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"We're all in this together"? A DSGE interpretation

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Abstract

The recent global economic downturn has resulted in hardship for many individuals and the unequal distribution of this hardship across agents is frequently debated. This paper constructs a small scale New Keynesian DSGE model to test whether individuals suffer to similar degrees during recessions: in effect testing the common political mantra ‘we’re all in this together’. It does this by including heterogeneity in the actions of households through their access to capital markets distinguishing those with full access (Ricardian agents) from those with no access (*rule-of-thumb* agents). In aggregate welfare movements as a result of recessionary shocks are small but this hides a big divergence with the credit constrained significantly losing. There is a redistribution of welfare from non-Ricardian to Ricardian households from the shock, and under reasonable calibrations, the latter are seen to gain at the expense of the former.

Key words: Cost of business cycles; rule-of-thumb consumers; welfare; heterogeneity.
JEL Classification: E32, I30, D63.

1 Introduction

The recent global economic downturn has resulted in hardship for many individuals in many countries; the unequal distribution of this hardship across agents is a topic of political significance, which is frequently debated. In autumn 2008, before the extent of the recession was known, The Economist (October 23, 2008) suggested that there was a desire for an ‘equality of sacrifice’ in the looming downturn with the fear that this equality would not be reached. A commentary in the Financial Times (March 25, 2011) reflecting upon the movements in share prices for different retail firms, inferred that the middle and lower classes in the US and UK were suffering more than the wealthiest in the current recession, supporting the concerns of the earlier Economist article.[1] Beyond this class and income divide there is clear anecdotal evidence that the current recession is impacting some more than others. Perhaps the most globally widespread, at least in Europe, are the high levels of youth unemployment. These debates and fears have entered the political rhetoric of the recession with calls of a ‘*squeezed* and *anxious* middle’ in the UK and US respectively.[2]

[1]The article compares the appreciation in shares in companies such as Tiffany and Saks against the depreciation in companies such as Walmart. Another article in the Financial Times came to similar conclusions using anecdotal evidence interviewing families in the US (Financial Times, July 30, 2010).

[2]The ‘anxious middle’ is an expression used in the US by Larry Summers former US Treasury Secretary; in the UK the leader of the opposition Ed Miliband has frequently used the phase the ‘squeezed middle’ (Financial Times, December 20, 2011).

Despite this there are continuing calls from ruling politicians that 'we're all in this together'.[3]

Lucas (2003) abstracted from these heterogeneity issues to propose that the impact of business cycles on aggregate welfare was negligible; through analysing movements in aggregate US consumption around trend, the welfare gain of removing economic fluctuations was calculated to be the utility equivalent of less than one-tenth a percentage point increase in average consumption. However, a number of papers have refuted this result by removing the representative agent assumption of Lucas' analysis. Krusell et al. (1999) claim that the welfare costs of business cycles are of interest because of the heterogeneous distribution of these costs: abstracting away from this abstracts away from the crux of the issue. Through introducing idiosyncratic agent productivity shocks combined with incomplete insurance markets Krusell et al. (1999) show that eliminating business cycles would be to the benefit of the poor and unemployed: fluctuations hurt these the most, something reflected in the anecdotal evidence of the recent downturn. Krusell et al. (2009) obtain comparable results showing that business cycle fluctuations disproportionately impact the poor and rich more than the middle classes. Mukoyama & Şahin (2006) perform a similar analysis to conclude that it is unskilled workers who suffer most from business cycles. Using a model including incomplete insurance markets and heterogeneity between agents across skill levels, employment status and discount factors, they reach similar conclusions as Krusell et al. (1999, 2009) through demonstrating that it is the unskilled (and therefore poor) who suffer the most because of their inability to self-insure.

