UNIVERSITY *of York*

Discussion Papers in Economics

No. 14/28

Default and Risk Premia in Microfinance Group Lending

P Simmons and N Tantisantiwong

Department of Economics and Related Studies
University of York
Heslington
York, YO10 5DD

Default and Risk Premia in Microfinance Group Lending

P Simmons & N Tantisantiwong
Department of Economics — School of Business
University of York, UK — University of Dundee, UK

Abstract

For a risk neutral lender and a group of borrowers facing identical revenue risks we compare individual loans and group lending. We stress the importance of group liquidity in defining the necessary risk premium. There are no welfare differences between the loan forms. However, the default rates and risk premia vary ambiguously between the loan forms. Simulations replicating empirical interest rates and default rates show that the group interest rate is lower for a larger group while the effect of group size on default risk is ambiguous. We then consider the case of identical correlated risks between borrowers. Positive correlation of projects gives a higher downward risk, so a higher group interest rate and a higher fraction of successes are required. Unlike independent group lending, the interest rate and the default risk are not lower in the larger group loan with correlated returns. Simulations using beta-binomial distributions are presented.

Key words: Group lending, default rate, interest rate, correlated outcomes

JEL No: G21; O16

1 Introduction

Group lending with joint liability of borrowers has been hailed as an innovation which will assist economic and social development in poorer regions (Ahlin et al., 2011; IMF, 2005). Its advantages are seen as a way of enabling excluded borrowers to access finance because it reduces the problems of lack of collateral, asymmetric information (ex-ante adverse selection and ex-post monitoring and audit costs), and administration costs of a large number of small loans. It can also overcome gender problems where often male and female adults have different risk and time preferences and different access to financial resources (Booth et al., 2011). In a rosy scenario a group of poor individuals with no collateral could successfully secure a joint loan with joint liability for repayment. Then once the group has a reputation for low default, it can enter the formal financial system and secure loans from banks or other financial institutions. The two driving forces which are seen so attractive are its role in reducing default risk by spreading liability across multiple risks and the peer pressure for good behaviour exerted by one borrower on another.

Empirical group sizes are quite varied. Historically one common pattern is for group lenders to start with group lending in the sense above: loans of small size per capita within the group, for small scale projects which borrowers see as key to improvement. A typical Grameen bank group loan in the 1990's had no more than five borrowers and often the first loan is to a group of just two borrowers. However, some groups have been much larger than this. Ghatak and Guinnane (1999) report that German rural credit cooperatives usually had 75-250 members. The Foundation for International Community Assistance (FINCA) establishes "village banks" , a form of microlending, with group sizes between 10 and 50 members and claims that they have acheived a high repayment rate[1]. Then, the loans evolve into increasing size of loans to individuals, or increasing interest rates for groups with low average loan sizes (Gonzalez, 2010). Vigenina and Kritikos (2004) find that the size of individual loans tends to be larger than that of group loans. Other writers find a small group size is important in determining group loan success (Devereux and Fishe, 1993). Using 124 institutions in 49 countries, Cull et al. (2007) find that an increase in loan size is associated with lower cost, leading to a higher rate of return on assets for individual-based lenders. In addition, Gonzalez (2010) finds that the smaller loans have a higher operation cost and so lenders set a higher interest rate premium.

There are issues of how to measure group lending performance. In the microfinance empirical literature, default rates, interest rates, cost efficiency together with measures of outreach (the coverage of the population) are criteria used to judge the performance of group lending (Cull et al., 2007; Cull et al., 2009; Ahlin et al., 2011; Hermes et al., 2011). The default record on group loans is mixed. Apart from individual characteristics of borrowers, other features that cause variation in the default rate are the form of monitoring the group undertakes, repayment frequency, the type of lender (private sector or not) (Deininger and Liu, 2009). Another important issue is the role of correlation of returns between borrowers (Ahlin and Townsend, 2007). If most group loans are to borrowers who are similar to one another (they usually live in the same area and the social pressure for good behaviour relies on the borrowers knowing and interacting with each other), then systemic shocks like local bad weather are likely to affect all borrowers, leading to positive correlation of revenues. This affects the possibilities for risk diversification between group members. Varian (1990) notes that borrowers' homogeneity can make the lenders worse off. However, the literature typically uses special representations of the correlation.

Many prior studies emphasise the effects of peer pressure in group lending to give incentives for

[1] Sorce: FINCA (www.finca.org)

repayment. But a prior question is how the liquidity of the group varies with the outcomes of group members incomes. If each individual has a two state probability distribution (income succeeds or fails), the percentage of sucesses necessary to cover group repayment is a key measure of group liquidity. Group liquidity will vary with the size of group and the nature of risks faced by group members. If there is a high chance of low group liquidity, then there is a high chance of group default. In the context of this paper we abstract from ex-ante asymmetric information and from enforcement problems. The lender knows as much as the borrowers at each stage so there is no deadweight loss of ensuring incentive compatibility or monitoring. However, the lender can seize all the available assets of borrower(s) in the event of nonrepayment. The parties are all risk neutral. Collectively and individually, the borrowers investment projects generate an expected surplus above their cost at the safe interest rate. The lender receives the same expected payoff equal to the safe rate of return on the aggregate of loans to all borrowers no matter how the loans are organised. Since there is no deadweight loss, collectively the borrowers get the same expected return with any loan arrangement. In ex-ante welfare terms all the loan schemes are equally efficient. But they differ substantially in the interest rate and the default rate on the loans. We study the effects of the form of loan on group liquidity, default rates and interest rates.

In this paper we concentrate on how the group interest rate, the minimum number of successes required for repayment, and the default rate on group lending varies with group size, the size of loan, the probability distribution of revenues of borrowers and the market interest rate. We find that with a risk neutral lender and risk neutral borrowers, under symmetric information, individual lending and group lending with averaging over individual loans are equivalent in terms of risk premia and default rates. However, the interest rates and default rates for an individual loan and a jointly liable group loan are different; for example, group loans should typically have lower interest rates than individual loans. This is true whether or not individual borrower risks are correlated.

For a group size of at least two, the group interest rate falls with the size of the group. We find that the difference between the group and individual loan interest rates and the increase in group liquidity may not be monotone in the group size. It depends on the parameters of the project and the group size.

Regarding the default rate, we find that it falls with increases in the mean profitability of the projects at a given group size, but it does not change monotonically with group size at a given project probability distribution. The simulations also show that with a low project probability of success, the default rate of the large group tends to be higher than that of the small group while the interest rate falls. This may explain why microfinance institutions (MFIs) e.g. Grameen bank tend to lend to small groups especially during periods of crisis.

There are clear comparative static results on the impact of borrower collateral, loan size and the safe interest rate on loan interest and default rates. The effects of group size or probability of success are less obvious; some of the complication arises from small numbers and the discreteness of the distribution of the number of project successes. For example with three borrowers the difference in group revenue and the chance of default by the group when one more project succeeds is substantial since it involves a large percentage increase in the revenue of the group. These small number effects are important for group sizes below about 20, at which point the underlying binomial distribution of the number of project successes can be well approximated by a normal distribution. In fact the importance of small numbers in the group lending scenario seems to have been overlooked in the literature.

For correlated borrower risks, to analyse loan interest and default rates we seem to be the first to use the beta binomial mixture distribution, which seems well suited to the situation of lending to a local pool of borrowers, who are subject to identical systemic risk but who also face idiosyncratic risk. While previous studies concentrated on the effect of the correlation of project returns on the default rate, we find that the correlation affects not only the default rate, but also the interest rate and the minimum number of successes required for repayment. We find that with a given group size, the group with higher positive correlation would be charged a higher interest rate, while the group loan default rate can rise or fall with the correlation of project returns depending on whether the mean probablity of success is high or low. In simulations we find that as the correlation between projects increases, the default risk is not lowered by increasing group size and actually increases if the projects have low mean returns. Therefore, group lending may have a higher default risk than individual lending if the project returns are low and correlated.

Our analytical and simulation results show that there are few reasons to prefer one loan form to another in terms of interest rates or default rates which much depend on the parameters of the risky projects under consideration. Again there is no robust analytical result on the group size in a group loan which will minimise either the default rate or the interest rate. This finding matches the diversity of group sizes supported by different MFIs.

The plan of the paper is organised as follows. Section 2 outlines a simple model of risks facing individuals to see the default rate and risk premium that a risk neutral lender would experience on individual and group loans. In this section, we assume the individual risks are uncorrelated and analyse the default rate and interest rate on individual and group loans. Next we analyse the factors determining the interest rate, the minimum number of successes required for repayment and the default rate and simulation results are also provided. Section 3 examines the case of correlated risks between lenders using a binomial beta distribution. Finally, we add some discussion of asymmetric information and administrative cost in the conclusion.

