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To Block or not to Block? Network Competition when Skype enters the Mobile Market

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To Block or not to Block? Network Competition when Skype enters the Mobile Market*†

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Abstract

Voice over Internet Protocol (VoIP) such as Skype that enables users to make free internet-based calls to other users has been seen as a threat to voice revenues by traditional network operators. While some mobile network operators (MNOs) attempt to block Skype's entry on their networks, some actually welcome it even if it apparently conflicts with their interests in making calling profits. In this paper we develop a Hotelling-style model of network competition between two MNOs to analyse their incentives to accommodate or block Skype. We find that accommodation is the dominant strategy of an MNO whenever its equilibrium voice market share is at least 29%. Furthermore, the overall Nash equilibium of the game can be either symmetric (where Skype's entry is either accommodated or blocked by both MNOs) or asymmetric (where only one has the incentive to accommodate) depending upon the consumers' preference for a certain network and the quality of Skype-based interconnection. In a symmetric accommodation equilibrium, the MNO with a lower (higher) customer valuation is better-off (worse-off) relative to the one where entry is blocked.

JEL Classification: D43, L13, L96.

Keywords: Mobile network competition; Hotelling model, Voice over IP and Skype; entry; voice and network market shares.

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1 Introduction.

One of the most prominent features of economic development in the last decade has been the transformation of the telecommunications industry. The shift toward packet-switched technologies made possible by the digital revolution has made different transmission technologies increasingly interchangeable. The impending arrival of internet telephony, often known as the voice over internet protocol (VoIP) represents the most prominent illustration of this phenomenon. By the end of 2004, mobile telephones and more advanced wireless technologies such as VoIP finally surpassed conventional wireline telephony as the leading platform for providing voice communications. In particular, Skype - a new VoIP provider that allows its users to talk to one another for free using the internet, has experienced a phenomenal growth in its operation since its launch in August 2003. With its ever-increasing popularity and given its technological specifications, Skype is considered by many as a 'revolutionary' service that can potentially threaten the current functioning of the entire telecommunication market.[1]

In March 2009, Skype first launched the Skype iPhone Application. Immediately following its launch on mobiles, several attempts to ban or restrict the use of Skype on mobile phones were made by several mobile network operators (MNOs) such as AT&T, Deutsche Telekom, T-mobile UK and T-mobile Germany. At the same time however some network carriers such as 3 in Britain, Austria, Ireland and Denmark accommodated Skype over their networks. There seems to be two sides to this story. Those MNOs that attempted to block or restrict the use of Skype[2] on their networks did so for the fear of loss of their calling profits as the introduction of Skype on mobile phones enables mobile users to bypass the MNO altogether. On the other hand, some MNOs welcomed it not only for strategic reasons but perhaps also due to the regulatory pressure from the government. In the former case, even though accommodation of Skype seems to conflict with MNOs' interests in making calling profits, by doing so, the network operators could actually provide better and diverse services to attract more subscriptions (possibly from their rivals) and thereby increase their overall profits. In the latter case also, recent regulatory measures to control prices have lowered the mobile termination rates substantially with obvious implications for MNOs' overall profits. It is therefore not unusual for the MNOs to look for other ways to increase their profits such as partnering with VoIPs to increase customer base.

[1] See Goncalves and Ribiero (2005) for an overview of Skype in the context of the European regulatory framework.

[2] For example, in Germany T-mobile and Vodaphone do not completely block Skype but impose considerable fees for the use of Skype on their network to undermine Skype's attraction.

Driven by the above phenomenon, the objective of this paper is to analyse under what conditions MNOs will have incentives to accommodate or block Skype's entry into the mobile telephony market. To examine this, we consider a three-stage game between two competing MNOs as follows: in the first stage, the networks decide whether to accommodate Skype on their networks or not; in the second stage MNOs compete against each other *à la* Hotelling choosing their pricing strategies based on their anticipated network market shares. The market shares get determined as consumers decide which network to subscribe to. Then, in the third stage, consumers decide whether to use Skype or voice to make a call conditional on the availability of Skype on that network. This determines the voice market share of an MNO. Following Laffont, Rey and Tirole [1998 (a), (b)] we assume that the networks are vertically and horizontally differentiated from the customers' point of view. We solve the entire game backwards starting with consumers' decisions about using Skype.

Several interesting results emerge. First of all we find that *if* a network accommodates Skype then it engages in a two-part pricing strategy whereby it sets a calling price equal to its marginal cost for the voice callers and sets a fixed fee that is directly proportional to its network market share for all of its subscribers i.e. for both voice-callers and Skype-users (proposition 1). This is because, as shown in proposition 5, an MNO has an incentive to accommodate Skype only if by doing so it can still maintain a voice market share of at least about 29%, if not then it will block Skype's entry in its network. In order to ensure a voice market share of at least 29%, the MNO attempts to make voice calling as attractive as possible by setting a calling price as low as possible which results in setting a calling price equal to its (total) marginal cost. It then sets a fixed fee that is directly proportional to its subscription market shares in order to extract rents from *all* of its network subscribers as otherwise it would make a loss on its overall profits. In contrast, if an MNO blocks Skype's entry altogether then network subscribers do not have any other alternatives except for making only voice calls. If so, then there is no need for the network to use a two-part tariff and therefore it sets an above-marginal-cost uniform calling price for its subscribers (proposition 2). We find that there can be different possible equilibrium outcomes for the overall entry game that can be either symmetric where both MNOs can accommodate or block Skype's entry; or asymmetric where one MNO has the incentive to accommodate while the other does not. Which equilibrium will prevail at the end depends very much upon customers' preference for a certain network (the vertical differentiation parameter) as well as on the quality of Skype-based interconnection as we show that equilibrium voice market shares are functions of parameters of consumers' intrinsic preference for a certain network and the quality of Skype-based calls. Further, we show in proposition 6 that in the symmetric 'accommodation' equilibrium (where both networks allow Skype's entry), the MNO with a *lower* customer valuation parameter is *better-off* compared to the symmetric equilibrium where both MNOs block Skype's entry; whilst the MNO with a *higher* customer valuation parameter is actually made *worse-off.* If on the other hand, the MNOs are not vertically differentiated with respect to the customers' preference parameters, then they both are equally well-off in either equilibrium as they each then earn standard Hotelling profit.

We believe our results resemble some of the phenomenon that are observed in the real-world network competition. For example, the customer valuation parameter can be interpreted as an indicator of a firm's reputation e.g. higher customer valuation parameter being associated with a more established network. Our result that 'smaller' networks are better-off in an accommodation equilibrium (relative to the 'block' equilibrium) is reminiscent of the situation when 3 unilaterally accommodated Skype on its network and thereby increased its payoff. Similarly, the fact that many MNOs attempted to ban or restrict Skype's usage on their network can be explained in terms of their fear of losing the voice market shares below a certain the threshold level as we have shown in this paper.

