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# Collateral Constraints and the Interest Rate

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**Abstract**

This paper establishes a transmission mechanism between credit constraints and persistently low real interest rates. In doing so, it establishes a link between two strands of macroeconomic literature that have become prominent since the financial crisis; financial frictions literature and zero lower bound literature. In order to analyse the credit constraints and interest rate interaction, the paper presents a perpetual youth overlapping generations model, which is extended to incorporate an endogenous financial friction in the form of a collateral constraint. Analytical expressions and elementary diagrams are presented to illustrate the results of the model. It is found that a tightening of the financial friction reduces the steady state rate of interest, and that non-linearities exist in this relationship. This result occurs endogenously in the model subsequent to a change in the constraint. The model also supports hypotheses from the secular stagnation literature by way of illustrating that population ageing and higher debt levels can leave an economy more likely to encounter episodes of persistent low interest rates.

# 1 Introduction

The substantial impact of the 2008 financial crisis and the subsequent prolonged period of adverse conditions, which prevailed across a wide range of economies, have motivated an expansion of research on a broad range of topics in macroeconomics. This paper focuses on the potential connection between two of the main strands of this research; the financial frictions literature and the zero lower bound (ZLB) literature. By linking these two strands of literature, this paper derives a model that will seek to explain one of the salient features of many economies in the years after the financial crisis; that of persistently low interest rates. This will be done by building an analytical framework which permits the real steady state interest rate to be influenced by changes in the credit market. In addition to this, the resultant model can also be used to look at the probability of an economy experiencing episodes of persistently low interest rates following a disruption to the credit market, due to disparate features of the economy, such as financial liberalisation and ageing. The paragraphs that follow will detail the aforementioned first and second strands of literature which are employed in developing the model.

The interaction of financial factors on macroeconomic aggregates has been the subject of a long established literature, thus providing a natural framework for the research motivated by the 2008 crisis. This broad financial frictions literature developed from a "credit view" of the economy which stressed the importance of credit market imperfections and variables, such as, net worth, debt and asset prices in enhancing the explanation of economic fluctuations (Iacoviello 2005). In this literature, the presence of imperfect information between agents results in credit market imperfections, leading to a distinct credit channel (Walsh 2010). A large number of papers have developed microfounded partial equilibrium models capturing the sources of imperfections. In order to quantify the macroeconomic impact of these credit market imperfections, some of these concepts were incorporated into general equilibrium macroeconomic models, with the imperfections being termed, "financial frictions" (Brunnermeier and Sannikov 2014). The resulting macroeconomic financial frictions literature has developed in line with a number of theoretical frameworks; notably the costly state verification approach of Bernanke, Gertler and Gilchrist (1999), the collateral constraints approach as per Kiyotaki and Moore (1997) and the financial intermediaries approach of Gertler and Kiyotaki (2011). In Bernanke, Gertler and Gilchrist (1999), it is found that agency costs in credit markets vary counter-cyclically due to the effect on firms balance sheets, and this exacerbates the initial impact of a shock. This amplification effect, termed the "financial accelerator mechanism", is a key result that is found in the financial frictions literature and across the modelling approaches mentioned. The financial accelerator in these models adds to the amplification and persistence of shocks, and provides a theoretical basis for explaining the substantial movements in macroeconomic variables during the financial crisis (Bernanke, Gertler and Gilchrist 1999), (Iacoviello 2005). Much of the focus of the financial frictions literature has had as its subject the magnitude and persistence of variables in response to shocks in the short run. This literature has recently expanded to cover a wide range of issues, such as the effect of financial frictions on open economy models, Taylor rules, risk premiums, macroprudential policy and in fitting models to national and regional data[1].

With respect to the second strand of research that is the subject of this paper, the zero lower bound literature has sought to explain the characteristics of an economy and the policy options in a situation where the policy interest rate is close to zero, or at zero. This is the situation which has prevailed in a number of economies for a prolonged period since the 2008 crisis (Williams 2014). The literature on the ZLB expanded rapidly in the 1990's due to the Japanese zero interest rate policy, and also later in the 2000's due to the low interest rates experienced in Europe and the US after the technology bubble (Buiter, 2009, Eggertsson and Woodford, 2003). The general conclusion of this earlier literature was that episodes of ZLB would be relatively infrequent and generally short lived; with estimates that monetary policy would be constrained by the ZLB five percent of the time, and that the typical episode would last just one year (Williams 2014). A broad literature review of this pre-financial crisis research on the ZLB is found in Yates (2004), and in summary states, "Overall the risks of being trapped at the zero bound to interest rates are

[1]For an overview, refer to Walsh (2010) and Brunnermeier and Sannikov (2014).

probably small, and probably overstated". As stated in Buiter (2009), the existence of the ZLB has long been acknowledged, but was, however, viewed mainly as a purely theoretical concern with, as mentioned above, a low probability of practical relevance. In the period after the 2008 financial crisis, this situation changed sharply, with The Federal Reserve, The Bank of England, The Bank of Japan and the European Central Bank all lowering their policy rates to their effective lower bounds (Williams 2014). The ZLB has thus been a binding constraint on Central Bank interest rate setting across a large number of economies. The tightness of this binding constraint is estimated to be substantial. Buiter (2009) notes that a number of studies have shown that by following the Taylor rule principal and abstracting for the existence of the ZLB, the official policy rate in the US in early 2009 would have been negative and in the range of -5 to -7.5 percent. This condition of the post-crisis economy called into question much of the predictions of the previous ZLB research; in particular the predictions that these episodes had a very low probability of occurrence, thus instigating a new wave of research. The ZLB is now seen as an issue that will constrain policy options well into the future (Williams 2014). The ZLB literature has also extensively analysed the policy options for further stimulus, such as quantitative and credit easing.[2]

Despite this large expansion in research in these two closely aligned fields, there have been comparatively few papers which integrate elements of these two literatures. The contribution of this paper is that it establishes an analytical link between the financial frictions literature and a feature of the zero lower bound literature; that of prolonged low interest rates after a crisis. It does this by building a model with endogenous collateral constraints; as per the financial frictions literature, into a benchmark macroeconomic model, which permits a varying steady state interest rate. In this model low interest rates occur endogenously as a result of a change in the strength of financial friction. This is different to much of the financial frictions literature wherein the models steady state rate of interest is generally fixed, and much of the zero lower bound literature wherein the interest rate fall is initiated by an exogenous change in the rate of time preference. As will be discussed in the subsequent section, the link between these two features of the crisis has been explored in some

[2] Refer to Buiter (2009), Eggertsson and Woodford (2003) and Williams (2014) for an overview.

other papers, however, in this paper the result is shown analyticity in a more standard benchmark model with an endogenous friction. The model that is derived also has implications for hypotheses on secular stagnation, as the relationship between the credit constraint and the steady state rate of interest is shown to be non-linear. The link between the model developed in this paper and this literature will also be explored. The paper is structured as follows; section 1.1 reviews the literature relevant to the development of the model. Section 2 describes the basic perpetual youth model with heterogeneous agents. Section 3 details the borrowing constraint in this model. Section 4 analyses the equilibrium and steady state of the model. In Section 5, the interaction of the financial friction and the interest rate is examined. Finally, Section 6 summarises the results, and Section 7 concludes.

### 1.1 Existing Literature

This potential link between the interest rate and financial frictions has been explored in some papers. In Guerrieri and Lorenzoni (2011), a Bewly-Huggett-Ayagari model is used which features an exogenous borrowing constraint and a fixed quantity of bonds in order to examine the economic response to an unexpected tightening of the credit constraint and the way the economy adjusts from an easy credit regime to a situation of tight credit. They find that a tightening of the constraint leads the economy to adjust to a lower steady state interest rate. This occurs as a tightening of the constraint increases deleveraging and increases savings, and thus the supply of lending in the economy increases. Equilibrium in the asset market means the interest rate falls (Guerrieri and Lorenzoni 2011). Guerrieri and Lorenzoni (2011) argue that the interest rate dynamics modelled in the paper are linked to the idea of the liquidity trap. They define the liquidity trap as, "a situation where the economy is in recession and the nominal interest rate is zero" (Guerrieri and Lorenzoni 2011). Guerrieri and Lorenzoni (2011) state that their model demonstrates that shocks to agents borrowing capacity are precisely the type of shocks that push down the economy's natural rate of interest, and thus trigger a liquidity trap situation. This provides an alternative to the ad-hoc changes to intertemporal preferences commonly used in the literature to push the economy into a liquidity trap (Guerrieri and Lorenzoni 2011). They further argue that, historically, liquidity trap episodes have typically followed disruptions in the banking system, and that their model establishes a natural connection between credit market shocks and the emergence of a liquidity trap (Guerrieri and Lorenzoni 2011). In relation to liquidity traps and the 2008 crisis, Hall (2013) notes, that the crisis saw a sharp decline in the safe interest rate. Eggertsson and Mehrotra (2014) also examine the link between financial frictions and the interest rate using an endowment economy three stage overlapping generations model with exogenous collateral constraints. This model attempts to explicitly construct a situation in which unemployment is high due to a permanent drop in the natural rate of interest. Eggertsson and Mehrotra (2014) referred to this as a secular stagnation. As per the collateral constraints literature, Eggertsson and Mehrotra (2014) state that much of the analysis in the literature on the financial crisis is based on models with a representative agent framework and, therefore, a fixed steady state interest rate. This creates a problem in these models, as changing the steady state interest rate to a new lower level can only be done by permanently changing the rate of time preference, with the resultant effect that the maximization problem of the household is no longer well defined (Eggertsson and Mehrotra 2014). It is found that by using an alternative to the representative agent structure, the natural rate of interest can become negative in response to a shock to the debt limit of the household sector, and this gives rise to secular stagnation (Eggertsson and Mehrotra 2014). In terms of the methodology employed, in the Guerrieri and Lorenzoni (2011) paper, the model is solved by numerical simulation and the credit constraint is tightened through calibration. Eggertsson and Mehrotra (2014) present a three stage model with an exogenous credit constraint. In the current paper, a model is constructed that attempts to capture the result of these other models in an analytical way using a benchmark model with an endogenous constraint.

