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# Does the Stochastic Specification Matter?

Konstantina Mari

Department of Economics and Related Studies
University of York
Heslington
York, YO10 5DD

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Konstantina Mari[†]

University of York

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## Abstract

This paper highlights the importance and significance of the stochastic assumptions underlying any empirical analysis. The stochastic assumptions in any data analysis are usually implicit, rather than explicitly, stated. For example, in a binomial option pricing model, we assume that we have a binary random variable. In this paper we examine the significance of the stochastic assumptions by looking at the statistical properties of stated allocations (in an allocation problem) and their relation to the optimal allocation. The message that emerges is an important one: that the stochastic specification underlying any statistical analysis matters for the interpretation of its results. Our results suggest that before doing any statistical analysis one should carry out extensive simulations.

**Keywords**: stochastic assumption, stochastic specification, modelling, statistics, econometrics, analysis, simulation

[†] University of York, Department of Economics and Related Studies, E-mail: km824@york.ac.uk, mari.konstantina@gmail.com

## 1. Introduction

Underlying any statistical test of any hypothesis or any estimation of any model is some stochastic specification. It is fair to say that many economists generally pay scant attention to this, usually assuming normality somewhere. This chapter explores the implications of this, both for the hypothesis under study and the parameters being estimated.

The context in which we do this exploration is the estimation of the risk-aversion of decision-makers. This is crucial to most theories of decision-making under risk, and to many policy issues. There are several experimental methods of eliciting risk-aversion indices, the most prominent being Holt-Laury price lists (Holt and Laury 2002), pairwise choice questions (Hey and Orme 1994), the Becker-Degroot-Marschak mechanism (Becker *et al* 1964) and allocation problems (Loomes 1991). We concentrate here on the latter method. Wilcox (2009) has done a similar analysis using the method of pairwise choice (which can be considered to be a sort of unstructured Holt-Laury price list); he concludes that the stochastic specification may well be more important than the functional specifications[1]. We do not have different functional specifications, so as to concentrate on the effect of the stochastic specification.

Like all methods of eliciting preferences, one can make a variety of stochastic assumptions, but these depend on the elicitation method. Here we use allocation problems. We describe these in Section 2. In Section 3 we describe what the DM *ought* to be doing. But in experiments there is noise in subjects' behaviour. When we use experimental data to estimate their risk-aversion we need to take this noise into account. It is the description of this noise that is our stochastic specification. In Section 4 we discuss possible stochastic specifications. In order to compare between specifications we carry out extensive simulations – generating data under a variety of stochastic specifications and then estimating under them. We discuss our simulation and estimation methods in Section 5. Our results are in Section 6 and Section 7 concludes.

## 2. The allocation method for eliciting risk-aversion

This is one of a variety of methods for eliciting risk-aversion. Its advantages are that it is simple to describe and simple for subjects to understand – in contrast, for example, with the Becker-DeGroot-

[1] The preference functional (Expected Utility or some other), the weighting function and the utility function.

Marschak mechanism. One possible disadvantage, in an experimental setting, is that it implies that subjects must optimise rather than just choose, which latter is the case if the pairwise choice method is used.

In its simplest form, with the allocation method subjects are presented with a number of problems, each of the same form. In each problem, subjects are endowed with a quantity of tokens which they are asked to allocate between two *ex ante* risky states, with specified probabilities for the two states, and with specified exchange rates between tokens and real money for each of the states. Usually the experiment is computerised and the computer records their chosen allocations on each problem. After they have responded to all the problems, one of them is chosen at random, and their allocation on that problem retrieved from the computer records. Then a random device is implemented and one of the two states is realised. The subject is paid the money value of their allocation of tokens to the realised state, using the specified exchange rate for that state.

Let us give a simple example. Suppose the two states are labelled 'Red' and 'Blue', and suppose the probabilities are respectively 0.4 and 0.6. Suppose the subject is given 100 tokens to allocate and is told that the exchange rates between tokens and money for Red is 0.8 tokens to a £1 (so that a token allocated to Red is worth £1.25) and for Blue is 1.25 tokens to a £ (so that a token allocated to Blue is worth 80p). The allocation that the subject makes is obviously dependent on his or her attitude to risk (this being the whole point of the exercise) and the exchange rates. Suppose that the subject decides to allocate 40 to Red and 60 to Blue. If this problem was randomly selected to be played out for real, and if the random device resulted in Red being selected, then the subject would be paid £(40/0.8) = £50; if the random device resulted in Blue being selected, then the subject would be paid £(60/1.25) = £48.

To estimate the level of risk-aversion – assuming that the subject obeys Expected Utility theory – a utility function should be specified and the parameter(s) of it would be estimated. To keep things simple in this simulation, we assume[2] a CRRA (Constant Relative Risk Aversion) utility function

$$u(x) = \frac{x^{1-r}}{1-r}$$

[2] An alternative would be the CARA (Constant Absolute Risk Aversion) function.

and estimate the parameter *r*. If this takes the value 0 the individual is risk-neutral; if positive, risk-averse and if negative, risk-loving; a higher value of *r* indicates a higher level of risk-aversion.

## 3. The optimal allocation

The decision made by each subject on each problem is an allocation of their endowment in that problem between the two states of the world. Let us call them State 1 and State 2. We normalise the endowment to 1, and denote the allocation to State 1 by *x* (so that the allocation to State 2 is *1-x*). Let us denote the probability of State 1 by *p* (so that the probability of State 2 is *1*-p), and the exchange rate between tokens and money in State 1 by *e* (normalising the exchange rate in State 2 to 1). As noted above, we assume that the DM is an Expected Utility maximiser with the CRRA utility function specified above. There is an *optimal* allocation given by the maximization of

$$EU(X) = p\frac{ex^{1-r}}{1-r} + (1-p)\frac{(1-x)^{1-r}}{1-r}$$

The solution if $r > 0$ (the DM is risk-averse) is

$$x^* = \frac{(pe)^{\frac{1}{r}}}{(pe)^{\frac{1}{r}} + (1-p)^{\frac{1}{r}} e}$$

Thus if $r>0$ one immediate implication is that $x^*$ is strictly bounded between 0 and 1 for non-zero *p* and *e*. Throughout this study we maintain this assumption. This has implications for our choice of one of the stochastic specifications.

If $r \leq 0$ (the DM is risk-neutral or risk-loving) the DM will want to allocate as much as possible to one of the two states (the one depending on the problem and the exchange rate). If the experiment puts bounds on the allocation (see below) the DM will want to allocate all or nothing.

## 4. Assumed stochastic specifications

While an optimal allocation exists for any given value of *r*, we assume that there is some noise in its implementation. We denote the *actual* allocation by *x*. We assume that the error is purely stochastic and that it has no systematic component (otherwise we would take that it into account in our preference functional). It is the specification of this error that is our stochastic specification. In principle, if perhaps not in practice, the economist ought to take into account behavioural considerations when deciding on the specification. Our stochastic specifications follow.

### 4.1 Additive normal (*an*)

We start with the standard assumption in the economist's toolbox: that the noise is in the calculation of the optimal allocation, and that noise is added to the optimal allocation. Furthermore this noise is normal. To avoid it having a systematic component, it is assumed that this normal distribution has a zero mean. Thus there is no bias in the implementation of the allocation: on average it is equal to the optimal allocation. However it does not have a zero variance. We characterise this specification as $x = x^* + \varepsilon$ where $\varepsilon$ is $N(0,1/s^2)$. Following convention, we refer to $s$, the inverse of the standard deviation, as the *precision* of the allocation. The larger is $s$ the lower is the magnitude of the error.

One problem with using the normal distribution is that it is unbounded. In an experiment, subjects would not be allowed to allocate more than the endowment to any one State, as this would imply a negative allocation to the other state, and hence imply the possibility of the subject losing money. So actual decisions have to be truncated at 0 and 1.

### 4.2 Beta (*b*)

An alternative specification, and one that takes the boundedness of the optimal allocation into account, is to assume that $x$ has a *Beta* distribution. This is bounded between 0 and 1, and has two parameters $\alpha$ and $\beta$. Its mean and variance are given by $\frac{\alpha}{\alpha+\beta}$ and $\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$ respectively. If we put $\alpha=x^*(s-1)$ and $\beta=(1-x^*)(s-1)$, then the mean and variance are given by $x^*$ and $\frac{x^*(1-x^*)}{s}$ respectively. This is an attractive specification: it means that x is unbiased, and that the noise reduces to zero as x* approaches the bounds (0 and 1). This is behaviourally plausible:

when the optimal allocation is around one-half the DM suffers the most uncertainty about it, but when it is close to 0 or 1 the DM is very sure about it. Figure 1 illustrates.

### 4.3 Random preferences normal (rpn)

This postulates that the noise is generated *before* the allocations are determined: the noise is in the risk-aversion parameter *r*. Here the story is that the DM is unsure about his or her value of *r* and it is this that is random. But, given any value for *r,* the DM implements the optimal allocation for that *r* without error. Notice that this is quite a different behavioural story.

Our first implementation of this random preferences story is to assume that *r* has a normal distribution with a given precision *s*. Clearly we have to specify the mean value of *r* (which might be termed the DM's true value) and the precision. Note that negative values of *r* could be generated with this specification; these would imply risk-loving behaviour and hence all-or-nothing allocations. Again the larger is *s* the lower is the magnitude of the error.

### 4.4 Additive logistic (al)

This is almost identical to the additive normal, the only difference being that $x = x^* + \varepsilon$ where $\varepsilon$ has now a *logistic* distribution with mean 0 and scale parameter $1/s$. Again we refer to *s* as the precision: the larger is *s* the lower is the magnitude of the scale of the distribution.

This distribution is very similar to the normal – but it has slightly heavier tails. Figure 2, in which the normal *pdf* is the blue dashed one, illustrates.

### 4.5 Random preferences lognormal (rpl)

This is similar to random preferences normal, though the similarities are less than between the additive normal and the additive logistic. The normal distribution is symmetrical while the log-normal is skewed to the right. Figure 3, in which the normal *pdf* is the blue dashed curve, illustrates.

## 5. The simulation and estimation program

We carried out an extensive simulation and estimation. The program, and the input files needed to run the program, can be found on the EXEC website. We ran 1000 simulations, a simulation corresponding to a decision-maker. In each simulation, we first used each of the specifications to generate some random allocations, and then we fitted under each of the specifications. For each simulation, fitting involved the estimation of either a parameter *r* (being the risk-attitude for that simulation for the additive normal, beta and additive logistic specifications) or a parameter[3] *r* (being the *mean* of the random preferences normal and lognormal specifications); plus a precision parameter *s*.

