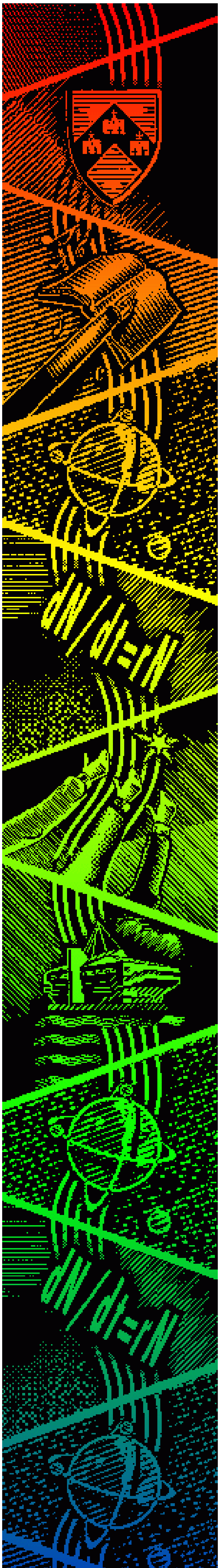

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Efficiency in strategic form games: A little trust can go a long way

Jianpei Li and Paul Schweinzer

Department of Economics and Related Studies
University of York
Heslington
York, YO10 5DD

Efficiency in strategic form games: A little trust can go a long way*

Jianpei Li† Paul Schweinzer‡

Abstract

We study the incentives of noncooperative players to play a cooperative game. That is, we look for individually rational, redistributive, pre-game agreements enacted in order to coordinate towards efficient equilibrium play. Contrasting with standard Nash equilibrium analysis, we assume that players can commit to the agreements they negotiate and that utility is verify- and transferable. We show that agreeing on a proportional-exponential redistribution rule is individually rational and implements the socially efficient outcome as Nash equilibrium. Moreover, we show that this class of redistributional contracts may be naturally obtained as the outcome of Nash bargaining. (JEL: *C72, D62, D71.* Keywords: *Redistribution, Efficiency, Social contract.*)

1 Introduction

In Prisoners' dilemma type of situations, players forego available, favourable outcomes of the game because individual decisions are taken independently and players cannot commit to taking their jointly profitable strategies. We are interested in whether or not players find it beneficial in such situations to jointly establish some redistributive agreement—a social contract—specifying some rule which redistributes social payoffs into individual utilities. A precondition for this to be possible is that players must be able to commit to later redistribute payoffs in the contracted way.

Consider a game in which, in the absence of an agreement, two players can reap the fruits of their *individual* labour. If they trust the other's guarantees, however, they can agree to share the added value of their *joint* surplus. If the latter, cooperative strategy is giving them a higher payoff than what they can get individually, then they can voluntarily forego their natural right in

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The payoff transformations implied by the redistributional contract can favour the weaker or stronger member of society. We define a proposal game in which an unanimously agreeable payoff transformation parameter is proposed. In this '*pre-game*,' a randomly chosen, perfectly rational and self-interested player proposes a single redistributional parameter which allows this player to maximise payoffs subject to (1) no deviation incentives from the efficient outcome by any player and (2) no incentives to revert to the inefficient Nash equilibrium of the original game. If the proposed parameter is rejected by at least one player, the original game is played. If the parameter is unanimously accepted, then the game proceeds to the '*redistribution*' stage at which the same social contract is applied for each outcome. That is, the joint payoffs in each outcome are redistributed according to the social contract. This redistribution transforms a wide range of strategic form games in a way that makes the socially efficient outcome implementable as a Nash equilibrium.

In the next section, after a brief discussion of the relation of our contribution to the existing literature, we present the model and our redistribution mechanism. In section 3, we derive the conditions under which the efficient outcome can be implemented as (subgame perfect) Nash equilibrium in various symmetric and (weighted) asymmetric settings. Our results include the case of incomplete information. The final section 4 provides an explicit bargaining foundation for the class of redistribution rules that we employ in this paper.

Literature

Coase (1960) states that if trade in an externality is possible and there are no transaction costs, bargaining will lead to an efficient allocation of resources regardless of the initial allocation of property. The present paper presents a mechanism which achieves this for a large class of strategic form games. To the best of our knowledge, the first paper to propose a formal pre-play negotiation or communication procedure as means to implementing efficiency in a game with externalities is

[1] The kind of commitment required is similar to that assumed in Jackson and Wilkie (2005) and Ellingsen and Paltseva (2013): players only commit to the use of a class of mechanisms if redistribution takes place. Every player can opt for no redistribution if so desired. For a wider discussion of this interpretation of trust see, for instance, Bacharach and Gambetta (2001).

Kalai (1981). Since then, investigation into pre-play negotiation has developed mainly along two lines: 1) payoff redistribution or side contracting and 2) commitment to certain strategies.

Along the first line, Jackson and Wilkie (2005) characterise the outcomes of games when players may make binding offers of strategy contingent payments before the game is played. They arrive at the anti-Coasian result that, despite complete information and costless contracting, unilateral promises of payoff transfers cannot always lead to efficient outcomes. In a recent paper, Ellingsen and Paltseva (2013) extend Jackson and Wilkie (2005) by considering not unilateral promises, but agreements which have to be accepted unanimously by both parties. They find that efficiency is implementable if it is compulsory for players to participate in contracting. If participation in contracting is voluntary, however, inefficiency may persist. Similarly, Yamada (2003) finds that for efficiency to obtain, the offers made by different players cannot be independent.[2]

Along the second line of commitment to certain strategies, Renou (2009) assumes that players can unilaterally pre-commit to playing some of their strategies and characterises the equilibrium payoffs of such commitment games. Bade, Haeringer, and Renou (2009) show that a strategy profile is implementable if and only if it is implementable by a simple commitment in which one of two players commits to a single action and the other commits to a subset of actions that contains his best-reply to the commitment of his opponent. In these contributions, commitment is assumed to be perfect, irrevocable and fully observable.

Instead of considering strategy contingent payoff transfers or commitments to a certain strategy set, we analyse a redistributive contract (or law) which enacts a particular redistribution of payoffs *whatever* strategies the players choose. Hence, this redistribution rule is not tailored to a particular outcome realisation but the same rule is applied to any realised outcome. As long as the players agree on the social contract and can commit to subsequently redistribute, the chosen outcome is played and payoffs are redistributed according to the contract. If the commitment is not upheld, then a legal machine is assumed to force the players in line with their commitments. If players do not agree on a contract, then the status quo equilibrium of the original game realises.

The particular formulation of the proportional-exponential redistribution rule that we use is borrowed from the contest literature. Corchón and Dahm (2010) interpret the ratio contest success function as sharing rule and establish its connection to a bargaining problem. In the contest literature, this Logit surplus sharing form has been axiomatised by Skaperdas (1996) and follows naturally from micro-foundations à la Fu and Lu (2012) and Jia (2008). We provide a Nash bargaining foundation for this rule in our payoff redistribution environment.

