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Abstract

It is well established in the empirical literature that venture capital (VC) plays an important role in the promotion of innovation at industry level and the professionalisation of firms at micro-level. Whilst the VC-to-success link has been well explored, the mechanism behind *how* and *why* certain venture-backed firms are apparently more successful is an important question that has been largely ignored within the majority of the literature. In this paper, we fill this gap by specifically analysing firms' pre- and post-VC investment decisions. By considering a two period, multi-stage game, we analyse whether VC spurs innovation (*i*) directly after being granted; (*ii*) indirectly by incentivising firms to increase initial research efforts to increase their chances of receiving VC funding and its associated benefits; or (*iii*) a combination of both. Our results show that VC has both direct and indirect effects on firms' innovation decisions regardless of whether the firm is successful in securing VC funding or not. Furthermore, we find that the commonly held assertion that venture capital spurs success is too simplistic: whilst venture capital spurs innovation amongst the *lucky*, chosen few, it unambiguously suppresses innovation of non-VC-backed firms, a result that has been overlooked in the empirical literature. The issue of 'who becomes the winner' in the final product market however is ultimately dependent upon the extent of heterogeneity amongst firms. Further, we show that VC

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funding, equity stake and value-adding services all have impacts upon firms' incentives to invest in the first stage.

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1 Introduction

It is now well established in the empirical literature that venture capital (henceforth VC) plays an important role in the promotion of innovation at industry level and the professionalisation of firms at micro-level (Da Rin *et al* (2013); Dessí and Yin (2010)). In spite of consistent empirical evidence that supports this VC-to-success link at the micro level, there exists a dearth of theoretical investigation that provides insight into an important and, as yet, unanswered question: how *exactly* does VC spur such success?[1] This question is not just of theoretical interest but has important implications for public policy in fostering an environment conducive to innovation. As Gompers and Lerner (1999, 2001) observe, some of the most successful high-tech innovators in the US, such as Microsoft and Apple Computers, have benefited from VC backing. Therefore, understanding the mechanisms behind *how* and *why* certain venture-backed firms are, apparently, more successful is important and has, to the best of our knowledge, been largely ignored within the majority of the literature.

In this paper, we attempt to fill this gap by focussing primarily on *firms'* perspectives. We ask three important questions: (*i*) what impact does VC have on a firm's incentives to invest in innovation?; (*ii*) how do rival, non-VC-backed firms respond?; and (*iii*) does the prospect of receiving VC funding in the future, and its associated benefits, spur innovation *ex ante*?

Whilst at the industry level, there exists a long established strong, positive relationship between VC and innovation,[2] at a firm level however, VC appears to have almost no link to innovation *per se* although it does appear to have other real impacts on a firm's potential for success. For example, Hellman and Puri (2000), using a selection of survey and commercially available data for 173 hand-picked Silicon Valley start-ups, observe that firms pursuing an innovator strategy are more likely to obtain VC funding and see a reduction in time needed to bring a product to market. Most intriguing, however, is their assertion that, "firms are

[1] So far the existing theoretical VC literature is generally focused on optimal contract theory.See Da Rin *et al* (2013) for an excellent review.

[2] Given our focus on a micro level model, we do not discuss industry level results here. However, for more information see Kortum and Lerner (2000), Hirukawa and Ueda (2008), Hirukawa and Ueda (2011), Popov and Roosenboom (2009), Popov and Roosenboom (2012), Faria and Barbosa (2013) and Geronikolau and Papachistou (2012).

more likely to consider VC a milestone event than obtaining financing from some other kind of financier" (Hellman and Puri, 2002, p.962). Though a reason for this is not given, all these findings are consistent with a venture capitalist possessing at least one of two skills: (*i*) a higher ability to seek out innovative firms *ex ante*; and (*ii*) offering benefits beyond those of traditional finance methods through the use value-adding services *ex post*.

Other work also has found remarkably similar results. Puri and Zarutskie (2012), using US firm level data between 1981-2005, compare VC- and non-VC-backed firms to examine relative growth rates. Whilst the results suggest that VC may be irrelevant in the creation of *new* firms (accounting for only 0.11% of new firms within the sample), they note consistently faster growth though this does not necessarily transfer to profitability. Peneder (2010), examining the impact of VC on 132 Austrian firms, found that such firms grew 70% quicker than equivalent non-VC-backed firms, though this growth did not extend to innovation. Chemmanur *et al* (2011), using US census data, adds that total factor productivity (TFP) is also an important signal to venture capitalists and is significantly higher both pre- and post-VC compared to non-funded firms. Da Rin and Penas (2007) find remarkably similar results using Dutch firm level data. Offering some additional insight into the growth of TFP, they suggest that venture capitalists push the firms they back into adopting more in-house R&D practices as well as investing in absorptive capacity.

To compare whether *ex ante* or *ex post* effects are more apparent, both Kaplan *et al* (2009) and Baum and Silverman (2004) examine the factors that are important for a firm to possess in order to receive VC backing. Kaplan *et al* (2009) examine whether venture capitalists are more likely to back "the horse" (the firm's business idea) or "the jockey" (the management team). They observe that whilst VC-backed firms do, indeed, grow much faster than those that did not receive such funding, the core business ideas also remained relatively consistent in comparison to management. Moreover, whilst management may make a firm more attractive, these are not related to post-VC performance.[3] In similar work, Baum and Silverman (2004), using data on 204 Canadian biotechnology start-ups and 407 incumbents, examine whether venture capitalists "pick" (*ex ante* selection) or "build" (*ex post* mentoring) their chosen firms. They find a combination of both effects with venture capitalists more likely to invest in firms that have already demonstrated some innovation (alliance participation or patents) and, thereafter, they perform better.

[3] In a related result, Wasserman (2003) finds that manager turnover is more likely when managers have succesfully developed a product rather than when they have performed poorly. The reason for this is that, once a firm has become a success, the skills that made the initial CEO so succesful in developing a product or idea may be less important once the firm faces a different scenario.

These results however do not come as a surprise given the active role that venture capitalists have been empirically demonstrated to play within a firm with respect to their value-adding services. As Bottazzi *et al* (2008, p.489) astutely stated, "the VC literature identifies a broad role for the investor, which goes beyond the simple provision of finance. Venture capitalists may engage in a number of value-adding activities, including monitoring, support, and control. Those activities are largely non-contractible, yet may have real consequences".

Monitoring is perhaps one of the most obvious, and empirically tested, of all of these value-adding services. Lerner's (1995) examination of biotechnology firms finds monitoring and control, as measured by venture capitalist board representation, were increasing in the need for oversight, as measured by CEO turnover. Gompers (1995) finds a similar relationship between agency costs and the monitoring within a sample of 794 VC-backed firms. More surprisingly, it appears that venture capitalists focus more on investment on early-stage projects for which information asymmetries are more pronounced.[4] However, monitoring a firm's activity is not a venture capitalists only value-adding service. Hellman and Puri (2002), analysing data on 170 young high-tech Silicon Valley start-ups, examine the impact of VC on the development of new firms. Similar to Chemmanur *et al* (2011), the results suggested that a venture capitalist's biggest impact was on the *professionalisation* of the firm. This impact is firm wide with benefits both at the top, by replacing the original founders with external CEOs, and at the bottom, by formulating HR policies and improving marketing strategies. Interestingly, this result of VC firms being more likely to replace founder CEOs with external candidates is supported by Wasserman (2003) who suggests founder CEOs skills are often outstripped by the rapid success that VC-backing offers.[5] Hochberg (2012) also finds evidence of stronger corporate governance within VC-backed firms and this result is made stronger when accounting for endogeneity. Finally, Bottazzi *et al* (2008), using survey data collected from 124 VCs across Europe, note that the aforementioned benefits may, in fact, be related to the prior business experience of the venture capitalist. To summarise their results, the more business experience a venture capitalist has, the more active it is within the firm.

Whilst empirical work has done well to shed some light on how venture cap-

[4]Dahiya and Ray (2011) observe a similar result to Gompers (1995). However, they add that venture capitalists may use staging as a screening tool to combat asymmetric information and abandon failing projects earlier. Hoenen *et al* (2012), evaluating 1500 US based technology firms, find that venture capitalists use other signals, for example patents, to screen weaker firms and offer stronger firms more investment. After initial round funding the impact of such signals diminishes - no further funding benefits - adding weight to a screening argument.

[5]Despite the apparent benefits of venture capitalists replacing existing CEOs, Kaplan et al (2012) find no performance difference between internal and external candidates once skills are accounted for.

italists add-value, little has been done with regards to "forward-looking selection effects". Simply put, the empirical literature assumes that the firm's *ex ante* actions are passive and that venture capitalists are the driving force behind the VC-to-success relationship. But why would such decisions by firms be passive? Caselli *et al*'s (2009) examination of 154 Italian IPOs (including 37 VC-backed firms) noted that VC was more likely to go to those firms that had already demonstrated some innovation and similar results have been demonstrated for the US (Hellman and Puri, 2000; Mann and Sager, 2007) and Germany (Engel and Keilbach, 2007). If so, then isn't it more likley that firms would change their strategic decisions knowing that the addition of VC-backing will improve their chances of success in the future?

In this paper, we turn the tables on the existing (empirical) literature by assuming that it is the firms who undertake a more active role regarding their innovation strategies than the venture capitalists, knowing that their ex ante investment decisions will likely have a strong impact on their probability of success in the future as securing VC backing (or not) is conditional on whether they innovate early enough. Thus the primary goal of this paper is to analyse the effects of VC on firms' incentives to innovate at every stage of the production process. In this paper, we assume that it is venture capitalists who have a rather passive role (just like the firms in the empirical literature). Nonetheless, we try not to lose any of the key features that VC possesses. Therefore, we assume VC funding is a package consisting of three things: (*i*) an equity stake in the firm; (*ii*) pecuniary funds; and (*iii*) value-adding services such as monitoring, implementing formal HR procedures or improved marketing.[6] To address the above issues, we consider a stylised two-period, multi-stage game in which innovation is uncertain and firms are of different innovative abilities. By examining both pre- and post-VC funding decisions, we analyse whether VC spurs innovation (*i*) directly after being granted; (*ii*) indirectly by incentivising firms to increase initial research efforts to increase their chances of receiving VC funding (and its associated benefits); or (*iii*) a combination of both. To our knowledge, this is the first paper of its kind to approach VC in this way.

We obtain a number of theoretical results that have not been observed before, not even empirically. First of all, we find that, in the post-VC stage, regardless of VC funding, "success breeds success" (propositions 3 and 10). That is to say, we show that a good predictor of the likelihood of future success is past success: *ceteris paribus*, a firm that innovates early is more likely to develop a high quality product. Nonetheless, the addition of VC has a profound impact on competition

[6] To an extent, one can think of an increase in funding and/or value-adding services as a proxy for the quality of the venture capitalist (see Bottazzi *et al* (2008)). However, we do not believe the specification of our model enables us to read too much into this.

directly after it has been granted. In essence, VC tips the balance of competition in favour of the firm that receives it, regardless of the firm's relative ability level. It does this by inducing the VC-backed firm to invest *more* and the rival firm *less* thereby improving the relative probability of success for the portfolio firm. Therefore, we suggest that the commonly held belief that VC spurs innovation is too simplistic since it overlooks the fact VC clearly damages the prospect of the firms it *does not* support: not only that VC spurs innovation amongst the "lucky", chosen few, but it unambiguously *suppresses* innovation of non-VC-backed firms; an idea that has been overlooked in the empirical literature. The magnitude of this result however is sensitive to the degree of heterogeneity between the firms. When firms are of relatively similar abilities, VC has a more pronounced impact on the composition of the final product market. In fact, it can single-handedly determine which firm is likely to be more innovative. In contrast, as firms become more heterogeneous, VC is unable to prevent the high ability firm from being the most likely innovator.

In the pre-VC stage, we observe two important results. First, firms may treat efforts as either strategic complements or substitutes, depending upon the relative sizes of expected *future* profits between subcases. When expected profits are relatively higher in the *symmetric* (duopoly) cases, the efforts of a rival are positively correlated with a firm's expected profits, inducing it to invest *more* when a rival does (efforts are strategic complements). In contrast, when *asymmetric* outcomes are more valuable, the firms "compete" in effort (efforts are strategic substitutes) (proposition 6). Second, and most important, we find that VC does impact on the firm's effort choices *indirectly*, by altering their future expected payoffs. The equity stake of the firm impacts on initial efforts in two ways: *i*) it directly reduces initial efforts by reducing expected future profits; and *ii*) it indirectly increases (decreases) efforts if the firms treat efforts as strategic substitutes (complements). Thus, the equity stake is *negatively* correlated with effort in the first stage if the firms treat efforts as strategic complements, and *ambiguously* correlated if efforts are treated as strategic substitutes (proposition 14). The impact of pecuniary funding and venture capitalist expertise are however ambiguous. This ambiguity though should not be misinterpreted as no effect. Rather, one should interpret our indirect effect results more broadly: given the specification, it is likely that future VC will have an impact on first period efforts, though it is not possible to say whether this impact is positive or negative.

The rest of the paper is as follows. In section 2 we specify the model in more detail. Section 3 analyses the benchmark, the no-VC, case. In section 4, we examine the impact of venture capital on the firms' effort decisions. Section 5 concludes.

