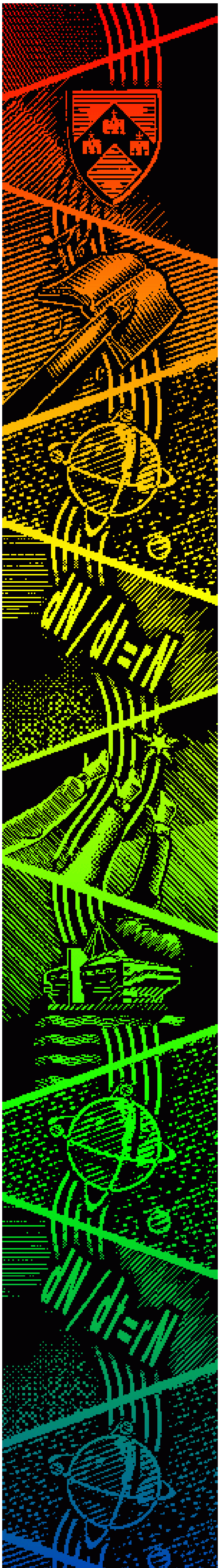

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Financial Markets Around the Great Recession: East Meets West

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Abstract

The 2007-2009 great recession saw sharp drops in equity values world wide and associated strong real effects. We develop an world CAPM approach, extended to allow for infinite risk/return opportunities, short sales constraints, borrowing and saving rate differentials. With MSCI monthly data, we use this to estimate tangent portfolios, standard deviations and market prices of risk in each country. We find short selling has a strong impact, in the crisis the net supply of equity finance vanished. If short selling is impossible, investors should have switched into cash. Postcrisis it rose but was still lower than precrisis.

Key words: Great recession, World CAPM, Supply of risky finance

JEL Nos: G01, G18, E44, E65

1 Introduction

The 2007-2009 great recession has been and still is amongst the sharpest in the last century with wider world effects than most preceding recessions. Its effects and transmission channels were different for different economies, depending partly on their initial economic position (Claessens et al., 2010) and also on their real and financial links (Obstfeld et al., 2009; Taylor, 2008). In a thumbnail sketch, the crisis initially started in the financial sectors in the US, then, since European financial institutions held the problematic mortgage backed securities, it spread to European and UK financial sectors. The fall in asset values of banks led to squeezed lending capacity for loans to both individuals and businesses (Mishkin, 2010). This had real effects depressing output and employment, changing expectations and increasing risk aversion so that generally the cost of capital for risky real investment in the West rose. The Western real recession led to a fall in demand for Asian imports and also a withdrawal of FDI from Asia which in turn caused real recessionary effects in Asia. Thus, the financial crisis spread from West to East through real sectors rather than financial sectors. The main purpose of this paper is to apply a CAPM framework to see how the tangent portfolio (TP), the associated amount of risk (measured by its standard deviation, σ, and the market price of risk (MPR) changed during and after the crisis. How did these changes in MPR, σ and in safe rates change the supply of risky investment?

CAPM provides an applicable simple framework which only requires means and covariance matrices of asset returns as inputs, alternatives would be more demanding in information and computation[1].We use a world CAPM with many risky assets serving to fund real risky investment and also a safe asset to investigate the impacts of the crisis on the efficient frontier EF, the TP and the amount of risky investment in China, Japan, US and UK with MSCI monthly data. Also largely based on our empirical evidence and previous studies, we assume that national equity markets are integrated and can be represented by a world efficient frontier (EF) but the safe asset markets are not integrated across countries, partly because of the behavior of monthly covered safe (borrowing/saving) rate parities and partly because there exist risk free arbitrage profits across countries.

However standard CAPM analysis does not adequately represent our empirical situation in which there is a spread between safe saving and borrowing rates, very high mean/risk portfolio returns are available in some instances, short selling opportunities on a very large scale are important and the equity holdings are dominated by institutions/corporations rather than individuals. So we develop a general theoretical

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[1]Second degree stocahstic dominance tests for portfolio efficiency exist (e.g. Post, 2003; Kopa, 2011) but typically require the solution of linear programmes and also have difficulties handling short sales and diversification.

extension of the CAPM model in each of these directions. The safe rate spread induces a three part EF with a region in which only risky assets are held. We know that the EF is increasing in both the mean and standard deviation of risky returns, we show that although it is unbounded (Merton, 1972) at high mean/risk returns, nevertheless it's limiting slope is finite. This can generate a TP with infinite mean and standard deviation. We extend the contributions of Ross(1977) on short sales in CAPM to encompass both short selling and constraints of no short sales in an increasing number of assets. We find that in general either there is a monotonically nested envelope structure of the efficient frontiers as more short sales constraints are added or more constrained EF's are strict subsets of those that are less constrained. Finally we argue that the importance of institutional investors means that the intertemporal consumption CAPM is less applicable than a zero consumption multiperiod CAPM in which we show that the single pricing factor each period is the excess return on the market portfolio of the period. That is, the heavy role of instutions means that the optimal risky portfolio will always be on the one period CAPM efficient frontier.

The ex-post data shows that with the onset of the crisis mean equity returns fell sharply, variances increased and the correlations (which are positive between all assets in all phases) increased substantially. The positive correlation of all equity returns means that short selling is important for risk diversification. Empirically, we split our sample period into three parts: pre-crisis, during the crisis and post crisis. For each sub-period, we estimate the expected returns and covariance matrix of equities using a VAR, based on the MSCI monthly data. Using the derived means and covariances, we calculate the EF, TP, MPR, and the optimal supply of risky investment funds in the three subperiods. We find that the combination of changes in the equity return distribution during the crisis both depressed, shifted right and also tilted the EF across the US, UK, China and Japan. However compared to pre-crisis, in the post crisis situation mean equity returns were largely restored, some variances also fell but the covariances and correlation remained relatively high. The effect is that post crisis the equity markets still have fairly high risks but mean returns at least equal to the pre crisis level. If short sales are unrestricted, there remain high risk diversification possibilities. If short selling is prohibited, the constraints reduce the extent of the EF.

Since all countries have safe borrowing above safe saving rates (which both differ by country), the concept of a capital market line becomes a three part portfolio locus corresponding to portfolios that are long or short in cash or cashless. Therefore, the capital market line and portfolio opportunities differ by country and investor group. Following this, we compute the MPR and asset shares in the TP for each phase: pre-crisis, during the crisis and post-crisis. If short selling is allowed, we find that investors are short in US national equities pre crisis and are short in Japanese national equities post crisis and have very high volumes of short selling the UK during the crisis. MPR during the crisis is very high and the tangent portfolios are "at infinity" with unbounded means and standard deviations of portfolio return. As we know in the background, investors have finite initial wealth and all risky assets have finite means and σ's. With the high positive correlations, investors can only get an unbounded portfolio mean and σ by selling high volumes of some risky assets and investing similar volumes in others. Institutionalised short selling is possible on several exchanges (hedgefundwriter, 2011) but generally not in the volumes we find to be optimal. Consequently, we provide a parallel analysis for the case where short sales are not permitted. In this case, in all phases for all investors, there are binding no short sales constraints. Precrisis, nearly all investors only hold UK & Japan & China. During the crisis, Japanese equities are the only feasible choices if there are no short sales but are actually dominated by cash. Postcrisis, most investors in most countries only hold US & Chinese equities. Finally we compute the optimal supply of risky investment funds in the three subperiods along the capital market line for given risk aversion. The supply of risky finance via equities collapses during the crisis, and partially recovers post-crisis but to a lower level than precrisis.

Our contributions are both theoretical and empirical. Theoretically we provide some general extensions of CAPM, of interest in themselves, but necessary to understand the empirical features of the data. Empirically we find large changes in the supply of risky investment funds over three subperiods and generally risk (even with worldwide diversification) is higher than precrisis.

The plan of the paper is first to develop and extend some theoretical features of the EF and the tangent portfolio that we need in the sequel (Section 2). In section 3, due to the empirical evidence and previous studies, we make two key assumptions for the theory. In Section 4 we use the MSCI monthly data and asset return moments derived from the VAR's to study the effects of the financial crisis on equity markets. For example we discuss the change in equity returns, the covariance matrices and correlations. We also compute the world EF, the nature of the capital market line, MPR and tangent portfolio including its asset shares with and without short sales in each of the four countries in the pre-crisis, crisis and post-crisis phases.

2 Multiperiod Institutional Investors

These markets have a strong presence of institutional investors for whom the standard intertemporal consumption CAPM is less appropriate since there is no obvious life cycle consumption element for institutions. For example, in 2010, domestic individuals owned only 11% of the market capitalisation in the UK (Office of National Statistics, 2010), whilst in the US (US Census Bureau, 2012)it was 36% and 20% in Japan (Tokyo Stock Exchange Statistics, 2012). In China, more than 60% of the market values of equities are held by state agencies or legal persons (which are predominantly private sector enterprises and coporations),(Qi D, Wu W and Zhang H, (1999), Tiana L and Estrin S,(2008)). Such institutions are more subject to shareholders withdrawing funds rather than maximising life cycle consumption.

If institutions have a mean variance objective based on a long holding horizon, Arditti & Levy (1977) show that efficient choices over the holding period must be efficient for one period when the per period return distribution is iid with zero one period skewness. They also show that if the n-period institutional objective is decreasing in the holding period variance but increasing in the holding period mean and skewness, then again a portfolio efficient over the holding period must be efficient each period so long as skewness increases with the mean return over the holding period. However this approach neglects the opportunities for rebalancing the portfolio period by period.

