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Abstract Applying Data Envelopment Analysis (DEA) to real-world public policy issues can raise many interesting complications beyond those considered in standard models of DEA. One of these complications arises if the funding levels of public service providers, and their ability to attract and retain clients and able staff, depend upon the quality of the output which they produce. This dependency introduces additional inter-relationships between inputs and outputs beyond the uni-directional Production Possibility Frontier (PPF) relationship considered by standard DEA models. The paper therefore analyses the multiplier effects which can be generated by these additional relationships, in which key resource inputs become endogenous variables subject to the external environmental variables which the public service provider faces across these different relationships. The magnitude of these multiplier effects can be captured by focussing DEA on the estimation of an Achievement Possibility Frontier, which reveals the wider set of opportunities which are available to a public service provider to improve its own output quality than that revealed by the estimation of the PPF associated with standard models of DEA. In doing so, the paper enables DEA to be still applied, but in modified form, to the estimation of the scope for improved output of any given public service provider in the presence of such resource endogeneity.

Keywords Data envelopment analysis, resource endogeneity, public services, output quality, frontier analysis.

JEL Classifications: C30, C61, D24, I23, I26, L30

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1 Introduction

In emphasising the need for "applications-driven theory" (see Banker and Kaplan 2014), William W. Cooper was well aware of the value of exploring directions in which public policy and other applied problems present additional features of reality which are not adequately addressed by existing analytical techniques as a way of stimulating productive theoretical and methodological developments in these available analytical techniques. One main area of application of Data Envelopment Analysis (DEA) has been with analysing the efficiency of public services, such as education and health care (see, for instance, Smith and Mayston 1987; Jesson et al. 1987; Johnes and Johnes 1995; Mayston 2003; Emrouznejad et al. 2008; Hollingsworth 2008). In many other contexts, DEA's main focus to date has been with identifying a Production Possibility Frontier (PPF) of the *quantities of output* that can be produced from a given set of inputs and the minimum level of resources that are required to produce a given vector of output quantities. However, evaluating the management of public services raises also important issues regarding the *quality* of service delivered. These quality issues are of concern both to the recipients of public services and to their funders. In an effort to stimulate greater efficiency and effectiveness in the delivery of public services, greater competition between public service providers, such as hospitals, schools and universities, has been introduced in recent years. The quality of the service delivered by a provider can therefore have important implications for their funding and available resources, which in turn introduces an additional inter-relationship beyond the simple *uni-directional relationship* between inputs and outputs considered by the PPF in the standard models of DEA. As a result, the provider's quality scores, in areas such as university research and teaching, have also become a major focus for managerial attention in recent years.

Such real-world complications underline the need to make methodological developments which will enable DEA in the presence of such additional inter-relationships to still adequately address questions such as (i) where is the feasible frontier that includes the quality of the output of any individual public service producer, given the constraints and opportunities which it faces? (ii) how much scope is there for an individual public service producer to improve its output quality, given the constraints and opportunities which it faces? and (iii) which individual public service producers are currently on the resultant quality achievement frontier? In Sections 2 and 3 below, we therefore examine several relevant additional inter-relationships beyond the uni-directional relationship between inputs and outputs which is considered in the PPF and beyond the associated *production function* relationship of standard micro-economic theory. At the same time, we examine the methodological developments which these additional considerations can give rise to in the application of DEA, which will enable DEA still to address the above questions in the presence of these complicating factors. Section 4 contains an application of our resultant modified DEA methodology to the interesting context of the achievable frontier of university teaching and research quality. Section 5 examines the relationship of our modified DEA methodology to the existing literature. Section 6 contains our conclusions.

2 Endogenous Resource Inputs

That additional inter-relationships between output quality and the availability of inputs can have important implications for the answers to questions (i) – (iii) above can be seen from the following example. We will consider the relatively simple case of local not-for-profit public service broadcasting where the quality of public service delivered by an individual local public service broadcaster matters to both its audience and its funders. We will denote by Q_i the quality of the service delivered by the local public service broadcaster i, where Q_i is measured by a survey of consumer satisfaction of the local residents on a continuous point-score basis. We will assume for simplicity that the local public service broadcaster i makes use of a single resource input x_i in its production process that involves a positive linear relationship between the maximum level Q_i^* of its quality rating it could achieve and its resource input x_i of the form:

$$Q_i^*(x_i) = \alpha_{10} + \alpha_{11} x_i \quad where\ \alpha_{11} > 0 \tag{1}$$

This linear relationship is mapped out by the line KL in Fig. 1 over a relevant range of variation of x_i. It corresponds to a relevant section of a *production function* for Q_i as the resource input x_i is varied. If DEA identifies broadcasters K and L, with input and output quality combinations (x_K, Q_K) and (x_L, Q_L) respectively in Fig. 1, as being the efficient DMUs with which to compare broadcaster J's result of (x_J, Q_J), the line KL would also be mapped out by considering all the interior convex combinations of K and L's achievements that DEA might consider.

We may note here that Eq. (1) indicates the maximum quality of output which producer i could produce with an input of x_i if it were production efficient. However, an individual broadcaster, such as $i = J$, may prove to be less than fully efficient in its production of output quality. Thus in Figure 1, the actual point J corresponding to (x_J, Q_J) for the public service producer $i = J$ is below the production frontier given by the line KL, with a shortfall of $\varepsilon_{JQ} = (Q'_J - Q_J) = JF$ in the quality score of Q_J which it did actually achieve, compared to the quality score $Q'_J \equiv Q^*_J(x_J)$ it could have achieved from its existing input level of x_J if it were fully productive efficient. We then have more generally from equation (1) that the actual quality score of broadcaster i equals:

$$Q_i = \alpha_{10} + \alpha_{11} x_i - \varepsilon_{iQ} \text{ where } \alpha_{11} > 0 \text{ and } \varepsilon_{iQ} \geq 0 \quad (2)$$

where ε_{iQ} is the extent of any shortfall for producer i in the output quality that it produces from its existing level of input x_i compared to the maximum that it could have achieved if it were fully production efficient.