The innovation of this paper is to address this question using a New Keynesian dynamic stochastic general equilibrium (DSGE) model. It does this through building upon the recent research using non-Ricardian or rule-of-thumb agents. These agents are assumed not to have access to credit and therefore consume 'hand-to-mouth' by spending their disposable income each period. This assumption means that these agents cannot smooth their consumption (unlike the traditional Ricardian agents who follow the permanent income hypothesis) and their inclusion in models is as a result of the empirical observation that for many individuals their spending closely tracks their income: see for example Campbell & Mankiw (1989).[4] We propose that these two agents (the Ricardian and non-Ricardian households) represent two clear distinct groups in society of empirically non-trivial proportions, and hypothesise that the latter will come from those individuals at the lower end of the income distribution; although the results are not sensitive to this hypothesis, the interpretations of these results are. Evidence from both micro and macro-econometric studies suggest that it is those agents with a lack of assets, and with it collateral, who cannot gain access to capital: see for example Jappelli & Pagano (1989), Zeldes (1989), Sarantis & Stewart (2003) and Sullivan (2008). The *ex-ante* justification for the inclusion of rule-of-thumb households in DSGE models is *ex-post* supported by the observation that these models better match empirical data: see for example Galí et al. (2007), Andrés et al. (2008), Graham (2008), Furlanetto & Seneca (2009), Boscá et al. (2011) and Furlanetto & Seneca (2012). Rule-of-thumb consumers also appear in many central banks and policy institutions DSGE models.[5] However, despite the growing use of these models, limited anal-

[3]This is most noticeably used by David Cameron in the UK but has also been used by others such as Barack Obama. The former in a speech given on the August 15 2011 (for example) and the latter in an interview with CBS News April 19, 2009.

[4]Campbell & Mankiw (1989) represents the seminal empirical work in identifying non-Ricardian households but the results have been replicated in further studies: see for example Jappelli & Pagano (1989), Evans & Karras (1996) and Sarantis & Stewart (2003). The results are shown to be robust to the specification, time period and country used.

[5]Iwata (2009) notes that the institutions that use such households include the European Central Bank (NAWM),

ysis currently exists on disaggregated variables. Although some papers do report on and display heterogeneous dynamics, these tend to only give more detail behind the aggregate variables and no in depth analysis of any frictions or divergences are discussed.

The contribution of this paper is to apply these now standard models to perform a detailed analysis of heterogeneous dynamics, with specific reference to adverse shocks: defined here as those that generate a negative output gap. We argue that these models are best suited towards explaining a situation where rule-of-thumb households would rather borrow than save, because it seems implausible that credit institutions would prohibit deposits from individuals. The model is similar to those used in Galí et al. (2007) and Bilbiie (2008). Compared to the former this research abstracts from physical capital accumulation and a fiscal authority; compared to the latter, it abstracts from fixed costs to production. Moreover, this model is differentiated from Bilbiie (2008) by allowing rule-of-thumb households to control their labour supply in order to optimise in any given time period. This is an important abstraction because it highlights the main transmission mechanism and intuition behind the aggregate and disaggregate results. Further, these abstractions are advantageous because the resulting simplicity of the model means that algebraic conditions in general equilibrium can be found, independent of parameter calibrations.

The results provided by the model present a clear and intuitive message. When an adverse shock strikes the economy Ricardian households are able to borrow from capital markets in order to insulate themselves. Non-Ricardian households, on the other hand, are unable to do this and are only left with their employment decisions from which to optimise, which they do through increasing their labour supply above that which would prevail if they had access to credit. This increases their disposable income and aggregate production however the former is not sufficient to purchase all of the latter and the surplus is consumed by the Ricardian agents who purchase this through additional dividends paid by firms through an improvement in profits from the supply of cheap non-Ricardian labour. The non-constrained agents both work less and consume more than their constrained counterparts; there is a redistribution of welfare from non-Ricardian to Ricardian households. These results are obtained through deriving algebraic properties in general equilibrium, dynamic simulations and moreover through observing welfare movements through the evaluation of a second order Taylor series expansion of the heterogeneous households homogeneous utility function. Not only is it shown that Ricardian households achieve higher levels of welfare from the adverse shocks than non-Ricardian households, they are also shown to experience positive welfare movements in the presence of these shocks: the Ricardian households *gain* compared to steady state values. The results support the conclusion of Lucas (2003) that in aggregate welfare losses are small, but this hides a disparity across agents with one set losing significantly whilst the other gain. These results are theoretically robust and are also supported by an empirical investigation performed showing a negative correlation between growth and income inequality.