We think of an institutional investor having mean variance preferences[2] each period t of an investment horizon $0, T$. With a single safe asset each period with return R_{Ft} and n risky assets each period with returns R_{it}, the investor objective is

$$\begin{aligned} & \Sigma_0^T \delta^t w_{t-1}(R_{Ft}a_{0t} + (1-a_{0t})c_t^{'} E_{t-1}R_t) - K c_t^{'} \Omega_t c_t) \\ \text{with } i' c_t &= 1 \end{aligned}$$

Here w_{t-1} is wealth at the start of period t, a_{0t} is the portfolio share in the safe asset,c_t is an $nx1$ column vectore of shares of total risky investment in each of the risky assets,$E_{t-1}R_t$ is a column vector of mean risky asset returns at t and Ω_t is the covariance matrix of these returns. δ is the long term investors discount rate. In the appendix we show that the value function has the form

$$V_t(w_{t-1}) = w_{t-1}B_t(R_{Ft}, R_{Ft+1}...R_{FT}) + A_t(E_t R_{t+1}, ..E_{T-1}R_T, \Omega_t, ..\Omega_T)$$

Hence Bellmans equation has the form

$$\begin{aligned} V_t(w_{t-1}) &= \max_{a_{0t}}[w_{t-1}(a_{0t}R_{Ft} + \max_{c_t}\{(1-a_{0t})\Sigma c_{it}E_{t-1}R_{it}) - K(1-a_{0t})^2 w_{t-1}^2 c_t'\Omega_t c_t + \delta E_{t-1}V_{t+1}(w_t)|\Sigma c_{it} = 1\}] \\ &= \max_{a_{0t}}[w_{t-1}(a_{0t}(1+\delta B_t)R_{Ft} + \max_{c_t}\{(1-a_{0t})(1+\delta B_t)\Sigma c_{it}E_{t-1}R_{it}) - K(1-a_{0t})^2 w_{t-1}^2 c_t'\Omega_t c_t + \delta E_{t-1}A_{t+1}|\Sigma c_i \end{aligned}$$

and the optimal choices a_{0t}, c_t are just one period choices solving

$$\max_{a_{0t}}[w_{t-1}(a_{0t}(1+\delta B_t)R_{Ft} + \max_{c_t}\{(1-a_{0t})(1+\delta B_t)\Sigma c_{it}E_{t-1}R_{it}) - K(1-a_{0t})^2 w_{t-1}^2 c_t'\Omega_t c_t|\Sigma c_{it} = 1\}]$$

Hence the optimal risky portfolios period by period must be on the one period efficient frontier[3].

This reinforces the arguments of Fama (1970), Elton & Gruber (1974) that if returns are iid and there are some preference restrictions, one period CAPM matches ICAPM. It justifies our concentration on computing one period efficient frontiers.

3 The N Asset EF

We start by presenting a succinct analysis of the main features of the CAPM model with n risky assets and then derive some extensions of this that we need to be able to understand the empirical experience within

[2] We can also think of per period preferences being quadratic.

[3] In more detail the efficient frontier is derived by fixing the mean of the risky asset portfolio at an arbitrary μ

$$\max_{a_0}\{w_{T-1}a_0 R_F + (1-a_0)w_{T-1}\max_{\mu}[\mu - K(1-a_0)\min_{c}[w_{T-1}c'\Omega c|\Sigma c_i = 1, \Sigma c_i ER_i = \mu]]\} \tag{1}$$

The best choice of c given μ, a_0 yields the EF, and then the joint choice of μ, a_0 gives both the tangent portfolio (and CML) and the best choice on it.

this framework. The key idea is that with n risky assets and investors who prefer higher mean (ER_P) but lower variance (σ^2) portfolios, the overall portfolio selection problem can be decomposed into firstly analysis of risky portfolios that cannot be dominated in terms of mean and variance, and secondly to the best mix between such portfolios and a safe asset. To derive the efficient frontier (EF) of risky portfolios which are undominated in mean and variance the standard approach solves

$$\sigma^2 = \min_a \{a^{'}\Omega a | a'i = 1, a'm = ER_P\}$$

where Ω is the covariance matrix of n assets; a is the column vector for the market investment share of those n assets; m is the column vector of expected asset returns $[ER_{1...}ER_N]$, ER_i is the mean return for risky asset i; ER_P is the mean value for the market return and i is the unit column vector. The solution was defined in Merton(1972), in the appendix we rewrite this in a form convenient for subsequent use (also see Brennan and Lo (2010)). This gives the efficient portfolio shares as

$$a = x_1 - b(ER_P)x_2$$

where

$$\begin{aligned} b(ER_P) &= \frac{m^{'}\Omega^{-1}i - ER_P(i^{'}\Omega^{-1}i)}{(i'\Omega^{-1}m)^2 - (m'\Omega^{-1}m)(i'\Omega^{-1}i)} \\ x_1 &= \frac{\Omega^{-1}i}{i'\Omega^{-1}i}; x_2 = [\frac{(i^{'}\Omega^{-1}m)\Omega^{-1}i}{i'\Omega^{-1}i} - \Omega^{-1}m] \end{aligned}$$

and the EF can be written as

$$\sigma = G(ER_P) = [x_1'\Omega x_1 + (b(ER_P))^2 x_2^{'}\Omega x_2]^{1/2} \tag{2}$$

It is well known that viewed as $\sigma = G(ER_P)$, then $G'() > 0, G^{''} > 0$. The first of these follows since in an efficient portfolio higher risk must be compensated by a higher mean return. The second holds by reductio ad absurdem (Elton at al., 2007).

3.1 The Tangent Portfolio

If there is a safe asset with return r_f, the two fund theorem holds and any optimal portfolio of safe and risky assets is a combination of the tangent portfolio (TP) and cash. If an interior tangent portfolio TP with mean ER_P and standard deviation of return ,σ and $MPR > 0$ exists it must satisfy the two equations

$$-\frac{\frac{\partial F}{\partial \sigma}}{\frac{\partial F}{\partial ER_P}} = \frac{ER_P - r_f}{\sigma} \tag{3}$$

$$F(\sigma, ER_P) = 0$$

at finite values of the variables σ, ER_P. The tangent portfolio is on the EF, and the tangent there passes through the safe rate of return point $(0, r_f)$.

By substituting out σ from the first equation, equality of the slopes of the capital market line and the EF becomes an equation solely in ER_P :

$$x_1^{'}\Omega x_1 + (\frac{m'x_1 - ER_P}{m'x_2})^2 x_2^{'}\Omega x_2 = -(ER_P - r_f)(m'x_1 - ER_P)\frac{x_2^{'}\Omega x_2}{(m'x_2)^2}$$

Some calculation (see details in Appendix) shows that if there is a finite positive solution to this equation then it is at.

$$ER_P = m'x_1 + \frac{x_1^{'}\Omega x_1 (m'x_2)^2}{x_2^{'}\Omega x_2(m'x_1 - r_f)} \tag{4}$$

3.2 The Capital Market Line and Market Price of Risk

Faced with a safe asset return and an EF, a mean variance investor can mix the safe asset and the TP in any proportions according to their risk preferences. The options are to hold only the safe asset, to mix the safe asset and the TP or to borrow the safe asset and invest the proceeds plus initial wealth in the TP[4]. The tangent itself (the capital market line), describes these investment opportunities. Its slope, the market price of risk (MPR), measures the equilibrium rate the market will offer for switching a unit of wealth from the safe asset to the TP. Using the definitions of ER_P and σ at the TP above

$$\begin{aligned} MPR &= \frac{ER_P - r_f}{\sigma} \\ &= \frac{(x_2'\Omega x_2(m'x_1 - r_f)^2 + x_1'\Omega x_1(m'x_2)^2)^{0.5}}{\left(x_1'\Omega x_1\right)^{0.5}(x_2'\Omega x_2)^{0.5}} \end{aligned} \tag{5}$$

(derivation of this formula is in the Appendix).

3.3 Extensions of the Theory

3.3.1 Infinite Mean and Risk TPs

In the appendix we show that along the EF, as ER_P tends to infinity so does σ. That is an infinitely high mean risky portfolio is available but only by exposure to unbounded risks. However EF does have a finite positive slope as ER_P tends to infinity. This allows us to define the idea of an asymptotic tangent (at) which is equal to the limiting value of $dER_P/d\sigma$ taken along the E, Fig 1.2. It is clear that if we take any tangent to the EF with a vertical intercept at x, by concavity of the frontier, if $r_f < x$ the TP must exist and have a lower mean and standard deviation than at B (a tangency must exist since we know that the slope of the EF is infinite at A). But if $r_f > x$ if there is a TP at a finite mean ER_P, it must occur on the EF above B as with r_f' in Figure 1.1. But suppose x itself was the intercept of the asymptotic tangent (at) (Fig 1.2). Then if $r_f > x$ the only possible TP involves an asymptotic tangency to the EF with (ER_P, σ) jointly tending to infinity. Note that when this case occurs there must be short selling of some of the risky securities, each security has a finite mean return and there is only a finite initial wealth to invest, so an unbounded mean return can only be realised by selling an unbounded amount of one security and buying an unbounded amount of another. This is more than an academic curiosity since in the countries we examine this case occurs, especially during the crisis.