2 A Simple Model with Identical Independent Risks

Our analysis is in the context of complete symmetric information, lenders and all borrowers know the distribution of returns to any borrower and can observe its realisation[2]. The risk neutral lender has access to a safe interest rate r. There are n borrowers. Each borrower has a project which requires finance B and which yields one of two returns: with probability p the project succeeds and yields high revenue H, with probability $1-p$ the project fails and yields low revenue L. For sure the borrower can always repay L and so we can also think of L as collateral that the borrower can post. The setup is equivalent to one in which the project yields 0 if it fails, but the borrowers collateral of L can then be seized. We assume that the projects are independent and there is no correlation between returns on projects. To ensure risk on the loan we assume that $H > (1+r)B > L$ so that only successes can repay individual loans and, with individual loans, fails default. We also assume $pH + (1-p)L > (1+r)B$ so that individual projects are

[2]In fact this could be weakened to the lender having a low audit cost in the event of default, see below.

socially desirable. If a loan defaults the lender knows that the borrower must have assets of at least L. It is well known that applying maximum punishment on false defaulters minimises the risk of false reporting and helps attain incentive compatibility under risk neutrality (see for example Border and Sobel (1987)). So we assume that on default, the contract stipulates that the lender can seize all of the assets of the borrower(s).

2.1 Individual Loans

The lender will offer an individual loan at an interest rate R^I satisfying

$$(1+r)B = p(1+R^I)B + (1-p)L$$

That is with probability p the project succeeds and the loan is repaid in full, with probability $1-p$ revenues do not cover the cost of the loan, the lender seizes all the available revenue of the borrower (L) and the borrower defaults. Thus R^I is set at

$$1 + R^I = \frac{(1+r)B - (1-p)L}{pB} = \frac{1+r}{p} - \frac{1-p}{p}\frac{L}{B} \tag{1}$$

The lender breaks even on average on each individual loan. The default probability on each loan is $1-p$ and the risk premium on loans is

$$(1+R^I) - (1+r) = R^I - r = \frac{1-p}{p}(1+r) - \frac{1-p}{p}\frac{L}{B} = \frac{1-p}{p}(1+r-\frac{L}{B}) > 0 \tag{2}$$

since $(1+r)B > L$. The low return on the project L plays the role of collateral, the lender is sure of getting at least this amount and so the higher L, the lower the default premium in the interest rate.

2.2 Averaging Over Individual Loans

Alternatively still with n loans of B the lender could set a common R^{II} on each loan with the aim of breaking even across the n loans. Each borrower still defaults with probability $1-p$ and in that case pays L, or succeeds and repays the loan with interest. Let k be the number of loans which succeed and deliver H revenue. That is, with independent chances of success or failure across projects, R^{II} is set by

$$(1+r)nB = E_k[k(1+R^{II})B + (n-k)L]$$

so

$$(1+R^{II}) = \frac{(1+r)nB - E_k[(n-k)L]}{BE_k k}$$

The default premium is

$$\begin{aligned} R^{II} - r &= (\frac{n}{E_k k} - 1)(1+r) - \frac{(n-E_k k)}{E_k k}\frac{L}{B} \\ &= \frac{(n-E_k k)}{E_k k}(1+r-\frac{L}{B}) > 0 \end{aligned}$$

because n must be no less than $E_k k$ and $n/E_k k > 1$.

With independent and identical risks $E_k k = np$,

$$R^{II} - r = \frac{(n-np)}{np}(1+r-\frac{L}{B}) = \frac{1-p}{p}(1+r-\frac{L}{B}) > 0 \tag{3}$$

With risk neutrality of the lender there is no gain in diversification when the risks are independent; the default premium is unchanged and the lender may as well treat each loan independently.

2.3 Group Lending With Independent Risks

In group lending, there is joint liability among members of the group. That is, if a member of the group defaults, then the other members have to repay not only their own loans but also the loan of that person in order not to default as a group.

A group formed by n borrowers borrows nB from a microfinance institution. Each borrower receives B to finance his/her project. Group revenue is $G(k) = kH + (n-k)L$ if there are k successes, and it is increasing in k since $H > L$.

The lender asks for a total repayment (including a risk premium) of P where $P = (1+R_n)nB$. The group has joint liability for repaying P or defaulting as a group. For a given P there may be a smallest group size k^* such that $P \leq G(k^*)$[3]. At k^* the repayment comes from the returns of the k^* successes and the $n-k^*$ fails. Then by definition for all $k > k^*, G(k) > P$. Thus, at any $k \geq k^*$ group revenue is sufficient to make the repayment so the group does not default and repays P. As a result, there is a

[3]Since k^* must be an integer number, there may be no integer number of successes which gives group revenue exactly equal to the required repayment. In that case, k^* is the lowest integer which gives the group revenue at least equal to the required repayment.

surplus for the group left, $G(k) - P$, after they have repaid P. Conversely, at $k < k^*$, $G(k) < P$ and group revenue cannot meet the repayment, so the group defaults and the whole of group revenue $G(k)$ is seized by the lender.

If the lender wants to cover expected losses arising from $k < k^*$ he chooses P so that

$$\begin{aligned}(1+r)nB &= \Pr(k \geqslant k^*)P + E[G(k)|k < k^*] \\ &= (1+R_n)nB\Pr(k \geqslant k^*) + E[Hk + (n-k)L|k < k^*] \end{aligned} \quad (4)$$

That is, the average lender return comes from full repayment by the group whenever $k \geq k^*$ and, if $k < k^*$, from the average value of group revenue in these states of group default. In a related paper, Baland et al. (2011) consider group lending when there is a zero return on individual projects if they fail so that effectively individuals and groups have no collateral, and also the lender cannot seize the available assets of the borrower(s) in the event of default. In their paper the lender cannot observe project outcomes but if an individual or group defaults then all borrowing parties pay a financial sanction. Repayment of a group loan when possible is shared equally between the project successes and to give each the incentive to repay, the loan size is capped at at a level which makes repayment when possible preferable to the sanction. In our approach since the lender can seize all borrower(s) assets in the event of nonrepayment, it is incentive compatible to repay.

When is the group loan viable? In general k^* may not exist i.e. even with n successes there may not be enough group revenue**.** But under our assumptions about project returns and cost, there is always a unique smallest k^*. With zero successes the group cannot afford to repay and hence must default. If all group members succeed then group revenue is $G(n) = nH > (1+r)nB$ and the group can certainly afford to repay. But group revenue is increasing in the number of successes, hence there must be a smallest critical number of successes k^* above which the group can repay and below which the group defaults.

This just requires $H > (1+r)B > L$. To see this formally

$$
\begin{aligned}
(1+r)nB &\leqslant \Pr(k \geq k^*)G(k^*) + E(G(k)|k < k^*) \\
&= \Pr(k \geq k^*)[(H-L)k^* + nL] + (H-L)E(k|k<k^*) + nL\Pr(k<k^*) \\
&= nL + (H-L)[\Pr(k \geq k^*)k^* + E(k|k<k^*)] \leq nL + (H-L)Ek
\end{aligned}
$$

yielding

$$(1+r)B \leq L + (H-L)\frac{Ek}{n}$$

so there is a lowest k^* so long as $[(1+r)B - L]/(H-L) \leq Ek/n$ the mean ratio of successes. The minimal k^* is unique since the group revenue is increasing in k.

It is clear that $R_n > r$. For $k < k^*$ the group cannot afford to repay $(1+R_n)nB$ and so if $R_n < r$, the lender will receive back less than the fair return for every value of k even when the group does not default. More formally,

$$1 + R_n = \frac{(1+r)nB - E(G(k)|k<k^*)}{nB\Pr(k \geqslant k^*)} \tag{5}$$

Note that (5) reduces to (1) of the individual loan when $n = 1$ i.e. $k^* = 1, \Pr(k=1)$ is the probability of success (p) and $E[k|k<k^*] = \Sigma_0^{k^*-1} kf(k) = 0$ where $f(k)$ is the binomial probability density function.

From (4) and (5),

$$
\begin{aligned}
R_n - r &= \frac{(1+r)nBp(k<k^*) - E(G(k)|k<k^*)}{nB\Pr(k \geqslant k^*)} \qquad (6) \\
&= \frac{[(1+R_n)nB\Pr(k \geqslant k^*) + E(G(k)|k<k^*)]\Pr(k<k^*) - E(G(k)|k<k^*)}{nB\Pr(k \geqslant k^*)} \\
&= \frac{[(1+R_n)nBp(k<k^*) - E(G(k)|k<k^*)]\Pr(k \geqslant k^*)}{nB\Pr(k \geqslant k^*)} \\
&= \frac{E((1+R_n)nB - G(k)|k<k^*)}{nB} > 0
\end{aligned}
$$

since at $k < k^*$ there is insufficient group revenue to repay the loan i.e. $G(k) < (1+R_n)nB \leqslant G(k^*)$.

