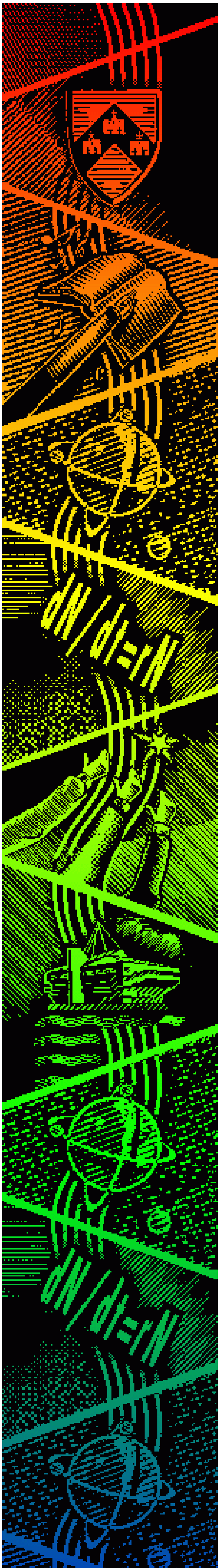

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Revisiting Jansen et al.'s Modified Cournot Model of the European Union Natural Gas Market[1]

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Abstract: In this paper we reconsider Jansen et al.'s (2012) Cournot model of the European Union natural gas market with three major suppliers Russian Gazprom, Norwegian Statoil, and Algerian Sonatrach. To reflect Russia's geopolitical consideration, we incorporate a relative market share to Gazprom's objective function. Compared with Jansen et al.'s use of standard market share, our study shows that the introduction of relative market share makes it not only possible to derive the same results in a more general environment, but also permits us to obtain clear-cut quantitative analysis results for equilibrium solution, consumer surplus, and social welfare. Our analysis also demonstrates for this modified Cournot model that by seeking a proper market share, Gazprom can achieve the same profits of a Stackelberg leader in a simultaneous move model as in the classical sequential move leader-follower model. When Gazprom pursues the control of market share besides profits, it will be good news for the EU's consumers but bad news for its rivals.

JEL classification: C62; C72; L13; L95; Q41

Key Words: Natural gas market; Cournot model; Stackelberg leader's advantage; Non-profit incentives; Relative market share; European Union

1 Introduction

The natural gas market has becoming increasingly important and popular as the world consumption of natural gas has risen significantly since IEA's record in early 1970s (IEA 2010) and because natural gas has considerably lower carbon dioxide emission than coal and oil, thus more conducive to environmental protection. This market usually displays regional characteristics over a given period of time due to transportation like pipelines and geographical constraints. Currently, the global natural gas market can be roughly divided

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into three major regional markets: North American, European, and Asian markets. In the current paper we focus our analysis on the European Union (EU) market. In this market, there are three major natural gas suppliers: Russian Gazprom, Norwegian Statoil, and Algerian Sonatrach. Unlike other markets, this market exhibits some unusual complex geopolitical features along with oligopolistic behavior. The EU gas market is currently dominated by the three players and will remain so for many years to come and its import is expected to expand to the level of 70% in 2030 (Lise and Hobbs, 2008).

As it is well recognized in the literature, in contrast to other players in the market which compete purely for commercial interest, Russia and its state monopolist Gazprom are not only pursuing profit but also seeking market power, presumably in the hope that this will enhance Russia's geopolitical influence on Europe; see e.g., Stern (2006), Paillard (2007), Finon and Locatelli (2008), Bilgin (2009), Boon von Ochssée (2010), Goldman (2010), Pirani et al. (2010), Smeenk (2010), and Yegorov and Wirl (2011). Russia is presently the biggest natural gas provider to the EU, accounting for more than 40 percentage of EU's natural gas import. Although unconventional gas development may potentially affect the current natural gas suppliers, uncertainty about such development is extremely high (McGlade et al. 2013) and the EU will still depend mainly on Russia's gas for a long period of time.

Due to imperfect competition, natural gas market is typically formulated as Cournot, Bertrand or Stackelberg models in which the goal or objective of every commercial enterprise (i.e., player) is to maximise its profit, nothing else. In a stricking analysis, Jansen et al. (2012) translate elegantly Russia's geopolitical consideration into a modified Cournot model in which Gazprom tries to maximise both profit and market share against two traditional profit maximizers: Norwegian Statoil and Algerian Sonatrach. Their major conclusion is that even if Gazprom may have a geopolitical agenda on top of profit, the outcome can be actually beneficial for consumers in the EU. Their use of standard market share, however, complicates their analysis for equilibrium price, production levels and welfare.[5] Consequently, it has tended to make their analysis less transparent and obscure a good understanding of their economic insights.

In this paper we reconsider Jansen et al.'s (2012) Cournot model of the European Union natural gas market. To capture Russia's geopolitical motive, we incorporate a relative market share to Gazprom's objective function. Compared with Jansen et al.'s use of standard market share, our study shows that the introduction of relative market share makes it not only possible to derive similar results in a more general environment, but also permits us to obtain rich and clear-cut quantitative analysis results for equilibrium solution,

[5]This involves a highly nonlinear equation with one unknown and makes it difficult to obtain an explicit formula for equilibrium price and production levels.

consumer surplus, and social welfare and gain fresh insights into the model. We prove that when Gazprom pursues the control of market share besides profits, it will be good news for the EU's consumers but bad news for its rivals, as Gazprom's behavior pushes total output up and brings prices down. Our analysis further demonstrates for our modified Cournot model that by seeking a proper market share, Gazprom can achieve the same profits of a Stackelberg leader in a simultaneous move oligopoly model as in the classical sequential move leader-follower oligopoly model, and that no matter how Gazprom might manoeuvre its influence on the market, its profit can never exceed that of the Stackelberg leader. This result is remarkably interesting and may have profound implications. It shows clearly that if a firm finds ways and means of controlling market share, it can reap the benefits of the Stackelberg leader even in a simultaneous move competition environment. This provides a novel and useful complement to the standard economic theory (of industrial organization) that a firm can attain the profits of a Stackelberg leader only in a sequential move environment where this firm makes its decision before all other firms (i.e., followers). Jansen et al. (2012, p. 283) seemed to have observed a similar result but were unsure about it by saying: "The corresponding profits for Russia appear to be the profits of a Stackelberg leader in a classical leader-follower model." Here we provide a solid theoretical foundation to confirm and validate their unverified observation.

The rest of this paper is arranged as follows. Section 2 briefly reviews a classical Cournot oligopoly model which will be used to compare new results obtained in later sections. Section 3 analyses our modified Cournot in which one player has a comprehensive objective combining profits with nonprofit strategic interests. Section 4 provides a comparative static analysis for the model. Section 5 concludes the paper.