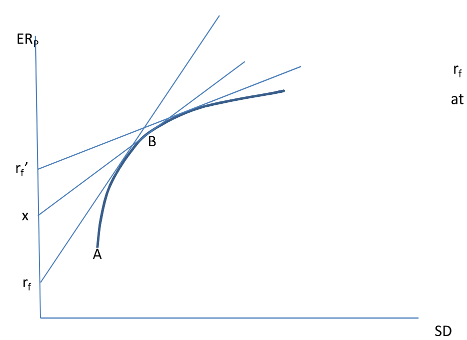

Figure 1.1 Tangent Portfolio

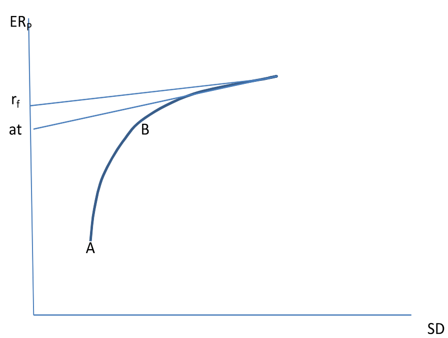

Figure 1.2 Tangent Portfolio at infinity

3.3.2 Safe Rate Differentials

Safe rates of interest often differ for borrowing and lending, and if the safe asset markets are not integrated, may also differ between economies. To avoid Ponzi game situations, the safe borrowing rate must be above the safe saving rate for each country (Figure 2.1).

[4] They will never short the tangent portfolio.

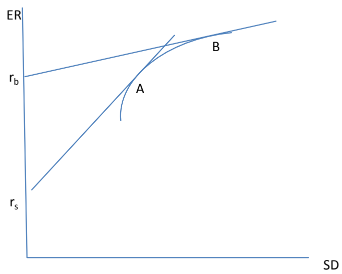

Figure 2.1 Safe rate differentials

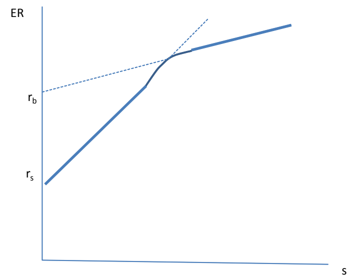

Figure 2.2 CML with different safes rates

As a consequence, the capital market locus has distinct segments (Elton et al., 2007). Taking the difference in interest rates into account an investor faces the bold line (Figure 2.2).

The investor may be relatively risk averse and invest in both cash and the TP A so that their overall portfolio is on the r_s linear section, if relatively risk loving they borrow in cash at the rate r_B and invest this and all their initial wealth in the TP B. Investors with intermediate risk aversion invest all their wealth in risky assets and are located at some point on the EF between A and B.

3.3.3 Structure of Efficient Portfolios

A portfolio on the EF may involve short selling one or more of the risky assets. We know also that the share of any individual asset in a risky efficient portfolio is monotonic in the mean of the risky portfolio, ER_P but may be increasing or decreasing, the result is

$$\frac{da}{dER_P} = \frac{i'\Omega^{-1}i}{(i'\Omega^{-1}m)^2 - m'\Omega^{-1}mi'\Omega^{-1}i}x_2$$

Short Sales Hence either along the EF an asset is never sold short, is sold short at all points on the frontier or the frontier divides into two segments at a point at which one asset has a zero holding. Since along the EF, each risky asset share in the TP is monotonic in ER_P, to one side of this point, along the frontier the asset is sold short, on the other side it is held long. To analyse this theoretically we can appeal to stepwise optimisation and the envelope theorem (see Appendix). In general the EF for $n-1$ assets must lie to the south east of the frontier for n assets since it provides fewer diversification possibilities. It lies wholly below that for n assets if all n assets are long (Fig 3.a). If one asset has a zero holding on the n asset frontier then, since the asset shares are monotonic in ER_P, that asset must be short to one side and long the other. Hence the $n-1$ frontier must touch the n asset frontier at this point and we have an envelope structure between the frontiers (Fig 3.b &3.c). Extending this argument it is possible that several assets have a zero share at a particular point on the n asset frontier, for example suppose the first two assets are not held at some point, then the n asset frontier divides again into two segments at that point with the two assets being either long or short to either side of the point. Then the two $n-1$ asset frontiers and the $n-2$ asset frontier must all be tangential at that point (Fig 3.d).

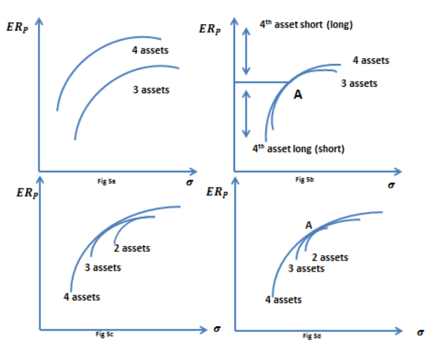

Fig 3 EF with short sales

With only two assets, if one asset has a zero share at a point on the EF, then the mean and standard deviation on the EF at that point are just the mean and standard deviation of the single long asset.

No Short Sale Constraints With no short sale constraints on risky assets, the available TP will typically consist of a continuous set of segments corresponding to $n, n-1, n-2..$ risky assets as successive short constraints bind on different assets as in Fig 4. If we move from two risky assets to just one due a binding short sale constraint on one of the assets the situation is slightly different. Depending on the risk free rate the TP can either contain both assets as with r_f or a single asset $r_f^{'}$.

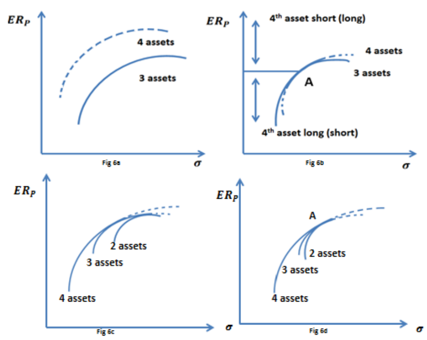

Fig 4 EF with no short sales

4 Assumptions of Theory

We make two key working assumptions: integrated equity markets and segregated safe markets, which we claim to be justified by the empirical data and previous studies. Each country has a wedge between its safe savings and its safe borrowing rate for each month in our data set. Moreover between countries there is no equality between safe saving rates or safe borrowing rates, allowing for exchange rate differences, using the spot and forward rates as appropriate. Thus a saver in China could for example risklessly save in the US and be better off than by saving in China by converting RMB into $ and at same time selling $ forward (a similar strategy is possible for borrowers). We call the resulting returns the safe covered interest parities, Fig 5 and 6 below show that for each of the savings and borrowing rates the monthly values of these differences are not zero. Similarly at any one data point there are risk free arbitrage profits to be made by borrowing in a cheap country and saving in a high yield country. Fig 7 shows the difference between the maximum saving rate and the minimum borrowing rate at each date expressed in $US at each time point. Throughout the sample these are positive, so if safe asset markets were integrated, we should observe specialisation of all safe saving in just one country and similarly for borrowing, and universally huge arbitrage profits are available to all. Of course we do not observe such profits being realised and so we assume that the safe asset markets are segregated.

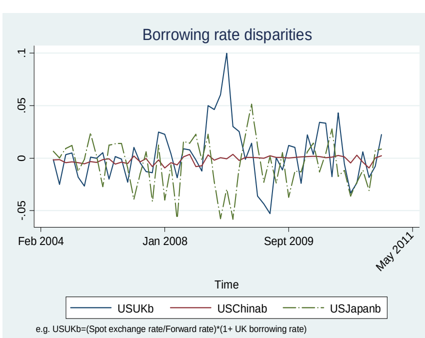

Fig 5 Borrowing rate disparities

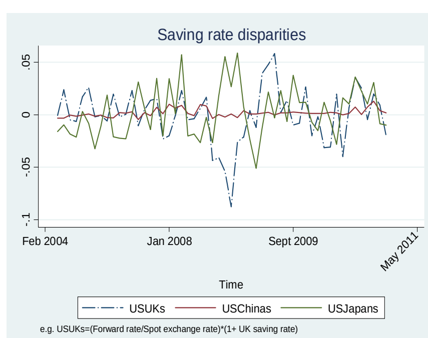

Fig 6 Saving rate disparities

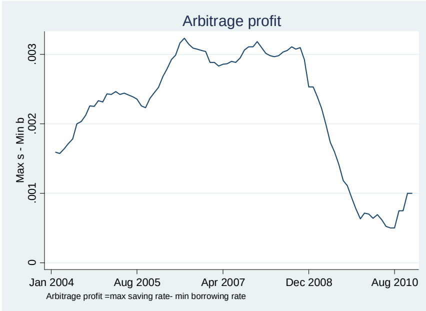

Fig 7 Arbitrage profit

Conversely we assume that equity markets are integrated across countries. A stream of past research indicates that there is some degree of integration and also that the extent of integration has increased especially in the last twenty years (Bekaert, 2005, 2011; Constantinides, 2003). In addition it is clear in our data that there were financial market spillovers from one exchange to another.

5 How did the Crisis Affect China, Japan, UK and US?

5.1 The Financial Sectors

5.1.1 The Equity Markets

For equity data we take monthly returns on the dollar denominated MSCI indices for UK, US, Japan and China (Fig 8-11). The crisis period is dated from September 2007, after BNP Paribas bank collapsed in Aug 2007 which manifested the onset of the crisis in Europe and the equity returns became negative. The crisis period ends in December 2008, the choice of end date is based on structural shifts in the monthly real returns on the different equity indices after each country's policy implementation to recover from the crisis Although the crisis had an international dynamic in its transmission between countries, there is surprisingly similar timing in the shifts of the mean of these series.