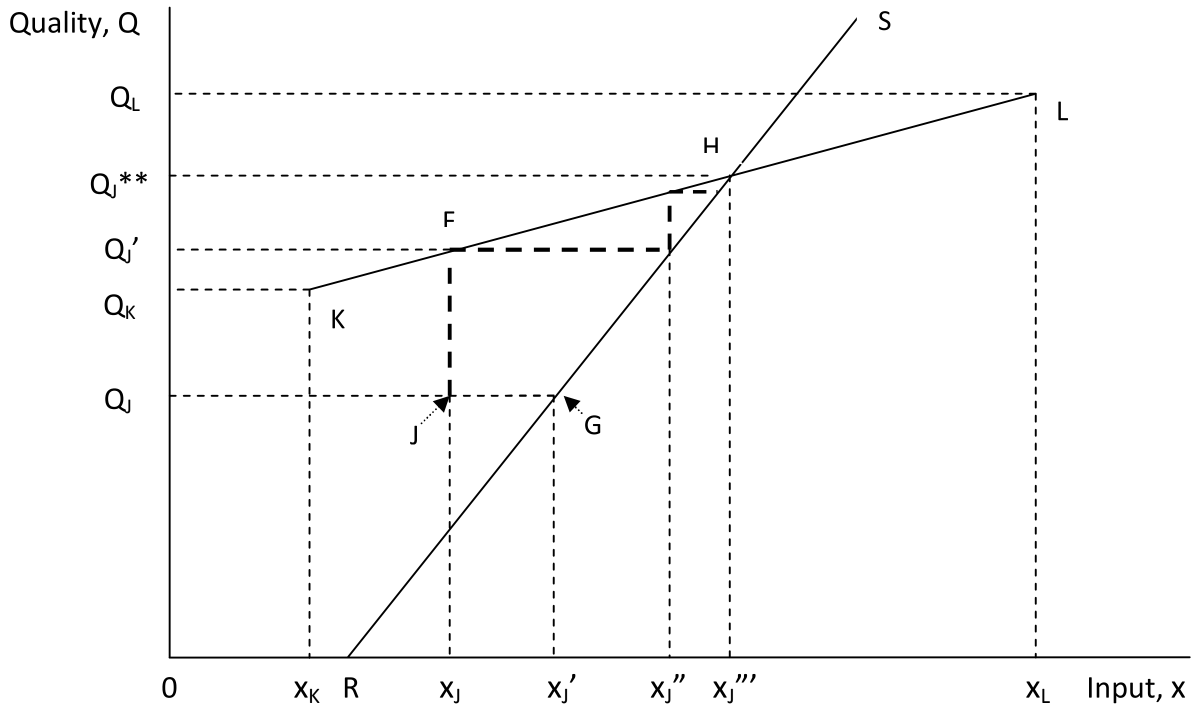

Fig. 1 The multiplier effect of output quality efficiency improvements

The value of ε_{iQ} for the broadcaster $i = J$ would indeed be correctly identified in this example by the application of the output-orientated Banker-Charnes-Cooper (BCC) form of DEA (Cooper et al. 2007, p.93), which would seek to find:

$$\theta^*_J = \max_{\theta_J, \lambda} \theta_J \ \ s.t. \ \theta_J Q_J \leq Q_K \lambda_K + Q_L \lambda_L, x_J \geq x_K \lambda_K + x_L \lambda_L, \lambda_K + \lambda_L = 1, \lambda_K \geq 0, \lambda_L \geq 0, \lambda \equiv (\lambda_K, \lambda_L) \quad (3)$$

and hence find the maximum feasible increase $\varepsilon_{iQ} = (\theta_J^* - 1)Q_J$ in the output quality Q_J that places it on the line KL of convex combinations of the efficient input-output vectors (x_i, Q_i) of producers $i = K, L$ at a point, given by $F = (x_J, Q'_J)$ in Fig.1, corresponding to broadcaster J's level of resource input x_J, with $Q'_J \equiv Q_J^*(x_J)$.

However, in addition to the linear production function relationship (1), we will assume that the not-for-profit broadcaster operates under a budget constraint in which the maximum resources x_i^* it could have available to it depend in a positive linear way on the population size z_i of its local area and on how satisfied local residents are with its output, as reflected in its Q_i rating, so that:

$$x_i^*(Q_i, z_i) = \alpha_{20} + \alpha_{21}Q_i + \alpha_{22}z_i \text{ where } \alpha_{21} > 0, \alpha_{22} > 0 \tag{4}$$

This additional *revenue generating function* relationship may arise because the not-for-profit broadcaster depends upon subscriptions from its audience whose size depends upon z_i, and whose willingness to pay depends in part upon how satisfied they are with its output. It may also arise because the size of any grant the public service broadcaster receives from local or central government is based upon its local population size and on its published satisfaction scores. In addition it may arise because any advertising revenue which the broadcaster receives depends upon advertisers' assessment of how popular the broadcaster is with its potential audience and the size of its potential audience. Such an additional inter-relationship between the broadcaster's output quality and their available inputs beyond the simple one-way relationship of the standard production function has important consequences for the value of the maximum achievable output quality that an initially inefficient producer could achieve.

Thus, for an individual broadcaster, such as $i = J$, Eq. (4) will map out another line, such as RS in Fig. 1, in (x_i, Q_i) space, holding constant the size of the local population $z_i = z_J$. Eq. (4) indicates the maximum level of resources producer i could secure when its output quality is Q_i and its population size is z_i if it were fully effective at revenue raising. We will assume in this example that the broadcasters K and L are themselves both production efficient and fully effective in their revenue raising for the size of their respective local populations, with $z_K < z_J < z_L$. The points K and L in Fig. 1 will therefore lie at the intersection points of the production frontier KL with the respective revenue generating lines parallel to RS corresponding to their respective values of z_K and z_L in Eq. (4).

However, broadcaster J in this example is less than fully effective at revenue raising. Thus in Fig. 1, the actual point J corresponding to (x_J, Q_J) is to the left of the revenue raising line RS for the given value of its local population size z_J, with a shortfall of $\varepsilon_{Jx} = (x'_J - x_J) = GJ$ in the resourcing level $x'_J \equiv x^*_J(Q_J, z_J)$ it could have achieved with its existing quality score of Q_J and its population size of z_J. We then have more generally from Eq. (4) that the actual resourcing level of broadcaster i equals:

$$x_i = \alpha_{20} + \alpha_{21}Q_i + \alpha_{22}z_i - \varepsilon_{ix} \text{ where } \alpha_{21} > 0, \alpha_{22} > 0 \text{ and } \varepsilon_{ix} \geq 0 \quad (5)$$

where ε_{ix} is the extent of any shortfall for producer i in the resources that it succeeds in raising with its existing output quality score and population size.

However, it is important to note that even if we had simply $\varepsilon_{Jx} = 0$ in Fig. 1, the answer to question (ii) raised in Sect. 1 above, of how much scope would there be for the public service producer J to increase its output quality, would here be not simply the amount of its existing quality shortfall ε_{iQ}. Instead if producer J did eliminate the existing shortfall in its output quality by the amount ε_{iQ}, so that is did achieve an output quality of $Q'_J \equiv Q^*_J(x_J)$ from Eq. 1, the existence of the additional revenue raising relationship (4) means that it could increase its input level to $x''_J \equiv x^*_J(Q'_J, z_J)$ in Fig. 1 if it were fully effective in its revenue raising. Moreover, this in turn would enable it to further increase its output quality beyond Q'_J, with a resultant *multiplier process* that has an equilibrium in Fig. 1 at the point H at which:

$$Q^*_i(x_i) = \alpha_{10} + \alpha_{11}x_i \text{ and } x_i = \alpha_{20} + \alpha_{21}Q^*_i + \alpha_{22}z_i \quad (6)$$

with both the efficient production function equation (1) and the effective revenue raising inter-relationship (4) holding simultaneously at the fully efficient and effective point $\text{H} = (x'''_i, Q^{**}_i)$ for $i = J$ in Fig.1. Eq. (6) in turn has a solution for the *maximum achievable output quality* for producer i, given by:

$$Q^{**}_i = \beta_0 + \beta_1 z_i \text{ where } \beta_0 \equiv \gamma(\alpha_{10} + \alpha_{11}\alpha_{20}),\ \beta_1 \equiv \gamma\alpha_{11}\alpha_{22} > 0, \gamma \equiv 1/(1-\alpha_{11}\alpha_{21}) > 1,\ \text{for } 0 < \alpha_{11}\alpha_{21} < 1 \quad (7)$$

As we note in Sect. 5 below, our approach parallels here that of deriving a *reduced form equation* in econometrics, in which the attainable equilibrium values of the endogenous variables are specified as functions of the exogenous (or pre-determined) variables.