2 Model

We consider a two-period, multi-stage, asymmetric duopoly model in which the quality of innovation is uncertain. We assume that two firms, i and j, have asymmetric "innovative" abilities, $a_i > 0, a_j > 0$ such that $a_i \geq a_j$ i.e. firm i is of higher ability than firm j. The structure of the game can be detailed as follows:

First period

At the beginning of the first period, given the above abilities, firms invest in effort in order to develop a prototype product that can either be of high quality (q_h) or low (q_l), the actual value of which becomes known only at the *end* of the first period. The probability of discovering a certain quality of prototype depends on a firm's ability as well as on its effort level. We denote the (unconditional) probability that firm i develops a high-quality output in a certain period by φ_i^t, $t = 1, 2$. This probability then is a function of firm i's effort level e_i^t in period t as well as its initial ability a_i i.e. $\varphi_i^t = \varphi_i^t(a_i, e_i^t)$.[7] Thus the probability that a firm develops a high or low quality prototype (q_h or q_l) in the first period is given by

$$\begin{aligned} \Pr[q_h | a_i, e_i^1] &= \varphi_i^1(a_i, e_i^1) \\ \Pr[q_l | a_i, e_i^1] &= 1 - \varphi_i^1(a_i, e_i^1) \end{aligned} \quad (1)$$

where e_i^1 is firm i's effort level in period one. The following assumptions characterise the function $\varphi_i^t(a_i, e_i^t)$.

A1. $\partial\varphi_i^t(a_i, e_i^t)/\partial e_i^t > 0; \partial^2\varphi_i^t(a_i, e_i^t)/\partial\left(e_i^t\right)^2 < 0;\ \varphi_i^t(a_i, 0) = 0; \partial\varphi_i^t(a_i, e_i^t)/\partial a_i > 0.$

A2. $\partial^2\varphi_i^t(a_i, e_i^t)/\partial e_i^t \partial a_i > 0.$

A1 says that the probability function is strictly concave in effort, that a firm can never develop a high-quality product if it puts in no effort, and that, for a given level of effort, the more able the firm is, the greater is its probability of success. Assumption A2, which states that a firm's marginal returns to effort are increasing in its ability, captures the idea that a more able firm is better able to target its effort along more effective research paths.

We assume that the marginal cost of effort, c, is constant in every period with $c > 0$. Firms choose their effort level, $e_i^1 \in [0, \infty)$, to maximise their expected profits. Output is then realised and the quality of the firms' prototypes are revealed to all players. There are now four possible scenarios to consider for the second period game:

[7] This probability function however may change in the second period, depending upon whether the firm discovers a high quality prototype or not - see below for the description of the second period game.

Case (i). $\left(q_l^i, q_l^j\right)$: When both firms develop *low* quality prototypes.

Case (ii). $\left(q_h^i, q_l^j\right)$: When firm i develops high quality prototype whilst firm j develops low.

Case (iii). $\left(q_l^i, q_h^j\right)$: When firm i develops low quality prototype whilst firm j develops high.

Case (iv). $\left(q_h^i, q_h^j\right)$: When both firms develop *high* quality prototypes.

At the beginning of the second period, given the above realisation about the quality of the prototypes, firms compete again with respect to their effort (investment) levels to produce output that can either be high (Q_h) or low (Q_l). The realisation of the second period output Q is uncertain *ex ante.* The quality of output Q discovered however determines a firm's future as follows: if only one firm innovates (i.e. develops a high quality good) whilst its rival does not, then that firm becomes a monopolist (e.g. through the grant of some kind of a patent right) and earns a monopoly profit M in the future period whilst its rival earns zero; if both firms innovate (i.e. if both develop Q_h) then both earn duopoly profits of D_H whereas if neither innovates (i.e. produce the low quality product Q_l) then each makes a duopoly profit of D_L in the next period. Without any loss of generality, we assume that

$$M > 2D_H > 2D_L$$

Obviously, firms aspire to become monopolists at the end of the second period and choose effort levels $e_i^2 \in [0, \infty)$ to maximise their expected payoffs at a marginal cost of c.

Our model incorporates a 'learning by doing' effect in the following sense: if a firm has been successful in discovering q_h, then even without any VC backing, this puts the firm in a better position to produce Q_h in the second period. We capture this idea by assuming that the probability of success function is now conditional on the discovery of q_h i.e.

$$\begin{aligned} \Pr[Q_h|q_h] &= \mu_i(a_i, e_i^2) \\ \Pr[Q_l|q_h] &= 1 - \mu_i(a_i, e_i^2) \end{aligned} \tag{2}$$

with

$$\begin{aligned} \lambda\mu_i(a_i, e_i^2) &= \varphi_i^1(a_i, e_i^2) \\ \lambda &\in (0, 1) \end{aligned} \tag{3}$$

Equation (3) then simply states that, at any level of effort, $e_i^2 \in [0, \infty)$, a firm that has developed a high quality prototype has a strictly higher success probability.[8] Consequently, assumptions similar to the ones made in A1 and A2 also hold for $\mu_i(a_i, e_i^2)$ and are summarised by A3 (i.e. $\mu_i(a_i, e_i^2)$ is a strictly concave function of e, is increasing in a_i and shows increasing marginal return to investment with respect to a_i).

A3. $\partial\mu_i(a_i, e_i^2)/\partial e_i^2 > 0; \partial^2\mu_i(a_i, e_i^2)/\partial\left(e_i^2\right)^2 < 0;\ \mu_i(a_i, 0) = 0; \partial\mu_i(a_i, e_i^2)/\partial a_i > 0$; and $\partial^2\mu_i(a_i, e_i^2)/\partial e_i^2\partial a_i > 0$.

Now, in this model we consider the possibility that a firm can obtain backing from a venture capitalist. The presence of a venture capitalist then substantially changes the above scenario. First of all, whether a firm receives any assistance from a venture capitalist depends entirely upon the fact whether it has developed a high quality prototype (q_h) in period 1 or not. Moreover, a VC packages is only offered to a *single* firm: where only one firm has developed a high quality prototype, the VC offering goes to that firm; if both firms developed q_h in the first period then each faces equal probability of securing VC funding (which ultimately is assigned 'randomly' or on the basis of certain outside criteria that are not considered in our model). Finally, VC comes in a *package* consisting of:

1. An equity stake in the firm, s: The equity stake that is required by the venture capitalist as compensation for its risk.
2. Pecuniary funding, F: This denotes the finance offered to the firm.
3. Value-adding services, E: This denotes the additional benefits a venture capitalist offers to the firm beyond finance such as mentoring and expert advice.

The above assumptions keep our modelling of VC in line with those of Bottazzi *et al* (2008) in so far as they imply a venture capitalist plays a far broader role in the firm than traditional financing methods.

How does the acquisition of a VC package affect the winning firm's probability of success? With VC funding, a firm's probability of success in producing Q_h is *further* enhanced over and above the one given by $\mu_i(a_i, e_i^2)$. The probability of innovation is now also a function of the amount of funding received, F, and the value-adding services, E. We denote this function as follows:

$$\begin{aligned} \Pr[Q_h|q_h, VC] &= \hat{\mu}_i(a_i, e_i^2) = \mu_i(a_i, e_i^2, E, F) \\ \Pr[Q_l|q_h, VC] &= 1 - \hat{\mu}_i(a_i, e_i^2) \end{aligned} \tag{4}$$

[8] Note that this assumption ensures that all the properties of φ_i^1 are also transferred to μ_i since λ is a scaler.

where, for any e_i^2 and a_i

$$\hat{\mu}_i(a_i, e_i^2) > \mu_i(a_i, e_i^2)$$

if E or F are positive. Consequently, assumptions similar to that given in A3 also apply here (and hence are not repeated). The following assumption now captures the specific benefits of receiving VC backing, namely, how mentoring and funding affect the probability of innovation[9].

A4. (i) $\partial\hat{\mu}_i(a_i, e_i^2)/\partial F > 0$; $(ii)\partial\, \hat{\mu}_i(a_i, e_i^2)/\partial E > 0$; and (iii) $\partial^2\hat{\mu}_i(a_i, e_i^2)/\partial e_i^2 \partial E > 0$.

A4 says that the impact of receiving mentoring and funding are strictly positive for the firm. Additionally, part (iii) of A4 highlights the indirect effect of mentoring via a firm's effort level: the more value-adding services that are offered by a venture capitalist, the better able a firm becomes at targeting its efforts and so the marginal returns to effort increase.

Finally, if a firm developed a low-quality prototype in the first period (i.e. q_h), then its probability of innovation remains exactly as is specified by the function φ_i^t i.e. it is given by $\varphi_i^2(a_i, e_i^2)$ in the second period.

The timing of the game can now be summarised as follows:

Stage 1: Start of first period. Firms choose effort levels, $e_i^1 \in [0, \infty)$ given their abilities a_i, a_j. Output is produced and the quality of the prototype $q_s, s \in \{h, l\}$, is revealed to all players. End of first period.

Stage 2: Start of second period. The VC package (F, E, s) is assigned to the winning player who then enjoys a probability of success given by $\hat{\mu}_i(.)$. If both have developed high quality prototypes then VC funding is offered to each of them with equal probability. If neither firm discovers q_h, neither receives VC backing. Players who do not receive VC funding have a probability of success given by $\varphi_i^2(.)$. Firms then invest in their effort levels. Output is realised at the end of period 2, and firms earn (future) payoffs according to their position in the market.

We solve the game using backward induction.

3 Benchmark: the no-VC case

In order to appreciate the impact of VC offering, we first consider the scenario where there is no possibility of receiving a VC package. If so, then the second period probability of innovation is given by (2).

[9] We use the reduced form, $\hat{\mu}_i(a_i, e_i^2)$, throughout.

3.1 Second stage equilibrium

First we compute the expected second stage profits corresponding to each of the cases (i)-(iv). Thus, the expected profit functions are

Case (i). $\left(q_l^i, q_l^j\right)$ – both firms develop *low* quality prototypes

$$\pi_{l,l}^i|_{NVC}^{t=2} = (1-\varphi_i^2)(1-\varphi_j^2)D_L + \varphi_i^2\varphi_j^2 D_H + \varphi_i^2(1-\varphi_j^2)M - ce_i^2 \ \forall \ i$$

Case (ii). (q_h^i, q_l^j) - firm i develops high quality prototype while firm j develops low

$$\begin{aligned} \pi_{h,l}^i|_{NVC}^{t=2} &= (1-\mu_i^2)(1-\varphi_j^2)D_L + \mu_i^2\varphi_j^2 D_H + \mu_i^2(1-\varphi_j^2)M - ce_i^2 \\ \pi_{h,l}^j|_{NVC}^{t=2} &= (1-\mu_i^2)(1-\varphi_j^2)D_L + \mu_i^2\varphi_j^2 D_H + \varphi_j^2(1-\mu_i^2)M - ce_j^2 \end{aligned}$$

Case (iii). (q_l^i, q_h^j) - firm i develops low quality prototype whereas firm j develops high

$$\begin{aligned} \pi_{l,h}^i|_{NVC}^{t=2} &= (1-\varphi_i^2)(1-\mu_j^2)D_L + \varphi_i^2\mu_j^2 D_H + \varphi_i^2(1-\mu_j^2)M - ce_i^2 \\ \pi_{l,h}^j|_{NVC}^{t=2} &= (1-\varphi_i^2)(1-\mu_j^2)D_L + \varphi_{ij}^{2\mu2} D_H + \mu_j^2(1-\varphi_i^2)M - ce_j^2 \end{aligned}$$

Case (iv). (q_h^i, q_h^j) - both firms develop *high* quality prototypes

$$\pi_{h,h}^i|_{NVC}^{t=2} = (1-\mu_i^2)(1-\mu_j^2)D_L + \mu_{ij}^{2\mu2} D_H + \mu_i^2(1-\mu_j^2)M - ce_i^2 \ \forall \ i$$

In the above notation for expected profits, the first superscript denotes which firm's profits we are discussing; the first subscript, x, y, denotes the case in which firm i has developed a prototype of quality $x \in \{h, l\}$ and j of quality $y \in \{h, l\}$; the second superscript denotes the period, $t \in \{1, 2\}$; and the second subscript denotes whether this is the benchmark case (NVC) or the VC case (VC).

In each of the above cases, firms maximise profits by choosing respective effort levels. With some manipulation of the relevant first order conditions, we obtain the following set of equations corresponding to each case:

Case (i). (q_l^i, q_l^j)

$$\frac{\partial\varphi_i^2}{\partial e_i^2} = \frac{c}{(M - D_L) - \varphi_j^2(M - D_L - D_H)} \ \forall \ i \qquad (5)$$

Case (ii). (q_h^i, q_l^j)

$$\frac{\partial \mu_i^2}{\partial e_i^2} = \frac{c}{(M - D_L) - \varphi_j^2(M - D_L - D_H)}$$
$$\frac{\partial \varphi_j^2}{\partial e_j^2} = \frac{c}{(M - D_L) - \mu_i^2(M - D_L - D_H)}$$

Case (iii). (q_l^i, q_h^j)

$$\frac{\partial \varphi_i^2}{\partial e_i^2} = \frac{c}{(M - D_L) - \mu_j^2(M - D_L - D_H)}$$
$$\frac{\partial \mu_j^2}{\partial e_j^2} = \frac{c}{(M - D_L) - \varphi_i^2(M - D_L - D_H)}$$

Case (iv). (q_h^i, q_h^j)

$$\frac{\partial \mu_i^2}{\partial e_i^2} = \frac{c}{(M - D_L) - \mu_j^2(M - D_L - D_H)} \forall i$$

The solutions to the above first order conditions then yield a firm's reaction function. The following proposition shows how the optimal effort level of a certain firm changes in response to its rival's.

