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**To defer or not defer? State Pension in a  
Lifecycle Model**

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# To defer or not defer? State Pension in a Lifecycle Model.

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## Abstract

The UK state pension (which depends only on age) includes an option to defer take up which yields either a subsequent lump sum or higher weekly pension. We analyse the joint decisions on pension deferral and intertemporal labour supply/participation in a life cycle setting. We show that deferral is purely a financial decision, but the impact of deferral on work decisions depends on preferences, wage rates, non-labour income and initial wealth. To exactly characterise this we use a quasilinear utility function, and provide calibrated simulations. We also discuss the choice between a lump sum or increased weekly pension.

JEL classification: J14, J18, J22 & J26.

Key words: Retirement, Labour Supply, Ageing, UK State Pension.

## 1 Introduction.

Aging populations and longevity raise issues of labour participation, savings and pensions especially amongst the elderly. These are also important issues for government fiscal balance since tax receipts, state pensions and work conditional benefits obviously vary with labour and capital incomes. The aim of state pension systems is to alleviate poverty in old age and in this sense it is an open ended government commitment. Governments respond by encouraging later retirement and/or raising the age of eligibility for receipt of a state pension. Eligibility for receipt of a UK state pension only depends on age, although the amount received depends on lifetime work and tax (national insurance contribution) history, in particular it is independent of current employment status.

The purpose of State Pensions can be thought of as two fold: (i) to avoid poverty in old age (Beveridges 1948 original aim) and (ii) to ensure a specified income replacement ratio for all eligible retired individuals.<sup>1</sup> The UK defines a statutory State Retirement Age (SRA) which serves two purposes: working individuals must pay National Insurance (NI) contributions at a % rate of their earnings until this age; it is also the age at which an individual first becomes

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<sup>1</sup>The description presented here draws from Bozio et al (2010).

eligible for receipt of a weekly state pension. The amount of the pension depends generally on past national insurance contributions (and so past earnings) although there is a guaranteed minimum state pension. As of April 2010 any man or woman who reaches SRA on or after 6<sup>th</sup> April 2010, is eligible to receive a full state pension if they have made 30 years of NI contributions, which replaces around 15% of the average labour income in 2008 (Coleman et al., 2008).

The 2013 White Paper introduced an entire overhaul of social security through the introduction of a flat rate pension from 2017, and for new retirees (post 2017) requires individuals to make 35 years of NI contributions. As with the existing rules, individuals who make less contributions will see an equivalent reduction in their state pension, whilst those individuals who make less than 10 years contributions will not receive any state pension.<sup>2</sup> The first date of eligibility for receipt of state pension will still only depend on age and will be independent of current or future employment status. Until recently the SRA had been 60 years for women and 65 for men but since 2012 there have been plans to bring the two closer together, by 2018 the female retirement age will be 65, equal to that of males and by 2020 the SRA for both men and women will be 66.

Since its inception in 1948 individuals who are eligible to claim have had the option to defer receipt of state pension, in exchange for an increased weekly pension when they do subsequently decide to claim.<sup>3</sup> Initially upon undeferral individuals could claim a higher weekly income for their retirement period, however since 2005 individuals can instead claim a lump sum on their missed weekly payments (which earns interest above the Bank of England base rate) and then continue to receive their usual weekly payment. Since April 2010 the government has committed the State Pension (SP) to a triple lock indexing policy, in doing so State Pensions are uprated in line with whichever is highest of: (1) September-September Consumer Price Index (2) average earnings or (3) 2.5%. For those who defer their pension, at the date of undeferral the rate of return earned in the lump sum option means that past indexed increases are accumulated in the lump sum. In addition all additional flows of the basic weekly SP are uprated each year. On the other hand under the deferred income option at the undeferral date, indexing only applies to the initial amount of the SP the individual was due to receive before deferring. The additional income earned per week has not been uprated since April 2010 (Thurley 2010).<sup>4</sup>

The possibility of State Pension deferral has implications for the planned savings and work pattern of individuals through changing their lifetime pattern of nonlabour income. What implications will deferral have for their work and savings patterns? Can deferral induce individuals to stay on longer in paid

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<sup>2</sup>Bozio et al. (2010) pp 13. For men before April 1945 and females born before 1950, UK State Pension legislation requires these individuals to make the equivalent of 44 years and 39 years of full contributions respectively, in order to be eligible for the maximum State Weekly Pension. For these individuals men must make at least 11 years of contributions and females 10 years, in order to be eligible for any state pension at all.

<sup>3</sup>This is a one shot choice and an individual can only defer their pension once. The length of defer initially had an upper bound however since 2005 this has been removed.

<sup>4</sup>This holds true since April 2010, prior to this, the additional income earned in the deferred income option was also uprated annually under the same rules as the BSP.

work? Disney and Smith (2002) formally analyse the effect of the abolition of the Earnings Rule (which effectively placed a very high marginal tax rate on individuals who wanted to claim their pension and continue working), and as a side issue also consider pension deferral. Their findings suggest that after abolition, male weekly hours (above SPA) rose by approximately 4 hours, whilst for women it rose by 2 hours. Disney and Smith (2002) do not explicitly consider the effects of pension deferral on labour supply. Farrar et al (2012) compare the two deferral options available under current State Pension legislation and conclude under most simulations that the incremental option (additional weekly state pension) generally tended to more lucrative.

Here we formally analyse the joint deferral and intertemporal labour supply and participation decisions in a life cycle setting. We find the deferral decision is independent of preferences, wage rates or wealth. It is a purely financial decision: choose to defer if it raises the present value of non-labour income. However the effect of deferral on intertemporal labour supply does depend on preferences, wage rates and wealth. In a general model we sketch the qualitative effects but to get analytical and empirically applicable results, we then specify preferences. After deriving analytical expression for the effects on reservation wages for different intertemporal labour participation patterns, we calibrate these to compute the size of the impacts. The present deferral scheme gives about a 2% increase in the reservation wage for full time work for 12 months of deferral. If an individual does defer, under the present system he can take the later rewards as either a lump sum or as an increase in the weekly payment. We analyse the choice between these, examining the effects of life expectancy/length of deferral and of interest rates.

In section 2 we lay out a general framework which encompasses the effects of pension deferral on optimal labour supply through the role of the present value of non labour income. In section 3 we show the effects of regime switches on the optimal labour supply, using a form of preferences used widely in the literature. Section 4 compares the two deferral options available under current UK State Pension legislation. Section 5 concludes.

## 2 The model.

With perfect capital markets and in a world of certainty, financial wealth can be transferred intertemporally by the consumer. So one would expect that the benefits of deferring a state pension will depend only on a comparison between the implicit interest rate used in the government set terms of deferral and the market interest rate. This is because individuals will only defer if it raises their disposable wealth at the date of deferral, through raising the present value of nonlabour income in the form of pension receipts. For individuals who defer we would expect optimal adjustment in consumption  $c$ , and leisure  $L$  as they intertemporally smooth the marginal utility of consumption. There will be wealth effects on present and future labour supply and consumption. Disney & Smith (2002) point out that there may be labour participation effects of

changes in the pension rules, or more specifically in the implicit wage income an individual can earn in the absence of an earnings rule. If we add uncertainty about other future income sources and especially about the remaining length of life, the decision to defer or not is much less clear. Similarly individuals who face borrowing constraints are less likely to defer when they have the opportunity.