Finally, the modern literature on the social contract was initiated by Hobbes (1651), Locke (1689), and Rousseau (1762) followed by Harsanyi (1953, 1955) and Rawls (1971). It is admirably set into a general game theoretic context by Binmore (1994).

[2] In two extensions to Jackson and Wilkie (2005), Qin (2008) shows that cooperation can be induced if players can commit to paying penalties and Goranko and Turrini (2013) consider a dynamic pre-play negotiation process in which players can make, reject, or withdraw strategy-contingent offers but still find that, under certain conditions, efficiency can not be achieved. The earlier Varian (1994) models the pre-game stage using a regulator levying a Pigouvian tax on the players in order to compensate for the social cost that player imposes on the other player through the choice of a certain action. Watson (2011) examines the issue of side-contracting in a more general network environment.

2 The model

We consider standard, finite, strategic form games composed of a player set, a joint strategy space, and a set of payoff functions $\{\mathcal{N}, S, u\}$, and assume that players can commit to a particular social contract to redistribute joint income. For ease of exposition we mainly consider generic two-player, two-strategy games in the main body of the paper but, as shown in the propositions, the extension to the general case is straightforward. Utilities are assumed to be positive and transferable among players in order to make payoff redistribution possible.[3] This also implies that utilities are interpersonally comparable.

We propose a pre-game in which the precise parameters of a redistributional social contract are proposed. At this pre-game stage, a randomly chosen player proposes a payoff transformation in order to maximise own payoffs subject to unanimous agreement. This take-it-or-leave-it proposal pre-game is played under full knowledge of the game and, in particular, player identities.

More specifically, we assume that a randomly selected player proposes an exponent $r \in \mathbb{R}$ in the class of redistribution rules (1) in order to maximise own expected payoffs.[4] Conditional on unanimous agreement on the proposed r, the second redistribution stage of the game is reached and payoffs are transformed as agreed under the proposed contract. The transformed payoff a player $i \in \mathcal{N}$ expects under the profile $\hat{s} = (\hat{s}_1, \cdots, \hat{s}_n) \in S$ is defined as[5]

$$\tilde{u}_i(\hat{s}) \;=\; \frac{\max\limits_{s'_i \neq \hat{s}_i} u_i(s'_i, \hat{s}_{-i})^r}{\sum\limits_{j \in \mathcal{N}} \max\limits_{s'_j \neq \hat{s}_j} u_j(s'_j, \hat{s}_{-j})^r} \sum_{j \in \mathcal{N}} u_j(\hat{s}), \;\text{ for } r \in \mathbb{R}. \tag{1}$$

This 'proportional-exponential' payoff sharing rule gives each player a share of the joint payoff proportional to the individual inside option realisable through unilateral deviation.[6] The range of r gives the set of all jointly and individually rational ways of sharing the available pie $\sum_{j \in \mathcal{N}} u_j(\hat{s})$, made feasible through playing profile $\hat{s}$. Note that, since only the players' maximum unilateral deviation payoff $\max\limits_{s'_i \neq \hat{s}_i} u_i(s'_i, \hat{s}_{-i})$, the inside option, is used in (1) apart from the profile under consideration, $\hat{s}$, any generic strategic form game can be rearranged into a form requiring only two payoffs per player. If the social contract is not upheld in the sense that players either do not agree on a contract in the first place or unanimously choose the outside option, then play reverts to the original game.

[3] Non-positive utilities can be easily accommodated by any increasing, linear transformation.

[4] The simple rule prescribing players' to 'split social payoffs evenly' in each outcome also implements the efficient outcome as a Nash equilibrium in symmetric games (as does the heuristic prescribing a split proportional to the players' outside options in every asymmetric strategic form game of the class we consider.) This equal sharing rule is a special case of the rule that we propose. Contrary to our rule, however, it is not endogenised in a proposal game among rational, self-interested players.

[5] This sharing rule seems to satisfy Aristotle's dictum that "what is just ... is what is proportional, and what is unjust is what violates the proportion." Part of a large literature on tournaments employs it to share available rents between contestants. The most popular version of (1) employed in the contest literature comes with an exponent of $r = 1$. It is there called Tullock or Lottery contest success function and is interpreted as specifying the players' effort-based winning probabilities. In the present context this function specifies individual shares of social payoffs.

[6] In (1), the inside option is defined on the highest payoff *deviating* strategy $s'_j \neq s_j$. Alternatively, and without changing our results qualitatively, it can be defined on the best payoff *including* s_j, given the opponents' strategies.

Definition 1. *We define the* <u>*socially efficient*</u> *outcomes of game* $\{\mathcal{N}, S, u\}$ *as the set of profiles* $s \in S$ *satisfying* $\sum_{i \in \mathcal{N}} u_i(s) \geq \sum_{i \in \mathcal{N}} u_i(s')$ *for all* $s' \in S$.

In order to avoid trivialities and because we are neither considering the problem of equilibrium selection nor equilibrium uniqueness, we restrict attention to the class of games in which the set of socially efficient outcomes is a singleton. This is not a critical assumption: in games with multiple efficient outcomes we could just pick one efficient outcome, perhaps randomly, and the analysis remains as it is. Note, moreover, that no renegotiation or pre-play communication is necessary if the efficient outcome is already a Nash equilibrium. Next, we define the equilibrium concept we employ for this analysis.

Definition 2. *We define a proposal* r^* *and a strategy profile* (s_i^*, s_{-i}^*) *to be a* <u>*subgame perfect Nash equilibrium*</u> *if (1) a randomly chosen player* i *proposes* r^* *which is accepted by all players* $j \neq i$ *in the pre-game; (2) given* r^* *agreed upon unanimously in the pre-game, it is a Nash equilibrium for the players to choose* (s_i^*, s_{-i}^*) *at the redistribution stage.*

3 Results

3.1 Symmetric games

By symmetric games, we refer to the class of games where two players' payoffs are identical if they exchange their identities, i.e., the player indices $i, j \in \mathcal{N}$. In such games, at least one Nash equilibrium is symmetric.[7] Consider the following symmetric, two-player game:[8]

	c	d
c	$a,\ a$	$g,\ h$
d	$h,\ g$	$b,\ b$

We make the assumption that $2a > \max\{2b, g+h\}$; this guarantees that (c,c) is the unique efficient outcome; it does not guarantee, however, that (c,c) is a Nash equilibrium. Consider a *symmetric* status quo Nash equilibrium of the game, in pure or mixed strategies, and denote its corresponding payoffs by $(\bar{u}, \bar{u})$, the players' outside options. Given that players agree on some r and the game proceeds to the redistribution stage, the resulting allocation is:

	c	d
c	$a,\ a$	$\frac{b^r}{a^r+b^r}(g+h),\ \frac{a^r}{a^r+b^r}(g+h)$
d	$\frac{a^r}{a^r+b^r}(g+h),\ \frac{b^r}{a^r+b^r}(g+h)$	$b,\ b$

[7] Asymmetric Nash equilibria of symmetric games are discussed in the section on asymmetric games.