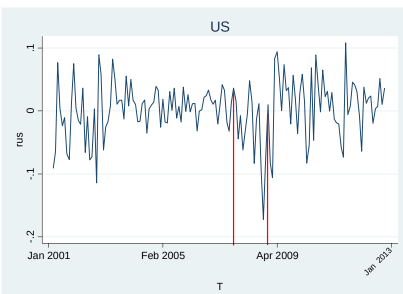

Fig 8 MSCI US

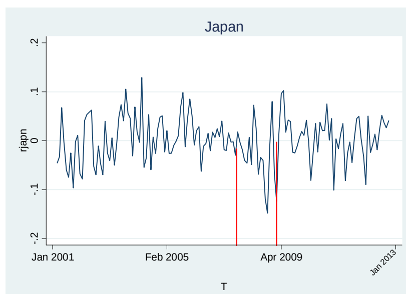

Fig 9 MSCI Japan

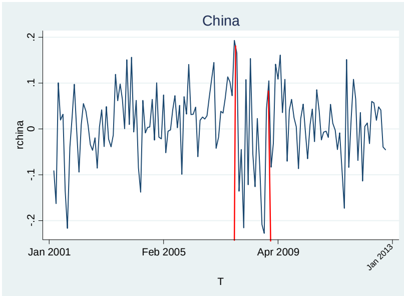

Fig 10 MSCI China

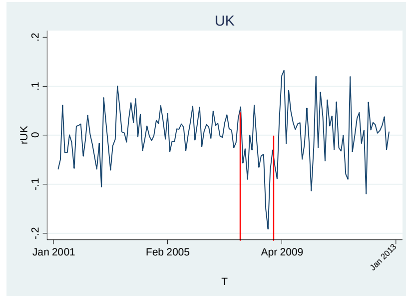

Fig 11 MSCI UK

We calculate the mean and covariance matrix of monthly real returns on each index for different subperiods of the sample (Table 1 Mean Equity Returns). For each phase we estimate a one lag VAR on the 4 asset returns, taking the mean predicted values for a phase as the mean returns, and the covariance matrix equal to that of the residuals from the VAR for that phase. The results are very similar to using historical data, essentially the VARs show little or no autocorrelation in returns but also are well specified statistical models according to specification tests for heteroscedasticity, omitted variables and autocorrelation (the Appendix gives details). Due to the breaks in the return series and the limited length of the crisis, we did not choose to use more explicit modelling of either adaptive expectations which are supported by some survey evidence of data (Greenwood and Shleifer (2013),or extrapolation of fundamentals (for example consumption/wealth ratios) of one type or another due to lack of data (see for example Goetzeman et al (2009)) although our approach is equivalent to using the information set of an investor at the end of a phase. It can also be thought of as a simple rational expectation of the return generating process in each phase by uninformed investors (Admati & Ross, 1985).

Before the crisis, all of the monthly mean equity real returns were positive for the period 2004-2007. During the crisis monthly mean equity returns fell sharply in all four markets After the crisis mean returns jumped back close to their pre-crisis levels for the period 2009-2010 but with a reduced spread between countries.

Variable	Pre crisis Mean	Crisis Mean	Post Crisis Mean
r_{UK}	.011	$-.046$	.018
r_{China}	.031	$-.041$	.023
r_{US}	.007	$-.032$	.016
r_{Japan}	.009	$-.026$	.008

Table 1 Mean Equity Returns

	UK	China	US	Japan
UK	1			
China	0.6126	1		
US	0.6299	0.4320	1	
Japan	0.2911	0.1146	0.2410	1

Table 2 (1) Pre-crisis correlation matrix

	UK	China	US	Japan
UK	1			
China	0.9309	1		
US	0.8617	0.7685	1	
Japan	0.7919	0.8075	0.7782	1

Table 2 (2) During crisis correlation matrix

	UK	China	US	Japan
UK	1			
China	0.7487	1		
US	0.8715	0.7056	1	
Japan	0.8310	0.6251	0.8530	1

Table 2 (3) Post crisis correlation matrix

Before the crisis there was reasonably strong positive correlation between all pairs of markets so the diversification possibilities between most pairs of markets depend partly on the possibility of short sales (Table 2 (1)-(3)).

	UK	China	US	Japan
UK	0.00057			
China	0.000878	0.003604		
US	0.000306	0.000528	0.000415	
Japan	0.000265	0.000262	0.000187	0.001455

Table 3 (1) Pre-crisis covariance matrix

	UK	China	US	Japan
UK	0.003175			
China	0.006673	0.016186		
US	0.002228	0.004486	0.002106	
Japan	0.002369	0.005454	0.001896	0.002819

Table 3 (2) During crisis covariance matrix

	UK	China	US	Japan
UK	0.004094			
China	0.003114	0.004227		
US	0.003194	0.002627	0.00328	
Japan	0.002511	0.001919	0.002307	0.00223

Table 3 (3) Post crisis covariance matrix

During the crisis the variances of monthly equity returns jumped up by 100% for all indexes (Tables 3 (1)-(3)). In absolute value terms the covariances of returns between pairs of countries also jumped upwards, significantly more than the variances. The correlations increase quite dramatically. In the crisis, the increased positive correlations implies that diversification depends more strongly on the possibility of short selling. After the crisis for all countries, except China, the variances and covariance remain virtually constant but the variance of China and its covariance with each other country falls.

One way of evaluating the change in risk between the phases is to see if the eigenvalues of one covariance matrix dominate those of another. Table 4 computes the eigenvalues of each covariance matrix arranged in descending order. This gives the clear impression that in the crisis there was more risk than before or after, but also that there was more risk post-crisis than pre-crisis. On the other hand the differences between the covariance matrices are not positive (or negative) definite for any pair of matrices.

Pre-crisis	Crisis	Post Crisis
.0402	.2236	.1164
.0017	.0018	.0012
.0046	.0064	.0052
.0240	.0011	.0156

Table 4 Eigenvalues of covariance matrix

5.1.2 Real Safe Rates

As safe interest rates for the UK we take the saving rate to be the monthly interest rate on time deposits with maturity less than 12 months, for the US the rate on CD's with maturity of at most 12 months, for China the published consumer saving rates and for Japan the deposit rate. For the borrowing rate we take the bank lending rate for non-housing loans for the UK, the US prime business loan rate, for China and for Japan the official consumer borrowing rate. All the safe rates are converted to dollar returns using spot exchange rates at relevant dates. Hence the safe rates can be negative only if the currency depreciated against dollar.

All the spreads for safe saving & borrowing rate are positive in all phases except for post crisis UK when the safe borrowing and saving rates are equal. Across countries, in average terms there are risk free arbitrage profits. For example, pre crisis, by borrowing in Japan and saving in the UK, during crisis by borrowing in the UK and saving in China and post crisis, borrowing in the US and saving in China (Table 5 (1)-(2)).

	Means	UK save	US save	China save	Japan save
Feb 2004-Aug 2007	pre-crisis	0.004	0.003	0.006	−0.0009
Sept 2007-Dec 2008	crisis	−0.0046	0.003	.003	−0.0086
Jan 2009-Dec 2010	post crisis	.001	.0005	.005	−0.001

Table 5 (1) Safe Saving rate

	Means	UK borrow	US borrow	China borrow	Japan borrow
Feb 2004-Aug 2007	pre-crisis	0.006	0.0051	0.009	0.0003
Sept 2007-Dec 2008	crisis	−0.0026	0.004	.0057	.014
Jan 2009-Dec 2010	post crisis	.001	.003	.007	.007

Table 5 (2) Safe Borrowing rate

5.1.3 Shifts in the world EF with short sales

Having briefly surveyed the national picture we then assemble the financial data on means and covariance matrices in a world CAPM type model, and examine how the "world" EF shifts before, during and after the crisis (Fig 12).

The result is shown in the diagram below where we plot the EFs corresponding to the three subperiods[5]. Note that these calculations assume that short sales of any index are possible. Each of the EFs has an asymptotic tangent of finite positive slope, pre-crisis the asymptotic tangent slope is 0.43 during the crisis this jumps to 1.6 and after the crisis is 0.45

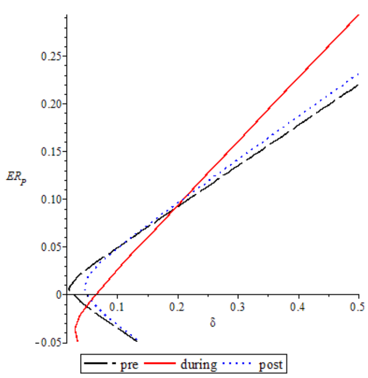

Fig 12 EF shift

The change in means and covariances of equity returns during the crisis caused a fall in the EF and also a tilting towards increased risk. The minimum variance portfolio entailed a substantial reduction in mean return and a small increase in risk.

After the crisis the EF lost some of its speculative opportunities, partly through the increased correlation between countries and partly through a fall in the variance of each country, all combined with increases in mean returns. The effect is that there are higher mean/risk opportunities postcrisis than precrisis but at low mean returns risk has increased. The minimum variance portfolio is now associated with a positive mean higher even than pre-crisis. But it is riskier than pre-crisis.

5.1.4 Shifts in the EFs with no short sales

Imposing no short sales, borrowers and savers in different countries may effectively be on different frontiers

[5]For clarity we show the full algebraic function although the EF corresponds just to the part to the right of the minimum variance portfolio.

according to the way that non-negativity constraints on asset holdings impact for the varying risk free rates. Pre-crisis and post-crisis we can plot the EF's corresponding to 4, 3,2 assets, these are in the Figs below where in each case when there is any short selling it occurs on the EF at high values of ER_P. During the crisis, all countries have TP at infinity and short selling does occur but involves implausibly high volumes. Pre-crisis we have Fig 13.1 whereas post-crisis the situation is Fig 13.2, a case in which Japanese equity is short at each point along EF.

