UNIVERSITY of York

Discussion Papers in Economics

No. 16/11

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**Andrew Clare, James Seaton,
Peter N. Smith, and Stephen Thomas**

Department of Economics and Related Studies
University of York
Heslington
York, YO10 5DD

Reducing sequence risk using trend following investment strategies and the CAPE

By

Andrew Clare*,

James Seaton*,

Peter N. Smith†

and

Stephen Thomas*

*Cass Business School, City University London

†University of York.

This Version: September 2016.

Abstract

Sequence risk is a poorly understood, but crucial aspect of the risk faced by many investors. Using US equity data from 1872-2015 we apply the concept of Perfect Withdrawal Rates to show how this risk can be significantly reduced by applying simple, trend following investment strategies. We also show that knowing the CAPE ratio at the beginning of a decumulation period is useful for predicting and enhancing the sustainable withdrawal rate.

Keywords: Sequence Risk; Perfect Withdrawal Rate; Decumulation; Trend-Following; CAPE

JEL Classification: G10, G11, G22.

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1. Introduction

In their book 'The Retirement Plan Solution' Ezra et al (2009) provide a careful dissection of 'The Reinvention of Defined Contribution' in pension savings and decumulation, a topic of growing importance in many parts of the world as companies retreat from defined benefit (DB) schemes leaving investment and withdrawal decisions to individuals. The book devotes a mere dozen pages (out of 200) to the topic of investing (though there are a few more pages on returns' history). Indeed, the topic is relegated to the authors' so-called "3rd dial" after Personal Spending' (i.e. withdrawal rates) and Longevity Protection. Similarly, economists increasingly focus on ever more creative decumulation strategies; for instance, combining deferred annuities, state benefits, guaranteed annuity-type income along with flexible income sourced from differing degrees of risky investment. But researchers are generally silent on the type of investment strategy needed for a successful decumulation experience with risky assets (for example see Merton (2014)), instead preferring to create a risk-free benchmark of index-linked bonds (see Sexaeur, Peskin and Cassidy (2012)), or utilising combinations of bonds and equities often in target date or glide-path commercial solutions which offer period-specific conclusions and ignore the diversification lessons of undergraduate finance. In our view, designing a savings and decumulation strategy without careful consideration of the investment strategy is like designing all of the necessary elements of a car – chassis, gear box, braking system, etc – except the engine.

Possibly inspired by Bengen's (1994) '4% rule', most of the literature focusses on the relation between withdrawal rates/drawdown strategies and the longevity of funds versus life expectancy, given some (usually arbitrary chosen) investment return series, or rate. With such analytical frameworks it is then a simple matter of identifying the sort of withdrawal rates that are consistent with different returns' experience and life expectancy/planning horizons – the ultimate great unknown for an individual at least! The literature has also experimented with changes to key parameters, which generally suggest that sustainable withdrawal rates lie between 3.5% and 4.5% (of initial balance) for individuals in countries such as the USA (for example see Finke et al (2013)). Empirical analysis in this area may focus on investment strategies based upon either a historical, 'real life' investment period, with the appropriate ex-post sustainable withdrawal experience. Alternatively, some researchers have made use of Monte Carlo methods to draw a large number of random investment returns from say, US capital market history, which allows for the construction of sustainable withdrawal rates for each drawing for a specified planning period and, given a large number of different draws, a probability distribution for the withdrawal rate itself. These distributions make probability statements about withdrawal rates possible and associated statistics regarding failure rates. Gerrard et al (2004) and Milevsky and Young (2004) consider the optimal choice of withdrawal rate (i.e. consumption) and financial investment decisions within a stylised world of asset return behaviour. Variations on this theme involve introducing a bequest motive, or allowing withdrawal rates to 'adapt' each year as new information becomes available such as investment return experience, or life expectancy (i.e. planning period) changes. Such adaptive rules can encompass a myriad of complex rules (see Spitzer et al (2008) and Mitchell (2009)). Finally, Waring and Siegel (2015) argue that the solution to the decumulation challenge is "annuity thinking", even if it is not an annuity, expanding on Waring and Whitney's (2009) principle of "periodic annuitisation".

In this paper we want to shift the focus back to investment strategy and employ the concepts of Perfect Withdrawal Rate and Sequence Risk to allow comparison of competing strategies in a systematic way:

- The *Perfect Withdrawal Rate* (PWR) is the withdrawal rate that effectively exhausts wealth at death (or at the end of a fixed, known period) if one had *perfect foresight* of all returns over that period. Note that a similar concept has been put forward by Blanchett et al (2012).

- *Sequence Risk* (sometimes called Sequential Risk) is the risk of experiencing bad investment outcomes at the *wrong time*: typically the *wrong time* is towards the *end* of the accumulation phase and at the *beginning* of the decumulation period, i.e. it is symmetric around the time of retirement.

This concept of sequence risk is of particular interest to the decumulation industry. Okusanya, (2015) and Chiappinelli and Thirukkonda (2015) both basically point out the importance of 'path dependency' of investment returns (i.e. the order in which returns occur). This concept is at least as important to the retirement journey as the total return earned by the investment, yet portfolio construction, both academic and practical, has typically focussed on total return and volatility, constructing Sharpe or similar performance statistics as a way of comparing strategies. Using simple arithmetic examples, studies typically show that higher withdrawal rates are always possible when the worst investment years occur later in the decumulation period (for *any given set* of returns). The natural reaction to Sequence Risk has therefore been to de-risk a portfolio as one approaches 'retirement' along the lines of 'glidepath' or similar strategies.

However this preference for de-risking is being challenged from a number of directions: the academic financial planning literature of around 20 years ago had discovered that de-risking by abandoning equities, the key element of the derisking strategies, could lead to far worse outcomes. As Cooley et al (1998), say:

> *'…… investors who expect long payout periods should choose an asset allocation that is at least 50% common stock and a lower withdrawal rate. Conversely, a higher withdrawal rate appears to be sustainable for shorter payout periods, such as 15 or 20 years, provided the portfolio has a substantial percentage of stocks. Investors who plan to inflation-adjust withdrawals should choose lower withdrawal rates and invest at least 50% of the portfolio in stocks. Finally, the lower withdrawal rates of 3% and 4% recommended by some analysts appear to be excessively conservative for portfolios with*

at least 50% stock, unless the investor wishes to leave a substantial portion of the initial retirement portfolio to his/her heirs.'

Blanchett et al (2012) show similar findings for 20 countries using 113 years of data. More recently, and very much from a practical financial planning/advisory perspective, Finametrica (2015) and others point out that with retirees living 30+ years that this de-risking may prove disastrous in terms of lost wealth and consequently lower drawdown rate experiences. In short, investors could be missing out on the upswings in equity markets. Finally, Ezra et al (2009) report that for the 12-month period ending 31st December 2008, the average return for the largest 3 target date US fund families for 2010 (the nearest to retirement) was -24%. While not as bad as the performance of equity markets over the same period, which fell around 43%, this would be cold comfort for an imminent retiree.

In this paper we use the concept of Perfect Withdrawal Rates (PWR) (Suarez et al, 2015) to investigate the decumulation experience since 1872 of a US investor with a 20-year investment horizon. We show how applying a simple (absolute), trend following investment strategy leads to a far better range of withdrawal outcomes relative to a long-only equity portfolio. Another question that we address here is whether indicators of equity market valuation are useful for predicting the withdrawal rates at any point in time? In other words, does, say, a high cyclically-adjusted Shiller PE ratio (CAPE) suggest an overvalued market followed by equity price falls and a bad sequence of returns, leading to subsequent lower future PWRs? We find clear evidence to suggest that CAPE can be used to help enhance withdrawal rates.

To summarise, we use PWR as a measure for comparing investment strategies and show that strategies with low maximum drawdown have superior sequencing experiences and higher PWR. Consistent with the findings of Blanchett et al (2015), we find evidence to suggest that the application of a simple trend following to an equity investment can help generate returns with low drawdowns, which reduces sequence risk leading to enhanced PWRs. We also find that knowledge of Shiller's CAPE ratio, can help enhance PWRs further. The rest of our paper is organised as follows. In section 2 we provide a review of the PWR literature; in Section 3 we present the derivation of the main measures used in our empirical analysis; in Section 4 we describe our approach to the construction of PWR distributions; in Section 5 we trend following and show how it can help to enhance PWRs; in Section 6 we gauge the impact of deferring the impact of regular withdrawal on PWRs; in Section 7 we introduce the CAPE ratio into the analysis; and finally we summarise our paper in Section 8.

2. Calculating Withdrawal Rates/Amounts (PWR/PWA)

The concept of PWR is a relatively new one to create withdrawal strategies from retirement portfolios and is based not on heuristics and/or empirical testing but on analytics. Suarez et al (2015) and also Blanchett et al (2012) construct a probability distribution for the PWR and apply it sequentially, deriving a new measure of *sequence risk* in the process. We use these ideas to show that a particular class of investment strategies (both simple and transparent) can offer superior (Perfect) Withdrawal Rates across virtually the whole range of return' environments. This smoothing of returns leads to a better decumulation experience across virtually all investing timeframes.

Retirees (here we focus on decumulation though sequencing risk applies similarly to savers in the accumulation phase) want to use their funds to support as high a standard of living as possible, but without depleting their wealth so quickly that their later years become difficult to finance: this is called 'failure risk'. The mirror image of this is withdrawing 'too little' money and hence leaving 'excessively high' balances at the end of the planning period (or, indeed, lifespan): this is called ‘surplus risk’, and implies an unnecessarily restricted standard of living throughout the decumulation years.

Researchers have typically derived ‘rules’ which determine the withdrawal amounts in each period based on the retiree’s age, portfolio of assets and preference for near-term consumption versus a later, higher potential failure rate: of course, inflation, taxes, liquidity requirements, precautionary balances etc, also influence choices but the former trio are generally considered of primary importance in driving the appropriate withdrawal rate (see Cooley et al (1998) Blanchett et al (2012)). This literature is essentially heuristic and empirical in nature, varying withdrawal rates, investment portfolios, almost always between equities and bonds rather than multi-assets, (and hence the pattern of returns) and time-horizons for consumption (usually age-related).

The literature on optimal withdrawals in retirement can be traced back to Bengen (1994), where he presents the concept of “the 4% rule”. Bengen shows that a 4% withdrawal rate from a retirement fund, adjusted for inflation, is ‘usually’ sustainable for ‘normal’ retirement periods. Cooley, Hubbard, and Walz (1998, 1999, 2003, and 2011) then confirmed this finding, with similar findings using overlapping samples of historical stock and bond returns.