2.4 Comparisons of Interest Rates and Default Rates between the 3 cases above

We can compare the risk premia and levels of default on individual and group loans. From (6),

$$R_n - r = (1 + r - \frac{L}{B})\frac{\Pr(k < k^*)}{\Pr(k \geqslant k^*)} - \frac{(H-L)}{nB}\frac{E(k \mid k < k^*)}{\Pr(k \geqslant k^*)}$$

Hence

$$\begin{aligned}(R^I - r) - (R_n - r) &= R^I - R_n \\ &= (\frac{1-p}{p} - \frac{\Pr(k < k^*)}{\Pr(k \geqslant k^*)})(1 + r - \frac{L}{B}) + \frac{(H-L)E(k \mid k < k^*)}{nB\Pr(k \geqslant k^*)} \qquad (7)\end{aligned}$$

With individual lending, the average percentage of defaults is $(1 - p)$. On the other hand, with group lending, the group defaults with probability $\Pr(k < k^*)$. The RHS is ambiguous in sign only because the first factor of the first term is of ambiguous sign.

$$\begin{aligned}\frac{1-p}{p} - \frac{\Pr(k < k^*)}{\Pr(k \geqslant k^*)} &= \frac{(1-p)\Pr(k \geqslant k^*) - p\Pr(k < k^*)}{p\Pr(k \geqslant k^*)} \\ &= \frac{1 - p - \Pr(k < k^*)}{p\Pr(k \geqslant k^*)} = \frac{1 - p - \Sigma_0^{k^*-1} f(k)}{p\Pr(k \geqslant k^*)}\end{aligned}$$

The expression $1 - \Sigma_0^{k^*-1} f(k)$ is decreasing in n. In the extreme case if $k^* = 1 < n, 1 - \Sigma_0^{k^*-1} f(k) = 1 - f(0) = 1 - (1-p)^n > p$. Therefore, the difference in risk premia is positive. On the other hand, if $k^* = n, 1 - \Sigma_0^{k^*-1} f(k) = f(n) = p^n < p$ so (7) is of ambiguous sign with the first term negative but the second positive.

The higher is k^*, the more likely that this term is negative and hence the ambiguity between individual and group default rates results. k^* depends on the group size, p, the possible project outcomes and the project cost. Even with complete information symmetry and a costless enforcement system on repayment, ambiguity in the default rates occurs because how group liquidity compares with individual liquidity is ambiguous. Without a costless enforcement system, successful borrowers can startegically default (Besley and Coate, 1995). A calibrated result is that the possibility that a successful borrower, who is due to repay his share of the group loan, can free ride on the repayments by other successful borrowers and renege on his own repayment makes the comparison between individual and group default rates ambiguous. The model of Besley and Coate (1995) has a continuous distribution of possible project outcomes where we only have two, but the ambiguity that we find even in a world of no reneging reinforces the point.

2.5 General Comparative Statics

The group lending contract is defined by two conditions which also give us the group default rate and the interest rate on the group loan

$$\begin{aligned}(1+r)nB &= (1+R_n)nB\Pr(k \geqslant k^*) + E[Hk+(n-k)L|k<k^*] \\ (1+r)nB &\leq G(k^*)\Pr(k \geqslant k^*) + E[Hk+(n-k)L|k<k^*]\end{aligned}$$

We can rewrite these as

$$(1+R_n) = \frac{1+r}{\Pr(k \geqslant k^*)} - \frac{(H-L)E[k|k<k^*] + nL\Pr(k<k^*)}{nB\Pr(k \geqslant k^*)} \tag{8}$$

$$z \leq K(p,n,k^*) \tag{9}$$

where

$$z = \frac{(1+r)B-L}{H-L}, K(p,n,k^*) = \frac{\Pr(k \geq k^*)k^* + E(k|k<k^*)}{n}$$

Two comparative static questions are of interest: how do R_n and k^* vary with the exogenous parameters B, H, L, r and particularly n and p? The key two endogenous variables are k^* and R_n. Since (9) involve an inequality, at best we can get bounds on how k^* and R_n vary with these parameters. If (9) is an equality then as z is monotonic in B, r, H and L, comparative static effects of these variables on k^* and R_n depend only on how K varies in p, n and k^*.

Recalling that we have a discrete distribution, $K(p,n,k^*)$ is increasing in k^* since

$$\begin{aligned}&K(p,n,k^*+1) - K(p,n,k^*) \\ &= \frac{(k^*+1)}{n}\Sigma_{k^*+1}^n f(k) + \Sigma_0^{k^*}\frac{k}{n}f(k) - \frac{k^*}{n}\Sigma_{k^*}^n f(k) - \Sigma_0^{k^*-1}\frac{k}{n}f(k) \\ &= \frac{1}{n}\left(k^*\Sigma_{k^*+1}^n f(k) + \Sigma_{k^*+1}^n f(k) + \Sigma_0^{k^*} kf(k) - k^*\Sigma_{k^*}^n f(k) - \Sigma_0^{k^*-1} kf(k)\right) \\ &= \frac{1}{n}\Sigma_{k^*+1}^n f(k) > 0 \end{aligned} \tag{10}$$

Thus if z rises then the smallest number of successes required for repayment (k^*) cannot fall. Hence the group default rate $\Pr(k < k^*)$ weakly rises as z rises. That is, a decrease in H or an increase in

r or B raise z which must then raise k^* and so raise the probability of default and the group interest rate. Steijvers and Voordeckers (2009) explain that a larger loan amount increases firm leverage and thus default risk. Similarly an increase in L reduces z which serves to reduce k^*, the default probability and the group interest rate. This is like a collateral effect: increasing L raises the amount that the lender can get for sure. This finding is supported by several empirical studies. For instance, Jimenez et al. (2006) find a negative relationship between collateral and default risk while Menkhoff et al. (2006) find that Thai banks use collateral to reduce risk rather than information asymmetry. Moreover, Menkhoff et al. (2012) studying microlending in Thailand find a negative relationship between collateral and the interest rate. In turn, from (9) an increase in L also allows B to rise.

The key questions are how p and n affect $K(p, n, k^*)$. If there were monotone effects of n and p on $K(p, n, k^*)$ then we could deduce the effect on k^* :

$$\frac{\triangle k^*}{\triangle p} = -\frac{\triangle K}{\triangle p} / \frac{\triangle K}{\triangle k^*}, \frac{\triangle k^*}{\triangle n} = -\frac{\triangle K}{\triangle n} / \frac{\triangle K}{\triangle k^*} \tag{11}$$

2.5.1 Comparative Statics in p

The effect of p can be seen by varying the $K(p, n, k^*)$ in p ie

$$\frac{\triangle K}{\triangle p} = \frac{\triangle \Sigma_{k^*}^n f(k) k^*}{n \triangle p} + \frac{\triangle \Sigma_0^{k^*-1} k f(k)}{n \triangle p} \tag{12}$$

The appendix shows that

$$\begin{aligned} \frac{\triangle K}{\triangle p} &= \frac{1}{n} \left(-\Sigma_0^{k^*-1} \frac{(k-np)}{p(1-p)} f(k) k^* + \Sigma_0^{k^*-1} k \frac{(k-np)}{p(1-p)} f(k) \right) \\ &= \Sigma_0^{k^*-1} (k-k^*) \frac{(k-np)}{np(1-p)} f(k) \end{aligned}$$

Certainly for every term in the sum $k - k^* < 0$. If $k^* - 1 \leq E(k) = np$ then every term $k - np$ is nonpositive, and thus $\triangle K / \triangle p > 0$. Thus an increase in p increases K; to compensate for this, k^* must fall. So as p rises, k^* falls.

Proposition 1 *If (9) is an equation and if $k^* - 1 < np$ then an increase in p leads to a fall in k^* and so also (weakly) a fall in the probability of group default,* $\Pr(k < k^*)$

Both the provisos in the proposition are important[4].