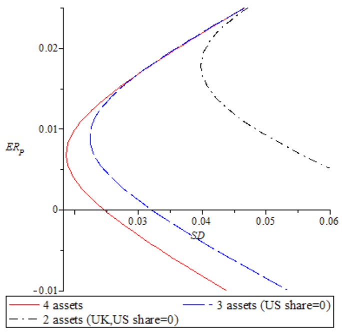

Fig 13.1 Precrisis EF shift

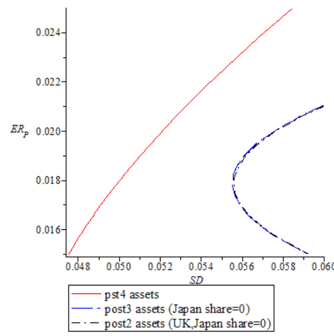

Fig 13.2 Postcrisis EF shift (no short sales)

5.1.5 The CML By Country in Phases

5.1.6 With short selling on tangent portfolio

With integrated world equity markets, all countries face a common world EF. However with differences between countries in safe saving and borrowing rates, each country will have its own capital market line and nationally based market prices of risk. Because the real savings rate is always below the real borrowing rate, we will typically be in the situation of Table 9 ((1)-(3)). There will generally be two TPs, one T_s corresponding to investors who are long in cash and the other T_b to investor who are short in cash[6].

Pre-crisis	T(s)		Market price	T(b)		Market price
	ER_P	σ	$\frac{ER_P - r_f}{\sigma}$	ER_P	σ	$\frac{ER_P - r_f}{\sigma}$
UK	.033	.066	.444	.125	.279	.427
US	.025	.046	.467	0.054	.111	.432
China	.116	.256	.427	**50**	**117**	**.426**
Japan	.015	.027	.593	.0164	.030	.554

Table 9 (1) Pre-Mean and σ of TP

During crisis all country's market price of risk at asymptotic tangent: Market price .664

Table 9 (2) Crisis MPR

Post-crisis	T(s)		Market price	T(b)		Market price
	ER_P	σ	$\frac{ER_P - r_f}{\sigma}$	ER_P	σ	$\frac{ER_P - r_f}{\sigma}$
UK	.113	.237	.456	.203	.437	.450
US	.088	.183	.462	.148	.316	.452
China	.199	.428	.449	**50**	**111**	**.447**
Japan	.052	.108	.492	.058	.121	.483

Table 9 (3)Post-Mean and σ of TP

When short sales are allowed pre and post crisis all investors have an interior tangent portfolio except for Chinese borrowers (shown in bold). The tangent portfolio for Chinese savers involves a much higher mean and σ of return than the other countries but the MPR's are fairly similar between countries. This is also

[6] The calculations below derive the CML and the supply of risky finance, $w_{t-1}(1 - a_{0t})$, using the current safe rate as the return on cash. Effectively this sets the investors discount rate equal to the safe rate.

true for borrowers although Chinese borrowers only achieve this MPR by very high volumes of short selling of the US.

5.1.7 Short Selling Volume in Tangent Portfolio

We can compute the shares of each risky asset in the TP of each country and each safe rate during the different phases. Table 6 (Table 6 Asset shares with short sales) documents the asset shares before and after the crisis in the four asset TPs allowing for short sales. Before the crisis safe savers and borrowers all short sell the US equity index and in quite sizeable amounts. The proceeds from short sales are then invested in a mix of the safe asset and the equity indices of each other country. Chinese safe borrowers have an asymptotic TP (Table 6 shows their asymptotic long and short holdings (in bold) at a point at which the difference between the slope of the EF and the asymptotic tangent is at most $|10^{-8}|$)[7]. Precrisis all other investor groups have a finite TP. During the crisis all country/safe rate combinations have MPR equal to the asymptotic tangent, so there is a single common TP for all country/safe rate combinations. The asset shares in the asymptotic tangent portfolio are approximately (in the sense above) $a_{uk} = -3669.14, a_{china} = 556.33, a_{US} = 1912.83, a_{japan} = 1200.98$ and so involve extreme positions with very high short sales of the UK equity. Post crisis the story is similar to the pre-crisis situation, except that now it is the Japanese equity index which is sold short and the volume of short selling is smaller than pre-crisis.

	Pre-crisis				Post-crisis			
	UK	China	US	Japan	UK	China	US	Japan
UKs	.436	1.051	−.676	.189	2.711	2.485	6.598	−10.794
UKb	.626	4.957	−5.210	.627	5.401	4.416	12.313	−21.134
Chinas	.606	4.540	−4.727	.580	5.274	4.323	12.038	−20.635
Chinab	**104**	**2130**	**−2471**	**238**	**1487**	**1067**	**3158**	**−5713**
USs	.417	.661	−.223	.146	1.976	1.958	5.038	−7.972
USb	.477	1.892	−1.653	.284	3.770	3.245	8.845	−14.860
Japans	.397	.253	.249	.101	.917	1.198	2.790	−3.904
Japanb	.399	.312	.181	.107	1.092	1.324	3.163	−4.580

Table 6 Asset shares with short sales

5.1.8 With no short sales

Pre-crisis if short selling is permitted, all risk free rates imply short sales of the US index. Imposing the constraint that the US asset share is zero (so moving to the 3 asset frontier), most country safe rate combinations have long holdings in the TP of each of the three equity indices. However savers and borrowers in China and borrowers in the UK would short sell the UK on the 3 asset frontier. Imposing that the UK has a zero share for these cases (moving to the two asset frontier) Chinese savers and UK borrowers are long in both Japanese and Chinese equities but Chinese borrowers are short in the equity of Japan. We conclude that pre-crisis Chinese borrowers only invest in Chinese equity when no short sales are permitted.

During the crisis if no short sales are imposed, the asymptotic TP with four risky assets is short in the UK (the asset shares are $a_{uk} = -3669.14, a_{china} = 556.33, a_{US} = 1912.83, a_{japan} = 1200.98$) so we move to the three asset frontier US, China, Japan. This also generates an asymptotic TP in which China and the US are both short.. In either of the two asset cases (China, Japan) or (US,Japan) there is again an asymptotic TP and respectively China or the US are short in this two asset TP. So under a no short sales constraint, during the crisis period all country/safe rate combinations will specialise in mixing just the Japanese equity with the relevant safe asset. To derive the best overall portfolio between the safe asset and Japanese equity, we can note that the Japanese mean equity return ($-.023$) is dominated by each safe rate but also carries some risk. Thus under the constraint of no short sales, all investors during the crisis just hold cash.

Repeating the exercise post-crisis we find that the zero short sales constraint impacts on a wider range of investors with all combinations of country/safe rate holding only US and Chinese equities long and with zero holdings of the UK and Japan, except for savers and borrowers in China and borrowers in the UK who each specialise in only Chinese equity (Table 7 Asset holding with no short sales).

[7] Asset shares are estimated when $ER_P = 50$ and $\sigma = 129.8$.

Long assets	4		3		2		1		1
	Pre	Post	Pre	Post	Pre	Post	Pre	Post	During
UKs			UK,J,C			US, C			J
UKb					J,C			C	J
Cs					J,C			C	J
Cb							C	C	J
USs			UK,J,C	J,C		US, C			J
USb					US,C			C	J
Js			UK,J,C			US, C			J
Jb			UK,J,C			US, C			J

Table 7 Asset holding with no short sales

5.2 The supply of risky investment

We can measure the supply of risky investment through the share of equities, λ, in an investor's portfolio and the investable wealth available,w. If λw rises so does the supply of entrepeneurial finance. Investors all face the opportunities of the CML ($x = MPR * y + r_f$ where x is the mean and y the standard deviation of an investors overall portfolio) but differ in their preferences and initial wealth. In the appendix we use a general utility function to derive the link between λw and MPR, σ but we illustrate here with a linear mean/variance tradeoff.

If investor h has initial wealth w_h and utility $u^h(x_h, y_h^2) = x_h - k_h y_h^2 = w_h MPR y_h + w_h r_f - k_h w_h^2 y_h^2/2$, where k_h is the risk aversion parameter, the optimal standard deviation of the portfolio is set at $y_h = (MRP/k_h w_h)$. If the investor a share λ_h of wealth in the TP, then $y_h = \lambda_h \sigma_P$ (where σ_P is the standard deviation of the TP) giving $\lambda_h = MPR/(k_h w_h \sigma)$. The aggregate amount of risky asset investment E is

$$E = \Sigma_h \lambda_h w_h = \frac{ER_P - r_f}{k\sigma_P^2} \tag{6}$$

The key result from (6) is that when MPR increases and/or the standard deviation of the TP falls, the demand by financial investors for risky assets increases, and hence the supply of investment funds available to firms for risky investment should increase.

5.2.1 With short selling

Allowing short sales and knowing the risk free rate (saving/borrowing), we can compute the TP, market price of risk and σ_P. From this, we can calculate the optimal risky investment (6) for those four countries. as $E = \frac{ER_P - r_f}{k\sigma^2}$:

$$\begin{aligned}
All &: E_{during} = \frac{0.008}{k} \\
UKs &: E_{pre} = \frac{6.72}{k}, E_{post} = \frac{1.92}{k'} \\
Chinas &: E_{pre} = \frac{1.67}{k}, E_{post} = \frac{1.06}{k'} \\
USs &: E_{pre} = \frac{10.152}{k}, E_{post} = \frac{2.52}{k'} \\
Japans &: E_{pre} = \frac{21.96}{k}, E_{post} = \frac{4.56}{k'}
\end{aligned}$$

$$\begin{aligned}
All &: E_{during} = \frac{0.008}{k} \\
UKb &: E_{pre} = \frac{1.68}{k}, E_{post} = \frac{0.98}{k'} \\
Chinab &: \\
USb &: E_{pre} = \frac{3.892}{k}, E_{post} = \frac{1.43}{k'} \\
Japanb &: E_{pre} = \frac{18.47}{k}, E_{post} = \frac{4.07}{k'}
\end{aligned}$$

We have no data on any crisis induced change in risk aversion, but conditional on this, we find that, during the crisis, the values for optimal risky investment for all countries and for both savers and borrowers are extremely small and are approximately zero. However, these tiny values for optimal risky investment for both safe saving and borrowing rates increase post crisis. Especially for Japanese saving & borrowing rates, the optimal values increase enormously and are raised by more than 2000%. For the UK and US saving & borrowing rates and Chinese saving rate, the optimal values are increased significantly as well. Risk aversion would have had to have increased by implausibly large amounts to counteract the financial market effects.