[8] For convenience, we always relabel strategy sets such that the efficient outcome is top-left.

Consider now a pre-game proposal by player 1. He proposes some $r \in \mathbb{R}$ such that

$$\begin{aligned}
\max_{r\in\mathbb{R}} \quad & \tilde{a}_1 = \frac{h^r}{h^r + h^r}(2a) = a \\
s.t.\ (\mathsf{DC}_1) \quad & \tilde{a}_1 = a \geq \frac{a^r}{a^r + b^r}(g+h), \\
(\mathsf{DC}_2) \quad & \tilde{a}_2 = a \geq \frac{a^r}{a^r + b^r}(g+h)
\end{aligned} \tag{2}$$

where the '*deviation constraints*' (DC_i) ensure that player $i = 1, 2$ has no incentives to deviate from (c,c) in the transformed game. If $g + h \leq a$, for any $r \in \mathbb{R}$, $\frac{a^r}{a^r + b^r} < 1$ and both deviation constraints hold. In this case, the set of all proposals r which implement the efficient outcome is $\mathcal{R} = \{r \mid r \in \mathbb{R}\}$. If $g + h > a$, the above program is satisfied by any r that is an element of

$$\mathcal{R} = \left\{ r \mid r \leq \frac{\log\left(\frac{g+h-a}{a}\right)}{\log(b) - \log(a)} \right\}. \tag{3}$$

Recall that by assumption $a > b$, $log(b) - log(a) < 0$. By assumption $2a > g + h$, $\frac{g+h-a}{a} \in (0, 1)$ and $\log\left(\frac{g+h-a}{a}\right) < 0$, thus the set $\mathcal{R}$ in (3) is nonempty and contains zero.

The players are symmetric, thus player 2's utility is maximised by whatever maximises player 1's utility and therefore she agrees to any proposed $r \in \mathcal{R}$. Note, however, that since player 1's maximisation problem above is satisfied by *any* $r \in \mathbb{R}$, both players are indifferent between any member of $\mathcal{R}$ and any $r \in \mathcal{R}$ ensures that the efficient outcome is a Nash equilibrium at the redistribution stage.

Hence, it is an equilibrium to suggest and accept redistribution under any $r \in \mathcal{R}$ for each player and the game progresses to the redistribution stage. Participation constraints at the redistribution stage are obviously satisfied by $r \in \mathcal{R}$ because of assumption $2a > \max\{g + h, 2b\}$. We arrive at the following lemma.

Lemma 1. *$\bar{\mathcal{R}}$, the set of r that ensures (c, c) to be a subgame perfect equilibrium, is nonempty in the symmetric, two-player game.*[9]

Proof. Immediate since at least $r = 0$ is an element of $\mathcal{R}$. □

The rationale is that players are contracting to *transfer* utility in inefficient states where their own incentives destroy the efficient outcome as an equilibrium. Players bind themselves in order to convince the opponents that they have no incentive to deviate from the efficient outcome. We only allow for a single social contract governing all redistribution in the game. Hence, reciprocal utility transfers in states where the opponents' incentives make the efficient outcome of the original game impossible are ensured. Note that the same procedure and outcome applies to symmetric n-player, two-strategy games.

Example 1: This example illustrates that the equilibrium implementing the efficient outcome is not necessarily unique even if the set of efficient outcomes is a singleton. Consider:

[9] Throughout the paper, we use $\mathcal{R}$ for the set of parameters r that satisfies all players' (DC), and $\bar{\mathcal{R}}$ for the set of r that satisfies (DC) and (PC) for all players. In the current symmetric case, $\bar{\mathcal{R}} = \mathcal{R}$.

	c	d
c	5,5	1,6
d	6,1	4,4

	c	d
c	5,5	$3^1/_2$,$3^1/_2$
d	$3^1/_2$,$3^1/_2$	4,4

under the right-hand side transformation with $r = 0$.[10] This transformed game has two (strict) Nash equilibria in pure strategies. But for sufficiently small r, efficiency can be uniquely implemented. $\triangleleft$

Proposition 1. *Any symmetric, n-player strategic form game $\{\mathcal{N}, S, u\}$ can be transformed through application of* (1) *to implement the socially efficient outcome as a Nash equilibrium.*

Proof. We first reduce the original player strategy sets S_i, $i \in \mathcal{N}$, to two elements each and then show that our equilibrium construction applies to any number of players. Find the unique socially efficient outcome and call it $(\mathsf{c}_i, \mathsf{c}_{-i})$. Now find the inside option (the best unilateral deviation strategy) for each player $i \in N$

$$\mathsf{d}_i = \underset{(s_i' \neq \mathsf{c}_i) \in S_i}{\text{argmax}} \; u_i(s_i', \mathsf{c}_{-i}), \tag{4}$$

or, any best deviation $\tilde{s}_i \in S_i$ among the set of strategies giving the highest deviation utility if the set on the right-hand side is not a singleton. Since we want to preserve player symmetry, we choose the same best deviation strategy for all players.

Call the set of all Nash equilibria of the original game N^*. Now, for every pure or mixed $(s_i^*, s_{-i}^*) \in N^*$, we define $u(\mathsf{d}_i, \mathsf{d}_{-i}) = u(s_i^*, s_{-i}^*)$. Each Nash equilibrium induces a reduction and each reduction gives subsequently rise to a transformation based on some outside option with $\bar{u}_i = u_i(s_i^*, s_{-i}^*)$; the efficient outcome pair is the same in each reduction (but the individual payoffs in the efficient outcome of the transformed game depend on the inequality inherent in the chosen outside option). If the efficient outcome can be implemented in an r-transformation of the reduced game, it can be implemented in the same r-transformation of the original game. In the set N^*, we pick a symmetric status quo Nash equilibrium as the basis for transformation such that the reduced game remains symmetric. Whenever we use an asymmetric status quo Nash equilibrium in a reduction, we turn the originally symmetric game into an asymmetric reduced game (which we discuss in the following subsections).

This procedure constructs a set of reduced symmetric games featuring a strategy set comprised of only two strategies per player. Since the game is symmetric, each player faces the same decision problem and, thus, no part of its analysis depends on the number of players. Therefore, step (2) in the above example applies in general, and we obtain the claimed transformation under $r \in \mathcal{R}$. Players' interim participation constraints are satisfied because (c,c_{-i}) is the efficient outcome and players's equilibrium payoffs in the reduced game must be no lower than their symmetric outside options. $\square$

[10] Our approach assumes that payoffs are verifiable so that redistribution of the payoffs is contractible. When payoffs are not contractible while side transfers agreed on by the players contingent upon actions are verifiable and contractible (as in, for example, Jackson and Wilkie (2005) and Ellingsen and Paltseva (2013)), the agreements among the players in our setup can be modified to be based on these terms. To see this in the present example we could, instead of agreeing on a contract specifying r, have players agree on a side transfer of $1^1/_2$ from player 1 to player 2 contingent on (d, c) being played and a side transfer of $1^1/_2$ from player 2 to player 1 contingent on (c, d) being played. The transformed game and its equilibrium is then exactly the same as ours.