A crucial distinguishing feature of these “first generation” papers is that they rely on a constant real withdrawal amount throughout the decumulation phase, with no ‘adaptive’ behaviour as circumstances change. A number of studies have introduced ‘adaptive’ rules: Guyton & Klinger (2006) manipulate the inflationary adjustment when return rates are too low, modifying the withdrawal amount, while Frank, Mitchell, and Blanchett (2011) use adjustment rules that depend on how much the rate of return deviates from the historical averages. Zolt (2013) similarly suggests curtailing the inflationary adjustment to the withdrawal amount in order to increase the portfolio's survival rate where appropriate. Basically these withdrawal rates 'adapt' to changing circumstances.

Of course an important addition is to treat the planning horizon length as a stochastic variable (instead of fixed). The aim here (quite sensibly!) is to ensure that the funds in the retirement account "outlive" the retiree: Stout and Mitchell (2006) use mortality tables to make sure that the uncertain retirement period is considered, while Stout (2008) decreases the withdrawal amount whenever the account balance falls below a measure of the present value of the withdrawals yet to be made and increases it when the balance is above this measure. Mitchell (2011) similarly uses thresholds to initiate such adjustments.

A more theoretically coherent approach treats the selection of withdrawal amounts as a lifetime-utility maximization problem. Milevsky and Huang (2011) consider the total discounted value of the utility derived across the entire retirement period, where this length of retirement is a stochastic variable and the subjective discount rate is a given. Williams and Finke (2011) use a similar model with more realistic portfolio allocations. Blanchett, Kowara, and Chen (2012) measure the relative efficiency of different withdrawal strategies by comparing the actual cash flows provided by each strategy to the flows that would have been feasible under perfect foresight.

3. The Perfect Withdrawal Amount (PWA)

The concept of PWA is introduced in a world of no taxes or inflation: annual withdrawals are made on the first day of each year and annual investment returns accrue on the last day of the year. For any given series of annual returns there is one and only one constant withdrawal amount that will leave the desired final balance on the account after *n* years (the planning horizon). The final balance could be a bequest or indeed zero. Suarez et al (2015) suggest that this is equivalent to finding the fixed-amount payment that will fully pay off a variable-rate loan after *n* years.

The basic relationship between account balances in consecutive periods is:

$$K_{i+1} = (K_i - w)\,(1+r_i) \qquad (1)$$

where K_i is the balance at the beginning of year i, w is the yearly withdrawal amount, and r_i is the rate of return in year i in annual percent. Applying Eq. (1) chain-wise over the entire planning horizon (n years), we obtain the relation between the starting balance KS (or K1) and the end balance KE (or K_n):

$$K_E = (\{[(K_S - w)\,(1+r_1) - w]\,(1+r_2) - w\}\,(1+r_3)\,\ldots - w)\,(1+r_n) \qquad (2)$$

And we solve equation (2) for *w* to get:

$$w = [K_S \prod_{i=1}^{n}(1 + r_i) - K_E] / \sum_{i=1}^{n} \prod_{j=i}^{n}(1 + r_j) \qquad (3)$$

Equation (3) provides the constant amount that will draw the account down to the desired final balance if the investment account provides, for example, a 5% return in the first year, 3% in the second year, minus 6% in the third year, etc., or any other particular sequence of annual returns. This figure is called the Perfect Withdrawal Amount (PWA).

Quite simply, if one knew in advance the sequence of returns that would come up in the planning horizon, one would compute the PWA, withdraw that amount each year, and reach the desired final balance exactly and just in time.

Numerous studies provide examples of a sequence of, say, 30 years of returns generated possibly with reference to an historical period or via Monte Carlo simulations, and offer the unique solution of the PWA. It involves withdrawing the same amount every year, giving the desired final balance with no variation in the income stream, no failure and no surplus. As we noted above, Blanchett, Kowara and Chen (2012) present a measure similar to PWA called Sustainable Spending Rate (SSR). Suarez et al (2015) point out that the PWA is a generalization of SSR, with SSR being the PWA when the starting balance is $1 and the desired ending balance is zero.

So every sequence of returns is characterised by a particular PWA value and hence the retirement withdrawal question is really a matter of "guessing" what the PWA will turn out to be (eventually) for each retiree's portfolio and objectives. So the problem now becomes how to estimate the probability distribution of PWAs from the probability distribution of the returns on the assets held in the retirement account.

Note that the analysis so far offers a number of useful insights into sequence risk measurement. First, Equation (3) can be restated in a particularly useful way since the term $\prod_{i=1}^{n}(1+r_i)$ in the numerator is simply the cumulative return over the entire retirement period, (call it *Rn*).

The *denominator*, in turn, can be interpreted as a measure of sequencing risk:

$$\sum_{i=1}^{n}\prod_{j=i}^{n}(1+r_i) = (1+r1)(1+r2)(1+r3)\ldots(1+rn) + (1+r2)(1+r3)\ldots(1+rn) + (1+r3)(1+r4)\ldots(1+rn) + \ldots. + (1+rn\text{-}1)(1+rn) + (1+rn) \qquad (4)$$

The interpretation of this is straightforward: for any given set of returns equation (4) is *smaller* if the *larger* returns occur *early* in the retirement period and *lower* rates occur at the *end*. This is because the later rates appear more often in the expression. Suarez et al (2015) suggest the use of the reciprocal of equation 4 to capture the effect of sequencing: so let $S_n = 1/\sum_{i=1}^{n}\prod_{j=i}^{n}(1+r_i)$. This rises as the sequence becomes more favourable, and even though one set of returns appearing in 2 different orders will have the same total return (i.e. R_n with different S_n values), so the PWA rates will be different.

We emphasised earlier that whereas in most finance contexts *total* return is the key variable, in both accumulation and decumulation the *order* of returns also matters. An example will make this clearer. Suppose we have 3 sets of returns in Table 1; clearly the mean, volatility and Sharpe (and indeed Maximum Drawdown) are the same, but the returns' sequence differ as is evidenced by the different values of Sequence Risk ($1/S_n$) with lower values of this metric associated with higher PWRs.

This allows a useful, highly intuitive simplification of Equation (3) in the form of Equation (5), such that the PWA depends positively on total return, R_n, the starting amount, K_n, and the measure of sequence risk, S_n, and negatively on the final amount, K_e:

$$w = (R_n K_s - K_E)\, S_n \qquad (5)$$

This representation emphasises that it is not simply the total return that matters but the order in which the component returns occur: if 'good' returns come early in the sequence then the PWA will be larger than if they occur later.

Other studies have tried to account for sequence risk (Frank and Blanchett, 2010; Frank, Mitchell, & Blanchett, 2011; Pfau, 2014), often developing proxy variables to measure the correction required due to the sequencing issue. Suarez et al (2015) suggest that equation (5) comes directly from the simplest, most natural interpretation of the problem, that is that S_n is not a proxy but a measure of what they term '*orientation'* (return rates going up, going down, up a little then down a lot, etc.), and this is the crucial concept for assessing sequencing.

Finally we should note that *w* (the PWA) can be transformed into a withdrawal *rate* by dividing equation (5) by K_s

$$w/K_s = R_n S_n - S_n(K_E / K_S) \qquad (6)$$

Note that if we have a bequest motive then we simply now need to know the fraction of the initial sum to be bequeathed to calculate the PWR: as Suarez et al (2015) point out in contrast to simplistic financial planning solutions, to set aside a bequest sum beforehand is not necessary as these funds can also generate returns and be used for consumption. Setting aside a sum is simply a special case of the above general form, equation (6). We show later that similarly setting aside cash for, say, 3 years of consumption, along with a subsequent withdrawal strategy, can be handled easily within the PWR framework.

Finally, how do we handle longevity risk? We conduct our analysis with a fixed planning period of 20 years to avoid unnecessary complication and allow us to focus on the investment process. The 20-year deferred annuity is our preferred longevity risk hedging tool (see Merton, 2014 and Sexauer et al, 2012). Laibson (2009) points out that cognitive function decline is well set in among over 50% of US adults by the time they are in their 80's and that there should be more help for people in making the right financial decisions, perhaps via enhanced automation: deferred annuities could help here. Our analysis can be extended to handle different longevity assumptions but at this stage will divert attention away from our main focus on investment strategy.

4. Constructing a Probability Distribution for PWRs for an all equity portfolio

Much of the financial planning literature aims to make probability statements regarding the chance of running out of funds given any particular withdrawal rate and planning horizon. So we now create a probability distribution for the PWR/PWA using a long-run of monthly equity returns extracted from the Shiller website[1]. This all-equity portfolio may be considered rather unlikely as an investment choice in practice but it serves to illustrate our key points regarding choice of investment strategy and adding, say, bonds to provide ‘de-risking’/diversification’, simply reinforces our findings. A surprising result may well be that a 100% equity portfolio is not such a bad idea *providing* one overlays it with trend-following.

Assuming that we have perfect foresight, what would the real PWR look like through time assuming a 20 year decumulation period? This is shown in Figure 1 where, as throughout this paper, we assume a zero bequest intention. We focus here on the [blue] line which shows the PWR generally varying between 8 and 12% but occasionally straying as low as 4% in 1930 and as high as 15% in 1949. Indeed for several years around 1980-1990 it is well 10%. This suggests two things. First, there is a huge variation in the ability to withdraw cash from a retirement pot depending on the accident of one’s birth date. Second, *all* of the rates are above 4%, giving very long term succour to Bengen’s (1994) 4% rule (at least over 20 year periods).

Now we know what the history of PWRs would look like with perfect foresight for the 100% S&P500 portfolio, we can construct a probability distribution for this particular investment strategy: we begin with 100% invested in this equity portfolio (while realising that taking more diversified portfolios will lead to a less dispersed distribution). We take the real returns on the S&P500 for the period 1872 through 2014 and use Monte Carlo techniques to draw 20 years of 12 monthly values at random from this set (240 as a sequence), one at a time with replacement. These were then interpreted as the monthly returns over a 20-year investment horizon, in the order in which they were drawn.

[1] http://www.econ.yale.edu/~shiller/data.htm

This process is repeated 20,000 times and we computed the cumulative return (R_n) and sequencing factor (S_n) for each series of returns obtained. This provided us with 20,000 (R_n, S_n) pairs. The [blue] line in Figure 2 presents the frequency distribution of the PWA formula (equation (5)) evaluated at each of these 20,000 (R_n, S_n) pairs, using $100,000 as a starting balance and with $0 as a desired termination balance. The first column in panel A of Table 2 contains the same information, but with cut-off points for various probability levels. Figure 2 and panel A of Table 2 are broadly comparable with Figures 3 and 4 in Suarez et al (2015), albeit with real PWRs and a 20 year horizon.

