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TIPS Liquidity Premium and Quantitative Easing

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Abstract

In the context of a state-space model for nominal and TIPS yields, we identify the liquidity premium in the TIPS market as the common component in TIPS yields unspanned by nominal yields. Using daily US yields, we find that the TIPS liquidity premium explains up to 22% of the variation in TIPS yields and that it sharply spiked during the recent financial crisis. A counterfactual exercise shows that the QE2 program had only limited effects on the liquidity premium in the TIPS market.

JEL classification codes: C33, C53, E43, G12.

Keywords: *TIPS; Liquidity Premium; Factor models; Quantitative Easing*

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1 Introduction

Treasury Inflation-Protected Securities (TIPS) have a smaller and less liquid market than US nominal Treasury bonds. Fleckenstein, Longstaff and Lustig (2014), D’Amico, Kim and Wei (2014) and Campbell and Viceira (2009), among others, find evidence of a liquidity premium component in TIPS, representing the compensation required by investors to hold a security that is less liquid than its nominal counterpart. However, despite the large literature supporting the existence of a liquidity premium component in TIPS, there is still little consensus about how to measure it.

In this paper, we propose a new measure of the liquidity premium in the TIPS market by disentangling the common variation in TIPS from the comovements in nominal and real rates. In the framework of a state-space model for nominal and TIPS yields, we identify the relative misspricing of TIPS with respect to nominal Treasury bonds as the common component in TIPS yields unspanned by nominal yields. This identifying assumption allows us to obtain a model-free measure of the liquidity premium in the TIPS market. The literature so far has not used this approach since in existing studies liquidity premia in TIPS are either computed using replicating portfolios (see Fleckenstein et al. (2014) and Christensen and Gillan (2013)), observable liquidity proxies (see Abrahams, Adrian, Crump and Moench (2013) and Pflueger and Viceira (2011)) or latent factors in the context of affine term structure models (see D’Amico et al. (2014)). Joint factor models for nominal and real yields that do not include a liquidity premium component have been proposed by Christensen, Lopez and Rudebusch (2010), Joyce, Lildholdt and Sorensen (2010) and Haubrich, Pennacchi and Ritchken (2012).

We estimate a joint state-space model for nominal and TIPS yields that treats the liquidity premium in the TIPS market as an unobservable component that we extract simultaneously with the yield curve factors. Our empirical model is a Dynamic Factor Model for nominal and TIPS yields with zero restrictions on the factor loadings of nominal yields on the TIPS liquidity factor. These zero restrictions capture the fact that the TIPS liquidity premium factor is unspanned by nominal yields. We obtain estimates of the TIPS liquidity premium factor by Quasi-Maximum

Likelihood using the Kalman filter and the EM algorithm.

Using daily US data from January, 2 2005 to December, 31 2014 we find that the TIPS liquidity factor explains up to 22% of the variation in short term TIPS yields and that it sharply spiked during the recent financial crisis. We also find that the liquidity premium in the TIPS market is Granger caused by measures of financial stress, such as market-wide illiquidity and corporate spreads. This implies that investors require a higher compensation for investing in TIPS rather than in nominal Treasuries when market conditions get worse, as during the recent financial crisis. In turn, we find that the liquidity premium in the TIPS market Granger causes the nominal yield curve factors and thus the nominal yield curve.

In the period from November, 23 2010 to June, 17 2011 the Federal Reserve implemented the Quantitative Easing (QE2) program which involved \$600 billion purchases of Treasury securities, of which \$26 billions were TIPS purchases. To assess the effect of the QE2 program on the liquidity premium in the TIPS market, we perform a counterfactual analysis. This is easily implementable in our framework regardless of the dimensionality of the conditioning variables, thanks to the use of the state-space representation and Kalman filtering. We construct a counterfactual path for the TIPS liquidity premium factor that does not incorporate the QE2 program. Results show that the counterfactual TIPS liquidity factor is on average higher than the realised TIPS liquidity factor, but the difference is only marginally significant when taking into account the accuracy of the counterfactual. We therefore conclude that the QE2 program had only a marginal effect on the liquidity premium in the TIPS market.

The paper is organised as follows. In Section 2, we define and identify the liquidity premium in the TIPS market. Section 3 outlines the estimation procedure and Section 4 describes how to perform counterfactual exercises in our framework. Section 5 introduces the data set. In Section 6 we report estimation results. Section 7 describes the Quantitative Easing programme and its effect on the liquidity premium in the TIPS market. Finally, Section 8 concludes and Appendix A contains some additional details about estimation.

2 Model

2.1 Decomposing nominal and TIPS yields

The yield of a nominal zero-coupon Treasury bond of any maturity can be decomposed into underlying real yield, inflation expectation and inflation risk premium. Denoting by $y_{t,\tau}^N$ the nominal yield with maturity τ , we can write

$$y_{t,\tau}^N = y_{t,\tau}^R + \pi_{t,t+\tau}^e + IP_{t,t+\tau} \quad (1)$$

where $y_{t,\tau}^R$ denotes the real yield with maturity τ , $\pi_{t,t+\tau}^e$ is the expected rate of inflation for the remaining life of the bond and $IP_{t,t+\tau}$ is the inflation risk premium.

The real yield $y_{t,\tau}^R$ can, in principle, be proxied by a Treasury Inflation Protected Security (TIPS) with the same maturity. However, while the market for nominal U.S. Treasuries is the most liquid debt market, TIPS only represent 10% of the outstanding U.S. Treasury debt. This implies that TIPS investors face liquidity risk due to the possibility that they may need to make portfolio adjustments after the initial auction or before maturity, being forced to buy or sell TIPS in the secondary market. For this reason, TIPS investors may demand a liquidity premium for holding an instrument that is less liquid than nominal Treasury securities. If we denote by $y_{t,\tau}^T$ the TIPS yield with maturity τ , we have

$$y_{t,\tau}^T = y_{t,\tau}^R + LP_{t,\tau} \quad (2)$$

where $LP_{t,\tau}$ is the liquidity premium at time t for the TIPS with maturity τ .

If the TIPS secondary market is sufficiently liquid, the liquidity premium is zero and Equations (1)-(2) imply that, all else equal, any change in TIPS yields should be reflected one-to-one into a change in nominal yields. The TIPS liquidity premium creates a wedge between TIPS yields and real interest rates. In particular, Equations (1)-(2) imply that any component that affects TIPS yields but not nominal Treasury yields is related to the TIPS liquidity premium. This observation allows us to identify the liquidity premium in the TIPS market as the common component in TIPS

yields unspanned by nominal yields.

2.2 Liquidity premium in the TIPS market

The liquidity premium in the TIPS market measures the liquidity premium in the entire TIPS market. As such, it can be measured as the common factor in the liquidity premia of TIPS with different maturities. If we denote by L_t the liquidity premium in the TIPS market, then

$$LP_{t,\tau} = a_\tau^L + B_\tau^L L_t + \varepsilon_{t,\tau}^L \quad (3)$$

where a_τ^L is the maturity-specific intercept, B_τ^L contains the factor loadings of the liquidity premium of the TIPS with maturity τ on the liquidity premium factor and $\varepsilon_{t,\tau}^L$ represents the maturity specific component of the liquidity premium of the TIPS with maturity τ .