2 The Classical Cournot Model

Following Jansen et al. (2012), we formulate the EU natural gas market as a Cournot model, as this market is dominated by homogeneous bulk goods. Russian Gazprom, Norwegian Statoil, and Algerian Sonatrach are the major gas suppliers, i.e., players. For ease of exposition, the subscripts $i = 1, 2, 3$ are used to represent Russia, Norway, and Algeria, respectively. Assume that marginal costs c_i are positive constants, and the inverse market demand function is given as

$$p(Q) = a - bQ, \tag{1}$$

where $Q = q_1 + q_2 + q_3$ and $a,\ b > 0$. Here a is the maximum price that any consumer is willing to pay, and b reflects the price elasticity. Production outputs q_i, $i = 1, 2, 3$, represent respectively the decision variables for Russia, Norway, and Algeria. The objective functions of three players are $\Pi_i(q_1, q_2, q_3) = (p - c_i)q_i,\ i = 1, 2, 3,$ with the marginal production costs satisfying $c_1 > c_2 > c_3$. Given the outputs of its opponents, each player i tries to maximise its profit

$$\max_{q_i} \Pi_i = (p - c_i)q_i = [a - b(q_1 + q_2 + q_3) - c_i]q_i. \tag{2}$$

Assume that the condition of interior solution is satisfied, i.e., $a - 3c_1 + c_2 + c_3 > 0$.[6] By the first order condition, we obtain the equilibrium output q_i^c for each player i, total output Q^c, and equilibrium price p^c:[7]

$$q_1^c = \frac{1}{4b}(a - 3c_1 + c_2 + c_3),\ q_2^c = \frac{1}{4b}(a + c_1 - 3c_2 + c_3),\ q_3^c = \frac{1}{4b}(a + c_1 + c_2 - 3c_3), \tag{3}$$

$$Q^c = \frac{1}{4b}(3a - c_1 - c_2 - c_3),\ \text{and } p^c = \frac{1}{4}(a + c_1 + c_2 + c_3). \tag{4}$$

In equilibrium, each player's profit Π_i^c and the consumer surplus CS^c are

$$\Pi_1^c = \frac{1}{16b}(a - 3c_1 + c_2 + c_3)^2 = b(q_1^c)^2, \tag{5}$$

$$\Pi_2^c = \frac{1}{16b}(a + c_1 - 3c_2 + c_3)^2 = b(q_2^c)^2, \tag{6}$$

$$\Pi_3^c = \frac{1}{16b}(a + c_1 + c_2 - 3c_3)^2 = b(q_3^c)^2, \tag{7}$$

$$CS^c = \frac{1}{32b}(3a - c_1 - c_2 - c_3)^2. \tag{8}$$

Let $\Pi^c = \Pi_1^c + \Pi_2^c + \Pi_3^c$ denote the total profits of all the suppliers and $W^c = \Pi^c + CS^c$ the social welfare. So far we have analysed the classical case in which all three players are profit maximisers. Our focus, however, will be the situation where Russia gas company Gazprom is not merely a profit maximiser.

We should point out that throughout the paper without loss of generality we concentrate on the model with three players, which is a fairly realistic description of the current EU gas market (Jansen et al. 2012, p. 281). The analysis can be easily extended to any finite number of players.[8]

[6]Notice that $a - 3c_1 + c_2 + c_3 > 0$ implies $a - 3c_2 + c_1 + c_3 > 0$ and $a - 3c_3 + c_1 + c_2 > 0$.

[7]In order to compare with the following modified version of Cournot competition where a nonprofit objective is involved, the superscript c is used here to represent the equilibrium solution of the traditional Cournot competition where all players use merely profit maximization as their objectives.

[8]We refer to Hobbs, Metzler, and Pang (2000), Pang and Sun (2006) for extensive studies on general Cournot-Nash equilibrium models.

3 A Modified Cournot Model with Relative Market Share

In the previous section we briefly reviewed the traditional Cournot model where profit maximization is the sole objective of every gas supplier. As discussed in Section 1, this traditional modelling does not capture the complex nature of the oligopoly competition in the EU natural gas market. This is because the Russia government through Gazprom seeks not only profits but also other strategic interests beyond profits. There are various nonprofit strategic interests or objectives (e.g., political influence and economic influence). Economic power is very often used as an effective means to achieve a political influence. For instance, economic sanction is a tool frequently used in the international politics. In general, economic power and political power are intertwined. Market power is one of the most fundamental devices to measure economic influence. Market share is a prominent index that is used to reflect the power and influence of a company on a market (Scherer and Ross 1990, Jansen et al. 2012). The higher market share a company possesses, the greater influence the company has on the market. In the following we adopt the market share to reflect Russia's nonprofit strategic objective.

We now reformulate the EU natural gas market into a more natural Cournot model where Algerian Sonatrach and Norwegian Statoil act as usual as (pure) profit maximisers but Russian Gazprom maximises both profits and its market share, which hopefully will improve its market power and hence its geopolitical influence. Formally we can write the decision problem of every player respectively as follows:

$$\max_{q_1} [\Pi_1 + wG(\frac{q_1}{q_2+q_3})] = [a - b(q_1+q_2+q_3) - c_1]q_1 + wG(\frac{q_1}{q_2+q_3}), \tag{9}$$

$$\max_{q_2} \Pi_2 = [a - b(q_1+q_2+q_3) - c_2]q_2, \tag{10}$$

$$\max_{q_3} \Pi_3 = [a - b(q_1+q_2+q_3) - c_3]q_3, \tag{11}$$

where $w \geq 0$ is a weight and $G : \mathbb{R} \to \mathbb{R}$ is Russia's non-profit objective function with its first-order derivative $G'(y) > 0$ and second-order derivative $G''(y) \leq 0$ for any real $y > 0$.

The key difference of our model from Jansen et al. (2012) is the use of market share measurement on Russia's nonprofit objective. We use $wG(\frac{q_1}{q_2+q_3})$ as Russia's nonprofit objective in (9), whereas Jansen et al. used $wM(\frac{q_1}{q_1+q_2+q_3})$ as a nonprofit objective. The ratio $\frac{q_1}{q_1+q_2+q_3}$ is called *standard market share* (SMS), whereas $\frac{q_1}{q_2+q_3}$ is called *relative market share* (RMS).[9] In our model, the weight w can be thought as the marginal utility of Russia's

[9]RMS is a method often used in industrial organization to measure market share. A company will measure its own market share with that of its competitors to determine RMS. In fact, the idea of RMS can be also used to refine some results in managerial economics, without changing the nature of the problems. For instance, we can apply RMS to Jansen *et al.*(2007, 2009) and simplify their analysis.

RMS. In Jansen et al.'s model, Russia's nonprofit objective is $wW(\frac{q_1}{q_1+q_2+q_3})$, which reflects that Russia's utility is an increasing function of its SMS. In fact, our introduction of RMS here suffices to capture this basic property, which is shown in the following simple but crucial lemma.

Lemma 1 *The relative market share* $\frac{q_1}{q_2+q_3}$ *is a strictly increasing function of the standard market share* $\frac{q_1}{q_1+q_2+q_3}$.

Proof: By dividing both the numerator and the denominator of the standard market share by (q_2+q_3), we have $q_1/(q_1+q_2+q_3) = [q_1/(q_2+q_3)]/[q_1/(q_2+q_3)+(q_2+q_3)/(q_2+q_3)]$. Let $x = q_1/(q_1+q_2+q_3)$ and $y = q_1/(q_2+q_3)$, and assume $q_i > 0$, then it is easy to see that $x \in (0,1)$ and $x = y/(y+1)$, which implies $y = x/(1-x)$. Define $f(x) = x/(1-x)$ for $x \in (0,1)$. Clearly, $f'(x) > 0$. In other words, RMS is a strictly increasing function of the traditional SMS. $\square$

It is clear from this lemma that if $w > 0$ and $G'(y) > 0$, then the utility from RMS $wG(f(x))$ is still a strictly increasing function of the SMS $x \in (0,1)$. In other words, RMS is a strict monotonic transformation of SMS and preserves the order of the numbers given by SMS. The great advantage of RMS over SMS lies in the fact that RMS does not have the variable q_1 in its denominator and thus can significantly simplify the analysis which involves the first and second derivatives, whereas SMS has the variable q_1 both its numerator and denominator which can complicate the analysis and obscure a good understanding of the problem.

3.1 Solving the Model with a General Nonprofit Objective

In this section we establish two fundamental properties for the modified Cournot model. The first property states that the Russia's production q_1 is a strictly increasing function of its nonprofit weight w and that for each given Russia's nonprofit weight w, the modified Cournot model has a unique Cournot-Nash equilibrium. This implies that if Russia puts more weight on its nonprofit objective $G(y)$, its sales and market share will increase. Compared with the classical Cournot case, because in this modified model the total output increases, the price will fall and thus yield more benefits to the consumers. In comparison with Jansen et al. (2012, Propositions 1 and 2), we obtain this result in a more general environment.