Proposition 1 *Second period efforts are strategic substitutes regardless of the quality of the prototypes discovered at the end of the first period.*
Proof. *See appendix 7.1* ■

According to proposition 1, second period effort levels are strategic substitutes: any *increase* in one firm's optimal effort level leads to a *decrease* in that of its rival's. The impetus for this result is the fact that, regardless of the prototypes developed by the firms, an increase in firm i's investment has two opposing effects on firm j's expected profits. First, it unambiguously decreases the chances that firm j will become a monopolist in the final product market and, consequently, reduces their expected returns to effort. Second, it increases the expected profits of becoming a duopolist by making it more likely that the firms will act as *high quality* duopolists in the final product market. However, given assumptions A1, A3 and $M > 2D_H > 2D_L$, it is trivial to demonstrate that it is the former of these effects that dominates. Therefore, should firm i's effort level increase, firm j's expected profits are strictly lower, at all levels of e_j^2, than they would have been otherwise. It is this reduction in the expected benefits of investment that drives firm j to cut its investment level in response to an increase by firm i.

The next proposition shows that regardless of the type of prototype discovered, the optimal effort level of a firm increases in its own ability but decreases in its rival's ability. Hence,

Proposition 2 *Regardless of the type of prototype discovered*

$$\frac{de_i^2}{da_i} > 0; \ \frac{de_j^2}{da_i} < 0$$

Proof. *See Appendix 7.2* ■

The importance of this proposition is that it suggests that a firm's ability level is positively correlated with its effort; *ceteris paribus*, a more able firm invests more. The rationale behind this is a consequence of assumptions A1 and A2. As a firm's ability increases, it is induced to invest more for two reasons. First, assumption A1 states that, for a given level of effort, the more able the firm, the greater its probability of success. Consequently, at all effort levels, each unit of investment yields a *higher* expected return which, in turn, induces the firm to increase its investment level. Second, assumption A2 implies that a firm's marginal returns to effort are increasing in its ability because the firm is better able to target its effort along more effective research paths. This further increases the returns to effort, once again spurring a firm to invest more. This increased investment of a more able firm, combined with proposition 1 thensuggests that whilst a higher ability firm will invest more, its rival will be induced to invest less.

Proposition 2 further implies that it is possible to determine, in every case, which firm will invest the most. In the symmetric cases, (ie cases (i) and (iv)), this is straightforward to detemine: assuming that, initially, the firms are symmetric with respect to their abilities ($a_i = a_j$), their equilibrium effort levels will be the same. If however firm i is then allowed to become the *high ability* firm, such that $a_i = a_j + \varepsilon$ where $\varepsilon > 0$ is of small value, then by propositions 1 and 2, this will imply that the effort level of firm i will be striclty highere whereas that of firm j will be strictly lower relative to the symmetric case where $a_i = a_j$. This on the other hand implies that in the cases in which the firms have developed prototypes of similar qualities, the high ability firm invariably invests more and is the more likely innovator. A similar result holds for the asymmetric case (ii).[10] We can

[10] To see that, note that the the relevant first order conditions (after applying equation (3)) are

$$\frac{\partial \varphi_i^2}{\partial e_i^2} = \frac{\lambda c}{(M - D_L) - \varphi_j^2(M - D_L - D_H)} \tag{6}$$

$$\frac{\partial \varphi_j^2}{\partial e_j^2} = \frac{\lambda c}{\lambda(M - D_L) - \varphi_i^2(M - D_L - D_H)} \tag{7}$$

therefore conclude, following propositions 1 and 2, that in all cases of (i), (ii) and (iv) the high-ability firm becomes the likely winner. In case (iv), where firm i (the high ability firm) invests strictly less than firm j,although it is not possible to conclude unambiguously which firm becomes the likey winner, we observe that if firm i still may become the likely winner by discovering Q_h in the final stage of the game if it is *sufficiently* able.

Finally, the following proposition shows that it is possible to order the second stage effort levels conditional on the type of prototype discovered in the first stage.

Proposition 3 *Regardless of the quality of a rival's prototype, a firm always invests strictly more effort when it has discovered a high quality prototype. More formally:*

$$e_i^2|_{q_h^i,q_s^j} > e_i^2|_{q_l^i,q_s^j}$$

for all $s \in \{l, h\}$.
Proof. *See Appendix 7.3* ■

Proposition 3 suggests that a firm will invest more, and be more likely to innovate, if it was successful in developing a high quality prototype at the end of the first stage. One important implication of proposition 3 then is that past success is a good indicator of the likeliness of future successes: once a firm has demonstrated an ability to successfully innovate, it becomes *more* likely to innovate in the future than if it had failed to innovate initially. Proposition 2 enables us to order the firms' profit levels as given by the following corollary.

Corollary 4 *Regardless of the quality of a rival's prototype, a firm's expected profits are higher when it has developed a high quality prototype. Formally,*

$$\pi_{h,s}^i|_{NVC}^{t=2} > \pi_{l,s}^i|_{NVC}^{t=2}$$

for all $s \in \{l, h\}$
Proof. *This proof is trivial and so it is omitted.* ■

Given the result in proposition 3, the implication of corollary 4 is straightforward: when a firm develops a high quality prototype, its expected returns to effort as measured in terms of both expected monopoly and duopoly profits, are strictly greater compared to the case where it invents a low quality protoptype.

Assuming both firms are of the same ability and so the investment levels being identical then yields

$$\frac{\partial \varphi_i^2}{\partial e_i^2} < \frac{\partial \varphi_j^2}{\partial e_j^2}$$

⇒ that a symmetric equilibrium cannot be supported.

Thus when a firm has been successful in developing a high quality prototype, each additional unit of effort yields *larger* increases in expected profits. These additional returns on investment induce a firm to increase their innovative efforts which, in turn, yield higher levels of expected profits an outcome that is *independent* of the quality of the rival's prototype implying that initial success has tangible consequences. To reiterate a previous point, these results suggest that past success is a good indicator of the likeliness of future successes.

3.2 First stage equilibrium

In the first period, the firms' expected profit functions are given by

$$\pi^i|^{t=1}_{NVC} = (1-\varphi^1_i)(1-\varphi^1_j)\pi^i_{l,l}|^{t=2}_{NVC} + \varphi^1_i\varphi^1_j\pi^i_{h,h}|^{t=2}_{NVC} + \varphi^1_i(1-\varphi^1_j)\pi^i_{h,l}|^{t=2}_{NVC} + (1-\varphi^1_i)\varphi^1_j\pi^i_{l,h}|^{t=2}_{NVC} - ce^1_i$$

$$\pi^j|^{t=1}_{NVC} = (1-\varphi^1_i)(1-\varphi^1_j)\pi^j_{l,l}|^{t=2}_{NVC} + \varphi^1_i\varphi^1_j\pi^j_{h,h}|^{t=2}_{NVC} + \varphi^1_i(1-\varphi^1_j)\pi^j_{h,l}|^{t=2}_{NVC} + (1-\varphi^1_i)\varphi^1_j\pi^j_{l,h}|^{t=2}_{NVC} - ce^1_j$$

With a little manipulation of the relevant first order conditions one obtains

$$\frac{\partial\varphi^1_i}{\partial e^1_i} = \frac{c}{(1-\varphi^1_j)(\pi^i_{h,l}|^{t=2}_{NVC} - \pi^i_{l,l}|^{t=2}_{NVC}) + \varphi^1_j[\pi^i_{h,h}|^{t=2}_{NVC} - \pi^i_{l,h}|^{t=2}_{NVC}]} \tag{8}$$

$$\frac{\partial\varphi^1_j}{\partial e^1_j} = \frac{c}{(1-\varphi^1_i)(\pi^j_{l,h}|^{t=2}_{NVC} - \pi^j_{l,l}|^{t=2}_{NVC}) + \varphi^1_i[\pi^j_{h,h}|^{t=2}_{NVC} - \pi^j_{h,l}|^{t=2}_{NVC}]} \tag{9}$$

These follow a similar functional form to those in the second stage but are now dependent on the second period's expected profits. However, as the following proposition demonstrates, each firm's first period efforts may be treated as *either* strategic substitutes *or* complements.

Proposition 5 *First period efforts can be treated as either strategic substitutes or complements. Furthermore, it is possible that one firm treats efforts as a strategic substitutes whilst the other treats them as complements.*
Proof. *See appendix 7.4* ■

Thus, in contrast to second period efforts, firms may treat effort either as strategic substitutes or complements. The above result can be explained as follows: (i) Firms treat efforts as strategic substitutes if and only if the expected profits of becoming the *sole* developer of a high quality prototype are sufficiently large. In this scenario, additional investment by one firm *strictly decreases* the probability that the rival firm will be able to become the sole developer of a high quality prototype. As this makes up a significant proportion of a firm's expected profits, relative to the other cases, an increase in the efforts of one firm significantly reduces the expected profits of the other. Therefore, investment by one firm reduces the incentives of its rival to invest in the first place and, consequently, the rival firm's effort level falls. (ii) In contrast, firms treat efforts as strategic complements when the expected profits of being the sole developer of a high quality prototype are smaller so that there is a greater emphasis on the expected payoffs in the *symmetric* (duopoly) cases. When these are sufficiently large, the investment of a rival actually *increases* the expected profitability of the firm. In essence, the profits of a firm are *positively* correlated with a rival firm's invesment. Therefore, when one firm increases its effort levels, this induces the other firm to do the same. (iii) An interesting third possibility, when firms have asymmetric abilities so that their expected profits are different, can also arise where both firms may treat effort differently such that one firm's reaction function slopes down whilst the other firm's slopes up. In essence, the firms' effort decisions becomes a game of "cat and mouse", with one firm trying to match the other, which is trying to get away.[11] In fact, this additional result may offer some theoretical grounding for the empirical observation that some firms adopt "innovator" strategies whilst others adopt "imitator" strategies (Hellman and Puri , 2000). In our model, the "innovators" are those firm that expect to make relatively large profits if they can innovate early (the firm that treats efforts as substitutes). In contrast, "imitators" are driven to invest not because they expect to be innovators alone, but because their expected profits are positively correlated with the efforts of their rival (the firm that treats efforts as complements). Therefore, in equilibrium, both firms are trying to balance two opposing forces. In the case of the "innovator", they wish to maximise their profits without attracting too much investment by an "imitator". In contrast, an "imitator" wishes to invest as much as possible, without suppressing too much innovative effort of the "innovator".

[11] Mathematically this is not problematic so long as the reaction functions allow for stability and uniqueness. To that end, we must ensure that firms do not "overreact" to a change in a rival's choice. Formally (Fudenberg and Tirole, 1991),

$$\left|R_i^{'}\right|\left|R_j^{'}\right| < 1$$

4 Effects of VC on firms' innovative incentives

Now consider the possibility that a firm can receive offerings from a venture capitalist. The possibility of securing VC backing then changes the above scenario substantially. Recall that a VC package, (s, E, F), is given to *only one* firm that has developed a high quality prototype where the winning firm now has a probability of innovation function given by equation (4). Further, recall that if both firms developed a high quality prototypes then each receives VC with equal probability, where the firm that is *not* successful in receiving the VC offering (despite the fact that it had developed a high-quality prototype) faces the probability $\mu_i(a_i, e_i^2)$. Finally, recall that the probability of success function for the firm that developed a *low* quality prototype remains unchanged i.e. it is given by $\varphi_i^2(a_i, e_i^2) = \lambda\mu_i(a_i, e_i^2, 0, 0)$ - see equation (3)

As in the No-VC case, we start our analysis with the second stage game.