We can also examine how R_n varies with p. Define the RHS of (4) as

$$\begin{aligned} Z &= (1+R_n)nB\Pr(k \geqslant k^*) + E[Hk+(n-k)L|k<k^*] \\ &= (1+R_n)nB\Sigma_{k^*}^n f(k) + (H-L)\Sigma_0^{k^*-1}kf(k) + nL\Sigma_0^{k^*-1}f(k) \end{aligned}$$

$$\frac{\partial Z}{\partial R_n} = nB\Sigma_{k^*}^n f(k) > 0$$

The appendix shows that

$$\frac{\partial Z}{\partial p} = \Sigma_0^{k^*-1}[k(H-L)+nL-(1+R_n)nB]\frac{(k-np)}{p(1-p)}f(k) > 0 \text{ if } k^*-1<np$$

Hence we deduce that

Proposition 2 *If $k^*-1<np$*

$$\frac{\partial R_n}{\partial p} = -\frac{\partial Z}{\partial p}/\frac{\partial Z}{\partial R_n} < 0$$

In other words, the greater the project risk $(1-p)$, the higher the group interest rate.

2.5.2 Comparative Statics in n

If n increases to $n+1, K(p,n,k_n^*)$ will change to $K(p,n+1,k_{n+1}^*)$. Therefore, the effect of n on K can be divided into two parts: the direct effect of n on K through changing the form of the distribution and the indirect effect via a change in k^*. Generally these effects can work in opposite directions.

$$K(p,n+1,k_{n+1}^*)-K(p,n,k_n^*) = [K(p,n+1,k_{n+1}^*)-K(p,n+1,k_n^*)]+[K(p,n+1,k_n^*)-K(p,n,k_n^*)] \quad (13)$$

The first part is the effect of an increase in k^* on K; normally, more successes are required when the group loan is larger. The second part is the effect of a changing probability distribution; the binomial distribution varies with the number of trials.

[4] According to the binomial distribution, $np+p$ is the mode and thus $f(k)$ is increasing when $k<(n+1)p$ and decreasing when $k>(n+1)p$. As $np+1>np+p$,

$$\Pr(k^*<np+1) > \Pr(k^*>np+1)$$
$$\Pr(k^*-1<np) > \Pr(k^*-1>np)$$

So there are chances that $k^*-1>np$. However, the chance is less than that of $k^*-1<np$

We know from (10) that $\Delta K/\Delta k^* \geq 0$ and so whenever an increase in n leads to an increase in k^*, the first term must at least weakly rise:

$$K(p, n+1, k^*_{n+1}) - K(p, n+1, k^*_n) \geq 0 \tag{14}$$

For the second part, the appendix shows that

$$\begin{aligned} K(p, n+1, k^*_n) - K(p, n, k^*_n) &= k^*_n \left(\frac{\Pr_{n+1}(k \geq k^*_n)}{n+1} - \frac{\Pr_n(k \geq k^*_n)}{n} \right) + \frac{E_{n+1}(k|k < k^*_n)}{n+1} - \frac{E_n(k|k < k^*_n))}{n} \\ &= k^*_n \Sigma^n_{k^*_n} \frac{k - np - 1}{(n+1-k)} \frac{f_n(k)}{n} + \Sigma^{k^*_n - 1}_{k=0} k \frac{k - np - 1}{(n+1-k)} \frac{f_n(k)}{n} \end{aligned} \tag{15}$$

since

$$f_{n+1}(k) = \frac{(n+1)(1-p)}{(n+1-k)} f_n(k) \text{ and so } \frac{f_{n+1}(k)}{n+1} - \frac{f_n(k)}{n} = \frac{k - np - 1}{(n+1-k)} \frac{f_n(k)}{n}$$

The values of k range from 0 to n. When $k = 0$, certainly $k - np - 1 < 0$. If $n < 1/(1-p)$, then $k - np - 1 < 0$ for all k and thus each term in (15) is negative. By contrast if $n > 1/(1-p)$ then $k - np - 1$ changes sign from negative to positive for an integer close to $np + 1$ in which case (15) is of ambiguous sign.

So the overall comparative static effect of n on K in (13) is ambiguous because although the effect of increasing k^* always weakly increases K, it is offset by the impact of the changing distribution on K which weakly decreases K if $n < 1/(1-p)$. Then changes in default probability and the mean of the lower part of the distribution due to the changing distribution work in the opposite way to the change in k^*. If $n > 1/(1-p)$ the effects of changes in the distribution as n changes have indeterminate sign so they may reinforce or offset the effects of n in changing K via changes in k^*. In both cases, combining (14) and (15), the effects of n on K are ambiguous which implies that the effect of n on R_n is also ambiguous. This is consistent with the ambiguity of $R^I - R_n$.

To find clear comparative static effects of p requires a condition involving an endogenous variable ($k^* < np + 1$). The effects of the group size on k^*, the group interest rate and the default rate are ambiguous. Therefore, we subsequently turn to simulations to establish some comparative static information.

2.6 Some Simulations

To see how this works in practice we have computed group default rates $\Pr(k < k^*)$ and interest rates R_n for varying group sizes with other parameters fixed at $H = 10; L = 1; r = 0.1; B = 4$. This leaves the group size n and probability of success as varying exogenous variables. Note that to ensure $pH + (1-p)L > (1+r)B$ with these values we need $p \geq 0.38$. Individual loans in this scenario would require an interest rate of $1 + r^I = 1.1/p - .25(1-p)/p$ and the individual default rate would be $1 - p$.

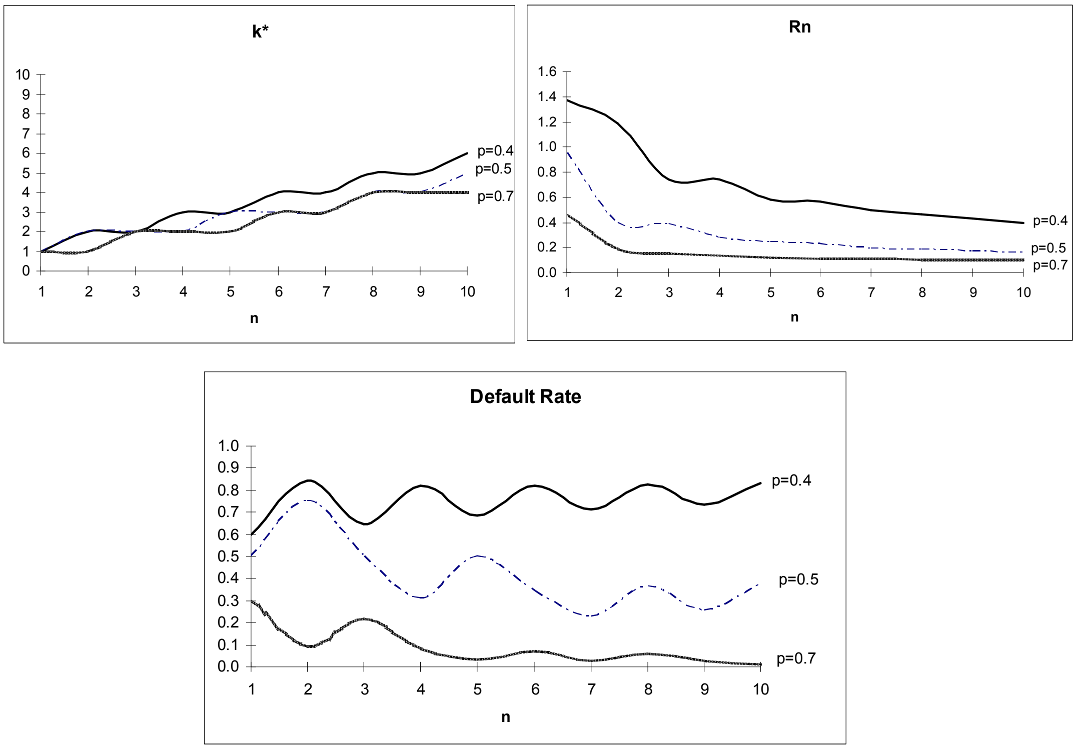

Figure 1: Simulation Results with Binomial Distribution

The simulation result shows that

(1) at a given n, an increase in p reduces k^*, R_n and $\Pr(k < k^*)$.

(2) at a given p, the larger group size tends to have higher k^* but lower k^*/n and R_n

(3) at a given p, $\Pr(k < k^*)$ doesn't change monotonically when n changes, but the change is smoother for a higher p. As discussed in the appendix, at high p the effect of changing k^* becomes smaller with n

(4) group interest rates are 10% - 40% for $p \geq 0.5$ and about 40% - 120% for $p < 0.5$.

(5) higher p allows the group to be bigger with a low interest and default risk; whereas, with low p default risk tends to increase when the group becomes larger.