Example 2: As an illustration of our construction, consider the following two-player game:

	L	R	M
L	5,5	1,6	3,2
R	6,1	1,1	2,2
M	2,3	2,2	4,4

In this example, we relabel (c,c)=(L,L), (d,c)=(R,L) and (c,d)=(L,R). For (d,d) we substitute the original (unique) Nash equilibrium (M,M). The single (symmetric) reduction is shown below, accompanied by the 0-transformation on the right-hand side.[11]

	c	d
c	5,5	1,6
d	6,1	4,4

	c	d
c	5,5	3.5,3.5
d	3.5,3.5	4,4

which implements the efficient outcome as a (subgame perfect) Nash equilibrium. ◁

Example 3: As an illustration involving mixed equilibria, consider the following two-player game:

	L	R	M
L	6,6	3,7	3,2
R	7,3	1,1	2,2
M	2,3	2,2	2,2

Again, we relabel (c,c)=(L,L), (d,c)=(R,L) and (c,d)=(L,R). Two of the three possible reductions are based on the original, pure strategy Nash equilibria (L,R) and (R,L). These two equilibria are asymmetric and not discussed here. The remaining reduction employs the symmetric, mixed-strategy equilibrium of the original game ($\frac{2}{3}$L+$\frac{1}{3}$R,$\frac{2}{3}$L+$\frac{1}{3}$R) with outcome (5,5). This is the payoff pair used in place of (d,d). The 0-transformation of this game is shown on the right-hand side:

	c	d
c	6,6	3,7
d	7,3	5,5

	c	d
c	6,6	5,5
d	5,5	5,5

and, again, it implements the efficient outcome as a Nash equilibrium. ◁

Example 4: As an illustration of the multiple-player case, consider the following three-player game:

	c	d
c	5,5,5	2,7,2
d	7,2,2	4,4,1

	c	d
c	2,2,7	1,4,4
d	4,1,4	2,2,2

[11] In all cases, if (d,d) is played in the reduced, transformed game, then the original game is invoked.

with the payoff table on the left-hand side for $s_3 = c$ and the payoff table on the right-hand side for $s_3 = d$, and the third element in each cell is player 3's payoff. Given the outside options $(2, 2, 2)$, the players unanimously agree on $r = 0$ and the redistributed payoffs are:

	c	d
c	5,5,5	$3^2/_3$,$3^2/_3$,$3^2/_3$
d	$3^2/_3$,$3^2/_3$,$3^2/_3$	3,3,3

	c	d
c	$3^2/_3$,$3^2/_3$,$3^2/_3$	3,3,3
d	3,3,3	2,2,2

where the payoff table on the left-hand side is for $s_3 = c$ and the payoff table on the right-hand side is for $s_3 = d$. In this transformed game, (c, c, c) is the unique equilibrium implementing the socially efficient outcome. ◁

Corollary. *The socially efficient outcome can be implemented as a Nash equilibrium through* 0*-transformation in any dominance-solvable (that is,* $a < h$ *and* $g < b$*), symmetric, two-player game. The efficient outcome is implementable in dominant strategies through* 0*-transformation if and only if* $g + h > 2b$.

Example 5: Suppose that randomly chosen player 1 proposes a fixed sharing rule $(\alpha, 1-\alpha)$, $\alpha \in [0, 1]$, of every outcome to player 2. The transformed payoff now becomes:

	c	d
c	$2a\alpha,\ 2a(1-\alpha)$	$(g+h)\alpha,\ (g+h)(1-\alpha)$
d	$(g+h)\alpha,\ (g+h)(1-\alpha)$	$2b\alpha,\ 2b\alpha$

Any $\alpha \in [\frac{b}{2a}, 1 - \frac{b}{2a}]$ is accepted by player 2, and for any α that is agreed upon, (c, c) is the equilibrium outcome. Since player 1's payoff strictly increases in α, player 1 proposes $\alpha = 1 - \frac{b}{2a}$. For b very small, the majority of social surplus goes to player 1 in equilibrium. This ad hoc sharing rule (which is a special case of our contract), however, is not supported naturally through Nash bargaining while our class is (see section 4). ◁

3.2 Asymmetric games

Consider the following asymmetric, two-player game where, for a notational convenience, we define $A = a_1 + a_2$, $B = b_1 + b_2$, $C = c_1 + c_2$, $D = d_1 + d_2$:

	c	d
c	$a_1,\ a_2$	$c_1,\ c_2$
d	$b_1,\ b_2$	$d_1,\ d_2$

Assumption. $A >$ *max*$\{B, C, D\}$ *so that (c,c) is the efficient action profile.*

Similar to the symmetric case, (c,c) is not necessarily a Nash equilibrium. We do not restrict the set of status quo equilibria to pure strategies. Choose a particular status quo Nash equilibrium and denote its payoffs as $(\bar{u}_1, \bar{u}_2)$. By the assumptions, we must have $A \geq \bar{u}_1 + \bar{u}_2$. Conditional on the acceptance of some proposed and agreed r-transformation, the resulting payoffs are as follows:

	c	d
c	$\frac{b_1^r}{b_1^r+c_2^r}A,\ \frac{c_2^r}{b_1^r+c_2^r}A$	$\frac{d_1^r}{d_1^r+a_2^r}C,\ \frac{a_2^r}{d_1^r+a_2^r}C$
d	$\frac{a_1^r}{a_1^r+d_2^r}B,\ \frac{d_2^r}{a_1^r+d_2^r}B$	$\frac{c_1^r}{c_1^r+b_2^r}D,\ \frac{b_2^r}{c_1^r+b_2^r}D$

At the pre-game stage, a randomly chosen player 1 proposes some $r \in \mathbb{R}$ such that

$$\begin{aligned} &\max_{r\in\mathbb{R}} && \tilde{a}_1 = \frac{b_1^r}{b_1^r + c_2^r}A \\ &s.t.\ (\mathrm{DC}_1) && \tilde{a}_1 = \frac{b_1^r}{b_1^r + c_2^r}A \geq \frac{a_1^r}{a_1^r + d_2^r}B = \tilde{b}_1, \\ &(\mathrm{DC}_2) && \tilde{a}_2 = \frac{c_2^r}{b_1^r + c_2^r}A \geq \frac{a_2^r}{a_2^r + d_1^r}C = \tilde{c}_2. \end{aligned} \tag{5}$$

Note that the set of proposals $\mathcal{R}$ satisfying (DC_1) and (DC_2) cannot be empty because both are always satisfied for $r = 0$ (by the definition of efficiency).