We know that in the crisis in this case the TP is at infinity and there is high volume short selling of the UK. From the approximate σ of the asymptotic TP and MPR we can infer that during the crisis the supply of risky funds for equities was negligible $(\lambda = .008)$.

Post crisis we find that there is a positive demand for equities for a wide range of risk aversion of investors which should raise the supply of risky finance. However we find that in all countries the supply of risky finance is lower post crisis than pre crisis.

5.2.2 With no short selling

With no short sales the EF will typically consist of sections of frontiers corresponding to a reducing number assets, culminating in a point at which only one asset is held[8].

With no short selling, during the crisis,we know that investors will hold only the safe asset. In Fig 20, the EF is shifted from the solid line in pre crisis to the circle point during the crisis, to the black long dashed line in post crisis (the dotted horizontal line is the point at which US equity becomes zero precrisis, the solid horizontal line is the point at which Japan equity becomes zero postcrisis). During the crisis the TP σ is below the risk in the minimum variance portfolio post crisis So post crisis both the mean returns of equities and σ increase. Depending on the particular value of the safe rate, optimal risky investment choices (λ) will vary but for sure $\lambda > 0$.

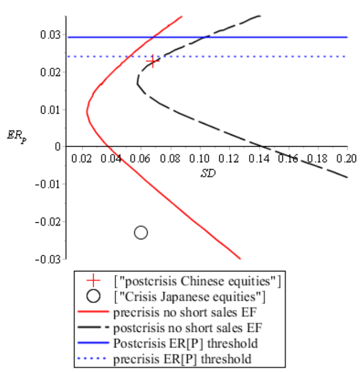

Fig 20 EF with no short sales

If no short sales is imposed during the crisis, equity holdings collapse to zero (the constraints imply any equity holding is specialised in just Japanese equity but the mean return on this is lower than on the safe asset).

6 Conclusions

The global financial crisis was relatively short lived (less than two years) but has had lasting real effects especially on some European economies, and also may have changed the risk return structure of financial assets around the world in a long run way. The transmission of the crisis/recession between countries had slightly different timings and different channels but the overall financial and real effects were very similar.

Having briefly derived some extensions to the CAPM approach that we need for our analysis (a convenient expression for the efficient frontier, an analysis of the theoretical impact of short sales constraints, the effect of an unbounded efficient frontier, the impact of differentials in safe borrowing and lending rates on tangent portfolios and the "capital market line"), we proceed to empirical analysis. Using monthly equity and safe

[8] Successive EFs may meet binding short sales constraints at different points.

rate data, all in $US, for June 2004-December 2010) we compute mean equity and safe saving and borrowing returns, and mean and covariances of equity returns from a one lag VAR for the three phases: pre-crisis, during the crisis and post crisis. From this we compute the efficient frontier for the three phases. During the crisis it shifts downward and to the right reflecting the fall in individual asset mean returns and increased risk, but also tilts so that there are more high risk-high mean opportunities (involving short equity sales) relatively to low risk-low mean opportunities. After the crisis it shifts upward again as mean equity returns rise but the level of aggregate equity risk remains above pre-crisis levels. Comparing the overall risk between the three phases by either the market price of risk or more fundamentally the eigenvalues of the covariance matrices, risk was highest during the crisis, fell post-crisis but was still higher than pre-crisis. The correlations between equity returns in each phase are all positive but they jump upwards and remain high during and after the crisis, suggesting that even more than before the crisis, diversification gains (if any exist) are to be found in short selling some equities but holding others long.

The implications of this are that pre-crisis, if short equity selling is possible, all investors in all countries short US equity. During the crisis the slump in equity returns radically reduces the net equity holdings by each investor group when the tangent portfolio is at infinity. If short selling equity is allowed, then investors will trade very high equity volumes typically being short in UK and long in the other three, but the net equity finance for entrepreneurs is negligible. If short selling is banned, then all investors in all countries shift into cash: the single dominant equity for Japan has a mean return below that on cash. This cannot be a CAPM equilibrium as there is excess supply of equities, so equity prices and hence returns will adjust.

The post crisis phase contains the effects of both this adjustment of returns and of the recovery policies undertaken by governments. Post crisis we find that there is a positive demand for equities for a wide range of risk aversion of investors which should raise the supply of risky finance. However we find that in each of the US, UK and Japan the supply of risky finance is lower post crisis than pre crisis, but that in China the supply is higher post crisis.

There are of caveats of course. We have used a world CAPM style model. Empirical work generally finds evidence of increasing but still imperfect integration and that often there is a role for regional effects as well as world effects. The world has more than four countries. In the way we have used it, CAPM depends basically on a one period investment horizon. It is not easy to identify suitable safe rates empirically. But it does give a simple applicable framework for analysing real and financial linkages and the ways in which policy can impact on them. It also highlights features like ways of measuring diversification possibilities giving a preference-free means of analysing efficient portfolios.

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A Extensions of CAPM

The Efficient Frontier

$$\sigma^2 = \min_a \{a'\Omega a |\ a'i = 1, a'm = ER_P\}$$

The first order conditions are (together with the constraints),

$$\Omega a = \lambda i + \delta m$$

Using the constraints to eliminate the multipliers λ, δ, gives

$$a = \frac{1}{i'\Omega^{-1}i}\Omega^{-1}i - \frac{(m'\Omega^{-1}i - ER_P i'\Omega^{-1}i)}{((i'\Omega^{-1}m)^2 - m'\Omega^{-1}mi'\Omega^{-1}i)}(\frac{i'\Omega^{-1}m}{i'\Omega^{-1}i}\Omega^{-1}i - \Omega^{-1}m)$$

Define

$$\begin{aligned} b(ER_P) &= \frac{(m'\Omega^{-1}i - ER_P i'\Omega^{-1}i)}{((i'\Omega^{-1}m)^2 - m'\Omega^{-1}mi'\Omega^{-1}i)} \\ x_1 &= \frac{1}{i'\Omega^{-1}i}\Omega^{-1}i, x_2 = (\frac{i'\Omega^{-1}m}{i'\Omega^{-1}i}\Omega^{-1}i - \Omega^{-1}m) \end{aligned}$$

Note that

$$\begin{aligned} x_1'\Omega x_2 &= \frac{i'\Omega^{-1}}{i'\Omega^{-1}i}\Omega[\frac{(i'\Omega^{-1}m)\Omega^{-1}i}{i'\Omega^{-1}i} - \Omega^{-1}m] \\ &= \frac{i'}{i'\Omega^{-1}i}[\frac{(i'\Omega^{-1}m)\Omega^{-1}i}{i'\Omega^{-1}i} - \Omega^{-1}m] \\ &= \frac{1}{i'\Omega^{-1}i}[\frac{(i'\Omega^{-1}m)i'\Omega^{-1}i}{i'\Omega^{-1}i} - i'\Omega^{-1}m] \\ &= 0 \end{aligned}$$

So

$$a = x_1 - b(ER_P)x_2$$

Note

$$m'x_1 = \frac{1}{i'\Omega^{-1}i}m'\Omega^{-1}i, m'x_2 = (\frac{i'\Omega^{-1}m}{i'\Omega^{-1}i}m'\Omega^{-1}i - m'\Omega^{-1}m) = (\frac{(i'\Omega^{-1}m)^2 - i'\Omega^{-1}im'\Omega^{-1}m}{i'\Omega^{-1}i})$$

and

$$\begin{aligned} b(ER_P) &= \frac{m'\Omega^{-1}i}{((i'\Omega^{-1}m)^2 - m'\Omega^{-1}mi'\Omega^{-1}i)} - \frac{ER_P i'\Omega^{-1}i}{((i'\Omega^{-1}m)^2 - m'\Omega^{-1}mi'\Omega^{-1}i)} \\ &= \frac{m'x_1}{m'x_2} - \frac{ER_P}{m'x_2} \end{aligned}$$

Hence

$$a = x_1 - (\frac{m'x_1 - ER_P}{m'x_2})x_2$$

Given ER_P the variance of a portfolio on the EF σ^2 is

$$\begin{aligned} \sigma^2 &= a'\Omega a \\ &= [x_1 - (\frac{m'x_1 - ER_P}{m'x_2})x_2]'\Omega[x_1 - (\frac{m'x_1 - ER_P}{m'x_2})x_2] \\ &= \left(x_1'\Omega x_1 + (\frac{m'x_1 - ER_P}{m'x_2})^2 x_2'\Omega x_2\right) \end{aligned}$$

and the efficient frontier has the equation

$$F(ER_P, \sigma) = \sigma - \left(x_1'\Omega x_1 + (\frac{m'x_1 - ER_P}{m'x_2})^2 x_2'\Omega x_2\right)^{1/2}$$

Tangent Portfolio

Locus of EF is $F(ER_P, \sigma) = 0$ where m is mean and s standard deviation of risky portfolio. The slope of the EF is $dER_P/d\sigma = (\partial F/\partial\sigma)/(\partial F/dER_P)$