The paper proceeds in the following way: Section 2 derives a model which includes a fraction of non-Ricardian (rule-of-thumb) households. Section 3 analyses the heterogeneous impact of adverse shocks on agents by deriving analytical expressions, through observing dynamic simulations and deriving a disaggregated welfare criterion. Section 4 tests the robustness of the results to the labour market assumption and Section 5 empirically tests the implication of the model. Section 6 performs further sensitivity analysis and Section 7 concludes.

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the European Commission (QUEST III), the Federal Reserve Board (SIGMA) and the International Monetary Fund (GIMF).

2 The model

The model presented below is a simple, cashless, DSGE model with sticky prices including five types of economic agents; a continuum of households split into two heterogeneous groups, a continuum of monopolistically competitive firms producing intermediate goods and a perfectly competitive firm producing the final good, and a monetary authority. The model is similar to Galí et al. (2007) but abstracts away from physical capital accumulation and a governmental sector. The model is simplified in order to isolate the transmission mechanisms involved and to analyse the algebraic properties of models that include non-Ricardian consumers.

2.1 Households

There is a continuum $[0, 1]$ of infinitely lived households, all of which consume the final good and supply labour to firms. A proportion of these households $(1 - \lambda)$ are 'Ricardian' who have access to capital markets and can trade in a full set of state contingent securities. The remaining proportion (λ) are 'non-Ricardian' who have no access to capital markets. The period utility function is assumed to be the same for both types of household and is given by:

$$U_t^i = \varepsilon_t^b \left(\frac{\left(C_t^i\right)^{1-\sigma}}{1-\sigma} - \frac{\left(N_t^i\right)^{1+\varphi}}{1+\varphi} \right) \tag{1}$$

where C_t and N_t are the amount of consumption and employment in period t and ε_t^b represents an exogenous shock to the discount rate which affects intertemporal substitution preferences of households. The parameter σ is the coefficient of relative risk aversion and φ is the inverse elasticity of work with respect to the real wage. Superscript i differentiates these variables between Ricardian $(i = R)$ and non-Ricardian $(i = NR)$ households. Households are assumed to supply labour in a perfectly competitive market with no frictions or time delays: sensitivity of the results to this labour market assumption is discussed in Section 4.

Ricardian households

Ricardian households receive income from their labour supply (at a wage rate W_t), from dividends, D_t^R, and from maturing one period bonds purchased in the previous time period, B_t^R. They use this income to reinvest in the bond market (at a given return R_t), purchase the consumption good (at a price P_t), and to pay any lump sum tax levied by the fiscal authority, T_t^R. This leaves a budget constraint for the Ricardian households given by:

$$P_t C_t^R + \frac{B_{t+1}^R}{R_t} \leq B_t^R + W_t N_t^R - P_t T_t^R + D_t^R \tag{2}$$

Ricardian households maximise expected lifetime utility (given by the sum of all occurrences of function (1) from $t = 0$ to $t = \infty$) discounting future periods of utility by a factor $\beta \in (0, 1)$, subject to the budget constraint (2) with respect to the consumption, employment and bond purchases where all prices are taken as given.