We can interpret the PWRs as follows: there is a 1% chance of a real PWR of 2.95% or less; a 10% chance of a real PWR of 5.01% or less; and a 50% chance of a real PWR of 8.64% or more. Given that the final amount is $0, any overshoot in withdrawing results in ruin: hence we could say that 50% of the Monte Carlo withdrawal runs produced real PWRs less than 8.64% so that failure risk for withdrawing over 8.64% is indeed 50%. Similarly failure risk for withdrawing over 5% p.a. is about 10% (i.e. 10% of runs produced real PWRs of over 5.01%).

Of course we could introduce a bequest motive which simply means a positive final balance target; Suarez et al (2015) show that this moves the PWR distribution to the right (implying a higher risk of failing to meet the bequest). The inverse of failure risk is surplus risk and this can be estimated by inverting the roles of PWR and the end balance. For a given end balance and PWR we can say that a surplus accrues a certain percentage of time reflecting the occurrence of PWRs greater than that chosen. In fact, in the Suarez et al example, with a nominal perfect withdrawal amount of $43,000 p.a. (i.e. a 4.3% withdrawal rule in their case), 74% of the Monte Carlo runs end up with more money than they began with; in 58% of the runs the final balance was double the starting balance; and it would have a 12% probability of ending up with 10 times the initial sum.

Of course in practice individuals do not maintain the constant withdrawal amount and update their behaviour in the face on new information: so it is assumed that we make a withdrawal at the beginning of retirement year 1 and live off this money for the next twelve months until, at the very end, the sum accrued during the previous year is actually credited to our account. Then the PWA/PWR distribution needs to be recalculated using equation (5), with the balance actually showing as the new starting balance and shortening the time horizon by one period. The shortening of the horizon is attained by substituting in a new set of 20,000 (Rn, Sn) pairs as before but this time drawing return rate sequences that are shorter by one year. From this new distribution we choose the withdrawal amount for year 2; we could change the risk-tolerance profile as necessary year by year. And this process is simply repeated every year.

So the process by which the withdrawal amount is selected is always the same, but not the withdrawal amount itself. PWA incorporates all new information into the set of data available for the next analysis, and this updating will create adjustment pressures that, in all likelihood, will end up with some modification to the withdrawal amount/rate. Of course, one can change the planning horizon to take on new information, say, regarding life expectancy. Hence, adjusting equation (6) for no bequests gives equation (7), which can be evaluated for various planning horizons, n, as desired:

$$w/K_s = R_n S_n \tag{7}$$

This calculation repeated for different planning horizons effectively creates a new distribution of PWRs, allowing the advisor to discuss appropriately updated PWAs year by year as new investment experiences take place and planning horizons (possibly) shift. A bad investment year will lead to an increase in the failure rate at the pre-existing PWR and hence possible recalibration of a desired PWR if the probability of failure at the previous rate is now unacceptable (to the client). This is the statistically correct approach since it recalculates the probability distribution of the PWA each year using all new information.

This is a simple approach, much more so than some of the adaptive rules found in the previous literature (Bernard, 2011; Blanchett & Frank, 2009; Guyton, 2004; Mitchell, 2011; Pye, 2000; Robinson, 2007; Stout & Mitchell, 2006). For example, in Guyton & Klinger (2006) we find that,

> *"withdrawals are to increase from year to year to make up for inflation, except that there is no increase after a year where the portfolio's total return is negative and when that year's withdrawal rate would be greater than the initial withdrawal rate"* (p. 5), or *"when a current year's withdrawal rate has risen more than 20 percent above the initial withdrawal rate, the current year's withdrawal is reduced by 10 percent; this rule expires 15 years before the maximum age to which the retiree wishes to plan"* (p. 7).

Of course if a bequest is required we return to equation (6):

$$w/K_S = R_n S_n - S_n(K_E/K_S) \qquad (6)$$

This now has a simple interpretation whereby the investment performance to date is reflected in the updated K_s and the withdrawal rate with no bequest (i.e. R_n, S_n) less the sequence measure (S_n) times the final (bequest) amount divided by the (updated) initial wealth: the larger the intended bequest, the lower the PWR.

A final introductory remark regarding PWR: when we discuss confidence ranges for PWR, since the process is updated (i.e. is adaptive), we now mean that there is a certain probability *not* that we will run out of funds but that one will *not have to reduce* the PWR in future to achieve a given bequest target, which may of course be zero.

5. Trend Following and Sequence Risk

What influences sequence risk? Clearly from equations (6) and (7) the sequence risk measure, S_n, influences the PWR directly: equation (6) shows that the more favourable sequencing, S_n, gives a higher PWR. More favourable sequencing is associated with relatively good returns early in the planning period (see Okusanya (2015)). In particular, the avoidance of heavy losses in the early phases of decumulation is crucial for high PWAs.

But if asset returns are fairly unpredictable, how can we secure a favourable S_n? One very straightforward solution is to acknowledge that while the *order* of returns cannot be predicted, it may be possible to produce investment strategies that offer substantially reduced return volatility or, more precisely, much reduced drawdown in returns since reduced volatility in itself is not enough to secure a high PWA. Indeed, while there is no precise mathematical relationship between maximum drawdown and sequence risk we suggest that a low maximum drawdown should be associated in practice with more favourable sequence outcomes.

While diversifying across asset classes should nudge portfolio returns in the desired direction with improved risk-return, and possibly lower maximum loss experiences, there is an even more powerful technique which can be applied to individual asset classes with dramatic effect: this is simple *'trend-following'*, whereby one invests in an asset when it is in an uptrend (defined as a current value above some measure of recent past average) and switch into cash when the current value is below such an average. This has a long history of application, for example see Hurst et al (2012), and is explored in a paper by Faber (2007) and more recently by Asness et al (2015) and by Clare et al (2016).

Our basic hypothesis is that applying a simple trend-following overlay to any series of asset returns dampens volatility, typically maintains or increases returns over longer periods, and substantially reduces maximum drawdown for that series: this is directly related to lower sequence risk. So far our empirical investment strategy in this paper has involved 100% US equity investment in the form of the S&P 500 (e.g. see Suarez et al (2015) for a similar equity portfolio example over a far shorter time period). We replace this with a simple-trend following adjustment to the 100% equity strategy based on comparing a month-end index value to a simple average of the previous 10 months' end-months' values. The results are not sensitive to the choice of trend definition (see Clare et al (2016)). To show the impact of this strategy Panel B of Table 2 presents the summary statistics for the performance of the S&P 500 with and without trend-following, in real terms. Clearly, the average return of 8.84% produced by the trend following strategy compares very favourably with the 6.82% from the buy-and-hold strategy. But from our point of view even more important is the one-third reduction in volatility from 14.29%pa to 9.86%pa, and the halving of maximum drawdown from 76.8% to 34.88%.

So how would the descriptive statistics above impact the distribution of PWRs if we now have a portfolio which is 100% S&P500 with a simple trend-following overlay? The [brown] line in Figure 2 shows the distribution of the PWRs with trend-following. There is a substantial shift to the right in the distribution compared with the distribution produced without trend following (represented by the blue line in Figure 2) and it is much more concentrated around its median value of around 9%. The final column in Panel A of Table 2 reinforces this conclusion with far higher PWRs around 90% of the time than those produced by investing in the buy-and-hold portfolio of equities (shown in the first column in Panel A). In fact at lower probability levels the PWRs are nearly double those for the 100% equity strategy. These results show that trend-following reduces both maximum loss and sequence risk, and this results in a noticeably higher PWR in virtually all cases except the relatively few high PWR instances (i.e. about 10% of the time).

6. What if the client wants three years of spending set aside?

One strategy that is popular with some financial advisors involves a recommendation for a client to take the first few years of withdrawals, typically three, from the accumulated pension sum at the point of retirement, which is then placed in a cash account to achieve a ‘certain’, precautionary objective. The remainder of the pension pot is then invested in risk assets and the decumulation begins when the initial cash withdrawals have been exhausted. We examine here how this strategy affects future withdrawals, and as such the standard of living, based on the two investment approaches described earlier. For the sake of simplicity, we assume that the initial cash withdrawals are held such that they maintain their real value while outside of the remaining investment (for instance, they could be invested in short-term index-linked government bonds).

Panel A of Table 3 shows the real remaining value of the investment pot after 3 years assuming that cash was withdrawn at the beginning of the decumulation period where the investment portfolio was either 100% equities or 100% equities with a trend following overlay[2]. We denote the *Deferred PWR* as being the perfect withdrawal rate after upfront cash withdrawals have been exhausted and more conventional decumulation has commenced. Table 3 shows different rates of upfront cash withdrawals as a proportion of the initial investment fund (0%, 15%, 30%, 45%, or 60%). Unsurprisingly, the more money that is withdrawn early the smaller the pot becomes at the end of this 3 year period. If very little cash is taken early (0% to 5%) then the investment pot is frequently larger after 3 years than when drawdown started. Large withdrawals such as 45% to 60% significantly eat into the pot with many years of decumulation remaining. Panel B of Table 3 shows the related deferred PWRs where the first column of the table shows the initial PWR which is the constant proportion of the initial pot money that can

[2] We have also produced deferred PWR values for 1, 2, 3, 4 and 5 year deferred periods that are available on request.

be withdrawn if no additional upfront cash withdrawals are made, and withdrawals are not deferred for three years.

The table shows that the Initial Pot and therefore PWR are both higher under the trend-following method than the standard buy-and-hold. It is also clear that taking small cash withdrawals upfront then allows for a higher withdrawal rate at a later date than the initial PWR. For example, a 5% withdrawal for 3 years using trend following leads to an average deferred PWR of 12% compared to an initial PWR of 10%. However, taking a lot of cash early, unsurprisingly, leads to substantially reduced future withdrawals. For example, three years of 20% withdrawals (i.e. putting 60% in cash at the start) and investing in US stocks with trend following results in an average deferred PWR of nearly 5.7% compared with an initial PWR of just over 10%.