The slope of the tangent through r_f to the EF is $(ER_P - \sigma)/\sigma$ at any point. This slope must equal the slope of the EF at the TP

$$\frac{\sigma}{ER_P - r_f} = -\frac{\partial F/\partial ER_P}{\partial F/d\sigma}$$

at the TP point ER_{PT}, σ_T From definition of EF

$$\frac{\sigma_T}{ER_{PT}-r_f} = \frac{\left(x_1^{'}\Omega x_1 + (\frac{m'x_1-ER_P}{m'x_2})^2 x_2^{'}\Omega x_2\right)^{1/2}}{ER_{PT}-r_f}$$

$$-\frac{\partial F/\partial ER_P}{\partial F/d\sigma} = -\frac{\partial F/\partial ER_P}{1} = -\left(x_1^{'}\Omega x_1 + (\frac{m'x_1-ER_{PT}}{m'x_2})^2 x_2^{'}\Omega x_2\right)^{-1/2}(\frac{m'x_1-ER_{PT}}{m'x_2})\frac{x_2^{'}\Omega x_2}{m'x_2}$$

Equating the 2 slopes

$$\left(x_1^{'}\Omega x_1 + (\frac{m'x_1-ER_{PT}}{m'x_2})^2 x_2^{'}\Omega x_2\right) = -(ER_{PT}-r_f)(\frac{m'x_1-ER_{PT}}{(m'x_2)^2})x_2^{'}\Omega x_2$$

This yields on rearranging

$$x_1^{'}\Omega x_1 \frac{(m'x_2)^2}{x_2^{'}\Omega x_2} + (m'x_1 - ER_P)[m'x_1 - r_f)] = 0$$

Solving analytically for ER_P

$$ER_P = m'x_1 + \frac{x_1^{'}\Omega x_1 (m'x_2)^2}{x_2^{'}\Omega x_2 (m'x_1 - r_f)}$$

Boundedness of EF and its slope

We know that at the minimum variance portfolio the slope of the EF is infinite, and that it is a concave function. The question is whether an interior TP exists and for this the boundedness of the frontier and its slope as $ER_P, \sigma \to \infty$ are important. The EF is given by

$$\sigma = \left(x_1^{'}\Omega x_1 + (m'x_1 - ER_P)^2 \frac{x_2^{'}\Omega x_2}{(m'x_2)^2}\right)^{1/2}$$

We know $x_1^{'}\Omega x_1 > 0$, $x_2^{'}\Omega x_2/(m'x_2)^2 > 0.m'x_1$ is of ambiguous sign. but still $(m'x_1 - ER_P)^2 > 0$ and tends to infinity with ER_P. Hence $\sigma \to \infty$ with ER_P. The EF is not bounded above.

$$\begin{aligned}\frac{d\sigma}{dER_p} &= -\left(x_1^{'}\Omega x_1 + (m'x_1 - ER_P)^2 \frac{x_2^{'}\Omega x_2}{(mx_2)^2}\right)^{-1/2}(m'x_1 - ER_P) \\ &= -\frac{m'x_1/ER_P - 1}{\left(\frac{x_1^{'}\Omega x_1}{ER_P^2} + (\frac{m'x_1}{ER_P} - 1)^2 \frac{x_2^{'}\Omega x_2}{(m'x_2)^2}\right)^{1/2}}\end{aligned}$$

Now the numerator $\to -1$ as $ER_P \to \infty$. The denominator tends $x_2^{'}\Omega x_2/(m'x_2)^2$. So as $ER_P \to \infty, \sigma \to (m'x_2)^2/(x_2^{'}\Omega x_2) > 0$ and finite. Hence the asymptotic tangent to the EF always exists.

Portfolios under no short sales constraints

Envelope structure of EF with varying numbers of assets

With 4 assets the EF is

$$\sigma_4 = F_4(ER_P) = \min_a\{a'\Omega a)|a'i = 1, m'a = ER_P\}$$

where a is 4x1. With only the first three assets the EF is

$$\sigma_3 = F_3(ER_P) = \min_a\{a'\Omega a)|a'i = 1, m'a = ER_P\}$$

Imposing one asset share is zero on $F_4(ER_P)$ yields $F_3(ER_P)$. Hence $F_4(ER_P) \leq F_3(ER_P)$ with equality only if at ER_P one asset optimally has a zero holding in the four asset EF. Hence the two frontiers touch at any point at which one asset share is zero. Since both are smooth convex functions, there must be an envelope relation between the two frontiers. Since all asset shares are monotonic in ER_P along the EF, either side of this point the asset in question must be surely long or short.

There cannot be one point on $F_4(ER_P)$ at which say $a_4 = 0$ and another point at which $a_3 = 0$. If there were the two corresponding three asset EF's would cross at a point below $F_4(ER_P)$ at which $a_3 = a_4 = 0$, with different slopes. But this crossing point must be on $F_2(ER_P^{'})$ which itself must be tangent to each of the three asset EF's, this is impossible.

B Investor Overall Portfolios

Investor h has utility $u^h(x_h, y_h^2)$ where x_h is the mean of the overall portfolio, y_h is the standard deviation of the overall portfolio (so y_h^2 is the variance). We assume that u^h is twice continuously .differentiable, increasing and concave in x and decreasing and convex in y_h^2. Thus (with subscripts for derivatives), $u_1^h > 0, u_{11}^h < 0, u_2^h < 0$ and $u_{22}^h > 0$. We also assume $u_{21}^h > 0$ so that the marginal utility of the mean return increases with uncertainty of the return.

The CML is $x = MPR * y + r_f$ where the investor can choose any point (x, y) on the CML and so, the investror solves

$$\max_{y_h} u(w_h(MPR * y_h + r_f), w_h^2 y_h^2$$

Under our assumptions for any finite MPR there is a unique solution $y^h(MPR, r_f)$.

$$\frac{dy^h}{dMPR} = -\frac{u_1^h[1+\varepsilon] - u_{11}^h w_h r_f + 2u_{21}^h w_h y^h}{MPR^2 w_h u_{11}^h + 4MPR w_h^2 u_{12}^h + 4y_h u_{22}^h + 2u_2^h y_h w_h^3} \tag{7}$$

where $\varepsilon = \frac{u_{11}^h}{u_1^h} w_h(MPRy_h + r_f)$.

The denominator is negative from the second order condition for a maximum so that y_h increases with MPR if the numerator is positive. This holds if the elasticity of the marginal utility of the mean portfolio return ε is greater than -1 and if $u_{21}^h > 0$[9] . If investor h has wealth w_h, his overall two fund portfolio is $P_h = w_h[(1-\lambda_h), \lambda_h a]$. Hence the standard deviation of his portfolio y_h is $\lambda_h \sigma$. The total amount of risky investment by h is then $w_h \lambda_h = w_h y_h(MPR, r_f)/\sigma$. For given σ, if $\lambda_h \sigma$ is increasing in MPR then the supply of risky funds to the market increases with MPR. If σ falls for given MPR, then the supply of risky funds to the market increases

Summary Statistics on the VAR

The VAR is a four equation system regressing each index on the first lag of each index and a constant. Typically the lagged returns in each regression are insignificant. R^2 and p values for the Breusch Pagan heteroscedasticity test (het), Ramsey reset test (Reset) and Breusch Godfrey autocorrelation test (AR) are given.

r^{uk}	Pre	During	Post	r^{china}	Pre	During	Post
R^2	.072	.22	.081	R^2	.091	.04	.075
Het	.75	.35	.79	Het	.003*	.25	.79
Reset	.71	.64	.74	Reset	.13	.41	.48
AR	.67	.36	.43	AR	.63	.02*	.85

r^{us}	Pre	During	Post	r^{japan}	Pre	During	Post
R^2	.083	.297	.04	R^2	.074	.187	.175
Het	.57	.28	.34	Het	.99	.21	.81
Reset	.98	.15	.55	Reset	.65	.02*	.15
AR	.95	.36	.57	AR	.83	.30	.02*

Institutional Investors[10]

With a time horizon of T, n risky one period assets with returns R_{it} (with mean returns μ_{it} and a covariance at t between risky returns of ω_{ijt}) and a single safe asset with return R_{Ft}, the typical investor wealth evolves according to

$$\begin{aligned} w_t &= w_{t-1}(a_{0t}R_{Ft} + \Sigma a_{it}R_{it}) = w_{t-1}(a_{0t}R_{Ft} + (1-a_{0t})\Sigma c_{it}R_{it}), \Sigma c_{it} = 1 \\ E_{t-1}w_t &= w_{t-1}(a_{0t}R_{Ft} + (1-a_{0t})\Sigma c_{it}E_t R_{it}) \\ var_t(w_t) &= (1-a_{0t})^2 w_{t-1}^2 var_t(\Sigma c_{it}R_{it}) = (1-a_{0t})^2 w_{t-1}^2 c_t' \Omega_t c_t \end{aligned}$$

Here a_{ot} is the share of wealth invested in cash and a_{it} the share invested in the ith risky asset at $t, a_{0t} + \Sigma a_{it} = 1$. c_{it} is the share of the overall risky portfolio invested in the ith risky asset, $\Sigma c_{it} = 1$.

The investor has preferences which are mean variance in the returns of each date

$$U_{0,T} = \Sigma_t \delta^t [E_t w_t - K var_t(w_t)$$

[9] This means that the marginal utility of mean return does not fall too fast.

[10] Full calculations of the solution are available on request.