The asymmetric game therefore progresses to the redistribution stage. In order to implement (c,c) as an equilibrium of the transformed game at the redistribution stage, it is necessary that player i has no incentive to deviate from playing 'c' given the other player playing 'c'. This is ensured by (DC_1) and (DC_2). Moreover, the players' payoffs in the transformed game must be larger than what they obtain from status quo equilibrium, the players' outside options, i.e., the players' participation constraints have to be fulfilled

$$(\mathrm{PC}_1) \quad \tilde{a}_1 \geq \bar{u}_1, \ (\mathrm{PC}_2) \quad \tilde{a}_2 \geq \bar{u}_2. \tag{6}$$

Any agreeable $r \in \bar{\mathcal{R}}$ that implements efficiency must satisfy (DC_1), (DC_2), (PC_1), and (PC_2) simultaneously.

Lemma 2. *$\bar{\mathcal{R}}$, the set of transformation parameters r that ensures that (c, c) is a subgame perfect equilibrium, is nonempty in the asymmetric, two-player game if max$\{\bar{u}_1, \bar{u}_2\} \leq \frac{1}{2}A$.*

Proof. Immediate since at least $r = 0$ is an element of $\bar{\mathcal{R}}$. □

Example 6: Efficiency can be implemented in the following game:

	c	d
c	5,5	1,6
d	7,2	4,4

P1	c	d
c	5.7,4.3	2.7,4.3
d	5.5,3.5	4,4

P2	c	d
c	4.0,6.0	4.5,2.5
d	3.2,5.8	4,4

If player 1 proposes (see payoff matrix P1), a satisfied (DC_2) implies that $r \leq 1.96$ while (DC_1) is satisfied for any $r \in \mathbb{R}$. Since his objective $\frac{b_1^r}{b_1^r + c_2^r} 10 = \frac{7^r}{7^r + 6^r} 10$ is increasing in r, player 1 will propose $r^* = 1.96$ which is accepted by player 2 and implements the efficient outcome (c,c) with payoffs (5.75, 4.25) which are better than the outside options (4,4) for both players.[12]

If player 2 proposes (see payoff matrix P2), (DC_1) is satisfied for any $r \in \mathbb{R}$. Since her objective is decreasing in r, player 2 picks $r^* = -2.63$ which makes (PC_1) bind. This implements the efficient outcome (c,c) with payoffs (4.0,6.0). Ex-ante payoff expectations are asymmetric. ◁

Example 7: An example in which the condition max$\{\bar{u}_1, \bar{u}_2\} \leq \frac{1}{2}A$ is violated and efficiency cannot be implemented:

	c	d
c	3, 3	1, 4
d	4.5, 1	4, 1

If player 1 proposes, (DC_1) requires that either $r \leq 0.19$, where (PC_1) fails, or $r \geq 20.36$, where (PC_2) fails. A proposal by player 2 must fail for the same reasons. Thus, there is no acceptable proposal in this asymmetric game. As we show in the following subsection, efficiency can be implemented in such cases if we modify the payoff allocation rule (1) with an asymmetric weighting function. ◁

3.3 An asymmetric weighting function

The symmetric weighting function (1) is restricted to assigning symmetric transformed payoffs if the players' inside options are symmetric. Thus, players' transformed payoffs only depend on their inside options. The idea of the asymmetric mechanism is that, if we are prepared to preserve the player's outside option (endowment) asymmetries also within the r-transformed game, then participation may be ensured in a larger set of cases. Therefore, assuming that $(\bar{u}_1, \bar{u}_2)$ are the players' payoffs in status quo equilibrium, we define a player's asymmetric share of transformed payoffs based on the players' outside options (endowments) as[13]

$$\alpha_i = \bar{u}_i / \sum_{j \in \mathcal{N}} \bar{u}_j. \tag{7}$$

The definition of weighted, transformed payoffs for strategy profile $\hat{s} \in S$ is then

$$\tilde{u}_i(\hat{s}) = \frac{\max\limits_{s_i' \neq \hat{s}_i} \alpha_i u_i(s_i', \hat{s}_{-i})^r}{\sum_{j \in \mathcal{N}} \max\limits_{s_j' \neq \hat{s}_j} \alpha_j u_j(s_j', \hat{s}_{-j})^r} \sum_{j \in \mathcal{N}} u_j(\hat{s}), \text{ for } r \in \mathbb{R}. \tag{8}$$

In the two players case, we can without loss of generality normalise $\tilde{\alpha}_1 = \alpha_1/\alpha_1 = 1$ and $\alpha = \tilde{\alpha}_2 = \alpha_2/\alpha_1$. Application of redistribution rule (8) to the asymmetric, two-player game leads to the following transformed game:

[12] We round off all numbers shown to two decimal places.

[13] This is an arbitrary weight assignment which could be further simplified or normalised.

	c	d
c	$\frac{b_1^r}{b_1^r+\alpha c_2^r}A,\ \frac{\alpha c_2^r}{b_1^r+\alpha c_2^r}A$	$\frac{d_1^r}{d_1^r+\alpha a_2^r}C,\ \frac{\alpha a_2^r}{d_1^r+\alpha a_2^r}C$
d	$\frac{a_1^r}{a_1^r+\alpha d_2^r}B,\ \frac{\alpha d_2^r}{a_1^r+\alpha d_2^r}B$	$\frac{c_1^r}{c_1^r+\alpha b_2^r}D,\ \frac{\alpha b_2^r}{c_1^r+\alpha b_2^r}D$

At the proposal stage of the weighted, asymmetric game, we consider player 1 proposing some $r \in \mathbb{R}$ such that

$$\begin{aligned} &\max_{r\in\mathbb{R}} && \tilde{a}_1 = \frac{b_1^r}{b_1^r + \alpha c_2^r}A \\ &s.t.\ (\mathrm{DC}_1) && \tilde{a}_1 = \frac{b_1^r}{b_1^r + \alpha c_2^r}A \geq \frac{a_1^r}{a_1^r + \alpha d_2^r}B = \tilde{b}_1, \\ &(\mathrm{DC}_2) && \tilde{a}_2 = \frac{\alpha c_2^r}{b_1^r + \alpha c_2^r}A \geq \frac{\alpha a_2^r}{\alpha a_2^r + d_1^r}C = \tilde{c}_2. \end{aligned} \tag{9}$$

Again, note that the set of agreeable proposals $\mathcal{R}$ is non-empty and contains zero. This set is not necessarily convex. The participation constraints are given by

$$(\mathrm{PC}_1) \quad \tilde{a}_1 \geq \bar{u}_1, \quad (\mathrm{PC}_2) \quad \tilde{a}_2 \geq \bar{u}_2. \tag{10}$$

We obtain the following lemma:

Lemma 3. *Redistribution parameter $r = 0$ satisfies both participation constraints.*

Proof. The two participation constraints (10) bind, respectively, at

$$r_1 = -\frac{\log(A - \bar{u}_1) - \log(\bar{u}_2)}{\log(b_1) - \log(c_2)} \quad \text{and} \quad r_2 = \frac{\log(A - \bar{u}_2) - \log(\bar{u}_1)}{\log(b_1) - \log(c_2)}. \tag{11}$$