4.1 Second stage equilibrium

In the presence of VC, the expected profits for each case are given by

Case (i). (q_l^i, q_l^j)

$$\pi_{l,l}^i|_{VC}^{t=2} = (1 - \varphi_i^2)(1 - \varphi_j^2)D_L + \varphi_i^2\varphi_j^2 D_H + \varphi_i^2(1 - \varphi_j^2)M - ce_i^2 \; \forall \; i$$

Case (ii). (q_h^i, q_l^j)

$$\begin{aligned} \pi_{h,l}^i|_{VC}^{t=2} &= (1 - s)\left[(1 - \widehat{\mu}_i^2)(1 - \varphi_j^2)D_L + \widehat{\mu}_i^2\varphi_j^2 D_H + \widehat{\mu}_i^2(1 - \varphi_j^2)M - ce_i^2\right] \\ \pi_{h,l}^j|_{VC}^{t=2} &= (1 - \widehat{\mu}_i^2)(1 - \varphi_j^2)D_L + \widehat{\mu}_i^2\varphi_j^2 D_H + \varphi_j^2(1 - \widehat{\mu}_i^2)M - ce_j^2 \end{aligned}$$

Case (iii). (q_l^i, q_h^j)

$$\begin{aligned} \pi_{l,h}^i|_{VC}^{t=2} &= (1 - \varphi_i^2)(1 - \widehat{\mu}_j^2)D_L + \varphi_i^2\widehat{\mu}_j^2 D_H + \varphi_i^2(1 - \widehat{\mu}_j^2)M - ce_i^2 \\ \pi_{l,h}^j|_{VC}^{t=2} &= (1 - s)\left[(1 - \varphi_i^2)(1 - \widehat{\mu}_j^2)D_L + \varphi_i^2\widehat{\mu}_j^2 D_H + \widehat{\mu}_j^2(1 - \varphi_i^2)M - ce_j^2\right] \end{aligned}$$

Case (iv). (q_h^i, q_h^j). Here,

(a) If firm i received VC

$$\begin{aligned} \pi_{h,h}^i|_{VC_i}^{t=2} &= (1 - s)\left[(1 - \widehat{\mu}_i^2)(1 - \mu_j^2)D_L + \widehat{\mu}_i^2\mu_j^2 D_H + \widehat{\mu}_i^2(1 - \mu_j^2)M - ce_i^2\right] \\ \pi_{h,h}^j|_{VC_i}^{t=2} &= (1 - \widehat{\mu}_i^2)(1 - \mu_j^2)D_L + \widehat{\mu}_i^2\mu_j^2 D_H + \mu_j^2(1 - \widehat{\mu}_i^2)M - ce_j^2 \end{aligned}$$

(b) If firm j received VC

$$\begin{aligned}\pi^i_{h,h}|^{t=2}_{VC_j} &= (1-s)\left[(1-\widehat{\mu}^2_i)(1-\mu^2_j)D_L + \widehat{\mu}^2_i\mu^2_j D_H + \widehat{\mu}^2_i(1-\mu^2_j)M - ce^2_i\right] \\ \pi^j_{h,h}|^{t=2}_{VC_j} &= (1-\widehat{\mu}^2_i)(1-\mu^2_j)D_L + \widehat{\mu}^2_i\mu^2_j D_H + \mu^2_j(1-\widehat{\mu}^2_i)M - ce^2_j\end{aligned}$$

where the (altered) second subscript VC_x in case (iv) now implies that firm $x \in \{i, j\}$ received VC when both firms were eligible.

Each firm now maximises their second period payoffs. Then, using the first order conditions - and with a little manipulation - we find that, for each of the cases (i) - (iv), the firms' effort level decisions are given by

Case (i). (q^i_l, q^j_l)

$$\frac{\partial \varphi^2_i}{\partial e^2_i} = \frac{c}{(M-D_L)-(1-\varphi^2_j)(M-D_L-D_H)} \; \forall \; i$$

Case (ii). (q^i_h, q^j_l)

$$\begin{aligned}\frac{\partial \hat{\mu}^2_i}{\partial e^2_i} &= \frac{c}{(M-D_L)-(1-\varphi^2_j)(M-D_L-D_H)} \\ \frac{\partial \varphi^2_j}{\partial e^2_j} &= \frac{c}{(M-D_L)-(1-\hat{\mu}^2_i)(M-D_L-D_H)}\end{aligned}$$

Case (iii). (q^i_l, q^j_h)

$$\begin{aligned}\frac{\partial \varphi^2_i}{\partial e^2_i} &= \frac{c}{(M-D_L)-(1-\hat{\mu}^2_j)(M-D_L-D_H)} \\ \frac{\partial \hat{\mu}^2_j}{\partial e^2_j} &= \frac{c}{(M-D_L)-(1-\varphi^2_i)(M-D_L-D_H)}\end{aligned}$$

Case (iv). (q^i_h, q^j_h)

$i)$ If firm i received VC

$$\begin{aligned}\frac{\partial \hat{\mu}^2_i}{\partial e^2_i} &= \frac{c}{(M-D_L)-(1-\mu^2_j)(M-D_L-D_H)} \\ \frac{\partial \mu^2_j}{\partial e^2_j} &= \frac{c}{(M-D_L)-(1-\hat{\mu}^2_i)(M-D_L-D_H)}\end{aligned}$$

$ii)$ If firm j received VC

$$\frac{\partial \mu_i^2}{\partial e_i^2} = \frac{c}{(M - D_L) - (1 - \hat{\mu}_j^2)(M - D_L - D_H)}$$
$$\frac{\partial \hat{\mu}_j^2}{\partial e_j^2} = \frac{c}{(M - D_L) - (1 - \mu_i^2)(M - D_L - D_H)}$$

The following proposition follows from the above equations.

Proposition 6 *Second period efforts are always strategic substitutes regardless of the quality of prototype developed.*
Proof. *The proof is identical in style to that of proposition 3 and so is omitted. Nonetheless, the result still hinges on the assumptions made in A1 - A3 and* $M > 2D_H > 2D_L$. ■

Thus, similar to proposition 3, proposition 6 also states that regardless of the prototypes developed by the firms, an increase in firm i's investment level will *decrease* firm j's effort level. The intuition behind this proposition is also similar to that of proposition 3: that is any increase in firm i's effort makes it less likely that firm j will become a monopolist, whilst it strictly increases the probability that firm j will become a high (and not low) quality duopolist. Given assumption A1, A3 and $M > 2D_H > 2D_L$, it can then be easily checked that the reduction in expected monopoly profits dominates which then incentivises firm j to cut back on its investment level.

Further, similar to the No-VC case, second period effort levels are determined by a firm's relative ability.

Remark 7 *Regardless of the type of prototype discovered*

$$\frac{de_i^2}{da_i} > 0; \ \frac{de_j^2}{da_i} < 0$$

Proof. *The proof is almost identical to that of proposition 2 and so is omitted. Nonetheless, the result still hinges on the assumptions made in A1 - A3 and* $M > 2D_H > 2D_L$. ■

The intuition behind remark 7 is identical to that of proposition 2 and is driven by assumptions A1 and A2. A higher ability makes a firm more likely to develop a high quality good *and* better able to target its efforts, increasing the expected returns to effort. Consequently, the firms are induced to invest more when they are of higher ability.

However, it is no longer just ability that plays a role in the determining the future successes of the firms. Instead, the VC package now plays crucial role. The following proposition demonstrates the impact VC has on the firms' incentives to innovate.

Proposition 8 *Assuming that firm i receives VC backing, we observe*[12]

$$\frac{de_i^2}{dF} > 0; \ \frac{de_j^2}{dF} < 0 \tag{10}$$

$$\frac{de_i^2}{dE} > 0; \ \frac{de_j^2}{dE} < 0 \tag{11}$$

$$\frac{de_i^2}{ds} = \frac{de_i^2}{ds} = 0 \tag{12}$$

Proof. *See Appendix 7.5* ■

The crucial element of proposition 8 is that VC unambiguously increases the probability of successful innovation for the firm that is chosen, by inducing it to invest *more*. In contrast, the non-VC backed firm that must compete against a VC-backed rival invests less and becomes *less* likely to develop a high quality good. Consequently, *VC tips the balance of competition in favour of the firm it backs.*

The two particular elements of the VC package that generate this result are the VC funding F, and the VC value-adding and mentoring services. First, the addition of pecuniary funding, F, makes a firm more likely to innovate at all levels of effort. Thus, a firm with financial backing is, in a sense, able to *buy* success as, regardless of their efforts or ability, the firm may now have access to new equipments or better quality materials. It is the addition of finance, and the greater likelihood that they innovate successfully, that makes effort more valuable and induces them to invest more. Second, a venture capitalist offers value-adding services ranging from simply mentoring firms and improving marketing strategies to overhauling corporate governance completely. Regardless of the extent of their involvement, venture capitalists' own efforts are likely to have two impacts: *i*) increases to E may simply raise the probability of success at all effort levels by allowing entrepreneurs more time to focus on innovation; or *ii*) a venture capitalist may use its expertise and market knowledge to channel the entrepreneurs efforts down more fruitful research pathways. In both cases, these strictly increase the returns to each additional unit of effort of the winning firm. Therefore, the existence of value adding services creates an environment that enables a firm to invest more.[13]

Given proposition 8, we are now able to determine whether a firm would invest more or less compared to the no-VC case, as the following corollary explains.

[12] Results for firm j can be derived by symmetry.

[13] Given the specification of the model it is not possible to determine which effect, E or F, is larger.

Corollary 9 *Compared to the benchmark case, a firm that has received VC funding invests strictly more than it would have done without VC-backing. Furthermore, a non-VC backed firm invests strictly* ***less****, compared to the benchmark case, when it faces a VC-backed rival.*

Proof. Follows directly from proposition 8. ■

The intuition behind this result follows directly from the benefits of funding and venture capitalist expertise: by increasing the returns to each additional unit of effort, venture capitalists induce VC-backed firms to invest more. If so, then by proposition (6) the non-winning firm invests strictly less.

One important implication of this corollary, which has so far been largely overlooked in the literature, is that whilst VC does spur innovation and increases the probability of success for the firm that receives it, there are also casualties. If a firm is competing against a VC backed rival it becomes *less* likely to develop a high quality final good than if no VC were present. Consequently, VC spurs innovation not only by incentivising (future) innovative efforts of an early innovator, but also by *suppressing* the efforts of firms that failed to innovate initially. This unique aspect of our result contributes significantly to the currently existing VC literature which only talks about the fact that VC *spurs success* but ignores completely the mechanism behind such success.

The following proposition highlights the imporatnce of early innovation.

Proposition 10 *Regardless of the quality of a rival's prototype, a firm invests strictly more if it has developed a high quality prototype. Moreover, in the cases in which a firm has developed a high quality prototype, it invests more if it recieves VC-funding than if does not.*

$$e_i^2|_{q_h^i,q_s^j|VC_i} > e_i^2|_{q_h^i,q_s^j|VC_j} > e_i^2|_{q_l^i,q_s^j}$$

Proof. *See Appendix 7.6* ■

The proposition therefore says whilst similar to the proposition 3 in the non-VC benchmark, a firm is always more likely to develop a high quality product if it has developed a high quality prototype regardless of the quality of its rival's prototype developed by a rival, securing VC backing further augments a firm's innovative process, by improving its likelihood of success. Thus, the fact that "success *still* breeds success" is even stronger in the presence of VC backing.

Finally, we also observe the following:

Corollary 11 *Regardless of the quality of a rival's prototype, a firm's expected profits are higher when it has received VC funding. More formally,*

$$\pi_{h,s}^i|_{VC_i}^{t=2} > \pi_{h,s}^i|_{VC_j}^{t=2} > \pi_{l,s}^i|_{VC}^{t=2}$$

Proof. *The proof is trivial and so it is omitted.* ■

Even though "success breeds success", the question is why? Given assumptions A1 - A4 and equation (4), it becomes apparent that, with no additional effort on the part of the entrepreneur, its expected profits are larger if it has successfully innovated a high quality prototype. Therefore, each additional unit of effort is more valuable and generates higher levels of marginal profit. This incentivises the firm to invest *more* and generates *larger* expected profits than if it had failed to innovate at the end of the first stage.[14]

Finally, it is important to understand which firm is most likely to develop a high quality final good.[15] Assuming that firm i is the *high* ability firm, $a_i > a_j$, it is obvious that in cases (ii) and (iv.i) (ie (q_h^i, q_l^j) and $(q_h^i, q_h^j | VC_i)$) cases firm i is more likely to succeed. This follows directly from remark 7 and proposition 8. However, there are also two ambiguous cases where firm j has received VC funding: (q_l^i, q_h^j) and $(q_h^i, q_h^j | VC_j)$. Intuitively, assuming that the firms are initially of equal ability, it must be that firm j invests more in the (q_l^i, q_h^j) and $(q_h^i, q_h^j | VC_j)$ cases, where it is VC-backed. However, by remark 7, an increase in firm i's ability will unambiguously increase e_i^2 and decrease e_j^2. This implies that for *any* VC package, $(s,\ E,\ F)$, as long as a_i is sufficiently large the more able firm is the more likely firm develop a high quality final product regardless of the quality of its prototype. However, as E and F increase this becomes harder and, therefore, less likely. For large values of E and F it is more probable that the likely winner is determined by who is chosen to receive VC funding. That is, the firm that receives the VC becomes, somewhat automatically, the stronger firm. Hence, depending the entrepreneurs' relative abilities and the specification of the VC package on offer, VC funding may have either a small or large impact on the likely *composition of the final product market.* One can therefore expect to see different impacts of VC funding across different industries depending upon the degree of heterogeneity amongst firms in different industries.

Given that the final outcome of 'who becomes the winner' depends heavily upon the firms' effort level in the first period, we now turn to analyse how the prospect of securing VC backing alters firms' initial effort levels.

[14] It is the shape of the probability functions that drives this result. As $\hat{\mu}$ and μ lie strictly above φ, firms are more likely to succeed, at any level of ability or effort, if they have demonstrated some initial innovative ability.

[15] We ignore the case in which both firms develop low quality prototypes as this is identical to the no-VC case.