For the modified Cournot model, the first-order conditions for a Cournot-Nash equilibrium are

$$[a - c_1 - 2bq_1 - b(q_2+q_3)] + w\frac{\partial G}{\partial q_1} = 0, \tag{12}$$

$$a - bq_1 - 2bq_2 - bq_3 - c_2 = 0, \tag{13}$$

$$a - bq_1 - bq_2 - 2bq_3 - c_3 = 0. \tag{14}$$

From (13) and (14), we have

$$q_2 + q_3 = \frac{1}{3b}(2a - 2bq_1 - c_2 - c_3). \tag{15}$$

Here, the requirement of $q_2 + q_3 > 0$ should be satisfied so as to make the equilibrium solution meaningful, which in turn needs the condition of $q_1 < \frac{2a-c_2-c_3}{2b}$. Solving w for equation (12) and substituting (15) into the resulting formula, we have

$$\begin{aligned} w &= -\frac{a + c_2 + c_3 - 3c_1 - 4bq_1}{3G'(y)/(q_2 + q_3)} \\ &= -\frac{(a + c_2 + c_3 - 3c_1 - 4bq_1)(2a - 2bq_1 - c_2 - c_3)}{9bG'(y)}. \end{aligned} \tag{16}$$

Let $L_1 = \frac{a-3c_1+c_2+c_3}{4b}$ which is equal to q_1^c being Russia's output (3) in the classical Cournot model. Let $L_2 = \frac{2a-c_2-c_3}{2b}$. It is easy to see that $L_1 < L_2$. So if $L_1 < q_1 < L_2$ and $G'(y) > 0$, then $w > 0$. With the definition of L_1 and L_2, (16) can be rewritten as

$$w = \frac{8b}{9}\frac{(q_1 - L_1)(L_2 - q_1)}{G'(y)}. \tag{17}$$

Similarly,

$$q_2 + q_3 = \frac{1}{3b}(2a - 2bq_1 - c_2 - c_3) = \frac{2}{3}(L_2 - q_1). \tag{18}$$

Then $y = q_1/(q_2 + q_3) = q_1/[2(L_2 - q_1)/3]$ and

$$\frac{dy}{dq_1} = \frac{\frac{2}{3}(L_2 - q_1) + \frac{2}{3}q_1}{\frac{4}{9}(L_2 - q_1)^2} = \frac{3}{2}\frac{L_2}{(L_2 - q_1)^2}. \tag{19}$$

With the help of (16), (19), and the definition of L_1 and L_2, we have

$$\begin{aligned} \frac{dw}{dq_1} &= \frac{8b}{9}d[\frac{(q_1 - L_1)(L_2 - q_1)}{G'(y)}]/dq_1 \\ &= \frac{8b}{9}\frac{(L_1 + L_2 - 2q_1)G'(y) - (q_1 - L_1)(L_2 - q_1)G''(y)\frac{dy}{dq_1}}{(G'(y))^2} \\ &= \frac{8b}{9}\frac{(L_1 + L_2 - 2q_1)G'(y) - \frac{3}{2}(q_1 - L_1)(L_2 - q_1)G''(y)\frac{L_2}{(L_2 - q_1)^2}}{(G'(y))^2}. \end{aligned} \tag{20}$$

It is clear that if $G'(y) > 0$, $G''(y) \leq 0$, and $L_1 < q_1 < (L_1 + L_2)/2$, then $\frac{dw}{dq_1} > 0$. Observe that $L_1 < q_1 < (L_1 + L_2)/2\,(< L_2)$ has a meaningful economic interpretation in the case of $c_1 = c_2 = c_3$ which results in $(L_1 + L_2)/2 = \frac{5}{4}(\frac{a-c_1}{2b})$. The number $\frac{a-c_1}{2b}$ corresponds to the optimal production when Russia is the monopolist supplier and acts as a profit maximiser in the EU natural gas market. So the condition $L_1 < q_1 < (L_1 + L_2)/2$ states that when Russia takes RMS as its non-profit objective, its production should be higher than the case of the classical Cournot competition but lower than 5/4 times of the monopolistic case.

We now prove that for each given Russia's nonprofit weight $w \geq 0$, the modified Cournot model has a unique equilibrium. Recall from the first-order condition that we know

$$w = -\frac{(a + c_2 + c_3 - 3c_1 - 4bq_1)(2a - 2bq_1 - c_2 - c_3)}{9bG'(y)}.$$

This shows that w can be written as a function of q_1, i.e., $w = F(q_1)$. We have also proved that $\frac{dw}{dq_1} > 0$, meaning that w is a strictly increasing function of w and thus has a unique inverse function F^{-1}. That is, for any given $w \in [0, \bar{w}]$, with $\bar{w}$ defined by $q_1(\bar{w}) = (L_1 + L_2)/2$, there is a unique q_1 such that $q_1 = F^{-1}(w)$. Clearly, q_2 and q_3 are uniquely determined by their respective first-order condition for the given w. Observe that the condition $L_1 < q_1 < (L_1 + L_2)/2$ is equivalent to $0 < w < \bar{w}$ because q_1 is a strictly increasing function of w.

Let $Q = q_1 + q_2 + q_3$ be the total equilibrium output. Then $Q = \frac{q_1}{3} + \frac{2L_2}{3}$ can be seen as a function of Russia's nonprofit weight w. Clearly, $\frac{dQ}{dw} = \frac{1}{3}\frac{dq_1}{dw} > 0$, meaning that the total production is a strictly increasing function of w. The more weight Russia puts on its nonprofit objective, the higher the total output, the lower the price and the better for the consumers. We summarise the above discussions in the following proposition.

Proposition 1 *Under the condition of $G'(y) > 0$ and $G''(y) \leq 0$ for every $y > 0$, the modified Cournot model has a unique equilibrium for each given nonprofit weight $w \in [0, \bar{w}]$. Furthermore Gazprom's output q_1 is a strictly increasing function of its nonprofit weight $w \in [0, \bar{w}]$ and so is the total output Q.*

It is worth pointing out that the above result has the same spirit as Propositions 1 and 2 in Jansen et al. (2012), but we achieve this result in a much more general environment in the sense that we dispense with Jansen et al.'s restrictive condition (d). The above result also provides a proper range for Gazprom to adjust its nonprofit weight and this range is bounded and cannot be unbounded. In Jansen et al.'s result, a similar requirement is also needed but missed.

Next we are going to show that by choosing a proper nonprofit weight w, Gazprom can attain the profits of a Stackelberg leader as in the classical leader-follower model. We also prove that no matter how Gazprom might manoeuvre its nonprofit objective, its profits can never exceed that of a Stackelberg leader. It is easy to derive that in the Stackelberg model when Gazprom acts as the leader and Statoil and Sonatrach behave as followers, and all three are profit-maximisers, Gazprom's equilibrium output is equal $\tilde{q}_1 = \frac{a-3c_1+c_2+c_3}{2b}$. Note that $\tilde{q}_1$ is equal to two times of Gazron's equilibrium output q_1^c in the classical Cournot competition. Recall that for the modified Cournot model, from the first-order conditions we have $q_2 + q_3 = \frac{1}{3b}(2a - c_2 - c_3) - \frac{2}{3}q_1$. We plug $q_2 + q_3$ into Gazprom's profit function

$$\Pi_1(q_1) = [a - b(q_1 + q_2 + q_3) - c_1]q_1 = \frac{b}{3}(\frac{a - 3c_1 + c_2 + c_3}{b} - q_1)q_1.$$