As shown in Figure 1, the default risk of individual lending is higher than the default risk of group lending for all $n > 1$ at high p. Therefore, at high p the first term of (7) is positive and unambiguously the risk premium on individual loans is above that on group loans. This matches with empirical evidence on the relative interest rates e.g. Gonzalez (2010) who finds that interest rate premium for individual lending is higher than group lending. The appendix shows that if $p < 0.5$, the effect of n on the default risk is ambiguous. The simulation shows that the default risk of individual lending can be lower than the default risk of group lending, but as discussed in Section 2.4 the individual lending's risk premium can still be higher if the negative first term of (7) is smaller than the positive second term of (7).

3 Correlated Risks: Binomial Mixture

Due to the localised nature of microfinance loans, group members may face some common risks and thus the success of projects may be correlated (Varian, 1990). Geographical proximity of group members means that area based shocks will impact on each group member. The dependence of outcomes violates the independence assumption of the binomial distribution.

A convenient way of modelling this dependence which also has empirical attractions is through a beta binomial mixture distribution. The idea is that the overall risk on a project is the joint effect of risk in the probability of success in a typical project, and a binomially distributed idiosyncratic risk on different projects for a given common risk For each project i there is a common constant probability p of success but p itself is a random variable $\widetilde{p}$ with mean p_0. As $\widetilde{p}$ varies randomly so does the chance of success. We can think of random variations in $\widetilde{p}$ as common shocks affecting all borrowers e.g. bad weather for geographically close farmers. Then given $\widetilde{p} = p$, some farmers will succeed and others fail with the number of successes conditional on p being a binomially distributed random variable. However $\widetilde{p}$ itself is a random variable following a beta distribution.

3.1 Beta Binomial Mixture

Let $k \sim Bin(n, \widetilde{p})$ where the binomial density function of k given $\widetilde{p} = p$ is

$$f(k \mid p) = \frac{n!}{k!(n-k)!} p^k (1-p)^{n-k},$$

and $\widetilde{p}$ has a beta distribution, with parameters α, β being positive non-integers i.e. $\widetilde{p} \sim B(\alpha, \beta)$. The probability density function of $\widetilde{p}$ is

$$w(p) = \frac{p^{\alpha-1}(1-p)^{\beta-1}}{B(\alpha, \beta)}$$

where

$$B(\alpha, \beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)} = \frac{(\alpha-1)!(\beta-1)!}{(\alpha+\beta-1)!} = \int_0^1 t^{\alpha-1}(1-t)^{\beta-1} dt$$

So the probability of k successes is

$$\begin{aligned} f(k) &= \int_0^1 f(k \mid p) w(p) dp \\ &= \frac{n!}{k!(n-k)!} \frac{1}{B(\alpha, \beta)} \int_0^1 p^{k+\alpha-1}(1-p)^{n-k+\beta-1} dp \\ &= \frac{n!}{k!(n-k)!} \frac{B(k+\alpha, n-k+\beta)}{B(\alpha, \beta)} \end{aligned}$$

The cumulative distribution function is

$$F(k^*) = \Sigma_0^{k^*-1} \frac{B(k+\alpha, n-k+\beta)}{(n+1)B(k+1, n-k+1)B(\alpha, \beta)}$$

Thus as in the binomial case $E(k|p) = np$, $var(k|p) = np(1-p)$ but $Ep = p_0 = \alpha/(\alpha+\beta), var(p) = \alpha\beta/[(\alpha+\beta)^2(1+\alpha+\beta)]$. The unconditional mean of k is $Ek = E_p np = nE_p p = n\alpha/(\alpha+\beta)$. The unconditional variance of k is

$$\begin{aligned} var(k) &= E_p var(k|p) + var(E_k(k|p)) \\ &= E_p np(1-p) + var(np) \\ &= np_0 - nE(p^2) + n^2[E(p^2) - (p_0)^2] \\ &= np_0(1-p_0) + n(n-1)(E(p^2) - p_0^2) \\ &= np_0(1-p_0) + n(n-1)p_0(1-p_0)\rho \\ &= np_0(1-p_0)(1+(n-1)\rho) \end{aligned} \tag{16}$$

where ρ is the common correlation between the returns on any two projects. The correlation between projects is $1/(\alpha+\beta+1) > 0$ and increases the lower are α, β. Replacing p_0 by its mean in (16), the unconditional variance is

$$\begin{aligned} var(k) &= n\frac{\alpha\beta}{(\alpha+\beta)^2}[1+(n-1)\rho] \\ &= \frac{n\alpha\beta(n+\alpha+\beta)}{(\alpha+\beta)^2(1+\alpha+\beta)} \end{aligned}$$

A special case of the betabinomial is that of heterogeneous binomial distributions of success, each borrower i has a fixed chance of success but these chances differ between borrowers. This has been used by Katzur and Lensink (2012).

Given that group loans are typically to geographically close borrowers, the idea of common systemic risk on a group loan is attractive. Also the tail probabilities (especially the lower tail) are likely to be higher than in the independent risks binomial case, since downside catastrophic risk is probably more common in the developing economy context in which group lending occurs. Particular events such as extreme weather (droughts, floods), geological events (earthquakes) and economic and political events (commodity price shocks, revolution) are likely to cause common high downside risk in the borrower group. And generally emerging economies have higher risk than developed ones (Harvey, 1995). Based on our arguments that there are high covariate shocks (high ρ) high return (high Ep) and high risk (high var(k)) in emerging economies i.e. α and β are low, $\alpha > \beta$ and α is close to β (See Figure 2)

To ensure viability of the projects we require that they have positive expected return taking into account both the chance that particular projects will succeed or fail and the systemic risk in p. Thus, financing a project is socially desirable if

$$E_p(\widetilde{p})H + (1 - E_p(\widetilde{p}))L = \frac{\alpha}{\alpha+\beta}H + \frac{\beta}{\alpha+\beta}L > (1+r))B > L$$

This also implies $H > (1+r)B > L$.

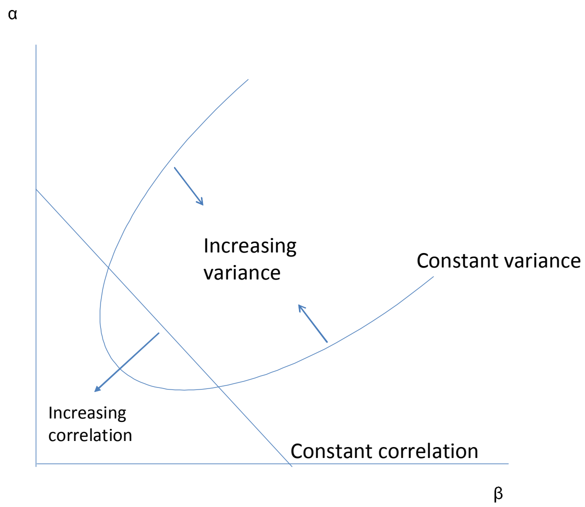

Figure 2: Correlation and Variance of p with varying α and β

3.2 Individual Loans: Each Must Break Even

This is identical to the uncorrelated case, except that $\widetilde{p}$ is now random. If $\widetilde{p}$ is beta distributed, the mean value of p is then $\alpha/(\alpha+\beta)$. So the average default chance is $E_p(1-\widetilde{p}) = \beta/(\alpha+\beta)$.

$$\begin{aligned}(1+r)B &= E_p(\widetilde{p}(1+R^I)B + (1-\widetilde{p})L) \\ &= \frac{\alpha}{\alpha+\beta}(1+R^I)B + \frac{\beta}{\alpha+\beta}L \end{aligned} \tag{17}$$

$$\begin{aligned}1+R^I &= \frac{(1+r)(\alpha+\beta)}{\alpha} - \frac{\beta}{\alpha}\frac{L}{B} \\ R^I - r &= \frac{\beta}{\alpha}(1+r-\frac{L}{B}) \end{aligned} \tag{18}$$

3.3 Lender Averages Over Group Loans

The lender makes n loans, each of which may fail with a random probability $1-\widetilde{p}$. If any individual loan fails, the lender gets L, if it succeeds he gets $(1+R^{II})B$ on that loan. With n loans the total repayment to the lender is $k(1+R^{II})B + (n-k)L$ if k loans succeed. The lender sets the interest rate so that on average he breaks even across the loans

$$\begin{aligned}(1+r)nB &= \Sigma_0^n[k(1+R^{II})B + (n-k)L]f(k) \\ &= [(1+R^{II})B - L]\Sigma_0^n kf(k) + nL\end{aligned}$$

Hence

$$(1+R^{II}) = \frac{(1+r)n + (L/B)\Sigma_0^n kf(k) - n(L/B)}{\Sigma_0^n kf(k)}$$

The mean of k is $n\alpha/(\alpha+\beta)$ so

$$\begin{aligned}
(1+R^{II}) &= \frac{(1+r)n - (L/B)n\beta/(\alpha+\beta)}{n\alpha/(\alpha+\beta)} \\
&= \frac{(1+r)(\alpha+\beta) - (L/B)\beta}{\alpha} \\
R^{II} - r &= \frac{\beta}{\alpha}(1+r-\frac{L}{B}) \qquad (19)
\end{aligned}$$

We have the same result again that averaging over multiple individual loans does not change the risk premium even if the loans are correlated.