In order to extract the TIPS liquidity premium factor L_t in Equation (3), we model nominal and TIPS yields using a dynamic factor model. We assume that the yield curve of nominal Treasuries is described by a few latent yield curve factors, while the TIPS yield curve is driven by both the yield curve factors and the TIPS liquidity factor. In practice, following Equations (1)–(2), we identify the liquidity premium in the TIPS market L_t as the common component in TIPS yields unspanned by nominal yields.

Formally, we assume that nominal yields with different time to maturities are driven by a vector of $r_X \times 1$ of common latent factors X_t as follows

$$y_{t,\tau}^N = a_\tau^N + B_\tau^N X_t + \varepsilon_{t,\tau}^N \quad (4)$$

where a_τ^N is the maturity-specific intercept, B_τ^N contains the factor loadings of the nominal yield with maturity τ on the latent factors (common across maturities) and $\varepsilon_{t,\tau}^N$ represent the maturity-specific component of the nominal yield with maturity τ . In the same way, for real yields we have

$$y_{t,\tau}^R = a_\tau^R + B_\tau^R X_t + \varepsilon_{t,\tau}^R \quad (5)$$

where a_τ^R is the maturity-specific intercept, B_τ^R contains the factor loadings and $\varepsilon_{t,\tau}^R$ is the maturity-specific component of the real yield with maturity τ .

Let the vector $y_t^N = (y_{t,\tau_1^N}^N, \dots, y_{t,\tau_n^N}^N)'$ collect the nominal yields with maturities $(\tau_1^N, \dots, \tau_n^N)$ and the vector $y_t^T = (y_{t,\tau_1^T}^T, \dots, y_{t,\tau_m^T}^T)'$ collect the TIPS yields with maturities $(\tau_1^T, \dots, \tau_m^T)$. Following equations (2)–(5), the joint model for nominal and TIPS yields can be written as

$$\begin{pmatrix} y_t^N \\ y_t^T \end{pmatrix} = \begin{pmatrix} a^N \\ a^T \end{pmatrix} + \begin{bmatrix} B^N & 0 \\ B^R & B^L \end{bmatrix} \begin{pmatrix} X_t \\ L_t \end{pmatrix} + \begin{pmatrix} \varepsilon_t^N \\ \varepsilon_t^T \end{pmatrix} \quad (6)$$

where $a^T = a^R + a^L$ and $\varepsilon_t^T = \varepsilon_t^R + \varepsilon_t^L$.

Equation 6 identifies two sets of latent factors: the yield curve factors and the TIPS liquidity factor. The yield curve factors are in line with the literature that exploits the high level of co-movement of yields with different maturities to provide a parsimonious representation of the yield curve. This literature has proven that yield curve factor models are very successful in fitting the yield curve of nominal interest rates, see Litterman and Scheinkman (1991), Nelson and Siegel (1987), Duffee (2002) and Coroneo, Nyholm and Vidova-Koleva (2011). As for the liquidity factor, by noticing that TIPS are less liquid than nominal Treasuries, see Gürkaynak and Wright (2012) and Fleckenstein et al. (2014), we are able to identify the TIPS liquidity premium factor L_t as the driver of the wedge between real and TIPS yields. We implement this identification condition through the zero factor loading restrictions in Equation (6), which imply that the liquidity factor does not affect the current nominal yield curve, as shown in Equations (1)–(2).

We allow the $(n + m)$ idiosyncratic components collected in $\varepsilon_t = [(\varepsilon_t^N)', (\varepsilon_t^T)']'$ to follow independent univariate AR(1) processes

$$\varepsilon_t = A\varepsilon_{t-1} + v_t, \quad v_t \sim N(0, R) \quad (7)$$

where A and R are diagonal matrices, implying that the common factors fully account for the joint correlation of the observations. The $(r_X + 1) \times 1$ vector of zero mean latent factors and a $w \times 1$

vector of zero mean observable variables W_t follow a VAR(1)

$$\begin{pmatrix} X_t \\ L_t \\ W_t \end{pmatrix} = \begin{bmatrix} \Phi_{XX} & \Phi_{XL} & \Phi_{XW} \\ \Phi_{LX} & \Phi_{LL} & \Phi_{LW} \\ \Phi_{WX} & \Phi_{WL} & \Phi_{WW} \end{bmatrix} \begin{pmatrix} X_{t-1} \\ L_{t-1} \\ W_{t-1} \end{pmatrix} + \begin{pmatrix} u_t^X \\ u_t^L \\ u_t^W \end{pmatrix}, \quad (8)$$

where the innovations $u_t = [(u_t^X)', (u_t^L)', (u_t^W)']'$ are normally distributed with zero mean and variance Q . The observable variables W_t in Equation (8) do not contribute to the identification of the TIPS liquidity factor but have predictive ability for it. If we do not have any observable variable then $W_t = \emptyset$.

Finally, the innovations driving the common factors, u_t in Equation (8), and the residuals to the idiosyncratic components of the individual variables, v_t in Equation (7), are normally distributed and mutually independent. This implies that the common factors are not allowed to react to variable specific shocks.

3 Estimation

The joint model for nominal and TIPS yields in Equations (6)–(8) is a restricted dynamic factor model with autocorrelated idiosyncratic components. In order to cast the model in a state-space form, we augment the vector of state variables with the vector of idiosyncratic components ε_t and an additional state variable c_t restricted to one at every period (by fixing its initial value to one

and the variance of its innovations to zero). We then rewrite the measurement equation as

$$\begin{pmatrix} y_t^N \\ y_t^T \\ W_t \end{pmatrix} = \begin{bmatrix} B^N & 0 & 0 & a^N & I_n & 0 \\ B^R & B^L & 0 & a^T & 0 & I_m \\ 0 & 0 & I_w & 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} X_t \\ L_t \\ W_t \\ c_t \\ \varepsilon_t^N \\ \varepsilon_t^T \end{pmatrix} + \begin{pmatrix} v_t^N \\ v_t^T \\ v_t^W \end{pmatrix} \quad (9)$$

where $((v_t^N)', (v_t^T)', (v_t^W)')' \sim N(0, \epsilon I_{n+m+w})$ and ϵ is a coefficient that we fix to 1^{-12} . In the same way, we write the state equation as

$$\begin{pmatrix} X_t \\ L_t \\ W_t \\ c_t \\ \varepsilon_t \end{pmatrix} = \begin{bmatrix} \Phi_{XX} & \Phi_{XL} & \Phi_{XW} & 0 & 0 \\ \Phi_{LX} & \Phi_{LL} & \Phi_{LW} & 0 & 0 \\ \Phi_{WX} & \Phi_{WL} & \Phi_{WW} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & A \end{bmatrix} \begin{pmatrix} X_{t-1} \\ L_{t-1} \\ W_{t-1} \\ c_{t-1} \\ \varepsilon_{t-1} \end{pmatrix} + \begin{pmatrix} u_t^X \\ u_t^L \\ u_t^W \\ \nu_t \\ v_t \end{pmatrix} \quad (10)$$

with $((u_t^X)', (u_t^L)', (u_t^W)', (\nu_t)', (v_t)')' \sim N(0, blkdiag(Q, \epsilon, R))$.