Bellmans equation is

$$\begin{aligned} V_t(w_{t-1}) &= \max_{a_{0t},c_{it}} [w_{t-1}(a_{0t}R_{Ft} + (1-a_{0t})\Sigma c_{it}E_tR_{it}) - K(1-a_{0t})^2 w_{t-1}^2 c_t'\Omega_t c_t + \delta E_t V_{t+1}(w_t)] \\ \text{st } \Sigma c_{it} &= 1 \end{aligned}$$

or

$$V_t(w_{t-1}) = \max_{a_{0t}}[w_{t-1}(a_{0t}R_{Ft} + \max_{c_t}\{(1-a_{0t})\Sigma c_{it}E_tR_{it}) - K(1-a_{0t})^2 w_{t-1}^2 c_t'\Omega_t c_t + \delta E_t V_{t+1}(w_t)|\Sigma c_{it} = 1\}]$$

Detailed calculation shows that at T

$$\max_{a_{0T}}[w_{T-1}(a_{0T}R_{FT} + \max_{c_T}\{(1-a_{0T})\Sigma c_{iT}E_TR_{iT}) - K(1-a_{0T})^2 w_{T-1}^2 c_T'\Omega_T c_T|\Sigma c_{iT} = 1\}]$$

has the solution

$$\begin{aligned} c_T &= (2Kw_{T-1})^{-1}(1-a_{0T})^{-1}[\Omega_T^{-1}ER_T - \frac{i'\Omega_T^{-1}ER_T - 2Kw_{T-1}(1-a_{0T})}{i'\Omega_t^{-1}i}\Omega_T^{-1}i] \\ a_{0T} &= 1 - \frac{i'\Omega_T^{-1}ER_T - i'\Omega_T^{-1}iR_{FT}}{2Kw_{T-1}} \end{aligned}$$

from which

$$\begin{aligned} V_T(w_{T-1}) &= w_{T-1}R_{FT} - \frac{R_{FT}}{2K}[i'\Omega_T^{-1}ER_T - \frac{1}{2}i'\Omega_T^{-1}iR_{FT}] + 2^{-1}(2K)^{-1}ER_T'\Omega_T^{-1}ER_T \\ &= w_{T-1}R_{FT} + A_T \\ A_T &= 2^{-1}(2K)^{-1}ER_T'\Omega_T^{-1}ER_T - \frac{R_{FT}}{2K}[i'\Omega_T^{-1}ER_T - \frac{1}{2}i'\Omega_T^{-1}iR_{FT}] \end{aligned}$$

At a generic period t, suppose we conjecture $V_{t+1}(w_t) = B_{t+1}w_t + A_{t+1}$

$$\begin{aligned} V_t(w_{t-1}) &= \max_{a_{0t}}[w_{t-1}(a_{0t}R_{Ft} + \max_{c_t}\{(1-a_{0t})\Sigma c_{it}E_tR_{it}) - K(1-a_{0t})^2 w_{t-1}^2 c_t'\Omega_t c_t + \delta E_t V_{t+1}(w_t)|\Sigma c_{it} = 1\}] \\ \text{where } V_{t+1}(w_t) &= B_{t+1}w_t + A_{t+1}, w_t = w_{t-1}[a_{0t}R_{Ft} + (1-a_{0t})\Sigma c_{it}R_{it}] \end{aligned}$$

$$\begin{aligned} &\max_{c_t}\{w_{t-1}(1-a_{0t})\Sigma c_{it}E_{t-1}R_{it}) - K(1-a_{0t})^2 w_{t-1}^2 c_t'\Omega_t c_t \\ &+\delta B_{t+1}(w_{t-1}(a_{0t}R_{Ft} + (1-a_{0t})\Sigma c_{it}E_{t-1}R_{it}))\} \end{aligned} \tag{8}$$

First choosing the optimal risky asset shares a lot of routine calculation yields

$$\begin{aligned} c_t = &(2K(1-a_{0t})w_{t-1})^{-1} \\ &[(1+\delta B_{t+1})\Omega_t^{-1}E_{t-1}R_t - \frac{(1+\delta B_{t+1})i'\Omega_t^{-1}E_{t-1}R_t - (2K(1-a_{0t})w_{t-1})}{i'\Omega_t^{-1}i}\Omega_t^{-1}i] \end{aligned}$$

To eliminate the optimal $c's$ from the value function it is convenient to derive the expected payoff from risky investments (again after a lot of routine calculation). The optimal payoff is

$$\begin{aligned} &w_{t-1}(1-a_{0t})[(1+\delta B_{t+1})\Sigma c_iE_{t-1}R_{it} - K(1-a_{0t})w_{t-1}c'\Omega_t c] \\ = &K^{-1}[(1+\delta B_{t+1})^2 E_{t-1}R_t'\Omega_t^{-1}E_{t-1}R_t - \frac{\{(1+\delta B_{t+1})i'\Omega_t^{-1}E_{t-1}R_t - 2K(1-a_0)w_{t-1}\}^2}{i'\Omega_t^{-1}i}] \end{aligned}$$

and the value function (conditional on safe asset investment) is

$$\begin{aligned} V_t(w_{t-1}) = &\max_{a_{0t}}[w_{t-1}a_{0t}(1+\delta B_{t+1})R_{Ft} \\ &+K^{-1}[(1+\delta B_{t+1})^2 E_{t-1}R_t'\Omega_t^{-1}E_{t-1}R_t - \frac{\{(1+\delta B_{t+1})i'\Omega_t^{-1}E_{t-1}R_t - 2K(1-a_0)w_{t-1}\}^2}{i'\Omega_t^{-1}i}] + \delta E_{t-1}A_{t+1} \end{aligned}$$

Choosing the risk free investment share, a_{0t}

$$R_{Ft}(1+\delta B_{t+1})i'\Omega_t^{-1}i = 2\{(1+\delta B_{t+1})i'\Omega_t^{-1}E_{t-1}R_t - 2K(1-a_{0t})w_{t-1}\}$$

$$a_{0t} = \frac{4Kw_{t-1} - 2(1+\delta B_{t+1})i'\Omega^{-1}ER + R_{Ft}(1+\delta B_{t+1})i'\Omega_t^{-1}i}{4Kw_{t-1}}$$

Substituting back into V_t

$$\begin{aligned}
V_t(w_{t-1}) &= w_{t-1}\frac{4Kw_{t-1} - 2(1+\delta B_{t+1})i'\Omega^{-1}ER + R_{Ft}(1+\delta B_{t+1})i'\Omega_t^{-1}i}{4Kw_{t-1}}(1+\delta B_{t+1})R_{Ft} \\
&\quad +K^{-1}[(1+\delta B_{t+1})^2 E_{t-1}R_t^{'}\Omega_t^{-1}E_{t-1}R_t - \frac{\{R_{Ft}(1+\delta B_{t+1})i'\Omega_t^{-1}i\}^2}{4i'\Omega_t^{-1}i}] \\
&\quad +\delta E_{t-1}A_{t+1} \\
&= (1+\delta B_{t+1})R_{Ft}w_{t-1} + K^{-1}[(1+\delta B_{t+1})^2 E_{t-1}R_t^{'}\Omega_t^{-1}E_{t-1}R_t - \frac{\{R_{Ft}(1+\delta B_{t+1})i'\Omega_t^{-1}i\}^2}{4i'\Omega_t^{-1}i}] \\
&\quad -\frac{2(1+\delta B_{t+1})i'\Omega^{-1}ER - R_{Ft}(1+\delta B_{t+1})i'\Omega_t^{-1}i}{4}(1+\delta B_{t+1})R_{Ft} + \delta E_{t-1}A_{t+1} \\
&= (1+\delta B_{t+1})R_{Ft}w_{t-1} + K^{-1}(1+\delta B_{t+1})^2[E_{t-1}R_t^{'}\Omega_t^{-1}E_{t-1}R_t - \frac{R_{Ft}^2 i'\Omega_t^{-1}i}{4}] \\
&\quad -\frac{2i'\Omega^{-1}E_{t-1}R_t - R_{Ft}i'\Omega_t^{-1}i}{4}(1+\delta B_{t+1})^2 R_{Ft} + \delta E_{t-1}A_{t+1} \\
&= B_t w_{t-1} + A_t
\end{aligned}$$

This gives the general recursion:

$$\begin{aligned}
B_t &= (1+\delta B_{t+1})R_{Ft} \text{ with } B_T = R_{FT} \\
A_t &= K^{-1}(1+\delta B_{t+1})^2[E_{t-1}R_t^{'}\Omega_t^{-1}E_{t-1}R_t - \frac{R_{Ft}^2 i'\Omega_t^{-1}i}{4}] \\
&\quad -\frac{2i'\Omega^{-1}E_{t-1}R_t - R_{Ft}i'\Omega_t^{-1}i}{4}(1+\delta B_{t+1})^2 R_{Ft} + \delta E_{t-1}A_{t+1} \\
\text{with } A_T &= -\frac{R_{FT}}{2K}[i'\Omega_T^{-1}ER_T - \frac{1}{2}i'\Omega_T^{-1}iR_{FT}] + 2^{-1}(2K)^{-1}ER_T'\Omega_T^{-1}ER_T
\end{aligned}$$

since at T

$$\begin{aligned}
V_T(w_{T-1}) &= w_{T-1}R_{FT} - \frac{R_{FT}}{2K}[i'\Omega_T^{-1}ER_T - \frac{1}{2}i'\Omega_T^{-1}iR_{FT}] + 2^{-1}(2K)^{-1}ER_T'\Omega_T^{-1}ER_T \\
&= w_{T-1}R_{FT} + A_T
\end{aligned}$$

The general solution for the slope of the value function has the form

$$B_t = \Sigma_{s=0}^{T-t}\delta^s \Pi_{k=0}^{s} R_{Fk+t}$$

which is like the discounter $T-t$ holding period safe return. The important point is that it is independent of the moments of current or future risky asset returns.