In order to generate the random allocations we need to work with some numerically-specified preference parameters and with some specific allocation problems. The former can be found in the file *PreferenceParameters.csv*. This contains 17 lines the first 9 being a guide to the remaining 8. The data can be summarised as follows:

| Parameter set | *r* for *an, b* and *al* | *s* for *an* | s for *b* | (mean) *r* for *rpn* | *s* for *rpn* | *s* for *al* | (mean) *r* for *rpl* | *s* for *rpl* |
|---|---|---|---|---|---|---|---|---|
| 1 | 0.5 | 25 | 40 | 0.5 | 10 | 20 | -0.7 | 5 |
| 2 | 0.5 | 50 | 40 | 0.5 | 20 | 40 | -0.7 | 9 |
| 3 | 0.5 | 25 | 40 | 0.5 | 10 | 20 | -0.7 | 5 |
| 4 | 0.5 | 50 | 40 | 0.5 | 20 | 40 | -0.7 | 9 |
| 5 | 1.5 | 25 | 80 | 1.5 | 10 | 20 | 0.4 | 5 |
| 6 | 1.5 | 50 | 80 | 1.5 | 20 | 40 | 0.4 | 9 |
| 7 | 1.5 | 25 | 80 | 1.5 | 10 | 20 | 0.4 | 5 |
| 8 | 1.5 | 50 | 80 | 1.5 | 20 | 40 | 0.4 | 9 |

It will be seen that we chose two values for the *r* for *an, b* and *al,* each combined with two values for the precision *s*. A similar pattern was chosen elsewhere.

The simulation also requires some allocation problems. These we took from an experiment (Zhou and Hey 2016) investigating different elicitation methods. One of the methods explored was the allocation method. In that experiment, subjects were presented with 81 allocation problems (with different *p*'s and *e*'s). Here we used these 81 problems. In addition, because we were interested in the effect of the *number of problems* on the *accuracy* of the estimation, we also took a subset of 41 of these 81 problems, and we also doubled them up – thus creating a file consisting of 162 problems.

[3] We trust that the use of the same notation will not confuse the reader.

In what follows, we refer to the 41 problems file as the *small* data set, the 81 problems file as the medium data set and the 162 problems file as the *large* data set.

The program was written in Matlab, and used Maximum Likelihood estimation, implemented with the Matlab procedure *patternsearch.* The program produces estimates of *r* and *s*, and reports the maximised log-likelihoods.

## 6. Results

We report our results in several ways. First, we report the maximised log-likelihoods and ask the question as to whether the true specification is identified in the estimation. Then we look at the estimates of *r*, then the standard deviation of the estimates of *r*, and finally the estimates of *s*.

Tables 1 report the maximised log-likelihoods – our measure of goodness-of-fit: tables 1.1s through to 1.8s for the small data set; tables 1.1m through to 1.8m for the medium data set; and tables 1.1l through to 1.8l for the large data set. On each page there are 8 tables – corresponding to the eight parameter sets[4]. The rows indicate the *true* specification – that generating the data; the columns indicate the *estimated* specification. What *should* be the case is that in each row the diagonal element should be the largest value in the row – indicating that the estimation has correctly identified the true specification. We indicate with (blue) shading the estimated specification with the highest log-likelihood in each row. It can be seen that for the small data set only 21 times out of 40 was the true specification correctly identified. Things were considerably better with the medium data set, with the true specification correctly identified 34 times out of 40; and they were marginally better again with the large data set – with 35 out of 40 correctly identified. Indeed, there is a greater separation of the log-likelihoods for the large data set. It should be noted that the log-likelihoods for *an* are generally very close to those for *al* – which is hardly surprising as the specifications are very close (see Figure 2). Interestingly the log-likelihoods for *b* when it is true are generally very close to those for *rpl*.

The message emerging from these tables is that, if the data set is large enough, the true specification is generally correctly identified, but that is not the case with the small data set.

[4] As there are minor differences between the results for the different parameter sets, we do not comment on them.

Tables 2 report the *mean* value of the estimated *r* values: tables 2.1s through to 2.8s for the small data set: tables 2.1m through to 2.8m for the medium data set; and tables 2.1l through to 2.8l for the large data set. As before, on each page there are 8 tables – corresponding to the eight parameter sets. The rows indicate the *true* specification – that generating the data; the columns indicate the *estimated* specification. The column headed 'True value' is the *r* value used for generating the data.

Before we look at these tables we should note something about the *rpl* specification. Here the *r* value reported for *rpl* in the table above is *not* the mean of the *r* values. Here *r* is lognormally distributed with parameters *r* and *s*. This means that *log(x)* is normally distributed with mean *r* and precision *s.* So the mean of *r* is $\exp(r+\frac{1}{2s^2})$ and its precision is the inverse of the square root of $\exp(\frac{1}{2s^2}-1)\exp(2r+\frac{1}{s^2})$. In constructing tables 2, we have taken this into account. This explains, for example, why the 'True value' for parameter set 1 with the small data set is 0.507, rather than the -0.7 in the table above.

The clear message that emerges from Tables 2 is that the mean *r* value is generally extremely precisely estimated. When the true *r* is 0.5, virtually all the mean estimates for the three data sets are between 0.49 and 0.51, and they are particularly close to 0.5 when the estimated specification is the true specification. When the true *r* is 1.5, virtually all the mean estimates for the three data sets are between 1.48 and 1.52, and they are particularly close to 1.5 when the estimated specification is the true specification. In other words it seems to be the case that in this context our maximum likelihood estimates are unbiased, which is not normally necessarily the case[5]. The message here seems to suggest that, if one is only interested in the mean value of *r* then the specification does not really matter.

However, we should take into account the standard deviation of the estimates of *r*. These are given in Tables 3: tables 3.1s through to 3.8s for the small data set; tables 3.1m through to 3.8m for the medium data set; and tables 3.1l through to 3.8l for the large data set. As before, on each page there are 8 tables – corresponding to the eight parameter sets. The rows indicate the *true* specification – that generating the data; the columns indicate the *estimated* specification. What *should* be the case is that in each row the smallest element should be along the main diagonal –

[5] It can be shown that in general maximum likelihood estimates are *consistent* but not necessarily unbiased. Our results could emerge if our sample sizes are considered to be 'close enough to infinity'.

where the estimated specification is the true specification. As in Tables 1 we highlight (in blue) where this is the case. It can be seen that for the small data set 29 times out of 40 this was the case; with the medium data set, 30 times out of 40; and with the large data set 32 times out of 40. Generally the standard deviations are very low (in comparison with the mean estimated *r* values) and importantly they decrease with the size of the problem set.

It should be remembered that the standard deviation indicates the accuracy of the estimates of *r*. While Tables 2 indicate that the *mean* estimates of *r* are very close to their true values, in practice one only has one data set (and not the 1000 in the simulation). Hence, when the standard deviation is high, individual estimates of *r* could depart quite significantly from the true value, but when the standard deviation is low, individual estimates of *r* will be generally closer to the true value. Looking at Tables 3, it is clear that the standard deviation of the *r* estimates decreases with the number of problems in the data set. So size does matter: with a larger set of problems one gets more precise estimates.

If one is interested in estimates of the *precision*, a different story emerges, and here our estimates are not even unbiased – unless the estimated specification is the true specification. Tables 4 report the average value of the estimated *s* values: tables 4.1s through to 4.8s for the small data set; tables 4.1m through to 4.8m for the medium data set; and tables 4.1l through to 4.8l for the large data set. As before, on each page there are 8 tables – corresponding to the eight parameter sets. The rows indicate the *true* specification – that generating the data; the columns indicate the *estimated* specification. The column headed 'True value' is the *s* value used for generating the data. If one looks down the main diagonal it will be seen that when the estimated specification is the true specification, the mean *s* estimate is close to its true value. However, when one departs from the main diagonal significant differences emerge. So, in general, the estimated precision is quite far (and in some cases very far) from the true precision. Yet, considering Tables 2, it does not seem to be the case that misestimating *s* affects the mean estimates of *r*[6].

[6] Incidentally this seems to be also true even if the maximum likelihood routine hits the bounds in the program – for example in Table 4.6s when *rpn* is the true specification and *b* the estimated specification.

## 7. Conclusions

We should first note, and as is clear from Figures 1, 2 and 3, that our chosen specifications are *very* close. As a consequence it may not be surprising that, whatever is the estimated specification, the parameter set and the size of the problem set, the *mean* estimates of the *r* parameter are close to their true values, In particular it seems to be the case that the stochastic specification does not matter when it comes to estimating the *mean* level of risk-aversion. Moreover, the effect of the size of the problem set on the *mean* of the estimates seems to be very low. However, when it comes to identifying the true specification, size does appear to have an effect: the bigger the problem set the better the identification. Size does also appear to have an effect on the standard deviation of the estimates of *r*: with a larger problem set one gets more precise estimates of *r*.

Furthermore, if one is interested in the estimates of *s* the precision, the specification and the size of the problem set do have a significant effect.

Does all this matter? Well, it depends on what use is to be made of the estimates: if one is going to use them for prediction of the optimal allocation with associated confidence intervals, then getting estimates correct is crucial. Figure 4 illustrates: this reports the distribution of 100,000 simulated allocations for particular preference parameters (set 1), a particular problem (number 22 from the small data set), and a particular true specification. The true specification here is *b*. The thick black curve is the (kernel[7]) density function for the implied distribution using the true parameters, and the thin black curve is the density function for the implied distribution using the estimated parameters with *b* estimated; the green curve is that with *an* estimated; the blue curve with *rpn* estimated; the yellow curve with *al* estimated; and the magenta curve with *rpl* estimated. The complete set (consisting of 1640[8] graphs), can be found on the EXEC website.

This Figure shows that when the estimated specification is the true specification, the distribution based on the estimated parameters is very close to that based on the true parameters, but when the estimated specification is *not* the true specification, the distribution can be quite different: when *rpn* is estimated the distribution is biased and skewed – leading to biased and skewed predictions; when *rpl* is estimated the distribution is also biased and skewed, though less so; with

[7] Kernel density estimation (KDE) is a non-parametric way to estimate the probability density function of a random variable. We did not want to impose particular functional forms on the density functions, particularly as the distributions are truncated.

[8] Composed of the combination of 8 parameter sets, 41 problems (the small data set) and 5 true models.

*an* and *al* the distributions are unbiased but have too small a spread. All the non-true distributions are quite different from the true distribution, and so will be any predictions and their associated confidence intervals.

So our conclusion must be: the stochastic specification *does* matter.

## References

Becker GM, DeGroot MH and Marschak J (1964), "Measuring utility by a single-response sequential method", *Behavioral Science*, *9*, 226-232.

Hey JD and Orme C (1994), "Investigating generalizations of expected utility theory using experimental data", *Econometrica, 62,* 1291-1326.

Holt CA and Laury SK (2002), "Risk aversion and incentive effects", *American Economic Review*, *92*, 1644-1655.

Loomes G (1991) "Evidence of a new violation of the independence axiom", *Journal of Risk and Uncertainty*, *4*, 91-108.

Wilcox (2008), "Stochastic Models for Binary Discrete Choice under Risk: A Critical Primer and Econometric Comparison", Risk Aversion in Experiments, Research in Experimental Economics, 12, 197–292.

Zhou W and Hey JD (2016), "Context Matters", Working Paper, University of York.