Non-Ricardian households

Non-Ricardian households do not have access to bonds markets and as such cannot intertemporally substitute consumption. Moreover they do not own company shares. They simply consume, period by period, their disposable income generated through their supply of labour; this provides the non-Ricardian consumption function:

$$P_t C_t^{NR} = W_t N_t^{NR} - P_t T_t^{NR} \tag{3}$$

where equality is given by assumed strictly positive marginal utility from consumption. Non-Ricardian households still optimise their period by period utility by making decisions on how much labour to supply at a given wage rate: maximisation of (1) subject to the budget constraint (3) with respect to consumption and employment where prices and wages are taken as given.

Steady state consumption, employment and utility

It is assumed that the government set lump sum taxes on the two types of households such that the increased income effect of the dividends for the Ricardian agents in steady state is eliminated. This is the only role of taxes and as such these will be equal to this constant steady state level ($T_t^{NR} = T^{NR}$ and $T_t^R = T^R$). This assumption combined with the identical utility function assumed for the two types of households leads to the same consumption and employment profile of the heterogeneous households in steady state: $C^{NR} = C^R = C$ and $N^{NR} = N^R = N$. This assumption simplifies the calculations but is not critical to the main results of the model.[6] However it provides the compelling benchmark whereby if the economy remains in steady state, both types of households will consume and work in identical proportions and consequently derive identical utilities.

2.2 Firms

The production sector is made up of a continuum of monopolistically competitive producers, $j \in [0, 1]$, who employ household labour in order to produce differentiated intermediate goods. These goods are then purchased by a perfectly competitive firm who makes the final good Y, consumed by households.

Final good firm

The agent which produces the final good is modelled as a single representative perfectly competitive firm which combines the intermediate goods using a standard Dixit-Stiglitz aggregator:

$$Y_t = \left(\int_0^1 Y_t(j)^{\frac{1}{1+\mu_t}} dj \right)^{1+\mu_t}$$

where Y_t represents the final good, $Y_t(j)$ represents the intermediate good quantity produced by firm j, and μ_t represents a (stochastic) time varying markup charged by intermediate good firms given by $\mu_t = \mu + \varepsilon_t^\mu$. This markup is possible because each intermediate firm produces a differentiated product with the elasticity of substitution across goods given by ϵ, such that the

[6] This assumption does go against the hypothesis discussed above that non-Ricardian households are poorer than Ricardian households. Sensitivity analysis of the results to this assumption is performed in Section 6.

steady state markup is given by $\mu = 1/(\epsilon - 1)$: ε_t^{μ} represents an AR(1) shock process in log linear form. Profit maximisation of the final good firm, taking all prices as given, yields the following standard demand schedules:

$$Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\frac{1+\mu_t}{\mu_t}} Y_t \quad \forall\, j \in [0,1] \tag{4}$$

where $P_t(j)$ is the price of the intermediate good $Y_t(j)$ and P_t is the price of the final good, in period t. The final good firm is perfectly competitive and as such makes zero profits which provides the following aggregate price index condition:

$$P_t = \left(\int_0^1 P_t(j)^{-\frac{1}{\mu_t}}\, dj\right)^{-\mu_t}$$

Intermediate good firms

A continuum of firms indexed $j \in [0,1]$ are assumed to produce the differentiated intermediate goods, $Y_t(j)$, subject to Cobb-Douglas technology where capital is fixed and normalised:

$$Y_t(j) = \varepsilon_t^a N_t(j)^{1-\alpha} \tag{5}$$

where $N_t(j)$ is the level of labour employment by firm j and ε_t^a represents an AR(1) productivity shock in log linear form. A Calvo (1983) pricing structure is assumed for intermediate goods, where firms in any period get the opportunity to reset prices at a probability $(1-\theta)$. This probability is fixed, exogenous, and independent of when the firm was last randomly selected to reset their prices. With assumed identical intermediate firms, all $(1-\theta)$ firms reseting their price in period t will do so at the same price, P_t^*. A firm that is able to reset their price in any given period will do so to maximise expected future profits given the new reset price, P_t^*, algebraically:

$$\max_{P_t^*} \sum_{k=0}^{\infty} E_t \left\{\theta^k Q_{t,t+k}\left(P_t^* Y_{t+k|t}(j) - \Psi_{t+k}\left(Y_{t+k|t}(j)\right)\right)\right\}$$

subject to the demand from the final goods firms (4) where $\Psi_{t+k}(.)$ is the nominal cost function, $Y_{t+k|t}(j)$ is the expected output in period $t+k$ for a firm who last reset its price in period t, and $Q_{t,t+k} = \beta^k E_t\left\{(C_{t+k}/C_t)^{-\sigma}(P_t/P_{t+k})\right\}$ is the stochastic discount function for nominal payoffs. Solving this problem provides the following first order condition:

$$\sum_{t=0}^{\infty} \theta^k E_t\left\{Q_{t,t+k} Y_{t+k|t}(j)\left(P_t^* - (1+\mu_t)\psi_{t+k|t}\right)\right\} = 0 \tag{6}$$

where $\psi_{t+k|t} = \Psi'_{t+k}$ denotes the nominal marginal cost in period $t+k$ for a firm who reset its price in period t. The remaining suppliers, θ, must maintain the price as they had in period $t-1$.

2.3 Monetary authority

For the conduct of monetary policy a simple log-linear Taylor rule is assumed where the nominal interest rate responds positively and contemporaneously to increases in inflation and output beyond their target level:

$$r_t = \varphi_\pi \pi_t + \varphi_y y_t + \varepsilon_t^r \tag{7}$$

Where the target rate of inflation and output is assumed to be zero and where ε_t^r represents policy error. Throughout the paper lower case letters represent log deviations of variables from their non-stochastic, zero inflation, steady state values; r_t is the log deviation in the nominal interest rate in period t, π_t the log inflation rate between period $t-1$ and t, and y_t the log deviation of output from steady state.

2.4 Market clearing, aggregation and equilibrium conditions

In equilibrium, aggregate consumption and aggregate employment is equal to the weighted average of the two variables across households:

$$C_t = (1-\lambda) C_t^R + \lambda C_t^{NR}, \ \ N_t = (1-\lambda) N_t^R + \lambda N_t^{NR} \tag{8}$$

Moreover, all output must be consumed by individuals:

$$Y_t = C_t \tag{9}$$

The model is solved by deriving the log-linear approximations of the key optimality and market clearing conditions around the non-stochastic steady state with zero inflation. Once all calculations have been performed the general equilibrium model can be expressed through the policy and non-policy blocks, the latter of which can be represented through the following two equations.

Aggregate demand

The aggregate demand relationship can be obtained by combining the log-linear versions of the goods market clearing condition (9), the production function (5), and the aggregate Euler equation obtained through combining the optimisation of the Ricardian and non-Ricardian household utility:[7]

$$\begin{aligned}
y_t &= E_t\{y_{t+1}\} - \Phi\Theta_A\left(r_t - E_t\{\pi_{t+1}\} - E_t\left\{\Delta\varepsilon_{t+1}^b\right\}\right) + \Phi\Theta_B E_t\left\{\Delta\varepsilon_{t+1}^a\right\} \qquad (10)\\
\Phi &= \frac{\Gamma^{-1}}{\Gamma^{-1} - \varphi\lambda(1+\varphi)}\\
\Theta_A &= (1-\lambda)\frac{1}{\sigma}\left(\varphi(1+\mu) + \sigma(1-\alpha)\right)\Gamma\\
\Theta_B &= \varphi\lambda(1+\varphi)\Gamma\\
\Gamma &= \left[\varphi(1+\mu) + \sigma(1-\alpha)\left[1-\lambda(1+\varphi)\right]\right]^{-1}
\end{aligned}$$

Demand is a function of monetary policy, as in a traditional DSGE model, the extent of which is dependent upon the share of non-Ricardian households in the economy. When $\lambda = 0$, the expression