The model in (9)–(10) is a restricted state-space model for which maximum likelihood estimators of the parameters are not available in closed form. Conditionally on the factors, the model reduces to a set of linear regressions. As consequence, we compute Maximum Likelihood estimates using the Expectation Maximization (EM) algorithm introduced by Shumway and Stoffer (1982) and Watson and Engle (1983). This estimator is feasible when the number of variables is large, and robust to non Gaussianity and to the presence of weak cross-sectional correlation among the idiosyncratic terms, see Doz, Giannone and Reichlin (2012). In addition, as shown in Coroneo, Giannone and Modugno (Forthcoming), using the Kalman filter and the EM algorithm allows us to easily impose linear restrictions on the parameters. For more details see Appendix A.

We also estimate a restricted version of the state-space model in (9)–(10), where we do not allow for any additional information, i.e. W_t is empty. This can be implemented in our framework by imposing the restrictions $\Phi_{WX} = \Phi_{WL} = \Phi_{XW} = \Phi_{LW} = 0$.

4 Counterfactual analysis

Let's now assume that at time t_0 a policy is implemented and we are interested in assessing the effect of this particular policy on the liquidity premium in the TIPS market. This can be done by comparing the realised path of the TIPS liquidity premium factor with a counterfactual TIPS liquidity factor that does not incorporate the policy.

The counterfactual TIPS liquidity premium factor is the forecast of the TIPS liquidity factor for $t \geq t_0$ conditional on past TIPS and nominal yields, as well as past and future values of suitable conditioning variables. A conditioning variable is suitable for the construction of a counterfactual path for the liquidity premium if it is informative about the liquidity premium in the TIPS market and it is invariant to the policy. In our framework, the first condition is satisfied if the conditioning variable Granger causes the TIPS liquidity premium, i.e. $\Phi_{LW} \neq 0$. The invariance condition instead is satisfied if the policy does not affect the conditioning variables, and the TIPS liquidity premium (that in principle can be affected by the policy) does not feedback into the conditioning variables, i.e. $\Phi_{WL} = 0$. Nominal yields can be included in the set of conditioning variables only if the yield curve factors satisfy the suitability conditions, i.e. $\Phi_{LX} \neq 0$, $\Phi_{XL} = 0$ and the yield curve factors are not affected by the policy.

If we collect the set of conditioning variables in W_t , we can denote the counterfactual TIPS liquidity factor as

$$L_{t|T}^* \equiv E[L_t | y_1^N, \dots, y_{t_0-1}^N, y_1^T, \dots, y_{t_0-1}^T, W_1, \dots, W_T], \quad t \geq t_0. \quad (11)$$

In practice, we are interested in extracting the TIPS liquidity factor for $t \geq t_0$ from a dataset of TIPS, nominal yields and conditioning variables, where the TIPS and nominal yields are unobserved

from t_0 . Notice from equation (9) that the TIPS liquidity premium factor is loaded only by TIPS yields. This implies that any conditioning variable included in the counterfactual does not contribute to the identification of the TIPS liquidity factor. Conditioning variables contribute to the construction of the counterfactual TIPS liquidity factor $L_{t|T}^*$ only through the state equation (10).

The full set of counterfactual yield curve and TIPS liquidity factors is given by

$$\begin{pmatrix} X_{t|T}^* \\ L_{t|T}^* \end{pmatrix} = E \left[\begin{pmatrix} X_t \\ L_t \end{pmatrix} \middle| y_1^N, \dots, y_{t_0-1}^N, y_1^T, \dots, y_{t_0-1}^T, W_1, \dots, W_T \right], \quad t \geq t_0. \quad (12)$$

with associated MSE

$$V_{t|T}^* = E \left[\left(\begin{pmatrix} X_t \\ L_t \end{pmatrix} - \begin{pmatrix} X_{t|T}^* \\ L_{t|T}^* \end{pmatrix} \right) \left(\begin{pmatrix} X_t \\ L_t \end{pmatrix} - \begin{pmatrix} X_{t|T}^* \\ L_{t|T}^* \end{pmatrix} \right)' \right], \quad t \geq t_0. \quad (13)$$

Both the counterfactual factors in (12) and their accuracy in (13) can be easily computed in our framework regardless of the dimensionality of the conditioning variables, thanks to the use of the state-space representation and Kalman filtering. This because we can use a modified state-space model where the dimensionality of the observation vector varies over time, see Durbin and Koopman (2012). In practice, after t_0 only the rows that refer to the conditioning variables will enter into the measurement equation.

The 95% confidence interval for the counterfactual TIPS liquidity factor can then be constructed as

$$CI(L_{t|T}^*) = \left(L_{t|T}^* - 1.96\sqrt{V_{L,t|T}^*}, \quad L_{t|T}^* + 1.96\sqrt{V_{L,t|T}^*} \right), \quad t \geq t_0 \quad (14)$$

where $V_{L,t|T}^*$ is the $[r_X+1, r_X+1]$ element of $V_{t|T}^*$, which denotes the accuracy of the counterfactual TIPS liquidity premium.

5 Data

We use end-of-day yield curve data spanning the period January, 2 2005 to December, 31 2014, for a total of 2,339 observations. Nominal yields and TIPS yields with maturity 3, 5, 8, 10, 13, 15 and 20 years are based on zero-coupon yield curves fitted at the Federal Reserve Board, see Gürkaynak, Sack and Wright (2007) and Gürkaynak, Sack and Wright (2010) for details.¹

To compare our estimates of the TIPS liquidity premium, we construct a liquidity proxy from inflation swap rates. We use mid-quotes of inflation swap rates with maturity 3, 5, 8, 10, 13, 15 and 20 years from Datastream converted to continuously compounded basis. Following Haubrich et al. (2012), we compute real rates as the difference between equivalent maturity nominal Treasury yields and inflation swap rates. We then construct a liquidity proxy as the average, across maturities, of the difference between equivalent maturity TIPS yields and real rates constructed using the inflation swaps.

As possible conditioning variables for the counterfactual analysis, we use the TED spread (defined as the spread between the three month LIBOR and the three month Tbill rates), the Chicago Board Options Exchange Volatility Index (VIX), the Cleveland Financial Stress Index, the corporate spread (defined as the spread between the Baa corporate and the ten year Treasury rates), the bid-ask spread on the three month Tbill and the noise measure of Hu, Pan and Wang (2013) which is a market-wide measure of illiquidity. Data for all variables is obtained from the FRED database, except for the illiquidity measure of Hu et al. (2013) which is obtained from the authors.²

In Table 1 we report the percentage of variance of nominal yields and of jointly nominal and TIPS yields explained by the first five principal components extracted, respectively, from nominal yields and jointly from nominal and TIPS yields. Results in Table 1 show that two factors fully explain the cross-section of nominal yields, but when considering nominal and TIPS yields jointly an additional factor should be included in the analysis. This indicates that TIPS yields are driven

¹We exclude the short end of the nominal yield curve to avoid the possible non-linearities associated with short maturities reaching the zero lower bound in the last part of the sample. Results including 3- and 6-month T-bill yields from the Federal Reserve Boards H.15 release and the nominal yield with maturity 1 year from Gürkaynak et al. (2007) are very similar to the ones presented in this paper and available upon request.