There is a substantial literature on network competition with regulatory issues (see e.g. Laffont and Tirole (1994, 1996), Peitz, Valletti, and Wright (2004), Armstrong (1998), (2002) among others). The papers that consider competition with VoIP are by Foros and Hansen (2001) and De Bijl and Peitz (2009). Foros and Hansen (2001) considers competition amongst internet service providers where the ISPs have incentives to strategically degrade the interconnection quality. De Bijl and Peitz (2009) analyses the effect of access regulation and retail price regulation of PSTN networks on the adoption of a new technology in the form of VoIP. However, we are not aware of any papers that deal with VoIP entry issues in the context of a mobile network market in a similar spirit to ours.

The paper is organised as follows. Section 1 lays out the basic framework of our model. In subsection 2.1, we analyse consumers' decision about using Skype determining the voice market share of an MNO. In subsection 2.2, we determine the market shares and pricing strategies of an MNO corresponding to cases where both or only one of the networks or neither accommodate Skype. We then determine the equilibrium values of network market shares, prices, fixed fees, and profits corresponding to each of the scenarios. In the subsection 2.3, we analyse MNOs incentives to accommodate or block Skype's entry and determine conditions for the overall Nash equilibrium of the entry game. Section 3 provides some concluding remarks. The appendix presented in section 4 contains some of the proofs.

2 The model.

Consider a mobile telecommunication model where mobile networks are differentiated *à la* Hotelling (1929). The preferences of the consumers are assumed to be uniformly distributed with density 1 on a segment [0,1]. There are two mobile network operators henceforth denoted by MNO_i, $i = 1, 2$ who are located at the extremities of the segment namely at $s^1 = 0$ and $s^2 = 1$ where s^i is the 'address' of network i.[3] We assume that the MNOs are symmetric with respect to their cost structures and each incurs a total marginal cost of c per call which incudes costs associated with originating and terminating a call (as well as any other costs in between).[4] We assume that customers have *unit* demands for calls. The MNO_i charges a price p_i per unit of a call (usage fee) and in addition can charge a fixed fee F_i for using the network. Net utility of a customer located at s connected to network i is therefore

$$U_i = v_i - x|s - s^i| - p_i - F_i \qquad i = 1, 2$$

where v_i denotes the fixed advantage of being connected to network i, x is the (horizontal) product differentiation parameter between two networks, and $x|s - s^i|$ represents the disutility from not being connected to the most preferred network type (similar to the transportation costs in a standard Hotelling model). Let $\Delta v = v_1 - v_2$. Hence $\Delta v = 0$ implies that there is no vertical differentiation whilst $\Delta v \neq 0$ implies that network services are vertically differentiated.[5] We make the following assumption about v_i.

Assumption 1. The fixed utility v_i is sufficiently large such that each customer always prefers to connect to a certain network.

The above assumption ensures that each of the consumer, located in the segment [0, 1], value the service sufficiently high such that they always prefer to subscribe to one network or the other.

In this paper, we consider the possibility of entry by Skype, a VoIP (Voice over Internet Protocol) into the mobile market, that enables users to make *free* internet-based calls to other users. Each MNO therefore has a decision to make: whether to accommodate Skype on their network or not. There are both advantages and disadvantages of accommodating Skype. On

[3]This is not to be confused with the network market share $s_i, i = 1, 2$, that we will introduce shortly and use throughout the paper i.e. supercsripts denote the address while subscripts denote the market share.

[4]In addition, serving a customer may involve a *fixed* cost $f \geq 0$. However, we set this fixed cost to zero for the moment as it will not change our analysis.

[5]Implications of Δv being positive or negative are explored in the sections below.

one hand, accommodating Skype implies foregoing calling profit that the MNO could have otherwise earned as using Skype to make calls is free for customers whereas voice calls are not (a disadvantage). On the other hand, by accommodating Skype on its network, an MNO can diversify its services and (potentially) increase its customer base thereby making more profits as the possibility of using Skype to make free internet based calls can be quite attractive to the customers (an advantage).

Given that the quality of Skype-based calls is usually lower than that for voice calls, we denote by parameter $\beta, 1 > \beta > 0$, the quality of Skype-based interconnection. Thus $\beta < 1$ means that the quality of Skype calls are always inferior to that made on voice where (1 - β) measures the extent by which Skype calls are inferior to voice calls. When $\beta \rightarrow 1$, the quality of a call made via Skype is almost as good as the voice one.[6] The quality parameter β directly affects a consumer's fixed utility v_i when using Skype on network i (see equation (1)).

Once a consumer has subscribed to the mobile network i, the customers' preference for using voice versus Skype is assumed to be uniformly distributed with density 1 on a line segment of unit length where MNO$_i$ is located at the end '0' and Skype at the end 1 on the *voice-Skype* segment. See Figure 1.

[Insert Figure 1 here]

Thus a typical consumer has two decisions to make that are inter-connected: She needs to decide which MNO to subscribe to, and then decide whether to use Skype or voice to make calls on that network. Obviously, a consumer's decision to choose a certain network is influenced not only by that network's prices but also whether she can use Skype on that network. MNOs know that consumers are going to behave this way. Thus the MNOs also have two decisions to make that are inter-connected: first they need to decide whether to accommodate Skype into their networks or not, and then they choose their pricing strategies based on their anticipated market shares (for both voice and network) that are determined by consumers' demands for a certain network and voice calls. Obviously MNOs' second stage pricing strategies influence their first stage decision of whether to accommodate Skype's entry and vice versa.

The timing of the above game therefore is as follows:

- Stage 1: MNO$_i$ decides whether to accommodate or block Skype's entry into its network.

[6] We assume β to be strictly positive and interpret $\beta = 0$ implies such bad Skype connection that consumers will not like to use Skype at all!

- Stage 2: Given its decision in stage 1, MNO$_i$ chooses its p_i and F_i (if any) in stage 2. Given the price vector chosen by MNO$_i$ and the location preference $s \in (0, 1)$ of the consumer, consumers decide which network to subscribe to that consequently determines the market share of MNO$_i$.

- Stage 3: Once the customer has chosen to join a certain network, she has to decide whether to use Skype or voice to make a call (if of course Skype is available on that network).

2.1 Stage 3: Consumers' decision about using Skype

Suppose an MNO$_i$ has accommodated Skype into its network. Then a customer located at address λ_i, $\lambda_i \in [0, 1]$ on the voice-Skype segment for network i, measured from network i's location (i.e. at point 0), chooses to make a voice call whenever $U_i \geq V_{i,s}$ where $V_{i,s}$, the net utility of the customer from making a Skype call on the network i, is given by

$$V_{i,s} = \beta v_i - y(1 - \lambda_i) - x|s - s^i| - F_i \quad i = 1, 2 \tag{1}$$

where y represents the degree of substitution between the voice call and the Skype call (equivalent to the 'transportation cost' in a standard Hotelling model). The term βv_i reflects the fact that using Skype calls reduces a caller's fixed utility depending upon the magnitude of β. Availability of Skype on network i implies that the net utility of a customer is now given by

$$U_i = v_i - y\lambda_i - x|s - s^i| - p_i - F_i \quad i = 1, 2 \tag{2}$$

The market share λ_i for the voice call for MNO$_i$ is therefore determined by solving the marginal condition $U_i = V_{i,s}$. Hence for the MNO$_i$, the market shares for the voice (λ_i) and Skype $(1 - \lambda_i)$ calls are respectively given by

$$\begin{aligned} \lambda_i &= \frac{1}{2} + \frac{1}{2y}[v_i(1 - \beta) - p_i], \quad \text{and} \\ (1 - \lambda_i) &= \frac{1}{2} - \frac{1}{2y}[v_i(1 - \beta) - p_i] \end{aligned} \tag{3}$$

Thus, higher the calling price p_i, lower is the MNO$_i$'s voice market share. Further, its market share for voice is influenced by the quality of Skype calls as shown by the following observation.