3.4 Group Lending

Again we have $G(k) = kH + (n-k)L$, which is still increasing in k. The group will default for any p and k such that

$$G(k) = kH + (n-k)L < (1+R_n)nB$$

The group loan is defined by k^* and R_n such that

$$\begin{aligned}
G(k^*) &\geq (1+R_n)nB \\
(1+r)nB &= E_p \Pr(k \geq k^*)(1+R_n)nB + E_p[G(k)|k < k^*]
\end{aligned}$$

If the variations in the binomial process and p jointly result in $k \geq k^*$ then $G(k) \geq G(k^*)$ and the group can repay. Conversely if it turns out that $k < k^*$ the group defaults and pays all revenue. The lender sets a single number k^* and R_n to break even over the random fluctuations in p and the binomial process in project success.

$$\begin{aligned}
1+R_n &= \frac{(1+r)nB - E_p[G(k)|k<k^*;p]}{nBE_p \Pr(k \geq k^*)} \\
&= \frac{1+r}{E_p \Pr(k \geq k^*)} - \frac{E_p[(H-L)k + nL|k<k^*;p]}{nBE_p \Pr(k \geq k^*)} \\
&= \frac{(1+r)nB - (H-L)\Sigma_0^{k^*-1}kf(k)) - nL\Sigma_0^{k^*-1}f(k)}{nBE_p \Pr(k \geq k^*)} \\
&= \frac{(1+r)}{E_p \Pr(k \geq k^*)} - \frac{(H-L)\Sigma_0^{k^*-1}kf(k) + nLE_p \Pr(k<k^*)}{E_p \Pr(k \geq k^*)nB} \qquad (20)
\end{aligned}$$

where

$$E_p(\Pr(k < k^*)) = \Sigma_0^{k^*-1} \frac{n!}{k!(n-k)!} \frac{B(k+\alpha, n-k+\beta)}{B(\alpha,\beta)} \tag{21}$$

$$E(k|k < k^*) = E_p(E(k|k < k^*;p)) = \Sigma_0^{k^*-1} \frac{n!k}{k!(n-k)!} \frac{B(k+\alpha, n-k+\beta)}{B(\alpha,\beta)} \tag{22}$$

(20) is similar to (5). However, the R_n in (5) is computed from the binomial distribution while the R_n in (20) is computed from the beta-binomial distribution.

The default premium with correlated risks and group lending is

$$R_n - r = (1 + r - \frac{L}{B}) \frac{E_p \Pr(k < k^*)}{E_p \Pr(k \geq k^*)} - \frac{(H-L)E_p[k|k < k^*;p]}{nBE_p \Pr(k \geq k^*)} > 0 \tag{23}$$

which can be demonstrated in exactly the same way as for independent risks (6)

The comparison between the group and individual interest rates works in a similar way to the case of independent risks

From (18) and (23),

$$\begin{aligned} R^I - R_n &= \frac{\beta}{\alpha}(1 + r - \frac{L}{B}) - (1 + r - \frac{L}{B}) \frac{E_p \Pr(k < k^*)}{E_p \Pr(k \geq k^*)} + \frac{(H-L)E_p[k|k < k^*;p]}{nBE_p \Pr(k \geq k^*)} \\ &= (1 + r - \frac{L}{B}) \left[\frac{\beta}{\alpha} - \frac{E_p \Pr(k < k^*)}{E_p \Pr(k \geq k^*)} \right] + \frac{(H-L)E_p[k|k < k^*;p]}{nBE_p \Pr(k \geq k^*)} \\ &= (1 + r - \frac{L}{B}) \left[\frac{1 - Ep}{Ep} - \frac{E_p \Pr(k < k^*)}{E_p \Pr(k \geq k^*)} \right] + \frac{(H-L)E_p[k|k < k^*;p]}{nBE_p \Pr(k \geq k^*)} \end{aligned} \tag{24}$$

(24) is similar to (7), but (24) depends on Ep and values of α and β. The argument about positiveness of (24) is similar to that using (7).

The comparative static effects of correlated risks on k^* and R_n can be obtained by applying the same approach as in Section 2.5 with

$$Z = \frac{(1+r)B - L}{H - L}; K(\alpha, \beta, n, k^*) = \frac{E_p \Pr(k \geq k^*)k^* + E_p(k|k < k^*;p)}{n}$$

where the distributions and moments are given by (21) and (22).

As in the case of independent risks, there are few robust analytic results on comparative statics. To show the magnitude (and in some case the direction) of such effects we turn to simulations.

4 Some Simulations

We simulate k^*, R_n and $E_p \Pr(k < k^*)$ for varying values of ρ and Ep which depend on α and β. The basic parameters on H, L, B and r are set to the same values as the independent risks case. In each of the graphs below, when $n = 1$ the outcome is identical between the independent and correlated risk cases.

Figure 3 shows main findings as follows:

(1) For a given n, k^* increases when the correlation increases. The correlation increases the chances of extreme numbers of project successes or failures. Hence, to ensure that the group has enough revenue to cover the default premium associated with the fatter lower tail, the group requires a higher number of successes

(2) With positive ρ, group loans with higher correlation are charged with higher R_n. High correlation of returns again gives a fatter low tail so the default premium generally increases.

(3) The difference between the correlated group loan's R_n and the independent group loan's R_n is larger when the mean probability of success is lower. Thus, the group with higher correlation of returns and lower mean probability of success is charged with a higher R_n and requires higher k^*.

(4) With high Ep, both R_n and the default rate are higher when the correlation increases. In contrast, with low Ep there is no conclusive relationship between ρ and the default rate.

(5) With the higher correlation, the increase in k^* and the change in default rate caused by a rise in n are smoother.

(6) Unlike the independent group loans, regardless of Ep increasing the group size does not lower the default risk and R_n especially when the correlation is high. So with a positive correlation between project returns, a smaller group can have a lower default risk..

Our simulation result for the case of high Ep is consistent with the empirical finding of Kurosaki and Khan (2012) that a higher default rate in a more homogeneous group can stem from covariate shocks. Like the case of independent project outcomes, the group interest rate decreases with a higher Ep and thus the group liquidity increases. With correlated project outcomes, a larger group may not have a lower interest rate and percentage of successes when the outcomes are highly correlated. If $\rho \to 1$ so that α and