[7] Specifically, the log linear non-Ricardian consumption function is combined with the Ricardian Euler equation to obtain dynamics of aggregate consumption which is equal to output in general equilibrium. The resulting equation will be a function of future consumption, employment and wages where the latter two can be substituted using the production function and an aggregate labour supply function, respectively. A full derivation in a similar model can be obtained from Galí et al. (2007).

becomes a simple log-linear Euler expression in terms of output, with intertemporal substitution between periods defined through the real interest rate. As λ increases so too does Φ, which is a coefficient reflecting that both employment and the real wage rate (which combine to give non-Ricardian households' disposable income) can be expressed as functions of output. Non-Ricardian consumption is a function of output which itself is a function of non-Ricardian consumption which creates a multiplier effect between demand and any initial stimulus.

New Keynesian Phillips curve

The production sector of the economy is independent of the proportion of rule-of-thumb agents and can be summarised by a typical New Keynesian Phillips curve:

$$\pi_t = \beta E_t \{\pi_{t+1}\} + \omega \left(mc_t^r + \varepsilon_t^\mu \right), \quad \omega = \frac{1-\alpha}{1-\alpha+\alpha\epsilon} \frac{(1-\theta)(1-\theta\beta)}{\theta} \tag{11}$$

$$mc_t^r = y_t \left[\sigma + \frac{\varphi+\alpha}{1-\alpha} \right] - \varepsilon_t^a \left[\frac{1+\varphi}{1-\alpha} \right]$$

where $E_t \{\pi_{t+1}\}$ is the expectation in the current time period of inflation in the next, and mc_t^r represents the (log deviations of) real marginal costs of monopolistically competitive firms in the current time period. Increases in output increase consumption in the economy and thus reduce labour supply as households substitute consumption for leisure, and increase the demand for labour in order to produce the extra output. Both of these factors push up the real wage and therefore increase the real marginal cost of production, mc_t^r, to firms.

Exogenous shocks and the closed system

The two equations above illustrate that the four exogenous shocks can be grouped into three separate categories: those which only enter aggregate demand (preference and interest rate shocks); those which only enter the Phillips curve (exogenous movements in the desired markup); and those which enter both (productivity shocks). Preference and interest rate shocks enter the aggregate demand condition because movements in both impact the consumption decisions of Ricardian households.[8] Exogenous increases in the desired markup charged by intermediate firms act as a cost push shock on the economy providing additional inflation given a certain level of output. Productivity increases, on the other hand, increase the productive capacity of the economy at no additional cost to inflation and moreover impact the labour demanded by firms. The former impacts the Phillips curve whilst the latter impacts aggregate demand through the consumption of non-Ricardian households.[9] Although the model is too simple to include the vast array of exogenous processes of larger DSGE models, these four shocks provide us with three possibilities that cover many further extensions. It is also important to highlight that the limited number of shocks in this small model makes direct analysis of the recent recession infeasible from an empirical perspective; however, the purpose of this paper is to provide a theoretically robust analysis of

[8] For the remainder of the analysis only interest rate shocks will be considered from these two exogenous demand processes. Given the identical way that interest rate and preference shocks enter the aggregate demand condition the analysis for the latter will be the equivalent of the former.

[9] Bilbiie (2008) also found that technology shocks enter additively in both the aggregate demand condition and New Keynesian Phillips curve.

general cases. Throughout the paper shocks are assumed to follow a first order autoregressive process with persistence ρ_i and an i.i.d error term η_t^i for $i = \{a, b, l, r\}$.