²The noise measure is available at <http://www.mit.edu/junpan/>

Table 1: Variances explained by principal components

	Nominal	Nom+TIPS
PC1	0.941	0.902
PC2	0.996	0.955
PC3	1.000	0.995
PC4	1.000	0.998
PC5	1.000	0.999

Note: this table report the percentage of variance of nominal yields (first column) and nominal and TIPS yields jointly (second column) explained by the first four principal components extracted from nominal (first column) and nominal and TIPS (second column) yields.

by a factor that is unspanned by nominal yields which accounts for the liquidity premium in the TIPS market. Accordingly, in our analysis we use two factors to explain the cross-section of nominal yields, i.e. $r_X = 2$, and one factor to explain the liquidity premium in the TIPS market.

6 Estimation Results

We first estimate the joint state-space model for nominal and TIPS yields in (9)-(10) with two yield curve factors, a liquidity factor and without using any additional information. Table 2 reports

Table 2: Model fit

Maturity	Nominal Yields		TIPS Yields	
	Mean	Std	Mean	Std
3	0.003	9.558	0.003	30.817
5	0.000	0.043	0.001	11.855
8	0.005	1.983	0.000	0.023
10	-0.002	0.097	-0.001	3.938
15	-0.007	7.516	-0.001	9.773
20	-0.004	15.288	-0.002	13.219

Note: this table reports the means and standard deviations of residuals for the model in (9)-(10) with $W_t = \emptyset$. All numbers are in basis points.

Table 3: Variance explained by the latent factors

Mat	Nominal Yields			TIPS Yields			
	X_1	X_2	Total	X_1	X_2	L	Total
3	0.850	0.146	0.996	0.722	0.005	0.229	0.956
5	0.947	0.053	1.000	0.810	0.001	0.179	0.991
8	0.999	0.000	0.999	0.841	0.017	0.141	1.000
10	0.991	0.009	1.000	0.840	0.032	0.125	1.000
15	0.945	0.048	0.993	0.809	0.078	0.098	0.988
20	0.901	0.058	0.960	0.744	0.135	0.088	0.969

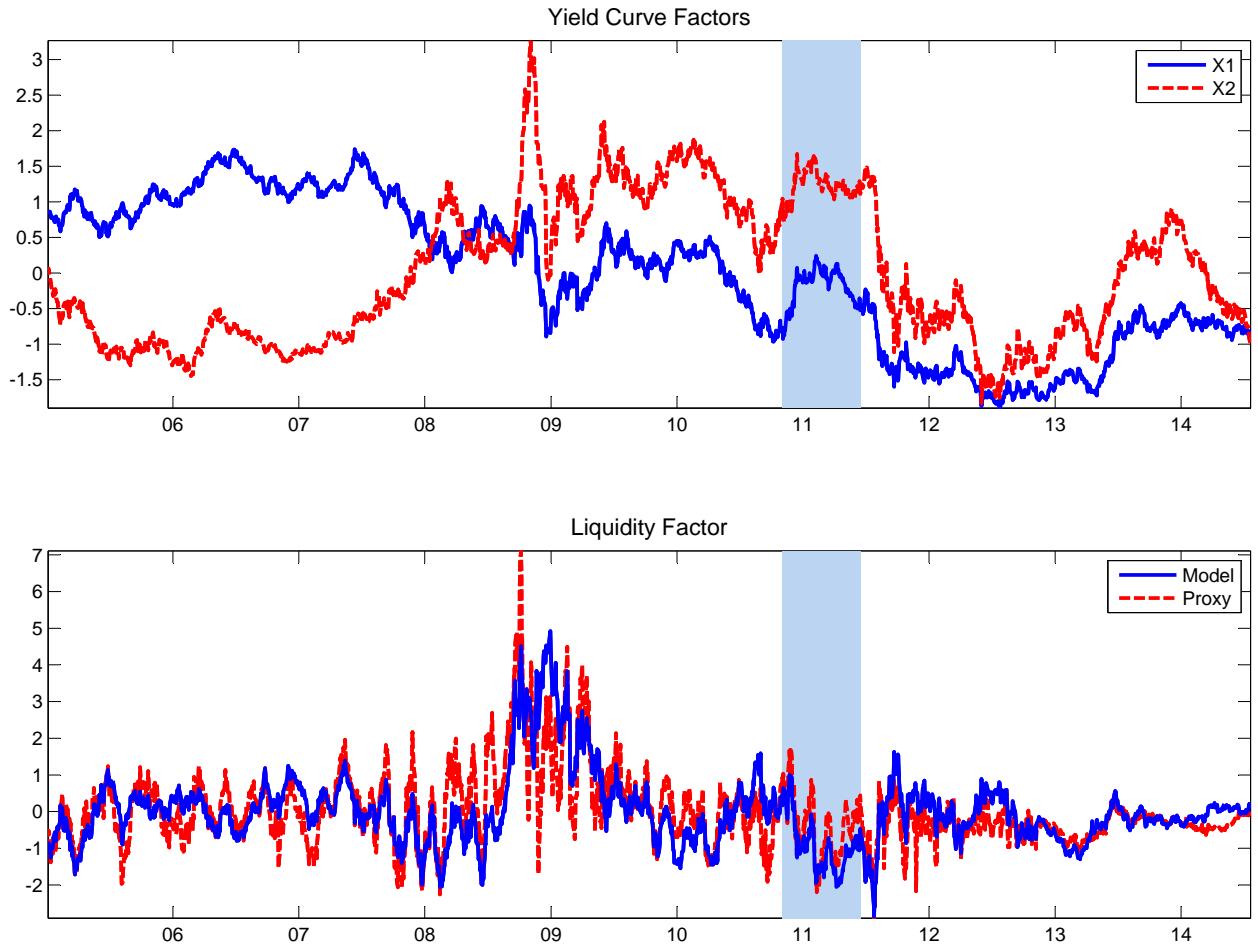
Note: this table reports the percentage of variance of nominal yields (left panel) and of TIPS yields (right panel) explained by the estimated latent factors of the model in (9)-(10) with $W_t = \emptyset$.

summary statistics for fitting errors of nominal and TIPS yields. The table shows that the model fits very well the cross-section of nominal and TIPS yields, as the errors are small for both nominal and TIPS yields. Table 3 reports the percentage of variance of nominal and TIPS yields explained by the estimated latent factors. The table confirms that the model has a good fit for both nominal and TIPS yields, with at least 96% of the variance of yields explained by the latent factors. The table also shows that the first latent factor explains the bulk of the variation in both nominal and TIPS yields, while the second yield curve factor explains up to 14% of the variance of nominal and TIPS yields.

The TIPS liquidity factor by construction does not affect nominal yields but has substantial explanatory power for TIPS yields. It explains up to 22% of the variance of TIPS yields and its explanatory power is higher for shorter maturities, implying that investors require higher compensation for holding shorter term TIPS rather than longer term ones. This might be due to the fact that growth of the TIPS markets has not occurred uniformly, see Shen (2006). For example, the Treasury has issued 10-year TIPS every year since the TIPS program began in 1997. On the contrary, the 5-year TIPS, were issued in 1997 and 1998, but then not again until 2005.