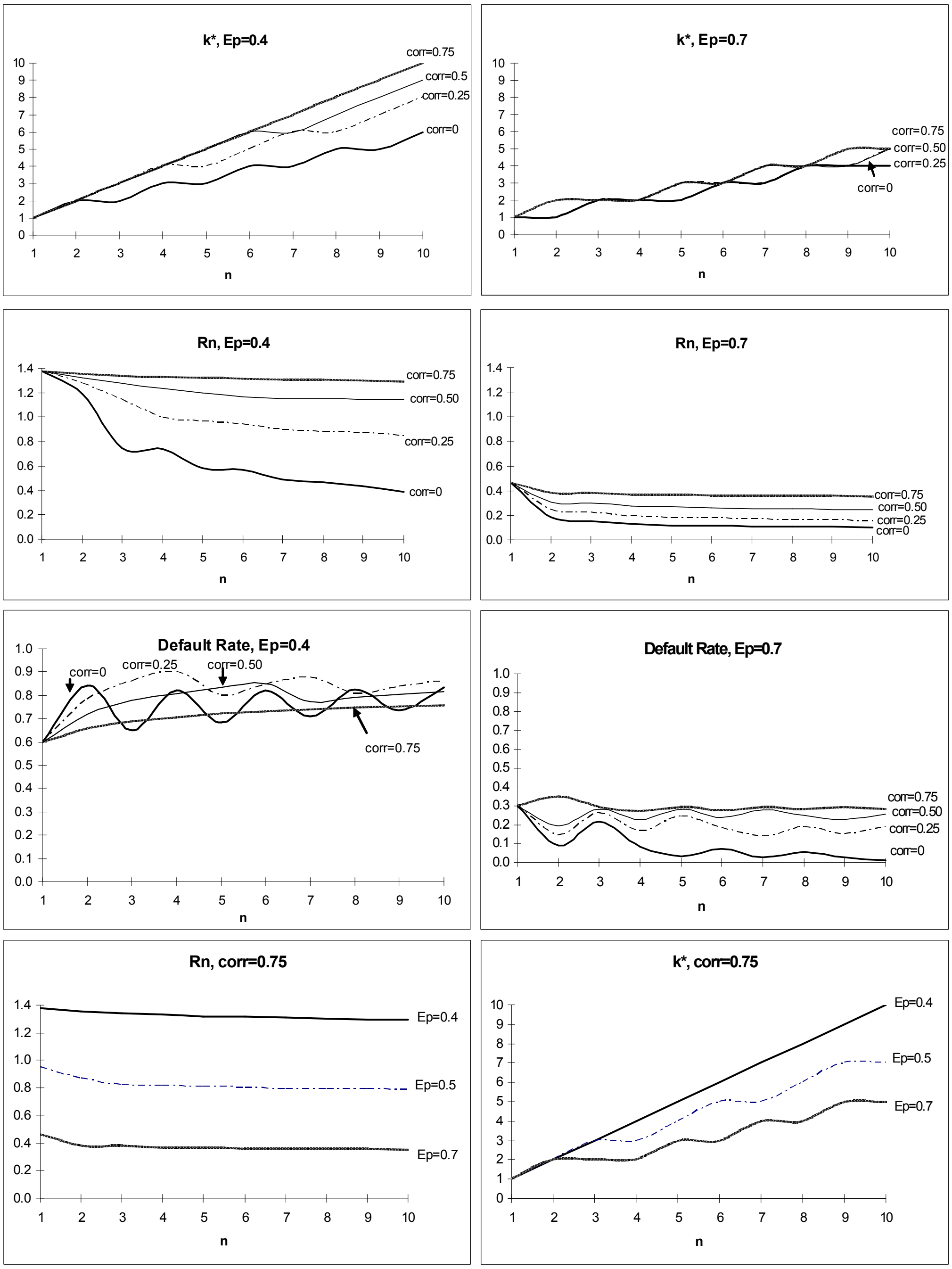

Figure 3: Simulation Results with Beta Binomial Distribution

β are close to zero, effectively either all projects succeeed or all fail. It is as if there is a single project with returns nH wp p or nL wp $1-p$ and a cost of nB. A loan on this project will only be repaid if all projects succeed so $k^* = n$ and $R_n \rightarrow R_I$.

5 Conclusions

To conclude, we find that for group lending with independent or positively correlated projects there is a unique minimum number of successful borrowers that enables the group to repay the loan with fair interest. The group lending rate is generally lower than the individual lending rate. In addition, the default rate changes non-monotonically when the group size increases. Group liquidity is higher when projects are safer. If project risks are independent, a higher probability of success allows the larger group to have the group loan interest rate lower and have lower default risk. However, for riskier projects the group default rate is higher in a larger group loan than a smaller one even though the interest rate is lower in a larger group loan.

If project risks are correlated, we use a mixed binomial distribution in which the chance of project success has a beta distribution. Compared with independent group lending, correlated group lending is charged a higher group interest rate, and solvency of the group requires a higher number of successes (k^*). In addition, correlated group lending has a higher default risk if the mean probability of success is high. Comparing correlated groups with different mean probability of success, the group with the lower mean probability of success requires a much higher risk premia and thus higher k^* when correlation is higher. We also find that the group should be small for low expected individual project returns if there is correlation between project returns. This may explain why, in the last decade, MFIs have moved away from group lending. It cannot be due to the repayment incentive and enforcement problems since most achieved repayment rates with group lending are in excess of 90%. It may be the fact that positive correlation between borrower outcomes actually causes the risk of group lending to exceed the risk of individual lending. It may also be difficulties in determining the best group size, which we find to have strong effects on group liquidity and the ability to repay.

We have abstracted from the compliance costs arising from asymmetric information. However, the form of group loan that we are using is a standard debt contract on the group and/or individual. That is, a fixed repayment is set and if this is not met, the lender seizes all the assets. Assuming the lender can observe the realised assets (maybe at a cost), it is incentive compatible for the individual or group to truthfully reveal their revenues (Townsend, 1979). However, if observation of borrower assets is costly, this does require the lender to commit to pay the cost to discover the borrower assets even though he knows that the borrowers are truthfully reporting. In the literature (especially Karlan (2007)) strong emphasis is put on the role of the group of borrowers in enforcing truthful revelation and repayment through peer pressure.

A way of modelling this would be to add either or both of an administrative cost of a loan and an observation/audit cost of verifying reports of failure by the contracted debtors. The administrative cost is probably lump sum, much of the costly state verification literature takes the audit cost as lump sum too. In which case the lender requires a payment of say $c_1 + c_2 \Pr(\text{default})$ per loan where c_1 is the administrative cost and c_2 the audit cost, both per loan. These costs enter the reservation constraint of the lender and so increase the required repayment on any loan. It is then possible that the individual projects can be ex-ante profitable $(pH + (1-p)L > (1+r)B)$ but fail to meet the administrative costs $(pH + (1-p)L < (1+r)B + c_1 + (1-p)c_2)$. On the other hand, the group loan could still be feasible if $n(pH + (1-p)L) < (1+r)nB + c_1 + \Pr(k \leq k^*)c_2$. In that eventuality, group lending will allow ex-ante profitable projects which cannot be undertaken on an individual loan basis.

We have used a simple setting of two state outcomes with individuals facing the same risks in their projects. This could obviously be extended to the continuous distribution case. But identity of risks within a group loan (even if they are correlated) and also small numbers of group borrowers represents the nature of these loans better than introducing deterministic heterogeneity.

A more important limit is the static nature of the analysis. The dynamics are primarily very important in analysing the compliance mechanisms in individual and group lending. However, abstracting from these highlights the determinants of default risk and interest rate for different group sizes and probability distributions of project returns.

Our findings are that group lending can indeed offer better risk diversification to borrowers than individual loans. The risk premium and interest rate on group lending can be below that on individual lending. This benefit, however, can be lower if the project returns are correlated.

A Appendix

A.1 Comparative statics in p :

(1) The effects on $K(p, n, k^*)$

$$\frac{\triangle K}{\triangle p} = \frac{\triangle \Sigma_{k^*}^n f(k)k^*}{n \triangle p} + \frac{\triangle \Sigma_0^{k^*-1} k f(k)}{n \triangle p}$$

where

$$\begin{aligned}
\frac{\triangle \Sigma_{k^*}^n f(k)k^*}{\triangle p} &= \Sigma_{k^*}^n \frac{n!}{k!(n-k)!} p^{k-1}(1-p)^{n-k-1}(k-np)k^* \\
&= \Sigma_{k^*}^n \frac{(k-np)}{p(1-p)} f(k)k^* \\
&= \Sigma_0^n \frac{(k-np)}{p(1-p)} f(k)k^* - \Sigma_0^{k^*-1} \frac{(k-np)}{p(1-p)} f(k)k^* \\
&= -\Sigma_0^{k^*-1} \frac{(k-np)}{p(1-p)} f(k)k^*
\end{aligned}$$

due to $\Sigma_0^n \frac{(k-np)}{p(1-p)} f(k)k^* = 0$, and

$$\begin{aligned}
\frac{\triangle \Sigma_0^{k^*-1} k f(k)}{\triangle p} &= \Sigma_0^{k^*-1} \frac{n!}{k!(n-k)!} p^{k-1}(1-p)^{n-k-1}(k-np)k \\
&= \Sigma_0^{k^*-1} \frac{(k-np)}{p(1-p)} f(k)k
\end{aligned}$$

(2) The effects on Z

$$\frac{\partial Z}{\partial p} = (1+R_n)nB \frac{\partial \Sigma_{k^*}^n f(k)}{\partial p} + (H-L) \frac{\partial \Sigma_0^{k^*-1} k f(k)}{\partial p} + nL \frac{\partial \Sigma_0^{k^*-1} f(k)}{\partial p}$$

$$\begin{aligned}
\frac{\partial \Sigma_{k^*}^n f(k)}{\partial p} &= \Sigma_{k^*}^n \frac{(k-np)}{p(1-p)} f(k) > 0 \\
\frac{\partial H \Sigma_0^{k^*-1} k f(k)}{\partial p} &= H \Sigma_0^{k^*-1} \frac{(k-np)}{p(1-p)} f(k)k < 0 \\
\frac{\partial \Sigma_0^{k^*-1} (nL - kL) f(k)}{\partial p} &= \Sigma_0^{k^*-1} (nL - kL) \frac{(k-np)}{p(1-p)} f(k)
\end{aligned}$$

if $k^* - 1 < np$

$$\begin{aligned}
\frac{\partial Z}{\partial p} &= (1+R_n)nB\Sigma_{k^*}^{n}\frac{(k-np)}{p(1-p)}f(k) + H\Sigma_0^{k^*-1}\frac{(k-np)}{p(1-p)}f(k)k \\
&+\Sigma_0^{k^*-1}(nL-kL)\frac{(k-np)}{p(1-p)}f(k) \\
&= (1+R_n)nB\left[\Sigma_0^{n}\frac{(k-np)}{p(1-p)}f(k) - \Sigma_0^{k^*-1}\frac{(k-np)}{p(1-p)}f(k)\right] \\
&+\Sigma_0^{k^*-1}[k(H-L)+nL]\frac{(k-np)}{p(1-p)}f(k) \\
&= \Sigma_0^{k^*-1}[k(H-L)+nL-(1+R_n)nB]\frac{(k-np)}{p(1-p)}f(k) > 0 \text{ if } k^*-1 < np
\end{aligned}$$