The first-order condition of maximising $\Pi_1(q_1)$ yields

$$q_1^* = \frac{a - 3c_1 + c_2 + c_3}{2b},$$

which equals $\tilde{q}_1$ being the equilibrium output of the Stackelberg leader. This means that when Gazprom has a nonprofit objective besides profits, its maximal profits can never exceed that of the Stackelberg leader. Whether Gazprom can attain this maximal profit depends on the nonprofit function $G(\frac{q_1}{q_2+q_3})$. We will show that under the conditions of $G'(y) > 0$ and $G''(y) \leq 0$, $L_1 < q_1 < (L_1 + L_2)/2$, it is possible to achieve the profits of the Stackelberg leader. To see this, recall from Gazprom's first-order condition that we have

$$w = -\frac{(a + c_2 + c_3 - 3c_1 - 4bq_1)(2a - 2bq_1 - c_2 - c_3)}{9bG'(y)}.$$

That is, w can be seen as a function of q_1, i.e., $w = F(q_1)$. Therefore there exists a $w^* > 0$ such that $w^* = F(q_1^*)$. Notice that $\frac{L_1}{2} < q_1^* < \frac{L_1+L_2}{2}$. In summary we have

Proposition 2 *Assume that $G'(y) > 0$ and $G''(y) \leq 0$ for all $y > 0$. For the modified Cournot model, no matter how Gazprom might exercise its market power, its profits can never exceed that of a Stackelberg leader as in the classical leader-follower model. It is, however, definitely possible to achieve the profits of the Stackelberg leader.*

As far as we know, this is the first formal result proving that by seeking a proper market share, a firm can also obtain the first mover advantage in a simultaneous decision-making oligopoly model, whereas the conventional wisdom is that a firm can attain the profits of a Stackelberg leader only in a sequential decision-making environment. As we said previously, Jansen et al.(2012, p.283) observed a similar result but were less uncertain about it.

4 Comparative Statics Analysis

In this section we focus on the very tractable case in which Gazprom's nonprofit objective is given by the relative market share $w\frac{q_1}{q_2+q_3}$. Obviously all results in the previous section easily apply to this case. The major advantage of dealing with this special case is that it will permit us to perform comparative statics analysis and derive clear-cut results for equilibrium, consumer surplus and social welfare. Such analysis would be impossible if one uses the standard market share $w\frac{q_1}{q_1+q_2+q_3}$.

For the modified Cournot model with Gazprom's nonprofit objective $w\frac{q_1}{q_2+q_3}$, the three first-order conditions are

$$[a - c_1 - 2bq_1 - b(q_2 + q_3)](q_2 + q_3) + w = 0, \tag{21}$$

$$a - bq_1 - 2bq_2 - bq_3 - c_2 = 0, \tag{22}$$

$$a - bq_1 - bq_2 - 2bq_3 - c_3 = 0. \tag{23}$$

From (22) and (23),we have

$$q_2 + q_3 = \frac{1}{3b}(2a - c_2 - c_3) - \frac{2}{3}q_1. \tag{24}$$

Substituting (24) into (21) and solving the quadratic equation yields

$$q_1 = \frac{1}{8b}[(5a - 3c_1 - c_2 - c_3) \pm 3\sqrt{(a + c_1 - c_2 - c_3)^2 - 8bw}]. \tag{25}$$

Obviously, the case of $w = 0$ should correspond to the case of the classical Cournot model. Notice further that we must have $q_1^c = \frac{a-3c_1+c_2+c_3}{4b}$ from (3). So, for the two possible solutions in (25), only the following one is valid,

$$q_1^w = \frac{1}{8b}[(5a - 3c_1 - c_2 - c_3) - 3\sqrt{(a + c_1 - c_2 - c_3)^2 - 8bw}]. \tag{26}$$

Here w should be no larger than $\frac{(a+c_1-c_2-c_3)^2}{8b}$ in order to make the square root meaningful. It follows from (22) and (23) that

$$q_2 = \frac{a - 2c_2 + c_3}{3b} - \frac{1}{3}q_1, \tag{27}$$

$$q_3 = \frac{a + c_2 - 2c_3}{3b} - \frac{1}{3}q_1. \tag{28}$$

Substituting (26) into (27) and (28) yields the equilibrium output for Norway and Algeria

$$q_2^w = \frac{1}{8b}[(a + c_1 - 5c_2 + 3c_3) + \sqrt{(a + c_1 - c_2 - c_3)^2 - 8bw}], \tag{29}$$

$$q_3^w = \frac{1}{8b}[(a + c_1 + 3c_2 - 5c_3) + \sqrt{(a + c_1 - c_2 - c_3)^2 - 8bw}]. \tag{30}$$

Then the total equilibrium output is

$$Q^w = q_1^w + q_2^w + q_3^w = \frac{1}{8b}[(7a - c_1 - 3c_2 - 3c_3) - \sqrt{(a + c_1 - c_2 - c_3)^2 - 8bw}], \tag{31}$$

and the equilibrium price is

$$p^w = a - bQ^w = \frac{1}{8}[(a + c_1 + 3c_2 + 3c_3) + \sqrt{(a + c_1 - c_2 - c_3)^2 - 8bw}]. \tag{32}$$

We summarise the above discussion in the following proposition.

Proposition 3 *Assume that $a - 3c_1 + c_2 + c_3 > 0$ and $c_1 > c_2 > c_3$ and Gazprom has a nonprofit objective $w\frac{q_1}{q_2+q_3}$. For each given weight w, $0 \le w \le \frac{(a+c_1-c_2-c_3)^2}{8b}$, the modified Cournot model has a unique equilibrium.*

4.1 What if Russia Maximises its Pure Profits

Now we examine how Russia's nonprofit weight w affects its profits. With the help of (26) and (32), we can calculate the total profits $\Pi_1(w)$ for Russia as

$$\begin{aligned} \Pi_1(w) = \frac{1}{8b}\{&\frac{1}{8}[(a + c_1 + 3c_2 + 3c_3) + \sqrt{(a + c_1 - c_2 - c_3)^2 - 8bw}] \\ &-c_1\}[(5a - 3c_1 - c_2 - c_3) - 3\sqrt{(a + c_1 - c_2 - c_3)^2 - 8bw}]. \end{aligned} \tag{33}$$

Clearly, Russia can choose the weight w to influence its profits. From the first-order condition of Russia's profit maximisation, i.e., $\partial\Pi_1(w)/\partial w = 0$, we can obtain the optimal weight w^* as[10]

$$w^* = \frac{1}{9b}(a + 3c_1 - 2c_2 - 2c_3)(a - 3c_1 + c_2 + c_3). \tag{34}$$

From (34), we see that the optimal weight w^* is affected not only by players' costs c_1, c_2, and c_3, but also by parameters a and b of the demand curve. It is obvious that $0 < w^* < \frac{1}{8b}(a + c_1 - c_2 - c_3)^2$ because $a - 3c_1 + c_2 + c_3 > 0$ and $c_1 \ge c_2, c_3$. When Russia takes the optimal weight w^*, the equilibrium outputs for Russia, Norway and Algeria will be given by[11]

$$q_1^* = \frac{1}{2b}(a - 3c_1 + c_2 + c_3), \tag{35}$$

$$q_2^* = \frac{1}{6b}(a + 3c_1 - 5c_2 + c_3), \tag{36}$$

$$q_3^* = \frac{1}{6b}(a + 3c_1 + c_2 - 5c_3). \tag{37}$$

Total equilibrium output and equilibrium price are

$$Q^* = \frac{1}{6b}(5a - 3c_1 - c_2 - c_3), \tag{38}$$

[10]The derivation is given in the Appendix 1.
[11]The detailed computation is given in the Appendix 2.

$$p^* = \frac{1}{6}(a + 3c_1 + c_2 + c_3). \tag{39}$$

Each player's profit is given as

$$\Pi_1^* = (p^* - c_1)q_1^* = \frac{1}{12b}(a - 3c_1 + c_2 + c_3)^2 = \frac{1}{3}b(q_1^*)^2, \tag{40}$$

$$\Pi_2^* = (p^* - c_2)q_2^* = \frac{1}{36b}(a + 3c_1 - 5c_2 + c_3)^2 = b(q_2^*)^2, \tag{41}$$

$$\Pi_3^* = (p * - c_3)q_3^* = \frac{1}{36b}(a + 3c_1 + c_2 - 5c_3)^2 = b(q_3^*)^2. \tag{42}$$