The ranges between maximum and minimum PWRs show considerable variation and reflect the varying returns and/or sequencing risk that has been experienced during the past century and more. Taking too much cash in early years reduces living standards in later periods. Taking small amounts early has the reverse effect which might encourage retirees to perhaps consider some part-time work in the first years of decumulation in order to preserve the investment pot. In general, taking upfront cash above the initial PWR leads to a lower standard of living in the future and vice versa. This is slightly complicated by the early returns on the market. Taking a little more cash than the initial PWR suggests is beneficial when the market has a negative real return but too much withdrawn depletes the pot. If the market has positive real returns then cash is underperforming and thus multi-year upfront cash withdrawals at the PWR lead to lower future amounts than the initial PWR taken annually.

7. Can Equity Valuation Measures help in securing higher withdrawals?

7.1 The relationship between CAPE and PWRs

If a simple trend-following investment strategy facilitates superior withdrawal rates most of the time, it is natural to ask whether other market timing or valuation indicators can help us choose withdrawal amounts to give similarly "improved" solutions? In particular, measures such the CAPE ratio (Shiller, 2001) have been shown to have some predictive power for longer-run equity returns[3]. Figure 3 shows the time-series plot of beginning period CAPE (right-hand axis) against the 20-year real PWR: if the earnings' yield is high (and hence CAPE is low) it is possibly indicative of future good equity returns and hence we would expect a higher perfect foresight PWR (for the subsequent 20-year period), and this is seen clearly in Figure 3, with low points for CAPE in 1920, 1930 and 1980 being associated with high (subsequent) PWRs. But does this casual observation carry over to precise calculations?

One way to assess the usefulness of knowing the CAPE ratio at any given moment of time is to examine the historical relation between the CAPE and associated PWR by a simple linear regression equation: this we do in Figure 4 which is a scatter diagram of all CAPE, PWR combinations. In Figure 5 we offer a slight variation on this by relating the CAPE values to the PWRs associated with the trend-adjusted S&P returns.

In passing we observe some interesting features of these Figures 4 and 5. First, there is a clear relation between higher CAPE values and lower subsequent PWRs. Second, the lower bound for trend-adjusted returns' PWRs for all CAPE values is 6% versus 4% for the unadjusted series; and finally, the scatter is much more concentrated for the trend-adjusted PWRs, suggesting a closer relation between valuation and PWRs. We estimate simple linear regressions (not shown here) to summarise the PWR/CAPE relations in our analysis of various sub-periods below.

7.2 PWRs, Trend-Following and the CAPE

[3] Blanchett et al (2012) introduce both bond yields and the CAPE ratio as indicators of market valuation.

A common feature of the financial planning literature is to take different periods of financial history and explore sustainable withdrawal rates in very different environments (e.g. Chatterjee, 2011). To this end we examine two very different historical periods, the 20 year period from 1973 and the 20 year period from 1995, to examine in more detail, first, the potential benefits of trend following and second, the possibility of integrating the "predictive" qualities of the CAPE.

We examine PWRs in three different environments with and without trend-adjusted equity returns:

i) Perfect foresight of returns

ii) Monte Carlo generated returns from our full data set up to whatever date is being considered.

iii) The PWR associated with the information from the simple regression lines summarising the relation between inverse CAPE and PWR in Figures 4 and 5, that is, to use the existing CAPE ratio at any moment in time to 'predict' the PWR from the simple regression lines in Figures 4 and 5.

With regard to using the CAPE the process was as follows: we "predict" the PWR for year t, by estimating a regression based on the past data, ten years' of CAPE and equity market data; then, knowing the CAPE at start of year t, plus the slope and constant from regression, we obtain a fitted value for PWR; this fitted PWR is then used to determine the withdrawal amount in subsequent year; this process is then repeated for year t+1, and so on.

7.3 The period 1995 to 2015

Table 4 contains the results for the 20 years beginning in 1995 for the buy-and-hold equity portfolio, while Table 5 contains the same results for the trend-adjusted returns. Beginning with the results in Table 4 we see from the second column that equity returns for the first 6 years were very high indeed, suggesting the likelihood of low sequencing risk; this is indeed the case and the perfect foresight real PWR is 10.781% giving a real PWA of $10,781 p.a. for each of the 20 years.

We now use the median of the distribution of Monte Carlo results for the PWR for each year updating the withdrawal rate sequentially with one less year each time. The table shows the *median* real PWR for planning horizons of 20 through to 1 year. Unsurprisingly the PWR converges on 100% as we start the final year (since what's left at the beginning of the final year is withdrawn). After the initial 5 years of good investment performance the investment pot reaches over $188,000 by the end of year 5 (i.e. 16 years left to go). Things then take a turn for the worse in 2008 with a 39% fall in the S&P, leading to a fall in the PWA from $10,629 to just under $6,000 for 2009 (with 6 years remaining).

The final set of results uses adaptive withdrawal but with the PWR associated with the CAPE at the beginning of each year given by the linear regression from Figure 4. The inverted CAPE values are given in column 3 ("headed EY"): the fairly low withdrawal rates in the early years, together with robust investment returns, leads to wealth reaching over $216,000 by the end of 1999. Together with the CAPE-driven PWRs, this leads to higher withdrawal amounts in the final years than those suggested by the Monte Carlo method. It would indeed appear that knowing today's CAPE ratio could lead to a superior withdrawal experience.

We now ask how the pattern of PWRs would differ if we repeat the calculations in Table 5 but with trend-adjusted equity returns as the "risk engine". First, we note a perfect foresight PWR of 12.308%, higher than the 10.781% when buy-and-hold equities provide the returns, not an unexpected result given our PWR distributions in Figure 2. Note how in column 2 the trend-adjusted strategy leads to far better returns of 1.1% compared to -39% in 2008 and -4.4%

compared to -22% in 2002. This facilitates a much higher withdrawal in the final 6 years relative to no trend-following, sometimes by several thousands of dollars per annum. An even more impressive withdrawal rate is achieved when we essentially overlay the information in the CAPE with trend following returns. For example the last three withdrawals are $12847, $13,188 and $15,868 which compare very favourably with the Monte Carlo results produced by the unadjusted raw equity returns in Table 4 of $6,941, $7,410 and $8,715 respectively. It would seem then that trend following combined with the predictive power of the CAPE together have the potential to produce a much better retirement experience in a period when raw investment returns are high in the early years.

7.4 The period 1973 to 1993

What happens if we now repeat the exercise over a period of financial history characterised by poor returns in the early years, for example the 20 years beginning 1973? The second column in Table 6 shows real US equity returns for each year from 1973 to 1992; 1973 and 1974 recorded real returns of 23% and 34% respectively, suggesting high sequencing risk. Although returns recovered later in the period the damage was done: the perfect foresight PWR, shown in the table, was only 4.591% for the 20 year period, emphasising that accidents of birth date can have a major bearing on one's income in retirement. Both the median Monte Carlo and CAPE valuation metric lead to substantial reduction in real withdrawals relative to those reported in Table 6. For example, Table 4 shows that the Monte Carlo Median approach gives a final withdrawal amount of $8,715, while the CAPE-based approach yields a final withdrawal value of $14,037; the equivalent values, shown in Table 6 for the 1973 to 1992 period, are $5,664 and $4,812 respectively.

But what if we now use trend-adjusted equity returns and repeat this exercise over this historical period? Table 7 contains these results for the trend-adjusted equity returns. First of all note the absence of really severe negative returns in column 2, which allows the perfect foresight PWR to rise by a third to over 6.148% pa. Similarly the Monte Carlo and CAPE-based results suggest much higher withdrawals are possible, particularly in the early years, relative to the trend-unadjusted returns reported in Table 6. However, Table 7 shows that the CAPE-based annual withdrawals are not as high as those produced by the Monte Carlo approach.

8. Concluding Thoughts

In this paper we have drawn attention to a number of key features of the much neglected investment aspects of retirement planning and execution. We have also seen how the accidents of birth date can dramatically impact retirement income. In particular, while the reduction of sequence risk is recognised by financial planning professionals as an important aspect of the decumulation journey, there is relatively little awareness of it in the mainstream asset management and investing strategy literature, possibly because there is no widely accepted measure of it in practice. The challenge of creating investing strategies for the decumulation phase beyond the risk-free TIPS portfolios of, say, Sexaeur et al (2012) has barely begun: the choice would seem to be between controlling tail-risk with derivatives (Milevsky and Posner, 2014), versus portfolio timing adjustments into and out of cash (Strub, 2013). This study is firmly in the latter camp. Certainly the vague notion of 'de-risking' via portfolio timing adjustment into cash using target-date or glide-path methods is largely untested and not rigorous.

We find that:

i) derisking is not necessarily a good idea as one approaches the point of retirement or indeed enters the decumulation phase;

ii) simple market-timing adjustments in the guise of trend-following overlays to a 100% equity portfolio can substantially enhance the feasible withdrawal rate;

iii) there is a practical, empirical relation between sequence risk and the maximum drawdown of an investment strategy(tail risk) and this should be a major concern when creating retirement investing strategies – large drawdowns early in the investment life destroy withdrawal rates; we have also explored measures of sequence risk;

iv) there is potentially useful information in market valuation measures, such as the CAPE ratio which can help assess ‘over/undervaluation’ of the equity market as a guide to future withdrawal rates.

v) additional research to include the bonds and equity portfolios along with multi-asset portfolios leads to similar conclusions: smoothing asset returns by simple trend-following offers substantially enhanced withdrawal rates relative to unadjusted portfolio strategies. Such strategies are more straightforward than options’ strategies and may be accessible by a wider array of investors.

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Table 1
Example of Sequence Risk

In this table we show the impact on Sequence risk ($1/S_n$) and the Perfect Withdrawal Rate (PWR) of three series of returns which have the same arithmetic mean (Mean), standard deviation (St. Dev.) and maximum drawdown (Max Draw).

Year	Return set 1	Return set 2	Return set 3
1	20%	-20%	0%
2	10%	-10%	10%
3	0%	0%	-10%
4	-10%	10%	-20%
5	-20%	20%	20%
Mean	0.00%	0.00%	0.00%
St. Dev.	15.8%	15.8%	15.8%
Max Draw	-20.0%	-20.0%	-20.0%
$1/S_n$	3.98	5.98	4.92
PWR	23.87%	15.90%	19.30%

Table 2

Real Perfect Withdrawal Rate Percentiles as a Percentage of Initial Balance

In Panel A of this table, in the column headed "S&P", we present the percentiles of Perfect Withdrawal Rates (PWRs) for an investor with a twenty year decumulation horizon, a starting investment balance of $100,000 and a desired investment balance of $0, based upon the total returns generated on a buy-and-hold investment in the S&P 500 from 1872 to 2014. Analogous results are presented in the column headed "S&P with trend following", where the returns have been generated by applying a trend following rule to real S&P returns as described in the text from 1872 to 2014. The distribution on which the percentiles were derived, were generated by Monte Carlo techniques which involved drawing 20 years of 12 monthly return values at random with replacement 20,000 times. Panel B of this Table presents the descriptive statistics of a buy-and-hold investment in the S&P (column headed "S&P") and for an investment in the S&P where returns have been generated by applying a trend following rule (column headed "S&P with trend following").