The model will need to contain two elements; a framework that permits a varying steady state interest rate and a financial frictions element. With respect to the financial frictions element, this paper utilises the collateral constraints approach. The collateral constraints literature has been used in a number of macroeconomic models to analyse a variety of questions on the role of financial frictions. Kiyotaki and Moore (1997) construct the benchmark collateral constraints model. Iacoviello (2005) translates a simplified version of the Kiyotaki-Moore model into a dynamic stochastic general equilibrium model (DSGE). In both Kiyotaki and Moore (1997) and Iacoviello (2005), the models are used to analyse the financial friction mechanism and the interaction of collateral effects and shocks on the models variables. Iacoviello and Neri (2010) use an extended version of Iacoviello (2010) to look at the volatility of house prices and housing investment. Monacelli (2009) uses a model with collateral constraints and durable and non-durable goods, and finds that collateral constraints help account for the co-movement of these two goods in response to a monetary policy shock. Gerali et al (2010) construct a DSGE model with an imperfectly competitive banking sector and collateral constraint type financial frictions. The model looks at the impact of financial friction on the propagation of shocks on macroeconomic variables. Brzoza-Brzezina and Makarski (2011) utilise a collateral constrains model featuring a banking sector in a small open economy setting to analyse the collateral effects on the amplification of monetary policy shocks, loan to value ratio shocks and shocks to the interest rate margin on the standard variables, amongst others. Campbell and Hercowitz (2006) construct an infinite horizon borrower-saver model with durable goods as collateral to analyse household borrowing, and, in particular, the potential impact of changes in equity requirements on the volatility of output, hours worked, debt and durable purchases. In all of these applications of the collateral constraints literature, the impact of the collateral constraint on a number of key variables in varying modelling specifications is explored. However, due to the modelling choices in these papers; specifically the infinite horizon representative household, the potential impact of a change in the financial friction on the steady state interest rate cannot be analysed.

With respect to the second element required in the model; the variation in the steady state rate of interest, a perpetual youth model is utilised. This model provides a straightforward synthesis of representative-agent and overlapping generations models, thus allowing features of both models to be employed in the analysis (Obstfeld and Roghoff 1996). These models have long been used in macroeconomic analysis, but this has been motivated by other questions unrelated to financial frictions. As stated in Ascari and Rankin (2007), the perpetual youth framework, like other overlapping generations models, implies that "Richardian Equivalence" does not hold, and thus it has been used extensively in analysing government debt and deficits, and growth and capital accumulation in the long run, such as in Blanchard (1985). Recently, this framework has become much more popular in short run models, as it allows a departure from the representative agent framework (Ascari and Rankin 2007). More recently, it has also been widely used to study interactions of fiscal and monetary policies, such as in Devereaux (2011), Annicchiarico et al (2008), Leith and Wren Lewis (2000) and the coordination of monetary and fiscal policy in Chadha and Nolan (2003). Ascari and Rankin (2013) use this model to show the sensitivity of the Ricardian equivalence result to the monetary policy rule.

Another advantage of the perpetual youth structure and a reason for its recent popularisation in short run macroeconomic models, is that in open economy models with a representative agent the level of net foreign assets is not restricted to a long run equilibrium level (Harrison et al 2005). The perpetual youth structure avoids this, as it ensures consumption will be stationary and thus net foreign assets will also be stationary (Harrison et al 2005). This feature has been used widely, especially in large models of particular economies that have been developed predominantly by Central Banks [3]. In relation to the topic of this paper, some of the aforementioned papers, such as Kumhoff et al (2010) and Almeida et al (2013), do introduce a concept of credit constraints into their models. This is done through having a subset of households behave as "Rule of Thumb" consumers (Gali et al 2004). As discussed in Beaton (2009), Cambell and Mankiw (1990) justify the use of this type of consumer by appealing to credit market imperfections and thus they can be thought of as credit constrained agents. For the purposes of this paper, however, these types of agents do not engage in borrowing and lending and don't possess an endogenous borrowing constraint that can be relaxed. In the model of Kumhoff et al (2010) and Almeida et al (2013), they are included mainly to enhance the non-Ricardian features of the models.

[3]Examples include; the LSM model for Luxembourg, Deák et al (2011), The Bank of England Quarterly Model (Harrison et al 2005), the PESSOA model of Portugal in Almeida et al (2013), the JEM model of the Japanese economy in Fujiwara et al (2005), Elbourne et al (2009) for the Dutch economy, the Euro area model of Smets and Woulters (2002) and the GIMF model of the global economy Kumhoff et al (2010).

There is a limited literature which uses overlapping generations models that do contain specific financial frictions. In the Kiyotaki and Moore (1997) paper, an overlapping generations version of the basic financial frictions model is outlined in an Appendix. This model contains two productive sectors differing in their production technologies. This basic model has been applied to study international asset market dynamics in Kasa (1998). In our model, however, the approach of Iacoviello (2005) is adopted. Iacoviello (2005) translates a simplified version of the Kiyotaki-Moore model into a standard macroeconomic model with one production sector and one household sector. A version of this approach is presented in Andrés et al (2013) who adopts the heterogeneous producers overlapping generations model of Kiyotaki-Moore to a perpetual youth setting with households and entrepreneurs. This paper looks at the effect of the presence of collateral constraints and banking competition on the trade-off between stabilization goals. Its structure does not, however, permit analysis of the financial friction and interest rate link as, given the structure used, the steady state interest rate is fixed. Some papers in the perpetual youth literature do present models that explore factors changing the interest rate aside from the standard increases in government debt. Vayanos and Vila (1999) find that when the transaction cost on financial assets falls, the interest rate for the illiquid asset rises in the perpetual youth model. These papers do not, however, look specifically at financial frictions.

In terms of the zero lower bound literature, large number of recent papers have been motivated by examining the size of fiscal multipliers when interest rates are very low[4]. Devereaux (2011) presents the interest rate and multiplier interaction in a perpetual youth setting, and in Christiano et al (2011) this interaction is presented in an infinite-horizon setting. In both papers, as in other papers in this literature such as Eggertsson and Woodford (2003), the concept used to lower the steady state of interest is a temporary, unanticipated rise in agents discount factor (Christiano et al 2011). As already stated; Eggertsson and Mehrotra (2014) state that this mechanism is problematic when modelling a prolonged slump in the interest rate.

In order to analyse the link between financial frictions and the rate of interest, this paper

[4] Refer to Christiano et al (2011) and Devereaux (2011) for an overview.

constructs a discreet time, perpetual youth model with durable goods and an endogenious financial friction in the form of a collateral constraint. We translate the friction into the model in a simplified way, as in Iacoviello (2005). The model developed in this paper has a link to the financial frictions models of Hall (2012) and Chari Kehoe and McGrattan (2007), which will be detailed in Section 5.

## 2 The Basic Model

The model constructed in this paper draws on a number of papers from different strands of literature. The financial frictions mechanism in the model is based on Iacoviello (2005). As stated previously, the Iacoviello (2005) model uses a simplified version of the Kiyotaki-Moore (1997) model, and it is this version of the Iacoviello model that is used for the friction in this paper. The model is simplified on the financial frictions side to the extent that, as in Iacoviello (2005), the borrowing constraint can be proved to bind in steady state only. However, without the additional assumptions of Kiyotaki-Moore, it cannot be proved to bind outside of the steady state. An advantage of taking the approach of Iacoviello as opposed to using two productive sectors (as in Kiyotaki and Moore (1997) and Kasa (1998)) is that it does not necessitate the use of the "net wealth net worth relation" employed in these papers, which requires restrictive assumptions on functional form.

In the model in this paper there is a population of consumers whose size is normalized to 1. This population will consist of two types of agents; a fraction $(1-\omega)$ of perpetual youth households and a fraction $(\omega)$ of entrepreneurs. The perpetual youth households will evolve according to a Blanchard-Yaari overlapping generations scheme. Agents are subject to stochastic finite lifetimes, and face a probability of dying $(1-\gamma), \gamma \in (0,1]$ at the end of each period. To make aggregation possible it is assumed that the probability of death is independent of age (Frenkel and Razin 1992). As in Almieda et al (2013), $(1-\gamma)$ can be interpreted as the relevant economic horizon behind agents decisions where the future is a period of lower economic relevance. The probability of death is the only source of uncertainty in the model. In each period a new cohort of agents $v$ will be born. The cohort is assumed to be large enough so that the probability of death $(1-\gamma)$ is also the rate at which the cohort size decreases through time. With this assumption each individual is still uncertain about their time of death, but the size of each cohort will decline deterministically through time (Almieda et al 2013). The population of households is held constant, implying that the number of agents born in each period is the same as the number of agents that die (Almieda et al 2013).

The financial frictions structure in the model differs from that used in Andrés et al (2013); where the credit flow relationship is between infinitely lived households, which are the patient agent, and perpetual youth entrepreneurs; the impatient agent. The patient agent in Andrés et al (2013), as in Kiyotaki and Moore (1997), fixes the interest rate in the model. This occurs because the patient agent in these models is an infinitely lived household. Therefore, the standard result; that the steady state interest rate is fixed by the rate of time preference of the households, holds. Within a borrower-lender financial frictions model, this situation can be thought of in terms of supply and demand in a credit market. The suppliers of credit, which in collateral constraints financial frictions models are typically households, have a perfectly elastic supply curve, and therefore, any changes in credit demand do not change the market clearing interest rate. In this paper, the households have a perpetual youth structure and the entrepreneurs are infinitely lived, and thus the steady state rate of interest in the model will not be determined by the rate of time preference alone. This facilitates an analysis of the potential effect of a change in the financial friction on the steady state interest rate as per Eggertsson and Mehrotra (2014). The change in structure allows the steady state rate to vary. In the remainder of this section the household sector is described, then the aggregation procedure and the aggregate euler equation are shown. Finally the entrepreneurs sector is presented.