To see how a decision to defer impacts on current and future labour supply as individual leisure preferences and wage rates vary needs a formal framework. We present this next. Individuals maximise a per period time additive concave utility function which depends on a single consumption good,  $c$ , and leisure  $L$  subject to their lifetime budget constraint:

$$\max_{c_{T-1}, c_T, L_{T-1}, L_T} u(c_{T-1}, L_{T-1}) + \delta u(c_T, L_T) \quad (1)$$

$$\text{st } rc_{T-1} + c_T = rA_{T-1} + ry_{T-1} + y_T + rw_{T-1}(1 - L_{T-1}) + w_T(1 - L_T) = x \quad (2)$$

$$0 \leq L_t \leq 1$$

Here  $r$  is the real interest factor,  $A_{T-1}$  is financial assets at the start of the penultimate period,  $y_{T-1}, y_T, w_{T-1}, w_T, L_{T-1}$  and  $L_T$  denote non labour income, wages and leisure respectively in periods  $T - 1$  and  $T$ . There is a fixed time endowment each period of one unit of time which can be used either for leisure or work. Nonlabour income includes any pension that is actually received in that period and so depends on the deferral decision.

Should an individual defer their pension from  $T - 1$  to  $T$ ? This depends on the present value of the stream of pension payments over the two periods with and without deferral. The individual will choose the option which has the higher present value. The pension flow available at  $T - 1$  is  $p$  per period. Thus if the individual has non-pension, non-labour income of  $y_{T-1}^0, y_T^0$  then without deferral they receive  $y_{T-1} = y_{T-1}^0 + p, y_T = y_T^0 + p$ . With deferral they receive  $y_{T-1} = y_{T-1}^0, y_T = y_T^0 + r_g p$  where  $r_g$  is the implicit interest rate set by the government in the terms of deferral.

If there is no uncertainty and no restrictions on borrowing or lending except that individuals cannot die in debt, only the present value of nonlabour income affects the maximum value of life cycle utility, and optimal labour market decisions depend only on nonlabour income through its present value. The implicit interest rate factor  $r_g$  is common to all individuals so variation amongst individuals in the decision to defer must be due to variation in the market interest rate available to individuals, and more generally to variation in borrowing constraints or other capital market imperfections, or to omitted issues like uncertainty over the length of life. The deferral decision only impacts achievable life cycle utility through affecting the present value of wealth  $x$  available from  $T - 1$  onwards. Deferral will be chosen iff it raises  $x = ry_{T-1} + y_T$ . Without deferral  $x = r(y_{T-1}^0 + p) + y_T^0 + p$  while with deferral  $x = ry_{T-1}^0 + y_T^0 + r_g p$ . The individual is better off deferring iff  $(1 + r) < r_g$ .

For an individual who does decide to defer, his life cycle wealth increases from the date of deferral. To explore the effects of this on intertemporal labour

supply and consumption we have to go further with solving the maximisation problem. Consumption each period must be interior:<sup>5</sup>

$$\begin{aligned} \frac{\partial u_{T-1}}{\partial c_{T-1}} &= r\delta \frac{\partial u_T}{\partial c_T} \\ rc_{T-1} + c_T &= x \end{aligned} \quad (3)$$

For fixed values of  $L_{T-1}, L_T$  this gives a semi-indirect utility  $v(L_{T-1}, L_T, x)$  which is increasing in all its arguments and also concave in the leisures of each period (see appendix A). The remaining problem for the individual is to choose optimal labour supply in each period:

$$\max_{L_T, L_{T-1}} v(L_{T-1}, L_T, x) \text{ st } 0 \leq L_i \leq 1$$

Our main focus is on the interaction between labour participation decisions, saving and pension deferral so we focus on just full time and zero work options for each time period.<sup>6</sup> There are four possible configurations of labour participation over the final two periods of life: full time work in both periods, zero work in both periods or full time work in one period and zero work in the other.

Define the life cycle full incomes at the start of  $T - 1$  corresponding to each lifetime pattern of labour participation (the subscripts refer to the amount of leisure in each period so e.g. 01 corresponds to full time work at  $T - 1$  but zero work at  $T$ ):

$$\begin{aligned} X_{11} &= rA_{T-1} + ry_{T-1} + y_T = Z \\ X_{00} &= rA_{T-1} + ry_{T-1} + y_T + rw_{T-1} + w_T = Z + rw_{T-1} + w_T \\ X_{01} &= rA_{T-1} + ry_{T-1} + y_T + rw_{T-1} = Z + rw_{T-1} \\ X_{10} &= rA_{T-1} + ry_{T-1} + y_T + w_T = Z + w_T \end{aligned}$$

We have a ranking of the full incomes  $X_{00} > X_{01} > X_{11}, X_{00} > X_{10} > X_{11}$ .

The possible payoffs corresponding to these labour participation patterns are then  $v(1, 1, X_{11}), v(0, 1, X_{01}), v(1, 0, X_{10})$  and  $v(0, 0, X_{00})$ . Note that if  $v(1, 1, X_{11}) > v(0, 1, X_{01}), v(1, 0, X_{10})$  then  $v(1, 1, X_{11}) > v(0, 0, X_{00})$  from the monotonicity of  $v()$  in all its arguments.

The only differences in the full incomes between participation patterns are in the value of the time endowment which arises in periods of work and depends on the wages of those periods. A suitable idea of the time profile of reservation wages between any two alternative profiles of labour participation is a pair

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<sup>5</sup> Assuming that the marginal utility of consumption in any period becomes arbitrarily high as consumption in that period becomes very small

<sup>6</sup> If we included interior solutions for labour participation there would be 9 configurations. The way of getting the "reservation wages" above would be similar eg suppose  $0 < L_{T-1} < 1$  and  $L_T = 0$ . Let  $L_{T-1}^*$  solve

$$\frac{dv(L_{T-1}^*, 0, x)}{dL_{T-1}} = 0 \text{ and then require } \frac{dv(L_{T-1}^*, 0, x)}{dL_T} < 0$$

$w_{T-1}, w_T$  giving indifference between the two patterns of labour participation. So with  $Z = rA_{T-1} + ry_{T-1} + y_T$ , we can define:

$$\begin{aligned}
V_{11} &= v(1, 1, Z) = v(1, 0, Z + w_T^{11,10}) = V_{10} \\
V_{01} &= v(0, 1, Z + w_{T-1}^{01,10}) = v(1, 0, Z + w_T^{01,10}) = V_{10} \\
V_{11} &= v(1, 1, Z) = v(0, 0, Z + rw_{T-1}^{00,10} + w_T^{00,10}) = V_{00} \\
V_{11} &= v(1, 1, Z) = v(1, 0, Z + w_{T-1}^{11,01}) = V_{01} \\
&\Rightarrow v(0, 0, Z + rw_{T-1} + w_{10}^{11}) < v(1, 0, Z + w_{10}^{11})
\end{aligned}$$