The total profit of three players is

$$\Pi^* = \Pi_1^* + \Pi_2^* + \Pi_3^* = \frac{1}{36b}[3(a - 3c_1 + c_2 + c_3)^2 + (a + 3c_1 - 5c_2 + c_3)^2 + (a + 3c_1 + c_2 - 5c_3)^2]. \tag{43}$$

The total consumer surplus is given as

$$CS^* = \frac{1}{2}(a - p^*)Q^* = \frac{1}{72b}(5a - 3c_1 - c_2 - c_3)^2. \tag{44}$$

Thus, the social welfare is given by

$$\begin{aligned} W^* &= \Pi^* + CS^* \\ &= \frac{1}{72b}[6(a - 3c_1 + c_2 + c_3)^2 + 2(a + 3c_1 - 5c_2 + c_3)^2 \\ &\qquad + 2(a + 3c_1 + c_2 - 5c_3)^2 + (5a - 3c_1 - c_2 - c_3)^2]. \end{aligned} \tag{45}$$

4.2 Comparison of Equilibrium Results

We now come to examine how equilibrium price, output, consumer surplus and social welfare change in response to Russia's weight w on nonprofit objective, compared with the classical Cournot model in Section 2. Recall that the classical Cournot model corresponds to the case of $w = 0$ in our modified Cournot model. From (31), (32), and (A2-13), we have total output change $\Delta Q = Q^w - Q^c > 0$, equilibrium price change $\Delta p = p^w - p^c < 0$, and consumer surplus change $\Delta CS = CS^w - CS^c > 0$ for any given $w \in (0, \bar{w}]$. Further, (26), (29), and (30) tell respectively that Russia's output change $\Delta q_1 = q_1^w - q_1^c > 0$, Norway's output change $\Delta q_2 = q_2^w - q_2^c < 0$, and Algeria's output change $\Delta q_3 = q_3^w - q_3^c < 0$. Similarly, (A2-14) and (A2-15) indicate Norway's profit change $\Delta\Pi_2 = \Pi_2(w) - \Pi_2^c < 0$ and Algeria's profit change $\Delta\Pi_3 = \Pi_3(w) - \Pi_3^c < 0$. Notice that Russia's profit change $\Delta\Pi_1 = \Pi_1(w) - \Pi_1^c$ has a single-peak at the optimal weight w^*. Because the total profit change of the three players is decreasing in general but the consumer surplus is increasing, the social welfare change $\Delta W = W^w - W^c$ appears to be less clear.[12] All these changes can be conveniently illustrated in Figure 1 and Figure 2.

[12] Several quantitative results for the case of $w = w^*$ are given in Appendix 3.

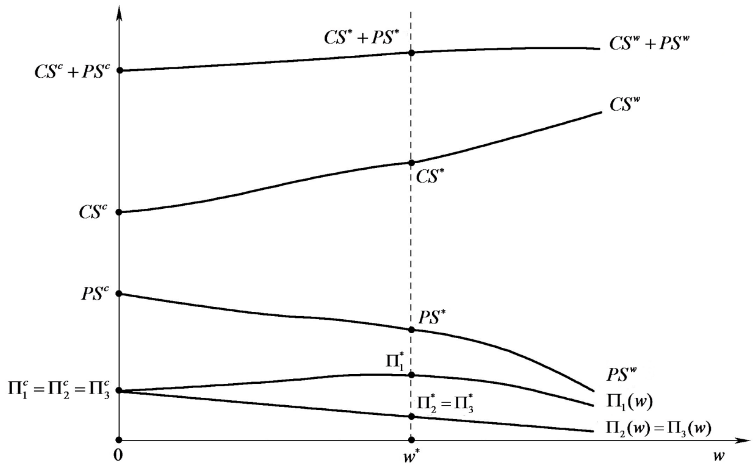

Figure 1: Trends in consumer surplus, producer surplus, and social welfare

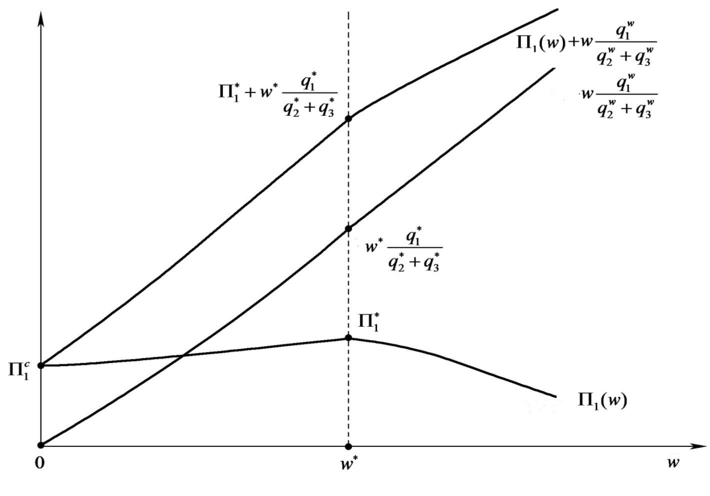

Figure 2: Trends in Russia's profit, nonprofit objective, and total objective

4.3 The Effect of Various Factors on the Optimal Weight

In the previous section we demonstrated how the change of Russia's nonprofit weight w influences quantity change of equilibrium price, outputs, profits, consumer surplus, and social welfare. In this section we will analyse the quantitative effect of various factors on Russia's optimal weight w^*. This analysis is made possible because the optimal weight w^* has been given explicitly in formula (34), i.e., $w^* = \frac{1}{9b}(a+3c_1-2c_2-2c_3)(a-3c_1+c_2+c_3)$. Let $k = \frac{c_1+c_2}{2}$ denote the average cost of Norway and Algeria. Then we have $w^* = \frac{1}{9b}(a+3c_1-4k)(a-3c_1+2k)$. We list four changes of the first-order derivatives in Table 1. For example, $\partial w^*/\partial c_1 < 0$ says that Russia's optimal weight w^* is a decreasing function of c_1. We know that w^* is defined to be the nonprofit weight that makes Russia achieve its maximal profit in the resulting Cournot game. Other things being equal, an increase in c_1 will decrease Russia's equilibrium production by the law of maximizing profit with marginal revenue and marginal cost. By (26), w is an increasing function of q_1, then a reduction in q_1 implies a reduction in w. This also shows that Russia's interest in RMS will be limited to its own cost level if it still cares about profit objective.

In fact, we can also obtain the second order derivatives. For example, it is not hard to see that $\partial^2 w^*/(\partial k \partial c_1) > 0$. From $\partial w^*/\partial c_1 < 0$, we have already known that the optimal weight is a decreasing function of Russia's cost. The cross partial derivative $\partial^2 w^*(\partial k \partial c_1)$ can be rewritten as $(\partial/\partial k)(\partial w^*/\partial c_1)$, which is more helpful to demonstrate its intuitive economic interpretation. $(\partial/\partial k)(\partial w^*/\partial c_1) > 0$ implies that, although a higher cost level of Russia has a negative effect on its optimal weight, the negative effect will become smaller with a cost increase of its rivals. Other cases can be analysed similarly and are therefore omitted.