Figure 1 reports the estimated yield curve (top panel) and the TIPS liquidity (bottom panel) factors. The first yield curve factor has a decreasing pattern in our sample due to a general decline in interest rates in this period. The second yield curve factor is more volatile than the first and

Figure 1: Estimated yields curve and TIPS liquidity factor



The figure displays the estimated latent factors from model (6)-(8) with $W_t = \emptyset$. The top panel displays the estimated yield curve factors X_1 and X_2 . The bottom panel displays the estimated TIPS liquidity factor L (continuous blue line) and a proxy for the TIPS liquidity factor constructed using inflation swaps (red dashed line). The blue shaded areas indicate the QE2 operation period.

is higher in the middle of the sample. The bottom plot of Figure 1 reports the TIPS liquidity factor and a proxy for the average liquidity in the TIPS market, constructed as the standardized average across maturities of the difference between TIPS yields and real rates computed using inflation swaps and nominal yields of the same maturity. The figure shows that the TIPS liquidity factor is highly correlated with its empirical proxy. The pairwise correlation coefficient is 75%. We can also notice that the estimated TIPS liquidity factor is more persistent than the proxy for the average liquidity in the TIPS market constructed using inflation swaps. This may be due to the fact that inflation swaps are also subject to liquidity frictions, see Fleming and Sporn (2013), and therefore the liquidity proxy constructed using inflation swaps measures the liquidity premium in both TIPS and inflation swaps, see Christensen and Gillan (2012). Figure 1 also shows that during the subprime crisis, the TIPS liquidity premium became more volatile and in September 2008, following the Lehman Brothers collapse, the TIPS liquidity factor increased substantially indicating a flight to liquidity in the US Treasury market. The TIPS liquidity premium factor remained at this higher level until mid-2009, when it returned to the pre-2008 level.

In Table 4 we report Granger causality tests for a set of possible conditioning variables that include the TED spread, the VIX index, the Cleveland Financial Stress Index, the noise measure of Hu et al. (2013), the corporate spread, the bid-ask spread on the three month Tbill and the yield curve factors. Results in Table 4 indicate that only the noise measure of Hu et al. (2013) and the corporate spread have significant predictive ability for the TIPS liquidity premium at 1% significance level and are not predicted by the TIPS liquidity premium. The noise measure of Hu et al. (2013) is a market-wide illiquidity measure that exploits the connection between the amount of arbitrage capital in the market and observed noise in U.S. Treasury bonds. Hu et al. (2013) show that this measure captures episodes of liquidity crises of different origins across financial markets. Corporate bond spreads measure default risk premium and liquidity premium in corporate bonds, see Longstaff, Mithal and Neis (2005). Therefore, results in Table 4 indicate that the TIPS liquidity premium is Granger caused by measure of financial stress but, given the size of the TIPS market, events in this market have limited ability to spread across financial markets. On the other hand,

Table 4: Granger causality tests

	Null: the variable does not Granger cause L							
	TED	VIX	CFSI	N	CS	BA	X ₁	X ₂
Fstat	2.375	4.331	2.295	11.262	6.266	0.005	2.199	0.804
pval	(0.12)	(0.04)	(0.13)	(0.00)	(0.01)	(0.94)	(0.14)	(0.37)
Null: L does not Granger cause the variable								
	TED	VIX	CFSI	N	CS	BA	X ₁	X ₂
Fstat	0.001	0.297	0.071	0.430	1.170	6.313	24.599	9.490
pval	(0.98)	(0.59)	(0.79)	(0.51)	(0.28)	(0.01)	(0.00)	(0.00)

Note: this table report likelihood ratio test statistics and p-values. All variance-covariance matrix are robust to autocorrelation and heteroskedasticity. All statistics refer to a univariate VAR(1) of the listed variable and the estimated TIPS liquidity factor L. TED refers to the TED spread (spread between the three month LIBOR and the three month Tbill rates), VIX refers to the Chicago Board Options Exchange Volatility Index, CFSI refers to the Cleveland Financial Stress Index, N refers to the noise measure of Hu et al. (2013), CS refers to the corporate spread (spread between the Baa corporate and the ten year Treasury rates), BA denotes the bid-ask spread on the three month Tbill, X_1 and X_2 are the yield curve factors

Table 4 shows that variables related to the yield curve of nominal Treasuries, i.e. the bid-ask spread on the three month Tbill and the yield curve factors, do not have any predictive ability for the TIPS liquidity premium factor. On the contrary, they are Granger caused by the TIPS liquidity premium. This indicates that changes in the TIPS liquidity premium affect nominal Treasuries.

7 Quantitative Easing

Following the 2007 financial crisis, the Federal Reserve conducted massive asset purchases known as quantitative easing (QE) to lower long-term interest rates and spur economic growth.

On November 25, 2009 the Federal Open Market Committee (FOMC) announced the first quantitative easing program (QE1) which would involve purchases in government-sponsored enterprises (GSEs) debt and in mortgage-backed securities (MBS). On March 18, 2009 the FOMC announced that the QE1 program would involve additional purchases in GSEs and MBS. It was also announced that the program would involve purchases of \$300 billion in long-term Treasury securities. The Treasury purchases ended on October 29, 2009 and involved \$6.1 billion in TIPS

purchases.

The second quantitative easing program (QE2) was announced on November 3, 2010 with the target of expanding the Federal Reserve's balance sheet by \$600 billions through Treasury security purchases over an eight-month period. The gross purchases of Treasury securities from November 3, 2010 until June 30, 2011 amounted to nearly \$750 billion, of which about \$26 billion were TIPS purchases.