Figure 1: Beta distributions of *x* for *x*=0.05* (green on the left)*, 0.25* red (in the middle) and *0.5* blue (on the right); *s=50*

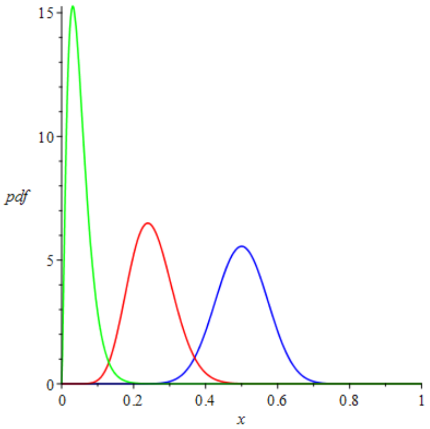



Figure 2: Normal (blue dashed) and logistic (red solid) distributions

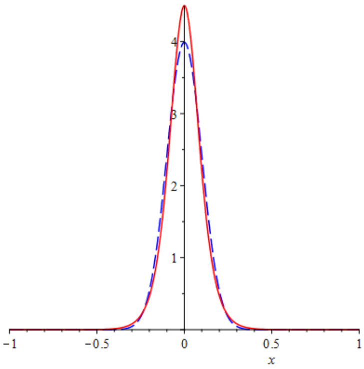



Figure 3: Normal (blue dashed) and lognormal (red solid) distributions

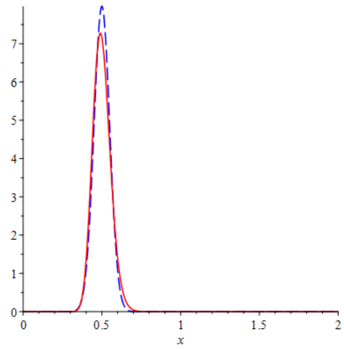
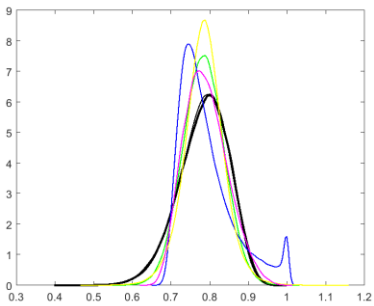



Figure 4: simulated allocations based on preference parameters 1 and problem number 29



Key: The true specification here is *b*, the parameter set number 1 and the problem number 22. The thick black curve is the (kernel) density function for the beta distribution using the true parameters, and the thin black curve is the function for the beta distribution using the estimated parameters; the green curve is that with *an* estimated; the blue curve with *rpn* estimated; the yellow curve with *al* estimated; the magenta curve with *rpl* estimated.

Table 1.1s: Log-Likelihoods parameter set 1, small data set

| Specification | | Estimated | | | | |
|---|---|---|---|---|---|---|
| | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | -110.89 | -111.56 | -114.33 | -111.11 | -108.87 |
| | *b* | -126.78 | -115.67 | -133.00 | -126.03 | -115.10 |
| | *rpn* | -106.47 | -101.21 | -91.07 | -104.69 | -92.09 |
| | *al* | -142.01 | -136.71 | -145.54 | -141.14 | -120.99 |
| | *rpl* | -105.77 | -100.06 | -91.66 | -104.09 | -90.81 |

Table 1.2s: Log-Likelihoods parameter set 2, small data set

| Specification | | Estimated | | | | |
|---|---|---|---|---|---|---|
| | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | -84.18 | -89.90 | -93.79 | -84.30 | -90.39 |
| | *b* | -126.73 | -115.87 | -144.76 | -125.75 | -114.75 |
| | *rpn* | -78.81 | -74.40 | -66.09 | -76.61 | -66.39 |
| | *al* | -115.48 | -115.08 | -118.89 | -114.75 | -111.01 |
| | *rpl* | -82.25 | -78.19 | -69.83 | -80.67 | -69.74 |

Table 1.3s: Log-likelihoods parameter set 3, small data set

| Specification | | Estimated | | | | |
|---|---|---|---|---|---|---|
| | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | -110.98 | -111.43 | -114.56 | -111.27 | -109.19 |
| | *b* | -126.84 | -115.84 | -130.55 | -125.85 | -115.06 |
| | *rpn* | -106.32 | -101.55 | -90.95 | -104.68 | -91.69 |
| | *al* | -141.78 | -136.87 | -149.03 | -141.19 | -121.10 |
| | *rpl* | -105.74 | -100.32 | -92.13 | -104.33 | -91.13 |

Table 1.3s: Log-likelihoods parameter set 4, small data set

| Specification | | Estimated | | | | |
|---|---|---|---|---|---|---|
| | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | -84.20 | -90.21 | -91.47 | -84.30 | -90.55 |
| | *b* | -126.62 | -115.86 | -139.78 | -125.85 | -114.66 |
| | *rpn* | -78.70 | -73.88 | -65.87 | -76.90 | -66.48 |
| | *al* | -115.55 | -114.66 | -117.21 | -114.67 | -110.81 |
| | *rpl* | -82.41 | -78.32 | -69.94 | -80.90 | -69.74 |

Table 1.5s: Log-Likelihoods parameter set 5, small data set

| Specification | | Estimated | | | | |
|---|---|---|---|---|---|---|
| | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | -113.78 | -114.56 | -152.90 | -114.64 | -115.66 |
| | *b* | -124.20 | -123.84 | -211.65 | -124.76 | -121.92 |
| | *rpn* | -63.96 | -65.85 | -53.09 | -62.77 | -52.82 |
| | *al* | -146.97 | -149.45 | -405.17 | -147.02 | -129.10 |
| | *rpl* | -107.42 | -109.41 | -92.42 | -105.61 | -91.63 |

Table 1.6s: Log-Likelihoods parameter set 6, small data set

| Specification | | Estimated | | | | |
|---|---|---|---|---|---|---|
| | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | -86.07 | -86.34 | -117.73 | -86.43 | -96.37 |
| | *b* | -124.29 | -123.67 | -546.80 | -124.63 | -121.92 |
| | *rpn* | -38.97 | -50.82 | -32.51 | -38.47 | -32.61 |
| | *al* | -119.18 | -119.54 | -384.26 | -118.42 | -118.35 |
| | *rpl* | -84.15 | -85.47 | -70.32 | -82.38 | -69.95 |

Table 1.7s: Log-Likelihoods parameter set 7, small data set

| Specification | | Estimated | | | | |
|---|---|---|---|---|---|---|
| | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | -114.21 | -114.46 | -142.21 | -114.68 | -115.53 |
| | *b* | -124.31 | -123.60 | -221.51 | -124.83 | -121.80 |
| | *rpn* | -64.02 | -65.98 | -52.81 | -62.88 | -53.06 |
| | *al* | -146.58 | -149.30 | -406.93 | -146.58 | -129.34 |
| | *rpl* | -107.22 | -109.62 | -92.57 | -105.69 | -91.69 |

Table 1.8s: Log-Likelihoods parameter set 8, small data set

| Specification | | Estimated | | | | |
|---|---|---|---|---|---|---|
| | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | -86.03 | -86.42 | -109.32 | -86.22 | -96.77 |
| | *b* | -124.32 | -124.03 | -518.55 | -124.50 | -122.06 |
| | *rpn* | -39.02 | -50.74 | -32.61 | -38.49 | -32.55 |
| | *al* | -118.92 | -120.04 | -369.84 | -118.42 | -117.80 |
| | *rpl* | -83.92 | -85.91 | -70.10 | -82.32 | -70.04 |

Key *an*: additive normal; *b*:beta; *rpn*: random preferences normal; *al*: additive normal; *rpn*: random preferences lognormal

Table 1.1m: Log-Likelihoods parameter set 1, medium data set

| Specification | | Estimated | | | | |
|---|---|---|---|---|---|---|
| | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | -220.69 | -222.91 | -230.90 | -221.02 | -224.16 |
| | *b* | -252.02 | -230.36 | -282.04 | -250.14 | -239.71 |
| | *rpn* | -213.20 | -204.68 | -181.86 | -208.80 | -184.11 |
| | *al* | -282.07 | -272.65 | -294.85 | -280.24 | -246.95 |
| | *rpl* | -211.69 | -201.26 | -183.42 | -207.99 | -181.92 |

Table 1.2m: Log-Likelihoods parameter set 2, medium data set

| Specification | | Estimated | | | | |
|---|---|---|---|---|---|---|
| | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | -167.56 | -180.21 | -183.18 | -167.90 | -189.31 |
| | *b* | -251.89 | -230.62 | -340.76 | -249.90 | -241.66 |
| | *rpn* | -157.64 | -149.79 | -132.88 | -153.87 | -133.42 |
| | *al* | -229.73 | -229.14 | -247.31 | -228.20 | -227.37 |
| | *rpl* | -165.52 | -157.22 | -140.64 | -161.48 | -140.13 |

Table 1.3m: Log-likelihoods parameter set 3, medium data set

| Specification | | Estimated | | | | |
|---|---|---|---|---|---|---|
| | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | -220.49 | -222.84 | -230.92 | -221.00 | -224.46 |
| | *b* | -252.39 | -230.88 | -279.27 | -250.07 | -239.41 |
| | *rpn* | -213.67 | -204.69 | -181.73 | -209.80 | -183.51 |
| | *al* | -281.77 | -273.29 | -291.37 | -280.41 | -247.65 |
| | *rpl* | -211.55 | -201.28 | -183.85 | -207.96 | -182.32 |

Table 1.3m: Log-likelihoods parameter set 4, medium data set

| Specification | | Estimated | | | | |
|---|---|---|---|---|---|---|
| | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | -167.42 | -179.89 | -182.95 | -167.80 | -188.14 |
| | *b* | -252.53 | -230.42 | -337.85 | -250.10 | -240.57 |
| | *rpn* | -158.25 | -149.71 | -132.55 | -153.90 | -133.18 |
| | *al* | -229.72 | -229.03 | -246.87 | -227.97 | -227.61 |
| | *rpl* | -165.74 | -157.08 | -140.66 | -162.20 | -140.28 |

Table 1.5m: Log-Likelihoods parameter set 5, medium data set

| Specification | | Estimated | | | | |
|---|---|---|---|---|---|---|
| | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | -226.20 | -227.38 | -290.60 | -227.29 | -241.29 |
| | *b* | -246.84 | -246.06 | -414.46 | -247.83 | -251.37 |
| | *rpn* | -127.70 | -131.54 | -104.82 | -124.81 | -104.85 |
| | *al* | -291.93 | -297.20 | -746.99 | -291.26 | -260.27 |
| | *rpl* | -213.82 | -218.20 | -184.05 | -209.88 | -181.84 |

Table 1.6m: Log-Likelihoods parameter set 6, medium data set

| Specification | | Estimated | | | | |
|---|---|---|---|---|---|---|
| | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | -170.86 | -171.73 | -225.68 | -171.56 | -209.29 |
| | *b* | -246.81 | -245.79 | -887.16 | -247.31 | -251.00 |
| | *rpn* | -77.66 | -100.40 | -63.97 | -76.11 | -64.09 |
| | *al* | -236.54 | -238.02 | -717.05 | -235.34 | -244.19 |
| | *rpl* | -167.41 | -170.63 | -139.84 | -163.32 | -139.22 |