Table 1: The effect of various factors on the optimal weight

	Derivative type	Derivative expression	Derivative sign
1	$\frac{\partial w^*}{\partial c_1}$	$\frac{1}{b}(-2c_1 + 2k)$	< 0
2	$\frac{\partial w^*}{\partial k}$	$\frac{2}{9b}(-a + 9c_1 - 8k)$	indefinite
3	$\frac{\partial w^*}{\partial a}$	$\frac{1}{9b}(2a - 2k)$	> 0
4	$\frac{\partial w^*}{\partial b}$	$-\frac{1}{9b^2}(a + 3c_1 - 4k)(a - 3c_1 + 2k)$	< 0

5 Conclusion

A number of authors have recently written on relations between the European Union countries and Russia with respect to natural gas. Their major concern is that unlike other profit maximising natural gas companies, Russia via Gazprom as the EU's major natural gas provider is driven not only by profit but also by its geopolitical aspiration. To capture Russia's geopolitical consideration and explore its impact on the market, we have developed a modified Cournot oligopoly model where Russia (Gazprom) maximizes its relative market share together with its profits to compete against two traditional profit maximisers Algeria (Sontrach) and Norway (Statoil). The relative market share (RMS) reflects Russia's nonprofit objective. In comparison with Jansen et al.'s (2012) approach, we have demonstrated that the use of RMS makes it not only possible to achieve the same results in a more general environment, but also permits us to derive rich and clear-cut quantitative analysis results for equilibrium price, outputs, consumer surplus, and social welfare. Our study has shown for this modified Cournot model that by seeking a proper market share, Gazprom can actually attain the same profits of a Stackelberg leader in a simultaneous decision-making model as in the classical leader-follower sequential decision-making model.

We have shown that in our modified Cournot model, Russia has a considerable rise both in production and in profit (even up a 100% rise in production and a 1/3 rise in profit; see(A3-9) in Appendix 3). The total production will go up and thus bring prices down. As a result, the consumer surplus will increase, which means good news for the EU's consumers. However, the production and profit for both Algeria and Norway will decrease, and the total profit of all three players will also decrease. This means that the change in social welfare is indefinite, and that if Russia does pursue a comprehensive goal of both profit and market share, it will indeed hurt Norway and Algeria in term of profit. One should, however, treat the possible increase of consumer surplus due to Russia's nonprofit objective with caution. In the long run, this will probably make Russia's market power even greater in European energy market, which in turn may further help Russia to enhance its geopolitical influence on Europe.

We have also shown that Russia's optimal weight is affected by many factors, such as cost, market capacity, and demand elasticity, and most of the effects can be analyzed in a quantitative way. For example, if all players' costs are equal, Russia could improve its standard market share by 80% via proper choice of optimal weight. In other words, the standard market share will increase from 1/3 in the classical Cournot model to 3/5 in our new model (see (A3-15) in the Appendix 3). We point out that, only one player - Russia is assumed to have nonprofit objective. An important precondition for this assumption is that Russia's rivals are limited by production capacity in the short run. In the long run, this game will be also affected by many other factors such as natural gas reserve stock and infrastructure investment. To pursue nonprofit objective, one has to investment heavily in its production and transportation. These investments are often irreversible. As other energy markets, the natural gas market is also highly affected by global economy, technology advancement, unconventional gas development, and international politics. All these uncertainties remind us that any huge natural gas investment should act with extreme caution and care.

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Appendix 1 Computation for the Optimal Weight w^*

In this appendix we derive the optimal weight w^* in the modified Cournot model.

It follows from (26) and (32) that

$$\begin{aligned}
\Pi_1(w) &= (p^w - c_1)q_1^w \\
&= (\frac{1}{8}[(a+c_1+3c_2+3c_3)+\sqrt{(a+c_1-c_2-c_3)^2-8bw}]-c_1) \\
&\quad \frac{1}{8b}[(5a-3c_1-c_2-c_3)-3\sqrt{(a+c_1-c_2-c_3)^2-8bw}] \\
&= \frac{1}{8}[(a+7c_1+3c_2+3c_3)+\sqrt{(a+c_1-c_2-c_3)^2-8bw}] \\
&\quad \frac{1}{8b}[(5a-3c_1-c_2-c_3)-3\sqrt{(a+c_1-c_2-c_3)^2-8bw}].
\end{aligned} \tag{A1-1}$$

We have the following first-order condition

$$\begin{aligned}
\frac{\partial \Pi_1(w)}{\partial w} &= \frac{1}{8}\frac{-8b}{2\sqrt{(a+c_1-c_2-c_3)^2-8bw}}\frac{(5a-3c_1-c_2-c_3)-3\sqrt{(a+c_1-c_2-c_3)^2-8bw}}{8b} \\
&\quad + \frac{(a-7c_1+3c_2+3c_3)+\sqrt{(a+c_1-c_2-c_3)^2-8bw}}{8}\frac{(-3)\cdot(-8b)}{16b\sqrt{(a+c_1-c_2-c_3)^2-8bw}} = 0.
\end{aligned} \tag{A1-2}$$

Rearrangement yields

$$-\frac{1}{16}\frac{(5a-3c_1-c_2-c_3)-3\sqrt{(a+c_1-c_2-c_3)^2-8bw}}{\sqrt{(a+c_1-c_2-c_3)^2-8bw}} \\ +\frac{3}{16}\frac{(a-7c_1+3c_2+3c_3)+\sqrt{(a+c_1-c_2-c_3)^2-8bw}}{\sqrt{(a+c_1-c_2-c_3)^2-8bw}}=0. \tag{A1-3}$$

It further reduces to

$$3\sqrt{(a+c_1-c_2-c_3)^2-8bw}=a+9c_1-5c_2-5c_3\ . \tag{A1-4}$$

By hypothesis that $a>c_i$ and $c_1>c_2>c_3$, we have $a>c_1>\frac{1}{2}(c_2+c_3)$. Then

$$a+9c_1-5c_2-5c_3>a+9(\frac{c_2+c_3}{2})-5c_2-5c_3=a-\frac{c_2+c_3}{2}>0 \tag{A1-5}$$

and

$$\begin{aligned} 8bw &= (a+c_1-c_2-c_3)^2-\frac{1}{9}(\ a+9c_1-5c_2-5c_3)^2 \\ &= [a+c_1-c_2-c_3+\frac{1}{3}(a+9c_1-5c_2-5c_3)][a+c_1 \\ &-c_2-c_3-\frac{1}{3}(\ a+9c_1-5c_2-5c_3)]. \end{aligned} \tag{A1-6}$$

The above equality gives the optimal weight

$$w^*=\frac{1}{9b}(a+3c_1-2c_2-2c_3)(a-3c_1+c_2+c_3). \tag{A1-7}$$

Appendix 2 Results Concerning Russia's Profit Maximisation

In this appendix we compute equilibrium results when Russia maximises its profit in the modified Cournot model.

Substituting(A1-7) into(26), (29), and (30), respectively, we obtain q_1^*, q_2^*, and q_3^*.

$$\begin{aligned} q_1^* =& \frac{1}{8b}[(5a-3c_1-c_2-c_3)-3\sqrt{(a+c_1-c_2-c_3)^2-8bw^*}] \\ =& \frac{1}{8b}[(5a-3c_1-c_2-c_3) \\ & -3\sqrt{(a+c_1-c_2-c_3)^2-\frac{8b}{9b}(a+3c_1-2c_2-2c_3)(a-3c_1+c_2+c_3)}] \\ =& \frac{1}{8b}[(5a-3c_1-c_2-c_3) \\ & -\sqrt{9(a+c_1-c_2-c_3)^2-8(a+3c_1-2c_2-2c_3)(a-3c_1+c_2+c_3)}] \\ =& \frac{1}{8b}[(5a-3c_1-c_2-c_3)-\sqrt{(a+9c_1-5c_2-5c_3)^2}] \\ =& \frac{1}{8b}[(5a-3c_1-c_2-c_3)-(a+9c_1-5c_2-5c_3)] \\ =& \frac{1}{2b}(a-3c_1+c_2+c_3). \end{aligned} \tag{A2-1}$$

The above fifth line and fourth line have respectively used (A1-5) and the following (A2-2).

