeping successively each of $m_0 m_{0,H} R_{0|HL}, m_0(1-m_{0,H})R_{0|H\cdot}$ constant. Thus $R_{0|HH} = R_{1|H} = 2H$, $R_{0|H\cdot} = R_{0|HL} = R_{0|LH} = H + L$.

We can moreover see that the right hand side of IC1 and IC3 is increasing in $m_{0,H}$, but this is neither in the objective nor in the participation constraint. So we can set $m_{0,H} = 1$, to slacken IC1 and IC3.

We can subsequently write the contract problem as:

$$\max E\Pi_J = p^2(2H - R_2) + 2p(1-p)\left\{H + L - m_1 R_{1|L} - (1-m_1)R_{1|\cdot}\right\}$$
$$+ (1-p)^2\left\{2L - m_0\left[m_{0,L}R_{0|LL} + (1-m_{0,L})R_{0|L\cdot}\right] - (1-m_0)R_{0|\cdot}\right\}$$

$$\text{st } E\Pi_L = p^2 R_2 + 2p(1-p)\left\{m_1\left(R_{1|L} - c\right) + (1-m_1)R_{1|\cdot}\right\}$$
$$+ (1-p)^2\left\{m_0\left[m_{0,L}\left(R_{0|LL} - c\right) + (1-m_{0,L})R_{0|L\cdot} - c\right] + (1-m_0)R_{0|\cdot}\right\} = 2I$$

$$\begin{aligned} R_2 &\leq m_0 2H + (1-m_0)R_{0|\cdot} \\ R_2 &\leq m_1 2H + (1-m_1)R_{1|\cdot} \end{aligned}$$

$$m_1 R_{1|L} + (1-m_1)R_{1|\cdot} \leq m_0\left[\frac{1}{2}(1+m_{0,L})(H+L) + \frac{1}{2}(1-m_{0,L})R_{0|L\cdot}\right] + (1-m_0)R_{0|\cdot}$$

1. The participation constraint must be binding. If it was slack, R_2 could be reduced, which would relax all IC's and increase the objective.

2. $R_{0|L\cdot} = R_{0|\cdot} = 2L$.
   If $R_{0|L\cdot} < 2L$ and $R_2 > 0$ we can reduce R_2 and raise $R_{0|L\cdot}$ so that $p^2 R_2 + (1-p)^2 m_0 (1-m_{0,L}) R_{0|L\cdot}$ stays constant, leaving both the objective function and the participation constraint unchanged. This slackens the first two incentive constraints and the right hand side of the third incentive constraint, again allowing a reduction in m_0. Similarly, we can reduce R_2 and raise $R_{0|\cdot}$ so that $p^2 R_2 + (1-p)^2 (1-m_0) R_{0|\cdot}$ stays constant, leaving both the objective function and the participation constraint unchanged, while slackening the three incentive constraints. We know $R_2 > 2L > 0$ since if $R_2 \leq 2L$ there is insufficient revenue to recoup the investment cost. Hence such reductions in R_2 are always possible. The result is $R_{0|L\cdot} = R_{0|\cdot} = 2L$.

3. $R_{0|LL} = 2L$
   With $R_{0|\cdot} = R_{0|L\cdot} = 2L$, the problem becomes:

$$\begin{aligned} E\Pi_J = {} & p^2 (2H - R_2) + 2p(1-p)\left\{H + L - m_1 R_{1|L} - (1-m_1) R_{1|\cdot}\right\} \\ & + (1-p)^2 \left\{2L - m_0 \left[m_{0,L} R_{0|LL} + 2(1-m_{0,L}) L\right] - 2(1-m_0) L\right\} \end{aligned}$$

$$\begin{aligned} \text{st } E\Pi_L = {} & p^2 R_2 + 2p(1-p)\left\{m_1 \left(R_{1|L} - c\right) + (1-m_1) R_{1|\cdot}\right\} \\ & + (1-p)^2 \left\{m_0 \left[m_{0,L} \left(R_{0|LL} - c\right) + 2(1-m_{0,L}) L - c\right] + 2(1-m_0) L\right\} = 2I \end{aligned}$$

$$\begin{aligned} R_2 &\leq m_0 2H + 2(1-m_0) L \\ R_2 &\leq m_1 2H + (1-m_1) R_{1|\cdot} \end{aligned}$$

$$m_1 R_{1|L} + (1-m_1) R_{1|\cdot} \leq m_0 \left[\frac{1}{2}(1+m_{0,L})(H+L) + (1-m_{0,L}) L\right] + 2(1-m_0) L$$

   $R_{0|LL}$ only appears in the objective function and the participation constraint. Lowering R_2 and raising $R_{0|LL}$ so as to keep $p^2 R_2 + (1-p)^2 m_0 m_{0,L} R_{0|LL}$ constant leaves both the objective and the PC unchanged, while slackening IC1 and IC2. So, also $R_{0|LL} = 2L$.