Table 1.7m: Log-Likelihoods parameter set 7, medium data set

| Specification | | Estimated | | | | |
|---|---|---|---|---|---|---|
| | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | -226.40 | -227.36 | -299.06 | -227.59 | -241.88 |
| | *b* | -247.28 | -245.55 | -382.28 | -247.98 | -251.54 |
| | *rpn* | -127.77 | -131.57 | -104.81 | -124.42 | -104.86 |
| | *al* | -291.30 | -297.72 | -758.16 | -291.00 | -261.37 |
| | *rpl* | -213.71 | -217.89 | -183.83 | -209.54 | -182.06 |

Table 1.8m: Log-Likelihoods parameter set 8, medium data set

| Specification | | Estimated | | | | |
|---|---|---|---|---|---|---|
| | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | -170.84 | -171.76 | -234.97 | -171.70 | -209.72 |
| | *b* | -246.42 | -245.92 | -952.76 | -247.53 | -250.88 |
| | *rpn* | -78.02 | -100.41 | -63.79 | -75.98 | -63.93 |
| | *al* | -236.07 | -239.05 | -689.76 | -235.53 | -245.12 |
| | *rpl* | -167.21 | -171.19 | -139.67 | -164.08 | -139.26 |

Key *an*: additive normal; *b*:beta; *rpn*: random preferences normal; *al*: additive normal; *rpn*: random preferences lognormal

Table 1.1l: Log-Likelihoods parameter set 1, large data set

| Specification | | Estimated | | | | |
|---|---|---|---|---|---|---|
| | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | -442.07 | -447.18 | -463.19 | -442.41 | -451.71 |
| | *b* | -505.66 | -461.76 | -550.58 | -501.42 | -484.20 |
| | *rpn* | -429.43 | -411.16 | -364.97 | -419.24 | -369.32 |
| | *al* | -565.48 | -547.53 | -592.59 | -561.18 | -496.64 |
| | *rpl* | -425.92 | -403.88 | -368.26 | -417.60 | -364.91 |

Table 1.2l: Log-Likelihoods parameter set 2, large data set

| Specification | | Estimated | | | | |
|---|---|---|---|---|---|---|
| | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | -335.95 | -361.26 | -369.68 | -337.08 | -383.33 |
| | *b* | -505.42 | -462.01 | -663.29 | -500.68 | -483.92 |
| | *rpn* | -317.02 | -301.66 | -266.32 | -309.06 | -267.72 |
| | *al* | -460.55 | -460.40 | -499.27 | -457.58 | -457.97 |
| | *rpl* | -333.17 | -315.49 | -282.31 | -324.56 | -281.50 |

Table 1.3l: Log-likelihoods parameter set 3, large data set

| Specification | | Estimated | | | | |
|---|---|---|---|---|---|---|
| | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | -442.03 | -447.00 | -466.02 | -443.07 | -452.59 |
| | *b* | -505.51 | -462.33 | -549.22 | -502.11 | -483.96 |
| | *rpn* | -429.93 | -411.89 | -364.54 | -420.32 | -368.61 |
| | *al* | -565.68 | -548.22 | -577.19 | -561.31 | -497.29 |
| | *rpl* | -425.66 | -404.24 | -368.58 | -417.58 | -365.24 |

Table 1.3l: Log-likelihoods parameter set 4, large data set

| Specification | | Estimated | | | | |
|---|---|---|---|---|---|---|
| | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | -336.00 | -361.56 | -372.51 | -336.77 | -382.46 |
| | *b* | -505.82 | -462.40 | -665.04 | -501.44 | -484.95 |
| | *rpn* | -317.99 | -300.97 | -266.43 | -309.47 | -267.56 |
| | *al* | -460.32 | -459.92 | -505.19 | -457.64 | -458.99 |
| | *rpl* | -332.73 | -315.62 | -282.33 | -325.16 | -281.56 |

Table 1.5l: Log-Likelihoods parameter set 5, large data set

| Specification | | Estimated | | | | |
|---|---|---|---|---|---|---|
| | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | -453.77 | -456.30 | -594.31 | -455.86 | -486.87 |
| | *b* | -494.67 | -492.52 | -809.77 | -496.35 | -506.74 |
| | *rpn* | -258.05 | -264.89 | -210.19 | -251.16 | -210.39 |
| | *al* | -584.58 | -596.53 | -1484.2 | -583.30 | -524.36 |
| | *rpl* | -429.10 | -437.85 | -369.22 | -421.34 | -364.97 |

Table 1.6l: Log-Likelihoods parameter set 6, large data set

| Specification | | Estimated | | | | |
|---|---|---|---|---|---|---|
| | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | -343.16 | -344.12 | -461.50 | -344.54 | -426.49 |
| | *b* | -494.55 | -492.75 | -1753.0 | -495.64 | -505.12 |
| | *rpn* | -156.94 | -201.20 | -129.59 | -153.40 | -128.75 |
| | *al* | -473.84 | -478.11 | -1429.5 | -471.95 | -492.07 |
| | *rpl* | -336.92 | -343.50 | -280.97 | -327.99 | -279.11 |

Table 1.7l: Log-Likelihoods parameter set 7, large data set

| Specification | | Estimated | | | | |
|---|---|---|---|---|---|---|
| | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | -453.83 | -455.97 | -597.50 | -455.74 | -487.70 |
| | *b* | -495.04 | -492.59 | -821.08 | -496.22 | -505.57 |
| | *rpn* | -257.79 | -265.00 | -210.40 | -250.06 | -210.81 |
| | *al* | -583.71 | -597.72 | -1496.1 | -582.29 | -525.17 |
| | *rpl* | -429.37 | -438.06 | -368.35 | -420.57 | -364.91 |

Table 1.8l: Log-Likelihoods parameter set 8, large data set

| Specification | | Estimated | | | | |
|---|---|---|---|---|---|---|
| | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | -342.74 | -344.44 | -470.41 | -344.33 | -428.16 |
| | *b* | -494.25 | -492.57 | -1765.0 | -496.30 | -504.76 |
| | *rpn* | -157.67 | -201.08 | -128.95 | -153.44 | -128.98 |
| | *al* | -473.79 | -479.22 | -1441.2 | -471.93 | -494.67 |
| | *rpl* | -336.94 | -343.43 | -280.40 | -328.83 | -279.47 |

Key *an*: additive normal; *b*:beta; *rpn*: random preferences normal; *al*: additive normal; *rpn*: random preferences lognormal

Table 2.1s: *r* estimates, parameter set 1, small data set

| | | True value | Estimated value | | | | |
|---|---|---|---|---|---|---|---|
| | | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | 0.500 | 0.503 | 0.497 | 0.524 | 0.503 | 0.523 |
| | *b* | 0.500 | 0.502 | 0.500 | 0.539 | 0.496 | 0.535 |
| | *rpn* | 0.500 | 0.494 | 0.495 | 0.500 | 0.499 | 0.499 |
| | *al* | 0.500 | 0.519 | 0.483 | 0.569 | 0.515 | 0.566 |
| | *rpl* | 0.507 | 0.499 | 0.501 | 0.506 | 0.499 | 0.505 |

Table 2.2s: *r* estimates, parameter set 2, small data set

| | | True value | Estimated value | | | | |
|---|---|---|---|---|---|---|---|
| | | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | 0.500 | 0.500 | 0.500 | 0.507 | 0.500 | 0.507 |
| | *b* | 0.500 | 0.501 | 0.500 | 0.539 | 0.497 | 0.535 |
| | *rpn* | 0.500 | 0.498 | 0.500 | 0.500 | 0.499 | 0.500 |
| | *al* | 0.500 | 0.504 | 0.496 | 0.526 | 0.503 | 0.526 |
| | *rpl* | 0.500 | 0.497 | 0.499 | 0.500 | 0.497 | 0.499 |

Table 2.3s: *r* estimates, parameter set 3, small data set

| | | True value | Estimated value | | | | |
|---|---|---|---|---|---|---|---|
| | | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | 0.500 | 0.505 | 0.496 | 0.524 | 0.502 | 0.522 |
| | *b* | 0.500 | 0.502 | 0.500 | 0.539 | 0.497 | 0.536 |
| | *rpn* | 0.500 | 0.494 | 0.494 | 0.500 | 0.499 | 0.499 |
| | *al* | 0.500 | 0.522 | 0.481 | 0.568 | 0.513 | 0.567 |
| | *rpl* | 0.507 | 0.499 | 0.501 | 0.505 | 0.498 | 0.506 |

Table 2.4s: *r* estimates, parameter set 4, small data set

| | | True value | Estimated value | | | | |
|---|---|---|---|---|---|---|---|
| | | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | 0.500 | 0.501 | 0.499 | 0.507 | 0.500 | 0.508 |
| | *b* | 0.500 | 0.500 | 0.500 | 0.540 | 0.496 | 0.533 |
| | *rpn* | 0.500 | 0.498 | 0.500 | 0.500 | 0.500 | 0.500 |
| | *al* | 0.500 | 0.504 | 0.495 | 0.527 | 0.503 | 0.527 |
| | *rpl* | 0.500 | 0.497 | 0.499 | 0.500 | 0.497 | 0.500 |

Table 2.5s: *r* estimates, parameter set 5, small data set

| | | True value | Estimated value | | | | |
|---|---|---|---|---|---|---|---|
| | | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | 1.500 | 1.502 | 1.503 | 1.587 | 1.501 | 1.580 |
| | *b* | 1.500 | 1.501 | 1.501 | 1.615 | 1.495 | 1.614 |
| | *rpn* | 1.500 | 1.496 | 1.496 | 1.500 | 1.498 | 1.500 |
| | *al* | 1.500 | 1.511 | 1.483 | 1.704 | 1.508 | 1.726 |
| | *rpl* | 1.522 | 1.483 | 1.478 | 1.524 | 1.489 | 1.522 |

Table 2.6s: *r* estimates, parameter set 6, small data set

| | | True value | Estimated value | | | | |
|---|---|---|---|---|---|---|---|
| | | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | 1.500 | 1.502 | 1.500 | 1.525 | 1.500 | 1.523 |
| | *b* | 1.500 | 1.503 | 1.502 | 1.587 | 1.500 | 1.616 |
| | *rpn* | 1.500 | 1.498 | 1.499 | 1.500 | 1.499 | 1.500 |
| | *al* | 1.500 | 1.501 | 1.495 | 1.585 | 1.497 | 1.597 |
| | *rpl* | 1.501 | 1.487 | 1.487 | 1.501 | 1.490 | 1.501 |

Table 2.7s: *r* estimates, parameter set 7, small data set

| | | True value | Estimated value | | | | |
|---|---|---|---|---|---|---|---|
| | | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | 1.500 | 1.503 | 1.499 | 1.593 | 1.503 | 1.584 |
| | *b* | 1.500 | 1.503 | 1.506 | 1.617 | 1.497 | 1.622 |
| | *rpn* | 1.500 | 1.495 | 1.494 | 1.499 | 1.498 | 1.500 |
| | *al* | 1.500 | 1.511 | 1.486 | 1.685 | 1.506 | 1.716 |
| | *rpl* | 1.522 | 1.481 | 1.477 | 1.521 | 1.486 | 1.521 |