$$
\begin{aligned}
&9(a+c_1-c_2-c_3)^2-8(a_1+3c_1-2c_2-2c_3)(a-3c_1+c_2+c_3)\\
&=9(a^2+c_1^2+c_2^2+c_3^2+2ac_1+2c_2c_3-2ac_2-2ac_3-2c_1c_2-2c_1c_3)\\
&\quad-8(a^2-3ac_1+ac_2+ac_3+3ac_1-9c_1^2+3c_1c_2+3c_1c_3-2ac_2\\
&\quad+6c_1c_2-2c_2^2-2c_2c_3-2ac_3+6c_1c_3-2c_2c_3-2c_3^2)\\
&=9(a^2+c_1^2+c_2^2+c_3^2+2ac_1+2c_2c_3-2ac_2-2ac_3-2c_1c_2-2c_1c_3)\\
&\quad-8(a^2-9c_1^2-2c_2^2-2c_3^2-ac_2-ac_3+9c_1c_2+9c_1c_3-4c_2c_3)\\
&=a^2+81c_1^2+25c_2^2+25c_3^2+18ac_1-10ac_2-10ac_3-90c_1c_2-90c_1c_3+50c_2c_3\\
&=(a+9c_1-5c_2-5c_3)^2.
\end{aligned}
\tag{A2-2}
$$

Similarly, we can get

$$
\begin{aligned}
q_2^* &= \frac{1}{8b}[(a+c_1-5c_2+3c_3)+\sqrt{(a+c_1-c_2-c_3)^2-8bw^*}]\\
&= \frac{1}{8b}[(a+c_1-5c_2+3c_3)\\
&\quad+\sqrt{(a+c_1-c_2-c_3)^2-\frac{8b}{9b}(a+3c_1-2c_2-2c_3)(a-3c_1+c_2+c_3)}]\\
&= \frac{1}{8b}[(a+c_1-5c_2+3c_3)+\frac{1}{3}\sqrt{(a+9c_1-5c_2-5c_3)^2}]\\
&= \frac{1}{24b}[3(a+c_1-5c_2+3c_3)+\sqrt{(a+9c_1-5c_2-5c_3)^2}]\\
&= \frac{1}{6b}(a+3c_1-5c_2+c_3)
\end{aligned}
\tag{A2-3}
$$

and

$$
\begin{aligned}
q_3^* &= \frac{1}{8b}[(a+c_1+3c_2-5c_3)+\sqrt{(a+c_1-c_2-c_3)^2-8bw^*}]\\
&= \frac{1}{8b}[(a+c_1+3c_2-5c_3)+\frac{1}{3}\sqrt{(a+9c_1-5c_2-5c_3)^2}]\\
&= \frac{1}{24b}[3(a+c_1+3c_2-5c_3)+\sqrt{(a+9c_1-5c_2-5c_3)^2}]\\
&= \frac{1}{6b}(a+3c_1+c_2-5c_3).
\end{aligned}
\tag{A2-4}
$$

Using (A2-1), (A2-3), and (A2-4), we obtain the total output

$$
\begin{aligned}
Q^* &= \frac{1}{2b}(a-3c_1+c_2+c_3)+\frac{1}{6b}(a+3c_1-5c_2+c_3)+\frac{1}{6b}(a+3c_1+c_2-5c_3)\\
&= \frac{1}{6b}(5a-3c_1-c_2-c_3).
\end{aligned}
\tag{A2-5}
$$

Combining (A2-5) and (1) leads to

$$p^*=a-bQ^*=a-\frac{b}{6b}(5a-3c_1-c_2-c_3)=\frac{1}{6}(a+3c_1+c_2+c_3). \tag{A2-6}$$

Using (A2-1), (A2-3), (A2-4), and (A2-6) we obtain the profit of each player as follows:

$$\Pi_1^* = \frac{1}{12b}(a-3c_1+c_2+c_3)^2=\frac{1}{3}b(q_1^*)^2, \tag{A2-7}$$

$$\Pi_2^* = \frac{1}{36b}(a + 3c_1 - 5c_2 + c_3)^2 = b(q_2^*)^2, \tag{A2-8}$$

and

$$\Pi_3^* = \frac{1}{36b}(a + 3c_1 + c_2 - 5c_3)^2 = b(q_3^*)^2. \tag{A2-9}$$

Thus, the total profit is given by

$$\begin{aligned}\Pi^* &= \Pi_1^* + \Pi_2^* + \Pi_3^* \\ &= \frac{1}{36b}[3(a - 3c_1 + c_2 + c_3)^2 + (a + 3c_1 - 5c_2 + c_3)^2 + (a + 3c_1 + c_2 - 5c_3)^2].\end{aligned} \tag{A2-10}$$

The consumer surplus and social welfare are given respectively

$$\begin{aligned}CS^* &= \frac{1}{2}(a - p^*)Q^* = \frac{1}{2b}(a - p^*)^2 \\ &= \frac{1}{2b}[a - \frac{1}{6}(a + 3c_1 + c_2 + c_3)]^2 = \frac{1}{72b}(5a - 3c_1 - c_2 - c_3)^2\end{aligned} \tag{A2-11}$$

and

$$\begin{aligned}W^* &= \Pi^* + CS^* \\ &= \frac{1}{36b}[3(a - 3c_1 + c_2 + c_3)^2 + (a + 3c_1 - 5c_2 + c_3)^2 + (a + 3c_1 + c_2 - 5c_3)^2] \\ &+ \frac{1}{72b}(5a - 3c_1 - c_2 - c_3)^2 \\ &= \frac{1}{72b}[6(a - 3c_1 + c_2 + c_3)^2 + 2(a + 3c_1 - 5c_2 + c_3)^2 + 2(a + 3c_1 + c_2 - 5c_3)^2 \\ &+ (5a - 3c_1 - c_2 - c_3)^2].\end{aligned} \tag{A2-12}$$

By (31) and (32), the consumer surplus can be written as a function of weight w

$$\begin{aligned}CS^w &= \frac{1}{2}(a - p^w)Q^w = \frac{1}{2}b(Q^w)^2 \\ &= \frac{1}{2}b\left\{\frac{1}{8b}[(7a - c_1 - 3c_2 - 3c_3) - \sqrt{(a + c_1 - c_2 - c_3)^2 - 8bw}]\right\}^2 \\ &= \frac{1}{128b}[(7a - c_1 - 3c_2 - 3c_3) - \sqrt{(a + c_1 - c_2 - c_3)^2 - 8bw}]^2.\end{aligned} \tag{A2-13}$$

By (29), (30) and (32), the profit functions of Norway and Algeria are given respectively by

$$\begin{aligned}\Pi_2(w) = \frac{1}{8b}\{&\frac{1}{8}[(a + c_1 + 3c_2 + 3c_3) + \sqrt{(a + c_1 - c_2 - c_3)^2 - 8bw}] \\ &- c_2\}[(a + c_1 - 5c_2 + 3c_3) + \sqrt{(a + c_1 - c_2 - c_3)^2 - 8bw}],\end{aligned} \tag{A2-14}$$

$$\begin{aligned}\Pi_3(w) = \frac{1}{8b}\{&\frac{1}{8}[(a + c_1 + 3c_2 + 3c_3) + \sqrt{(a + c_1 - c_2 - c_3)^2 - 8bw}] \\ &- c_3\}[(a + c_1 + 3c_2 - 5c_3) + \sqrt{(a + c_1 - c_2 - c_3)^2 - 8bw}].\end{aligned} \tag{A2-15}$$

From (A2-13), consumer surplus is an increasing function of w, so $\Delta CS = CS^w - CS^c > 0$ for any given $w \in (0, \bar{w}]$, with $\bar{w}$ defined by $q_1(\bar{w}) = (L_1 + L_2)/2$. Similarly, from (A2-14) and (A2-15), it is easy to see that both $\Pi_2(w)$ and $\Pi_3(w)$ are strictly decreasing with w. Then $\Delta\Pi_2 = \Pi_2(w) - \Pi_2^c = \Pi_2(w) - \Pi_2(0) < 0$ and $\Delta\Pi_3 = \Pi_3(w) - \Pi_3^c = \Pi_3(w) - \Pi_3(0) < 0$. But for Russia, its profit change is indefinite by (33). So the change in total profit of the three players is undetermined and so is the social welfare.