Note that $\min\{r_1, r_2\} < 0 < \max\{r_1, r_2\}$ and, depending on the ranking of b_1 and c_2, the expressions change signs. Consider the case when $r_1 < 0$, $r_2 > 0$, i.e., when $c_2 < b_1$. Then $\tilde{a}_1$ is increasing in r and $\tilde{a}_2$ is decreasing. Hence, $r = 0$ satisfies both (PC_i) in this case. If $r_1 > 0$, $r_2 < 0$, i.e., when $c_2 > b_1$, then $\tilde{a}_1$ is decreasing in r and $\tilde{a}_2$ is increasing and, again, $r = 0$ satisfies both (PC_i). If $c_2 = b_1$, both (PC_i) hold for any r, and the result is immediate. □

Observe that the set $[\min\{r_1, r_2\}, \max\{r_1, r_2\}]$ is convex and contains zero. The set $\bar{\mathcal{R}} = \mathcal{R} \bigcap [\min\{r_1, r_2\}, \max\{r_1, r_2\}]$ which satisfies all participation and deviation constraints is convex and contains zero.

Lemma 4. *Both players can agree on either $\min\{\bar{\mathcal{R}}\}$ or $\max\{\bar{\mathcal{R}}\}$.*

Proof. If $\tilde{a}_i$, $i = 1, 2$, is increasing (decreasing) in r, then a proposing player i will choose $\max\{\bar{\mathcal{R}}\}$ ($\min\{\bar{\mathcal{R}}\}$) as redistribution parameter r. If $\tilde{a}_i$ is independent of r (i.e., inside options are symmetric), then player i is indifferent between all $r \in \bar{\mathcal{R}}$. □

Thus, only in perfectly symmetric reduced games, interior points of $\bar{\mathcal{R}}$ may be proposed. In general, only the extreme points of $\bar{\mathcal{R}}$ will be put forward. Again, note that the procedure developed for the two-player, two-strategy game applies to an arbitrary n-player, two-strategy game.

Example 8: Continuing the previous example we get the following transformations:

	c	d
c	3,3	1,4
d	4.5,1	4,1

P1		c	d
	c	4.87,1.13	4.13, 0.87
	d	4.87,0.63	4,1

P2		c	d
	c	4.63,1.37	3.63,1.37
	d	2.50,3.00	4,1

If player 1 proposes, a satisfied (DC_1) implies that $r \leq 0.59$; at this value, both (PC_1) and (DC_2) are satisfied. Since his objective $\frac{b_1^r}{b_1^r + c_2^r} A = \frac{(9/2)^r}{(9/2)^r + 4^r} 6$ is increasing in r, player 1 will propose $r^* = 0.59$ which is accepted by player 2 and implements the efficient outcome (c,c) as shown in the top transformation on the right-hand side.

If player 2 proposes, (DC_2) is satisfied for any $r \geq -1.43$; at this value, all other constraints are satisfied. Since her objective is decreasing in r, player 2 picks $r^* = -1.43$ which implements (c,c) as shown in the bottom transformation on the right-hand side. $\triangleleft$

Proposition 2. *Any n-player strategic form game $\{\mathcal{N}, S, u\}$ can be transformed through application of* (8) *to implement the socially efficient outcome as a Nash equilibrium.*

Proof. As in the proof of proposition 1, we first reduce the original player strategy sets S_i, $i \in \mathcal{N}$, to two elements each and then show that our equilibrium construction applies to any number of players. We label the unique socially efficient outcome $(\mathsf{c}_i, \mathsf{c}_{-i})$. Now, as in the symmetric case, we find the best asymmetric unilateral deviation strategy for each player $\mathsf{d}_i = \underset{(s_i' \neq \mathsf{c}_i) \in S_i}{\text{argmax}} \; u_i(s_i', \mathsf{c}_{-i})$.

If there is more than one best deviation $\tilde{s}_i \in S_i$ for player i, then we choose either best deviation with equal probability. Call the (nonempty) set of all Nash equilibria of the original game N^*. Now, for every pure or mixed $(s_i^*, s_{-i}^*) \in N^*$, we define $u(\mathsf{d}_i, \mathsf{d}_{-i}) = u(s_i^*, s_{-i}^*)$. Each Nash equilibrium induces a reduction and each reduction gives subsequently rise to a transformation based on some outside option with $\bar{u}_i = u_i(s_i^*, s_{-i}^*)$; the efficient outcome pair is the same in each reduction (but the individual payoffs in the efficient outcome of the weighted, transformed game depend on the inequality inherent in the chosen outside option). Then, in the set N^*, we pick a status quo Nash equilibrium as the basis for transformation and apply the r-weighted transformation on the reduced game as specified in (7) and (8).

This procedure constructs a set of reduced asymmetric games featuring a strategy set comprised of only two strategies per player. By construction, for each player, this set contains the strategy leading to the efficient outcome and a best deviation from that outcome. Thus, for each possible outcome, every player faces the problem of making an identity-independent proposal as in (9).

As these problems are all identical for any number of players, the problem does not depend on the set of players. $\square$

Example 9: An example in which efficiency cannot be implemented by a *single* transformation for *all* status quo equilibria:[14]

	c	d
c	4,4	2,5
d	5,2	1,1

	c	d
c	4,4	2,5
d	5,2	2,5

	c	d
c	2.3,5.7	2,5
d	2,5	2,5

	c	d
c	4,4	2,5
d	5,2	3,3

	c	d
c	4,4	3.5,3.5
d	3.5,3.5	3,3

	c	d
c	4,4	2,5
d	5,2	5,2

	c	d
c	5.7,2.3	5,2
d	5,2	5,2

In this example, the two pure status quo equilibria are (c,d) and (d,c) and the mixed status quo equilibrium is ($\frac{1}{2}$c+$\frac{1}{2}$d,$\frac{1}{2}$c+$\frac{1}{2}$d) with outcome (3,3). The three reductions based on the three equilibria are shown in the middle. The efficient outcome has symmetric inside options and therefore the proposer is indifferent between all parameters r which satisfy (weighted) participation and deviation constraints. For simplicity, consider $r = 0$. The corresponding transformations by the asymmetric weighting mechanism are shown on the right-hand side. The efficient, transformed outcome cannot be simultaneously individually rational for both pure status quo Nash equilibria and the mixed strategy equilibrium of the original game. $\triangleleft$

Hence, redistributive contracts must be specified with respect to the status quo equilibrium and not to the whole original game. This problem cannot occur in games with a unique status quo Nash equilibrium, i.e., in dominance-solvable games.

Corollary. *If there is a unique status quo equilibrium then there is a unique transformation through* (8) *which implements the socially efficient outcome as a Nash equilibrium and satisfies participation constraints with respect to the status quo.*

3.4 Incomplete information

Consider the following asymmetric, two-player game with incomplete information on the side of player 1. We assume that, at the pre-play stage, player 2 is privately informed on the realisation of $\varepsilon \in \{-1, +1\}$, $\Pr(\varepsilon = 1) = p$ while player 1 only knows the distribution p. At the redistribution stage, the realisation of ε is common knowledge. (We keep the notation employed previously but now adopt $C = c_1 + c_2 + \varepsilon$.)