## 2.1 Households

In the perpetual youth household sector a new cohort of agents are born in each period of time. The timing convention that is adopted for the model is that of Frenkel and Razin (1986). Using this timing convention, the expected lifetime utility function of an agent alive at period $t$ who was born in period $s$ is:

$$E_t U_{s,t} = E_t \sum_{v=t}^{\infty} (\beta)^{v-t} U_{s,v} = \sum_{v=t}^{\infty} (\gamma\beta)^{v-t} U_{s,v} \tag{1}$$

In the equation above $E_t$ is the expectation operator, $\beta \in (0,1)$ is the discount factor and $U_{s,v}$ is the flow utility in period $v$. Utility is in the form of expected utility to take account of the probability of survival from one period to the next. In this modelling structure the probability of surviving any $n$ periods will be a joint probability (Wickens 2012). The expression adopted for utility on the left hand side of equation one follows Frenkel and Razin (1992), and is termed the "certainty equivalent utility function" as it takes into account the probability of survival. It can be seen that the infinite horizon utility function is a special case of the perpetual youth model when ($\gamma = 1$). Therefore, the presence of the survival probability, with $\gamma < 1$, has the effect of reducing the utility of future consumption, and thus it makes individuals more impatient relative to the infinite horizon model (Frenkel and Razin 1992). This tilting of consumption towards the present relative to the future can be seen as the probability of death reduces the effective discount factor relative to an infinite horizon model (Frenkel and Razin 1992). Households will optimize utility with respect to consumption of durable and non-durable goods, and leisure. The form of instantaneous utility $U_{s,v}$ is assumed to be:

$$U_{s,v} = ln(c_{s,v}^{1-\theta} X_{s,v}^{\theta} (1 - H_{s,v})^{k}) \tag{2}$$

In the equation above $\kappa$, is a taste parameter on leisure and $H_{s,v}$ is hours worked as a fraction of the total time endowment in each period. As noted in Ascari and Rankin (2007), to maintain the wealth-independence of labour supply the hours worked must enter the utility function in this way. The function used for utility is a constant relative risk aversion utility function (CRRA) with the coefficient of relative risk aversion equal to one. The CRRA utility function is homogeneous, and any function that is homogeneous is also homothetic (Varian 1992). That the utility function is homothetic is a property that is required for aggregation across cohorts. In line with Deák et al (2011) and Ascari and Rankin (2007), $\theta$, is a taste parameter related to the expenditure share of durable and non-durable consumption [5]. Using this formulation the parameter can change the marginal rate of substitution between durable and non-durable consumption, but not the elasticity of substitution. $c_{s,v}$ is consumption of cohort $s$ in period $v$ in real terms. $X_{s,v}$ is the stock of the durable good held by cohort $s$ in period $v$.

Households maximization is subject to the following budget constraint which is expressed in nominal terms:

$$P_v c_{s,v} + B_{s,v} + Q_v X_{s,v} = W_v H_{s,v} + \frac{1}{\gamma}\left((1 + i_{v-1})B_{s,v-1} - Q_v X_{s,v-1}\right) \tag{3}$$

In this equation, $P_v$ is the price level in period $v$, $B_{s,v}$ is the stock of households financial assets, $W_v$ is the money wage in period $v$, $Q_v$ is the price of the durable good and $(i_{v-1})$ is the nominal interest rate on loans between periods $(v-1)$ and $v$.

It can be seen from the budget constraint that, in addition to the nominal interest rate on bonds, each individual agent also receives an additional payment equal to a fraction of their total asset level. This occurs because in the Blanchard-Yaari model the existence of a non-profit life insurance company is assumed (Frenkel and Razin 1992). In a situation in which individuals maximize expected lifetime utility with no bequest motive, and in which there is uncertainty about the time of death, and in addition to which one assumes there is a large enough number of agents born in each new cohort such that the frequency of those who survive is equal to the survival probability $\gamma < 1$, there is no aggregate uncertainty about the time of death, and thus insurance is possible (Blanchard and Fischer 1989). This paper follows the approach to insurance of Hu (1994). The Hu (1994) paper describes the case of both assets and durable goods. In the perpetual youth situation, the insurance contract stipulates that the individuals wealth; which includes both financial assets and the durable stock, are transferred to the life insurance company. The insurance company agrees in return to pay a fraction of each agents wealth to them, in the form of a premium

[5] The durable good in this model is a dual purpose good that households get utility from and that firms use as a factor of production. It is a perfect substitute on the sellers side and heterogeneous on the buyers side. It is thought of as land in Kiyotaki and Moore (1997) and as real estate in Iacoviello (2005)

payment, whilst the agent is still alive. This allows households to increase utility to a level higher than if there were no insurance company. Again, the assumption that the conditional expectation of future life is independent of wealth and the large population turnover ensures that income receipts and payouts for the insurance company will match in every period (Acemoglu 2009). Free entry of insurance companies is assumed, and therefore, with zero profits the fraction of wealth which forms the premium payment is actuarially fair and is equal to $1/\gamma$ (Almieda et al 2013).

In this paper, credit allocation is of interest and this institutional arrangement for insurance in the form of a premium payment does not impact the credit market, as all agents will borrow and lend at the same rate. It should be noted that the alternative institutional arrangement of insurance companies; that of modelling their presence in the form of a direct surcharge on loans as outlined in Frenkel and Razin (1992), is not suitable in the presence of durable assets.

As this is a heterogeneous agent model with production controlled by entrepreneurs, and is also without nominal rigidities, households do not gain profit receipts from firms, and thus these transfers do not enter the household budget constraint. This has the benefit of avoiding unintended household wealth effects due to some income streams being subject to insurance and others being uninsured, as discussed in Harrison (2005).

Households optimising utility subject to the flow budget constraint with the choice variables as $C_{s,v}$, $H_{s,v}$, $X_{s,v}$ and $B_{s,v}$ yield the following first order conditions:

$$\frac{1}{c_{s,v}} = \gamma\beta \left[ \frac{(1+i_v)P_v}{c_{s,v+1}P_{v+1}\gamma} \right] \tag{4}$$

$$\frac{\kappa}{(1-H_{s,v})} = \frac{W_{s,v}}{C_{s,v}P_v} \tag{5}$$

$$\frac{\theta}{X_{s,v}} = \frac{q_v}{c_{s,v}} - \gamma\beta \left[ \frac{q_{v+1}}{c_{s,v+1}\gamma} \right] \tag{6}$$

In the equation above lower-case letters indicate values in real terms with the price of the non-durable good being the numeraire.

As mentioned previously, an essential requirement of the perpetual youth model is that the agents consumption should constitute a linear function of their lifetime wealth. This facilitates aggregation across cohorts, such that aggregate consumption can be written as a function of aggregate wealth (Ascari and Rankin 2007). This consumption function is the key behavioural equation in perpetual youth models (Almieda et al 2013). To arrive at this expression, individual consumption as a function of wealth at the cohort level is first derived. Firstly, the budget constraint is written in real terms, and is given as;

$$c_{s,v} + b_{s,v} + q_v X_{s,v} = w_v H_{s,v} + \frac{1}{\gamma}\left((1 + r_{v-1})b_{s,v-1} - q_v X_{s,v-1}\right) \tag{7}$$

In the equation above $r_t$ is defined as in Devereaux (2011) as $r_t = \frac{i_t P_t}{P_{t+1}}$ and inflation is defined as $P_{t+1}/P_t$. The real flow budget constraint is first solved forwards, with the no-ponzi game condition imposed, to obtain the present value inter-temporal budget constraint. The inter-temporal budget constraint is as follows:

$$WT_{s,t} = \sum_{v=t}^{\infty} \gamma^{v-t}\frac{\alpha_v}{\alpha_t}\left(c_{s,v} + f_v X_{s,v}\right) = \sum_{v=t}^{\infty} \gamma^{v-t}\frac{\alpha_v}{\alpha_t}\left(w_v H_{s,v}\right) + \frac{1 + r_{t-1}}{\gamma} b_{s,t-1} + \frac{q_t}{\gamma} X_{s,t-1} \tag{8}$$

In the above $WT_{s,t}$ is the wealth of an agent in cohort $s$ at time $t$. $WT_{s,t}$ is the present value of all future consumption, or equally the present value of all future income from period $t$ onwards plus initial wealth. As in Deák et al (2011), the variable $f_v$ is defined as the user cost of the durable good;

$$f_v = \left[q_v - \frac{q_{v+1}}{1 + r_v}\right] \tag{9}$$

$\frac{\alpha_v}{\alpha_t}$ is the present value factor (Frenkel and Razin 1992). It is the one period rate of interest compounded from period zero up to period $t - 1$, such that:

$$\alpha_3 = (1 + r_0)^{-1}(1 + r_1)^{-1}(1 + r_2)^{-1} \tag{10}$$

In reaching the expression for the present value inter-temporal budget constraint, the no-ponzi game condition was imposed. Without this condition, the expression would contain a term for present value of future debt. If this was positive, the agent would be consuming in excess of the present value of income plus initial wealth by an amount that never converges to zero (Obstfeld and Roghoff 1996). The no ponzi-game condition thus requires borrowing to be negative in the limit. Equally, the household does not want the net present value of consumption to be less than income plus initial wealth, as this would not be optimizing lifetime utility (Obstfeld and Rhoghoff 1996). Therefore, in the inter-temporal budget constraint the limit of borrowing and saving is set equal to zero.