The condition $\alpha_{11}\alpha_{21} < 1$ in Eq. (7) is here a *stability condition* that ensures that the *feedback effect* $\alpha_{21} = (\partial x_i / \partial Q_i)$ of a unit improvement in a producer's output quality on its resource availability in Eq.

(4), when multiplied by the feasible additional output quality $\alpha_{11} = (\partial Q_i / \partial x_i)$ that the DMU could achieve with an additional unit of the resource input in Eq. (1), does not exceed the initial unit increase in Q_i, so that the successive iterations in the multiplier process in Fig. 1 grow smaller and converge to an equilibrium point.

The overall feasible increase in output quality for producer J under its given population size of z_J is therefore here $Q_J^{**} - Q_J$, which in Fig. 1 substantially exceeds the increase $Q_J' - Q_J$ that the standard DEA program (3) would indicate as being feasible. Recognising that there is not only a *production* side to a DMU's operations, but also a *demand* side which influences consumers' willingness to pay for its output and available resources can therefore make a substantial difference to an assessment of the DMU's feasible scope for output quality increases.

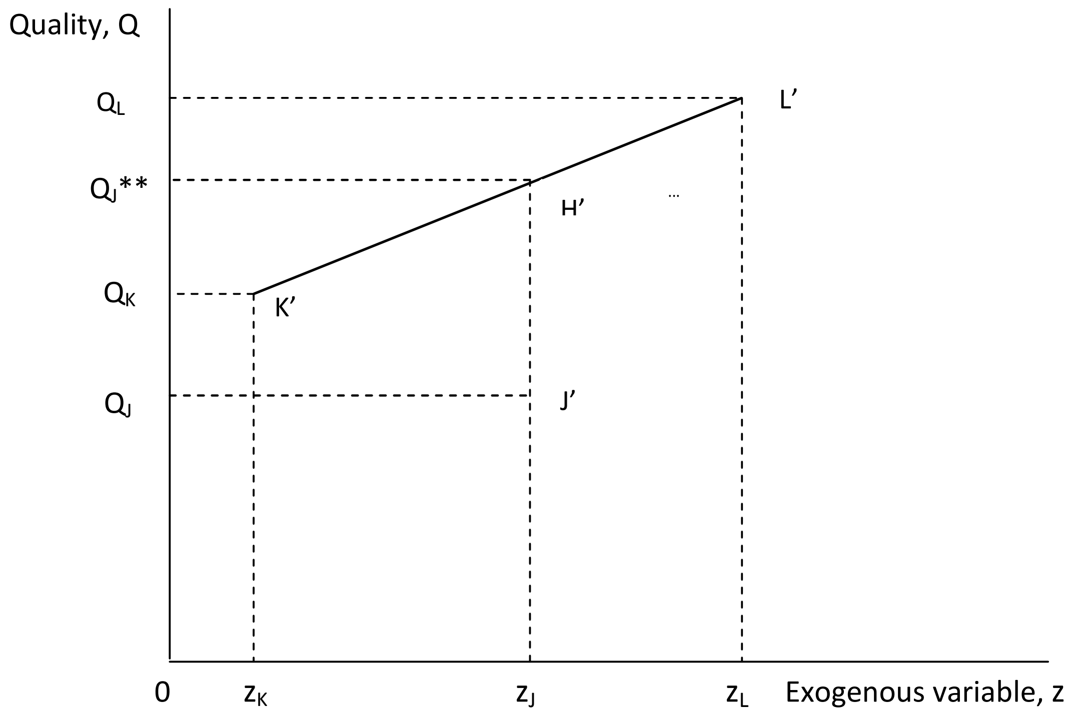

Fig. 2 A linear section of the Achievement Possibility Function

When the efficient production-side relationship (1) holds simultaneously with the effective revenue raising function (4), the result is a set of two simultaneous equations which yield a solution for the maximum achievable output quality for producer i in Eq. (7) which depend upon its local population size z_i. Under our above assumption that the comparator DMUs K and L are both production efficient and fully effective in their revenue raising, Eq. (7) defines here a new locally linear relationship between

Q_i and the exogenous variable z_i that maps out feasible convex combinations of the end points of $K' = (z_K, Q_K)$ and $L' = (z_L, Q_L)$ in the relevant new (z_i, Q_i) space in Fig. 2. *Thus even though the exogenous variable z_i does not directly enter into the production relation in Eq. (1), it plays an important part in determining the achievable level of output quality which is feasible in Eq. (7) if the DMU is both technically efficient and fully effective in increasing its resource income in response to feasible improvements in its output quality*. For the case of a single output variable, Eq. (7) therefore defines a linear facet of what we can call an *Achievement Possibility Function* in the relevant (z_i, Q_i) space, in contrast to the conventional *production function* in the standard (x_i, Q_i) space.

Equations (2), (5) and (7) imply that the actual output quality of producer i equals:

$$Q_i = \beta_0 + \beta_1 z_i - \eta_i \; where \; \eta_i = \gamma(\varepsilon_{iQ} + \alpha_{11}\varepsilon_{ix}) \geq 0 \tag{8}$$

Equation (8) in turn implies *performance multiplier effects* from reductions in the production efficiency shortfall ε_{iQ} and the revenue-raising effectiveness shortfall term ε_{ix} that are given by:

$$\psi_1 \equiv (\Delta Q_i / -\Delta\varepsilon_{iQ}) = \gamma > 1, \psi_2 \equiv (\Delta Q_i / -\Delta\varepsilon_{ix}) = \gamma\alpha_{11} > 0, \; for \; 1 > \alpha_{11}\alpha_{21} > 0 \tag{9}$$

We are now in a position to extend the application of DEA to more fully answer questions (i), (ii) and (iii) of Sect. 1. in the above context. The overall value of the shortfall η_J in Eq. (8) for any DMU that is less than fully efficient and effective can be estimated here using DEA by modifying the output-orientated BCC form of DEA so that the exogenous variable z_i, such as local population size in the above example, replaces the endogenous resource input x_i in its formulation. We then have:

$$\rho_J^* \equiv \max_{\rho_J, \varpi} \rho_J \;\; s.t. \;\; \rho_J Q_J \leq Q_K\varpi_K + Q_L\varpi_L, z_J \geq z_K\varpi_K + z_L\varpi_L, \varpi_K + \varpi_L = 1, \varpi_K \geq 0, \varpi_L \geq 0 \tag{10}$$

$$with \;\; Q_J^{**} = \rho_J^* Q_J = Q_J + \eta_J, with \, \eta_J = (\rho_J^* - 1)Q_J, \kappa_J \equiv (Q_J / Q_J^{**}) = 1/\rho_J^*, \varpi \equiv (\varpi_K, \varpi_L) \tag{11}$$

In the absence of slacks, the modified output-orientated DEA program (10) finds the convex combination of z_K and z_L that replicates z_J, together with the corresponding convex combination of the output qualities Q_K and Q_L that identifies the point H' on the line K'L' in the (z_i, Q_i) space in Fig.2, and hence the maximum feasible increase J'H' = $\eta_J = (\rho_J^* - 1)Q_J$ in Q_J in (8) and (10).