In general there may not exist finite positive wages ensuring these indifferences. But the general pattern of how life cycle labour participation is determined is clear. For the pattern  $ij$  to be optimal (i.e. participation state  $i$  in period  $T - 1$  and  $j$  in  $T$ ) to be optimal we require that  $V_{ij} > V_{kl}$  for each other possible participation profile  $kl$ . How the optimal participation profile varies with  $Z$  and current wages depends on the form of the utility. There are some basic results just from monotonicity of  $v()$  in its arguments. Thus if  $V_{11} = V_{10}$  then  $V_{00} < V_{10}$ . In general for a given  $Z$  and utility function, there will be a region of high wages in both periods where it is optimal to work full time in both periods (corresponding to  $V_{00} > V_{10}, V_{01}, V_{11}$ ). Similarly there will be a region of low wages in both periods where it is not optimal to work in either period (corresponding to  $V_{11} > V_{10}, V_{01}, V_{00}$ ). And finally there will be two regions: one with high wages in  $T - 1$  but low wages in  $T$  (corresponding to  $V_{01} > V_{10}, V_{00}, V_{11}$ ), where it is optimal to work full time in  $T - 1$  but not work at all at  $T$ , and conversely a region of high wages at  $T$  but low wages at  $T - 1$  where it is optimal to stay out of the labour market at  $T - 1$  but work full time at  $T$  (corresponding to  $V_{10} > V_{00}, V_{01}, V_{11}$ ). With given preferences,  $Z$  and wage rates of each period, the optimal profile of labour participation over the two periods is determined.

How will introduction of the deferral option affect the optimal participation profile? Deferral is only taken up if it raises the present value of nonlabour income including the pension stream. This change in wealth changes the demand for leisure in each period. If leisure is a normal good, an increase in wealth increases the demand for leisure in each period. So we would generally expect a drop in work hours in each period when an individual prefers to defer. If an individual was planning full time work in each period in the absence of deferral but chooses to defer, then if their wage rates were close to the reservation wage in one of the periods (as computed above), with deferral his optimal profile may switch into zero work in that period. Disney and Smith (2002) consider the effects of relaxation of the earnings rule on labour supply participation of older workers in the UK. Their empirical results indicate that increasing generosity of work incentives, such as reducing the marginal tax rate on earnings for older workers increases the number of hours worked.<sup>7</sup> This suggests strong income

<sup>7</sup>This may not hold true for all workers depending on whether their income is above or below the earnings rule threshold.



effects are at work, whereas in our model deferral has a direct wealth effect and under standard assumptions would act to increase the amount of leisure consumed.

To see the impact of pension deferral on life cycle labour force participation we need to know more about the wage regions corresponding to different labour participation patterns and how these vary with  $Z$ . To determine this we have to resort to a specification of preferences which allows us to explicitly compute the labour participation areas and the ways in which they vary with  $Z$ . From this we can predict which parts of the intertemporal wage rate distribution will lead to a switch to zero hours of work in either or both of periods  $T-1, T$  on introduction of the pension deferral option. We can then also see how deferral will impact on consumption and savings in different parts of the wage rate distribution.

### 3 Quasilinear utility.

In this section we take a commonly used specification for the utility function (Gustman and Steinmeier (2010), Blau (2012)), in which consumption,  $c$ , is isoelastic and labour,  $L$ , is quasilinear. We derive optimal saving and labour supply regimes in each case. We find the channels through which pension deferral affects optimal labour supply. In this specification, remaining lifetime preferences are given by

$$u(c_{T-1}, L_{T-1}) + \delta u(c_T, L_T) = \frac{C_{T-1}^\alpha}{\alpha} + h_{T-1}L_{T-1} + \delta\left(\frac{C_T^\alpha}{\alpha} + h_T L_T\right) \quad (4)$$

In appendix B we derive savings function  $A_T$  as

$$A_T = \frac{x_{T-1} - (\delta r)^{1/(\alpha-1)}(y_T + w_T(1 - L_T))}{1 + r(\delta r)^{1/(\alpha-1)}}$$

where  $x_{T-1} = rA_{T-1} + y_{T-1} + w_{T-1}(1 - L_{T-1})$ .

The resulting value function is

$$v(K, w_{T-1}, w_T) = \frac{(K + rw_{T-1}(1 - L_{T-1})) + w_T(1 - L_T))^\alpha}{\alpha} D + h_{T-1}L_{T-1} + \delta h_T L_T$$

where

$$K = r(rA_{T-1} + y_{T-1}) + y_T, D = ((\delta r)^{\alpha/(\alpha-1)} + \delta)$$

The value function  $v$ , is isoelastic in disposable wealth at  $T-1$  and linear in present and future leisures. Quasilinearity in leisure given the wealth effect of pension deferral, means that the income effects fall solely on participation.

The maximal utilities obtained from the life cycle labour force regime (define by the subscript notation) are defined as

$$V_{00}(K, w_{T-1}, w_T) = \frac{(K + rw_{T-1} + w_T)^\alpha}{\alpha} D$$

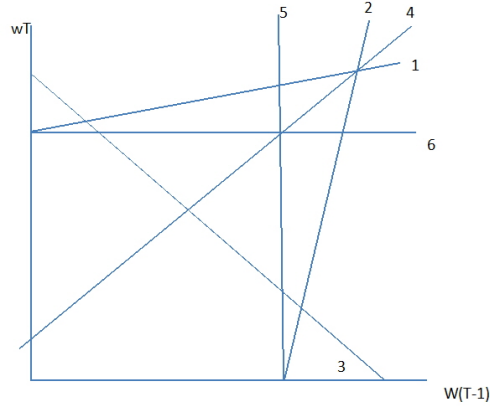
$$\begin{aligned}
V_{01}(K, w_{T-1}) &= \frac{(K + rw_{T-1})^\alpha}{\alpha} D + \delta h_T \\
V_{10}(K, w_T) &= \frac{(K + w_T)^\alpha}{\alpha} D + h_{T-1} \\
V_{11}(K) &= \frac{K^\alpha}{\alpha} D + h_{T-1} + \delta h_T
\end{aligned}$$

This allows us to define six combinations of wages  $w_{T-1}^i, w_T^i$   $i = 1..6$  which give indifference between pairs of maximal utility levels

- (1)  $V_{00}(K, w_{T-1}^1, w_T) = V_{01}(K, w_{T-1}^1)$
- (2)  $V_{00}(K, w_{T-1}^2, w_T^2) = V_{10}(K, w_T^2)$
- (3)  $V_{00}(K, w_{T-1}^3, w_T^3) = V_{11}(K)$
- (4)  $V_{01}(K, w_{T-1}^4) = V_{10}(K, w_T^4)$
- (5)  $V_{01}(K, w_{T-1}^5) = V_{11}(K)$
- (6)  $V_{10}(K, w_T^6) = V_{11}(K)$

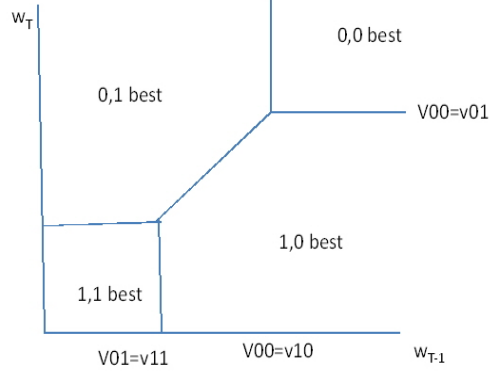
Using the detailed expressions for the various value functions appendix B shows that the critical wage combinations are related as depicted in figure 1 below, and that all the intersections of regions exist at finite positive wages. Each of the lines labeled in figure 1 correspond to the reservation wages  $w_{T-1}^i, w_T^i$   $i = 1..6$  giving indifference between pairs of maximal utility levels.