	S&P	**S&P with trend following**
	Panel A	
Percentile	%	%
1	2.95	5.57
5	4.20	6.61
10	5.01	7.21
20	6.11	8.03
30	7.00	8.67
40	7.85	9.23
50	8.64	9.80
60	9.43	10.38
70	10.38	11.01
80	11.47	11.81
90	13.05	13.00
95	14.37	14.04
99	16.87	16.22
	Panel B	
Annualized Real Return (%)	6.82	8.84
Annualized Real Volatility (%)	14.29	9.86
Maximum Real Drawdown (%)	76.8	34.88

Table 3
Deferring regular withdrawals for three years

Panel A of this table presents some descriptive statistics for the final value of an investment belonging to an investor who had a twenty year decumulation horizon, and who started with an investment balance of $100,000, and where this investor withdrew a lump sum amount from the value of their initial investment balance at the start of the twenty year decumulation horizon. For simplicity, we assume that the initial cash withdrawals are held such that they maintain their real value while outside of the remaining investment. The descriptive statistics in the Table were generated by Monte Carlo techniques which involved drawing 20 years of 12 monthly return values at random with replacement 20,000 times for different initial withdrawals. We present results based on buy-and-hold investment in the S&P ("S&P") and for an investment in the S&P where returns have been generated by applying a trend following rule ("S&P with trend following"). Panel B presents the descriptive statistics for the Deferred PWRs achievable following the initial withdrawal of capital. In addition, for ease of reference, the column headed "Initial PWR" in Panel B shows the descriptive statistics for PWR as a proportion of the initial pot of money that can be withdrawn if no additional upfront cash withdrawals are made, and if withdrawals are not deferred for three years. Finally, in each panel results have been generated using buy-and-hold investment in the S&P (headed "S&P") and for an investment in the S&P where returns have been generated by applying a trend following rule (headed "S&P with trend following").

		Withdrawn amount as proportion of initial balance (%)				
		0	**15**	**30**	**45**	**60**
Panel A: Investment balance after three years as proportion of initial balance (%)						
S&P						
Mean		127.44	108.33	89.21	70.09	50.98
Median		123.35	104.84	86.34	67.84	49.34
Maximum		223.80	190.23	156.66	123.09	89.52
Minimum		49.64	42.19	34.75	27.30	19.86
S&P with trend following						
Mean		132.68	112.78	92.88	72.97	53.07
Median		129.87	110.39	90.91	71.43	51.95
Maximum		217.58	184.95	152.31	119.67	87.03
Minimum		71.78	61.02	50.25	39.48	28.71
Panel B: Deferred Real Perfect Withdrawal Rates (%)						
	Initial PWR					
S&P						
Mean	8.85	12.09	10.28	8.46	6.65	4.84
Median	8.73	11.59	9.85	8.11	6.37	4.64
Maximum	15.06	24.37	20.71	17.06	13.40	9.75
Minimum	4.28	5.02	4.27	3.52	2.76	2.01
S&P with trend following						
Mean	10.01	14.14	12.02	9.90	7.78	5.66
Median	9.81	13.38	11.37	9.37	7.36	5.35
Maximum	16.86	28.48	24.21	19.93	15.66	11.39
Minimum	5.81	7.05	5.99	4.94	3.88	2.82

Table 4
20-Year Decumulation Starting in 1995, based on buy-and-hold S&P 500 real returns

In this Table we present statistics for an investor beginning a 20-year decumulation period beginning in 1995, where investment returns are all driven from a buy-and-hold investment in the S&P500. The second column in the table presents the annual, real return achieved from investing in a buy-and-hold S&P 500 equity portfolio. The third column in the table presents the 1/CAPE value (EY) at the start of each decumulation year. The columns under the heading "Perfect Foresight PWA", present the value of the investment fund at the start of each year, the annual, perfect foresight withdrawal amount, and the value of the investment fund at the end of each year respectively. The columns under the heading "Monte Carlo Median PWA" present: the value of the investment fund at the start of each year; the annual, perfect foresight withdrawal amount as a proportion of the fund; the cash withdrawal amount; and the value of the investment fund at the end of each year respectively, where the Median PWA has been determined by the Monte Carlo technique described in the text which is applied using data up to the start of the next withdrawal year. The columns under the heading "CAPE-Based PWA" present: the value of the investment fund at the start of each year; the annual, perfect foresight withdrawal amount as a proportion of the fund; the cash withdrawal amount; and the value of the investment fund at the end of each year respectively, where the PWA has been determined at the start of each year by the CAPE regression described in the text.

			Perfect Foresight PWA			Monte Carlo Median PWA				CAPE-Based PWA			
Start year	Real Ret (%)	EY Start	Value start ($)	Withdrawal ($)	Value end ($)	Start ($)	Withdrawal (%)	Withdrawal ($)	End ($)	Start ($)	Withdrawal (%)	Withdrawal ($)	End ($)
1995	35.0	5.02	100,000	10,781	120,439	100,000	8.55	8,545	123,458	100,000	6.91	6,910	125,666
1996	19.6	4.00	120,439	10,781	131,140	123,458	8.92	11,013	134,473	125,666	6.55	8,233	140,437
1997	29.6	3.61	131,140	10,781	155,943	134,473	9.18	12,340	158,242	140,437	6.59	9,256	169,966
1998	23.5	3.03	155,943	10,781	179,278	158,242	9.58	15,153	176,717	169,966	6.55	11,140	196,154
1999	18.4	2.58	179,278	10,781	199,446	176,717	9.99	17,647	188,287	196,154	6.60	12,956	216,848
2000	-8.8	2.26	199,446	10,781	171,980	188,287	10.44	19,654	153,721	216,848	6.79	14,734	184,241
2001	-14.2	2.68	171,980	10,781	138,385	153,721	10.82	16,625	117,693	184,241	7.51	13,828	146,295
2002	-22.0	3.28	138,385	10,781	99,480	117,693	11.19	13,167	81,489	146,295	8.43	12,333	104,437
2003	20.0	4.33	99,480	10,781	106,445	81,489	11.63	9,476	86,421	104,437	9.71	10,143	113,160
2004	9.3	3.76	106,445	10,781	104,522	86,421	12.46	10,772	82,653	113,160	10.24	11,582	110,983
2005	3.5	3.68	104,522	10,781	97,067	82,653	13.22	10,929	74,269	110,983	11.27	12,504	101,974
2006	11.4	3.78	97,067	10,781	96,153	74,269	14.34	10,652	70,892	101,974	12.55	12,797	99,375
2007	2.1	3.67	96,153	10,781	87,203	70,892	15.70	11,131	61,043	99,375	13.84	13,753	87,458
2008	-39.3	3.85	87,203	10,781	46,397	61,043	17.41	10,629	30,607	87,458	15.53	13,582	44,851
2009	26.6	6.51	46,397	10,781	45,084	30,607	19.57	5,989	31,162	44,851	19.11	8,572	45,924
2010	12.3	4.92	45,084	10,781	38,534	31,162	22.83	7,115	27,014	45,924	21.61	9,922	40,443
2011	-0.8	4.47	38,534	10,781	27,521	27,014	27.82	7,515	19,337	40,443	26.20	10,595	29,600
2012	14.8	4.87	27,521	10,781	19,218	19,337	35.90	6,941	14,230	29,600	34.41	10,187	22,286
2013	27.8	4.71	19,218	10,781	10,781	14,230	52.07	7,410	8,715	22,286	50.71	11,302	14,037
2014	15.0	4.02	10,781	10,781	0	8,715	100.00	8,715	0	14,037	100.00	14,037	0

Table 5

20-Year Decumulation Starting in 1995, based on real S&P 500 returns with trend following overlay

In this Table we present statistics for an investor beginning a 20-year decumulation period beginning in 1995, where investment returns are all driven by the real return on the S&P500 with a trend following overlay. The second column in the table presents the annual, real return achieved from investing in a buy-and-hold S&P 500 equity portfolio. The third column in the table presents the 1/CAPE value (EY) at the start of each decumulation year. The columns under the heading "Perfect Foresight PWA", present the value of the investment fund at the start of each year, the annual, perfect foresight withdrawal amount, and the value of the investment fund at the end of each year respectively. The columns under the heading "Monte Carlo Median PWA" present: the value of the investment fund at the start of each year; the annual, perfect foresight withdrawal amount as a proportion of the fund; the cash withdrawal amount; and the value of the investment fund at the end of each year respectively, where the Median PWA has been determined by the Monte Carlo technique described in the text which is applied using data up to the start of the next withdrawal year. The columns under the heading "CAPE-Based PWA" present: the value of the investment fund at the start of each year; the annual, perfect foresight withdrawal amount as a proportion of the fund; the cash withdrawal amount; and the value of the investment fund at the end of each year respectively, where the PWA has been determined at the start of each year by the CAPE regression described in the text.