4.2 First stage equilibrium

The first stage expected profits are given by

$$\pi^i|_{VC}^{t=1} = (1-\varphi_i^1)(1-\varphi_j^1)\pi_{l,l}^i|_{VC}^{t=2} + \frac{1}{2}\varphi_i^1\varphi_j^1\left[\pi_{h,h}^i|_{VC_i}^{t=2} + \pi_{h,h}^i|_{VC_j}^{t=2}\right] + \varphi_i^1(1-\varphi_j^1)\pi_{h,l}^i|_{VC}^{t=2} + (1-\varphi_i^1)\varphi_j^1\pi_{l,h}^i|_{VC}^{t=2} - ce_i^1$$

$$\pi^j|_{VC}^{t=1} = (1-\varphi_i^1)(1-\varphi_j^1)\pi_{l,l}^j|_{VC}^{t=2} + \frac{1}{2}\varphi_i^1\varphi_j^1\left[\pi_{h,h}^j|_{VC_i}^{t=2} + \pi_{h,h}^j|_{VC_j}^{t=2}\right] + \varphi_i^1(1-\varphi_j^1)\pi_{h,l}^j|_{NVC}^{t=2} + (1-\varphi_i^1)\varphi_j^1\pi_{l,h}^j|_{NVC}^{t=2} - ce_j^1$$

Where the first order conditions yield

$$\frac{\partial\varphi_i^1}{\partial e_i^1} = \frac{c}{\left\{\begin{array}{c}(1-\varphi_j^1)\left[\pi_{h,l}^i|_{VC}^{t=2} - \pi_{l,l}^i|_{VC}^{t=2}\right] \\ +\varphi_j^1\left[\frac{1}{2}\left(\pi_{h,h}^i|_{VC_i}^{t=2} + \pi_{h,h}^i|_{VC_j}^{t=2}\right) - \pi_{l,h}^i|_{VC}^{t=2}\right]\end{array}\right\}} \tag{13}$$

$$\frac{\partial\varphi_j^1}{\partial e_j^1} = \frac{c}{\left\{\begin{array}{c}(1-\varphi_i^1)\left[\pi_{l,h}^j|_{VC}^{t=2} - \pi_{l,l}^j|_{VC}^{t=2}\right] \\ +\varphi_i^1\left[\frac{1}{2}\left(\pi_{h,h}^j|_{VC_i}^{t=2} + \pi_{h,h}^j|_{VC_j}^{t=2}\right) - \pi_{h,l}^j|_{VC}^{t=2}\right]\end{array}\right\}} \tag{14}$$

Similar to the No-VC case, first period efforts are determined by expected future profits. However, as the following proposition demonstrates, the reaction functions of each firm can be *either* upward *or* downward sloping.

Proposition 12 *In the VC case, first period efforts can be treated as either strategic substitutes or complements. Additionally, one firm may treat efforts as a strategic substitutes whilst another treats efforts as complements.*
Proof. *See Appendix 7.7* ■

Similar to the mechanism to proposition 5, proposition 12 suggests firms consider efforts as strategic substitutes if and only if the expected profits of becoming the *sole* developer of a high quality prototype are sufficiently large. The addition of venture capital, which unambiguously increases (decreases) expected profits for the VC-backed (non-VC-backed) firm, does not alter this intuition. Thus, when the expected gains are disproportionately large in the case in which only one firm develops a high quality prototype, any increase in effort by a rival firm *significantly* reduces a firm's expected profits. Consequently, an increase in one firm's effort reduces the incentive for the other to invest, regardless of whether that firm is VC-backed or not. In contrast, when the expected profits from symmetric innovation are relatively large, the expected profits of one firm become positively correlated with the effort of its rival. Therefore, when a rival firm invests more, a firm is incentivised to invest more too.

These results suggest that, whilst VC funding clearly has an impact on effort levels by influencing expected second period profits, the mechanisms by which the firms compete remain unchanged. Both firms may still treat effort as strategic complements, substitutes or a combination of the two, but they still act in a similar way to the no-VC case. This is, perhaps, one of venture capitals greatest strengths: whilst it does influence the *outcome*, it does not affect the *mechanism*.[16]

An important question to ask then is: how does the lure of VC impact firms' effort levels in the first period? Firstly, as the following remark shows, whilst the pecuniary funding F and venture capitalists' effort E do have an impact on firms' first period effort level, the magnitudes of those cannot (yet) be quantified. [17]

Remark 13 *The impact of pecuniary funding, F, and venture capitalist effort, E, on first period effort is ambiguous, regardless of whether firms treat effort as strategic substitutes or complements.*
Proof. *See Appendix 7.8* ■

Despite that, Proposition 14 demonstrates the impact of equity stake s on firms' first period incentives to invest.

Proposition 14 *When firms treat effort as strategic complements, the higher the equity stake in the firm, the lower the effort level, or*

$$\frac{de_i^1}{ds} < 0 \ \forall \ i$$

However, when firms treat efforts as strategic substitutes the effect of the equity stake can be either positive or negative.
Proof. *See Appendix 7.9* ■

What proposition 14 reveals is that the venture capitalist's equity stake has both a *direct* and *indirect* impact on a firm's effort choice. The *direct* effect is unambiguously negative: as the venture capitalist's equity stake becomes larger, a firm will want to invest less in the first period. Intuitively, as a the venture

[16] It is the use of an equity stake that is the reason for this observation. This is, perhaps, why venture capitalists use equity shares and not traditional methods, so as to avoid altering the incentives of the firms (see Brander and Lewis (1986)).

[17] This is mainly because the interplay of numerous factors (see the proof) makes it difficult to gauge which effect domintates which.

capitalist's share of *future* profits become larger, there is less incentive for the firm to invest because the expected profits of innovation are reduced. In contrast, the *indirect* effect accounts for a firm's reaction to a fall in a rival's investment *caused* by an increase in the equity stake. The indirect effects therefore depends upon whether first period efforst are strategic substitutes or complements. Where the firms treat efforts as strategic substitutes, one firm will *increase* its research efforts in response to a *reduction* in a rival's. Therefore, as an increase in the equity stake unambiguously puts downward pressure on the investment decisions of both firms, this indirectly induces the firms to invest more; *a positive indirect effect*. Consequently, whether firms actually invest more or less is determined by the balance of these opposing forces. In contrast, where firms treat efforts as strategic complements, a fall in a rival's investment will induce a firm to invest less too; *a negative indirect effect*. The reason behind this is that, a firm's expected profits are positively correlated with the effort level of its rival. Therefore, as a higher equity stake reduces the rival firm's incentives to invest, this leads a firm to reduce their effort levels too. Thus, in the strategic complements case both the direct and indirect effects act in the *same* direction, and it must be that increasing the equity stake reduces investments.

5 Conclusion

This paper set out to address a notable imbalance in the VC literature, that venture capitalists were the sole force behind the VC-to-success link. This link however is incomplete as it does not address the mechanism through which VC alters firms' strategic investment decisions that can lead to innovation. In this paper we have we examined the VC-to-success link from the *firms'* perspectives, in order to understand the exact route through which VC can lead to successful innovation. Specifically, we have examined the pre- and post-VC funding decisions to determine whether VC-funding spurs innovation (*i*) directly after being granted; (*ii*) indirectly by incentivising firms to increase initial research efforts to increase their chances of receiving VC funding (and its associated benefits); or (*iii*) a combination of both. Our analysis shows that VC has both direct (*ex post*) *and* indirect (*ex ante*) implications for a firm's investment decisions in both cases where a firm has been successful in securing VC funding and where it has not.

Our second stage results clearly demonstrated a *direct* link between VC and innovation/success. First, it appears that "success breeds success", in so far as a predictor of future innovation appears to be past innovative success. Second, and most important, the addition of venture-backing to a firm *tips* the balance of competition in its favour. The addition of funding enables firms to, in essence, "buy success", by spending money on better equipment or materials. The addition of value-adding services is important for two reasons: it (*i*) directly increases the likelihood of success by enabling an entrepreneur to *focus* on innovation; and *ii*) a venture capitalist's expertise may indirectly benefit a firm if it is able to direct it along more fruitful research pathways. We show that whilst the addition of VC benefits the firm that receives it, it unambiguously reduces the likelihood of innovation for *all other firms* that did not receive VC backing. Therefore, the commonly held belief that VC spurs innovation is only a partial truth. Whilst it is true that VC can indeed spur innovation (and does indeed do so in many circumstances) by increasing the likelihood of success of the *winning* firm and lowering that for the 'loser' firms, the ultimate effect however is dependent upon the extent of heterogeneity amongst firm. The more symmetric the firms are, stronger is the impact of VC funding on the VC-backed firm's probability of success, in which case VC becomes an important factor in determining the ultimate composition of the final product market.

We show that VC also impacts firms' initial effort levels ie *before* VC is offered to firms. It does so by altering firms' future expected profit levels. In this context, we have shown that venture capitalists' equity stake plays a prominent role where it impacts firms' intitial effort levels in two ways: it (*i*) directly reduces initial efforts by reducing expected future profits; and it (*ii*) indirectly increases (decreases) efforts if the firms treat efforts as strategic substitutes (complements). Finally, we have shown that the extent of VC finance and the value-adding services also have an impact upon firms' intial effort levels although it was not possible to determine the exact magnitudes of such effects without assigning specific functional forms (and this remains as a future plan of work).

We believe, our paper is one the first theoretical analyses that examines the VC-to-success link at a micro level, from firms' perspectives. Analysing such mechanisms are of utmost importance as they have serious public policy implications for fostering environments conducive to economic growth and innovation.

There are several ways the model can be further extended. Recently, a relatively new body of the empirical literature has started to uncover additional benefits derived from the interactions between firms backed by a *single* venture capitalist *i.e.* the issue of strategic alliances.[18] It would be useful to see whether such interactions would further enhance a firm's chances of innovation or whether they would still compete in efforts in post-VC stage.

[18] For evidence, see Lindsey (2008) and Ozmel *et al* (2013)

6 Bibliography

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7 Appendix

7.1 Proof of Proposition 1

Proof. The slope of the reaction functions are given by

$$R_i^{'} = -\frac{\frac{\partial^2 \pi^i}{\partial e_i^2 \partial e_j^2}}{\frac{\partial^2 \pi^i}{\partial (e_i^2)^2}}$$

Consider case (i) - $\left(q_l^i, q_l^j\right)$ first. Then observe that

$$\frac{\partial^2 \pi_{l,l}^i|_{NVC}^{t=2}}{\partial (e_i^2)^2} = \frac{\partial^2 \varphi_i^2}{\partial (e_i^2)^2}\left[(M - D_L) - \varphi_j^2 (M - D_L - D_H)\right] < 0 \tag{15}$$

$$\frac{\partial^2 \pi_{l,l}^i|_{NVC}^{t=2}}{\partial e_i^2 \partial e_j^2} = \frac{\partial \varphi_i^2}{\partial e_i^2}\frac{\partial \varphi_j^2}{\partial e_j^2}[D_L + D_H - M] < 0 \tag{16}$$

by assumption A1 (concavity) and $M > 2D_H > 2D_L$. As both probability functions, $\varphi_i^2(a_i, e_i^2)$ and $\mu_i^2(a_i, e_i^2)$, possess similar properties (by A1 and A3) it then follows immediately that that the other cases yield the same result. ■

7.2 Proof of Proposition 2

Proof. The proof holds for all cases (i) - (iv) given the assumptions in A1 - A3. Consequently, we only prove this for the case (q_l^i, q_l^j), but similar proofs exist for all other cases. We solve this comparative static using Cramer's rule where

$$Ax = b$$

$$\begin{bmatrix} \frac{\partial^2 \pi^i_{l,l}|^{t=2}_{NVC}}{\partial (e_i^2)^2} & \frac{\partial^2 \pi^i_{l,l}|^{t=2}_{NVC}}{\partial e_i^2 \partial e_j^2} \\ \frac{\partial^2 \pi^j_{l,l}|^{t=2}_{NVC}}{\partial e_i^2 \partial e_j^2} & \frac{\partial^2 \pi^j_{l,l}|^{t=2}_{NVC}}{\partial (e_j^2)^2} \end{bmatrix} \begin{bmatrix} \frac{de_i^2}{dx} \\ \frac{de_j^2}{dx} \end{bmatrix} = \begin{bmatrix} -\frac{\partial^2 E\pi_i}{\partial e_i^2 \partial x} \\ -\frac{\partial^2 E\pi j}{\partial e_{j2} \partial x} \end{bmatrix}$$