4. With $R_{0|\cdot} = R_{0|L\cdot} = R_{0|LL} = 2L$, the problem becomes:

$$E\Pi_J = p^2 (2H - R_2) + 2p(1-p)\left\{H + L - m_1 R_{1|L} - (1-m_1) R_{1|\cdot}\right\}$$

$$\begin{aligned} \text{st } E\Pi_L = {} & p^2 R_2 + 2p(1-p)\left\{m_1 \left(R_{1|L} - c\right) + (1-m_1) R_{1|\cdot}\right\} \\ & + (1-p)^2 \left\{2L - m_0 (1+m_{0,L}) c\right\} = 2I \end{aligned}$$

$$\begin{aligned} R_2 &\leq m_0 2H + 2(1-m_0) L \\ R_2 &\leq m_1 2H + (1-m_1) R_{1|\cdot} \end{aligned}$$

$$m_1 R_{1|L} + (1-m_1) R_{1|\cdot} \leq \frac{1}{2} m_0 (1+m_{0,L})(H-L) + 2L$$

5. $m_0 > 0$ and IC3 binding.
If $m_0 = 0$, the first and third IC's would be $R_2, R_{1|L}, R_{1|\cdot} \leq 2L$. Then the available revenue to repay $2I$ is no higher than $2L - 2p(1-p)m_1 c$, which is less than $2I$. So we must have $m_0 > 0$. Moreover, IC3 must be binding. Suppose not. It would then be possible to lower $m_{0,L}$ slackening the PC, thus allowing a reduction in R_2.

$$E\Pi_J = p^2(2H - R_2) + 2p(1-p)\left[H + L - m_1 R_{1|L} - (1-m_1)R_{1|\cdot}\right] \tag{33}$$

$$\text{st } E\Pi_L = p^2 R_2 + 2p(1-p)\left[m_1\left(R_{1|L} - c\right) + (1-m_1)R_{1|\cdot}\right] + (1-p)^2\left[2L - m_0(1+m_{0,L})c\right] = 2I$$

$$\begin{aligned} R_2 &\leq 2m_0(H-L) + 2L \\ R_2 &\leq m_1 2H + (1-m_1)R_{1|\cdot} \end{aligned}$$

$$m_1 R_{1|L} + (1-m_1)R_{1|\cdot} = \frac{1}{2}m_0(1+m_{0,L})(H-L) + 2L$$

6. $m_1 = 0$
The variables are $R_2, R_{1|\cdot}, R_{1|L}, m_{0,L}, m_1, m_0$. We know that the participation constraint and IC3 must bind and also that $m_0 > 0, R_2 > 2L$ to provide sufficient expected revenue to repay the debt. So we can eliminate these two variables using binding PC and IC3:

$$m_0 = 2\frac{(1-m_1)R_{1|\cdot} + m_1 R_{1|L} - 2L}{(1+m_{0,L})(H-L)}$$

$$\begin{aligned} R_2 &= \frac{2(1-p)\left\{(1-p)\left[m_1 R_{1|L} + (1-m_1)R_{1|\cdot} - 2L\right] + m_1 p(H-L)\right\}c}{p^2(H-L)} \\ &\quad - \frac{2(1-p)\left[m_1 R_{1|L} + (1-m_1)R_{1|\cdot}\right]}{p} + \frac{2\left[I - (1-p)^2 L\right]}{p^2} \end{aligned}$$

Substituting out m_0 and R_2 in the objective function and in IC1 and IC2 leaves the variables $R_{1|\cdot}, R_{1|L}, m_{0,L}, m_1$. Starting from any feasible position in the variables, we can locally vary all the variables but $m_{0,L}$ in ways which keep each constraint unchanged (thus requiring $dICI = dIC2 = 0$) and see which directions of change will improve the objective function. This requires the variations to satisfy

$$\begin{aligned} \frac{\partial IC_1}{\partial R_{1|\cdot}}dR_{1|\cdot} + \frac{\partial IC_1}{\partial R_{1|L}}dR_{1|L} + \frac{\partial IC_1}{\partial m_1}dm_1 &= 0 \\ \frac{\partial IC_2}{\partial R_{1|\cdot}}dR_{1|\cdot} + \frac{\partial IC_2}{\partial R_{1|L}}dR_{1|L} + \frac{\partial IC_2}{\partial m_1}dm_1 &= 0 \end{aligned}$$