The certainty equivalent utility function is now optimized with respect to the present value inter-temporal budget constraint. The first order conditions for non-durable and durable consumption are:

$$c_{s,v} = \lambda^{-1}(1-\theta)\frac{\alpha_t}{\alpha_v}\beta^{v-t} \tag{11}$$

$$\frac{\theta(\gamma\beta)^{v-t}}{X_{s,v}} + \frac{\lambda}{\gamma}q_{v+1} - \lambda\gamma^{v-t}\frac{\alpha_v}{\alpha_t}f_v = 0 \tag{12}$$

In the above $\lambda$ is the multiplier on the present value inter-temporal budget constraint. The result is the same as in a standard infinite horizon model in that agents cannot increase lifetime utility by shifting consumption across time. As in Fujiwara et al (2005), combining the first order conditions 11 and 12 yields an expression for durable goods stock in terms of non-durable goods:

$$X_{s,v} = \frac{\theta}{1-\theta}c_{s,v}f_v^{-1} \tag{13}$$

Using these conditions, and combining the expression for wealth and the first order condition for durable and non-durable consumption at period ($v = t$) results in the expressions:

$$c_{s,t} = (1-\beta\gamma)WT_{s,t} \tag{14}$$

$$c_{s,t} = (1-\beta\gamma)\left(\frac{1+r_{t-1}}{\gamma}b_{s,t-1} + \frac{q_t}{\gamma}X_{s,t-1} + \sum_{v=t}^{\infty}\gamma^{v-t}\frac{\alpha_v}{\alpha_t}\left(w_v H_{s,v}\right)\right) \tag{15}$$

The above represents the consumption of an individual as a linear function of wealth. The term

$(1 - \beta\gamma)$ is the marginal propensity to consume out of wealth. All individuals of the same cohort will have the same consumption as they will consume the same proportion of their total wealth in each period. This allows for aggregation, as the saving propensity is independent of age, and thus there is no within age variation for individuals born at the same time. Consumption thus depends on total wealth and the propensity to consume. As discussed in Frenkel and Razin (1992), given the assumptions made about the functional form of utility, which yield a log utility function, the consumption propensity is a constant over time. In line with Frenkel and Razin (1992), with respect to the second set of brackets on the right hand side of expression 15, the first two terms constitute non-human wealth and the sum of wages is termed "agents human wealth".

## 2.2 Aggregation

Up to the present point in this paper, the expressions derived have been at the cohort level. Aggregate consumption will now be derived. This is defined as the sum of consumption for all individual cohorts alive at date $t$. Given that the probability of survival is $\gamma$ and the proportion of the total population of consumers that are households is $(1 - \omega)$, aggregate per capita consumption of the household sector is given by:

$$c_t = (1 - \omega)(1 - \gamma) \sum_{s=-\infty}^{t} \gamma^{t-s} c_{s,t} \tag{16}$$

This is interpreted as a per capita expression due to the timing convention used. In the remainder of this paper, aggregate and aggregate per capita are used interchangeably. The aggregation formulation follows that of Frenkel and Razin (1986), such that $s$ is the period in which a cohort was born (and the cohort can be born at any time to minus infinity). An alternative specification used in Harrison et al (2005) is to define cohorts by age. The sum is then from $s$ (defined as age) to positive infinity. In both cases, time extends back to minus infinity, therefore, in the latter case, agents age can extend to infinity. The aggregate consumption is written as:

$$c_t = (1 - \beta\gamma) \left( (1 + r_{t-1}) b_{t-1} + q_t X_{t-1} + \sum_{v=t}^{\infty} \gamma^{v-t} \frac{\alpha_v}{\alpha_t} \left( w_v H_v \right) \right) \tag{17}$$

Equation 17 above is the optimal aggregate consumption rule of the overlapping generations households. In perpetual youth models it expresses current aggregate consumption of households as a function of their real aggregate human and non-human wealth and their propensity to consume (Kumhoff and Laxton 2010).

### 2.3 The Aggregate Euler Equation

In deriving the aggregate euler equation, the aggregate consumption function is used along with the dynamic equation for the evolution of human capital:

$$HC_t = \frac{1}{\gamma} \left(1 + r_{t-1}\right) \left[HC_{t-1} - w_{t-1} H_{t-1}\right] \tag{18}$$

In the equation above $HC_t$ is the human component of wealth at time $t$. The aggregate budget constraint is also required for the derivation of the aggregate euler equation, and, as in Piergallini (2004), it is given by the expression:

$$c_t + b_t + q_t X_t = w_t H_t + \left(1 + r_{t-1}\right) b_{t-1} + q_t X_{t-1} \tag{19}$$

As in Piergallini (2004) there is no stream of payments from the insurance company in this budget constraint as these payments are between individuals and so on aggregate payments will cancel with losses. Combining the expression for wealth, the aggregate budget constraint and then using the equation for the evolution of human capital yield an expression for the aggregate euler equation. After some manipulation the aggregate euler equation can be written as:

$$c_t = \beta(1 + r_{t-1}) c_{t-1} - \left( \frac{(1-\theta)(1-\beta\gamma)(1-\gamma)}{\gamma} \right) \left( \tilde{b}_{t-1} + q_t X_{t-1} \right) \tag{20}$$

As in Ascari and Rankin (2007), the level of debt is set in real terms inclusive of the interest rate:

$$\tilde{b}_{t-1} = (1 + r_{t-1})b_{t-1} \tag{21}$$

This euler equation will collapse into a standard euler equation if ($\gamma = 1$), and thus the probability of death is zero and agents have an infinite horizon. As noted in Heijdra and Ligthart (2000) and Ascari and Rankin (2007), the aggregate euler equation contains additional terms which are not present in the individual euler equation. The euler equation now contains a term that is negatively related to financial wealth and the stock of durable goods. This is due to the generational turnover effect. As can be seen from the aggregate euler equation, the interest rate will be greater than the discount factor, and therefore, it can be seen from the individual euler equations that each cohort will have an increasing path of lifetime consumption. Agents in the prepetual youth model are thus always saving and accumulating assets (Leith et al 2011). This occurs as agents use financial assets to fill the gap between their income profile and their desired level of consumption (Ascari and Rankin 2007). As outlined in Heijdra and Ligthart (2000), this means that the share of human wealth in total wealth of a cohort is decreasing in the age of the cohort, which is to say very old agents will have a large stock of financial assets. Given the conditions of cohorts consuming out of their wealth, new agents having no wealth (apart from their human wealth) and given the death of a random sample of agents in each period, aggregate consumption between dates $t$ and $t+1$ will be reduced. This is termed the "generational turnover effect" (Heijdra and Ligthart 2000). Therefore, aggregate consumption will have a negative term, and will be lower than the infinite horizon case in proportion to the size of the stock of wealth, the propensity to consume and the probability of death.

As can be seen from the aggregate euler equation, the generational turnover effect also effects the interest rate. In the individual euler equation the interest rate is at a level that makes individuals unable to increase utility by shifting consumption across time, given preferences. The death of a random sample of agents that have higher consumption than the newly born agents will increase this interest rate. The aggregate interest, in line with the individual case, must be so that agents cant shift consumption across time and increase utility, and thus with the probability of death and the higher consumption of older agents, is higher than in the infinite horizon.

### 2.4 Entrepreneurs

The fraction $(1-\omega)$ of entrepreneurs in the population use the durable good and labour to produce a composite good $y$ from a Cobb-Douglas, constant returns to scale production function:

$$y_t = A(X_{t-1}^{'})^{\nu}(H_t^{'})^{1-\nu} \tag{22}$$

In the above $A$ is the constant technology parameter, $X^{'}$ is the entrepreneurs holding of the durable good and $H^{'}$ is their labour demand. The parameter $\nu \in (0,1)$ is the output elasticity of the durable good. As constant returns to scale have been assumed, $(1-\nu)$ is the output elasticity of labour. There is a one period lag on the production of output from the durable good.

Entrepreneurs seek to optimize lifetime utility and their utility is a function of their level of consumption $c'$. The functional form of deterministic utility at time $t$ is given by:

$$U_t' = \sum_{t=0}^{\infty} \varphi^t \ln c_t' \tag{23}$$

The parameter $\varphi \in (0,1)$ is the entrepreneurs discount factor and $c'$ is their level of consumption. As in the collateral constraints literature of Kiyotaki and Moore (1997) and Iacoviello (2005), an assumption will be required for the value of $\varphi$ so that financial frictions play a role in the model. This assumption will be outlined in Section three. The entrepreneurs flow of funds constraint in real terms is given by:

$$y_t + b_t' = c_t' + q_t \Delta X_t' + (1+r_{t-1})b_{t-1}^{'} + w_t H_t^{'} \tag{24}$$

$b_t'$ is the entrepreneur's net stock of financial liabilities. The constraint states that income from production plus borrowing must equal expenditure plus accumulated debt. Accumulated debt is divided by inflation, $P_t/P_{t-1}$, as it is assumed that debt contracts are set in nominal terms, and thus changes in the price level will affect realized real interest rates, as per Iacoviello (2005).

In the model in this paper, entrepreneurs are also assumed to face a borrowing constraint that is tied to the value of their collateral; in this case the value of their durable goods holdings for the next period. This is the same borrowing constraint concept as detailed in Kiyotaki and Moore (1997) and Iacoviello (2005). As in Kiyotaki and Moore (1997), it is assumed that there is a threat of debt repudiation, and thus there is the possibility of mutually beneficial debt write-down negotiations between the agents. Creditors in the model know that this threat exists, and so wont allow the size of borrowing to exceed the collateral value of the durable good, as per Kiyotaki and Moore (1997). Lenders will only lend up to the value of the reprocessable asset, thus imposing a limit on borrowing. The borrowing constraint is given as:

$$b'_t \leqslant \frac{m(q_{t+1}X'_t)}{R_t} \tag{25}$$

It is assumed, as in Iacoviello (2005), that there is a proportional transaction cost $(1-m)$ on lenders repossessing borrowers assets. The parameter $m \in (0,1)$ enters the borrowing constraint, as this scales the value of the durable goods stock that can be used as collateral. An increase in $m$ is a reduction in the transaction cost. This parameter change, that can be interpreted as a loosening of the borrowing constraint, will be used later in the analysis. The value of the durable good holding is divided by the interest rate $R_t = (1 + r_t)$ in the borrowing constraint to take account of interest payments. The total amount that can be borrowed is thus the value of the durable good holding less the interest payment and the repossession transaction cost.

Entrepreneurs in the model optimize utility with respect to the budget and collateral constraints. The first order conditions for the entrepreneur are as follows:

$$\frac{1}{c'_t} = \varphi\left(\frac{R_t}{c'_{t+1}}\right) + \Lambda_t R_t \tag{26}$$

$$\frac{q_t}{c_t'} = \left(\frac{\varphi}{c_{t+1}'}\left(\nu\frac{y_{t+1}}{X_t'} + q_{t+1}\right) + \Lambda_t m q_{t+1}\right) \tag{27}$$

$$w_t^{'} = (1-\nu)y_t/H_t^{'} \tag{28}$$

The inequality constraint that arises from the borrowing constraint in optimization gives rise to the standard complementary slackness condition:

$$\Lambda_t[m(q_{t+1}X_t') - b_t'R_t] = 0 \tag{29}$$

$\Lambda_t$ is the multiplier on the borrowing constraint. This condition states that either the constraint binds with equality, or it does not, and drops out of the optimization condition. The conditions under which the borrowing constraint binds will be outlined in Section 3. For now, a binding credit constraint is assumed. For the entrepreneur this implies that:

$$u^{'}(c_t') > \varphi\left(R_t u^{'}(c_{t+1}')\right) \tag{30}$$

The marginal utility of consumption in period $t$ is greater than the marginal utility of consumption in $t+1$, and, as a result, as in the model of Monacelli (2009), the financial friction means resources cannot be shifted optimally across time.