Using this, we can obtain $\frac{de_{i2}}{da_i}$ by substituting $a_i = x$ and using

$$\frac{de_i^2}{da_i} = \frac{\left|A_{e_i^2 a_i}\right|}{|A|} = \frac{\begin{vmatrix} -\frac{\partial^2 \pi^i_{l,l}|^{t=2}_{NVC}}{\partial e_i^2 \partial a_i} & \frac{\partial^2 \pi^i_{l,l}|^{t=2}_{NVC}}{\partial e_i^2 \partial e_j^2} \\ -\frac{\partial^2 \pi^i_{l,l}|^{t=2}_{NVC}}{\partial e_{j2} \partial a_i} & \frac{\partial^2 \pi^j_{l,l}|^{t=2}_{NVC}}{\partial (e_j^2)^2} \end{vmatrix}}{\begin{vmatrix} \frac{\partial^2 \pi^i_{l,l}|^{t=2}_{NVC}}{\partial (e_i^2)^2} & \frac{\partial^2 \pi^i_{l,l}|^{t=2}_{NVC}}{\partial e_i^2 \partial e_j^2} \\ \frac{\partial^2 \pi^j_{l,l}|^{t=2}_{NVC}}{\partial e_i^2 \partial e_j^2} & \frac{\partial^2 \pi^j_{l,l}|^{t=2}_{NVC}}{\partial (e_j^2)^2} \end{vmatrix}}$$

which yields

$$|A| = \frac{\partial^2 \varphi_i^2}{\partial (e_i^2)^2} \frac{\partial^2 \varphi_j^2}{\partial (e_j^2)^2} \left\{ \begin{array}{l} [(M - D_L) - \varphi_i^2 (M - D_L - D_H)] \\ [(M - D_L) - \varphi_j^2 (M - D_L - D_H)] \end{array} \right\} - \left(\frac{\partial \varphi_i^2}{\partial e_i^2} \right)^2 \left(\frac{\partial \varphi_j^2}{\partial e_j^2} \right)^2 (M - D_H - D_L)^2$$

$$\left|A_{e_i^2 a_i}\right| = -\frac{\partial^2 \varphi_i^2}{\partial e_i^2 \partial a_i} \frac{\partial^2 \varphi_j^2}{\partial (e_j^2)^2} \left\{ \begin{array}{l} [(M - D_L) - \varphi_i^2 (M - D_L - D_H)] \\ [(M - D_L) - \varphi_j^2 (M - D_L - D_H)] \end{array} \right\} + \frac{\partial \varphi_i^2}{\partial e_i^2} \frac{\partial \varphi_i^2}{\partial a_i} \left(\frac{\partial \varphi_j^2}{\partial e_j^2} \right)^2 (M - D_H - D_L)^2$$

Signing these equations is quite simple. First, $\left|A_{e_i^2 a_i}\right|$ is strictly positive given the assumptions A1 and A2. The sign of $|A|$ is harder to interpret. However, assuming uniqueness and stability holds, or

$$\frac{1}{|R_i'|} > \left|R_j'\right| \tag{17}$$

we observe

$$\frac{\partial^2 \varphi_i^2}{\partial (e_i^2)^2} \frac{\partial^2 \varphi_j^2}{\partial (e_j^2)^2} \begin{pmatrix} [(M - D_L) - \varphi_i^2(M - D_L - D_H)] \\ [(M - D_L) - \varphi_j^2(M - D_L - D_H)] \end{pmatrix}$$
$$> \left(\frac{\partial \varphi_i^2}{\partial e_i^2}\right)^2 \left(\frac{\partial \varphi_j^2}{\partial e_j^2}\right)^2 (M - D_H - D_L)^2$$

Consequently, $|A| > 0$ and $\frac{de_i^2}{da_i} > 0$.

The case for $\frac{de_j^2}{da_i}$ is similar and we obtain

$$\left|A_{e_j^2 a_i}\right| = -\frac{\partial^2 \varphi_i^2}{\partial (e_i^2)^2} \frac{\partial \varphi_i^2}{\partial a_i} \frac{\partial \varphi_j^2}{\partial e_j^2} \begin{pmatrix} [(M - D_L) - \varphi_i^2(M - D_L - D_H)] \\ (D_H + D_L - M) \end{pmatrix}$$
$$+ \frac{\partial^2 \varphi_i^2}{\partial e_i^2 \partial a_i} \frac{\partial \varphi_i^2}{\partial e_i^2} \frac{\partial \varphi_j^2}{\partial e_j^2} [(M - D_L) - \varphi_j^2(M - D_L - D_H)](D_H + D_L - M)$$

which is strictly negative. Consequently $\frac{de_j^2}{da_i} < 0$ ■

7.3 Proof of Proposition 3

Proof. The proof for proposition 3 is similar for both firms. Therefore, we only present the proof for firm i.

First, comparing $e_i^2|_{q_h^i, q_l^j}$ and $e_i^2|_{q_l^i, q_l^j}$, and recalling $\lambda \mu_i(a_i, e_i^2) = \varphi_i(a_i, e_i^2)$, we observe

$$\frac{\partial \varphi_i^2}{\partial e_i^2}|_{q_h^i, q_l^j} = \frac{\lambda c}{(M - D_L) - \varphi_j^2(M - D_L - D_H)}$$
$$\frac{\partial \varphi_i^2}{\partial e_i^2}|_{q_l^i, q_l^j} = \frac{c}{(M - D_L) - \varphi_j^2(M - D_L - D_H)}$$

Assuming across both cases firm i invests symmetrically, we observe

$$\frac{\partial \varphi_i^2}{\partial e_i^2}|_{q_h^i, q_l^j} < \frac{\partial \varphi_i^2}{\partial e_i^2}|_{q_l^i, q_l^j}$$

Given the assumptions made in A1 and A3, it must be that this implies that firm i would like to: i) invest strictly more where it has developed a high quality prototype; *ii*) invest strictly less where it has developed a low quality prototype; or *iii*) some combination of *i*) and *ii*). Either way, when firm j has developed a low quality prototype, firm i invests more when it has developed a high quality prototype.

Second, comparing $e_i^2|_{q_h^i,q_h^j}$ and $e_i^2|_{q_l^i,q_h^j}$ we observe

$$\begin{aligned}\frac{\partial\varphi_i^2}{\partial e_i^2}|_{q_h^i,q_h^j} &= \frac{\lambda^2 c}{\lambda(M-D_L)-\varphi_j^2(M-D_L-D_H)}\\ \frac{\partial\varphi_i^2}{\partial e_i^2}|_{q_l^i,q_h^j} &= \frac{\lambda c}{\lambda(M-D_L)-\varphi_j^2(M-D_L-D_H)}\end{aligned}$$

Again, a symmetric level of effort cannot be observed. More formally, we find

$$\frac{\partial\varphi_i^2}{\partial e_i^2}|_{q_h^i,q_h^j} < \frac{\partial\varphi_i^2}{\partial e_i^2}|_{q_l^i,q_h^j}$$

which implies $e_i^2|_{q_h^i,q_h^j} > e_i^2|_{q_l^i,q_h^j}$. Therefore, regardless of the prototype developed by firm j, firm i always invests strictly more when it has developed a high quality prototype. ■

7.4 Proof of Proposition 5

Proof. The slope of firm i's reaction function depends upon

$$\begin{aligned}\frac{\partial^2\pi^i|_{NVC}^{t=1}}{\partial(e_i^2)^2} &= \frac{\partial^2\varphi_i^1}{\partial(e_i^1)^2}\left[\begin{array}{c}(1-\varphi_j^1)(\pi_{h,l}^i|_{NVC}^{t=2}-\pi_{l,l}^i|_{NVC}^{t=2})\\ +\varphi_j^1[\pi_{h,h}^i|_{NVC}^{t=2}-\pi_{l,h}^i|_{NVC}^{t=2}]\end{array}\right]\\ \frac{\partial^2\pi^i|_{NVC}^{t=1}}{\partial e_i^1\partial e_j^1} &= \frac{\partial\varphi_i^1}{\partial e_i^1}\frac{\partial\varphi_j^1}{\partial e_j^1}\left[\begin{array}{c}\left(\pi_{h,h}^i|_{NVC}^{t=2}+\pi_{l,l}^i|_{NVC}^{t=2}\right)\\ -\left(\pi_{h,l}^i|_{NVC}^{t=2}+\pi_{l,h}^i|_{NVC}^{t=2}\right)\end{array}\right]\end{aligned}$$

Given assumptions A1, A3 and corollary 4 it must be that the former of these equations is strictly negative. In contrast, corollary 4 is not sufficient to determine the sign of the second order cross partial derivative. However, we are able to determine that the second order cross partial derivative of firm i is negative, and so efforts are treated as strategic substitutes, if and only if

$$\pi_{h,l}^i|_{NVC}^{t=2}-\pi_{l,l}^i|_{NVC}^{t=2} > \pi_{h,h}^i|_{NVC}^{t=2}-\pi_{l,h}^i|_{NVC}^{t=2} \quad (18)$$

Given this, it is trivial to note that in the case of firm j effort is treated as strategic substitutes if and only if

$$\pi_{l,h}^j|_{NVC}^{t=2}-\pi_{l,l}^j|_{NVC}^{t=2} > \pi_{h,h}^j|_{NVC}^{t=2}-\pi_{h,l}^j|_{NVC}^{t=2} \quad (19)$$

Where both of these equations are met both firms act as strategic substitutes. If neither of these are met then both firms act as strategic complements. Interestingly, it is possible that one treats effort as a strategic substitute whilst the other strategic compliments. ■

7.5 Proof of Proposition 8

Proof. This proof is done for the case (q_h^i, q_l^i) but holds for all other cases due to assumptions A1, A3, A4 and $M > 2D_H > 2D_L$. Recall that this implies firm i has received VC funding as the sole developer of a high quality prototype. We solve the comparative statics using Cramer's rule, or

$$Ax = b$$

$$\begin{bmatrix} \frac{\partial^2 \pi^i_{h,l}|^{t=2}_{NVC}}{\partial (e_i^2)^2} & \frac{\partial^2 \pi^i_{h,l}|^{t=2}_{NVC}}{\partial e_i^2 \partial e_j^2} \\ \frac{\partial^2 \pi^j_{h,l}|^{t=2}_{NVC}}{\partial e_i^2 \partial e_j^2} & \frac{\partial^2 \pi^j_{h,l}|^{t=2}_{NVC}}{\partial (e_j^2)^2} \end{bmatrix} \begin{bmatrix} \frac{de_i^2}{dx} \\ \frac{de_j^2}{dx} \end{bmatrix} = \begin{bmatrix} -\frac{\partial^2 E\pi_i}{\partial e_i^2 \partial x} \\ -\frac{\partial^2 E\pi j}{\partial e_{j2} \partial x} \end{bmatrix}$$

where x could represent either venture capitalist effort, E, or pecuniary funding, F.

First, solving the comparative statics with respect to F, we find

$$\frac{de_i^2}{dF} = \frac{\left|A_{e_i^2 F}\right|}{|A|} = \frac{\begin{vmatrix} -\frac{\partial^2 \pi^i_{h,l}|^{t=2}_{VC}}{\partial e_i^2 \partial F} & \frac{\partial^2 \pi^i_{h,l}|^{t=2}_{VC}}{\partial e_i^2 \partial e_j^2} \\ -\frac{\partial^2 \pi^i_{h,l}|^{t=2}_{VC}}{\partial e_{j2} \partial F} & \frac{\partial^2 \pi^j_{h,l}|^{t=2}_{VC}}{\partial (e_j^2)^2} \end{vmatrix}}{\begin{vmatrix} \frac{\partial^2 \pi^i_{h,l}|^{t=2}_{VC}}{\partial (e_i^2)^2} & \frac{\partial^2 \pi^i_{h,l}|^{t=2}_{VC}}{\partial e_i^2 \partial e_j^2} \\ \frac{\partial^2 \pi^j_{h,l}|^{t=2}_{VC}}{\partial e_i^2 \partial e_j^2} & \frac{\partial^2 \pi^j_{h,l}|^{t=2}_{VC}}{\partial (e_j^2)^2} \end{vmatrix}}$$

where

$$|A| = \frac{\partial^2 \hat{\mu}_i^2}{\partial \left(e_i^2\right)^2} \frac{\partial^2 \varphi_j^2}{\partial \left(e_j^2\right)^2} \left[(1-\hat{\mu}_i^2)(M-D_L) + \hat{\mu}_i^2 D_H\right] \left[(1-\varphi_j^2)(M-D_L) + \varphi_j^2 D_H\right] - \left(\frac{\partial \hat{\mu}_i^2}{\partial e_i^2}\right)^2 \left(\frac{\partial \varphi_j^2}{\partial e_j^2}\right)^2 [M - D_L - D_H]^2$$

and

$$\left|A_{e_i^2 F}\right| = \frac{\partial \hat{\mu}_i^2}{\partial e_i^2} \frac{\partial \hat{\mu}_i^2}{\partial F} \left(\frac{\partial \varphi_j^2}{\partial e_j^2}\right)^2 [M - D_L - D_H]^2$$

Given assumptions A1, A3, A4 and $M > 2D_H > 2D_L$, it is trivial that $\left|A_{e_i^2 F}\right| > 0$ but $|A|$ is not so trivial to sign. However, it is possible to observe that a unique and stable equilibrium exists if

$$\frac{1}{|R_i'|} > \left|R_j'\right|$$

holds. It turns out this this is the case if

$$\frac{\partial^2 \hat{\mu}_i^2}{\partial \left(e_i^2\right)^2} \frac{\partial^2 \varphi_j^2}{\partial \left(e_j^2\right)^2} \left\{ \begin{array}{c} \left[(1-\hat{\mu}_i^2)(M-D_L)+\hat{\mu}_i^2 D_H\right] \\ \left[(1-\varphi_j^2)(M-D_L)+\varphi_j^2 D_H\right] \end{array} \right\} > \left(\frac{\partial \hat{\mu}_i^2}{\partial e_i^2}\right)^2 \left(\frac{\partial \varphi_j^2}{\partial e_j^2}\right)^2 \left[M-D_L-D_H\right]^2$$

Therefore, $|A|$ is strictly positive too and therefore $\frac{de_i^2}{dF} > 0$.