The term κ_J in (11) defines what we can call a *cumulative coefficient of effectiveness*, being inversely related to the overall performance shortfall η_J, which from Eq. (8) is a weighted sum of the DMU's efficiency and effectiveness shortfalls ε_{iQ} and ε_{ix}, where the weights are the corresponding multiplier effects in Eq. (9), with $\gamma > 1$. It therefore more fully answers question (ii) of Sect. 1 of how much scope there is for a producer, such as J, to increase its output quality, once there are additional revenue raising relationships involved. It also addresses questions (i) and (iii) by identifying the fully efficient and effective DMUs K and L and the feasible frontier between them that is formed by the convex combinations of their exogenous variable and output-quality vectors in (10), with such convexity implied by the underlying linear relationships (1), (4) and (7) in the above example.

3 Additional Inter-relationships

We can extend the above analysis by considering the general form of the BCC output-orientated DEA program (see Cooper et al. 2007, p.93):

$$\max_{\varphi_J,\mu} \varphi_J \quad s.t. \;\; \varphi_J Y_J \le Y\mu, X_J \ge X\mu, e\mu = 1, \mu \ge 0 \;\; for\; e \equiv (1,1,..,1) \tag{12}$$

where $X_i = (x_{11},...,x_{mi})'$ and $Y_i = (Q_{1i},...,Q_{ri})'$ in our present context are the input and output quality vectors of providers $i = 1,...,n$, with $X = (X_1,...,X_n)$ and $Y = (Y_1,...,Y_n)$. In the presence of endogenous resource inputs, the problem with the standard DEA formulation (12) is that it takes provider J's input vector X_J as being fixed independently of any feasible expansion of provider J's achieved output quality vector by a factor such as φ_J.

In contrast, recognition of endogeneity amongst the resource inputs would involve permitting producer J's input vector X_J to expand in response to *positive feedback* from relevant improvements in its output quality vector. Any such expansion in X_J in (12) would in turn permit those input vectors X_i in X that are given positive weights μ_i in (12) in defining a relevant comparison group C_J for the DMU J to be greater than previously in some relevant directions. Such increases in the comparison input vectors X_i would have associated with them greater output quality vectors Y_i that efficient DMUs can produce with these increased input vectors. An increase in the Y_i that receive positive weights μ_i in (12) in turn facilitates a feasible increase in the expansion factor φ_J in DMU J's achievable output quality vector. This situation is illustrated in Fig. 3 below where the frontier $Y_1\,Y_2\,Y_3\,Y_4$ represents the feasible Production Possibility Frontier (PPF) based upon the original input vector X_J. It implies a corresponding feasible proportional expansion factor for DMU J of OE_{J1}/OT_J in its original output quality vector here at point T_J. However, under positive resource endogeneity, an improvement in DMU J's output quality vector would increase its available input vector, making a new reference set of DMUs with greater output quality vectors, such as N_2 and N_3 in Fig. 3, admissible as comparators, with a correspondingly higher Achievement Possibility Frontier $N_1N_2N_3N_4$ for different output mixes and a greater feasible expansion factor for DMU J at point P_J of OE_J^*/OT_J in Fig.3 .

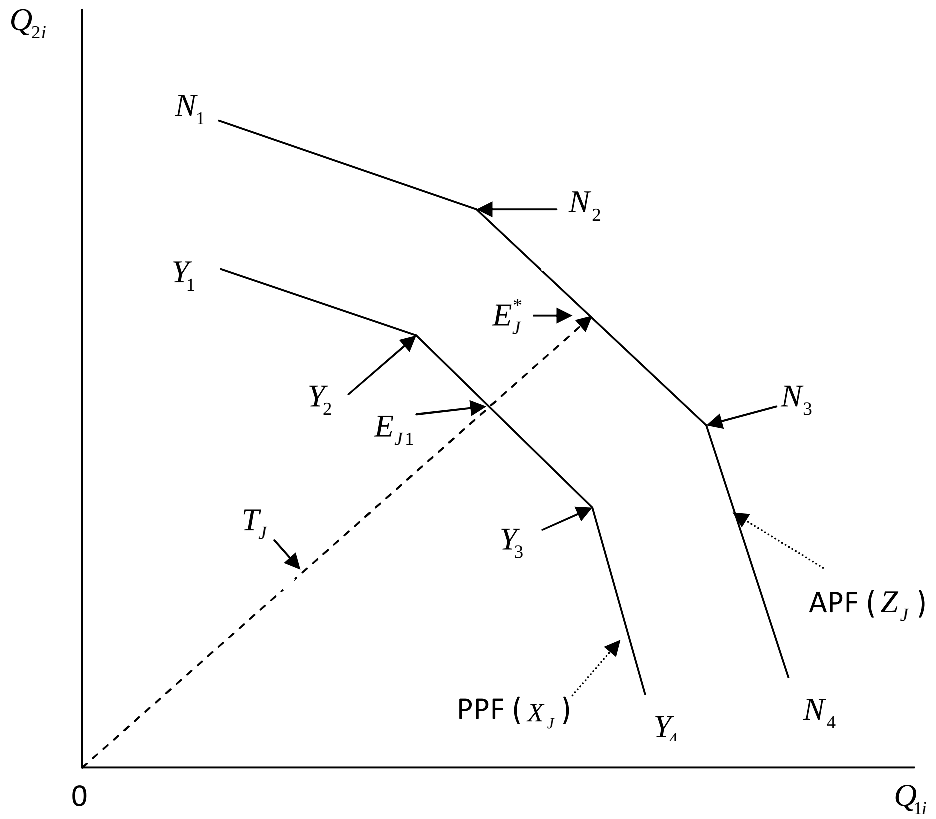

Fig 3: Attaining a higher possibility frontier under positive resource endogeneity

The highest possible frontier that is achievable by DMU J when resources are endogenous can be identified by considering the achievements of those public service providers which are fully production efficient and fully effective at raising revenue, in attracting able staff and at other activities which can increase their available resources. As a multi-dimensional generalisation of the endogenous resourcing