The optimization condition for the durable good is also changed with the imposition of the financial friction. In equation 27, the first two terms on the right hand side are standard for a durable good, in that the utility from the durable good; which the entrepreneur uses for production, is equal to the consumption utility value of the discounted marginal product in terms of the composite final good, plus the consumption utility value of the resale value in the next period. These first two terms and the left hand side yield the standard result of equalised marginal utility between the two variables at the optimum level. The difference caused by the presence of the financial friction is that the durable good has a collateral value, as shown in the borrowing constraint. There is now an additional benefit to acquiring the durable good; in that it allows a relaxation of the binding borrowing constraint (Andrés et al 2013).

This collateral effect gain for the durable good will be positive with a binding constraint. It can be seen that the tighter the constraint, the greater the marginal utility of the durable good. This occurs because a tighter constraint increases the gain in total utility from expanded borrowing and thus reallocating consumption across time. The last optimization condition for the entrepreneur, equation 28, yields the standard labour demand condition.

## 3 The Borrowing Constraint

As discussed in Leith et al (2011), in a non-Richardian economy, such as in an overlapping generations model, the steady state rate of interest will be above the rate of time preference. This gap between the steady state real rate of interest and the rate of time preference is positively related to the ratio of financial assets to consumption (Ascari and Rankin 2013). In a typical perpetual youth model, this feature gives the model the property of a failure of Richardian equivalence, as an increase in government debt increases the household stock of financial assets. Thus there is a consumption response from the households and so the increase in debt is non-neutral (Ascari and Rankin 2013). This general feature of overlapping generations models in which the steady state real interest rate is not fixed, as in infinite horizon models, but is an increasing function of household asset holdings, has implications for the lender borrower framework as presented in this paper. As discussed, in the models of Kiyotaki and Moore (1997) and Iacoviello (2005), the rate of time preference of the most patient agent in the model fixes the level of the real interest rate. As a result of the impatient agent/entrepreneur having a higher discount factor, the real interest rate required for this agent to be indifferent between consumption now and consumption in the future is higher than that of the patient agent, and thus that which prevails in the model. This difference creates the motivation to borrow in these models. The other assumption regarding repudiation provides a basis for a borrowing constraint.

In order for the complementary slackness condition for the borrowing constraint to bind, it must be the case that $\Lambda_t > 0$. The conditions for this to hold in the steady state of the model will be analysed below. The expression for $\Lambda$ in steady state is derived from the entrepreneurs Euler equation:

$$\Lambda = \frac{1}{c'}\frac{1}{R} - \frac{1}{c'}\varphi \tag{31}$$

In a typical infinite horizon collateral constraints model this expression would depend on the relative rates of time preference of the agents. However, in the overlapping generations framework $R$ is a function of variables and is not fixed by the rate of time preference alone. What is required for a binding constraint is an implied $R$ for the impatient agent that is higher than that of the patient agent. This should be possible as given $\varphi \in (0, 1)$, the implied $R$ for the entrepreneur can be calibrated to be arbitrary large. However, in order to provide an analytical solution for the conditions required to guarantee that the borrowing constraint binds in the steady state of the model an expression for an upper bound value for $R$ is employed. In order to obtain this upper bound, the discreet time version of a result derived in continuous time in Rankin (2012) is utilised. The result in Rankin (2012) is stated as a Lemma. It states that in a closed economy version of the perpetual youth model, a steady state equilibrium cannot exist unless the interest rate is less than a critical value; a ceiling value. This ceiling is determined by the pure rate of time preference and the probability of death of the household sector (Rankin 2012). This result is derived under the condition that aggregate consumption in steady state is finite; that aggregate consumption cannot be increasing in time Rankin (2012). As outlined in Rankin (2012), this is because a high interest rate implies consumption increases rapidly with age. In aggregate, this increase would be offset by the death rate and aggregate consumption would be finite. However, if $R$ is too large, aggregate consumption would explode with time (Rankin 2012).

The result is derived by solving the cohort level household euler equation. As noted in Mallik (1998), the closed form solution to a first order difference equation with variable coefficients is known. Using this solution, the household euler equation at $v = t$ is given as:

$$C_{s,t} = \beta^{t-s} C_{s,s} \prod_{k=s}^{t-1} R_k \tag{32}$$

This is consumption of an individual born in period $s$ at time $t$. Aggregate consumption is thus:

$$c_t = (1-\omega)(1-\gamma)C_{s,s}\sum_{s=-\infty}^{t}(\gamma\beta)^{t-s}\prod_{k=s}^{t-1}R_k \tag{33}$$

As in Rankin (2012), it is assumed that $C_{s,s}$ is independent of $s$. It can be seen that the first three terms on the right hand side of this formulation of aggregate consumption are finite, and thus whether aggregate consumption is finite in $t$, or not, will depend on the second two terms. Aggregate consumption will be finite in steady state if:

$$R < \frac{1}{\gamma\beta} \tag{34}$$

Therefore, under the condition of finite aggregate consumption in the steady state $R$ has a ceiling. In order for the borrowing constraint to be binding in steady state is sufficient to assume that:

$$\varphi \leqslant \gamma\beta \tag{35}$$

This expression contains the condition, as noted in Kiyotaki and Moore (1997), wherein, unlike in the infinite horizon case, in an overlapping generations model, it is not necessary to assume a difference in the rate of time preference between agents in order for the borrowing constraint to bind in steady state. The entrepreneurs will then always want to borrow more at the prevailing interest rate in the model. This is a condition used in the financial frictions models of Kiyotaki and Moore (1997) and Iacoviello (2005). Taking this assumption; which implies an imposition on the rate of time preference of the entrepreneur, the borrowing constraint will bind with equality in the steady state. The subsequent section will analyse the steady state of the model, as it is the long run effects of a permanent change in the model that are of interest.

The restriction on the value of $R$ can also be seen by rewriting the aggregate euler equation as:

$$c_t = (1-\theta)(1-\beta\gamma)(1-\gamma)H + (1+r)\beta\gamma c_{t-1} \tag{36}$$

Using this equation for expositional purposes as in Frenkel and Razin (1992), it is assumed that the path of human wealth is stationary, $H$, and the interest rate is at its steady state value. The human wealth term is positive, and therefore, this dynamic consumption equation has a positive intercept. The slope depends on $R\beta\gamma$. This yields the same condition as derived previously, as the consumption dynamics equation will only meet the conditions for the existence of a steady state equilibrium in a one dimensional first order system if $R\beta\gamma < 1$ (Galor 2010). If this condition did not hold, the economy would not converge to a long run steady state, but would instead be unstable and consumption would increase without bound (Frenkel and Razin 1992). The condition used here as derived in Rankin (2012) does not require the assumption of stationary human capital to arrive at the result.

## 4 Equilibrium and Steady State

To determine the general equilibrium of the model market clearing is imposed. In the labour market, the wage rate will clear the market, the supply wage from household optimization will equal the demand wage from entrepreneurs optimization and labour supply will equal labour demand (Wickens 2012).

$$(1-\omega)H_t = \omega H_t^{'} \tag{37}$$

In the goods market the supply of goods is equal to the demand for goods, and equilibrium is given as:

$$y_t = (1-\omega)c_t + \omega c_t^{'} \tag{38}$$

The stock of the durable good is fixed, and its size is normalised to one:

$$\bar{X} = 1 = (1-\omega)X_t + \omega X_t^{'} \tag{39}$$

To achieve equilibrium in the credit market, total borrowing must equal total lending:

$$(1-\omega)b_t = \omega b_t^{'} \tag{40}$$

The steady state occurs when all variables in the model are at their stationary value. It is assumed that in steady state of this model inflation is zero ($\pi = 1$), and there is a constant rate of technological progress, ($A = 1$). The steady state equations in the model are given as [6]:

$$c + b = wH + Rb \tag{41}$$

$$c = \beta Rc - \left(\frac{(1-\theta)(1-\beta\gamma)(1-\gamma)}{\gamma}\right)\left(\tilde{b} + qX\right) \tag{42}$$

$$\frac{\kappa}{1-H} = \frac{w}{c} \tag{43}$$

$$\frac{\theta}{X} = \frac{q}{c} - \beta\left[\frac{q}{c}\right] \tag{44}$$

$$y = A(X^{'})^{\nu}(H^{'})^{1-\nu} \tag{45}$$

$$\tilde{b}' = mqX' \tag{46}$$

$$\frac{1}{c'} = \varphi\left(\frac{R}{c'}\right) + \Lambda R \tag{47}$$

$$\frac{q}{c'} = \left(\frac{\varphi}{c'}\left(\nu\frac{y}{X'} + q\right) + \Lambda mq\right) \tag{48}$$

$$w' = (1-\nu)y/H' \tag{49}$$

[6] The equations are the aggregate household budget constraint, the aggregate euler equation, the first order condition for aggregate household labour supply, the aggregate household first order condition for durable goods holding, the production function, the borrowing constraint, the entrepreneurs consumption euler equation, the entrepreneurs durable goods first order condition and the entrepreneurs demand for labour condition respectively.