Similarly, the effects of pecuniary funding on firm j can simply be derived from

$$\left|A_{e_j^2 F}\right| = -\frac{\partial^2 \hat{\mu}_i^2}{\partial \left(e_i^2\right)^2} \frac{\partial \hat{\mu}_i^2}{\partial F} \frac{\partial \varphi_j^2}{\partial e_j^2} \left[(1-\varphi_j^2)(M-D_L)+\varphi_j^2 D_H\right] \left[D_H+D_L-M\right]$$

which is again strictly negative given assumptions A1, A3, A4 and $M > 2D_H > 2D_L$. Note that since the sign of $|A|$, positive, remains unchanged and so $\frac{de_j^2}{dF} < 0$.

Determining the impact of venture capitalist effort is derived in a similar way, with the sign on $|A|$ still unchanged. For the sake of brevity, we simply state

$$\left|A_{e_i^2 E}\right| = \frac{\partial \widehat{\mu}_{i2}}{\partial e_{i2}} \frac{\partial \widehat{\mu}_{i2}}{\partial E} \left(\frac{\partial \varphi_{j2}}{\partial e_{j2}}\right)^2 \left[D_H+D_L-M\right]^2 - \frac{\partial^2 \widehat{\mu}_{i2}}{\partial e_{i2} \partial E} \frac{\partial^2 \varphi_{j2}}{\partial e_{j2}^2} \left\{ \begin{array}{c} \left[(1-\widehat{\mu}_{i2})(M-D_L)+\widehat{\mu}_{i2} D_H\right] \\ \left[(1-\varphi_{j2})(M-D_L)+\varphi_{j2} D_H\right] \end{array} \right\} > 0$$

$$\left|A_{e_j^2 E}\right| = -\frac{\partial^2 \widehat{\mu}_{i2}}{\partial e_{i2}^2} \frac{\partial \widehat{\mu}_{i2}}{\partial E} \frac{\partial \varphi_{j2}}{\partial e_{j2}} \left\{ \begin{array}{c} \left[(1-\varphi_{j2})(M-D_L)+\varphi_{j2} D_H\right] \\ \left[D_H+D_L-M\right] \end{array} \right\} + \frac{\partial^2 \widehat{\mu}_{i2}}{\partial e_{i2} \partial E} \frac{\partial \widehat{\mu}_{i2}}{\partial e_{i2}} \frac{\partial \varphi_{j2}}{\partial e_{j2}} \left\{ \begin{array}{c} \left[(1-\varphi_{j2})(M-D_L)+\varphi_{j2} D_H\right] \\ \left[D_H+D_L-M\right] \end{array} \right\} < 0$$

Again the signs of these equations are determined by assumptions A1, A3, A4 and $M > 2D_H > 2D_L$. Given that $\left|A_{e_i^2 E}\right|$ ($\left|A_{e_j^2 E}\right|$) is strictly positive (negative), it is easy to observe both $\frac{de_i^2}{dE} > 0$ and $\frac{de_j^2}{dE} < 0$.

As it has already been noted, assumptions A1 - A4 cover all the possible functional forms that may be present but assume that, whilst they are not identical, they all act in a similar way. Consequently, the result of this case extends to all other VC cases.

It is trivial to demonstrate that the equity stake, s, has no impact given the first order conditions are independent of s. Consequently, equity does not impact upon the optimal investment decision and has no impact on either e_i^2 or e_j^2. ■

7.6 Proof of Proposition 10

Proof. The proof of proposition 10 is complex simply because there are a large number of cases to examine. However, given the almost identical nature of the proofs, we only derive the result for the case in which $s = h$, or the rival firm has developed a high quality prototype.

First, assume that firm j has developed a high quality prototype such that the effort ordering becomes

$$e_i^2|_{q_h^i, q_h^j VC_i} > e_i^2|_{q_h^i, q_h^j | VC_j} > e_i^2|_{q_l^i, q_h^j}$$

Comparing $e_i^2|_{q_h^i, q_h^j VC_i}$ to $e_i^2|_{q_h^i, q_h^j | VC_j}$ we observe

$$\frac{\partial \hat{\mu}_i^2}{\partial e_i^2}|_{q_h^i, q_h^j, VC_i} = \frac{c}{(M - D_L) - (1 - \mu_j^2)(M - D_L - D_H)}$$

$$\frac{\partial \mu_i^2}{\partial e_i^2}|_{q_h^i, q_h^j, VC_j} = \frac{c}{(M - D_L) - (1 - \hat{\mu}_j^2)(M - D_L - D_H)}$$

When $a_i = a_j$ and $E = F = 0$, it is obvious given assumptions A1 - A4 that the effort levels are equal, $e_i^2 = e_j^2 = e^*$. Furthermore, given the results derived in remark 7 and proposition 8, it is known that should a firm receive VC backing, *any* increases in venture capitalist effort or funding will strictly increase that firm's investment levels and decrease that of its rival. Therefore, starting from a purely symmetric case, $a_i = a_j$ and $E = F = 0$, when firm i receives venture capital, holding abilities constant, it must be that $e_i^2 > e^* > e_j^2$. Similarly, were firm j to receive funding, $e_j^2 > e^* > e_i^2$. Therefore, it must be that a firm invests *more* when it is VC-backed rather than its rival. The addition of asymmetric abilities does nothing to alter this result.

Second, in the other relevant case, $e_i^2|_{q_h^i, q_h^j | VC_j} > e_i^2|_{q_l^i, q_h^j}$, and after using equation (3) we observe

$$\frac{\partial \varphi_i^2}{\partial e_i^2}|_{q_h^i, q_h^j, VC_j} = \frac{\lambda c}{(M - D_L) - (1 - \hat{\mu}_j^2)(M - D_L - D_H)}$$

$$\frac{\partial \varphi_i^2}{\partial e_i^2}|_{q_l^i, q_h^j} = \frac{c}{(M - D_L) - (1 - \hat{\mu}_j^2)(M - D_L - D_H)}$$

It is obvious that, for any given level of effort by firm j, it must be that

$$\frac{\partial \varphi_i^2}{\partial e_i^2}|_{q_h^i, q_h^j, VC_j} < \frac{\partial \varphi_i^2}{\partial e_i^2}|_{q_l^i, q_h^j}$$

Therefore, it is the case in which firm i has developed a high quality prototype, but not received VC funding, that yields a greater level of investment. ■

7.7 Proof of Proposition 12

Proof. The proof is determined by the relevant first and second order equations, given by

$$\frac{\partial^2 \pi^i|_{NVC}^{t=1}}{\partial(e_i^2)^2} = \frac{\partial^2 \varphi_i^1}{\partial(e_i^1)^2}\left\{(1-\varphi_j^1)\left[\pi_{h,l}^i|_{VC}^{t=2} - \pi_{l,l}^i|_{VC}^{t=2}\right] + \varphi_j^1\left[\frac{1}{2}\left(\pi_{h,h}^i|_{VC_i}^{t=2} + \pi_{h,h}^i|_{VC_j}^{t=2}\right) - \pi_{l,h}^i|_{VC}^{t=2}\right]\right\}$$

$$\frac{\partial^2 \pi^i|_{NVC}^{t=1}}{\partial e_i^1 \partial e_j^1} = \frac{\partial \varphi_i^1}{\partial e_i^1}\frac{\partial \varphi_j^1}{\partial e_j^1}\left[\left(\frac{1}{2}\left(\pi_{h,h}^i|_{VC_i}^{t=2} + \pi_{h,h}^i|_{VC_j}^{t=2}\right) + \pi_{l,l}^i|_{VC}^{t=2}\right) - \left(\pi_{h,l}^i|_{VC}^{t=2} + \pi_{l,h}^i|_{VC}^{t=2}\right)\right]$$

Given the profit ordering in corollary 11, it is obvious that the former of these equations is negative. Consequently, the slope of the reaction function is determined by the second order cross partial derivative. However, corollary 11 is not sufficient to determine the sign in this case. Given assumptions A1 - A4, it is obvious that it is trivial to observe that the sign is determined by

$$\left(\frac{1}{2}\left(\pi_{h,h}^i|_{VC_i}^{t=2} + \pi_{h,h}^i|_{VC_j}^{t=2}\right) + \pi_{l,l}^i|_{VC}^{t=2}\right) - \left(\pi_{h,l}^i|_{VC}^{t=2} + \pi_{l,h}^i|_{VC}^{t=2}\right)$$

which is negative, for both firms i and j, if and only if

$$\pi_{h,l}^i|_{VC}^{t=2} > \frac{1}{2}\left(\pi_{h,h}^i|_{VC_i}^{t=2} + \pi_{h,h}^i|_{VC_j}^{t=2}\right) + \pi_{l,l}^i|_{VC}^{t=2} - \pi_{l,h}^i|_{VC}^{t=2}$$

$$\pi_{l,h}^j|_{VC}^{t=2} > \frac{1}{2}\left(\pi_{h,h}^j|_{VC_i}^{t=2} + \pi_{h,h}^j|_{VC_j}^{t=2}\right) + \pi_{l,l}^j|_{VC}^{t=2} - \pi_{h,l}^j|_{VC}^{t=2}$$

Where both of these conditions are met, both firms treat effort as strategic substitutes. ■

7.8 Proof of Remark 13

Proof. Solving these comparative statics by Cramer's rule obtains

$$\frac{de_i^1}{dx} = \frac{\left|A_{e_i^1 x}\right|}{|A|} = \frac{\begin{vmatrix} -\frac{\partial^2 \pi^i|_{VC}^{t=1}}{\partial e_i^2 \partial x} & \frac{\partial^2 \pi^i|_{VC}^{t=1}}{\partial e_i^2 \partial e_j^2} \\ -\frac{\partial^2 \pi^i|_{VC}^{t=1}}{\partial e_{j2} \partial x} & \frac{\partial^2 \pi^j|_{VC}^{t=1}}{\partial (e_j^2)^2} \end{vmatrix}}{\begin{vmatrix} \frac{\partial^2 \pi^i|_{VC}^{t=1}}{\partial (e_i^2)^2} & \frac{\partial^2 \pi^i|_{VC}^{t=1}}{\partial e_i^2 \partial e_j^2} \\ \frac{\partial^2 \pi^j|_{VC}^{t=1}}{\partial e_i^2 \partial e_j^2} & \frac{\partial^2 \pi^j|_{VC}^{t=1}}{\partial (e_j^2)^2} \end{vmatrix}}$$

where $x = E, F$. In both cases

$$|A| = \left(\frac{\partial^2\varphi_i^1}{\partial(e_i^1)^2}\right)\left(\frac{\partial^2\varphi_j^1}{\partial(e_j^1)^2}\right)\left[\begin{array}{c}\left\{\begin{array}{c}(1-\varphi_j^1)\left[\pi_{h,l}^i|_{VC}^{t=2} - \pi_{l,l}^i|_{VC}^{t=2}\right] \\ +\varphi_j^1\left[\frac{1}{2}\left(\pi_{h,h}^i|_{VC_i}^{t=2} + \pi_{h,h}^i|_{VC_j}^{t=2}\right) - \pi_{l,h}^i|_{VC}^{t=2}\right]\end{array}\right\} \\ \left\{\begin{array}{c}(1-\varphi_i^1)\left[\pi_{l,h}^j|_{VC}^{t=2} - \pi_{l,l}^j|_{VC}^{t=2}\right] \\ +\varphi_i^1\left[\frac{1}{2}\left(\pi_{h,h}^j|_{VC_i}^{t=2} + \pi_{h,h}^j|_{VC_j}^{t=2}\right) - \pi_{h,l}^j|_{VC}^{t=2}\right]\end{array}\right\}\end{array}\right]$$

$$- \left(\frac{\partial\varphi_i^1}{\partial e_i^1}\right)^2\left(\frac{\partial\varphi_j^1}{\partial e_j^1}\right)^2\left[\begin{array}{c}\left\{\begin{array}{c}\left(\frac{1}{2}\left(\pi_{h,h}^i|_{VC_i}^{t=2} + \pi_{h,h}^i|_{VC_j}^{t=2}\right) + \pi_{l,l}^i|_{VC}^{t=2}\right) \\ -\left(\pi_{h,l}^i|_{VC}^{t=2} + \pi_{l,h}^i|_{VC}^{t=2}\right)\end{array}\right\} \\ \left\{\begin{array}{c}\left(\frac{1}{2}\left(\pi_{h,h}^j|_{VC_i}^{t=2} + \pi_{h,h}^j|_{VC_j}^{t=2}\right) + \pi_{l,l}^j|_{VC}^{t=2}\right) \\ -\left(\pi_{h,l}^j|_{VC}^{t=2} + \pi_{l,h}^j|_{VC}^{t=2}\right)\end{array}\right\}\end{array}\right]$$