Figure 1: Indifferences between participation profiles



Using monotonicity of the value function expressions in the wage rates, we can deduce regions of the wage space in which different intertemporal labour participation patterns are optimal as shown in figure 2 below. The boundaries between the regions in figure 2 correspond to the relevant parts of the lines in Figure 1: 1, 2 giving lower bounds on full time work, 5, 6 giving upper bounds on the zero work region and 4 giving the division between working either just in  $T - 1$  or in  $T$ .

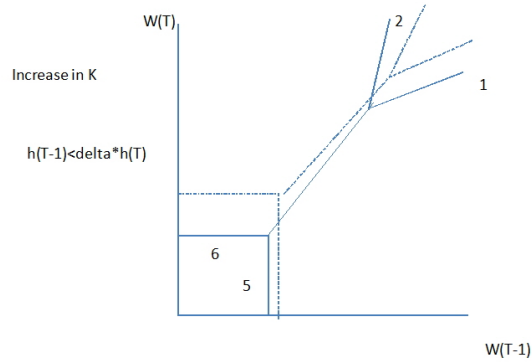
Figure 2: Optimal lifecycle participation profiles.



### 3.1 The effect of pension deferral on labour force participation.

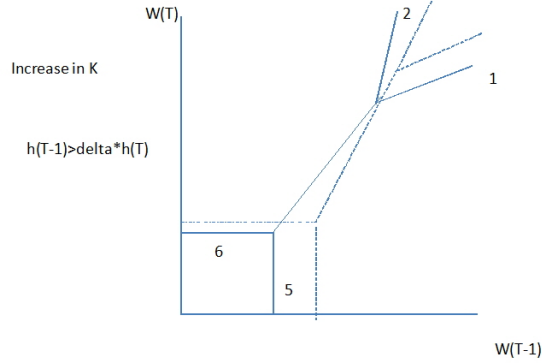
To examine the impact of pension deferral which raises the present value of non-labour income on life cycle labour participation, we show how figure 2 changes with  $K$ . Appendix C shows that the effect on the optimal labour participation profile of an increase in  $K$  depends on whether the utility value of leisure is higher in  $T - 1$  or  $T$ . In both cases the wage region with zero work in both periods expands and that with full time work in both periods contracts. But if the value of leisure is higher in period  $T$  than  $T - 1$ , the wage region with full time work only at  $T - 1$  expands at the expense of the wage region with full time work only in  $T$  (as in figure 3 below). Conversely if the value of leisure is higher in  $T - 1$  than  $T$ , the wage region with full time work at  $T$  expands at the expense of the wage region with full time work only in  $T - 1$  (as in figure 4 below).

Figure 3: Increase in non labour income  $h_{T-1} < \delta h_T$ .



If the option to defer is suddenly introduced or taken up, or is made more

Figure 4: Increase in non labour income  $h_{T-1} > \delta h_T$ .



generous, the present value of non-labour income increases. We can deduce the likely effects on life cycle participation profiles. If leisure is more valuable in period  $T - 1$ , the increase in  $K$  will tend to reduce full time work in  $T - 1$ . A proportion of those individuals who were working full time in both periods may switch to only working in period  $T$  and some of those who previously only worked in  $T - 1$  may switch to only working in  $T$ . But some who previously only worked in  $T$  may switch into inactivity in both periods. Thus with leisure more valuable in  $T - 1$ , the increase in the value of the deferred pension unambiguously reduces the number of full time workers in  $T - 1$ , but may raise or lower it in period  $T$ . If the value of leisure is higher in period  $T$ , the opposite effects occur: the number of full time workers in  $T$  unambiguously falls while the number of full time workers in  $T - 1$  may fall or rise depending on the distribution of the life cycle wages  $w_{T-1}, w_t$  in the population.

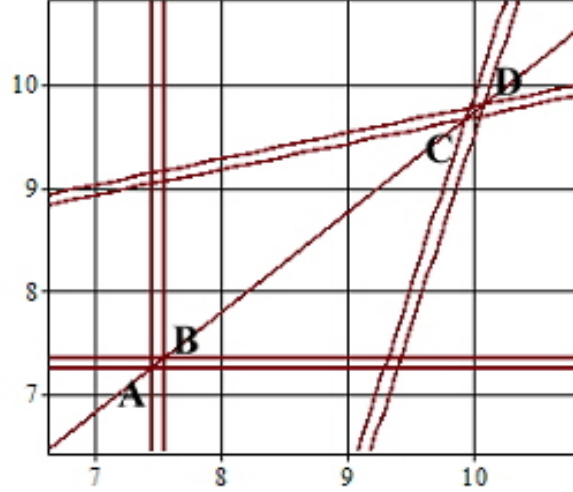
The wealth change caused by deferral has participation effects on individuals close to the reservation wage in one period at least. However labour force participation is unaffected by the presence of pension deferral for those who earn sufficiently above the relevant critical wage defining full time work. In the next subsection we simulate the effect of pension deferral implied by our framework.

### 3.1.1 Stylised simulation: defer or not defer?

Having considered the theoretical effects of pension deferral within our framework, we turn to a numerical simulation. In order to calibrate our model we use a mixture of assumed parameters available in the literature and those inferred from secondary data. Following the work of Attanasio et al. (2008) we set the relative risk parameter  $\alpha$  to  $-0.5$ , we assume annual (non housing) wealth holding of £1500, weekly total non labour income (for example the total of private and state pension) of £350 under no deferral and £365 under deferral. We assume individuals work 40 hours per week, can earn an annual rate of return of 3% in the free market and have a discount rate of 0.95. We set the marginal value of leisure in the penultimate period and terminal period of 0.006 and

0.00630 respectively (an increase of 5%).<sup>8</sup> In doing so we replicate the effects of figure 3 more clearly i.e. assuming  $h_{T-1} < \delta h_T$  as shown figure 5 below:

Figure 5: Wage co-ordinates defining zero and full time work.



By deferring one period the required wages to be in a given labour supply regime increase in each period. In the case of zero work this is shown by the curves shifting from point *A* to point *B*, whereas for full time work the corresponding loci shifts from point *C* to point *D*. Under the no deferral option the wage rate required to be in zero work at  $T - 1, T$  is (£7.25, £7.45) respectively, whilst under deferral it rises to (£7.34, £7.54). Similarly for full time work at  $T - 1, T$  the corresponding wage rates are (£11.75, £10.95), under deferral these increase to (£12.02, £11.21). The effect of pension deferral therefore raises the full time reservation wage by around 1.5% assuming the above parameters. In various simulations the average rise in the full time reservation wage for a 12 month deferral is around 2%, the particular example given has a zero work reservation wage close to NMW. Implicitly in the calibration the change in reservation wage on deferral is for deferral lasting one year. As the period of deferral increases beyond this, the reservation wage difference will rise. The relative slopes of the participation regime boundaries and their shifts principally depends on the difference between the value of non labour income by deferring and the difference in the marginal value of leisure in each period.