Observation 1 As the quality of Skype-based calls increases, MNO$_i$'s market share for the voice decreases i.e.$\partial\lambda_i/\partial\beta < 0$.

Proof. Follows immediately as $\partial\lambda_i/\partial\beta = -v_i/2y < 0$. ■

The above result is quite intuitive: as the quality of Skype calls increases and gets closer to the voice-based calls, $V_{i,s}$ increases and customers naturally prefer Skype to make calls as using Skype to make calls is free. So the voice market share of MNO$_i$ declines. In the extreme case when the quality of the Skype call is almost as good as the voice call, network i can maintain a positive market share only by pricing its call sufficiently low (i.e. below y) in order to retain some customers. We make the following assumption to ensure reasonable equilibrium values of voice market share for the overall game.

Assumption 2. $y > c$; and $y \geq \max v_i - c; i = 1, 2$.

Finally note that above are expressions for the market shares for the voice and Skype *if* Skype's entry into network i has been *accommodated*. If however, the MNO$_i$ has *blocked* Skype's entry, then the consumers do not have the option to make a decision about using Skype implying that the market shares for the voice and Skype *by default* will be $\lambda_i = 1$ and (1-λ_i) = 0 respectively.

2.2 Stage 2. Network market share and price competition

In this stage, the MNOs decide on their calling prices and fixed fees based on anticipated values of their market shares s_i in Hotelling style. Now, the demand and hence the network market share of MNO$_i$ is directly affected by whether the customers are able to use Skype on its network or not and therefore by its voice market share in stage 3, which of course is conditional on the MNO$_i'$s decision to accommodate Skype in stage 1. Denote the network market share of MNO$_i$ by $s_i, i = 1, 2$. The MNOs then incorporate these (anticipated) market shares s_i into their profit maximisation problem to decide on its pricing strategy. We determine market shares and pricing strategies in turn as follows.

■ **Market share of an MNO:** If MNO$_i$ has accommodated Skype into its network, then the consumer's expected utility from joining MNO$_i$ is given by

$$EU_i = \int_0^{\lambda_i} [v_i - y\theta - p_i]d\theta + \int_{\lambda_i}^1 [\beta v_i - y(1-\theta)]d\theta - xs_i - F_i$$

where θ is the preference distribution parameter of the consumer. Thus the above expected utility now has a slightly different specification from that given by (2).The first term in the above expression represents the expected utility derived from making voice calls whereas the second term represents the expected utility derived from making Skype calls. Note that the two networks' market shares are $s_1 = s$ and $s_2 = (1-s)$ since both networks have full coverage.

♦ *Market shares when both MNOs accommodate Skype.*

If both MNOs have accommodated Skype into their networks, then market share s_1 for MNO_1 is determined by the indifference condition: $EU_1 = EU_2$ i.e.

$$\int_0^{\lambda_1} [v_1 - y\theta - p_1]d\theta + \int_{\lambda_1}^1 [\beta v_1 - y(1-\theta)]d\theta - xs - F_1 =$$
$$\int_0^{\lambda_2} [v_2 - y\theta - p_2]d\theta + \int_{\lambda_2}^1 [\beta v_2 - y(1-\theta)]d\theta - x(1-s) - F_2$$

which yields, after simplification, the following value of s :

$$s_1 = s = \frac{1}{2} + \frac{1}{2x}[\beta\Delta v + y\left(\lambda_1^2 - \lambda_2^2\right) + (F_2 - F_1)] \tag{4}$$

and so the market share of MNO_2 is given by

$$s_2 = 1 - s = \frac{1}{2} - \frac{1}{2x}[\beta\Delta v + y\left(\lambda_1^2 - \lambda_2^2\right) + (F_2 - F_1)] \tag{5}$$

The above equations show how an MNO's market share is influenced by its pricing strategies: whilst fixed fee F_i directly reduces network i's market share, the calling price p_i impacts its market share *indirectly* by altering the demand for its voice calls i.e. by impacting its voice market share λ_i.

♦ *Market shares when both MNOs block Skype's entry.*

If both MNOs have blocked Skype's entry into their networks then consumers have no other options than using the network to make voice calls $\Rightarrow \lambda_1 = \lambda_2 = 1$. Market shares s and $(1-s)$ are then simply obtained by solving the following indifference condition:

$$v_1 - xs - p_1 - F_1 = v_2 - x(1-s) - p_2 - F_2$$
$$\Rightarrow \quad s_1 = \frac{1}{2} + \frac{1}{2x}[(\Delta v + (p_2 - p_1) + (F_2 - F_1)] \quad \text{and} \tag{6}$$
$$s_2 = \frac{1}{2} - \frac{1}{2x}[(\Delta v + (p_2 - p_1) + (F_2 - F_1)] \tag{7}$$

Obviously, in this case, prices and fixed fees affect network market shares directly.

♦ *Market shares when one MNO accommodates Skype while the other blocks.*

Suppose MNO_1 accommodates Skype whilst MNO_2 blocks it so that $\lambda_2 = 1$ whilst $\lambda_1 = \frac{1}{2} + \frac{1}{2y}[v_1(1-\beta) - p_1]$. In that case, the markets shares are obtained by solving the following marginal condition:

$$\int_0^{\lambda_1} [v_1 - y\theta - p_1]d\theta + \int_{\lambda_1}^{1} [\beta v_1 - y(1-\theta)]d\theta - xs - F_1 = v_2 - x(1-s) - p_2 - F_2$$

so that the following are the expressions for market shares:

$$s_1 = s = \frac{1}{2} + \frac{1}{2x}[\beta v_1 - v_2 + y\lambda_1^2 - \frac{y}{2} + F_2 - F_1 + p_2] \quad (8)$$

$$s_2 = 1 - s = \frac{1}{2} + \frac{1}{2x}[v_2 - \beta v_1 - y\lambda_1^2 + \frac{y}{2} + F_1 - F_2 - p_2] \quad (9)$$

Similarly, if MNO_1 blocks Skype's entry while MNO_2 accommodates it, then by symmetry, the market shares will be as follows:

$$s_1 = s = \frac{1}{2} + \frac{1}{2x}[v_1 - \beta v_2 - y\lambda_2^2 + \frac{y}{2} + F_2 - F_1 - p_1] \quad (10)$$

$$s_2 = 1 - s = \frac{1}{2} + \frac{1}{2x}[\beta v_2 - v_1 + y\lambda_2^2 - \frac{y}{2} + F_1 - F_2 + p_1] \quad (11)$$

2.2.1 Pricing strategies

Given the above market shares, the network i chooses p_i and (possibly) F_i to maximise its profits. Since the profit expressions will differ depending upon whether the network has accommodated Skype or not, we consider each of these cases separately.