Eq. (4), we will assume that for DMUs which are within any given comparison group C of fully efficient and effective DMUs, we have the resources of type k which are available to DMU i are given by:

$$x_{ki} = \sum_{\substack{h=1 \\ h \neq k}}^{m} a_{kh} x_{hi} + \sum_{\tau=1}^{r} b_{k\tau} Q_{\tau i} + \sum_{\ell=1}^{p} d_{k\ell} z_{\ell i} \quad for\ k = 1,...,m\ and\ i \in C \tag{13}$$

with each $a_{kh} \geq 0, b_{k\tau} \geq 0$ and $d_{k\ell} \geq 0$, and $b_{k\tau} > 0$ for some k and τ and $d_{k'\ell} > 0$ for some k' for each ℓ. The logic of the endogenous resourcing equations given by (13) can be illustrated by their application to the case where the public service provider is a university operating under a budget constraint:

$$x_{1i} = x_{2i} + x_{3i} + z_i \tag{14}$$

where x_{1i} is the university's total expenditure budget, z_i is here its base level of exogenous government funding, x_{2i} is any additional tuition fee income that it raises from students outside those specified in its base level of government funding, and x_{3i} is the level of additional research grants which it attracts. Both x_{2i} and x_{3i} we assume to be in turn dependent upon the attractiveness of the university to students and to research grant-awarding bodies. Such attractiveness is determined by the quality of the university's teaching and research (which we denote by Q_{1i} and Q_{2i} respectively), by the ability of its staff in teaching and research (which we denote by the variables x_{4i} and x_{5i} respectively), and by its total expenditure level, x_{1i}, on staff, equipment and other facilities, as in the relationships:

$$x_{ki} = a_{k1} x_{1i} + a_{k4} x_{4i} + a_{k5} x_{5i} + b_{k1} Q_{1i} + b_{k2} Q_{2i} \quad for\ k = 2,3\ where\ each\ a_{kh} \geq 0, b_{k\tau} \geq 0 \tag{15}$$

Similarly, we will assume that the ability of the university to attract able staff depends upon its academic reputation, as reflected in the quality of the university's teaching and research, and upon its total expenditure budget, as in the relationships:

$$x_{ki} = a_{k1} x_{1i} + b_{k1} Q_{1i} + b_{k2} Q_{2i} \quad for\ k = 4,5\ where\ each\ a_{kh} \geq 0, b_{k\tau} \geq 0 \tag{16}$$

The resultant input vector $X_i = (x_{1i},...,x_{5i})'$ is here *endogenous* because it depends upon the output quality levels Q_{1i} and Q_{2i} via the relationships (14) – (16). These inter-relationships exist *in addition to* the one-way *production correspondence* that maps X_i into $Y_i = (Q_{1i}, Q_{2i})$ which the standard DEA program (12) seeks to reflect as a representation of the production *supply-side* of output quality.

However, the additional relationships (14) – (16) reflect important additional considerations that affect its resource availability and output quality, such as the *demand* by students for places at the university, *the willingness* of grant-awarding bodies *to pay* research grants to the university, and the *labour market* willingness of able staff to work for the university.

The general form of (13), which takes into account such additional inter-relationships in a linear way, can be written in the matrix form:

$$X^C = AX^C + BY^C + DZ^C \tag{17}$$

For $I - A$ non-singular, the input vectors for a fully efficient and effective DMU in the comparison group C then become linear functions of its output quality vector and its exogenous variables (such as the base level of its government funding) of the form:

$$X^C = (I - A)^{-1} BY^C + (I - A)^{-1} DZ^C \tag{18}$$

For any given output quality vector Y_J'', DMU J is assumed to face similar inter-relationships less any shortfall, given by a vector ε_{JX}, in the resources it actually secures compared to what it could have achieved if it were fully efficient and effective, so that we have:

$$X_J = AX_J + BY_J'' + DZ_J - \varepsilon_{JX} \quad where \;\; \varepsilon_{JX} \geq 0 \tag{19}$$

The *maximum possible output quality* vector Y_J^{**} which producer J could produce is given by:

$$Y_J^{**} = \varphi_J^{**} Y_J + s_J = Y^C \mu^C \quad where \; s_J \geq 0 \tag{20}$$

for an appropriate choice of the comparison group C and associated vector μ^C, where Y_J in (20) is producer J's initial output quality vector. s_J is the non-negative vector of output quality slacks that producer J could have achieved if it were fully efficient and effective in addition to the maximum proportional expansion $\varphi_J^{**} Y_J$ in this initial quality vector that is implied by the convex combination $Y^C \mu^C$ of the achieved output quality vectors Y^C of DMUs in the comparison set C. However, the actual output quality vector Y_J'' which producer J does achieve is given by Y_J^{**} *less* a vector of quality shortfalls ε_{JY}, so that:

$$Y_J'' = Y_J^{**} - \varepsilon_{JY} = Y^C \mu^C - \varepsilon_{JY} \tag{21}$$

From (19) and (21), we have:

$$X_J = (I-A)^{-1} BY^C \mu^C + (I-A)^{-1} DZ_J - (I-A)^{-1} \varepsilon_{J0} \; where \; \varepsilon_{J0} = B\varepsilon_{JY} + \varepsilon_{JX} \tag{22}$$

In a similar way to the parameter γ in Eq. (9) for the case of one input and one output, the matrix $(I-A)^{-1}$ provides a set of *multiplier effects*, here for the impact on producer J's available input vector X_J of reductions in its vector of overall effectiveness shortfalls ε_{J0}. We can illustrate the strength of these multiplier effects by considering the case given by Eqs. (14) – (16) in which $m = 5$, using the following numerical values:

$$(I-A) = \begin{bmatrix} 1 & -1 & -1 & 0 & 0 \\ -0.2 & 1 & 0 & -0.3 & -0.4 \\ -0.3 & 0 & 1 & 0 & -0.5 \\ -0.4 & 0 & 0 & 1 & 0 \\ -0.1 & 0 & 0 & 0 & 1 \end{bmatrix} \text{ and hence } (I-A)^{-1} = \begin{bmatrix} 3.45 & 3.45 & 3.45 & 1.03 & 3.10 \\ 1.24 & 2.24 & 1.24 & 0.67 & 1.52 \\ 1.21 & 1.21 & 2.21 & 0.36 & 1.59 \\ 1.38 & 1.38 & 1.38 & 1.41 & 1.24 \\ 0.34 & 0.34 & 0.34 & 0.10 & 1.31 \end{bmatrix} \tag{23}$$

Once the above interactions in Eqs. (19) – (21) are taken into account, a unit reduction in simply the first element of ε_{JX} would increase the availability of the first input by 3.45 times the initial reduction in its shortfall. Similarly, a unit reduction in its shortfall each element of the effectiveness shortfall vector ε_{J0} would here increase producer J's availability of the first resource input by (3.45+3.45+3.45+1.03+3.10) = 14.48 units. At the same time, it would similarly increase the availability of the other four inputs by 6.91, 6.38, 6.79 and 2.43 units respectively. Recognising the endogeneity of the resource inputs can therefore make a substantial difference to the resourcing level, and the associated output quality outcomes which can be used as feasible targets, for producer J if it does become fully efficient and effective.