			Perfect Foresight PWA			Monte Carlo Median PWA				CAPE-Based PWA			
Year	Real Ret (%)	EY Start	Start ($)	Withdrawal ($)	End ($)	Start ($)	Withdrawal (%)	Withdrawal ($)	End ($)	Start ($)	Withdrawal (%)	Withdrawal ($)	End ($)
1995	32.4	5.02	100,000	12,308	116,082	100,000	9.75	9,746	119,473	100,000	8.21	8,213	121,503
1996	19.6	4.00	116,082	12,308	124,104	119,473	10.03	11,988	128,541	121,503	7.85	9,534	133,903
1997	29.6	3.61	124,104	12,308	144,850	128,541	10.35	13,301	149,311	133,903	7.91	10,587	159,775
1998	10.6	3.03	144,850	12,308	146,622	149,311	10.67	15,927	147,554	159,775	7.91	12,636	162,770
1999	11.0	2.58	146,622	12,308	149,052	147,554	10.97	16,186	145,782	162,770	8.03	13,064	166,132
2000	-4.1	2.26	149,052	12,308	131,198	145,782	11.42	16,649	123,896	166,132	8.28	13,753	146,199
2001	1.8	2.68	131,198	12,308	121,034	123,896	11.70	14,492	111,376	146,199	8.99	13,136	135,462
2002	-4.4	3.28	121,034	12,308	103,967	111,376	12.17	13,555	93,539	135,462	9.84	13,326	116,789
2003	20.9	4.33	103,967	12,308	110,861	93,539	12.63	11,818	98,841	116,789	10.98	12,821	125,749
2004	1.8	3.76	110,861	12,308	100,296	98,841	13.37	13,217	87,137	125,749	11.53	14,498	113,217
2005	-0.1	3.68	100,296	12,308	87,916	87,137	14.16	12,338	74,738	113,217	12.54	14,195	98,940
2006	9.0	3.78	87,916	12,308	82,437	74,738	15.11	11,290	69,179	98,940	13.78	13,638	93,007
2007	1.1	3.67	82,437	12,308	70,931	69,179	16.36	11,317	58,524	93,007	15.09	14,032	79,878
2008	1.3	3.85	70,931	12,308	59,367	58,524	18.08	10,579	48,554	79,878	16.84	13,448	67,274
2009	18.2	6.51	59,367	12,308	55,612	48,554	20.20	9,806	45,789	67,274	20.01	13,464	63,588
2010	8.0	4.92	55,612	12,308	46,748	45,789	23.40	10,714	37,865	63,588	22.66	14,410	53,089
2011	-6.1	4.47	46,748	12,308	32,332	37,865	28.12	10,646	25,552	53,089	27.33	14,510	36,217
2012	9.6	4.87	32,332	12,308	21,938	25,552	36.02	9,203	17,912	36,217	35.47	12,847	25,605
2013	27.8	4.71	21,938	12,308	12,308	17,912	51.98	9,311	10,991	25,605	51.51	13,188	15,868
2014	15.0	4.02	12,308	12,308	0	10,991	100.00	10,991	0	15,868	100.00	15,868	0

Table 6

20-Year Decumulation Starting in 1973, based on buy-and-hold S&P 500 real returns

In this Table we present statistics for an investor beginning a 20-year decumulation period beginning in 1973, where investment returns are all driven from a buy-and-hold investment in the S&P500. The second column in the table presents the annual, real return achieved from investing in a buy-and-hold S&P 500 equity portfolio. The third column in the table presents the 1/CAPE value (EY) at the start of each decumulation year. The columns under the heading "Perfect Foresight PWA", present the value of the investment fund at the start of each year, the annual, perfect foresight withdrawal amount, and the value of the investment fund at the end of each year respectively. The columns under the heading "Monte Carlo Median PWA" present: the value of the investment fund at the start of each year; the annual, perfect foresight withdrawal amount as a proportion of the fund; the cash withdrawal amount; and the value of the investment fund at the end of each year respectively, where the Median PWA has been determined by the Monte Carlo technique described in the text which is applied using data up to the start of the next withdrawal year. The columns under the heading "CAPE-Based PWA" present: the value of the investment fund at the start of each year; the annual, perfect foresight withdrawal amount as a proportion of the fund; the cash withdrawal amount; and the value of the investment fund at the end of each year respectively, where the PWA has been determined at the start of each year by the CAPE regression described in the text.

			Perfect Foresight PWA			Monte Carlo Median PWA				CAPE-Based PWA			
Year	Real Ret (%)	EY Start	Start ($)	Withdrawal ($)	End ($)	Start ($)	Withdrawal (%)	Withdrawal ($)	End ($)	Start ($)	Withdrawal (%)	Withdrawal ($)	End ($)
1973	-23.5	5.36	100,000	4,591	72,972	100,000	8.84	8,844	69,719	100,000	7.48	7,477	70,765
1974	-34.2	7.41	72,972	4,591	44,987	69,719	8.86	6,178	41,802	70,765	8.79	6,222	42,461
1975	29.1	12.06	44,987	4,591	52,147	41,802	8.87	3,707	49,177	42,461	11.39	4,837	48,568
1976	16.8	9.76	52,147	4,591	55,563	49,177	9.20	4,523	52,172	48,568	10.56	5,130	50,752
1977	-12.2	8.62	55,563	4,591	44,759	52,172	9.62	5,019	41,405	50,752	10.33	5,245	39,960
1978	-1.1	10.33	44,759	4,591	39,728	41,405	9.87	4,089	36,907	39,960	11.57	4,623	34,949
1979	4.3	11.10	39,728	4,591	36,646	36,907	10.25	3,782	34,548	34,949	12.41	4,336	31,928
1980	15.7	11.43	36,646	4,591	37,102	34,548	10.76	3,717	35,684	31,928	13.11	4,185	32,110
1981	-10.5	10.65	37,102	4,591	29,102	35,684	11.42	4,076	28,293	32,110	13.31	4,273	24,918
1982	14.8	12.77	29,102	4,591	28,142	28,293	12.09	3,422	28,555	24,918	15.24	3,796	24,250
1983	18.7	11.81	28,142	4,591	27,948	28,555	13.02	3,719	29,474	24,250	15.56	3,774	24,299
1984	0.7	10.19	27,948	4,591	23,532	29,474	14.14	4,168	25,493	24,299	15.57	3,783	20,669
1985	26.6	10.42	23,532	4,591	23,970	25,493	15.45	3,939	27,278	20,669	16.92	3,496	21,732
1986	22.8	8.55	23,970	4,591	23,793	27,278	17.21	4,695	27,725	21,732	17.48	3,799	22,016
1987	-4.3	7.10	23,793	4,591	18,367	27,725	19.62	5,438	21,318	22,016	18.93	4,167	17,073
1988	13.8	7.47	18,367	4,591	15,674	21,318	22.82	4,864	18,718	17,073	22.33	3,813	15,085
1989	24.4	6.80	15,674	4,591	13,790	18,718	27.72	5,189	16,833	15,085	26.85	4,050	13,731
1990	-8.0	5.67	13,790	4,591	8,466	16,833	35.86	6,036	9,937	13,731	34.37	4,719	8,293
1991	18.4	6.31	8,466	4,591	4,591	9,937	51.87	5,154	5,664	8,293	51.01	4,231	4,812
1992	12.2	5.42	4,591	4,591	0	5,664	100.00	5,664	0	4,812	100.00	4,812	0

Table 7

20-Year Decumulation Starting in 1973, based on real S&P 500 returns with trend following overlay

In this Table we present statistics for an investor beginning a 20-year decumulation period beginning in 1973, where investment returns are all driven by the real return on the S&P500 with a trend following overlay. The second column in the table presents the annual, real return achieved from investing in a buy-and-hold S&P 500 equity portfolio. The third column in the table presents the 1/CAPE value (EY) at the start of each decumulation year. The columns under the heading "Perfect Foresight PWA", present the value of the investment fund at the start of each year, the annual, perfect foresight withdrawal amount, and the value of the investment fund at the end of each year respectively. The columns under the heading "Monte Carlo Median PWA" present: the value of the investment fund at the start of each year; the annual, perfect foresight withdrawal amount as a proportion of the fund; the cash withdrawal amount; and the value of the investment fund at the end of each year respectively, where the Median PWA has been determined by the Monte Carlo technique described in the text which is applied using data up to the start of the next withdrawal year. The columns under the heading "CAPE-Based PWA" present: the value of the investment fund at the start of each year; the annual, perfect foresight withdrawal amount as a proportion of the fund; the cash withdrawal amount; and the value of the investment fund at the end of each year respectively, where the PWA has been determined at the start of each year by the CAPE regression described in the text.

			Perfect Foresight PWA			Monte Carlo Median PWA				CAPE-Based PWA			
Year	Real Ret (%)	EY Start	Start ($)	Withdrawal ($)	End ($)	Start ($)	Withdrawal (%)	Withdrawal ($)	End ($)	Start ($)	Withdrawal (%)	Withdrawal ($)	End ($)
1973	-15.3	5.36	100,000	6,148	79,522	100,000	10.20	10,203	76,086	100,000	8.81	8,806	77,270
1974	-4.0	7.41	79,522	6,148	70,429	76,086	10.25	7,800	65,545	77,270	10.13	7,830	66,652
1975	8.3	12.06	70,429	6,148	69,592	65,545	10.39	6,810	63,587	66,652	12.80	8,535	62,918
1976	13.0	9.76	69,592	6,148	71,671	63,587	10.63	6,757	64,199	62,918	11.86	7,462	62,648
1977	-4.9	8.62	71,671	6,148	62,304	64,199	10.96	7,036	54,354	62,648	11.58	7,258	52,669
1978	-5.5	10.33	62,304	6,148	53,075	54,354	11.20	6,087	45,619	52,669	12.78	6,731	43,417
1979	-2.4	11.10	53,075	6,148	45,820	45,619	11.54	5,266	39,402	43,417	13.53	5,874	36,658
1980	13.4	11.43	45,820	6,148	44,994	39,402	11.96	4,713	39,341	36,658	14.13	5,180	35,699
1981	-3.9	10.65	44,994	6,148	37,328	39,341	12.49	4,913	33,083	35,699	14.27	5,096	29,408
1982	20.5	12.77	37,328	6,148	37,579	33,083	13.17	4,357	34,621	29,408	15.92	4,681	29,801
1983	18.7	11.81	37,579	6,148	37,300	34,621	14.02	4,854	35,325	29,801	16.22	4,835	29,627
1984	-1.3	10.19	37,300	6,148	30,760	35,325	15.10	5,334	29,614	29,627	16.29	4,825	24,491
1985	26.6	10.42	30,760	6,148	31,147	29,614	16.32	4,832	31,362	24,491	17.51	4,288	25,567
1986	22.8	8.55	31,147	6,148	30,692	31,362	18.07	5,667	31,546	25,567	18.29	4,677	25,646
1987	11.6	7.10	30,692	6,148	27,386	31,546	20.28	6,399	28,059	25,646	19.97	5,123	22,899
1988	2.5	7.47	27,386	6,148	21,764	28,059	23.49	6,590	22,000	22,899	23.27	5,329	18,006
1989	24.4	6.80	21,764	6,148	19,430	22,000	28.11	6,185	19,677	18,006	27.84	5,012	16,167
1990	-10.8	5.67	19,430	6,148	11,845	19,677	36.16	7,114	11,203	16,167	35.49	5,738	9,301
1991	7.9	6.31	11,845	6,148	6,148	11,203	51.99	5,824	5,805	9,301	51.75	4,813	4,842
1992	9.5	5.42	6,148	6,148	0	5,805	100.00	5,805	0	4,842	100.00	4,842	0