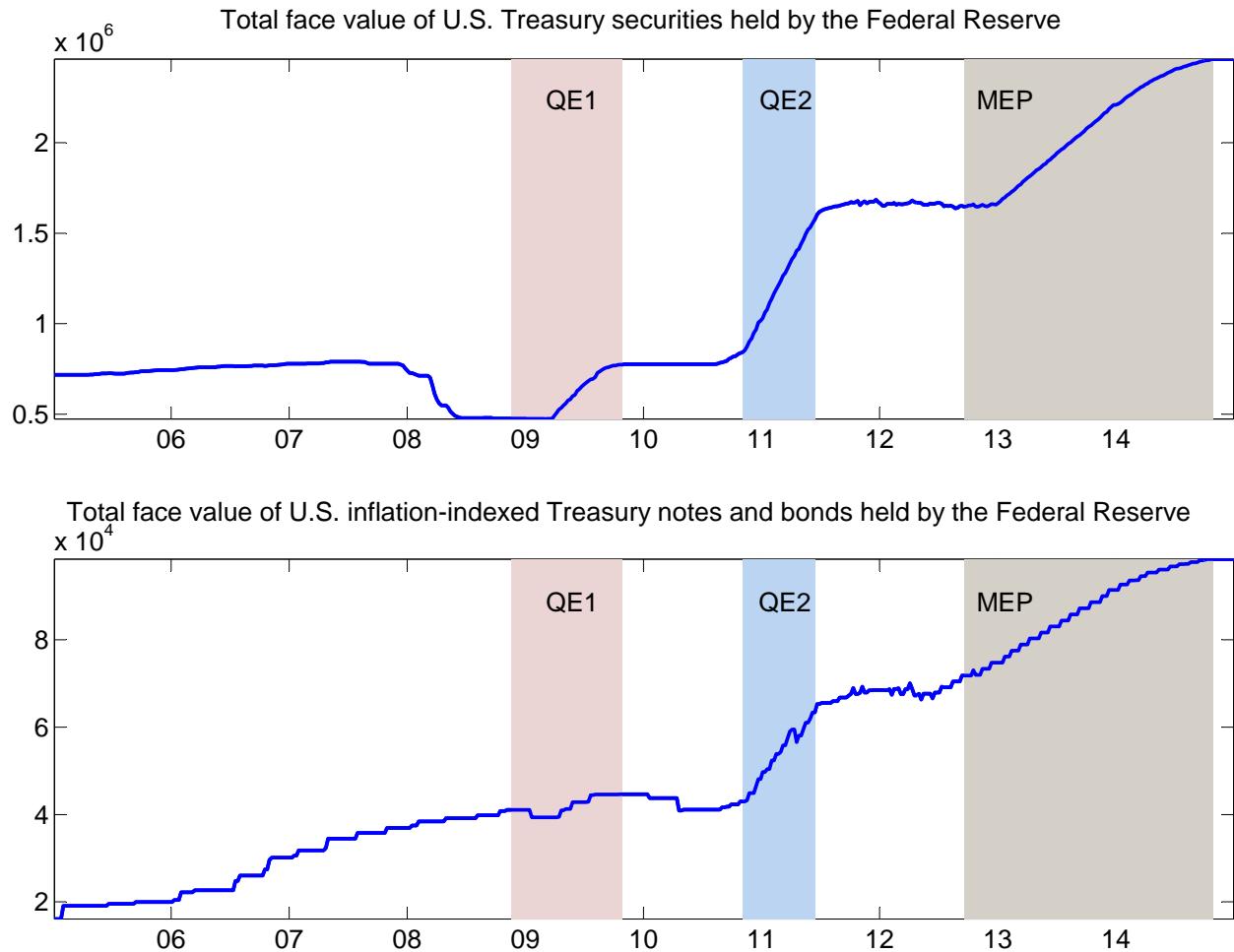
On September 21, 2011 the FOMC announced the third QE program, known as maturity extension program (MEP), which would involve Fed purchases of \$400 billion in long-term Treasuries and equivalent sales in short-term Treasuries. On June 20, 2012 the FOMC announced that purchases of long-term bonds and the sales of short-term bonds would continue through 2012 and would involve a total of \$600 billion in purchases and sales of securities. On December 12, 2012 the Fed announced that it would continue to purchase \$45 billion in long-term Treasuries per month but without the sale of short-term Treasuries to sterilize purchases. The MEP involved TIPS purchases for a total of \$27.1 billion, all in TIPS with more than 6 years to maturity. The TIPS sales within the MEP totaled \$13.4 billion and only included TIPS with less than 3.25 years to maturity.

In Figure 2 we report the Federal Reserve's outright holdings of Treasury securities and TIPS. The figure shows that the Federal Reserve's outright holdings of both nominal Treasuries and TIPS sharply increased during the QE2 program. This because the Fed purchases of Treasuries within the QE2 program have been larger and concentrated in a shorter time span with respect to the QE1 and the MEP programs.

Such massive purchases of Treasuries may have reduced the liquidity premia required by investors to buy TIPS, as the Federal Reserve's purchases may have made it less costly for investors to sell TIPS. However, this effect of the QE2 program on the TIPS liquidity premia depends on the Federal Reserve's flow of purchases and, therefore, we expect it to be limited to the duration of the QE2 program. On the other hand, the FED's TIPS purchases have reduced the stock of TIPS available to investors and this may have instead increased the liquidity premium in the TIPS market.

Table 5 contains the exact dates of the TIPS purchases along with the amount and the average

Figure 2: Federal Reserve's outright holdings of Treasuries



The figure displays the total face value of the Federal Reserve's outright holdings of Treasury securities (top graph) and inflation-indexed Treasury notes and bonds (bottom plot). The shaded areas indicates the QE1, QE2 and the MEP operation periods. Data are weekly.

Table 5: QE2 TIPS purchase

	Dates	Amount (Mill.)	Average Maturity
0	03-Nov-10		
1	23-Nov-10	\$1,821	9.43
2	08-Dec-10	\$1,778	8.88
3	21-Dec-10	\$1,725	16.09
4	04-Jan-11	\$1,729	16.98
5	18-Jan-11	\$1,812	14.64
6	01-Feb-11	\$1,831	13.58
7	14-Feb-11	\$1,589	14.16
8	04-Mar-11	\$1,589	11.37
9	18-Mar-11	\$1,653	17.77
10	29-Mar-11	\$1,640	18.29
11	20-Apr-11	\$1,729	23.17
12	04-May-11	\$1,679	13.62
13	16-May-11	\$1,660	20.49
14	07-Jun-11	\$1,589	14.3
15	17-Jun-11	\$2,129	5.98
	Average	\$1,730	14.58

Note: this table reports the QE2 TIPS purchase operations dates along with the amount (in millions) and the (weighted) average maturity.

maturity. As a preliminary assessment of the effect of the QE2 program on the liquidity in the TIPS market, we report in Table 6 the changes in the estimated TIPS liquidity factor around the days of the QE2 announcement (Nov. 3, 2010) and operations (from Nov. 23, 2010 to Jun. 17, 2011) for different window sizes (from one day change to five days change). The table shows that on the day of the QE2 program announcement the liquidity factor declined. In addition, on eleven out of fifteen TIPS operation dates of the QE2 program, the liquidity factor declined either the same day or the following day. The average and median of the changes of the liquidity factor in the QE2 dates are negative, indicating that the program may have lowered the liquidity premium required by market participants in order to invest in TIPS. To formally asses the impact of the QE2 program on the TIPS liquidity premium factor, in the next section we perform a counterfactual analysis.

Table 6: Cumulative changes in the TIPS liquidity factor around QE2 events

		1	2	3	4	5
0	03-Nov-10	-0.262	0.082	0.172	0.108	-0.231
1	23-Nov-10	0.191	-0.306	0.163	0.146	0.120
2	08-Dec-10	-0.272	-0.006	-0.420	-0.287	-0.864
3	21-Dec-10	-0.081	-0.193	-0.297	-0.092	-0.163
4	04-Jan-11	0.168	-0.199	-0.234	0.057	0.099
5	18-Jan-11	0.069	0.298	0.083	0.317	0.314
6	01-Feb-11	-0.235	-0.457	-0.856	-1.141	-1.255
7	14-Feb-11	0.223	0.087	0.037	0.119	0.022
8	04-Mar-11	0.284	0.310	0.199	0.505	0.728
9	18-Mar-11	-0.042	0.021	-0.020	-0.126	-0.291
10	29-Mar-11	-0.053	-0.101	-0.174	-0.227	-0.069
11	20-Apr-11	-0.137	-0.144	0.180	0.145	0.261
12	04-May-11	0.146	0.337	0.294	0.241	0.092
13	16-May-11	-0.079	0.044	-0.137	-0.121	-0.088
14	07-Jun-11	0.043	-0.039	-0.305	-0.163	-0.066
15	17-Jun-11	-0.096	-0.214	-0.264	-0.475	-0.438
	Mean	-0.008	-0.030	-0.099	-0.062	-0.114
	Median	-0.047	-0.022	-0.078	-0.017	-0.068

Note: this table reports the cumulative changes in the TIPS liquidity factor around the days of the QE2 announcement (Nov. 3, 2010) and operations (from Nov. 23, 2010 to Jun. 17, 2011) for different window sizes (from one day change to five days change). The changes in the TIPS liquidity factor are computed from the day before the event, i.e. the two day change for the QE2 announcement is the difference in the liquidity factor between November 4, 2010 and November 2, 2010.