The *maximum feasible value* X_J^* *for the resource inputs and the associated output quality* for producer J can be found by setting its overall effectiveness shortfall vector ε_{J0} equal to zero in Eq. (22). When resources are endogenous, the original constraint $X_J \geq X\mu$ in the DEA program (12) that involves a fixed input vector X_J can be replaced by one that requires that producer J could have at least as much of each input if it were fully efficient and effective as a relevant convex combination of the input vectors of other DMUs, i.e. by $X_J^* \geq X\mu$. Since the vector μ will include only positive weights on the input vectors of DMUs that are in the comparison group C of fully efficient and effective DMUs and zero weights on all others, we will have $X\mu = X^C \mu^C$, where X^C includes only the input vectors of DMUs that are in C, and μ^C is the vector of the corresponding positive elements of μ. Using Eqs. (18) and (22), the constraint $X_J^* \geq X\mu = X^C \mu^C$ now becomes:

$$(I-A)^{-1}BY^C\mu^C + (I-A)^{-1}DZ_J \geq (I-A)^{-1}BY^C\mu^C + (I-A)^{-1}DZ^C\mu^C \tag{24}$$

and hence
$$(I-A)^{-1}D(Z_J - Z^C\mu^C) \geq 0 \tag{25}$$

When all the elements of $(I-A)^{-1}$ are positive, as in (23), and all the elements of the matrix D in Eq. (13) are non-negative with at least one positive in each column, all the elements of the matrix $(I-A)^{-1}D$ will also be positive. These conditions guarantee that the maximum attainable value of each input for DMU J within X_J^* is an increasing function of each element of its vector Z_J of exogenous variables. From (24) and (25), the constraint $X_J^* \geq X\mu = X^C\mu^C$ will then be satisfied *for all such positive values* of the elements of $(I-A)^{-1}D$ and for any given comparison group C by requiring that:

$$Z_J \geq Z^C\mu^C \text{ where } Z^C\mu^C = Z\mu \tag{26}$$

i.e. the exogenous variables which DMU J faces are no worse than the convex combination of those faced by DMUs in the comparison group C. Under such circumstances, DMU J could have attained at least as much of each input if it were fully efficient and effective as the relevant convex combination of the input vectors of other DMUs.

As in Eq. (7), also relevant are the *stability conditions* which ensure a stable solution to the multiplier process in (22), and which can be shown to require that the principal minors of the matrix $I-A$ are all positive (see Quirk and Saposnik, 1968). Under such stability conditions, it follows from Morishima (1963, p. 15) that whenever $a_{kk} = 0$ and $a_{k\ell} > 0$ for all $k \neq \ell = 1,...,m$, all the elements of the inverse matrix $(I-A)^{-1}$ will indeed be positive. Moreover even if we relax this condition to simply $a_{k\ell} \geq 0$ for all $k,\ell = 1,...,m$, it follows from Morishima (1963, p. 15) that it will be sufficient for the elements of the matrix $(I-A)^{-1}$ to all still be positive under such stability that the A matrix is indecomposable, i.e. cannot be transformed, by permutations of the same rows and columns, to a matrix of the form $\begin{bmatrix} A_1 & A_2 \\ 0 & A_3 \end{bmatrix}$ where A_1 and A_3 are square sub-matrices on the main diagonal.

When we replace the constraint $X_J \geq X\mu$ under a fixed input vector X_J within the conventional DEA program (12) with the less restrictive constraint $Z_J \geq Z\mu$ that permits input vectors to vary endogenously subject to the exogenous parameters which the DMUs face, the associated DEA program becomes:

$$\max_{\phi_J, \mu_0} \phi_J \;\; s.t. \;\; \phi_J Y_J \leq Y\mu_0, Z_J \geq Z\mu_0, e\mu_0 = 1, \mu_0 \geq 0 \;\; for \; e \equiv (1,1,..,1) \tag{27}$$

with the positive elements in the optimal value of the vector μ_0 defining the relevant comparison group C of DMUs for producer J under this less restrictive formulation. In a parallel way to our 2-dimensional case of Sect. 2 above, our multi-dimensional exploration of the implications of resource endogeneity here yields a well-defined modified DEA program (27) in the space of the (Z_i, Y_i) vectors, rather than in the (X_i, Y_i) space of the conventional DEA program (12). The new DEA program (27) therefore defines a multi-dimensional *Achievement Possibility Frontier* (APF), which maps out the frontier of output qualities in each relevant direction which producer J could achieve if it became fully production efficient and fully effective at boosting its available resources, given the external exogenous factors which it faces.

We will denote by $\mu_0^*(Z_J, Y_J)$ the optimal value of the vector μ_0 for the given values of Z_J and Y_J in (27). If producer J does become fully efficient and effective, so that $\varepsilon_{J0} = 0$ in (22), we have the associated optimal resource vector which producer J could achieve given by

$$X_J^*(Z_J, Y_J) = (I - A)^{-1} BY\mu_0^*(Z_J, Y_J) + (I - A)^{-1} DZ_J \tag{28}$$

where the vector $\mu_0^*(Z_J, Y_J)$ places positive weights on the output vectors in the sub-matrix Y^C of Y for DMUs in the corresponding comparison group C, and zero weights on DMUs outside this reference set. The optimal resource vector $X_J^*(Z_J, Y_J)$ in (28) can be regarded as the multi-dimensional generalisation of the point x_J''' in Fig. 1, being the equilibrium outcome of a multiplier process from efficiency improvements that result in improved output quality and hence also greater resource availability when resourcing levels are endogenous. The point E_J^* in Fig. 3 on the APF facing producer J therefore corresponds to the point along the ray $0E_J^*$ through Y_J at T_J that lies on the PPF which producer J could attain if it did secure the optimal resource vector $X_J^*(Z_J, Y_J)$.

4 Application

For empirical analyses, differences in the production processes and associated cost functions across science, arts, medical and engineering Departments within universities make university Departments covering more specific subject areas a more suitable focus for efficiency analysis than an analysis at university level, particularly when different universities involve different subject mixes. We will therefore illustrate how DEA can be used empirically to explore the quality frontier between teaching and research for a single subject category, namely that of Economics and Econometrics, based upon our above analysis. In order to keep our illustration relatively straightforward, we will focus upon a recent period of time in which there was a major exogenous component to government funding for individual universities in the UK. This was the period before 2012-13 when individual UK universities were subject to strict externally determined controls on the total number of funded home and EU undergraduate and taught Masters students which they could admit, with standardised national fee remuneration based upon these controlled student numbers determining the associated block government grant to the university. The partial relaxation of these student number controls from 2012-13 onwards (see DBIS, 2011), and the accompanying freedom of individual universities in England to compete with each other, in large part on the basis of their teaching and research quality scores, for additional well-qualified home and EU students, and freedom to determine their own tuition fees, add further complexities to the scope for endogeneity, including of home and EU student numbers from 2012-13 onwards, that we will examine in a later paper.

Before 2012-13, the latest available comprehensive quality assessment of the research output in individual university subject areas in the UK was that of the Research Assessment Exercise (RAE) that was carried out in 2008, based upon publications in the previous five years submitted to the assessment panels by the census date of 31st October 2007. The relative quality weights of 0, 1, 3 and 9 were placed by the Higher Education Funding Council for England (HEFCE, 2010a) on its assessment of the relative importance of the different quality grades 1*, 2*, 3* and 4* on individual publications. The average quality-weighted score, which ranges from 0 to 9, for each university's submitted publications in a given subject area for this period provides the quantitative research quality measure Q_{iR} used in our empirical application of the modified DEA program (27).