Figure 1

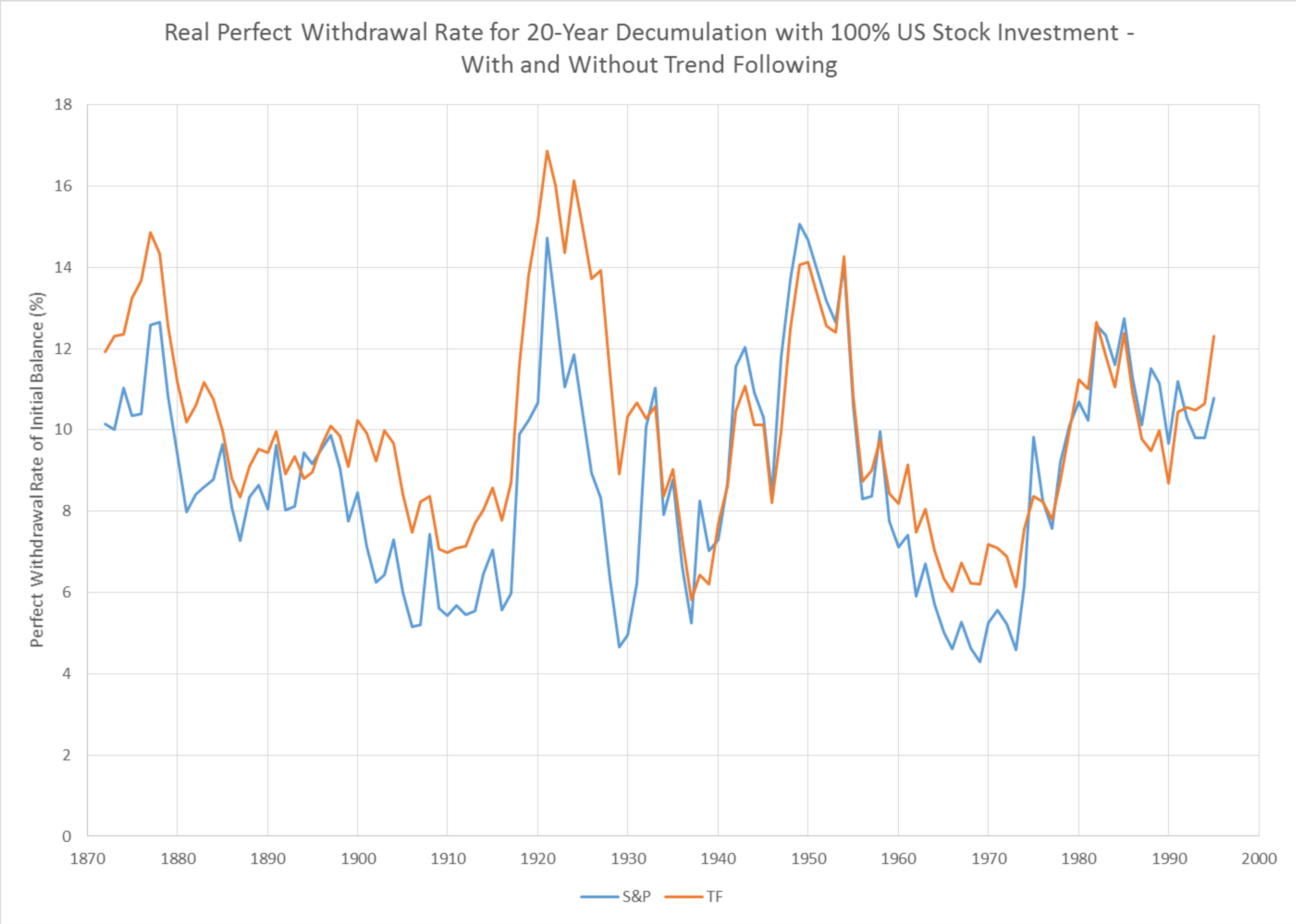

Real Perfect Withdrawal Rate for 20-Year Decumulation with 100% US Stock Investment - With and Without Trend Following
Perfect Withdrawal Rate of Initial Balance (%)
0
2
4
6
8
10
12
14
16
18
1870
1880
1890
1900
1910
1920
1930
1940
1950
1960
1970
1980
1990
2000
S&P
TF

Figure 2

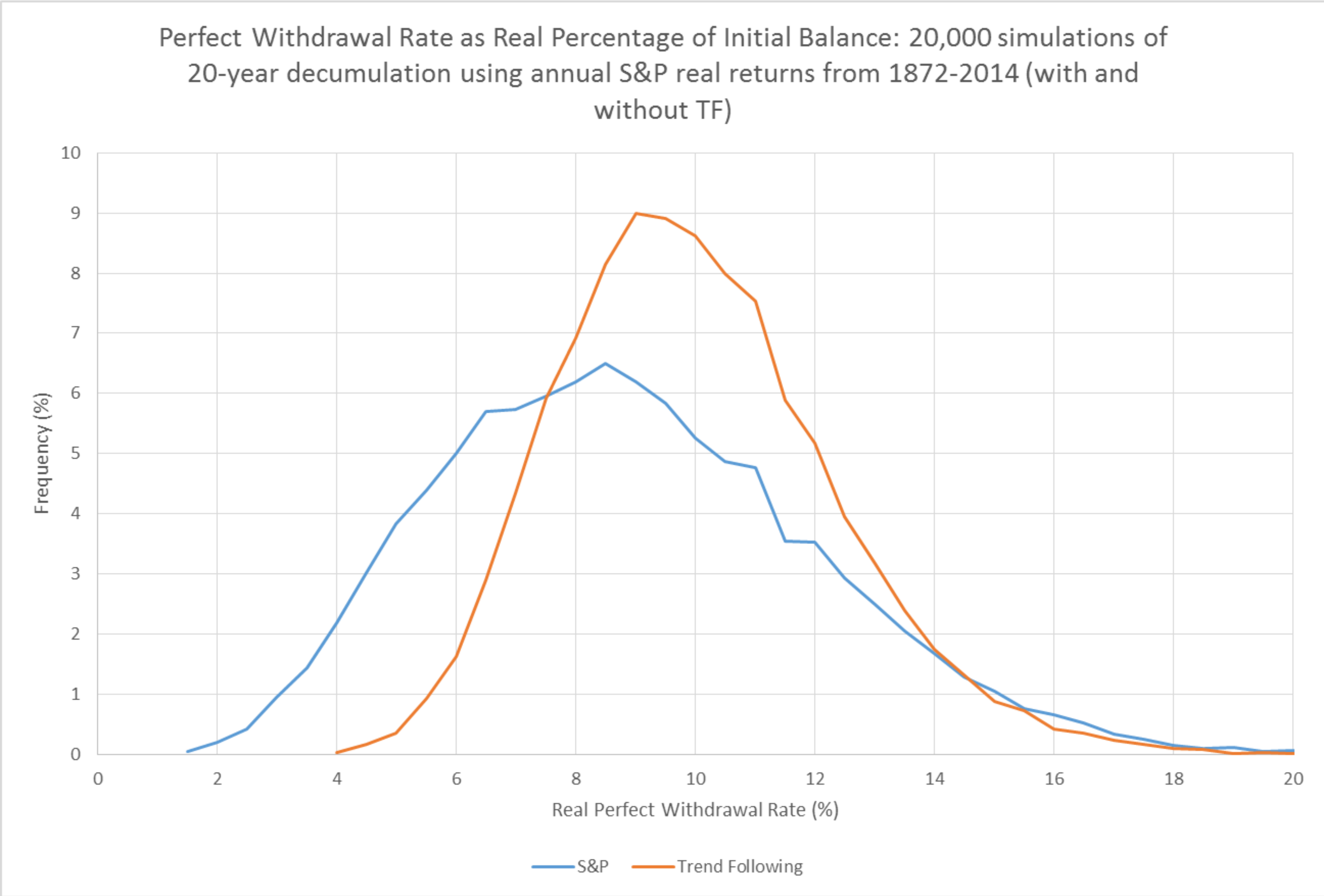
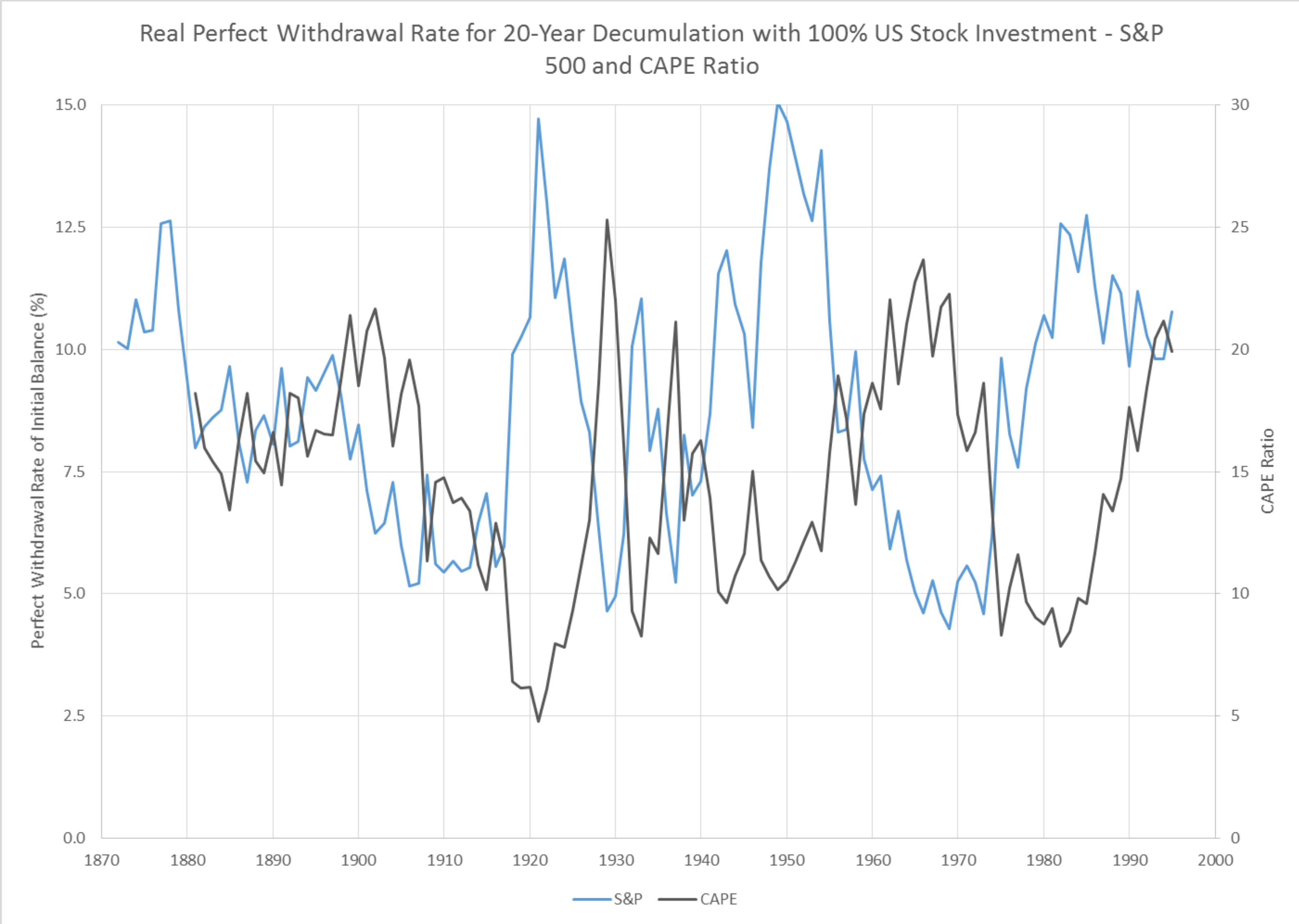
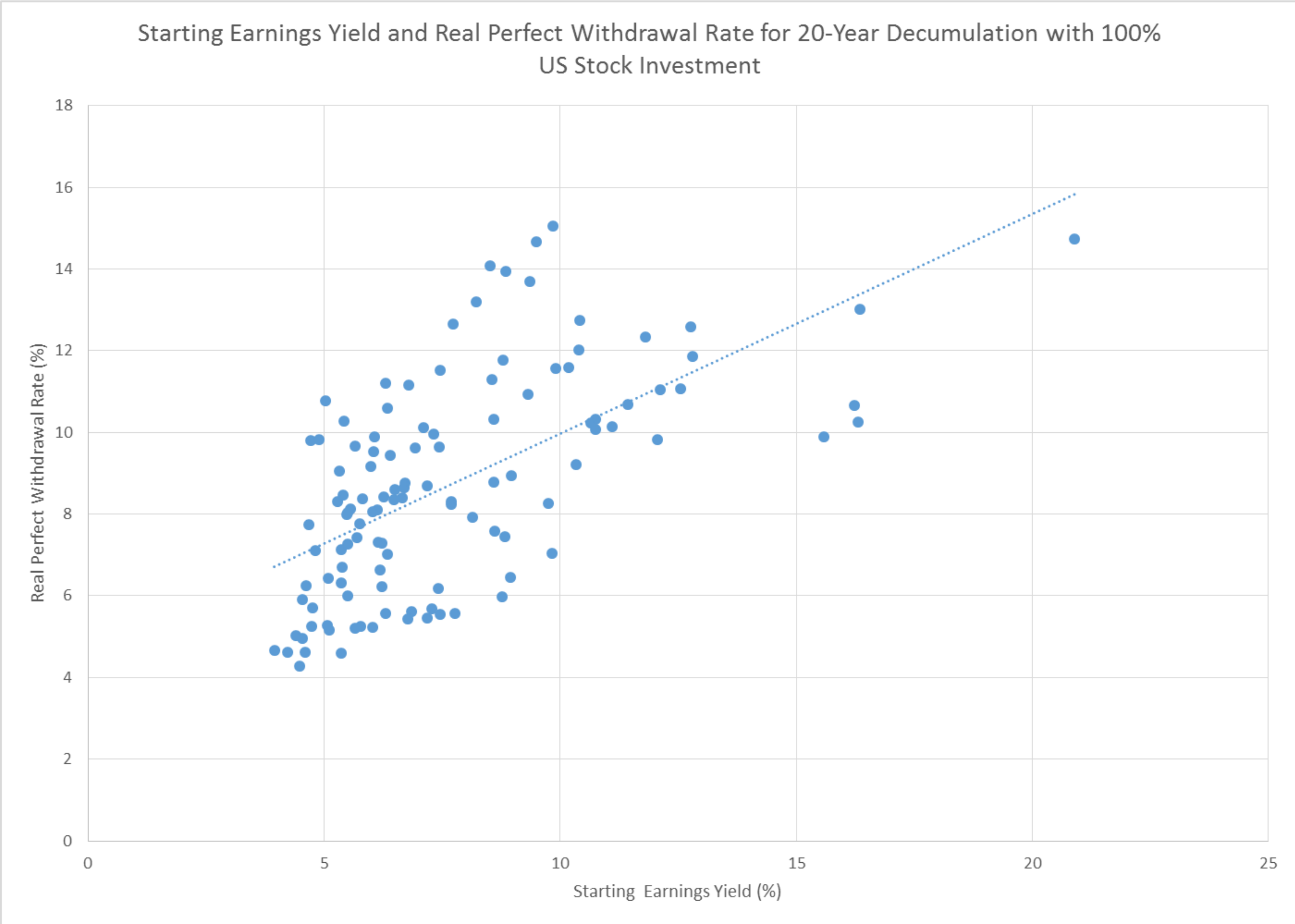