## 5 The Borrowing Constraint and the Interest Rate

The change in the interest rate resulting from a change in the tightness of the borrowing constraint is analysed through the steady state of the model. Analysing the steady state of the model allows the long run effect of permanent changes to the model to be seen (Herr and Mauner 2009). Following on from the entrepreneurs steady state first order condition with respect to the durable good, the multiplier on the borrowing constraint can be written as:

$$\Lambda = \frac{1}{c'm} - \frac{\varphi \nu y}{c'mqX'} - \frac{\varphi}{c'm} \tag{50}$$

Given the presence of the assumptions regarding the discount factors, the borrowing constraint binds, $\Lambda > 0$, it can be seen that in terms of a partial derivative the change in $\Lambda$ with respect to $m$ is negative. As $(1 - m)$ is the transaction cost, an increase in $m$ reduces this cost, thus loosening the borrowing constraint, and thus it would be expected that the financial friction in the model would be reduced. Following Pinheriro et al (2013), a change in the parameter $m$ is interpreted as a summary of potential factors constraining borrowing and lending between agents. From the borrowing constraint below it can be seen that borrowing rises with a change in $m$ in proportion to the value of the entrepreneurs holding of the durable good:

$$\tilde{b}' = mqX' \tag{51}$$

From market clearing as the entrepreneurs borrowing increases, the lending of the household sector will also increase. Also, combining the entrepreneurs first order condition for the durable good, with their first order condition for non-durable consumption, it can be shown using the assumptions regarding the maximum value of $R$ and the entrepreneurs discount factor, that the value of the entrepreneurs holding of the durable good is decreasing in $m$. This result is noted in Andrés et al (2013); that in the presence of a collateral constraint entrepreneurs over-invest in the durable goods in order to smooth consumption. When $m$ increases and the constraint is loosened, the durable goods collateral value is reduced, and thus entrepreneurs reduced their holding of the good.

In terms of the context in the broader literature it is worth noting that this result of a reduction in the holding of a production input with a relaxation of the borrowing constraint is in contrast to another strand in the financial frictions literature, as outlined in Hall (2012) and (2013) which follows Chari, Kehoe and McGrattan (2007). In this literature the concept used for the financial friction is one of combining frictions from a number of potential sources and the entering of these frictions into the model as a wedge between the return to businesses from the use of physical capital and the risk free interest rate (Hall 2013). In Hall (2012) and (2013), this concept is modelled as a tax. This is on the same conceptual basis as the investment wedge approach as used in Chari, Kehoe and McGrattan (2007). In Hall (2012) and (2013), the analysis in relation to the financial friction is empirically focussed, and borrowing limits and the investment wedge are assumed and are exogenous. It can, however, still be compared to the model in this paper. If, in the entrepreneurs sector, utility from consumption is assumed to be linear and friction is set to zero, $\Lambda_t = 0$, then the optimization conditions yield:

$$R_t = \frac{\varphi}{q_t}\left(\nu \frac{y_{t+1}}{X'_t} + q_{t+1}\right) \tag{52}$$

This, aside from the absence of depreciation, is the same as one of the key expressions in the model with financial frictions set equal to zero, as presented in Hall (2012). The difference in the reaction of the holding of the productive asset to a relaxation of the borrowing constraint in the Hall (2012) and (2013) papers and the model in this paper, is due to the differing effect of a positive financial friction between the two models in the above expression. In Hall (2012) and (2013), the friction, as defined by an investment wedge, means that the return to capital is higher than the real interest rate. In this paper, as stated, and as in Iacoviello (2005) and Andrés et al (2013), the presence of the friction adds to the value of the productive asset in the form of a collateral value, and thus the return to capital will be lower than the interest rate. Therefore, the holding of this asset will respond in opposite directions to a relaxation of the constraint in each modelling approach. It will be shown that, given the assumptions of the model, this difference is not relevant to the interest rate analysis.

In relation to other approaches to modelling financial frictions, another finding of Chari, Kehoe and McGrattan (2007) is that financial frictions, as modelled by a costly state verification approach in Bernanke, Gertler and Gilchrist (1999) or a credit market frictions model such at Carlstrom and Fuerst (1997), are equivalent to their model; a growth model with an investment wedge. This finding links these approaches to the Hall (2012) and (2013) approach and the collateral constraints framework employed in this paper.

## 5.1 The Interest Rate

Turning to the interest rate, the aggregate euler equation gives an expression for its steady state level.

$$R = \frac{1}{\beta} + \frac{1}{\beta}\left(\frac{(1-\theta)(1-\beta\gamma)(1-\gamma)}{\gamma}\right)\left(\frac{\tilde{b}}{c} + \frac{qx}{c}\right) \tag{53}$$

As already noted in the infinite horizon case, with the probability of survival equal to one, this equation reduces to the standard condition that the steady state interest rate is equal to the reciprocal of the discount factor. From this equation, it can be seen that the steady state interest rate will depend on the real amount of borrowing and the holding of durable goods per unit of consumption. With the presence of a durable good the intercept value is different from that found in more standard perpetual youth models. The intercept will be larger for a positive holding of the durable good, and will be a function of more parameters than just the household discount factor, as is the case in standard models.

From the aggregate version of the households first order condition for the durable good, it can be shown that:

$$\frac{qx}{c} = \left(\frac{\theta}{1-\beta}\right) \tag{54}$$

The ratio of the value of durable to non-durable consumption is fixed by the preference parameter and the discount factor. Substituting this result into the aggregate euler equation simplifies the analysis of a change in the financial friction on the steady state level of the interest rate. As has been shown, the entrepreneurs holding of the durable good changes in the parameter $m$, and this change could have introduced another dynamic to the aggregate euler equation, through the market for durable goods, that would have changed the interest rate. This fixed ratio simplifies the analysis.

Another implication of this feature of the model is related to another concept of the financial friction as used by Hall (2014) and Chari Kehoe and McGrattan (2007). As already noted, this literature gives the opposite prediction for the change in a factor of production in response to a change in the financial friction. That the ratio between the value of durable and non-durable consumption is fixed also avoids this complication, and allows a focus on the change in borrowing only. This permits the result to be thought of in more general terms as the main impact of a tightening of the borrowing constraint in a broad range of financial frictions models is that borrowing will decrease, the issue of changes in the holding of the durable good is abstracted from.

The change in the steady state interest rate in the model from a change in $m$ will, therefore, be determined by the ratio of households financial assets to consumption only. From the steady state conditions:

$$c = wH + mqX' \frac{\omega}{1-\omega} (1 - 1/R) \tag{55}$$

$$\tilde{b} = mqX' \frac{\omega}{1-\omega} \tag{56}$$

In the model, households cannot hold negative amounts of the durable good, and will not on aggregate borrow in steady state. Therefore, the lower bound value on R in steady state is the reciprocal of the discount factor, $\beta < 1$. Given that the ratio of the real level of borrowing to consumption in steady state will increase in $m$, this suggests that $R$ will increase with an increase in $m$. Combining the above two equations with the other equations, in steady state, an expression for the ratio of households financial assets to consumption as a function of the interest rate can be derived as:

$$\frac{\tilde{b}}{c} = \frac{m}{\left(\frac{1-\nu}{\varphi\nu}\right)(1-\varphi-m(1/R-\varphi))+m(1-1/R)} \tag{57}$$

From this equation, it can be shown that the change in the ratio of households financial assets to consumption is negative with respect to an increase in the interest rate $R$. This result, given a reasonable range of values for $R$, does not rely on any additional parameter restrictions in addition to those already stated. It is also the case that this equation confirms the positive relationship between a change in $m$ and a change in the assets to consumption ratio. This relationship again is not reliant on any further assumptions on parameters.

In addition to analysing the sign of the relationship between the variables and parameters, it is also of interest to examine changes in magnitudes of these relationships. The relationship between $\frac{\tilde{b}}{c}$ and $m$ is positive. Using the quotient rule, it can also be shown that given the restrictions on parameter values required to ensure a non-explosive steady state; as already discussed, there is a positive and convex relationship between these two variables for realistic values (non-negative) of $R$. It should be noted that as $R = 1 + r$, this permits a broad range of values for the real interest rate. The convex relationship in $m$ is shown in Figure 1.

The result indicates that, given the parameter assumptions made in the model, changes in $m$ have a greater impact on the financial assets to consumption ratio at higher levels of $m$. Given that an increase in $m$ reduces the financial friction, this result indicates that economies with an initial condition of a looser financial friction will experience a sharper fall in $\frac{\tilde{b}}{c}$, and thus in the interest rate with a tightening of the financial friction.

Turning again to the relationship between $R$ and the financial assets to consumption ratio, it can be shown that this relationship; whilst negative, is strictly convex for a reasonable non-negative range of values for the interest rate. It is also of interest to note that the limit of the ratio as R tends to infinity is a positive constant:

$$\lim_{R\to\infty} \frac{\tilde{b}}{c} > 0 \tag{58}$$

This limit result applies without any additional parameter restrictions. These results can be used along with the steady state aggregate euler equation to sketch the relations among variables. This is shown in Figure 2:

In Figure 2, the equation relating the interest rate to the financial assets to consumption ratio is labelled $BC$, and the aggregate euler equation is labelled $EE$. In order to be consistent with a non-explosive steady state, this line ($EE$) will have an upper bound. It should be noted that the intercept value of the aggregate euler equation is determined by a restricted set of parameter values, as some value choices will violate the restriction on the value of R required for steady state. In particular, it is sensitive to the choice of taste parameter on durable and non-durable consumption. While this taste parameter, $\theta$, must not be large, the steady state condition does not restrict it to an unrealistic range of values. The restriction on the level of this variable is stated to be reasonable, as selecting a value for this parameter at the calibrated value chosen in Iacoviello (2005), $\theta = 0.1$, satisfies the condition. The diagram shows a reduction in parameter $m$. This is interpreted as a tightening of the financial friction which will reduce the financial assets to consumption ratio for any given interest rate and thus lead to a lower interest rate. This is shown in Figure 2 as a shift in the curve from $BC$ to $BC^*$, and a change in equilibrium from $A$ to $A^*$.

The non-linear relationship between $\frac{\tilde{b}}{c}$ and $m$ has another implication in that for the same observed interest rate, there could be a scenario where, if there are factors which reduce the slope and intercept of the EE curve, shifting down and flattening the line, then in a situation of low financial friction, this may increase the probability of the occurrence of problems of a persistent low interest rate due to the convex relationship highlighted in Figure 1. This scenario is presented in Figure 3. In this figure, this downward shift of the $EE$ curve to $EE^*$ could occur due to an increase in the probability of the survival parameter. If this occurs simultaneously with a reduction in the financial friction; a shift to the right of the $BC$ curve to $BC^*$, it is possible to arrive at a new equilibrium point to the right of the original equilibrium; point $A$, with the same interest rate, shown as $R^*$, but with a higher debt level. At this new point, a bigger shift in the interest rate would occur from a change in $m$ at the same initial interest rate. This could be thought of as two different economies with the same interest rate, but one would experience a bigger fall in the steady state interest rate given an increased financial friction.