The quality of teaching and associated facilities in UK universities has been assessed in this period by an annual National Student Survey (NSS) of final-year undergraduates, with the proportion of student who agree, or strongly agree, in response to Q22, taken to provide an overall summary of the degree of satisfaction of students with the quality of their course (see HEFCE, 2010b) in a given subject area. We will therefore use this proportion as our quantitative measure Q_{iT}, for the Economics subject area for the academic year 2006-7 as the latest available for such final-year students for the period in question.

In our DEA study of the quality frontier between teaching and research quality, we combine this proportion of satisfied students with the available RAE research quality measure for the Economics and Econometrics Unit of Assessment 34 for research in this subject area during the period up to 31st October 2007 as our two output quality variables. There were a total of 50 universities which took part in the NSS for Economics for the academic year 2006-7. There were also a few universities, such as the University of Cambridge, which declined to take part in the NSS for that year, even though they took part in the RAE. Rather than substitute a score of zero for their teaching quality assessment, these universities were excluded from the sample. However, within the 50 universities which took part in the NSS for Economics for the academic year 2006-7, there were 21 which made no submission to the RAE 2008 for Economics and Econometrics. Since a positive outcome from a RAE 2008 submission would have been to their financial advantage and enhanced their academic reputation, a non-submission is taken to imply a lack of confidence in a positive assessment, with these universities given a zero score for their associated Q_{iR} measure in the analysis.

TABLE 1: The distribution of effectiveness scores across DMUs

Effectiveness Score	**DMUs**	**No of DMUs**
$=1.0$	2(9), 6(0),22(1),27(6),28(2),36(1),42(15),45(36),47(4)	9
≥ 0.9 & < 1.0	4, 7, 12, 13, 14, 15, 18, 19, 23, 24, 26, 29, 33, 35, 39,41,43, 44, 46, 48	20
≥ 0.8 & < 0.9	1, 9, 10, 11, 17, 32, 37, 38, 40, 49, 50	11
≥ 0.7 & < 0.8	3, 5, 8, 16, 21, 25, 30, 31, 34	9
≥ 0.6 & < 0.7	20	1

The exogenous variable which is used in the empirical application of our modified DEA program (27) as our single input variable z_i for each Department i is that of the total home and EU student numbers for undergraduates and taught postgraduate students in Economics for the academic year 2006-7, which determines the associated level of the base government funding to the university for students in this subject area. The two output variables used were the research and teaching quality scores Q_{iR} and Q_{iT} specified above. The results of this analysis are shown in Tables 1 and 2. Table 1 shows that there are 9 of the 50 relevant DMUs which have overall effectiveness scores of 1.0. However, since the 21 universities which did not make an RAE submission for Economics and Econometrics in 2008 are labelled 1 – 21, it can be seen that only two of these, namely DMUs 2 and 6, have such a score. The figures in the brackets in the second line of Table 1 indicate how many comparison groups for other DMUs the respective DMU enters into. Thus, whilst DMU 2 entered into 9 such comparison groups, DMU 6 failed to enter into any, so that those DMUs which concentrated their efforts on teaching rather than research are in general not shown as being outstandingly effective at achieving output quality. Eight of the 20 DMUs which had an overall effectiveness score of between 0.9 and 1.0 were, however, amongst those that concentrated on teaching. At the same time the DMU with the lowest overall effectiveness score, and 5 of the 9 DMUs with an efficiency score between 0.7 and 0.8, were amongst those that concentrated on teaching. Of the 7 DMUs which did have positive RAE submissions and are assessed as being fully effective, DMU 45 enters into by far the largest number, namely 36, of comparison groups of other DMUs.

TABLE 2: Average scores and slacks for the two groups of DMUs

	DMUs 1 – 50	**DMUs 1 – 21**	**DMUs 22- 50**
Average Effectiveness Score	0.8926	0.8648	0.9126
Average z slack	76.681	57.537	90.543
Average Q_R slack	0.976	2.170	0.111
Average Q_T slack	0.000	0.000	0.000
Average Q_R score	2.139	0.000	3.688
Average Q_T score	0.826	0.808	0.839
Average z value	314.33	224.32	379.51

As in Table 2, the average value of the overall effectiveness score for those DMUs that concentrated on teaching was below the corresponding average for those that had positive scores in the RAE. This is despite the fact that the output-orientated DEA analysis allows each DMU to choose its own output mix, and then estimates the proportionate feasible increases in its outputs for this given output mix. Table 2 also shows a lower average NSS score for the DMUs that concentrated on teaching, and larger average slacks for potential research quality, when compared with those DMUs that also made positive RAE submissions. Even though the DMUs that concentrated on teaching had on average smaller intakes of home and EU students, the analysis also revealed decreasing returns to scale at all points along the quality frontier, except for the points corresponding to DMUs 6 and 36, which exhibited constant returns to scale.

We can thus obtain useful empirical insights from the modified DEA program (27). Moreover, this is true even though comprehensive detailed data are not available at an individual subject area or Departmental level for universities across the UK for the period in question for the important expenditure and staffing input variables which would need to enter into the estimation of a standard DEA program of the form (12).

5 Relationship to Existing Literature

As noted in Sect. 1 above, there are numerous existing applications of DEA seeking to identify the conventional PPF for public service providers. In the case of universities, for instance, Johnes and Johnes (1995) use DEA to identify those DMUs which are on or below a PPF that involves different categories of research publications as outputs, with research grants classified as one of the key *inputs*. In contrast, Izadi et al. (2002) use the value of research grants and contracts received as a key *output* in their efficiency analysis. The difficulty in categorising research grant income as either an input or an output is arguably better resolved by explicitly recognising it as an *endogenous resource input* which contributes towards the production of research publications, but with the ability to attract such research grant income also dependent upon the quality of research being produced by the DMU.

The extent of the bias in the parameter estimates of a conventional production function, and its associated PPF, which the presence of endogeneity for public service providers in sectors such as education can produce when single-equation regression-based econometric techniques, such as Ordinary Least Squares (OLS), are deployed is discussed in detail, for instance, in Mayston (1996, 2007, 2009). However, "because additional demand-side relationships can systematically change the set of observed points, in ways which a production frontier alone cannot adequately model, DEA is itself not immune from endogeneity bias, even in the case of multiple outputs" (Mayston 2003). Yet, as recently stressed by Cordera et al. (2013), "the potential distortions that endogeneity may cause in the measurement of technical efficiency using nonparametric techniques have received much less attention in the literature" than is the case for econometric models. Following earlier contributions by Orme and Smith (1996), Bifulco and Bretschneider (2001, 2003), Ruggiero (2003a, 2003b) and Johnson and Ruggiero (2011), Cordera et al. (2013) have recently concluded from their detailed simulation study that "a high positive endogeneity level, i.e., a high positive correlation between one input and the true efficiency level, severely biases DEA performance". As we have discussed above, such positive correlations may well exist in sectors such as education. Therefore, rather than attempting to use DEA to produce biased estimates of the position of a PPF in the conventional (X_i, Y_i) input-output space, our above approach uses DEA to estimate an Achievement Possibility Frontier in the space of the (Z_i, Y_i) vectors, where the variables in Z_i are explicitly chosen to be exogenous, and therefore uncorrelated with the true efficiency levels of the individual DMUs. It therefore provides a positive way forward in response to Cordera et al. (2013)'s plea that "a technique should be developed to deal with endogeneity in order to improve DEA estimations".