Table 2.8s: *r* estimates, parameter set 8, small data set

| | | True value | Estimated value | | | | |
|---|---|---|---|---|---|---|---|
| | | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | 1.500 | 1.499 | 1.500 | 1.526 | 1.500 | 1.526 |
| | *b* | 1.500 | 1.503 | 1.504 | 1.601 | 1.495 | 1.618 |
| | *rpn* | 1.500 | 1.498 | 1.499 | 1.500 | 1.499 | 1.500 |
| | *al* | 1.500 | 1.504 | 1.495 | 1.570 | 1.504 | 1.596 |
| | *rpl* | 1.501 | 1.489 | 1.488 | 1.501 | 1.488 | 1.501 |

<u>Key</u> *an*: additive normal; *b*:beta; *rpn*: random preferences normal; *al*: additive normal; *rpn*: random preferences lognormal

Table 2.1m: *r* estimates, parameter set 1, medium data set

| | | True value | Estimated value | | | | |
|---|---|---|---|---|---|---|---|
| | | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | 0.500 | 0.502 | 0.496 | 0.517 | 0.502 | 0.519 |
| | *b* | 0.500 | 0.503 | 0.499 | 0.542 | 0.496 | 0.541 |
| | *rpn* | 0.500 | 0.495 | 0.495 | 0.500 | 0.498 | 0.500 |
| | *al* | 0.500 | 0.518 | 0.483 | 0.571 | 0.511 | 0.578 |
| | *rpl* | 0.507 | 0.500 | 0.501 | 0.506 | 0.499 | 0.506 |

Table 2.2m: *r* estimates, parameter set 2, medium data set

| | | True value | Estimated value | | | | |
|---|---|---|---|---|---|---|---|
| | | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | 0.500 | 0.501 | 0.500 | 0.502 | 0.500 | 0.502 |
| | *b* | 0.500 | 0.502 | 0.500 | 0.544 | 0.496 | 0.543 |
| | *rpn* | 0.500 | 0.498 | 0.500 | 0.500 | 0.499 | 0.500 |
| | *al* | 0.500 | 0.504 | 0.496 | 0.520 | 0.503 | 0.523 |
| | *rpl* | 0.500 | 0.498 | 0.499 | 0.499 | 0.497 | 0.499 |

Table 2.3m: *r* estimates, parameter set 3, medium data set

| | | True value | Estimated value | | | | |
|---|---|---|---|---|---|---|---|
| | | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | 0.500 | 0.503 | 0.495 | 0.516 | 0.502 | 0.521 |
| | *b* | 0.500 | 0.502 | 0.501 | 0.542 | 0.496 | 0.541 |
| | *rpn* | 0.500 | 0.494 | 0.494 | 0.500 | 0.499 | 0.500 |
| | *al* | 0.500 | 0.519 | 0.484 | 0.571 | 0.511 | 0.580 |
| | *rpl* | 0.507 | 0.499 | 0.502 | 0.506 | 0.499 | 0.507 |

Table 2.4m: *r* estimates, parameter set 4, medium data set

| | | True value | Estimated value | | | | |
|---|---|---|---|---|---|---|---|
| | | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | 0.500 | 0.501 | 0.500 | 0.502 | 0.500 | 0.503 |
| | *b* | 0.500 | 0.502 | 0.499 | 0.541 | 0.496 | 0.543 |
| | *rpn* | 0.500 | 0.498 | 0.500 | 0.500 | 0.499 | 0.500 |
| | *al* | 0.500 | 0.504 | 0.495 | 0.520 | 0.503 | 0.525 |
| | *rpl* | 0.500 | 0.497 | 0.499 | 0.499 | 0.497 | 0.500 |

Table 2.5m: *r* estimates, parameter set 5, medium data set

| | | True value | Estimated value | | | | |
|---|---|---|---|---|---|---|---|
| | | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | 1.500 | 1.503 | 1.498 | 1.618 | 1.501 | 1.623 |
| | *b* | 1.500 | 1.500 | 1.502 | 1.639 | 1.495 | 1.667 |
| | *rpn* | 1.500 | 1.496 | 1.495 | 1.500 | 1.498 | 1.499 |
| | *al* | 1.500 | 1.514 | 1.477 | 1.737 | 1.501 | 1.799 |
| | *rpl* | 1.522 | 1.479 | 1.473 | 1.521 | 1.488 | 1.520 |

Table 2.6m: *r* estimates, parameter set 6, medium data set

| | | True value | Estimated value | | | | |
|---|---|---|---|---|---|---|---|
| | | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | 1.500 | 1.501 | 1.500 | 1.539 | 1.501 | 1.542 |
| | *b* | 1.500 | 1.500 | 1.502 | 1.659 | 1.497 | 1.665 |
| | *rpn* | 1.500 | 1.498 | 1.499 | 1.500 | 1.499 | 1.500 |
| | *al* | 1.500 | 1.501 | 1.496 | 1.634 | 1.499 | 1.635 |
| | *rpl* | 1.501 | 1.487 | 1.486 | 1.501 | 1.489 | 1.501 |

Table 2.7ms: *r* estimates, parameter set 7, medium data set

| | | True value | Estimated value | | | | |
|---|---|---|---|---|---|---|---|
| | | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | 1.500 | 1.501 | 1.499 | 1.616 | 1.500 | 1.622 |
| | *b* | 1.500 | 1.499 | 1.503 | 1.643 | 1.498 | 1.670 |
| | *rpn* | 1.500 | 1.494 | 1.494 | 1.500 | 1.498 | 1.500 |
| | *al* | 1.500 | 1.512 | 1.473 | 1.721 | 1.502 | 1.795 |
| | *rpl* | 1.522 | 1.479 | 1.473 | 1.522 | 1.487 | 1.522 |

Table 2.8m: *r* estimates, parameter set 8, medium data set

| | | True value | Estimated value | | | | |
|---|---|---|---|---|---|---|---|
| | | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | 1.500 | 1.500 | 1.501 | 1.538 | 1.500 | 1.539 |
| | *b* | 1.500 | 1.500 | 1.504 | 1.632 | 1.495 | 1.665 |
| | *rpn* | 1.500 | 1.498 | 1.499 | 1.500 | 1.498 | 1.500 |
| | *al* | 1.500 | 1.500 | 1.498 | 1.628 | 1.502 | 1.634 |
| | *rpl* | 1.501 | 1.489 | 1.486 | 1.502 | 1.489 | 1.501 |

Key *an*: additive normal; *b*:beta; *rpn*: random preferences normal; *al*: additive normal; *rpn*: random preferences lognormal

Table 2.1l: *r* estimates, parameter set 1, large data set

| | | True value | Estimated value | | | | |
|---|---|---|---|---|---|---|---|
| | | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | 0.500 | 0.502 | 0.496 | 0.514 | 0.502 | 0.521 |
| | *b* | 0.500 | 0.502 | 0.500 | 0.538 | 0.496 | 0.546 |
| | *rpn* | 0.500 | 0.494 | 0.495 | 0.500 | 0.498 | 0.500 |
| | *al* | 0.500 | 0.518 | 0.483 | 0.565 | 0.510 | 0.584 |
| | *rpl* | 0.507 | 0.500 | 0.502 | 0.506 | 0.499 | 0.507 |

Table 2.2l: *r* estimates, parameter set 2, large data set

| | | True value | Estimated value | | | | |
|---|---|---|---|---|---|---|---|
| | | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | 0.500 | 0.501 | 0.500 | 0.502 | 0.501 | 0.503 |
| | *b* | 0.500 | 0.502 | 0.500 | 0.546 | 0.497 | 0.545 |
| | *rpn* | 0.500 | 0.498 | 0.500 | 0.500 | 0.500 | 0.500 |
| | *al* | 0.500 | 0.504 | 0.495 | 0.518 | 0.503 | 0.526 |
| | *rpl* | 0.500 | 0.497 | 0.499 | 0.499 | 0.497 | 0.499 |

Table 2.3l: *r* estimates, parameter set 3, large data set

| | | True value | Estimated value | | | | |
|---|---|---|---|---|---|---|---|
| | | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | 0.500 | 0.503 | 0.496 | 0.515 | 0.502 | 0.523 |
| | *b* | 0.500 | 0.502 | 0.500 | 0.538 | 0.497 | 0.545 |
| | *rpn* | 0.500 | 0.494 | 0.495 | 0.500 | 0.499 | 0.500 |
| | *al* | 0.500 | 0.518 | 0.483 | 0.565 | 0.512 | 0.585 |
| | *rpl* | 0.507 | 0.499 | 0.502 | 0.506 | 0.499 | 0.507 |

Table 2.4l: *r* estimates, parameter set 4, large data set

| | | True value | Estimated value | | | | |
|---|---|---|---|---|---|---|---|
| | | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | 0.500 | 0.501 | 0.500 | 0.502 | 0.500 | 0.504 |
| | *b* | 0.500 | 0.501 | 0.500 | 0.545 | 0.497 | 0.547 |
| | *rpn* | 0.500 | 0.498 | 0.499 | 0.500 | 0.500 | 0.500 |
| | *al* | 0.500 | 0.504 | 0.495 | 0.519 | 0.503 | 0.527 |
| | *rpl* | 0.500 | 0.497 | 0.499 | 0.500 | 0.497 | 0.500 |

Table 2.5l: *r* estimates, parameter set 5, large data set

| | | True value | Estimated value | | | | |
|---|---|---|---|---|---|---|---|
| | | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | 1.500 | 1.501 | 1.497 | 1.616 | 1.502 | 1.635 |
| | *b* | 1.500 | 1.499 | 1.503 | 1.669 | 1.496 | 1.686 |
| | *rpn* | 1.500 | 1.496 | 1.494 | 1.500 | 1.498 | 1.500 |
| | *al* | 1.500 | 1.509 | 1.477 | 1.801 | 1.501 | 1.826 |
| | *rpl* | 1.522 | 1.480 | 1.473 | 1.522 | 1.486 | 1.521 |

Table 2.6l: *r* estimates, parameter set 6, large data set

| | | True value | Estimated value | | | | |
|---|---|---|---|---|---|---|---|
| | | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | 1.500 | 1.500 | 1.500 | 1.538 | 1.501 | 1.550 |
| | *b* | 1.500 | 1.499 | 1.502 | 1.734 | 1.497 | 1.685 |
| | *rpn* | 1.500 | 1.498 | 1.499 | 1.500 | 1.499 | 1.500 |
| | *al* | 1.500 | 1.502 | 1.496 | 1.681 | 1.500 | 1.648 |
| | *rpl* | 1.501 | 1.488 | 1.486 | 1.501 | 1.489 | 1.501 |

Table 2.7l: *r* estimates, parameter set 7, large data set

| | | True value | Estimated value | | | | |
|---|---|---|---|---|---|---|---|
| | | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | 1.500 | 1.501 | 1.498 | 1.622 | 1.500 | 1.638 |
| | *b* | 1.500 | 1.501 | 1.501 | 1.660 | 1.496 | 1.684 |
| | *rpn* | 1.500 | 1.495 | 1.494 | 1.500 | 1.498 | 1.500 |
| | *al* | 1.500 | 1.507 | 1.473 | 1.787 | 1.500 | 1.825 |
| | *rpl* | 1.522 | 1.479 | 1.472 | 1.523 | 1.487 | 1.522 |