## 6 Results

In the model presented the steady state interest rate is shown to be determined by fixed parameters and ratios of variables; household real lending/financial assets to consumption and the value of households durable goods holdings to consumption. Given the specification of the utility function and the separability of durable and non-durable consumption in the steady state, the second ratio is fixed, and thus the fact that changes in the financial friction impact the market for durable goods in the model has no effect on the households ratio of durable to non-durable consumption. The variable ratio of interest is thus household real lending/financial assets to consumption.

With an increase in $m$, entrepreneurs in the model have the opportunity to borrow more, and given the assumptions made regarding discount factors, they will increase their borrowing when possible. With credit market clearing households lending/financial assets increase, and as demonstrated previously, the ratio increases. This increases the steady state interest rate. The interest rate is bounded from above under the condition of the existence of a stable steady state level of consumption. Given that the relaxation of the borrowing constraint raises borrowing in the model, and this raise in borrowing raises the interest rate, the result is similar to that of Ascari and Rankin (2007) in that there are real effects in a non-Richardian economy; although from a different source. In the aforementioned model, it is a government debt increase that raises the interest rate by adding to households net wealth (Ascari and Rankin 2007). The direction of the response of the steady state interest rate to the borrowing constraint also matches that in the Bewley model of Guerrieri and Lorenzoni (2011), and the three generation overlapping generations model of Eggertsson and Mehrotra (2014). A summary of the model dynamics is similar to the supply and demand, asset market equilibrium diagram presented for the basic model of Eggertsson and Mehrotra (2014). In typical collateral constraints models, the credit market, in the steady state, can be thought of as one characterised by an perfectly elastic supply curve. In the Eggertsson and

Mehrotra (2014) model, as in the model presented here, there is a supply and demand for credit and a tightening of the constraint leads to a drop in demand for credit. This shifts the demand curve downwards. In equilibrium, the steady state real interest rate will be lower. This market structure is presented in Figure 4.

The model also indicates that a tightening of the financial friction leads to a bigger fall in the interest rate when the economy is in a situation where the financial friction is relatively low. This result parallels a result from more short run analysis as in Pinheiro et al (2013), which finds that the magnitude of the response to shocks can rise and fall depending on the level of financial frictions present.

As per Figure 3, the model indicates that it is possible to have a scenario where, if there was simultaneously a relaxation of the financial friction in the economy; for example from a process of financial development (as in Pinheiro et al 2013), and a process of a reduction in the probability of death; for example population ageing, this would lead to a situation of an economy with the same steady state interest rate, but with a higher debt level, which would be vulnerable to a greater fall in the interest rate from tightening of the financial friction. This scenario involving a change in the probability of survival holding down the interest rate is in line with a hypothesis promulgated in Summers (2014). It may also point to a factor to be considered in the literature on the probability of the occurrence of persistent low interest rates, as discussed in Williams (2014).

# 7 Conclusions

This paper begins with the observation that, although a vast literature has developed, particularly since the financial crisis, that analyses the role of financial frictions in macroeconomic models and the features of macroeconomic models where the policy interest rate is low, there are comparatively few papers which analyse the link between financial frictions and the interest rate. The main contribution of this paper is to show that, using a basic and tractable benchmark macroeconomic model and a benchmark financial frictions model, there is an analytical link between these two phenomenon.

To this end, in this paper, a model has been proposed that provides an endogenous theoretical framework for the impact of financial frictions on the steady state interest rate. The model presented is a perpetual youth type overlapping generations model with financial frictions in the form of collateral constraints. This model replicates the basic result of more complex and numerically simulated models, and demonstrates that a tightening of the financial friction generates a fall in the steady state interest rate. The model also highlights a non-linearity in this relationship, and discusses how this result had relevance for the literature on the ZLB and secular stagnation.

Guerrieri and Lorenzoni (2011) state that this basic result may explain why recessions driven by financial market trouble are more likely to be accompanied by zero lower bound issues. Given that the model proposed in this paper sees a fall in the interest rate from financial frictions, and that a major consideration of the zero lower bound literature is the size of fiscal multipliers in a situation where the interest rate is low, a promising direction for future research could be to provide an analytical solution to the size of fiscal multipliers in the model.

# References

[1] Daron Acemoglu. *Introduction to Modern Economic Growth*. Princeton University Press, January 2009.

[2] Vanda Almeida, Gabriela Lopes de Castro, Ricardo Mourinho Flix, Paulo Jlio, and Jos R. Maria. Inside PESSOA - A Detailed Description of the Model. Technical report, 2013.

[3] Javier Andrés, scar Arce, and Carlos Thomas. Banking competition, collateral constraints, and optimal monetary policy. *Journal of Money, Credit and Banking*, 45(s2):87–125, 2013.

[4] Guido Ascari and Neil Rankin. Perpetual youth and endogenous labor supply: A problem and a possible solution. *Journal of Macroeconomics*, 29(4):708–723, December 2007.

[5] Guido Ascari and Neil Rankin. The effectiveness of government debt for demand management: Sensitivity to monetary policy rules. *Journal of Economic Dynamics and Control*, 37(8):1544–1566, 2013.

[6] Annicchiarico Barbara, Marini Giancarlo, and Piergallini Alessandro. Monetary Policy and Fiscal Rules. *The B.E. Journal of Macroeconomics*, 8(1):1–42, February 2008.

[7] Kimberly Beaton. Credit Constraints and Consumer Spending. Technical report, 2009.

[8] Ben S. Bernanke, Mark Gertler, and Simon Gilchrist. The financial accelerator in a quantitative business cycle framework. In J. B. Taylor and M. Woodford, editors, *Handbook of Macroeconomics*, volume 1 of *Handbook of Macroeconomics*, chapter 21, pages 1341–1393. Elsevier, 1999.

[9] Olivier J Blanchard. Debt, Deficits, and Finite Horizons. *Journal of Political Economy*, 93(2):223–47, April 1985.

[10] Olivier Jean Blanchard and Stanley Fischer. *Lectures on Macroeconomics*, volume 1 of *MIT Press Books*. The MIT Press, August 1989.

[11] Markus K. Brunnermeier and Yuliy Sannikov. A Macroeconomic Model with a Financial Sector. *American Economic Review*, 104(2):379–421, February 2014.

[12] Micha Brzoza-Brzezina and Krzysztof Makarski. Credit crunch in a small open economy. *Journal of International Money and Finance*, 30(7):1406 – 1428, 2011.

[13] Willem H. Buiter. Negative nominal interest rates: Three ways to overcome the zero lower bound. *The North American Journal of Economics and Finance*, 20(3):213–238, December 2009.

[14] Jeffrey R. Campbell and Zvi Hercowitz. The Role of Collateralized Household Debt in Macroeconomic Stabilization. NBER Working Papers 11330, National Bureau of Economic Research, Inc, May 2005.

[15] John Y Campbell and N Gregory Mankiw. Permanent Income, Current Income, and Consumption. *Journal of Business & Economic Statistics*, 8(3):265–79, July 1990.

[16] Charles T. Carlstrom and Timothy S. Fuerst. Agency costs, net worth, and business fluctuations: A computable general equilibrium analysis. *The American Economic Review*, 87(5):pp. 893–910, 1997.

[17] J.S. Chadha and C. Nolan. On the Interaction of Monetary and Fiscal Policy. Cambridge Working Papers in Economics 0303, Faculty of Economics, University of Cambridge, January 2003.

[18] V. V. Chari, Patrick J. Kehoe, and Ellen R. McGrattan. Business Cycle Accounting. *Econometrica*, 75(3):781–836, 05 2007.

[19] Szabolcs Dek, Lionel Fontagn, Marco Maffezzoli, and Massimiliano Marcellino. LSM: A DSGE model for Luxembourg. *Economic Modelling*, 28(6):2862–2872, 2011.

[20] Michael B. Devereux. Fiscal Deficits, Debt, and Monetary Policy in a Liquidity Trap. In Luis Felipe Cspedes, Roberto Chang, and Diego Saravia, editors, *Monetary Policy under Fi-*

*nancial Turbulence*, volume 16 of *Central Banking, Analysis, and Economic Policies Book Series*, chapter 10, pages 369–410. Central Bank of Chile, July 2011.

[21] Gauti B. Eggertsson and Neil R. Mehrotra. A Model of Secular Stagnation. NBER Working Papers 20574, National Bureau of Economic Research, Inc, October 2014.

[22] Gauti B. Eggertsson and Michael Woodford. The Zero Bound on Interest Rates and Optimal Monetary Policy. *Brookings Papers on Economic Activity*, 34(1):139–235, 2003.

[23] Adam Elbourne, Sander Muns, and Sander van Veldhuizen. Towards a DSGE model for policy analysis in the Netherlands. CPB Discussion Paper 227, CPB Netherlands Bureau for Economic Policy Analysis, August 2009.

[24] Jacob. A. Frenkel and Razin Assaf. *Fiscal Policies and the World Economy.* MIT Press, 2nd edition, 1992.

[25] Jacob A. Frenkel and Assaf Razin. Budget Deficits and Rates of Interest in the World Economy. NBER Working Papers 1354, National Bureau of Economic Research, Inc, August 1986.

[26] Ippei Fujiwara, Naoko Hara, Yasuo Hirose, and Yuki Teranishi. The Japanese Economic Model (JEM). *Monetary and Economic Studies*, 23(2):61–142, May 2005.

[27] Jordi Gali, J. David Lopez-Salido, and Javier Valles. Rule-of-Thumb Consumers and the Design of Interest Rate Rules. NBER Working Papers 10392, National Bureau of Economic Research, Inc, March 2004.

[28] Oded Galor. *Discrete Dynamical Systems.* Springer-Verlag Berlin Heidelberg, 2010.