By looking at the maximum feasible output quality that a DMU can achieve given the exogenous environmental variables which it faces, our approach parallels the specification of a *reduced form equation* in econometrics (see e.g. Gujarati and Porter 2010, p. 352) which can be used to produce unbiased estimates of the impact of changes in the stochastic disturbance terms within a system of simultaneous equations on the equilibrium values of the endogenous variables. For an application of this approach to the address the endogeneity problem in Stochastic Frontier Analysis, see Mayston (2015). The separate identification and estimation of the parameters of the underlying structural inter-relationships are not necessary for the unbiased estimation of the reduced form parameters.

A basic assumption of DEA models, such as the BCC output-orientated model (12), is that of convexity of the associated production possibility set facing any given DMU i given by:

$$\Phi_i = \{X_i, Y_i : X_i \geq 0 \text{ and } Y_i \in P(X_i) \subset R_+^r\} \tag{29}$$

where $P(X_i)$ is the set of outputs which it is feasible to produce from an input vector of X_i under existing technology, and R_+^r is the non-negative domain of r-dimensional Euclidian space. A feasible combination (Z_i, Y_i) in our above model is one such that:

$$X_i = AX_i + BY_i + DZ_i - \varepsilon_{iX} \quad where \;\; \varepsilon_{iX} \geq 0 \;\; \text{and} \;\; (X_i, Y_i) \in \Phi_i \tag{30}$$

If follows from (30) that if (Z_i', Y_i') and (Z_i'', Y_i'') are both feasible combinations, then so too is (Z_i''', Y_i'''), where:

$$Z_i''' = \omega Z_i' + (1-\omega)Z_i'', \, Y_i''' = \omega Y_i' + (1-\omega)Y_i' \;\; and \;\; X_i''' = \omega X_i' + (1-\omega)X_i'' \quad where \; 1 \geq \omega \geq 0 \tag{31}$$

when Φ_i is a convex set. Convexity of Φ_i in the (X_i, Y_i) space, as the BCC model (12) assumes, therefore implies here convexity of the feasible set in the (Z_i, Y_i) space for the DEA program (27).

It should be noted that Z_i in our above model does not directly enter the production process, but instead is a vector of exogenous variables that influences the input vector X_i via the inter-relationships given by (30), and therefore affects the maximum feasible output quality which any given DMU can attain given the exogenous environment that it faces. Our above model therefore differs here from those of Banker and Morey (1986) and of Ruggiero (1996) in which environmental variables enter directly into the production process, with Ruggiero (1996) relaxing the convexity condition which Banker and Morey (1986) retained for their direct influence in the production process. Here convexity of the feasible set in the (Z_i, Y_i) space follows directly from the basic DEA assumption of convexity of the feasible set in the (X_i, Y_i) space, under the linear endogeneity relationships in (30).

As stressed above, our main focus in this paper is with addressing questions (i) – (iii) of Sect. 1, when additional inter-relationships exist between inputs and output quality beyond those of the uni-directional production correspondence assumed by standard DEA models. We have shown that answers to these questions can be obtained by adopting a modified form of DEA in which the exogenous variables facing individual DMUs determine the underlying constraints within which their inputs may be endogenously varied. How well individual DMUs do in achieving output quality subject to the exogenous variables which they face is then the key to answering questions (i) – (iii). This approach is both powerful and efficient in its data demands. It does not require detailed data on the input expenditure patterns of individual DMUs, which, as our above application illustrates, may not be readily available. It does not require detailed quantitative knowledge of the parameters of the additional underlying structure inter-relationships beyond the general linearity assumptions involved in (30) and associated stability assumptions. Whilst these structural parameters influence the overall outcome, the data one needs to estimate the Achievement Possibility Frontier under the modified DEA program (27) are simply the resultant observable output quality outcomes for the individual DMUs and the exogenous variables they face in achieving them. This frontier is not the same as the PPF for the current input vector of any inefficient DMU, since the APF recognises that improvements in the efficiency of such a DMU in boosting its output quality can in turn attract a higher level of resources and shift out the relevant PPF, as in Fig. 3 above.

The extent to which an individual DMU could improve its output quality subject to the exogenous variables which it faces is revealed in the modified DEA program (27) by a comparison of the DMU's current output quality with a convex combination of the output qualities currently attained by efficient and effective DMUs in its comparison group who have had the opportunity to maximise their output quality subject to the exogenous variables which they face. While the extent of the feasible improvement is shown diagrammatically in Fig. 1 as occurring sequentially as a series of steps, the DEA program (27) identifies the final outcome of this multiplier process of feasible improvement, whether it is made in one step or many. It is indeed this final outcome which is relevant to answering questions (i)-(ii) of Sect. 1.

Rather than viewing the current output qualities of efficient DMUs as a result of their equilibrium achievements under the exogenous variables which they face, an alternative approach would be to model the world as being in a state of flux, involving the dynamic analysis over time of the inter-dependencies between output quality and resource availability. Some progress can be made in this direction by using past levels of output quality as pre-determined variables within the relevant Z_i vectors in the efficiency analysis. However, if they are to be truly exogenous, possible inter-temporal correlations in the efficiency levels of individual DMUs may need to be excluded. An alternative approach would be that of network DEA (see e.g. Cook et al. 2010; Cook and Zhu 2014) in which a two-stage DEA model is used in which the outputs from the first stage can form part of the inputs for the second stage. In comparison, our above approach is essentially a multi-stage multiplier approach in which efficient DMUs have converged on a stable equilibrium outcome for their given exogenous variables. If these equilibrium outcomes form the available database for efficient DMUs, then our modified DEA program (27) provides a direct way of assessing the overall performance of individual DMUs. However, if individual efficient DMUs are yet to converge on such an equilibrium outcome and sufficient additional data on detailed resource inputs are available, a multi-stage version of network DEA, rather than simply a two-stage version, may provide an interesting comparison with the results of our above modified DEA program.

6 Conclusions

By considering the additional inter-relationships which may exist for a public service provider between its output quality and its available resource inputs, we have been able to make theoretical and methodological developments which enable DEA to move beyond the consideration of the standard uni-directional relationship between inputs and outputs that is inherent in its estimation of a conventional PPF. Instead our analysis has focussed on the estimation of a more general Achievement Possibility Frontier that incorporates important multiplier effects from improvements in a given DMU's technical efficient and/or in its effectiveness in raising finance and in attracting and retaining clients and able staff. We have examined applications of this analysis to both local public broadcasting and higher education, in which public service output quality has important interactions with resource availability. Our resultant modified DEA methodology is efficient in its data needs and productive in indicating the wider set of opportunities which are available to public service producers once resource inputs are no longer taken as fixed, but are rather are endogenous to their own output quality achievements.

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