Table 2.8l: *r* estimates, parameter set 8, large data set

| | | True value | Estimated value | | | | |
|---|---|---|---|---|---|---|---|
| | | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | 1.500 | 1.501 | 1.500 | 1.539 | 1.501 | 1.550 |
| | *b* | 1.500 | 1.502 | 1.501 | 1.744 | 1.496 | 1.679 |
| | *rpn* | 1.500 | 1.498 | 1.499 | 1.500 | 1.499 | 1.500 |
| | *al* | 1.500 | 1.499 | 1.497 | 1.665 | 1.502 | 1.651 |
| | *rpl* | 1.501 | 1.488 | 1.486 | 1.501 | 1.489 | 1.501 |

<u>Key</u> *an*: additive normal; *b*:beta; *rpn*: random preferences normal; *al*: additive normal; *rpn*: random preferences lognormal

Table 3.1s: Standard deviation of r estimates, parameter set 1, small data set

| Specification | | Estimated | | | | |
|---|---|---|---|---|---|---|
| | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | 0.019 | 0.023 | 0.035 | 0.019 | 0.055 |
| | *b* | 0.028 | 0.024 | 0.045 | 0.027 | 0.065 |
| | *rpn* | 0.023 | 0.021 | 0.017 | 0.021 | 0.033 |
| | *al* | 0.041 | 0.050 | 0.074 | 0.041 | 0.103 |
| | *rpl* | 0.023 | 0.021 | 0.017 | 0.022 | 0.031 |

Table 3.2s: Standard deviation of r estimates, parameter set 2, small data set

| Specification | | Estimated | | | | |
|---|---|---|---|---|---|---|
| | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | 0.009 | 0.013 | 0.016 | 0.009 | 0.029 |
| | *b* | 0.029 | 0.025 | 0.046 | 0.028 | 0.064 |
| | *rpn* | 0.011 | 0.010 | 0.008 | 0.010 | 0.017 |
| | *al* | 0.021 | 0.026 | 0.036 | 0.020 | 0.062 |
| | *rpl* | 0.012 | 0.011 | 0.009 | 0.012 | 0.018 |

Table 3.3s: Standard deviation of r estimates, parameter set 3, small data set

| Specification | | Estimated | | | | |
|---|---|---|---|---|---|---|
| | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | 0.018 | 0.024 | 0.033 | 0.019 | 0.055 |
| | *b* | 0.028 | 0.025 | 0.044 | 0.027 | 0.065 |
| | *rpn* | 0.022 | 0.021 | 0.016 | 0.022 | 0.034 |
| | *al* | 0.043 | 0.049 | 0.068 | 0.041 | 0.104 |
| | *rpl* | 0.022 | 0.020 | 0.017 | 0.021 | 0.034 |

Table 3.4s: : Standard deviation of r estimates, parameter set 4, small data set

| Specification | | Estimated | | | | |
|---|---|---|---|---|---|---|
| | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | 0.009 | 0.012 | 0.016 | 0.009 | 0.031 |
| | *b* | 0.029 | 0.025 | 0.044 | 0.027 | 0.063 |
| | *rpn* | 0.011 | 0.010 | 0.008 | 0.010 | 0.017 |
| | *al* | 0.021 | 0.025 | 0.036 | 0.020 | 0.060 |
| | *rpl* | 0.012 | 0.011 | 0.009 | 0.012 | 0.018 |

Table 3.5s: Standard deviation of r estimates, parameter set 5, small data set

| Specification | | Estimated | | | | |
|---|---|---|---|---|---|---|
| | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | 0.054 | 0.056 | 0.118 | 0.054 | 0.062 |
| | *b* | 0.067 | 0.064 | 0.153 | 0.071 | 0.075 |
| | *rpn* | 0.023 | 0.023 | 0.018 | 0.022 | 0.011 |
| | *al* | 0.119 | 0.133 | 0.317 | 0.117 | 0.110 |
| | *rpl* | 0.064 | 0.069 | 0.051 | 0.063 | 0.031 |

Table 3.6s: Standard deviation of r estimates, parameter set 6, small data set

| Specification | | Estimated | | | | |
|---|---|---|---|---|---|---|
| | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | 0.027 | 0.027 | 0.056 | 0.028 | 0.035 |
| | *b* | 0.068 | 0.064 | 0.221 | 0.066 | 0.078 |
| | *rpn* | 0.011 | 0.012 | 0.010 | 0.011 | 0.006 |
| | *al* | 0.062 | 0.065 | 0.207 | 0.055 | 0.066 |
| | *rpl* | 0.038 | 0.038 | 0.027 | 0.036 | 0.019 |

Table 3.7s: Standard deviation of r estimates, parameter set 7, small data set

| Specification | | Estimated | | | | |
|---|---|---|---|---|---|---|
| | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | 0.055 | 0.053 | 0.125 | 0.056 | 0.061 |
| | *b* | 0.065 | 0.064 | 0.150 | 0.069 | 0.077 |
| | *rpn* | 0.022 | 0.024 | 0.017 | 0.021 | 0.012 |
| | *al* | 0.124 | 0.132 | 0.352 | 0.121 | 0.112 |
| | *rpl* | 0.064 | 0.066 | 0.049 | 0.061 | 0.033 |

Table 3.8s: Standard deviation of r estimates, parameter set 8, small data set

| Specification | | Estimated | | | | |
|---|---|---|---|---|---|---|
| | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | 0.026 | 0.026 | 0.060 | 0.027 | 0.036 |
| | *b* | 0.068 | 0.067 | 0.264 | 0.068 | 0.075 |
| | *rpn* | 0.012 | 0.012 | 0.010 | 0.011 | 0.006 |
| | *al* | 0.061 | 0.063 | 0.184 | 0.057 | 0.067 |
| | *rpl* | 0.036 | 0.037 | 0.027 | 0.034 | 0.019 |

Key *an*: additive normal; *b*:beta; *rpn*: random preferences normal; *al*: additive normal; *rpn*: random preferences lognormal

Table 3.1m: Standard deviation of r estimates, parameter set 1, medium data set

| Specification | | Estimated | | | | |
|---|---|---|---|---|---|---|
| | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | 0.013 | 0.017 | 0.023 | 0.014 | 0.044 |
| | *b* | 0.021 | 0.017 | 0.038 | 0.019 | 0.050 |
| | *rpn* | 0.016 | 0.015 | 0.011 | 0.015 | 0.024 |
| | *al* | 0.030 | 0.034 | 0.052 | 0.027 | 0.079 |
| | *rpl* | 0.016 | 0.014 | 0.012 | 0.015 | 0.023 |

Table 3.2m: Standard deviation of r estimates, parameter set 2, medium data set

| Specification | | Estimated | | | | |
|---|---|---|---|---|---|---|
| | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | 0.007 | 0.009 | 0.011 | 0.007 | 0.026 |
| | *b* | 0.020 | 0.018 | 0.055 | 0.019 | 0.052 |
| | *rpn* | 0.008 | 0.007 | 0.006 | 0.007 | 0.012 |
| | *al* | 0.015 | 0.018 | 0.024 | 0.014 | 0.047 |
| | *rpl* | 0.009 | 0.008 | 0.006 | 0.009 | 0.013 |

Table 3.3m: Standard deviation of r estimates, parameter set 3, medium data set

| Specification | | Estimated | | | | |
|---|---|---|---|---|---|---|
| | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | 0.013 | 0.016 | 0.023 | 0.014 | 0.043 |
| | *b* | 0.019 | 0.017 | 0.037 | 0.019 | 0.048 |
| | *rpn* | 0.016 | 0.015 | 0.011 | 0.015 | 0.025 |
| | *al* | 0.030 | 0.035 | 0.054 | 0.029 | 0.083 |
| | *rpl* | 0.016 | 0.014 | 0.012 | 0.015 | 0.023 |

Table 3.4m: Standard deviation of r estimates, parameter set 4, medium data set

| Specification | | Estimated | | | | |
|---|---|---|---|---|---|---|
| | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | 0.006 | 0.009 | 0.011 | 0.007 | 0.026 |
| | *b* | 0.020 | 0.017 | 0.052 | 0.020 | 0.053 |
| | *rpn* | 0.008 | 0.007 | 0.006 | 0.008 | 0.012 |
| | *al* | 0.015 | 0.018 | 0.025 | 0.014 | 0.047 |
| | *rpl* | 0.009 | 0.007 | 0.007 | 0.008 | 0.013 |

Table 3.5m: Standard deviation of r estimates, parameter set 5, medium data set

| Specification | | Estimated | | | | |
|---|---|---|---|---|---|---|
| | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | 0.040 | 0.040 | 0.122 | 0.040 | 0.049 |
| | *b* | 0.048 | 0.047 | 0.201 | 0.049 | 0.060 |
| | *rpn* | 0.016 | 0.017 | 0.013 | 0.015 | 0.008 |
| | *al* | 0.085 | 0.102 | 0.363 | 0.081 | 0.084 |
| | *rpl* | 0.044 | 0.046 | 0.035 | 0.045 | 0.024 |

Table 3.6m: Standard deviation of r estimates, parameter set 6, medium data set

| Specification | | Estimated | | | | |
|---|---|---|---|---|---|---|
| | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | 0.020 | 0.019 | 0.058 | 0.020 | 0.032 |
| | *b* | 0.049 | 0.046 | 0.302 | 0.049 | 0.062 |
| | *rpn* | 0.008 | 0.009 | 0.007 | 0.008 | 0.005 |
| | *al* | 0.045 | 0.045 | 0.242 | 0.041 | 0.054 |
| | *rpl* | 0.026 | 0.026 | 0.020 | 0.026 | 0.014 |

Table 3.7m: Standard deviation of r estimates, parameter set 7, medium data set

| Specification | | Estimated | | | | |
|---|---|---|---|---|---|---|
| | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | 0.039 | 0.039 | 0.118 | 0.038 | 0.053 |
| | *b* | 0.049 | 0.047 | 0.176 | 0.049 | 0.061 |
| | *rpn* | 0.016 | 0.017 | 0.012 | 0.015 | 0.009 |
| | *al* | 0.085 | 0.097 | 0.312 | 0.082 | 0.086 |
| | *rpl* | 0.045 | 0.048 | 0.035 | 0.044 | 0.024 |

Table 3.8m: Standard deviation of r estimates, parameter set 8, medium data set

| Specification | | Estimated | | | | |
|---|---|---|---|---|---|---|
| | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | 0.019 | 0.019 | 0.050 | 0.019 | 0.032 |
| | *b* | 0.048 | 0.047 | 0.238 | 0.050 | 0.060 |
| | *rpn* | 0.009 | 0.008 | 0.007 | 0.008 | 0.005 |
| | *al* | 0.042 | 0.046 | 0.247 | 0.042 | 0.055 |
| | *rpl* | 0.026 | 0.026 | 0.020 | 0.025 | 0.013 |

Key *an*: additive normal; *b*:beta; *rpn*: random preferences normal; *al*: additive normal; *rpn*: random preferences lognormal