Appendix 3 Comparison of Equilibrium Results

In this appendix we compare a variety of changes in equilibrium when Russia maximises its profits in the modified Cournot model with the classical Cournot model.

Using (A2-1), (A2-3), (A2-4), and (3), we can compute the production difference between the two models as follows:

$$\Delta q_1^* = q_1^* - q_1^c = \frac{1}{2b}(a - 3c_1 + c_2 + c_3) - \frac{1}{4b}(a - 3c_1 + c_2 + c_3) = \frac{1}{4b}(a - 3c_1 + c_2 + c_3) > 0, \quad \text{(A3-1)}$$

$$\Delta q_2^* = q_2^* - q_2^c = \frac{1}{6b}(a + 3c_1 - 5c_2 + c_3) - \frac{1}{4b}(a + c_1 - 3c_2 + c_3) = -\frac{1}{12b}(a - 3c_1 + c_2 + c_3) < 0, \quad \text{(A3-2)}$$

$$\Delta q_3^* = q_3^* - q_3^c = \frac{1}{6b}(a + 3c_1 + c_2 - 5c_3) - \frac{1}{4b}(a + c_1 + c_2 - 3c_3) = -\frac{1}{12b}(a - 3c_1 + c_2 + c_3) < 0. \quad \text{(A3-3)}$$

According to (A2-5) and (4), the total production difference is

$$Q^* - Q^c = \frac{1}{6b}(5a - 3c_1 - c_2 - c_3) - \frac{1}{4b}(3a - c_1 - c_2 - c_3) = \frac{1}{12b}(a - 3c_1 + c_2 + c_3) > 0. \quad \text{(A3-4)}$$

By (A2-6) and (4), the equilibrium price difference is given by

$$p^* - p^c = \frac{1}{6}(a + 3c_1 + c_2 + c_3) - \frac{1}{4}(a + c_1 + c_2 + c_3) = -\frac{1}{12}(a - 3c_1 + c_2 + c_3) < 0. \quad \text{(A3-5)}$$

Then from (A2-7)-(A2-9) and (5)-(7), the profit changes have the following properties.

$$\begin{aligned} \Delta\Pi_1^* = \Pi_1^* - \Pi_1^c &= \frac{1}{12b}(a - 3c_1 + c_2 + c_3)^2 - \frac{1}{16b}(a - 3c_1 + c_2 + c_3)^2 \\ &= \frac{1}{48b}(a - 3c_1 + c_2 + c_3)^2 > 0, \end{aligned} \quad \text{(A3-6)}$$

$$\begin{aligned} \Delta\Pi_2^* = \Pi_2^* - \Pi_2^c &= \frac{1}{36b}(a + 3c_1 - 5c_2 + c_3)^2 - \frac{1}{16b}(a + c_1 - 3c_2 + c_3)^2 \\ &= -\frac{1}{144b}(5a + 9c_1 - 19c_2 + 5c_3)(a - 3c_1 + c_2 + c_3) < 0, \end{aligned} \quad \text{(A3-7)}$$

$$\begin{aligned}\Delta\Pi_3^* = \Pi_3^* - \Pi_3^c = &\ \frac{1}{36b}(a+3c_1+c_2-5c_3)^2 - \frac{1}{16b}(a+c_1+c_2-3c_3)^2 \\ = -&\frac{1}{144b}(5a+9c_1+5c_2-19c_3)(a-3c_1+c_2+c_3) < 0.\end{aligned} \tag{A3-8}$$

To derive (A3-7) and (A3-8), (A1-5) has been used. Furthermore, combining (A3-6) and (5) leads to

$$\frac{\Delta\Pi_1^*}{\Pi_1^c} = \frac{1}{3}. \tag{A3-9}$$

It is not hard to see that, the reason for the decrease in profits of Algeria and Norway is the reduction for both of their outputs and the market price. The increase in Russia's profit attributes mainly to the property that the "decreasing effect" from price reduction is lower than the "increasing effect" from its own production rise. From (A3-6)-(A3-8), the total profit change is

$$\begin{aligned}\Delta\Pi^* &= \Pi^* - \Pi^c = \Delta\Pi_1^* + \Delta\Pi_2^* + \Delta\Pi_3^* \\ &= \frac{1}{48b}(a-3c_1+c_2+c_3)^2 + \frac{1}{144b}(5a+9c_1-19c_2+5c_3) \\ &\quad (-a+3c_1-c_2-c_3) + \frac{1}{144b}(5a+9c_1+5c_2-19c_3)(-a+3c_1-c_2-c_3) \\ &= \frac{1}{144b}(-7a-27c_1+17c_2+17c_3)(a-3c_1+c_2+c_3) < 0.\end{aligned} \tag{A3-10}$$

We have used the following (A3-11) in the last line of (A3-10).

$$7a+27c_1-17c_2-17c_3 > 7a+27\frac{c_2+c_3}{2}-17c_2-17c_3 = \frac{7}{2}(a-c_2)+\frac{7}{2}(a-c_3) > 0. \tag{A3-11}$$

According to (A2-11) and (8), the consumer surplus difference is given by

$$\begin{aligned}\Delta CS^* &= CS^* - CS^c \\ &= \frac{1}{72b}(5a-3c_1-c_2-c_3)^2 - \frac{1}{32b}(3a-c_1-c_2-c_3)^2 \\ &= \frac{1}{288b}(19a-9c_1-5c_2-5c_3)(a-3c_1+c_2+c_3) > 0\end{aligned} . \tag{A3-12}$$

Note that the last line in (A3-12) has used the following result

$$19a-9c_1-5c_2-5c_3 = 9(a-c_1)+5(a-c_2)+5(a-c_3) > 0. \tag{A3-13}$$

By (A3-10) and (A3-12), we have

$$\begin{aligned}\Delta W^* = &\ \Delta\Pi^* + \Delta CS^* \\ = &\ \frac{1}{144b}(-7a-27c_1+17c_2+17c_3)(a-3c_1+c_2+c_3) \\ &+ \frac{1}{288b}(19a-9c_1-5c_2-5c_3)(a-3c_1+c_2+c_3) \\ = &\ \frac{1}{288b}(5a-63c_1+29c_2+29c_3)(a-3c_1+c_2+c_3).\end{aligned} \tag{A3-14}$$

If costs of the three players are equal, then it follows from (A2-1), (A2-3), (A2-4), and (3) that

$$\frac{q_1^*}{q_1^* + q_2^* + q_3^*} - \frac{q_1^c}{q_1^c + q_2^c + q_3^c} = \frac{3}{5} - \frac{1}{3} = \frac{4}{15} \approx 26.67\%. \tag{A3-15}$$

So, Russia's market share will rise by $\frac{4}{15}/(1/3) = 80\%$. Its optimal weight can be rewritten as

$$\begin{aligned} w^* &= \frac{1}{9b}(a + 3c_1 - 2c_2 - 2c_3)(a - 3c_1 + c_2 + c_3) \\ &= \frac{1}{9b} 3b(q_2^* + q_3^*) 2bq_1^* \\ &= \frac{2}{3} b(q_2^* + q_3^*) q_1^*. \end{aligned} \tag{A3-16}$$

By (A2-1), (A2-3), (A2-4), (A2-7), and (A3-16), Russia's objective $\Pi_1^* + w^* \frac{q_1^*}{q_2^* + q_3^*}$ can be reformulated as

$$\Pi_1^* + w^* \frac{q_1^*}{q_2^* + q_3^*} = \frac{1}{3} b(q_1^*)^2 + \frac{2}{3} b(q_2^* + q_3^*) q_1^* \frac{q_1^*}{q_2^* + q_3^*} = b(q_1^*)^2. \tag{A3-17}$$

Notice that Russia's objective value $b(q_1^*)^2$ given by (A3-17) surprisingly resembles Russia's profit $\Pi_1^c = b(q_1^c)^2$ given by (5) in the classical Cournot model.