7.1 Counterfactual Results

Our objective is to assess the effect of the QE2 program on the liquidity premium in the TIPS market. We do this by comparing the realised path of the TIPS liquidity premium factor with a counterfactual TIPS liquidity premium factor that does not incorporate the QE2 program.

As explained in Section 4, conditioning variables can be used in the constructions of the counterfactual, as long as they Granger cause the TIPS liquidity premium, they are not Granger caused by the TIPS liquidity premium and they are not affected by the policy. Table 4 indicates that only the noise measure of Hu et al. (2013) and the corporate spread satisfy the first two conditions. As

Table 7: Median cumulative changes around QE2 events

	L	CS	N
1 day	-0.047	-0.011	-0.028
2 days	-0.022	-0.021	0.003
3 days	-0.079	-0.026	-0.038
4 days	-0.017	-0.021	0.002
5 days	-0.041	0.000	0.012
6 days	-0.095	-0.016	0.005
7 days	-0.159	-0.011	-0.005
8 days	-0.148	0.005	-0.032

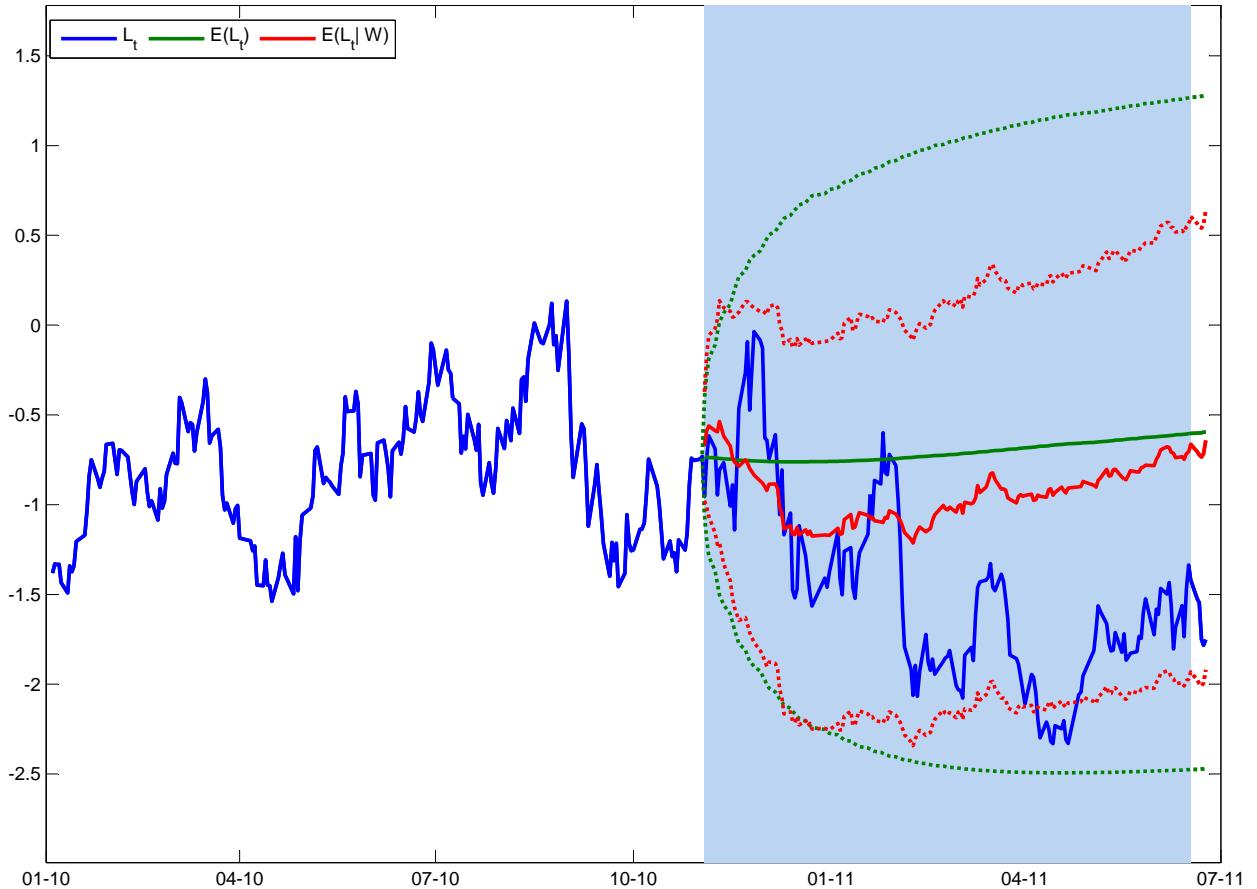
Note: this table reports the median cumulative changes in the TIPS liquidity factor (L), the standardized corporate spread (CS) and the standardized noise measure of Hu et al. (2013) (N) around the days of the QE2 announcement and operations for different window sizes (from one day change to eight days change).

for the last condition, in Table 7 we report the median responses of the TIPS liquidity premium, the standardized corporate spread and the standardized noise measure of Hu et al. (2013) around the dates of the QE2 events, for different window sizes. The table indicates that both the corporate spread and the noise measure were not directly affected by the QE2 operations, as opposed to the TIPS liquidity premium that instead had large declines for any window size. We can therefore conclude that both the noise measure and the corporate spread are suitable conditioning variables.

We construct the counterfactual TIPS liquidity premium factor by estimating model parameters in (6)-(8) using observations up to November, 2 2010, i.e. the day before the announcement of the QE2 purchases. This implies that the counterfactual keeps historical pre-QE2 relations among the variables. We then assume that nominal and TIPS yields are only observed until November, 2 2010, and that conditioning variables are always observed. The counterfactual TIPS liquidity premium and the corresponding accuracy are then computed, respectively, as in (11) and (13), where $t_0 = \text{November, 3 2010}$.

We perform two counterfactual exercises. In the first, we do not use any conditioning variable,

Figure 3: TIPS liquidity premium factor: realised vs counterfactuals



The figure displays the realised liquidity premium (continuous blue line) and two counterfactuals in which nominal and TIPS yields are observed only until November, 2 2010, defined in (11). The green line is the counterfactual TIPS liquidity premium factor that does not use any conditioning information, i.e. $W = \emptyset$. The red line refers to the counterfactual TIPS liquidity premium factor that conditions on the noise measure of Hu et al. (2013) and the corporate spread. The dashed lines delimitate the 95% confidence intervals of the counterfactuals, computed as in (14). The blue shaded area indicates the QE2 operation period.

i.e. $W_t = \emptyset$. In the second, we use as conditioning variables the noise measure of Hu et al. (2013) and the corporate spread, i.e. $W_t = [N_t, CS_t]$. Results in Figure 3 show that the two counterfactual liquidity premia are on average higher than the realised TIPS liquidity premium. The figure also shows that using conditioning variables has two effects on the counterfactual. First, when using conditioning information, the counterfactual path for the TIPS liquidity factor gets on average closer to the realised liquidity premium. This indicates that part of the observed reduction of the TIPS liquidity premium in this period is due to a general reduction in illiquidity and risk premia in financial markets. Second, the accuracy of the counterfactual increases as more conditioning information is used. This because the conditioning variables provide additional information.