■ **Pricing strategies of network i when it accommodates Skype's entry.**

Consider the profit maximisation problem for MNO_i. MNO_i's profit is

$$\pi_i = s_i[(p_i - c)\lambda_i + F_i] \qquad i = 1, 2$$

where λ_i is given by (3) and s_i is given either by equation (4) or (8) depending upon what the other MNO does. MNO$_i^\prime$s problem is to

$$\max_{p_i,F_i} \; s_i[(p_i-c)\lambda_i+F_i]$$

The first order conditions are:

$$\frac{\partial \pi_i}{\partial p_i}=s_i[(p_i-c)\frac{\partial \lambda_i}{\partial p_i}+\lambda_i]+[(p_i-c)\lambda_i+F_i]\frac{\partial s_i}{\partial p_i}=0 \tag{12}$$

and

$$\frac{\partial \pi_i}{\partial F_i}=s_i+[(p_i-c)\lambda_i+F_i]\frac{\partial s_i}{\partial F_i}=0 \tag{13}$$

Proposition 1 summarises pricing strategies of MNO$_i$.

Proposition 1 *When an MNO accommodates Skype into its network, it practises a two-part pricing policy whereby it sets its calling price equal to the (total) marginal cost i.e.* $p_i^*=c$, *and sets a fixed fee* F_i *which is directly proportional to the network market share* s_i *such that* $F_i=2xs_i$.

Proof: see appendix.

Proposition 1 says when a network accommodates Skype into its network, it engages in a two-part pricing strategy whereby it sets its calling price p_i equal to the (total) marginal cost and then uses a *fixed fee* to extract as much surplus as possible where the magnitude of fixed fee it sets is directly proportional to its (network) market share s_i. Thus when the MNO makes its service attractive to customers by allowing Skype on its network, it simply charges a flat fee to all its customers for the network service. At the same time, the MNO attempts to make voice calls attractive for its customers, by charging as low a price as possible and hence sets $p_i=c$. The ability to set F_i at a certain level however depends on how its market share s_i is affected by Skype accommodation which on the other hand is sensitive to the quality of Skype calls β and consumers' preference parameter v_i (see the analysis in the section on equilibrium).

Remark 1. With the above pricing strategy, MNO$_i$'s equilibrium voice market share is given by $\lambda_i^*=\frac{1}{2}+\frac{1}{2y}[v_i(1-\beta)-c]$.

Remark 2 As long as $v_1\neq v_2$ (i.e. $\Delta v\neq 0$), $\lambda_1^*\neq\lambda_2^*$, and $\lambda_1^*\gtrless\lambda_2^*$ according as $v_1\gtrless v_2$.

It follows from above that, given $\beta < 1$, network i will have larger voice market share than the other network whenever consumers derive more satisfaction from subscribing to it. One can, for example, interpret higher value of v being associated with the network's reputation e.g. a more established network is more likely to generate a higher value of v whereas a relatively 'new' (into the market) network is likely to give rise to a lower value of v.

Henceforth, to save on notation, the equilibrium values of voice market shares will simply be denoted by $\lambda_{i.}$

■ **Pricing strategies of network i when it blocks Skype's entry.**

If MNO_i has blocked Skype's entry then its voice market share λ_i equals 1, and so the profit maximisation problem of MNO_i is now

$$\max_{p_i, F_i} \pi_i = s_i[(p_i - c) + F_i]$$

where s_i is given by equations (6) or (10) [or (11)] depending upon which firm we are considering. The first order conditions therefore are:

$$\frac{\partial \pi_i}{\partial p_i} = s_i + [(p_i - c) + F_i]\frac{\partial s_i}{\partial p_i} = 0 \tag{14}$$

and

$$\frac{\partial \pi_i}{\partial F_i} = s_i + [(p_i - c) + F_i]\frac{\partial s_i}{\partial F_i} = 0 \tag{15}$$

Proposition 2 *When an MNO blocks Skype's entry into its network, it adopts a uniform pricing policy whereby it sets a calling price above the (total) marginal cost such that* $\widetilde{p}_i = c + 2xs_i$.

Proof. First note from equations (10), (11) and (6) that $\frac{\partial s_i}{\partial F_i} = \frac{\partial s_i}{\partial p_i} = -1/2x < 0$. Therefore, equations (14) and (15) are exactly identical implying that there is practically no difference between p_i and F_i, as they both serve exactly the same purpose $\Rightarrow$ any fixed fee is 'as if' included within p_i. If so, then denote that uniform price by $\widetilde{p}_i$.The FOCs (as given by equations (14) or (15)) then imply

$$\widetilde{p}_i = c + 2xs_i$$

The second order conditions are satisfied since $\frac{\partial^2 s_i}{\partial p_i^2} = \frac{\partial^2 s_i}{\partial F_i^2} = -1/x < 0$. ■

Note that given that these market shares are (strictly) positive, an MNO prices its calls above the marginal costs where how high it can set its price is directly proportional to its (equilibrium) network market shares. In equilibrium, network market shares s_i on the other hand will be determined depending upon the above pricing strategies of *both* MNOs. Given that all network users can only make voice calls when an MNO blocks Skype's entry, there is no need to set a separate fixed fee in order to extract surplus from non-voice users in contrary to the entry accommodation case. Whilst the second term $2xs_i$ in the price expression is similar to the expression for fixed fee in the accommodation case, the main difference though is that the price $\widetilde{p}_i$ is now paid by *all* network i users, whereas under the accommodation case, the Skype users pay only the fixed fee $2xs_i$.However, given that the values of market shares will be different under different situations, these values themselves will be different as we shall see below.

■ **Equilibrium**

Given the above pricing strategies, there are now four possible equilibrium configurations corresponding to cases where (a) both accommodate; (b) both block, and (c) two cases where one MNO accommodates whilst the other blocks. We analyse each of these cases in turn.

Case (a) - Both MNOs accommodate Skype's entry into their networks.

Note, in this case whilst the equilibrium calling price for both MNOs are: $p_1^* = p_2^* = c$, the equilibrium values of market shares and hence fixed fees will generally be different depending upon the exogenous parameter values of v_i where $\Delta v = v_1 - v_2$.We will analyse the implications of Δv being zero, positive or negative reflecting the consumers' preference for a certain network, shortly.

- *Equilibrium network market shares s_i^*.*

Substituting for $F_i = 2xs_i$ in equations (4) and (5) and simplifying, we obtain the following equilibrium values of the market shares:

$$\begin{aligned} s_1^* &= s^* = \frac{1}{2} + \frac{1}{6x}[\beta\Delta v + y\left(\lambda_1^2 - \lambda_2^2\right)] \quad \text{and} \\ s_2^* &= 1 - s^* = \frac{1}{2} - \frac{1}{6x}[\beta\Delta v + y\left(\lambda_1^2 - \lambda_2^2\right)] \end{aligned}$$

where $\lambda_i = \frac{1}{2} + \frac{1}{2y}[v_i(1-\beta) - c]$, $i = 1, 2$ is the equilibrium value (i.e. λ_i^*).

- *Equilibrium values of fixed fees* F_i^*.