Perfect Withdrawal Rate as Real Percentage of Initial Balance: 20,000 simulations of 20-year decumulation using annual S&P real returns from 1872-2014 (with and without TF)
Frequency (%)
10
9
8
7
6
5
4
3
2
1
0
0
2
4
6
8
10
12
14
16
18
20
Real Perfect Withdrawal Rate (%)
S&P
Trend Following

Figure 3



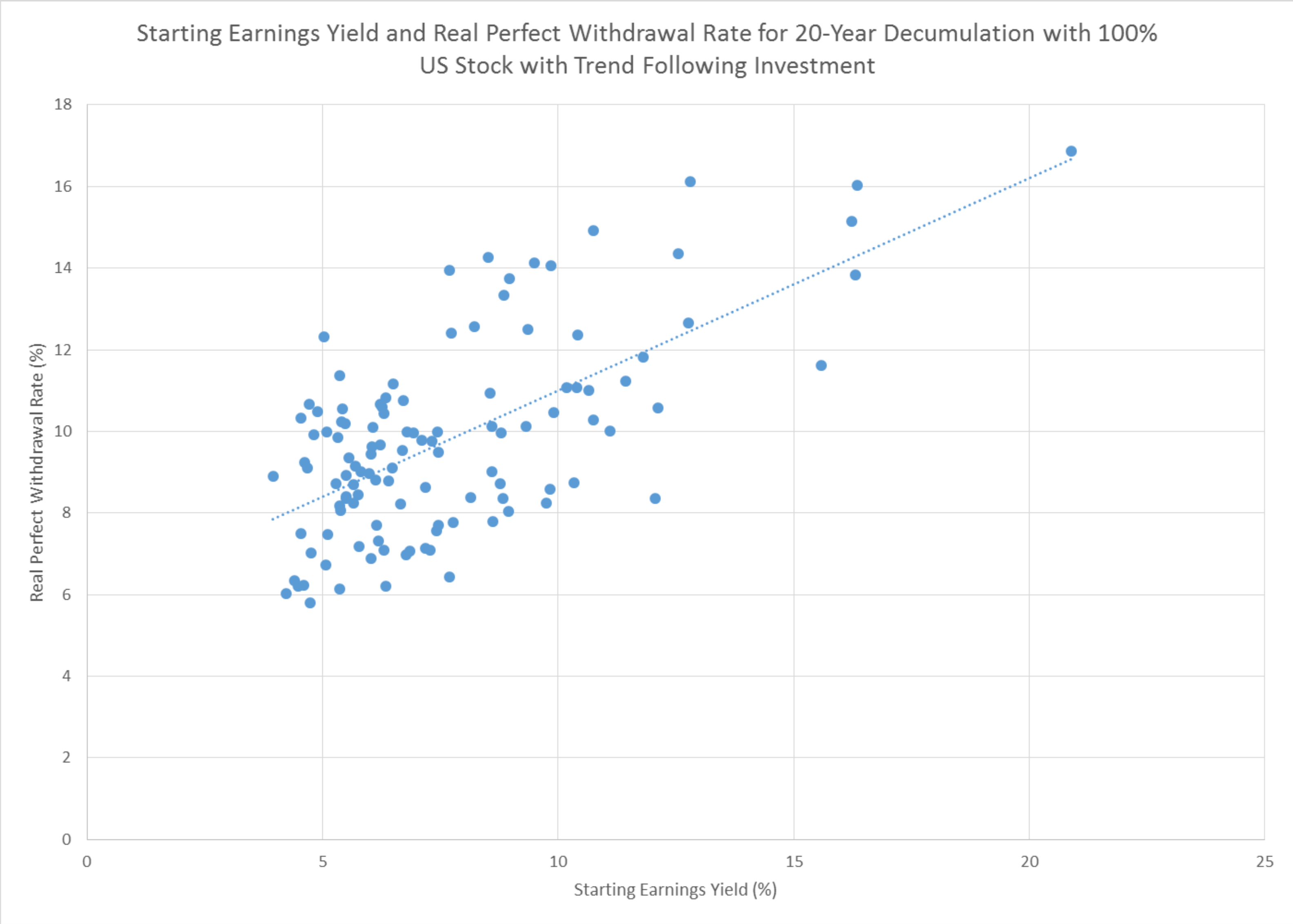
Real Perfect Withdrawal Rate for 20-Year Decumulation with 100% US Stock Investment - S&P 500 and CAPE Ratio
Perfect Withdrawal Rate of Initial Balance (%)
CAPE Ratio
15.0
12.5
10.0
7.5
5.0
2.5
0.0
30
25
20
15
10
5
0
1870
1880
1890
1900
1910
1920
1930
1940
1950
1960
1970
1980
1990
2000
S&P
CAPE

Figure 4



Starting Earnings Yield and Real Perfect Withdrawal Rate for 20-Year Decumulation with 100% US Stock Investment
Real Perfect Withdrawal Rate (%)
0
2
4
6
8
10
12
14
16
18
0
5
10
15
20
25
Starting Earnings Yield (%)

Figure 5



Starting Earnings Yield and Real Perfect Withdrawal Rate for 20-Year Decumulation with 100% US Stock with Trend Following Investment
Real Perfect Withdrawal Rate (%)
18
16
14
12
10
8
6
4
2
0
0
5
10
15
20
25
Starting Earnings Yield (%)