We use this to express local variations in $R_{1|\cdot}, R_{1|L}$ in terms of the variations in m_1, holding $m_{0,L}$ constant. Finally we see the effect on the objective:

$$dobj = dobjR_{1|L}dR_{1|L} + dobjR_{1|\cdot}dR_{1|\cdot} + dobjm_1 dm_1.$$

Substituting in the variations in $dR_{1|\cdot}$ and $dR_{1|L}$ which insure that IC1 and IC3 hold, we get:

$$\frac{dobj}{dm_1} = \frac{-2p^2(1-p)c\left[1+m_{0,L}+p\left(1-m_{0,L}\right)\right](H-L)}{p\left[1+m_{0,L}+p\left(1-m_{0,L}\right)\right](H-L)-\left(1+m_{0,L}\right)(1-p)^2 c}$$

The above expression is negative. To see this, notice that the numerator is always negative, while the denominator term is positive. This can be seen by noticing that the derivative of the denominator with respect to $m_{0,L}$, $(1-p)\left[p\left(H-L\right)-(1-p)\,c\right]$, is always positive, and at $m_{0,L}=0$ the value of the denominator is positive.
Thus, the objective function can be increased by reducing m_1 as far as possible, whilst preserving feasibility.
A solution will then have $m_1 = 0$ so long as the implied $R_{1|\cdot}, R_{1|L}, R_2 \geq 0, R_2 < 2H, R_{1|L}, R_{1|\cdot} \leq H+L, 0 < m_0 < 1$, and there are sufficient revenues to repay the debt cost.[5] This amounts to say that condition (1) is satisfied.
Setting $m_1 = 0$ allows pooling the repayments in the top two states, thus removing any cheating incentives. So audit costs can be minimised.
With $m_1 = 0$, the problem reduces to

$$E\Pi_J = p^2\left(2H-R_2\right)+2p\left(1-p\right)\left(H+L-R_{1|\cdot}\right)$$

$$\text{st } E\Pi_L = p^2 R_2 + 2p\left(1-p\right)R_{1|\cdot} + \left(1-p\right)^2\left\{2L - m_0\left(1+m_{0,L}\right)c\right\} = 2I$$

$$\begin{aligned} R_2 &\leq 2m_0\left(H-L\right)+2L \\ R_2 &\leq R_{1|\cdot} \end{aligned}$$

$$R_{1|\cdot} = \frac{1}{2}m_0\left(1+m_{0,L}\right)\left(H-L\right)+2L$$

7. $R_2 = R_{1|\cdot}$.
From IC2, because of monotonicity of repayments, we deduce that $R_2 = R_{1|\cdot}$. Also since $m_1 = 0$, $R_{1|L}$ is never paid and it can be set to any value between 0 and $H+L$. Thus, IC1 becomes

$$\begin{aligned} R_{1|\cdot} &\leq m_0 2H + 2\left(1-m_0\right)L \\ &\leq 2m_0\left(H-L\right)+2L \end{aligned}$$

By comparing IC1 and IC3 we see that if IC3 is satisfied, then certainly IC1 is. So, we can ignore IC1 and the contract problem becomes:

$$E\Pi_J = p^2\left(2H-R_2\right)+2p\left(1-p\right)\left(H+L-R_{1|\cdot}\right)$$

$$\text{st } E\Pi_L = p^2 R_2 + 2p\left(1-p\right)R_{1|\cdot} + \left(1-p\right)^2\left\{2L - m_0\left(1+m_{0,L}\right)c\right\} = 2I$$

$$R_{1|\cdot} = \frac{1}{2}m_0\left(1+m_{0,L}\right)\left(H-L\right)+2L$$

[5] Notice this holds for any non-negative fixed value of $m_{0,L}$.