Equilibrium values of the fixed fees are given by

$$\begin{aligned} F_1^* &= x + \frac{1}{3}[\beta\Delta v + y\left(\lambda_1^2 - \lambda_2^2\right)] \quad \text{and} \\ F_2^* &= x - \frac{1}{3}[\beta\Delta v + y\left(\lambda_1^2 - \lambda_2^2\right)] \end{aligned}$$

- *Equilibrium values of network profits* π_i^*.

Network profits are now $\pi_i^* = s_i^* F_i^*$ as $p_i^* = c$. Hence the equilibrium profits are:

$$\begin{aligned} \pi_1^*|_{A,A} &= \frac{1}{18x}[3x + \{\beta\Delta v + y\left(\lambda_1^2 - \lambda_2^2\right)\}]^2 \quad \text{and} \\ \pi_2^*|_{A,A} &= \frac{1}{18x}[3x - \{\beta\Delta v + y\left(\lambda_1^2 - \lambda_2^2\right)\}]^2 \end{aligned}$$

where the subscripts (A, A) denote networks' profits when both accommodate.

Remark 3. $\Delta v > (<) 0$ then implies $s_1^* > (<)\ s_2^*$; $F_1^* > (<)\ F_2^*$ and $\pi_1^*|_{A,A} > (<)\ \pi_2^*|_{A,A}$ whereas when $\Delta v = 0$, both networks split the market equally i.e. $s_1^* = s_2^* = 1/2$ and earn Hotelling profit $x/2$.

Effect of β.

When the quality of Skype calls increases, voice market shares for both MNOs decline (see observation 1). However whether this implies an increase or decrease in s_i^* of an MNO depends very much on how the decline in the *voice* market share of the *rival* firm affects the *network* market share of this firm as s_i^* is a function of both λ_1 and λ_2. Interestingly, as the proposition below shows, how this two opposing effects play out depends to a large extent on whether consumers have any intrinsic preference for a certain network.

Proposition 3 *There exists a* $\widehat{\beta}$ $(= 1 - \frac{2y+c}{v_1+v_2})$ *such that (i) when* $v_1 > v_2, \partial F_1/\partial\beta \geq 0$*;* $\partial s_1/\partial\beta \geq 0; \partial\pi_1/\partial\beta \geq 0$ *for* $\beta \geq \widehat{\beta}$ *whilst* $\partial\ F_1/\partial\beta \leq 0$*;* $\partial s_1/\partial\beta \leq 0; \partial\pi_1/\partial\beta \leq 0$ *for* $\beta \leq \widehat{\beta}$*; whereas (ii) when* $v_1 < v_2$*, the opposite holds.*

Proof: See the appendix.

The above proposition illustrates that when an MNO accommodates Skype, the effect of change of the quality parameter β on the MNOs' market shares, prices, and profits can be ambiguous: an MNO is able to increase its network market share and set higher fixed fee thereby making more profits only if the initial quality of Skype connection is higher than a certain threshold level provided its customer valuation is higher than its rival's. This happens due to opposing effects - direct and indirect. On one hand, an increase in the quality of Skype connection reduces the network's own voice market share (the direct effect); on the other hand, higher Skype quality helps the present network to attract more customers away from its rival (the indirect/strategic effect). If the latter effect dominates the former then the market share and profits will increase unambiguously. But if the former dominates the latter, then they will decrease. Whether the latter effect dominates the former however depends ultimately on consumers' preference for a certain network as increase in β implies that the quality of Skype connection is improved on *both* networks: if consumers prefer network 1 more over network 2 (i.e. if $v_1 > v_2$) then the consumers would rather make Skype calls on their preferred network implying an increase in the network's market share and profits.

One implication of the proposition then is that MNOs may have incentives to invest in the quality of Skype-based interconnection (for example by improving its own quality with a spill-over effect on Skype-based connection) in order to raise profits if it accommodates this VoIP.

Case (b): Both MNOs block Skype's entry into their networks.

In this case, $\lambda_i = 1$ for both and the prices are given by $\widetilde{p}_i^* = c + 2xs_i^*, i = 1, 2$. Hence the equilibrium values are as follows:

- *Equilibrium network market shares* s_i^*.

$$s_1^* = s^* = \frac{1}{2} + \frac{\Delta v}{6x} \text{ and } s_2^* = 1 - s^* = \frac{1}{2} - \frac{\Delta v}{6x}$$

- *Equilibrium prices* $\widetilde{p}_i^*$.

$$\widetilde{p}_1^* = c + x + \frac{\Delta v}{3} \text{ and } \widetilde{p}_2^* = c + x - \frac{\Delta v}{3}$$

- *Equilibrium values of network profits* π_i^*. Since networks' profits are now given by $\pi_i^* =$

$s_i^*[\widetilde{p}_i^* - c]$, therefore

$$\pi_1^*|_{B,B} = \frac{(3x + \Delta v)^2}{18x} \text{ and } \pi_2^*|_{B,B} = \frac{(3x - \Delta v)^2}{18x}$$

where the subscripts (B, B) denote network' profits when both block.

Obviously, β has no effect on equilibrium values as the entry of Skype has been blocked. Finally, we note that

Observation 2. $\Delta v > (<) \ 0$ implies $s_1^* > (<) \ s_2^*$; $\widetilde{p}_1^* > (<) \ \widetilde{p}_2^*$ and $\pi_1^*|_{B,B} > (<) \ \pi_2^*|_{B,B}$ whereas when $\Delta v = 0$, both networks split the market equally i.e. $s_1^* = s_2^* = 1/2$ and earn Hotelling profit $= x/2$.

Case (c): One network accommodates whilst the other blocks.

There are now two subcases where (i) MNO_1 accommodates but MNO_2 blocks and (ii) where MNO_1 blocks but MNO_2 accommodates. Assume MNO_i is the one who accommodates and MNO_j is the one who blocks, where $i, j = 1, 2$, and $i \neq j$.Therefore, $\lambda_i = \frac{1}{2} + \frac{1}{2y}[v_i(1 - \beta) - c]$, $\lambda_j = 1$; $p_i^* = c$ and $F_i^* = 2xs_i^*$; and $\widetilde{p}_j^* = c + 2xs_j^*$. In the following the first subscript denotes i's strategy and the second $j's$. Thus we have:

- *Equilibrium network market shares.*

$$\begin{aligned} s_i^*|_{\text{A, B}} &= \frac{1}{2} + \frac{1}{6x}[\beta v_i - v_j + y(\lambda_i^*)^2 - \frac{y}{2} + c] \quad \text{and} \\ s_j^*|_{\text{B, A}} &= \frac{1}{2} + \frac{1}{6x}[v_j - \beta v_i - y(\lambda_i^*)^2 + \frac{y}{2} - c] \qquad \text{for } i \neq j, i = 1, 2; j = 1, 2 \end{aligned}$$

where the first subscript denotes the MNO's own strategy and the second denotes its opponent's strategy. Equilibrium values of prices and fixed fees then follow immediately by substituting the above values of the market shares into their respective expressions.