<sup>8</sup>This value generates an optimal labour income/asset ratio of about 30%, reservation wages for zero work at a little above the UK minimum wage and for switching from part-time to full time of about one and a half times the minimum wage.

**Empirical relevance of pension deferral.** The stylised simulation showed that deferral is financially beneficial for all full time workers who earn more than approximately £11 per hour. We pool four waves of data from the Labour Force Survey between the years 2008 and 2013 to determine the wage distributions (conditional on being in work) for women aged between 60 and 65, and men aged between 65 and 70.<sup>9</sup> We restrict our sample to these age ranges as they cover the state retirement age and hence the period when individuals make the deferral decision. It is important to note that the decision to work and the deferral decision are independent (except for the implications on income tax). Our final sample consists of 483 individuals. Figures 6 and 7 depict their wages.

Figure 6: Female wage distribution.

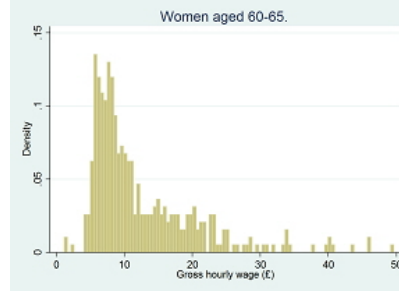
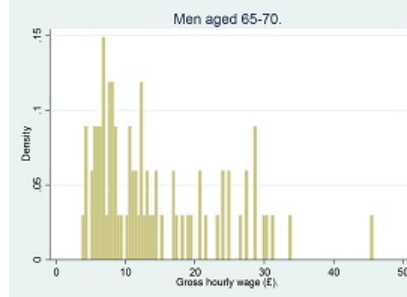


Figure 7: Male wage distribution.



It is clear that a significant proportion, around 40% of females and more than 50% males in our sample earn more than £11 per hour, therefore deferral policy is an important component of the labour supply decision for a large

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<sup>9</sup>We ensure there is no overlap in the surveys to ensure our sample does not contain any repeated observations. We include individuals working below full time hours to boost sample size, noting that the mean wage for full time and part time workers in this age category are roughly equal.

proportion of older workers. Indeed a DWP report in 2008 suggested that individuals tend to coordinate their labour supply and deferral decision. They found 79% of deliberate deferrers were in paid work and tended to maintain their preretirement hours, primarily full time, after deferral. Family decisions were important so a partner continuing in work or uncertainty of life expectation made deferral more likely, as did the desire to avoid liability for a higher income tax rate. Given that state pension deferral is likely to become increasingly important in the face of population ageing and longevity, then it is of equal importance to analyse the choice between an increment in weekly state pension or a lump sum payment, we turn to this in the next section.

## 4 UK State Pension & Deferral.

### 4.1 Which Deferral Option is Best

In a multiperiod setting the decision becomes one of choosing both if to defer and, if so, for how many years. In this section we simulate the present value of an individuals state pension pot at the date of undeferral, under both the incremental and lump sum option for deferral over a varying number of years.

On reaching SRA an individual can choose whether to take up the state pension or defer it from that date. They do not have to precommit to a length of deferral but at any future date can ask for their pension to start from then on.<sup>10</sup> If an individual chooses to defer their pension, then current rules mean that for every five weeks an individual defers, their weekly State Pension increases by 1%, this is equivalent to a 10.4% rate of return for each full year of deferral. Alternatively an individual may also defer their State Pension and receive a lump sum payment.<sup>11</sup> <sup>12</sup> If an individual chooses to take the latter option, the lump sum they receive is the value of their past deferred weekly pension payments accumulated at an interest rate of at least 2% above the Bank of England base rate.<sup>13</sup> Depending on the life expectancy of the individual there is no clear answer as to which option is more lucrative, however given the increasing life expectancy observed in the past 30 years, it is generally considered (see Farrar et al (2012)) that the incremental option offers a higher rate of return.

At the point of reinstatement of a deferred pension  $S$ , the present value of the

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<sup>10</sup>This not true for the lump sum option, in which case the individual must defer for at least 52 weeks.

<sup>11</sup>Extra State Pension and lump sum payment are both taxed. In addition if you choose to defer then this will impact means tested benefits, whereas if you choose to receive a lump sum, this will not affect certain means tested benefits.

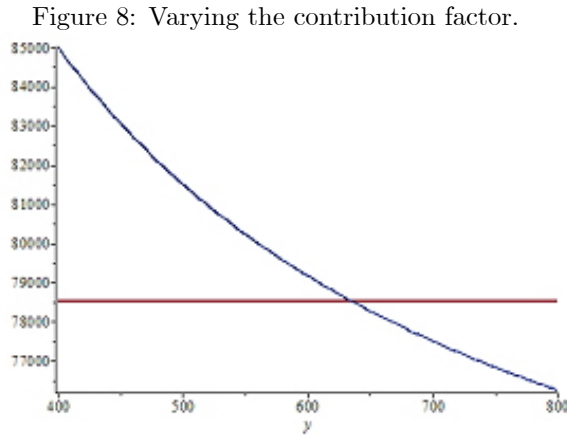
<sup>12</sup>Since its inception there has been various changes to legislation regarding how the rate of return on the deferral option is formulated, and the introduction of the lump sum option in 2006. For a more detailed description of these changes see Bozio et al (2010).

<sup>13</sup>In terms of pension deferral one of the biggest changes of the move to a single tier pension is that the lump sum option will be scrapped and only the incremental option will be available to those who defer (White Paper 2013). At the time of writing the actual generosity of the incremental option is yet to be decided, however is believed to be in the region of half its current generosity (FT, September 2013).

extra weekly payment coming from the deferral is  $x(1+1.01+1.01^2+..1.01^\tau)(1+(1+r)^{-1}+..(1+r)^{T-S})$  where  $\tau$  is the number of months for which the pension has been deferred between SRA and age at  $S$ ,  $x$  is the original weekly pension payable at SRA,  $r$  is a constant market interest rate and  $T$  is the date of death. On the other hand the lump sum payable at  $S$  is  $x(1+1+\rho+(1+\rho)^2+..(1+\rho)^\tau)$  where  $\rho$  is at least 2% above bank base rate.

We plot the present value under each option in figures 8 and 9 below. In figure 8 we vary the generosity of the incremental option, i.e. the length of time it takes to earn a 1% increase in an individuals weekly state pension. Whilst in figure 9 we vary the length of the period from the date of undeferral to death. It is these two factors which to a large extent dictate the PV of the deferred pension. To show this we set all other parameter values as follows: initial weekly state pension of £100, weekly interest rate on lump sum option equal to  $\frac{0.05}{52}$ , post undeferral weekly net rate of return equal to  $\frac{0.02}{52}$  and deferral period equal to two years.

**(i) Varying rate of return on incremental option** Figure 8 shows the effect of changing the rate of return or relative generosity, assuming an individual lives for 15 years following the date of undeferral. The sloping curve represents the deferred income option whilst the flat curve corresponds to the lump sum option.