As for the comparison of the realised TIPS liquidity premium with the counterfactuals, Figure 3 shows that the realised TIPS liquidity premium is within the 95% confidence bands of the two counterfactuals most of the times. The only exception is in April 2011, when the realised TIPS liquidity factor is significantly lower than the counterfactual constructed using the illiquidity measure of Hu et al. (2013) and the corporate spread. However, this effect is only temporary and the realised TIPS liquidity factor reverts back within the 95% confidence bands in May 2011. We therefore conclude that the QE2 program had only a marginal effect on the liquidity premium in the TIPS market.

8 Conclusion

In this paper, we identify the liquidity premium in the TIPS market as the common component in TIPS yields that is unspanned by nominal yields. Within a joint factor model representation for nominal and TIPS yields, we use quasi-maximum likelihood and Kalman filter to extract the liquidity premium in the TIPS market. Using daily data from January, 2 2005 to December, 31 2014, we find that the TIPS liquidity factor explains up to 22% of the variance in TIPS yields and that it sharply increased during the recent financial crisis.

We use our setup to perform a counterfactual analysis of the liquidity premium in the TIPS

market during the QE2 program. This is easily implementable in our framework regardless of the dimensionality of the conditioning variables, thanks to the use of the state-space representation and Kalman filtering. We construct a counterfactual path for the TIPS liquidity premium factor that does not incorporate the QE2 program but that conditions on suitable conditioning information. We define suitable conditioning variables for a counterfactual as variables that are invariant to the policy but informative about the variable on which we want to construct the counterfactual. Our empirical results indicate that a measure of market-wide illiquidity and the corporate spread are suitable conditioning variables for the construction of a counterfactual TIPS liquidity premium.

The counterfactual exercises show that using suitable conditioning variables improves the quality of the counterfactual. However, we only find mild evidence for an effect of the QE2 program in the liquidity premium in the TIPS market.

A Estimation

The model in (9)–(10) can be written in compact notation as

$$\begin{aligned} z_t &= B^* F_t^* + v_t^*, \quad v_t^* \sim N(0, R^*) \\ F_t^* &= \Phi^* F_{t-1}^* + u_t^*, \quad u_t^* \sim N(0, Q^*) \end{aligned}$$

where

$$\begin{aligned} \bullet \quad z_t &= \begin{pmatrix} y_t^N \\ y_t^T \\ W_t \end{pmatrix}, \quad B^* = \begin{bmatrix} B^N & 0 & 0 & a^N & I_n & 0 \\ B^R & B^L & 0 & a^T & 0 & I_m \\ 0 & 0 & I_w & 0 & 0 & 0 \end{bmatrix}, \quad F_t^* = \begin{pmatrix} X_t \\ L_t \\ W_t \\ c_t \\ \varepsilon_t^N \\ \varepsilon_t^T \end{pmatrix}, \quad R^* = \epsilon I_{n+m+w}; \\ \bullet \quad \Phi^* &= \begin{bmatrix} \Phi & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & A \end{bmatrix}, \quad u_t^* = \begin{bmatrix} u_t \\ \nu_t \\ v_t \end{bmatrix}, \quad Q^* = \begin{bmatrix} Q & 0 & 0 \\ 0 & \epsilon & 0 \\ 0 & 0 & R \end{bmatrix} \\ \bullet \quad \epsilon &\text{ is a coefficient that we fix to } 1^{-12}. \end{aligned}$$

We can write the restrictions on the factor loadings B^* and on the transition matrix Φ^* as

$$H_1 \text{ vec}(B^*) = q_1, \quad H_2 \text{ vec}(\Phi^*) = q_2, \quad (15)$$

where H_1 and H_2 are selection matrices, and q_1 and q_2 contain the restrictions.

We assume that $F_1^* \sim N(\pi_1, V_1)$, and define $z = [z_1, \dots, z_T]$ and $F^* = [F_1^*, \dots, F_T^*]$. Then denoting the parameters by $\theta = \{B^*, \Phi^*, Q^*, \pi_1, V_1\}$, we write the joint loglikelihood of z_t and F_t ,

for $t = 1, \dots, T$, as

$$\begin{aligned}
L(z, F^*; \theta) &= -\sum_{t=1}^T \left(\frac{1}{2} [z_t - B^* F_t^*]' (R^*)^{-1} [z_t - B^* F_t^*] \right) + \\
&\quad -\frac{T}{2} \log |R^*| - \sum_{t=2}^T \left(\frac{1}{2} [F_t^* - \Phi^* F_{t-1}^*]' (Q^*)^{-1} [F_t^* - \Phi^* F_{t-1}^*] \right) + \\
&\quad -\frac{T-1}{2} \log |Q^*| + \frac{1}{2} [F_1^* - \pi_1]' V_1^{-1} [F_1^* - \pi_1] + \\
&\quad -\frac{1}{2} \log |V_1| - \frac{T(p+k)}{2} \log 2\pi + \lambda'_1 (H_1 \text{vec}(B^*) - q_1) + \\
&\quad + \lambda'_2 (H_2 \text{vec}(\Phi^*) - q_2)
\end{aligned}$$

where λ_1 and λ_2 contain the lagrangian multipliers associate with the constraints in (15).

The EM algorithm allows to obtain maximum likelihood estimates of the parameters and the latent factors by alternating Kalman filter extraction of the factors to the maximization of the likelihood. In practice, at the j -th iteration the algorithm we perform two steps:

1. In the Expectation-step, we compute the expected log-likelihood conditional on the data and the estimates from the previous iteration

$$\mathcal{L}(\theta) = E[L(z, F^*; \theta^{(j-1)})|z]$$

which depends on three expectations

$$\begin{aligned}
\hat{F}_t^* &\equiv E[F_t^*; \theta^{(j-1)}|z] \\
P_t &\equiv E[F_t^*(F_t^*)'; \theta^{(j-1)}|z] \\
P_{t,t-1} &\equiv E[F_t^*(F_{t-1}^*)'; \theta^{(j-1)}|z]
\end{aligned}$$

These expectations can be computed, for given parameters of the model, using the Kalman filter.

2. In the Maximization-step, we update the parameters maximizing the expected log-likelihood

with respect to the parameters θ :

$$\theta^{(j)} = \arg \max_{\theta} \mathcal{L}(\theta)$$

Given that the restrictions in (15) are linear. This step can be implemented taking the corresponding partial derivative of the expected log likelihood, setting to zero, and solving.

In order to start the algorithm, we initialize the yield curve factors with the principal components extracted from nominal yields. We then project TIPS yields on these yield curve factors and use principal components of the residuals of this regression to initialize the liquidity factor. We then initialize all parameters with the OLS estimates obtained using the initial guesses of yield and liquidity factor. Using this set of initial parameters, we use the EM algorithm to obtain first estimates the latent factors.

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