- *Equilibrium values of network profits π_i^*.*

If MNO_i has accommodated, then

$$\begin{aligned} \pi_i^*|_{\text{A, B}} &= s_i^* F_i^* \\ &= \frac{1}{18x}[3x + \{\beta v_i - v_j + y(\lambda_i^*)^2 - \frac{y}{2} + c\}]^2 \end{aligned}$$

and if MNO$_j$ has blocked Skype's entry, then its equilibrium profit will be

$$\begin{aligned}\pi_j^*|_{\text{B, A}} &= s_j^*(\widetilde{p}_j^* - c) \\ &= \frac{1}{18x}[3x + \{v_j - \beta v_i - y(\lambda_i^*)^2 + \frac{y}{2} - c\}]^2\end{aligned}$$

Effect of β on MNO$_i$'s profits.

In this case any increase in the quality of Skype connection unambiguously increases MNO$_i'$s market share and hence its profits via the increase in fixed fee as shown by the following proposition.

Proposition 4 *If MNO$_i$ accommodates Skype whilst MNO$_j$ blocks it, then higher β means MNO$_i$ can increase its market share, charge higher fixed fee and thereby increase its profits i.e.* $\partial s_i/\partial\beta > 0$; $\partial F_i/\partial\beta > 0$ *and* $\partial\pi_1/\partial\beta > 0$.

Proof. Letting *i=1*,and $j = 2$, $\frac{\partial s_1}{\partial\beta} = \frac{1}{6x}[v_1 + 2y\lambda_1\frac{\partial\lambda_1}{\partial\beta}] = \frac{1}{6x}[v_1 - v_1\lambda_1] = \frac{1}{6x}[v_1(1 - \lambda_1)] > 0$. Hence, $\partial F_i/\partial\beta = 2x\frac{\partial s_1}{\partial\beta} > 0$ and $\partial\pi_1/\partial\beta = (F_1 + 2x)\frac{\partial s_1}{\partial\beta} > 0$. ■

Thus, in contrary to the result of proposition 3 where the increase or decrease on market share, fixed fees and profits depended upon the initial value of β and customers' preference for a certain network, here they all increase unambiguously regardless of the value of β and customers' preference: Given that the rival network has blocked Skype's entry, any increase in Skype quality directly helps the MNO to attract customers which in turn enables the MNO to increase its market share, fixed fee and profits unambiguously.

2.3 Stage 1: MNOs decisions: To block or not to block?

This is the first stage of the game where the MNOs need to decide whether or not to block Skype on their networks, keeping in mind all possible equilibrium configurations as described above. Given that each network has two (pure) strategies: {accommodate, block}, the game can be described in a strategic form as follows:

[Insert Figure 2 here]

where player i's strategies are $\{A_i, B_i\}, i = 1, 2,$ (A: accommodate, B: block). Proposition 5 shows that unilateral accommodation is the dominant strategy of MNO$_i$ whenever by doing so it can maintain a voice market share above a certain threshold level (approximately 29%).

Proposition 5 *MNO_i prefers to accommodate Skype regardless of what its rival does, whenever its voice market share $\lambda_i^* \geq 0.29$ (approximately) for $i = 1, 2$.*

Proof: See the appendix.

The proposition 5 implies that an MNO will have incentives to accommodate Skype if it anticipates that through its pricing strategy it will be able to maintain at least a significant level of its voice market share i.e. it will not lose its entire network subscription to Skype. Implications of the proposition 5 are as follows. Since 'accommodate' is the dominant strategy of both MNOs for $\lambda_i^* \geq 0.29, i = 1, 2$, the Nash equilibrium of the game will be where both MNOs accommodate[7]. On the other hand, if $\lambda_i < 0.29$ is true for both firms, then the Nash equilibrium of the game will be where both MNOs block Skype's entry. Thus there can be two possible symmetric equilibria $\{A_1, A_2\}$; and $\{B_1, B_2\}$. On the other hand, for cases where $\lambda_1 \geq 0.29 > \lambda_2$, or $\lambda_2 \geq 0.29 > \lambda_1$ is true, there can be two (asymmetric) Nash equilibria: $\{A_1, B_2\}$ and $\{B_1, A_2\}$ respectively.

Corollary If MNOs are not vertically differentiated then only a symmetric equilibrium can prevail where $\pi_i^*|_{\text{A, A}} = \pi_i^*|_{\text{B, B}}$ where both earn Hotelling profits.

Proof. If firms are not vertically differentiated then $\Delta v = 0 \Rightarrow v_1 = v_2$ and hence $\lambda_1 = \lambda_2$ (see remark 2). Given that firms are now completely symmetric, the only possible equilibria are the symmetric equilibria $\{\text{A}_1,\text{A}_2\}$ or $\{\text{B}_1,\text{B}_2\}$. However,

$$\begin{aligned}
\pi_i^*|_{\text{A, (A)}} &= \frac{1}{18x}[3x + \{\beta\Delta v + y\left(\lambda_1^2 - \lambda_2^2\right)|_{A,A}\}]^2 \\
&= \frac{1}{18x}[3x + \beta\Delta v]^2 = \frac{x}{2} = \pi_i^*|_{\text{B, B}}
\end{aligned}$$

■

The above corollary implies when firms are completely symmetric with respect to vertical differentiation parameters then it does not matter whether they accommodate or block regardless of the value of λ_i as they make exactly the same Hotelling profit in either case. Hence any of the equilibrium $\{A_1, A_2\}$ or $\{B_1, B_2\}$ is possible. If however the firms are vertically differentiated so that $\Delta v \neq 0$ then λ_i s are different, and hence depending on the particular value of λ_i,equilibrium can be any of the four possibilities (see figure 2). Which one of the

[7] Strictly speaking, there will be mixed strategy equilibrium if the condition holds with strict equality. However, we focus mainly on pure strategy equilibria.

four possible outcomes will emerge as the equilibrium outcome therefore depends upon the magnitudes of β and v_i as, given that MNOs are now vertically differentiated, it is mainly the combination of β and v_i that determine whether $\lambda_i \gtreqqless (<)$ 0.29: for both networks, higher the value of β, higher would the value of v_i be needed in order to maintain λ_i above 0.29 if the MNOs were to accommodate Skype's entry. Note that even if an MNO accommodates Skype's entry for strategic reasons, that does not necessarily guarantee that the MNOs will be able to increase their payoffs by improving the quality parameter β. To see that, for a given v_i let β^* denote the value of β such that $\lambda_i^* = 0.29$ i.e. $\beta^* = \{1 - \frac{c-0.42y}{v_i}\}$. Then $\lambda_i^* \geq 0.29$ whenever $\beta^* \geq \beta$ so that MNOs have the incentives to accommodate Skype. Suppose $\Delta v > 0$. From Proposition 3, improving Skype connection can help an MNO raise its profits (by attracting new customers) only if $\beta \geqq \widehat{\beta}$. So if β is such that $\beta^* \geq \widehat{\beta} > \beta$ then even if the MNO has incentives to accommodate Skype, it cannot increase its profits by doing so as the quality of Skype connection is not high enough, although it can if $\Delta v < 0$. Moreover, as the following proposition shows, even when MNOs accommodate Skype for strategic reasons, so that the resulting equilibrium is indeed $\{\mathrm{A}_1, A_2\}$, they are not necessarily better-off compared to the $\{\mathrm{B}_1, B_2\}$ equilibrium. This equilibrium therefore resembles the equilibrium of a classic prisoners' dilemma game. Proposition 6 shows that the MNO with a higher v is in fact worse-off whereas the MNO with lower v is better-off in the $\{A_1, A_2\}$ equilibrium compared to the $\{B_1, B_2\}$ equilibrium.