8. Determination of $R_{1|\cdot}$ and $m_0(1+m_{0,L})$
   Notice that the remaining monitoring probabilities only enter the constraints and only through the composite variable $m_0(1+m_{0,L})$. There is then a redundancy of instruments.
   In the participation constraint $R_{1|\cdot}$ needs to be high enough to cover the investment plus audit cost, but in the incentive constraint low enough to make cheating unprofitable between 1 and 0 zero successes. Similarly, the audit probability $m_0(1+m_{0,L})$ has to be high enough to control the cheating incentive but low to minimise the audit cost. The balance between the two comes from solving the remaining binding constraints for m_0 and $R_{1|\cdot}$. We get $m_0(1+m_{0,L}) = \dfrac{4(I-L)}{p(2-p)(H-L)-2(1-p)^2 c}$, $R_{1|\cdot} = \dfrac{2(I-L)(H-L)}{p(2-p)(H-L)-2(1-p)^2 c} + 2L$.
   We next verify that $m_0(1+m_{0,L}) \leq 2$. Indeed, since there are many combinations of monitoring a zero success report, the condition that guarantees that any combination is feasible is that $m_0(1+m_{0L}) = \dfrac{4(I-L)}{p(2-p)(H-L)-2(1-p)^2 c} \leq 2$. For this, we need
   $$0 < 2(I-L) < p(2-p)(H-L) - 2(1-p)^2 c,$$
   which certainly holds under Assumption (1). When the condition (1) holds with equality or is violated, then $m_0(1+m_{0L}) \geq 2$

9. We next verify that $R_{1|\cdot} \leq H+L$.

$$\begin{aligned} R_{1|\cdot} - H - L &= \frac{1}{2} m_0(1+m_{0,L})(H-L) + 2L - H - L \leq 0 \\ \frac{1}{2} m_0(1+m_{0,L})(H-L) &\leq H-L \\ m_0(1+m_{0,L}) &\leq 2 \end{aligned}$$

   which is non-positive iff $m_0(1+m_{0,L}) \leq 2$. Thus, $R_{1|\cdot} \geq H+L$ when condition (1) holds with equality or is violated.
   Hence this is then a feasible solution which also maximises the borrowers objective.

■

Proof of Proposition 4. To prove this, let us consider programme $\mathcal{P}_{weak}$. From that we see that there are two mutually exclusive cases: ■

- $m_1 = 0$
  In this case, from (28) $R_2 = H+L$. Moreover, if $m_1 = 0$, $R_{1|L}$ is never paid. Using $m_1 = 0$, $R_2 = R_{1|\cdot} = H+L$ in the participation constraint gives
  $$\left[p^2 + 2p(1-p)\right](H+L) + (1-p)^2 2(L-c) = 2I.$$

- $R_{1|L} = H + L$
  In this case the contract problem becomes

$$E\Pi_J = p^2 (2H - R_2)$$

$$\text{st } E\Pi_L = p^2 R_2 + 2p(1-p)(H + L - m_1 c) + (1-p)^2 2(L - c) = 2I$$

$$R_2 \leq m_1 2H + (1 - m_1)(H + L)$$

  Solving PC and IC2 for m_1 and R_2,**(show binding)**

$$m_1 = \frac{2(I - L) - p(2 - p)(H - L) + 2(1 - p)^2 c}{p^2 (H - L) - 2p(1 - p) c} > 0$$

  which is positive (because condition (1) is violated) and less than one:

$$m_1 - 1 = -2\frac{pH + (1 - p) L - I - (1 - p) c}{p[p(H - L) - 2(1 - p) c]} < 0$$

$$\begin{aligned} R_2 &= 2\frac{-(H - L)[pH + (1 - p) L - I - (1 - p) c] + H[p^2 (H - L) - 2p(1 - p) c]}{p^2 (H - L) - 2p(1 - p) c} \\ &= 2H - 2(H - L)\frac{[pH + (1 - p) L - I - (1 - p) c]}{p[p(H - L) - 2(1 - p) c]} \end{aligned}$$

  which is smaller than $2H$. It is also larger than $H + L$. Indeed, $R_2 - (H + L)$ can also be written as

$$H + L + \left\{ \frac{2(I - L) - p(2 - p)(H - L) + 2(1 - p)^2 c}{p[p(H - L) - 2(1 - p) c]} \right\} (H - L)$$

  Substituting out in the objective function $p^2 (2H - R_2)$ gives:

$$2[pH + (1 - p) L - I] - 2p(1 - p)\frac{2(I - L) - p(2 - p)(H - L) + 2(1 - p)^2 c}{p^2 (H - L) - 2p(1 - p) c} c - 2(1 - p)^2 c$$