Whilst this looks rather unpleasant and impossible to sign, the condition for uniqueness and stability, $\left|R_i^{'}\right|\left|R_j^{'}\right| < 1$, yields

$$\left(\frac{\partial^2\varphi_i^1}{\partial(e_i^1)^2}\right)\left(\frac{\partial^2\varphi_j^1}{\partial(e_j^1)^2}\right)\left[\begin{array}{c}\left\{\begin{array}{c}(1-\varphi_j^1)\left[\pi_{h,l}^i|_{VC}^{t=2} - \pi_{l,l}^i|_{VC}^{t=2}\right] \\ +\varphi_j^1\left[\frac{1}{2}\left(\pi_{h,h}^i|_{VC_i}^{t=2} + \pi_{h,h}^i|_{VC_j}^{t=2}\right) - \pi_{l,h}^i|_{VC}^{t=2}\right]\end{array}\right\} \\ \left\{\begin{array}{c}(1-\varphi_i^1)\left[\pi_{l,h}^j|_{VC}^{t=2} - \pi_{l,l}^j|_{VC}^{t=2}\right] \\ +\varphi_i^1\left[\frac{1}{2}\left(\pi_{h,h}^j|_{VC_i}^{t=2} + \pi_{h,h}^j|_{VC_j}^{t=2}\right) - \pi_{h,l}^j|_{VC}^{t=2}\right]\end{array}\right\}\end{array}\right]$$

$$> \left(\frac{\partial\varphi_i^1}{\partial e_i^1}\right)^2\left(\frac{\partial\varphi_j^1}{\partial e_j^1}\right)^2\left[\begin{array}{c}\left\{\begin{array}{c}\left(\frac{1}{2}\left(\pi_{h,h}^i|_{VC_i}^{t=2} + \pi_{h,h}^i|_{VC_j}^{t=2}\right) + \pi_{l,l}^i|_{VC}^{t=2}\right) \\ -\left(\pi_{h,l}^i|_{VC}^{t=2} + \pi_{l,h}^i|_{VC}^{t=2}\right)\end{array}\right\} \\ \left\{\begin{array}{c}\left(\frac{1}{2}\left(\pi_{h,h}^j|_{VC_i}^{t=2} + \pi_{h,h}^j|_{VC_j}^{t=2}\right) + \pi_{l,l}^j|_{VC}^{t=2}\right) \\ -\left(\pi_{h,l}^j|_{VC}^{t=2} + \pi_{l,h}^j|_{VC}^{t=2}\right)\end{array}\right\}\end{array}\right]$$

Consequently, the sign on both comparative statics, E and F, depend on $|A_{e_iE}|$, $\left|A_{e_jE}\right|$, $|A_{e_iF}|$, $\left|A_{e_jF}\right|$.

For the sake of brevity, we simply offer the equations here:

$$\begin{aligned}|A_{e_iE}| &= -\left(\frac{\partial\pi^j|_{VC}^{t=1}}{\partial(e_j^1)^2}\right)\left\{\frac{\partial\varphi_i^1}{\partial e_i^1}\left(\begin{array}{c}(1-\varphi_j^1)\frac{\partial\pi_{h,l}^i|_{VC}^{t=2}}{\partial E} \\ +\varphi_j^1\left[\frac{1}{2}\left(\frac{\partial\pi_{h,h}^i|_{VC_i}^{t=2}}{\partial E} + \frac{\partial\pi_{h,h}^i|_{VC_j}^{t=2}}{\partial E}\right) - \frac{\partial\pi_{l,h}^i|_{VC}^{t=2}}{\partial E}\right]\end{array}\right)\right\} \\ &\quad + \left(\frac{\partial\pi^i|_{VC}^{t=1}}{\partial e_i^1\partial e_j^1}\right)\left\{\frac{\partial\varphi_j^1}{\partial e_j^1}\left(\begin{array}{c}(1-\varphi_i^1)\frac{\partial\pi_{l,h}^j|_{VC}^{t=2}}{\partial E} \\ +\varphi_i^1\left[\frac{1}{2}\left(\frac{\partial\pi_{h,h}^j|_{VC_i}^{t=2}}{\partial E} + \frac{\partial\pi_{h,h}^j|_{VC_j}^{t=2}}{\partial E}\right) - \frac{\partial\pi_{h,l}^j|_{VC}^{t=2}}{\partial E}\right]\end{array}\right)\right\}\end{aligned}$$

$$
\begin{aligned}
\left|A_{e_jE}\right| \;=\; & -\left(\frac{\partial \pi^i|_{VC}^{t=1}}{\partial (e_i^1)^2}\right)\left\{\frac{\partial \varphi_j^1}{\partial e_j^1}\left(\begin{array}{c}(1-\varphi_i^1)\frac{\partial \pi^j_{l,h}|_{VC}^{t=2}}{\partial E} \\ +\varphi_i^1\left[\frac{1}{2}\left(\frac{\partial \pi^j_{h,h}|_{VC_i}^{t=2}}{\partial E}+\frac{\partial \pi^j_{h,h}|_{VC_j}^{t=2}}{\partial E}\right)-\frac{\partial \pi^j_{h,l}|_{VC}^{t=2}}{\partial E}\right]\end{array}\right)\right\} \\
& +\left(\frac{\partial \pi^j|_{VC}^{t=1}}{\partial e_j^1 \partial e_i^1}\right)\left\{\frac{\partial \varphi_i^1}{\partial e_i^1}\left(\begin{array}{c}(1-\varphi_i^1)\frac{\partial \pi^i_{h,l}|_{VC}^{t=2}}{\partial E} \\ +\varphi_i^1\left[\frac{1}{2}\left(\frac{\partial \pi^i_{h,h}|_{VC_i}^{t=2}}{\partial E}+\frac{\partial \pi^i_{h,h}|_{VC_j}^{t=2}}{\partial E}\right)-\frac{\partial \pi^i_{l,h}|_{VC}^{t=2}}{\partial E}\right]\end{array}\right)\right\}
\end{aligned}
$$

$$
\begin{aligned}
\left|A_{e_iF}\right| \;=\; & -\left(\frac{\partial \pi^j|_{VC}^{t=1}}{\partial (e_j^1)^2}\right)\left\{\frac{\partial \varphi_i^1}{\partial e_i^1}\left(\begin{array}{c}(1-\varphi_j^1)\frac{\partial \pi^i_{h,l}|_{VC}^{t=2}}{\partial F} \\ +\varphi_j^1\left[\frac{1}{2}\left(\frac{\partial \pi^i_{h,h}|_{VC_i}^{t=2}}{\partial F}+\frac{\partial \pi^i_{h,h}|_{VC_j}^{t=2}}{\partial F}\right)-\frac{\partial \pi^i_{l,h}|_{VC}^{t=2}}{\partial F}\right]\end{array}\right)\right\} \\
& +\left(\frac{\partial \pi^i|_{VC}^{t=1}}{\partial e_i^1 \partial e_j^1}\right)\left\{\frac{\partial \varphi_j^1}{\partial e_j^1}\left(\begin{array}{c}(1-\varphi_i^1)\frac{\partial \pi^j_{l,h}|_{VC}^{t=2}}{\partial F} \\ +\varphi_i^1\left[\frac{1}{2}\left(\frac{\partial \pi^j_{h,h}|_{VC_i}^{t=2}}{\partial F}+\frac{\partial \pi^j_{h,h}|_{VC_j}^{t=2}}{\partial F}\right)-\frac{\partial \pi^j_{h,l}|_{VC}^{t=2}}{\partial F}\right]\end{array}\right)\right\}
\end{aligned}
$$

$$
\begin{aligned}
\left|A_{e_jF}\right| \;=\; & -\left(\frac{\partial \pi^i|_{VC}^{t=1}}{\partial (e_i^1)^2}\right)\left\{\frac{\partial \varphi_j^1}{\partial e_j^1}\left(\begin{array}{c}(1-\varphi_i^1)\frac{\partial \pi^j_{l,h}|_{VC}^{t=2}}{\partial E} \\ +\varphi_i^1\left[\frac{1}{2}\left(\frac{\partial \pi^j_{h,h}|_{VC_i}^{t=2}}{\partial E}+\frac{\partial \pi^j_{h,h}|_{VC_j}^{t=2}}{\partial E}\right)-\frac{\partial \pi^j_{h,l}|_{VC}^{t=2}}{\partial E}\right]\end{array}\right)\right\} \\
& +\left(\frac{\partial \pi^j|_{VC}^{t=1}}{\partial e_j^1 \partial e_i^1}\right)\left\{\frac{\partial \varphi_i^1}{\partial e_i^1}\left(\begin{array}{c}(1-\varphi_j^1)\frac{\partial \pi^i_{h,l}|_{VC}^{t=2}}{\partial E} \\ +\varphi_j^1\left[\frac{1}{2}\left(\frac{\partial \pi^i_{h,h}|_{VC_i}^{t=2}}{\partial E}+\frac{\partial \pi^i_{h,h}|_{VC_j}^{t=2}}{\partial E}\right)-\frac{\partial \pi^i_{l,h}|_{VC}^{t=2}}{\partial E}\right]\end{array}\right)\right\}
\end{aligned}
$$

Unfortunately, given that it is not possible to determine the magnitude of the first order conditions with respect to E or F, it is not possible to sign these equations.
■

7.9 Proof of Proposition 14

Proof. Using Cramer's rule we observe

$$
\frac{de_i^1}{ds}=\frac{\left|A_{e_i^1 s}\right|}{|A|}=\frac{\left|\begin{array}{cc}-\frac{\partial^2 \pi^i|_{VC}^{t=1}}{\partial e_i^2 \partial s} & \frac{\partial^2 \pi^i|_{VC}^{t=1}}{\partial e_i^2 \partial e_j^2} \\ -\frac{\partial^2 \pi^i|_{VC}^{t=1}}{\partial e_{j2} \partial s} & \frac{\partial^2 \pi^j|_{VC}^{t=1}}{\partial (e_j^2)^2}\end{array}\right|}{\left|\begin{array}{cc}\frac{\partial^2 \pi^i|_{VC}^{t=1}}{\partial (e_i^2)^2} & \frac{\partial^2 \pi^i|_{VC}^{t=1}}{\partial e_i^2 \partial e_j^2} \\ \frac{\partial^2 \pi^j|_{VC}^{t=1}}{\partial e_i^2 \partial e_j^2} & \frac{\partial^2 \pi^j|_{VC}^{t=1}}{\partial (e_j^2)^2}\end{array}\right|}
$$

where $|A|$ is identical to that in remark 13. Thus, it is only the signs on $\left|A_{e_i^1 s}\right|$ and $\left|A_{e_j^1 s}\right|$ that are important. With come manipulation we obtain

$$\begin{aligned}
|A_{e_i s}| &= -\left(\frac{\partial \pi^j|_{VC}^{t=1}}{\partial (e_j^1)^2}\right)\left\{\frac{\partial \varphi_i^1}{\partial e_i^1}\left((1-\varphi_j^1)\frac{\partial \pi_{h,l}^i|_{VC}^{t=2}}{\partial s}+\frac{1}{2}\varphi_j^1\frac{\partial \pi_{h,h}^i|_{VC_i}^{t=2}}{\partial s}\right)\right\} \\
&\quad +\left(\frac{\partial \pi^i|_{VC}^{t=1}}{\partial e_i^1 \partial e_j^1}\right)\left\{\frac{\partial \varphi_j^1}{\partial e_j^1}\left((1-\varphi_i^1)\frac{\partial \pi_{l,h}^j|_{VC}^{t=2}}{\partial s}+\frac{1}{2}\varphi_i^1\frac{\partial \pi_{h,h}^j|_{VC_j}^{t=2}}{\partial s}\right)\right\}
\end{aligned}$$

$$\begin{aligned}
\left|A_{e_j s}\right| &= -\left(\frac{\partial \pi^i|_{VC}^{t=1}}{\partial (e_i^1)^2}\right)\left\{\frac{\partial \varphi_j^1}{\partial e_j^1}\left((1-\varphi_i^1)\frac{\partial \pi_{l,h}^j|_{VC}^{t=2}}{\partial s}+\frac{1}{2}\varphi_i^1\frac{\partial \pi_{h,h}^j|_{VC_j}^{t=2}}{\partial s}\right)\right\} \\
&\quad +\left(\frac{\partial \pi^j|_{VC}^{t=1}}{\partial e_j^1 \partial e_i^1}\right)\left\{\frac{\partial \varphi_i^1}{\partial e_i^1}\left((1-\varphi_j^1)\frac{\partial \pi_{h,l}^i|_{VC}^{t=2}}{\partial s}+\frac{1}{2}\varphi_j^1\frac{\partial \pi_{h,h}^i|_{VC_i}^{t=2}}{\partial s}\right)\right\}
\end{aligned}$$

where

$$\frac{\partial \pi_{x,y}^i|_{VC}^{t=2}}{\partial s} < 0 \; \forall \; x, y \in \{l, h\}$$

Therefore, when firms treat effort as strategic complements, or

$$\frac{\partial^2 \pi^i|_{NVC}^{t=1}}{\partial (e_i^2)^2} < 0; \;\; \frac{\partial^2 \pi^i|_{NVC}^{t=1}}{\partial e_i^1 \partial e_j^1} > 0$$

we find $|A_{e_i s}|$ and $\left|A_{e_j s}\right|$ are both strictly negative. Thus,

$$\frac{de_i^1}{ds} < 0$$

■