Proposition 6 *For $\triangle v \neq 0$, (i) if $v_1 > v_2$ then MNO_1 is worse-off while MNO_2 is better-off in equilibrium by accommodating compared to the equilibrium where they both block; (ii) if $v_2 > v_1$ then MNO_1 is better-off while MNO_2 is worse-off in equilibrium by accommodating compared to the equilibrium where they both block.*

Proof: See the appendix.

3 Conclusion.

In this paper, we have examined the incentives for mobile network operators to block or accommodate Skype, a VoIP with a huge popularity as it enables customers to make free internet-based calls, into mobile networks. We have modelled this as a three-stage game between two competing networks *à la* Hotelling where firms first decide whether to accommodate Skype or not and then compete in prices; consumers then decide whether to make voice or Skype calls in the third stage of the game. We have shown that unless the MNOs can maintain a voice market share of at least about 29%, they will not have incentives to accommodate Skype over their networks. Whether they can maintain a voice market share of at least 29% depends not just on their pricing strategy but more importantly on the consumers' preference parameter for a certain network (measuring the degree of vertical differentiation) and the quality of Skype-based interconnection. Further, we found that in a symmetric accommodation equilibrium, the MNO with a lower customer valuation parameter (i) is better-off relative to the equilibrium where entry is blocked and (ii) can increase its market share and profit by improving the quality of Skype connection whenever that quality is below a certain threshold level. This then implies that there maybe an argument for investment in overall improvement for the quality of internet-based call connection. This, we believe, will have important policy implications about regulatory measure as not only can this increase consumers' welfare (as internet-based calls are either free or very cheap to make), it can also lower market concentration in the telecommunication industries by promoting less established or newer firms.

Despite the simplicity of our model, we believe our results can explain several real-world phenomena that took place when Skype first launched its iPhone application in 2009 and can provide economic justification as to when and why some MNOs restrict Skype's entry while some don't. Finally, there are various ways our model can be extended. For example, we have assumed that the quality parameter β is the same for both firms. A more realistic scenario would be to consider heterogenous values of β as the quality of connection is likely to differ from one network to another. We have also considered symmetric marginal costs for both MNOs. Relaxing these assumptions will be useful for future research.

4 Appendix.

Proof of Proposition 1. From equation (3), $\frac{\partial \lambda_i}{\partial p_i} = -1/2y < 0$. From (4) and (8), $\frac{\partial s_i}{\partial p_i} = \frac{y\lambda_i}{x}\frac{\partial \lambda_i}{\partial p_i} = -\frac{\lambda_i}{2x} < 0$, and $\frac{\partial s_i}{\partial F_i} = -1/2x < 0$. Equation (13) then implies

$$s_i = \frac{1}{2x}[(p_i - c)\lambda_i + F_i]$$

Substituting the above value of s_i into equation (12), and simplifying obtain:

$$\frac{1}{2x}[(p_i-c)\lambda_i+F_i][(p_i-c)\frac{\partial\lambda_i}{\partial p_i}+\lambda_i]-[(p_i-c)\lambda_i+F_i]\frac{\lambda_i}{2x}=0$$

$$\text{or,}\qquad (p_i-c)\frac{\partial\lambda_i}{\partial p_i}+\lambda_i=\lambda_i \Rightarrow \quad (p_i-c)\frac{\partial\lambda_i}{\partial p_i}=0$$

$$\text{Hence it must be that } p_i^*=c \quad \text{as } \frac{\partial\lambda_i}{\partial p_i}=-1/2y<0$$

Therefore, in equilibrium

$$s_i=\frac{1}{2x}F_i \Rightarrow \;\; F_i=2xs_i$$

The second order conditions for the maximisation problem are:

$$\frac{\partial^2\pi_i}{\partial p_i^2}=2s_i\frac{\partial\lambda_i}{\partial p_i}+2\frac{\partial s_i}{\partial p_i}[(p_i-c)\frac{\partial\lambda_i}{\partial p_i}+\lambda_i]+[(p_i-c)\lambda_i+F_i]\frac{\partial^2 s_i}{\partial p_i^2} \tag{i}$$

and

$$\frac{\partial^2\pi_i}{\partial F_i^2}=2\frac{\partial s_i}{\partial F_i}+[(p_i-c)\lambda_i+F_i]\frac{\partial^2 s_i}{\partial F_i^2} \tag{ii}$$

As $p_i^*=c$, (i) yields

$$\begin{aligned}\frac{\partial^2\pi_i}{\partial p_i^2} &= 2s_i\frac{\partial\lambda_i}{\partial p_i}+2\frac{\partial s_i}{\partial p_i}\lambda_i+F_i\frac{\partial^2 s_i}{\partial p_i^2}\\ &= -\frac{1}{2xy}F_i-\frac{1}{x}\lambda_i^2+\frac{1}{4xy}F_i\\ &= -\frac{1}{x}\lambda_i^2-\frac{1}{4xy}F_i<0\end{aligned}$$

and (ii) yields

$$\frac{\partial^2\pi_i}{\partial F_i^2}=-1/x<0$$

Hence the equilibrium exists and is also unique. ■

Proof of Proposition 3. Straight forward differentiation yields $\frac{\partial F_1}{\partial\beta}=\frac{1}{3}[\Delta v+y\{2\lambda_1\frac{\partial\lambda_1}{\partial\beta}-$

$2\lambda_2 \frac{\partial \lambda_2}{\partial \beta}\}]$. Using $\lambda_i = \frac{1}{2} + \frac{1}{2y}[v_i(1-\beta) - c], i = 1, 2$ and simplifying obtain,

$$\begin{aligned}
\frac{\partial F_1}{\partial \beta} &= \frac{1}{3}[\Delta v + \{v_2\lambda_2 - v_1\lambda_1\}] \\
&= \frac{1}{3}[\Delta v + \frac{1}{2}\{-\Delta v + \frac{1}{y}[c\Delta v - (1-\beta)(v_1^2 - v_2^2)]\}] \\
&= \frac{\Delta v}{6}[1 + \frac{1}{2y}\{c - (1-\beta)(v_1 + v_2)\}]
\end{aligned}$$

The critical value $\widehat{\beta}$ is found by solving $\frac{\partial F_1}{\partial \beta} = 0 \Rightarrow \widehat{\beta} = 1 - \frac{(2y+c)}{v_1+v_2}$. Hence for $\Delta v > 0$, it follows immediately that for $\beta \geq \widehat{\beta}, \partial\ F_1/\partial\beta \geq 0$; $\partial s_1/\partial\beta \geq 0; \partial\pi_1/\partial\beta \geq 0$ whilst for $\beta \leq \widehat{\beta}$, $\partial\ F_1/\partial\beta \leq 0$; $\partial s_1/\partial\beta \leq 0; \partial\pi_1/\partial\beta \leq 0$. It is then easily verified that the opposite holds when $\Delta v < 0$. ∎

Proof of Proposition 5. Consider network 1. There are two situations to consider.