Table 3.1l: Standard deviation of r estimates, parameter set 1, large data set

| Specification | | Estimated | | | | |
| --- | --- | --- | --- | --- | --- | --- |
| | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | 0.009 | 0.012 | 0.015 | 0.010 | 0.031 |
| | *b* | 0.014 | 0.012 | 0.025 | 0.014 | 0.037 |
| | *rpn* | 0.011 | 0.011 | 0.008 | 0.010 | 0.017 |
| | *al* | 0.021 | 0.024 | 0.040 | 0.020 | 0.057 |
| | *rpl* | 0.011 | 0.010 | 0.009 | 0.010 | 0.016 |

Table 3.2l: Standard deviation of r estimates, parameter set 2, large data set

| Specification | | Estimated | | | | |
| --- | --- | --- | --- | --- | --- | --- |
| | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | 0.005 | 0.006 | 0.008 | 0.005 | 0.019 |
| | *b* | 0.014 | 0.013 | 0.076 | 0.014 | 0.036 |
| | *rpn* | 0.006 | 0.005 | 0.004 | 0.005 | 0.009 |
| | *al* | 0.011 | 0.013 | 0.017 | 0.010 | 0.033 |
| | *rpl* | 0.006 | 0.006 | 0.005 | 0.006 | 0.009 |

Table 3.3l: Standard deviation of r estimates, parameter set 3, large data set

| Specification | | Estimated | | | | |
| --- | --- | --- | --- | --- | --- | --- |
| | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | 0.009 | 0.011 | 0.016 | 0.010 | 0.031 |
| | *b* | 0.014 | 0.012 | 0.025 | 0.014 | 0.035 |
| | *rpn* | 0.012 | 0.011 | 0.008 | 0.011 | 0.017 |
| | *al* | 0.021 | 0.024 | 0.037 | 0.020 | 0.057 |
| | *rpl* | 0.011 | 0.010 | 0.008 | 0.011 | 0.016 |

Table 3.3l: Standard deviation of r estimates, parameter set 4, large data set

| Specification | | Estimated | | | | |
| --- | --- | --- | --- | --- | --- | --- |
| | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | 0.005 | 0.006 | 0.008 | 0.005 | 0.019 |
| | *b* | 0.014 | 0.012 | 0.080 | 0.013 | 0.037 |
| | *rpn* | 0.006 | 0.005 | 0.004 | 0.005 | 0.009 |
| | *al* | 0.011 | 0.013 | 0.042 | 0.010 | 0.034 |
| | *rpl* | 0.006 | 0.005 | 0.005 | 0.006 | 0.009 |

Table 3.5l: Standard deviation of r estimates, parameter set 5, large data set

| Specification | | Estimated | | | | |
| --- | --- | --- | --- | --- | --- | --- |
| | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | 0.027 | 0.029 | 0.115 | 0.028 | 0.034 |
| | *b* | 0.035 | 0.033 | 0.236 | 0.034 | 0.042 |
| | *rpn* | 0.011 | 0.011 | 0.009 | 0.010 | 0.006 |
| | *al* | 0.060 | 0.070 | 0.402 | 0.059 | 0.059 |
| | *rpl* | 0.032 | 0.033 | 0.026 | 0.032 | 0.017 |

Table 3.6l: Standard deviation of r estimates, parameter set 6, large data set

| Specification | | Estimated | | | | |
| --- | --- | --- | --- | --- | --- | --- |
| | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | 0.014 | 0.014 | 0.036 | 0.014 | 0.022 |
| | *b* | 0.034 | 0.034 | 0.368 | 0.034 | 0.042 |
| | *rpn* | 0.006 | 0.006 | 0.005 | 0.006 | 0.003 |
| | *al* | 0.031 | 0.032 | 0.301 | 0.029 | 0.039 |
| | *rpl* | 0.019 | 0.019 | 0.014 | 0.018 | 0.009 |

Table 3.7l: Standard deviation of r estimates, parameter set 7, large data set

| Specification | | Estimated | | | | |
| --- | --- | --- | --- | --- | --- | --- |
| | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | 0.027 | 0.027 | 0.145 | 0.028 | 0.037 |
| | *b* | 0.033 | 0.032 | 0.214 | 0.034 | 0.043 |
| | *rpn* | 0.011 | 0.011 | 0.009 | 0.011 | 0.006 |
| | *al* | 0.059 | 0.069 | 0.398 | 0.058 | 0.060 |
| | *rpl* | 0.031 | 0.034 | 0.025 | 0.030 | 0.016 |

Table 3.8l: Standard deviation of r estimates, parameter set 8, large data set

| Specification | | Estimated | | | | |
| --- | --- | --- | --- | --- | --- | --- |
| | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | 0.013 | 0.013 | 0.040 | 0.014 | 0.022 |
| | *b* | 0.035 | 0.032 | 0.371 | 0.034 | 0.043 |
| | *rpn* | 0.006 | 0.006 | 0.005 | 0.006 | 0.003 |
| | *al* | 0.030 | 0.032 | 0.281 | 0.031 | 0.039 |
| | *rpl* | 0.019 | 0.019 | 0.014 | 0.018 | 0.009 |

Key *an*: additive normal; *b*:beta; *rpn*: random preferences normal; *al*: additive normal; *rpn*: random preferences lognormal

Table 4.1s: *s* estimates, parameter set 1, small data set

| | | True value | Estimated value | | | | |
|---|---|---|---|---|---|---|---|
| | | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | 25.00 | 27.95 | 56.23 | 5.30 | 49.33 | 1.67 |
| | *b* | 40.00 | 18.98 | 42.68 | 4.51 | 34.92 | 1.38 |
| | *rpn* | 10.00 | 31.43 | 91.84 | 10.30 | 60.21 | 2.94 |
| | *al* | 20.00 | 13.10 | 19.62 | 2.80 | 24.05 | 0.73 |
| | *rpl* | 2.92 | 31.87 | 96.00 | 10.12 | 60.79 | 3.04 |

Table 4.2s: *s* estimates, parameter set 2, small data set

| | | True value | Estimated value | | | | |
|---|---|---|---|---|---|---|---|
| | | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | 50.00 | 54.06 | 169.37 | 10.48 | 95.47 | 3.21 |
| | *b* | 40.00 | 19.00 | 42.26 | 4.59 | 35.15 | 1.40 |
| | *rpn* | 20.00 | 62.33 | 375.08 | 20.69 | 120.33 | 6.18 |
| | *al* | 40.00 | 25.05 | 48.38 | 4.99 | 45.73 | 1.53 |
| | *rpl* | 5.41 | 57.18 | 293.34 | 18.68 | 108.81 | 5.60 |

Table 4.3s: *s* estimates, parameter set 3, small data set

| | | True value | Estimated value | | | | |
|---|---|---|---|---|---|---|---|
| | | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | 25.00 | 27.89 | 56.39 | 5.26 | 49.19 | 1.65 |
| | *b* | 40.00 | 18.96 | 42.30 | 4.48 | 35.07 | 1.39 |
| | *rpn* | 10.00 | 31.52 | 89.56 | 10.36 | 60.21 | 2.97 |
| | *al* | 20.00 | 13.17 | 19.51 | 2.80 | 24.01 | 0.73 |
| | *rpl* | 2.92 | 31.90 | 95.70 | 10.02 | 60.49 | 3.01 |

Table 4.4s: *s* estimates, parameter set 4, small data set

| | | True value | Estimated value | | | | |
|---|---|---|---|---|---|---|---|
| | | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | 50.00 | 54.05 | 165.03 | 10.53 | 95.58 | 3.20 |
| | *b* | 40.00 | 19.05 | 42.38 | 4.49 | 35.11 | 1.41 |
| | *rpn* | 20.00 | 62.51 | 384.39 | 20.83 | 119.45 | 6.16 |
| | *al* | 40.00 | 25.01 | 49.02 | 4.92 | 45.85 | 1.53 |
| | *rpl* | 5.41 | 56.91 | 291.35 | 18.61 | 107.88 | 5.60 |

Table 4.5s: *s* estimates, parameter set 5, small data set

| | | True value | Estimated value | | | | |
|---|---|---|---|---|---|---|---|
| | | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | 25.00 | 26.00 | 136.08 | 1.67 | 44.96 | 0.49 |
| | *b* | 80.00 | 20.15 | 85.83 | 1.38 | 35.13 | 0.37 |
| | *rpn* | 10.00 | 91.07 | 1394.94 | 10.38 | 168.50 | 3.47 |
| | *al* | 20.00 | 11.59 | 24.63 | 1.10 | 20.59 | 0.19 |
| | *rpl* | 1.06 | 30.56 | 179.74 | 3.44 | 58.48 | 1.10 |

Table 4.6s: *s* estimates, parameter set 6, small data set

| | | True value | Estimated value | | | | |
|---|---|---|---|---|---|---|---|
| | | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | 50.00 | 51.55 | 546.74 | 3.17 | 90.36 | 1.00 |
| | *b* | 80.00 | 20.09 | 86.60 | 2.03 | 35.24 | 0.36 |
| | *rpn* | 20.00 | 182.72 | 1600.00 | 21.35 | 330.73 | 7.10 |
| | *al* | 40.00 | 22.85 | 109.21 | 2.07 | 41.50 | 0.44 |
| | *rpl* | 1.99 | 54.40 | 567.33 | 6.24 | 104.10 | 2.07 |

Table 4.7s: *s* estimates, parameter set 7, small data set

| | | True value | Estimated value | | | | |
|---|---|---|---|---|---|---|---|
| | | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | 25.00 | 25.72 | 137.08 | 1.65 | 44.92 | 0.49 |
| | *b* | 80.00 | 20.11 | 87.02 | 1.38 | 35.05 | 0.36 |
| | *rpn* | 10.00 | 90.75 | 1390.86 | 10.47 | 168.15 | 3.45 |
| | *al* | 20.00 | 11.69 | 24.91 | 1.11 | 20.85 | 0.19 |
| | *rpl* | 1.06 | 30.70 | 177.50 | 3.43 | 58.33 | 1.10 |

Table 4.8s: *s* estimates, parameter set 8, small data set

| | | True value | Estimated value | | | | |
|---|---|---|---|---|---|---|---|
| | | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | 50.00 | 51.60 | 544.50 | 3.13 | 90.90 | 0.99 |
| | *b* | 80.00 | 20.09 | 85.29 | 2.03 | 35.37 | 0.36 |
| | *rpn* | 20.00 | 182.69 | 1600.00 | 21.29 | 329.97 | 7.12 |
| | *al* | 40.00 | 23.01 | 105.95 | 2.09 | 41.55 | 0.44 |
| | *rpl* | 1.99 | 54.68 | 557.47 | 6.28 | 104.08 | 2.07 |

<u>Key</u> *an*: additive normal; *b*:beta; *rpn*: random preferences normal; *al*: additive normal; *rpn*: random preferences lognormal