[14] Note that this example is originally symmetric but requires an asymmetric weighting transformation to ensure participation.

	c	d
c	$a_1,\, a_2$	$c_1,\, c_2+\varepsilon$
d	$b_1,\, b_2$	$d_1,\, d_2$

	c	d
c	$\frac{b_1^r}{b_1^r+\alpha(c_2+\varepsilon)^r}A,\ \frac{\alpha(c_2+\varepsilon)^r}{b_1^r+\alpha(c_2+\varepsilon)^r}A$	$\frac{d_1^r}{d_1^r+\alpha a_2^r}C,\ \frac{\alpha a_2^r}{d_1^r+\alpha a_2^r}C$
d	$\frac{a_1^r}{a_1^r+\alpha d_2^r}B,\ \frac{\alpha d_2^r}{a_1^r+\alpha d_2^r}B$	$\frac{c_1^r}{c_1^r+\alpha b_2^r}D,\ \frac{\alpha b_2^r}{c_1^r+\alpha b_2^r}D$

Example 10: Assume that $p = 1/2$. Note that in the equally probable events of $\varepsilon < 0$, (c,c) is efficient and if $\varepsilon > 0$, (c,d) is efficient. The ratio of outside options is $\alpha = 1/2$. The right-hand-side gives the weighted 0-transformation of the game.

	c	d
c	$3,\ 3$	$3,\ 3+\varepsilon$
d	$4,\ 1$	$3.5,\ 1.75$

	c	d
c	$4,\ 2$	$4+\frac{\varepsilon}{2},\ 2+\frac{\varepsilon}{2}$
d	$\frac{10}{3},\ \frac{5}{3}$	$3.5,\ 1.75$

The question is, whether there exists some r that can be agreed on and implements the efficient outcome in the transformed game. Consider the case for a proposing player 1:

1. Assume first that $\varepsilon = -1$ implying that (c,c) is efficient. Following calculations similar to those in the previous example, we obtain $\bar{\mathcal{R}}_{\varepsilon<0} = [-0.25, 0.14]$ with $\tilde{a}_1$ increasing in r.[15]
2. Next, assume that $\varepsilon = +1$ implying that (c,d) is efficient. Then, $\bar{\mathcal{R}}_{\varepsilon>0} = [-6, 3.76]$ with $\tilde{c}_1$ increasing in r.

Define $\bar{\mathcal{R}} = \bar{\mathcal{R}}_{\varepsilon<0} \bigcap \bar{\mathcal{R}}_{\varepsilon>0} = [-0.25, 0.14]$. (It is worthwhile noting that both sets necessarily include zero and, hence, the above shown weighted 0-transformation is always feasible and satisfies all DC and PC.) If player 1 proposes, he chooses

$$\max_r \frac{1}{2}\frac{b_1^r}{b_1^r+\alpha(c_2-1)^r}(a_1+a_2) + \frac{1}{2}\frac{d_1^r}{d_1^r+\alpha a_2^r}(c_1+c_2+1)$$

which is increasing. If, perhaps because of ex-post contract scrutiny by a court with penalising powers, player 1 were restricted to propose only $r \in \bar{\mathcal{R}}$, then this problem would yield $r = 0.14$ which player 2 always would accept as is shown below:

	c	d
c	3.96,2.04	5.37,2.63
d	3.42,1.58	3.50,1.75

But in our setup, player 1 is *not* restricted to choosing proposals from $\bar{\mathcal{R}}$ and he will in general speculate on a high realisation to improve his payoffs. To see this, imagine that $p \to 1$; then, the chances for improving payoffs through betting on (c,d) are very high and player 1 will virtually disregard the case of $\varepsilon = -1$ and propose the extreme point of $\bar{\mathcal{R}}_{\varepsilon>0}$. But this will not always implement the efficient outcome. $\triangleleft$

[15] Note that our (DC) are now true incentive compatibility constraints.

Thus, the above example shows that, in general, the efficient outcome cannot be implemented through a proposal by the uninformed player. (Therefore, the same is also true for two-sided incomplete information.) In the light of the Myerson-Satterthwaite theorem on efficient trade under incomplete information, this conclusion is not surprising. We now consider the same example for the informed player 2 in the role of proposer.

Example 11: By assumption, player 2 is perfectly informed on the game and its payoffs. We analyse the game from the back. At the redistribution stage, because of (PC_2), she prefers player 1 choosing c over d. Hence, she is constrained to making offers from the 'true' $\bar{\mathcal{R}}_\varepsilon$. But since this is the same problem as under perfect information, we know that the efficient outcome can be implemented as Nash equilibrium. $\lhd$

We conclude that offers by the privately informed player will indeed implement the efficient outcome by the same argument as in the previous subsection.

4 Bargaining foundation

This section provides a cooperative foundation for the rent-seeking redistribution rules we use in (1) and its asymmetrically weighted form in (8).[16] Specifically, we want to implement these rules as equilibrium of a well-specified bargaining game. Throughout this section, we restrict attention to the two-player, two-strategy game forms analysed in the main body of the paper.

In our present analysis we want to find the bargaining solution for each individual outcome of a two-player, asymmetric game. For instance, sharing the efficient outcome (a_1, a_2) according to the Nash bargaining solution, with relative bargaining powers of $(\beta_A, 1-\beta_A)$, $\beta_A \in (0,1)$, implies that players choose $(\hat{a}_1, \hat{a}_2 = A - \hat{a}_1)$ such as to maximise $\omega(\beta_A)$ defined as

$$\max_{\hat{a}_1} \ \omega(\beta_A) = (\hat{a}_1 - b_1)^{\beta_A}((A - \hat{a}_1) - c_2)^{1-\beta_A} \tag{12}$$

resulting, after maximisation, in

$$\hat{a}_1^* = A\beta_A + b_1 - (b_1 + c_2)\beta_A. \tag{13}$$

Repeating the process for all outcomes of the game, the Nash bargaining payoffs are:

	c	d
c	$A\beta_A + b_1 - (b_1 + c_2)\beta_A$, $A(1-\beta_A) - b_1 + (b_1 + c_2)\beta_A$	$C\beta_C + d_1 - (d_1 + a_2)\beta_C$, $C(1-\beta_C) - d_1 + (d_1 + a_2)\beta_C$
d	$B\beta_B + a_1 - (a_1 + d_2)\beta_B$, $B(1-\beta_B) - a_1 + (a_1 + d_2)\beta_B$	$\bar{u}_1, \bar{u}_2$

[16] A well-known general noncooperative implementation of the cooperative Nash bargaining solution is Hart and Mas-Colell (1996). Kalai and Kalai (2013) recently define a new solution concept, the co-co value, to unify existing various bargaining solutions.