(i) If MNO_2 has **accommodated** Skype, then MNO_1 will accommodate Skype if and only if $\pi_1^*|_{A,\,A} \geq \pi_1^*|_{B,\,A}$; and (ii) if MNO_2 has **blocked** Skype, then MNO_1 will accommodate Skype if and only if $\pi_1^*|_{A,\,B} \geq \pi_1^*|_{B,\,B}$. Consider (i) first. MNO_1 accommodates when MNO_2 does, whenever the following holds:

$$\pi_1^*|_{A,\,A} \geq \pi_1^*|_{B,\,A}$$

Or, whenever the following holds:

$$\frac{1}{18x}[3x + \{\beta\Delta v + y\left(\lambda_1^2 - \lambda_2^2\right)|_{A,(A)}\}]^2 \geq \frac{1}{18x}[3x + \{v_i - \beta v_2 - y(\lambda_2|_{B,(A)})^2 + \frac{y}{2} - c\}]^2$$

Or,

$$\beta\Delta v + y\left(\lambda_1^2 - \lambda_2^2\right)|_{A,(A)} \geq v_1 - \beta v_2 - y(\lambda_2|_{B,(A)})^2 + \frac{y}{2} - c$$

Now note that equilibrium voice market share λ_i when one or both MNOs accommodate(s) is same regardless of what its rival has done, i.e. $\lambda_i = \frac{1}{2} + \frac{1}{2y}[v_i(1-\beta) - c]$ always. Hence substituting this in above and simplifying, obtain $\pi_1^*|_{A,\,A} \geq \pi_1^*|_{B,\,A}$ whenever

$$y\lambda_1^2 \geq v_1(1-\beta) - c + \frac{y}{2}$$

Now, $v_1(1-\beta) - c = [2\lambda_1 - 1]y$. Therefore, substituting in above obtain $\pi_1^*|_{A,\,A} \geq \pi_1^*|_{B,\,A}$

whenever the following holds

$$\lambda_1^2 - 2\lambda_1 + \frac{1}{2} \geq 0$$

Solving $\lambda_1^2 - 2\lambda_1 + \frac{1}{2} = 0$ yields the two roots of $\lambda_1 = \frac{2 \pm \sqrt{2}}{2}$. Given that $1 \geq \lambda_1$, only the value $1 - \frac{\sqrt{2}}{2}$ is acceptable. Hence, given MNO_2 has accommodated, MNO_1 will too whenever $\lambda_1 \geq 1 - \frac{\sqrt{2}}{2} \approx 0.29$.

Now consider the other case (ii): If MNO_2 has blocked then MNO_1 will accommodate Skype if and only if $\pi_1^*|_{\mathrm{A,\ B}} \geq \pi_1^*|_{\mathrm{B,\ B}}$. Or, whenever the following holds:

$$\frac{1}{18x}[3x + \{\beta v_1 - v_2 + y(\lambda_1^*)^2 - \frac{y}{2} + c\}]^2 \geq \frac{(3x + \Delta v)^2}{18x}$$

Or, after simplifying, yields the following:

$$y\lambda_1^2 \geq v_1(1-\beta) - c + \frac{y}{2}$$

which is the same condition as above. Therefore, it is easily verified that in this case too, MNO_1 will accommodate Skype whenever its voice market share $\lambda_1 \geq 0.29$. Likewise, it can be easily verified that MNO_2 will accommodate Skype, regardless of what MNO_1 does whenever MNO_2's voice market share λ_2 exceeds 0.29. ∎

Proof of Proposition 6. First of all note when $\triangle v \neq 0$, $(\lambda_1 - \lambda_2) = \frac{(1-\beta)\triangle v}{2y}$ which can be either positive or negative depending upon whether $\triangle v \gtrless 0$. When both firms accommodate (i.e. $\lambda_i \geq 0.29\ \forall i$), MNO_1 will be better-off in $\{A_1, A_2\}$ compared to $\{\mathrm{B}_1, B_2\}$ if and only if the following holds:

$$\begin{aligned}
&\pi_1^*|_{A,A} = \frac{1}{18x}[3x + \{\beta\Delta v + y\left(\lambda_1^2 - \lambda_2^2\right)\}]^2 \geq \pi_1^*|_{B,B} = \frac{(3x+\Delta v)^2}{18x} \qquad \text{i.e. } \textit{iff} \\
&\beta\Delta v + y\left(\lambda_1^2 - \lambda_2^2\right) \geq \Delta v \qquad \text{or, } \textit{iff} \\
&y\left(\lambda_1^2 - \lambda_2^2\right) \geq (1-\beta)\Delta v \\
&\text{or,} \quad y(\lambda_1 + \lambda_2)(\lambda_1 - \lambda_2) \geq (1-\beta)\Delta v \\
&\text{or, } (\lambda_1 + \lambda_2)\frac{(1-\beta)\triangle v}{2} \geq (1-\beta)\Delta v
\end{aligned}$$

Similarly, MNO_2 is better-off in $\{A_1, A_2\}$ compared to $\{\mathrm{B}_1, B_2\}$ if and only if the following holds

$$\pi_2^*|_{A,A} = \frac{1}{18x}[3x - \{\beta\Delta v + y\left(\lambda_1^2 - \lambda_2^2\right)\}]^2 \geq \pi_2^*|_{B,B} = \frac{1}{18x}[3x - \Delta v]^2$$

$$\text{or },\ \triangle v(1-\beta) - y(\lambda_1+\lambda_2)\frac{(1-\beta)\triangle v}{2y} \geq 0$$

(i) If $v_1 > v_2$ so that $\Delta v > 0$, then for MNO_1, the condition $(\lambda_1+\lambda_2)\frac{(1-\beta)\triangle v}{2} \geq (1-\beta)\Delta v$ cannot hold as it implies $(\lambda_1+\lambda_2) \geq 2$ which is not possible since the maximum value of λ_i is 1. Hence, it must be true that $\pi_1^*|_{A,A} \leq \pi_1^*|_{B,B}$ i.e. MNO_1 is worse-off in the $\{A_1, A_2\}$ relative to $\{B_1, B_2\}$.For MNO_2 on the other hand, $\triangle v(1-\beta) - y(\lambda_1+\lambda_2)\frac{(1-\beta)\triangle v}{2y} \geq 0 \Rightarrow 2 \geq (\lambda_1+\lambda_2)$ which is satisfied $\Rightarrow \pi_2^*|_{A,A} \geq \pi_2^*|_{B,B}$ i.e. the MNO_2 is better-off in the $\{A_1, A_2\}$ equilibrium compared to the $\{\mathrm{B}_1, B_2\}$ equilibrium.

(ii) If $v_2 > v_1 \Rightarrow \Delta v < 0$, then for MNO_1, the condition becomes $2 \geq (\lambda_1+\lambda_2)$ which is satisfied $\Rightarrow \pi_1^*|_{A,A} \geq \pi_1^*|_{B,B}$. On the other hand, for MNO_2, the above inequality implies $(\lambda_1+\lambda_2) \geq 2$ which is not possible to hold $\Rightarrow$ MNO_2 is now worse-off. ■

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