Table 4.1m: *s* estimates, parameter set 1, medium data set

| | | True value | Estimated value | | | | |
|---|---|---|---|---|---|---|---|
| | | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | 25.00 | 27.28 | 53.24 | 5.25 | 48.23 | 1.48 |
| | *b* | 40.00 | 18.52 | 41.26 | 3.95 | 34.29 | 1.15 |
| | *rpn* | 10.00 | 30.09 | 82.22 | 10.15 | 58.29 | 2.89 |
| | *al* | 20.00 | 12.78 | 18.63 | 2.48 | 23.52 | 0.62 |
| | *rpl* | 2.92 | 30.61 | 89.52 | 9.94 | 58.65 | 2.98 |

Table 4.2m: *s* estimates, parameter set 2, medium data set

| | | True value | Estimated value | | | | |
|---|---|---|---|---|---|---|---|
| | | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | 50.00 | 53.02 | 160.04 | 10.41 | 93.50 | 2.87 |
| | *b* | 40.00 | 18.55 | 41.12 | 4.00 | 34.40 | 1.12 |
| | *rpn* | 20.00 | 60.36 | 350.26 | 20.38 | 115.67 | 6.09 |
| | *al* | 40.00 | 24.42 | 46.70 | 4.79 | 44.67 | 1.37 |
| | *rpl* | 5.41 | 54.53 | 283.12 | 18.26 | 104.97 | 5.52 |

Table 4.3m: *s* estimates, parameter set 3, medium data set

| | | True value | Estimated value | | | | |
|---|---|---|---|---|---|---|---|
| | | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | 25.00 | 27.35 | 53.17 | 5.24 | 48.27 | 1.47 |
| | *b* | 40.00 | 18.44 | 41.02 | 3.88 | 34.32 | 1.16 |
| | *rpn* | 10.00 | 29.91 | 81.73 | 10.20 | 57.62 | 2.91 |
| | *al* | 20.00 | 12.83 | 18.44 | 2.51 | 23.48 | 0.62 |
| | *rpl* | 2.92 | 30.66 | 89.44 | 9.88 | 58.68 | 2.97 |

Table 4.4m: *s* estimates, parameter set 4, medium data set

| | | True value | Estimated value | | | | |
|---|---|---|---|---|---|---|---|
| | | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | 50.00 | 53.10 | 161.42 | 10.41 | 93.66 | 2.93 |
| | *b* | 40.00 | 18.41 | 41.20 | 4.09 | 34.31 | 1.14 |
| | *rpn* | 20.00 | 59.83 | 350.39 | 20.47 | 115.74 | 6.10 |
| | *al* | 40.00 | 24.43 | 46.54 | 4.81 | 44.84 | 1.37 |
| | *rpl* | 5.41 | 54.37 | 284.13 | 18.27 | 104.01 | 5.50 |

Table 4.5m: *s* estimates, parameter set 5, medium data set

| | | True value | Estimated value | | | | |
|---|---|---|---|---|---|---|---|
| | | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | 25.00 | 25.47 | 131.56 | 1.43 | 44.39 | 0.38 |
| | *b* | 80.00 | 19.72 | 82.35 | 1.23 | 34.43 | 0.28 |
| | *rpn* | 10.00 | 88.77 | 1402.67 | 10.19 | 167.83 | 3.38 |
| | *al* | 20.00 | 11.31 | 23.06 | 1.04 | 20.32 | 0.15 |
| | *rpl* | 1.06 | 29.78 | 166.95 | 3.35 | 57.33 | 1.08 |

Table 4.6m: *s* estimates, parameter set 6, medium data set

| | | True value | Estimated value | | | | |
|---|---|---|---|---|---|---|---|
| | | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | 50.00 | 50.84 | 530.53 | 2.69 | 89.09 | 0.76 |
| | *b* | 80.00 | 19.73 | 82.99 | 2.00 | 34.65 | 0.29 |
| | *rpn* | 20.00 | 178.10 | 1600.00 | 20.60 | 331.01 | 6.87 |
| | *al* | 40.00 | 22.44 | 102.50 | 2.02 | 40.67 | 0.35 |
| | *rpl* | 1.99 | 53.27 | 549.07 | 6.13 | 102.82 | 2.03 |

Table 4.7m: *s* estimates, parameter set 7, medium data set

| | | True value | Estimated value | | | | |
|---|---|---|---|---|---|---|---|
| | | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | 25.00 | 25.40 | 131.73 | 1.44 | 44.21 | 0.38 |
| | *b* | 80.00 | 19.61 | 83.56 | 1.23 | 34.35 | 0.28 |
| | *rpn* | 10.00 | 88.62 | 1401.38 | 10.19 | 168.71 | 3.38 |
| | *al* | 20.00 | 11.40 | 22.75 | 1.04 | 20.40 | 0.15 |
| | *rpl* | 1.06 | 29.82 | 168.26 | 3.36 | 57.55 | 1.08 |

Table 4.8m: *s* estimates, parameter set 8, medium data set

| | | True value | Estimated value | | | | |
|---|---|---|---|---|---|---|---|
| | | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | 50.00 | 50.87 | 530.24 | 2.72 | 88.99 | 0.77 |
| | *b* | 80.00 | 19.82 | 82.69 | 2.00 | 34.57 | 0.29 |
| | *rpn* | 20.00 | 177.34 | 1600.00 | 20.65 | 331.73 | 6.90 |
| | *al* | 40.00 | 22.58 | 101.97 | 2.02 | 40.57 | 0.35 |
| | *rpl* | 1.99 | 53.37 | 541.65 | 6.14 | 101.78 | 2.02 |

Key *an*: additive normal; *b*:beta; *rpn*: random preferences normal; *al*: additive normal; *rpn*: random preferences lognormal

Table 4.1l: *s* estimates, parameter set 1, large data set

| | | True value | Estimated value | | | | |
|---|---|---|---|---|---|---|---|
| | | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | 25.00 | 27.11 | 52.11 | 4.99 | 48.05 | 1.42 |
| | *b* | 40.00 | 18.29 | 40.82 | 3.46 | 33.99 | 1.08 |
| | *rpn* | 10.00 | 29.40 | 79.21 | 10.06 | 57.60 | 2.85 |
| | *al* | 20.00 | 12.64 | 18.11 | 2.27 | 23.37 | 0.59 |
| | *rpl* | 2.92 | 30.02 | 87.28 | 9.82 | 57.93 | 2.95 |

Table 4.2l: *s* estimates, parameter set 2, large data set

| | | True value | Estimated value | | | | |
|---|---|---|---|---|---|---|---|
| | | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | 50.00 | 52.65 | 156.67 | 10.14 | 92.60 | 2.73 |
| | *b* | 40.00 | 18.32 | 40.66 | 3.75 | 34.13 | 1.09 |
| | *rpn* | 20.00 | 59.43 | 335.47 | 20.25 | 114.44 | 6.03 |
| | *al* | 40.00 | 24.20 | 45.10 | 4.55 | 44.25 | 1.31 |
| | *rpl* | 5.41 | 53.63 | 277.30 | 18.10 | 103.74 | 5.45 |

Table 4.3l: *s* estimates, parameter set 3, large data set

| | | True value | Estimated value | | | | |
|---|---|---|---|---|---|---|---|
| | | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | 25.00 | 27.12 | 52.18 | 4.98 | 47.87 | 1.41 |
| | *b* | 40.00 | 18.31 | 40.54 | 3.48 | 33.84 | 1.09 |
| | *rpn* | 10.00 | 29.31 | 78.38 | 10.10 | 57.27 | 2.87 |
| | *al* | 20.00 | 12.62 | 18.03 | 2.30 | 23.37 | 0.60 |
| | *rpl* | 2.92 | 30.07 | 86.68 | 9.81 | 57.97 | 2.95 |

Table 4.4l: *s estimates,* parameter set 4, large data set

| | | True value | Estimated value | | | | |
|---|---|---|---|---|---|---|---|
| | | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | 50.00 | 52.62 | 156.13 | 10.12 | 92.78 | 2.75 |
| | *b* | 40.00 | 18.27 | 40.47 | 3.74 | 33.98 | 1.08 |
| | *rpn* | 20.00 | 59.08 | 338.60 | 20.24 | 114.24 | 6.04 |
| | *al* | 40.00 | 24.23 | 45.19 | 4.59 | 44.28 | 1.31 |
| | *rpl* | 5.41 | 53.78 | 276.93 | 18.09 | 103.32 | 5.45 |

Table 4.5l: *s* estimates, parameter set 5, large data set

| | | True value | Estimated value | | | | |
|---|---|---|---|---|---|---|---|
| | | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | 25.00 | 25.21 | 128.11 | 1.37 | 43.95 | 0.36 |
| | *b* | 80.00 | 19.56 | 81.49 | 1.16 | 34.22 | 0.26 |
| | *rpn* | 10.00 | 86.87 | 1406.74 | 10.12 | 166.22 | 3.35 |
| | *al* | 20.00 | 11.23 | 22.16 | 1.01 | 20.19 | 0.14 |
| | *rpl* | 1.06 | 29.42 | 162.04 | 3.32 | 56.66 | 1.07 |

Table 4.6l: *s* estimates, parameter set 6, large data set

| | | True value | Estimated value | | | | |
|---|---|---|---|---|---|---|---|
| | | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | 50.00 | 50.29 | 522.46 | 2.53 | 88.16 | 0.70 |
| | *b* | 80.00 | 19.58 | 81.22 | 2.00 | 34.37 | 0.27 |
| | *rpn* | 20.00 | 174.87 | 1600.00 | 20.16 | 327.71 | 6.79 |
| | *al* | 40.00 | 22.28 | 98.52 | 2.00 | 40.29 | 0.33 |
| | *rpl* | 1.99 | 52.39 | 529.54 | 6.06 | 101.66 | 2.01 |

Table 4.7l: *s* estimates, parameter set 7, large data set

| | | True value | Estimated value | | | | |
|---|---|---|---|---|---|---|---|
| | | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | 25.00 | 25.20 | 128.65 | 1.36 | 43.99 | 0.35 |
| | *b* | 80.00 | 19.52 | 81.44 | 1.16 | 34.24 | 0.27 |
| | *rpn* | 10.00 | 87.00 | 1407.58 | 10.10 | 167.45 | 3.34 |
| | *al* | 20.00 | 11.29 | 21.84 | 1.01 | 20.32 | 0.14 |
| | *rpl* | 1.06 | 29.38 | 161.53 | 3.33 | 56.93 | 1.07 |

Table 4.8l: *s* estimates, parameter set 8, large data set

| | | True value | Estimated value | | | | |
|---|---|---|---|---|---|---|---|
| | | | *an* | *b* | *rpn* | *al* | *rpl* |
| True | *an* | 50.00 | 50.43 | 520.29 | 2.53 | 88.30 | 0.69 |
| | *b* | 80.00 | 19.62 | 81.39 | 2.00 | 34.23 | 0.27 |
| | *rpn* | 20.00 | 173.88 | 1600.00 | 20.28 | 327.60 | 6.78 |
| | *al* | 40.00 | 22.29 | 98.27 | 2.00 | 40.31 | 0.32 |
| | *rpl* | 1.99 | 52.38 | 530.06 | 6.08 | 101.10 | 2.01 |

Key *an*: additive normal; *b*:beta; *rpn*: random preferences normal; *al*: additive normal; *rpn*: random preferences lognormal