In case Nash bargaining fails, the game reverts to the original one and the players receive their status quo payoffs $(d, d) \to (\bar{u}_1, \bar{u}_2)$. We now equate player 1's bargaining power in each outcome with the bargaining power we use in our redistributive contract (1), i.e., we set

$$\beta_A = \frac{b_1^r}{b_1^r + c_2^r},\ \beta_B = \frac{a_1^r}{a_1^r + d_2^r},\ \beta_C = \frac{d_1^r}{d_1^r + a_2^r} \tag{14}$$

in proportion to the players' marginal contribution in the outcome concerned. Solving now for the exponent r which equates the Nash bargaining payoffs with those of the redistributive rule (1), we obtain $r = 1$ as unique solution for each outcome.

We now generalise the Nash bargaining solution as defined above slightly by introducing an exponent s on the bargaining power parameter β, i.e., we redefine the bargaining problem (12) as

$$\max_{\hat{a}_1}\ \omega(\beta_A) = (\hat{a}_1 - b_1)^{(\beta_A)^s}((A - \hat{a}_1) - c_2)^{1-(\beta_A)^s}. \tag{15}$$

Going through the same steps as above, we can derive the following correspondence between this form of 'Nash power bargaining' and our general redistributive rule. For any r in (1), the exponent s which implements the outcome as Nash power bargaining outcome is given by[17]

$$s_A = \frac{\log\left(\frac{b_1^r(a_1+a_2-b_1)-b_1c_2^r}{\left(b_1^r+c_2^r\right)(a_1+a_2-b_1-c_2)}\right)}{\log\left(\frac{b_1^r}{b_1^r+c_2^r}\right)},\ s_B = \frac{\log\left(\frac{a_1^{r+1}+a_1^r(-(b_1+b_2))+a_1d_2^r}{\left(a_1^r+d_2^r\right)(a_1-b_1-b_2+d_2)}\right)}{\log\left(\frac{a_1^r}{a_1^r+d_2^r}\right)},\ s_C = \frac{\log\left(\frac{d_1a_2^r-d_1^r(c_1+c_2-d_1)}{\left(a_2^r+d_1^r\right)(a_2-c_1-c_2+d_1)}\right)}{\log\left(\frac{d_1^r}{a_2^r+d_1^r}\right)}.$$

The different bargaining powers s, however, are only equal across outcomes for the simple case of $r = 1$. Nevertheless, the Nash bargaining solution indeed provides an axiomatic justification for the redistribution rule we use throughout the paper where the relative size of players inside options are used as their individual bargaining powers.

Concluding remarks

We consider players' incentives to devise and propose a social contract in full knowledge of all aspects of the game including their identities. We find that, if players are able to choose from and commit to a member of a certain class of proportional-exponential redistributional contracts, then players will always find it profitable to unanimously agree to and honour such a contract capable of implementing the efficient outcome.

We claim that our pre-game—the game in which payoff transformations are defined—is a *reasonable* device to achieve this. By reasonable, we not only mean that the pre-game is the result of a well-defined utility maximisation problem of the players, we would also like to point out that the proportional-exponential rule which we use can be seen as fair precisely because it is grounded on the players' individual alternatives to cooperation. Moreover, we show that this class of redistributional contracts is natural in the sense that the corresponding settlements can be micro-founded using the Nash bargaining solution.

[17] This formulation uses bargaining powers of, e.g., $\beta_A = \left(\frac{a_1^r}{a_1^r+d_2^r}\right)^s$; a similar result obtains for $\beta_A = \left(\frac{a_1}{a_1+d_2}\right)^s$.

References

Bacharach, M., and D. Gambetta (2001): "Trust in Signs," in *Trust in Society*, ed. by K. S. Cook, pp. 148–84.

Bade, S., G. Haeringer, and L. Renou (2009): "Bilateral commitment," *Journal of Economic Theory*, 144, 1817–31.

Binmore, K. (1994): *Game Theory and the Social Contract, Vol. 1: Playing Fair*. MIT Press, Cambridge, Mass.

Coase, R. H. (1960): "The Problem of Social Cost," *Journal of Law and Economics*, 3, 1–44.

Corchón, L. C., and M. Dahm (2010): "Foundations for contest success functions," *Economic Theory*, 43, 81–98.

Ellingsen, T., and E. Paltseva (2013): "A Model of Contract Negotiations," *Working paper*, Department of Economics, Stockholm School of Economics.

Fu, Q., and J. Lu (2012): "Micro foundations of multi-prize lottery contests: a perspective of noisy performance ranking," *Social Choice and Welfare*, 38, 497–517.

Goranko, V., and P. Turrini (2013): "Preplay negotiations in non-cooperative games," *Working paper*, http://arxiv.org/abs/1208.1718v3.

Harsanyi, J. C. (1953): "Cardinal utility in welfare economics and in the theory of risk-taking," *Journal of Political Economy*, 61, 434–35.

——— (1955): "Cardinal welfare, individualistic ethics, and interpersonal comparison of utility," *Journal of Political Economy*, 63, 309–21.

Hart, S., and A. Mas-Colell (1996): "Bargaining and Value," *Econometrica*, 64, 357–80.

Hobbes, T. (1651): *Leviathan: Or the Matter, Forme, and Power of a Common-Wealth Ecclesiasticall and Civil*. London.

Jackson, M. O., and S. Wilkie (2005): "Endogenous Games and Mechanisms: Side Payments Among Players," *Review of Economic Studies*, 72, 543–66.

Jia, H. (2008): "A stochastic derivation of the ratio form of contest success functions," *Public Choice*, 135, 125–30.

Kalai, A. T., and E. Kalai (2013): "Cooperation in Strategic Games Revisited," *Quarterly Journal of Economics*, 128, 917–966.

Kalai, E. (1981): "Preplay Negotiations and the Prinsoner's Dilemma," *Mathematical Social Science*, 1(4), 375–379.

Locke, J. (1689): *Second Treatise of Government*. Awnsham and John Churchill, London.

Qin, C.-Z. (2008): "Inducing Cooperation by Self-Stipulated Penalties," *Annals of Economics and Finance*, 9(2), 385–395.

Rawls, J. (1971): *A Theory of Justice*. Belknap Press, Cambridge, Mass.

Renou, L. (2009): "Commitment games," *Games and Economic Behavior*, 66, 488–505.

Rousseau, J.-J. (1762): *Du Contrat Social; ou, Principes du Droit Politique*. Marc Michel Rey, Amsterdam.

Skaperdas, S. (1996): "Contest Success Functions," *Economic Theory*, 7(2), 283–90.

Varian, H. (1994): "A Solution to the Problem of Externalities when Agents are Well-Informed," *American Economic Review*, 84(5), 1278–1293.

Watson, J. (2011): "Contractual Chains: Decentralized Contracting with Externalities," *Working paper*, Department of Econonics, University of California at San Diego.

Yamada, A. (2003): "Efficient Equilibrium Side Contracts," *Economic Bulletin*, 3(6), 1–7.