A.2 Comparative Statics in n

$$K(p,n,k^*) = \frac{\Sigma_{k^*}^{n} f_n(k)k^* + \Sigma_0^{k^*-1}kf_n(k)}{n}$$

where

$$f_n(k) = \frac{n!}{k!(n-k)!}p^k(1-p)^{n-k}$$

$$\begin{aligned}
f_{n+1}(k) &= \frac{(n+1)!}{k!(n+1-k)!}p^k(1-p)^{n+1-k} \\
&= \frac{(n+1)(1-p)}{(n+1-k)}\frac{n!}{k!(n-k)!}p^k(1-p)^{n-k} \\
&= \frac{(n+1)(1-p)}{(n+1-k)}f_n(k)
\end{aligned}$$

(1) The effect of n on default probability is

$$\begin{aligned}
\Pr_{n+1}(k &< k^*_{n+1}) - \Pr_n(k < k^*_n) = \Pr_{n+1}(k^*_n \le k < k^*_{n+1}) + \Pr_{n+1}(k < k^*_n) - \Pr_n(k < k^*_n) \\
&= \Sigma_{k=k^*_n}^{k^*_{n+1}-1} f_{n+1}(k) + \Sigma_{k=0}^{k^*_n-1}\left(\frac{k-(n+1)p}{(n+1-k)}\right)f_n(k)
\end{aligned}$$

The overall effect is ambiguous because although the effect of increasing k^* always weakly increases default probability, it can be offset by the impact of the changing distribution on default probability. The first positive term becomes smaller with n at high p. However, at low p the change of the first term is not monotonic in n. The second term is the effect of n on $\Pr(k < k^*)$ for given k^*which is negative if $k^* - 1 < np < (n+1)p$ (the mode) for any n or if $p > \frac{n}{n+1}$ for any k^*. Higher p raises the bound of n that has each term in the sum negative. If $p < 0.5$, this condition fails and then the second term can be positive or its negativity is smaller.

Thus, if $p > \frac{n}{n+1}$, the change in default probability due to the changing distribution works in the opposite way to the change in k^*. If $p < 0.5$,the sign of the second term is ambiguous. In both cases of p, the overall effect of n on default probability are ambiguous.

(2) The effect of n on $K(p, n, k^*)$,

$$
\begin{aligned}
K(p, n+1, k_n^*) - K(p, n, k_n^*) &= k_n^* \left(\frac{\Pr_{n+1}(k \geq k_n^*)}{n+1} - \frac{\Pr_n(k \geq k_n^*)}{n} \right) + \frac{E_{n+1}(k|k < k_n^*)}{n+1} - \frac{E_n(k|k < k_n^*))}{n} \\
&= k_n^* \left(\frac{\Sigma_{k_n^*}^n f_{n+1}(k)}{n+1} - \frac{\Sigma_{k_n^*}^n f_n(k)}{n} \right) + \frac{E_{n+1}(k|k < k_n^*)}{n+1} - \frac{E_n(k|k < k_n^*))}{n} \\
&= k_n^* \Sigma_{k_n^*}^n \frac{k - np - 1}{(n+1-k)} \frac{f_n(k)}{n} + \Sigma_{k=0}^{k_n^*-1} k \frac{k - np - 1}{(n+1-k)} \frac{f_n(k)}{n} = b_1 + b_2
\end{aligned}
$$

References

[1] Ahlin, C., Lin, J. and Maio, M., 2011. Where does microfinance flourish? Microfinance institution performance in macroeconomic context, *Journal of Development Economics* 95, 105-120

[2] Ahlin, C. and Townsend, R., 2007. Using repayment data to test across models of joint liability lending, *Economic Journal* 117, F11-F51

[3] Besley, T. and Coate, S., 1995. Group lending, repayment incentives and social collateral, *Journal of Development Economics* 46, 1-18

[4] Baland, J., Somanathan, R. and Wahhaj, Z., 2011. Repayment incentives and distribution of gains from group lending, *CEPR Discussion Paper* no. 8197

[5] Booth, A. Sosa, L.C. and Nolen, P., 2011. Gender differences in risk aversion: Do single-sex environments affect their development?, *CEPR Discussion Paper* no. 8690

[6] Border K.C. and Sobel J., 1987. Samurai accountant: A thoery of auditing and plunder, *Review of Economic Studies* 54, 525-540

[7] Cull, R., Demirguc-Kunt, A. and Morduch, J., 2007. Financial performance and outreach: A global analysis of leading microbanks, *Economic Journal* 117, F107-F133

[8] Cull, R., Demirguc-Kunt, A. and Morduch, J., 2009. Microfinance meets the market, *Journal of Economic Perspectives* 23, 167-192

[9] Deininger, K. and Liu, Y., 2009. Determinants of repayment performance in indian micro-credit groups, *World Bank Policy Research Working Paper* no. 4885

[10] Devereux, J. and Fishe, R., 1993. An economic analysis of group lending programs in developing countries, *The Developing Economies* 31(1), 102-121

[11] Ghatak, M. and Guinnane, T.W., 1999. The economics of lending with joint liability: Theory and practice, *Journal of Development Economics* 60, 195-228

[12] Gale, D. and Hellwig, M., 1985. Incentive-compatible debt contracts: The one-period problem *Review of Economic Studies* 52(4), 647-663

[13] Gonzalez, A., 2010. Analyzing microcredit interest rates: A review of the methodology proposed by Mohammed Yunus, *MIX Data Brief* no. 4

[14] Hald, A., 1968. The mixed binomial distribution and the posterior distribution for p for a continuous prior distribution, *Journal of the Royal Statistical Society (Series B)* 30, 359-367

[15] Harvey, C.R., 1995. Predictable risk and return in emerging markets, *Review of Financial Studies* 8, 773-816

[16] Hermes, N., Lensink, R. and Meesters, A., 2011. Outreach and efficiency of microfinance institutions, *World Development* 39, 938-948

[17] International Monetary Fund, 2005. *Microfinance: A View from the Fund*

[18] Jimenez, G., Salas-Fumas, V. and Saurina, J., 2006. Determinants of collateral, *Journal of Financial Economics* 81, 255-281

[19] Karlan, D., 2007. Social connections and group banking, *Economic Journal* 117, F52-F84

[20] Katzur, T. and Lensink, R., 2012. Group lending with correlated project outcomes, *Economics Letters* 117, 445-447

[21] KurosakiI, T. and Khan, H. U., 2012. Vunerability of microfinance to strategic default and covariate shocks: Evidence from Pakistan, *The Developing Economies* 50, 81-115

[22] Ladd, D.W., 1975. An algorithm for the binomial distribution with dependent trials, *Journal of the American Statistical Association* 70, 333-340

[23] Menkhoff, L., Neuberger, D. and Rungruxsirivorn, O., 2012. Collateral and its substitutes in emerging markets' lending, *Journal of Banking and Finance* 36, 817-834

[24] Menkhoff, L., Neuberger, D. and Suwanaporn, C., 2006. Collateral-based lending in emerging markets: Evidence from Thailand, *Journal of Banking and Finance* 30, 1-21

[25] McLachlan, G. and Peel, D., 2000. *Finite Mixture Models*, John Wiley & Sons, Inc.

[26] Steijvers, T. and Voordeckers, W., 2009. Collateral and credit rationing: A review of recent empirical studies as a guide for future research, *Journal of Economic Surveys* 23, 924-946

[27] Townsend R., 1979. Optimal contracts and competitive markets with costly state verification, *Journal of Economic Theory* 21, 265-293

[28] Varian, H.R., 1990. Monitoring agents with other agents, *Journal of Institutional and Theoretical Economics* 146, 153-174

[29] Vigenina, D. and Kritikos, A.S., 2004. The individual micro-lending contract: Is it a better design than joint-liability? Evidence from Georgia, *Economic Systems* 28, 155-176