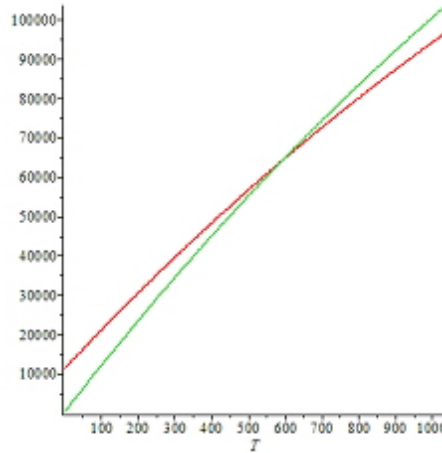
The break even of point for the PV of the pension is at a rate of return of about 1% for every 6.25 weeks deferred. Under existing rules the current rate of return is a 1% increment for every 5 weeks deferred, and therefore in this example it is worth approximately £3000 to the individual to choose the deferred income option. However is if the individual was credit constrained then it could



be the case they require the lump sum to clear some debt, e.g. an outstanding mortgage. What is also clear is that during the 1970's when the contribution rate was approximately 1% for every 7-8 weeks deferred, and individuals had a shorter life span (see figure 10), the lump sum option would have been more lucrative had it been available. A recent government announcement to scrap the lump sum option and reduce the generosity of the incremental option by half would leave individuals worse off, however the proposed rate of return on the incremental option is still in excess of the free market rate.

**(ii) Varying life span from undeferral date** Figure 9 shows the effect of increasing longevity under the incremental option (green) and lump sum option (red), assuming parameters of the current legislation.

Figure 9: Varying individuals life expectancy.



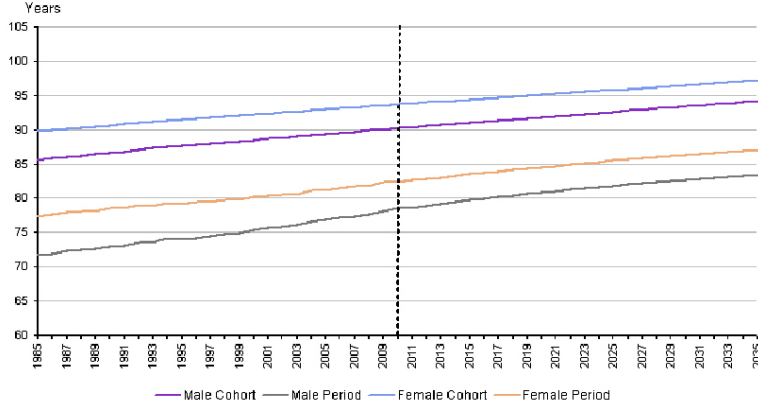
Intuitively, the PV for those who only live a short period after they undefer are much better off choosing the lump sum option. However it is clear that the deferred income option is more lucrative provided an individual lives for approximately 12 years or more after they undefer.

The Office for National Statistics (2011) published current and projected life expectancy tables by gender in the UK covering the period 1985 to 2035. Over this period it is quite clear life expectancy has increased substantially, for both cohort and period groups. Period life expectancy refers to the life expectancy for those individuals in a given calendar year (ONS, 2011). Hence in 2013 females are expected to live until 83 years of age on average. In contrast, cohort life expectancy at birth is calculated using age-specific mortality rates which allow for known or projected changes in mortality throughout a person's life (ONS, 2011). Which implies a female born in 2013 is expected to live until 94 years of age.

Supposing an individual reaches their life expectancy, figure 10 implies the

deferred income option is more lucrative for both current and future retirees, so long as the relative generosity of this option is not changed.<sup>14</sup>

Figure 10: Cohort and period life expectancy men and women.



Source: ONS (2011).

Farrar et al. (2012) compare the two undeferral options relative to not deferring and investing at the market rate, in most policy simulations deferral of any kind is preferred over non deferral.<sup>15</sup> Similar to Disney and Smith (2002) and our own model, Farrar et al. (2012) assume individuals face no borrowing constraints, deferral would not be optimal if individuals could not borrow against their future income. Assuming individuals live to their life expectancy, then the incremental option tended to offer a higher rate of return in most simulated examples, the post-tax deferral state pension income stream. Their results indicate the 10.4% interest payment substantially exceeded the break even interest rate required for the incremental and lump sum option to be of equal PV.

Coleman et al (2008) analyse the characteristics of deferrers versus those who claim state pension at SPA. Their results suggest deferrers are mainly high earners who had good financial knowledge of the deferral option (hence the majority of them chose the deferred income option), and either they or their partner tended to continue engaging with paid work post SPA. These individuals reported they were financially comfortable during the deferral period. This suggests the employment and deferral decision may well be jointly determined, and it is unlikely deferrers are from credit constrained households. More recent data from waves four and five from the English Longitudinal Study of Ageing (ELSA), spanning the years 2008-2013 also contain information on state pension deferral, despite small sample sizes those who do defer tend to have worked in

<sup>14</sup> Along with the introduction of the single tier pension, the White Paper (2013) also notes potential changes to the relative generosity of the incremental income option. The Financial Times reported the implied interest rate is likely to be half the current annual rate of 10.4% (FT, September 2013).

<sup>15</sup> In their paper the authors worked in continuous time and do not consider a formal model of labour force participation.

professional, managerial or skilled non manual occupations. These individuals are more likely to choose the deferred income option, and tend to defer their state pension for between 1 and 5 years.<sup>16</sup>

## 4.2 Prevalence of pension deferral.

A recent DWP statement showed between September 2009-2010 approximately 66,300 individuals deferred their pension.<sup>17</sup> Of this total roughly just over one third took the increment option, whilst nearly half took the lump sum option, the remainder took a mixture of the two.<sup>18</sup> Of the total number of individuals eligible to claim their state pension, roughly 1 in 10 chose to defer their pension. This suggests the prevalence of pension deferral should not be understated.

Coleman et al (2008) surveyed individuals who were approaching or had reached SPA, and found only a low level of respondents, 65%, knew of the option to defer. This proportion only increased slightly after SPA. The main reasons cited were due time constraints and it being the ‘spouses responsibility’, lack of interest or confidence in financial matters. Therefore despite roughly 1 in 10 individuals deferring it is likely with increased awareness (one of the central aims of the 2013 White Paper) that this proportion should continue to grow.<sup>19</sup>

## 5 Conclusion and policy implications

In this paper we develop a lifecycle to model the joint decision of pension deferral and intertemporal labour supply. Contrary to the policy aim of pension deferral which is to extend working lives, our theoretical model indicates pension deferral acts to raise the reservation wage and reduce the likelihood of labour force participation. The exact direction in which labour force changes in a two period framework depends on the marginal value of leisure in each period and its change over time. There are clear qualitative effects, depending on wage profile, non-labour wealth and preferences, introduction of a pension deferral scheme can tilt labour participation towards the present or future.

Our numerical simulation and empirical evidence suggest that the deferral policy affects a large proportion of the older working population. As a ballpark

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<sup>16</sup>Occupation data is fed forward to wave one of ELSA from the Health Survey for England data, from which the original ELSA sample is derived.

<sup>17</sup>Freedom of Information request catalogue number (2773/2011).