[29] Andrea Gerali, Stefano Neri, Luca Sessa, and Federico M. Signoretti. Credit and Banking in a DSGE Model of the Euro Area. *Journal of Money, Credit and Banking*, 42(s1):107–141, 09 2010.

[30] Mark Gertler and Peter Karadi. A model of unconventional monetary policy. *Journal of Monetary Economics*, 58(1):17–34, January 2011.

[31] Veronica Guerrieri and Guido Lorenzoni. Credit Crises, Precautionary Savings, and the Liquidity Trap. NBER Working Papers 17583, National Bureau of Economic Research, Inc, November 2011.

[32] Robert E. Hall. Quantifying the forces leading to the collapse of gdp after the financial crisis. Stanford University, 2012.

[33] Robert E. Hall. Financial frictions. *International Journal of Central Banking*, 9(2):155–163, 2013.

[34] Richard Harrison, Kalin Nikolov, Meghan Quinn, and Alasdair Scott. The Bank of England Quarterly Model. Working papers, Bank of England, 2005.

[35] Burkhard Heer and Alfred Mauner. *Dynamic General Equilibrium Modelling: Computational Methods and Applications.* Springer Publishing Company, Incorporated, 2nd ed. 2009. 2nd printing edition, 2009.

[36] Ben J Heijdra and Jenny E Ligthart. The Dynamic Macroeconomic Effects of Tax Policy in an Overlapping Generations Model. *Oxford Economic Papers*, 52(4):677–701, October 2000.

[37] Sheng Cheng Hu. Transactions cost and durable consumption: A general-equilibrium approach. *Journal of Macroeconomics*, 16(3):383–402, 1994.

[38] Matteo Iacoviello. House Prices, Borrowing Constraints, and Monetary Policy in the Business Cycle. *American Economic Review*, 95(3):739–764, June 2005.

[39] Matteo Iacoviello and Stefano Neri. Housing Market Spillovers: Evidence from an Estimated DSGE Model. *American Economic Journal: Macroeconomics*, 2(2):125–64, April 2010.

[40] Kenneth Kasa. Borrowing constraints and asset market dynamics: evidence from the Pacific Basin. Technical report, 1998.

[41] Nobuhiro Kiyotaki and John Moore. Credit Cycles. *Journal of Political Economy*, 105(2):211–48, April 1997.

[42] Douglas Laxton, Susanna Mursula, Michael Kumhof, and Dirk Muir. The Global Integrated Monetary and Fiscal Model (GIMF); Theoretical Structure. IMF Working Papers 10/34, International Monetary Fund, February 2010.

[43] Campbell Leith, Ioana Moldovan, and Simon Wren-Lewis. Debt stabilization in a Non-Ricardian economy. Working Paper 23, Business School, Economics, University of Glasgow, October 2011.

[44] Campbell Leith and Simon Wren-Lewis. Interactions between Monetary and Fiscal Policy Rules. *Economic Journal*, 110(462):C93–108, March 2000.

[45] Ranjan K Mallik. Solutions of linear difference equations with variable coefficients. *Journal of Mathematical Analysis and Applications*, 222(1):79 − 91, 1998.

[46] Maurice Obstfeld and Kenneth S. Rogoff. *Foundations of International Macroeconomics*, volume 1 of *MIT Press Books*. The MIT Press, August 1996.

[47] Alessandro Piergallini. Real Balance Effects, Determinacy and Optimal Monetary Policy. MPRA Paper 59832, University Library of Munich, Germany, January 2004.

[48] Tiago Pinheiro, Francisco Rivadeneyra, and Marc Teignier. Financial Development and the Volatility of Income. Technical report, 2013.

[49] Neil Rankin. Maximum Sustainable Government Debt In The Perpetual Youth Model. *Bulletin of Economic Research*, 66(3):217–230, 07 2014.

[50] Lawrence H Summers. U.S. Economic Prospects: Secular Stagnation, Hysteresis, and the Zero Lower Bound. *Business Economics*, 49(2):65–73, April 2014.

[51] Carl E. Walsh. *Monetary Theory and Policy, Third Edition*, volume 1 of *MIT Press Books*. The MIT Press, June 2010.

[52] M. Wickens. *Macroeconomic Theory: A Dynamic General Equilibrium Approach (Second Edition)*. Princeton University Press, 2012.

[53] John C. Williams. Monetary Policy at the Zero Lower Bound: Putting Theory into practice. *Hutchins Center on Fiscal and Monetary policy at Brookings*, January 2014.

[54] Tony Yates. Monetary Policy and the Zero Bound to Interest Rates: A Review. *Journal of Economic Surveys*, 18:427–481, 07 2004.

# 8 Figures

Figure 1: $\frac{\tilde{b}}{c}$ and $m$

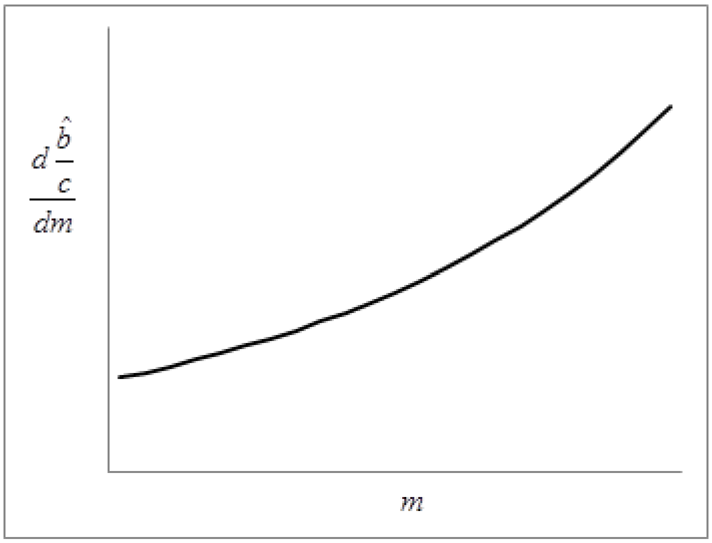


Figure 2: Interest rate and Assets

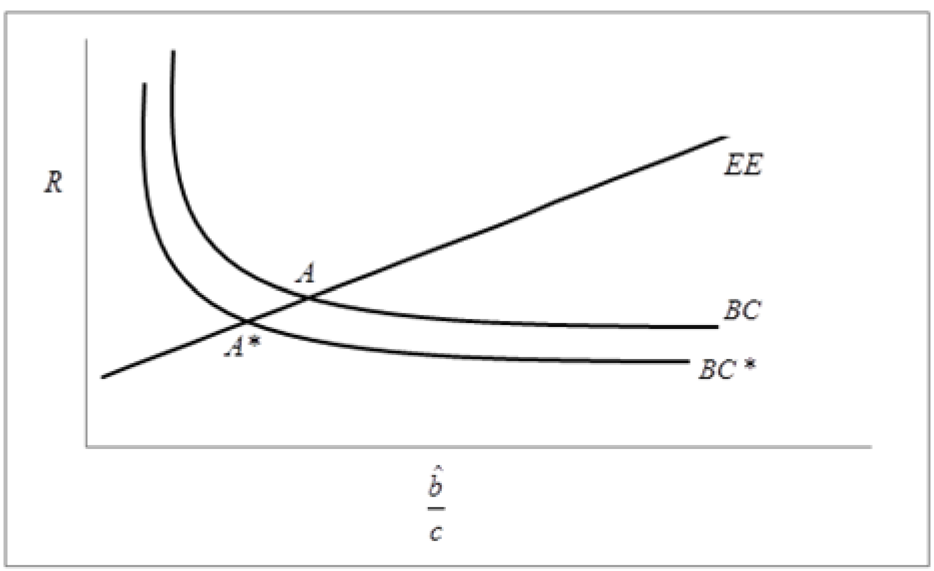

Figure 3: Change of Equilibrium

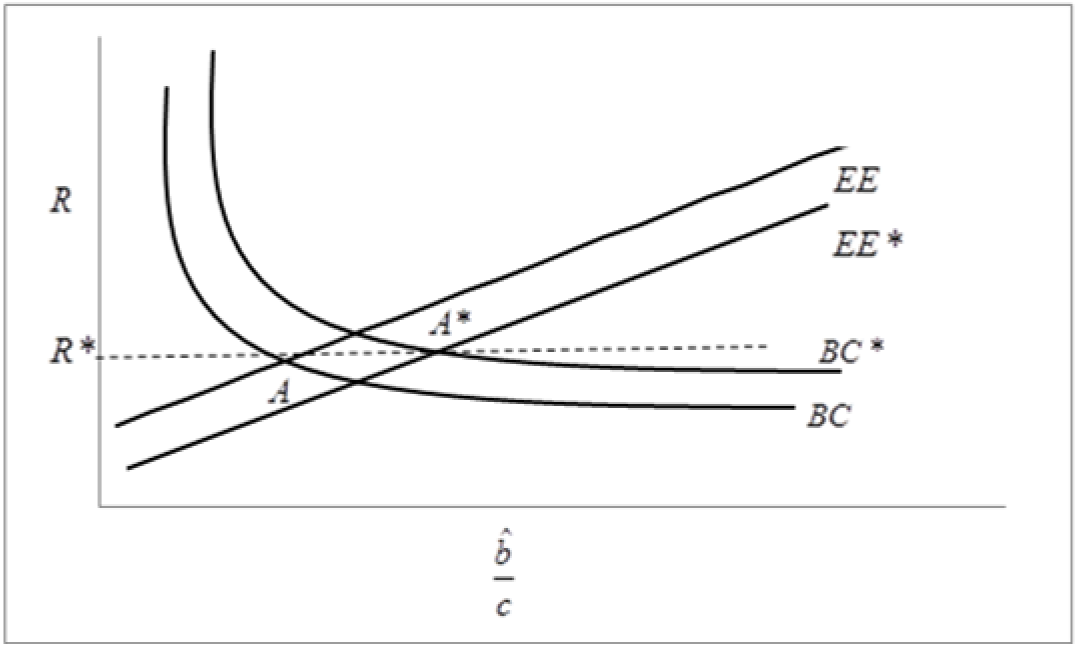


Figure 4: Equilibrium in the Asset Market