<sup>18</sup>FOI DWP (2011) pp.2 notes: New rules for deferral came into effect in April 2005 and lump payments became available from April 2006. A person who deferred their State Pension before April 2005 would qualify for increments for the period up to April 2005 and may have a choice of either a lump sum payment or an increment for the period of deferral from April 2005. This means some people may have both an increment and a lump sum payment. The lump sum option is only available to those who have deferred continuously for at least 12 months. The numbers do not include those who deferred for less than 12 months and opted for simple arrears instead of increments.

<sup>19</sup>Options to allow increased flexibility of deferring and undefering multiple times are also being considered by the DWP (White Paper 2013).

figure the option changes the reservation wages by about 2.5%. Moreover similarly to Farrar et al. (2012) our results indicate (1) pension deferral is optimal in the absence of credit constraints and (2) of the deferral options available, the incremental income option is more lucrative. Combined with the results of Coleman et al. (2008) the existing body of evidence suggests deferral take up is concentrated amongst higher income groups, who jointly determine the decision to defer and retire.

The recent UK announcement of a move to a single tier pension system will have a number of financial implications for those approaching retirement and future generations (see Crawford et al. 2013). This includes changes to the rules governing pension deferral, the most significant of which relate to the abolition of the lump sum option and reduction in the generosity of the incremental option (the implied annual interest rate on deferrals will halve from 10.4% to 5.2%).

Notwithstanding this with one in ten retirees choosing to defer their state pension, twinned with the focus of extending working lives and increasing longevity, the potential importance of understanding the effects of pension deferral from a theoretical and policy viewpoint are of crucial importance.

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## A Semi-indirect utility

Individuals optimally choose consumption in each period for fixed values of  $L_{T-1}, L_T$ . This gives semi-indirect utility  $v(L_{T-1}, L_T, x)$  :

$$\frac{\partial v}{\partial L_{T-1}} = \frac{\partial u_{T-1}}{\partial c_{T-1}} \frac{\partial c_{T-1}}{\partial L_{T-1}} + \delta \frac{\partial u_T}{\partial c_T} \frac{\partial c_T}{\partial L_{T-1}} + \frac{\partial u_{T-1}}{\partial L_{T-1}} \quad (5)$$

$$= \delta \frac{\partial u_T}{\partial c_T} \left[ r \frac{\partial c_{T-1}}{\partial L_{T-1}} + \frac{\partial c_T}{\partial L_{T-1}} \right] + \frac{\partial u_{T-1}}{\partial L_{T-1}} \quad (6)$$

From (above) we can infer:

$$r \frac{\partial c_{T-1}}{\partial L_{T-1}} + \frac{\partial c_T}{\partial L_{T-1}} = \frac{\partial x}{\partial L_{T-1}} = -rw_{T-1}$$

so

$$\frac{\partial v}{\partial L_{T-1}} = -\delta rw_{T-1} \frac{\partial u_T}{\partial c_T} + \frac{\partial u_{T-1}}{\partial L_{T-1}}$$

Similarly

$$\frac{\partial v}{\partial L_T} = \delta \frac{\partial u_T}{\partial c_T} \left[ r \frac{\partial c_{T-1}}{\partial L_T} + \frac{\partial c_T}{\partial L_T} \right] + \frac{\partial u_T}{\partial L_T}$$

and

$$r \frac{\partial c_{T-1}}{\partial L_T} + \frac{\partial c_T}{\partial L_T} = \frac{\partial x}{\partial L_T} = -w_T$$

The remaining problem for the individual is to choose optimal labour supply in each period:

$$\max_{L_T, L_{T-1}} v(L_{T-1}, L_T, x) \text{ st } 0 \leq L_i \leq 1$$

If each  $u(\cdot)$  is concave (due to time additivity) then  $v(\cdot)$  is also.

## B The Value Function for QuasiLinear-Isoelastic Preferences

Defining  $A_T$  as the financial wealth carried forward from period  $T-1$  to period  $T$ , we can substitute out the lifetime budget constraint to write  $c_{T-1}$  in terms of initial wealth minus savings and leave the problem

$$U = \frac{(rA_{T-1} + y_{T-1} + w_{T-1}(1 - L_{T-1}) - A_T)^\alpha}{\alpha} + h_{T-1}L_{T-1} \\ + \delta \left( \frac{(rA_T + y_T + w_T(1 - L_T))^\alpha}{\alpha} + h_T L_T \right)$$

Maximising  $U$  wrt  $A_T$  gives

$$A_T = \frac{x_{T-1} - (\delta r)^{1/(\alpha-1)}(y_T + w_T(1 - L_T))}{1 + r(\delta r)^{1/(\alpha-1)}}$$

where  $x_{T-1} = rA_{T-1} + y_{T-1} + w_{T-1}(1 - L_{T-1})$  and putting this back into  $U$  gives

$$U = \frac{(rA_{T-1} + y_{T-1} + w_{T-1}(1 - L_{T-1}) - \left[ \frac{x_{T-1} - (\delta r)^{1/(\alpha-1)}(y_T + w_T(1 - L_T))}{1 + r(\delta r)^{1/(\alpha-1)}} \right])^\alpha}{\alpha} + h_{T-1}L_{T-1} \\ + \delta \left( \frac{\left( r \left[ \frac{x_{T-1} - (\delta r)^{1/(\alpha-1)}(y_T + w_T(1 - L_T))}{1 + r(\delta r)^{1/(\alpha-1)}} \right] + y_T + w_T(1 - L_T) \right)^\alpha}{\alpha} + h_T L_T \right)$$

The value function is then

$$v = \frac{(r(rA_{T-1} + y_{T-1} + w_{T-1}(1 - L_{T-1})) + y_T + w_T(1 - L_T))^\alpha}{\alpha} ((\delta r)^{\alpha/(\alpha-1)} + \delta) \\ + h_{T-1}L_{T-1} + \delta h_T L_T$$

which can be rewritten as

$$v = \frac{(K + r w_{T-1}(1 - L_{T-1})) + w_T(1 - L_T))^\alpha}{\alpha} D + h_{T-1}L_{T-1} + \delta h_T L_T$$

where:

$$K = r(rA_{T-1} + y_{T-1}) + y_T, D = ((\delta r)^{\alpha/(\alpha-1)} + \delta)$$

## C The Wage Profiles Giving Indifferent Participation Profiles

Each pairwise utility combinations is defined as follows:

- (i)  $v_{00} = v_{01}$

$$\begin{aligned}
(K + rw_{T-1} + w_T)^\alpha &= (K + rw_{T-1})^\alpha + \frac{\alpha}{D} \delta h_T \\
w_T^1 &= \left( (K + rw_{T-1})^\alpha + \frac{\alpha}{D} \delta h_T \right)^{1/\alpha} - K - rw_{T-1}^1
\end{aligned}$$

$$(ii) \ v_{00} = v_{10}$$

$$\begin{aligned}
\frac{(K + rw_{T-1} + w_T)^\alpha}{\alpha} D &= \frac{(K + w_T)^\alpha}{\alpha} D + h_{T-1} \\
rw_{T-1}^2 &= \left( (K + w_T)^\alpha + \frac{\alpha}{D} h_{T-1} \right)^{1/\alpha} - K - w_T^2
\end{aligned}$$

$$(iii) \ v_{00} = v_{11}$$

$$\begin{aligned}
\frac{(K + rw_{T-1} + w_T)^\alpha}{\alpha} D &= \frac{K^\alpha}{\alpha} D + h_{T-1} + \delta h_T \\
K + rw_{T-1}^3 + w_T^3 &= (K^\alpha + \frac{\alpha}{D} (h_{T-1} + \delta h_T))^{1/\alpha}
\end{aligned}$$

$$(iv) \ v_{01} = v_{10}$$

$$\begin{aligned}
(K + rw_{T-1}^4)^\alpha &= (K + w_T^4)^\alpha + \frac{\alpha}{D} (h_{T-1} - \delta h_T) \\
rw_{T-1}^4 &= \left( (K + w_T^4)^\alpha + \frac{\alpha}{D} (h_{T-1} - \delta h_T) \right)^{1/\alpha} - K
\end{aligned}$$

$$(v) \ v_{01} = v_{11}$$

$$rw_{T-1}^5 = (K^\alpha + \frac{\alpha}{D} h_{T-1})^{1/\alpha} - K$$

$$(vi) \ v_{10} = v_{11}$$

$$w_T^6 = (K^\alpha + \frac{\alpha}{D} \delta h_T)^{1/\alpha} - K$$

For convenience we repeat the indifference relations here, but setting wages on the RHS to zero:

$$\begin{aligned}
(1) &= V_{00} - V_{01} : w_T^1 = \left( K^\alpha + \frac{\alpha}{D} \delta h_T \right)^{1/\alpha} - K \\
(2) &= V_{00} - V_{10} : rw_{T-1}^2 = \left( K^\alpha + \frac{\alpha}{D} h_{T-1} \right)^{1/\alpha} - K \\
(3) &= V_{00} - V_{11} : rw_{T-1}^3 + w_T^3 = (K^\alpha + \frac{\alpha}{D} (h_{T-1} + \delta h_T))^{1/\alpha} - K \\
(4) &= V_{10} - V_{01} : rw_{T-1}^4 = (K^\alpha + \frac{\alpha}{D} (h_{T-1} - \delta h_T))^{1/\alpha} - K \text{ (ref } w_{T-1}) \\
(5) &= V_{11} - V_{01} : rw_{T-1}^5 = (K^\alpha + \frac{\alpha}{D} h_{T-1})^{1/\alpha} - K \\
(6) &= V_{10} - V_{11} : w_T^6 = (K^\alpha + \frac{\alpha}{D} \delta h_T)^{1/\alpha} - K
\end{aligned}$$

Comparing the loci we see that for wages such that  $V_{10} = V_{01}$  and  $V_{00} = V_{01}$  we must also have  $V_{00} = V_{10}$ , in terms of figure 1 the loci (1),(2) must cross each other on the locus (4). Similarly the loci (5),(6) ( $V_{11} = V_{01}$  and  $V_{11} = V_{10}$ ) must cross on the locus (4) ( $V_{01} = V_{10}$ ). For similar reasons loci (1),(3),(5) must intersect at a common point; and so must loci (2),(3),(6).

The indifference relations  $V_{11} = V_{01}$ ,  $V_{11} = V_{10}$  and  $V_{00} = V_{11}$  are all linear in the wage rates with the last being negatively sloped and the other two respectively horizontal and vertical. Relation (4), (1) and (2);  $V_{10} - V_{01}$ ,  $V_{00} = V_{01}$  and  $V_{00} = V_{10}$  respectively are all positively sloped. For example differentiating  $v_{00} - v_{01}$  implicitly

$$\alpha(K + rw_{T-1} + w_T)^{\alpha-1}(rdw_{T-1} + dw_T) = \alpha(K + rw_{T-1})^{\alpha-1}rdw_{T-1}$$

$$\frac{dw_T}{dw_{T-1}} = r \frac{(K + rw_{T-1})^{\alpha-1} - (K + rw_{T-1} + w_T)^{\alpha-1}}{(K + rw_{T-1} + w_T)^{\alpha-1}}$$

$\alpha < 1$  so  $(K + rw_{T-1})^{\alpha-1} > (K + rw_{T-1} + w_T)^{\alpha-1}$  and the slope of locus (1) is always positive at any  $w'$ s. The same logic applies to locus (2):

$$\alpha(K + rw_{T-1} + w_T)^{\alpha-1}(rdw_{T-1} + dw_T) = \alpha(K + w_T)^{\alpha-1}(rdw_T)$$

$$\frac{dw_T}{dw_{T-1}} = r \frac{(K + rw_{T-1})^{\alpha-1} - (K + rw_{T-1} + w_T)^{\alpha-1}}{(K + rw_{T-1} + w_T)^{\alpha-1}}$$

Comparing the intercept of the loci: those of (1) and (6) are equal as are those of (2) and (5). But the intercept of locus (1) is below that of locus (3) on the  $w_T$  axis, and of locus (2) is below that of locus (3) on the  $w_{T-1}$  axis. Combining this information gives figure 1 in the text.

### C.1 The intersections of the loci all exist

Assets,  $y$ 's and  $h$ 's may be such that not all the intersections happen at strictly positive wages. But a finite positive solution must exist: both sides continuous in  $w_T$ , at  $w_T = 0$  LHS greater than RHS

$$[K^\alpha + \frac{\alpha h_{T-1}}{DD}]^{1/\alpha} > \{K^\alpha + \frac{\alpha(h_{T-1} - \delta h_T)}{DD}\}^{1/\alpha}$$

and as  $w_T \rightarrow \infty$

$$\lim_{w_T \rightarrow \infty} \{[(K + w_T)^\alpha + \frac{\alpha h_{T-1}}{DD}]^{1/\alpha} - [(K + w_T)^\alpha + \frac{\alpha(h_{T-1} - \delta h_T)}{DD}]^{1/\alpha}\} < \lim_{w_T \rightarrow \infty} w_T$$

A possible problem is that  $w_{T-1}$  where  $v_{01} = v_{10}$  may not be positive: e.g. looking at (??) above if  $\delta h_T$  is huge compared with  $h_{T-1}$  may give  $w_{T-1} < 0$  where they cross.



## D Comparative statics of the optimal life cycle labour participation regimes

In order to ascertain the effect of pension deferral on participation one must notice that the critical wage expressions (as a function of  $K$ ) with the exception of  $w^4$  all take the form:

$$w = (K^\alpha + z)^{1/\alpha} - K$$

where  $z > 0$ . Differentiate wrt  $K$

$$\begin{aligned} \frac{dw}{dK} &= (K^\alpha + z)^{(1-\alpha)/\alpha} K^{\alpha-1} - 1 \\ &= (1 + zK^{-\alpha})^{(1-\alpha)/\alpha} - 1 > 0 \text{ if } z > 0 \end{aligned}$$

In the case of  $w^4$  we have  $(h_{T-1} - \delta h_T)$  so if  $h_{T-1} > \delta h_T$  then  $dw/dK > 0$  but if  $h_{T-1} < \delta h_T$  then  $dw/dK < 0$ . So if  $h_{T-1} > \delta h_T$  the wage region with full time work at  $T$  expands at the expense of the wage region with full time work only in  $T - 1$ . Or vice versa if  $h_{T-1} < \delta h_T$ .