The economy can be closed through combining the aggregate demand condition (10), the New Keynesian Phillips curve (11) and the Taylor rule (7). Substituting the latter into the former provides the following system:

$$E_t \{u_{t+1}\} = \mathbf{A} u_t + \mathbf{B} \varepsilon_t \tag{12}$$

where $u_t = [y_t, \pi_t]'$, $\varepsilon_t = [\varepsilon_t^r, E_t \{\Delta \varepsilon_{t+1}^b\}, \varepsilon_t^a, \varepsilon_{t+1}^a, \varepsilon_t^\mu]'$ and the matrices $\mathbf{A}$ and $\mathbf{B}$ are given by:

$$\mathbf{A} = \begin{bmatrix} 1 + \Theta_A \Phi \varphi_y + \beta^{-1} \omega_y \Theta_A \Phi & \Theta_A \Phi (\varphi_\pi - \beta^{-1}) \\ -\beta^{-1} \omega_y & \beta^{-1} \end{bmatrix},$$

$$\mathbf{B} = \begin{bmatrix} \Phi \Theta_A & -\Phi \Theta_A & \Phi \Theta_B - \beta^{-1} \Phi \Theta_A \omega_\varepsilon & -\Phi \Theta_B & \beta^{-1} \omega \Phi \Theta_A \\ 0 & 0 & \beta^{-1} \omega_\varepsilon & 0 & -\beta^{-1} \omega \end{bmatrix}$$

where $\omega_y = \omega [\sigma + (\varphi + \alpha)/(1 - \alpha)]$ and $\omega_\varepsilon = \omega(1 + \varphi)/(1 - \alpha)$. Determinacy of this system, as highlighted by Bilbiie (2008), depends critically on the calibration of λ. From the definition of the coefficients in the aggregate demand condition (10) it is possible to observe that Φ is a rectangular hyperbola in λ, which results in a critical value (denoted in Bilbiie as λ^*) around which the properties of the model are significantly altered. For function (10), this critical value is given by:

$$\lambda^* = \frac{\varphi(1 + \mu) + \sigma(1 - \alpha)}{(1 + \varphi) [\varphi + \sigma(1 - \alpha)]} \tag{13}$$

With calibrations in the region of $\lambda < \lambda^*$, the Taylor principal ($\varphi_\pi > 1$) provides determinacy and 'normal' demand relationships are observed: an increase in interest rates suppresses demand. However, at calibrations of $\lambda > \lambda^*$, demand relationships are inverted and determinacy is only provided with passive or very-aggressive monetary policy.[10] Although this critical value and its implications contributes additional insights, this paper will restrict itself to the case where $\lambda < \lambda^*$: this is empirically appropriate (see discussion below).

3 Heterogeneous impacts of adverse shocks

This section uses the model derived in Section 2 to address the main question of this paper: what are the heterogeneous impacts of adverse shocks across households? First, this question is addressed by reviewing algebraic properties; the model is sufficiently simple to allow for these properties to be identified which permits the question to be answered independent of calibration. Next, the question is reviewed by performing simulations on the benchmark model to observe disaggregated impulse response functions in the presence of adverse shocks. Finally, a welfare criterion is derived from which these simulations can be converted into quantifiable comparisons of utilities of the two households.

[10] The conditions for this model are similar to that of Bilbiie (2008): determinacy when $\lambda > \lambda^*$ is given by:

$$\varphi_\pi \in \min \left\{ 1, (\Phi \Theta_A)^{-1} \left(\frac{\beta - 1}{\omega_y} \right), - \left[1 + (\Phi \Theta_A)^{-1} \left(\frac{2(\beta + 1)}{\omega_y} \right) \right] \right\} \cup \max \left\{ 1, - \left[1 + (\Phi \Theta_A)^{-1} \left(\frac{2(\beta + 1)}{\omega_y} \right) \right] \right\}$$

when φ is set equal to zero.

3.1 Algebraic properties

Disaggregate relationships

The modelling assumption for rule-of-thumb agents makes it possible to track their employment and consumption behaviour relative to output. These, in unison with the market clearing conditions, can be used in turn to infer Ricardian agents' disaggregated dynamics. This task is straight forward in the absence of productivity shocks as in such a scenario movements in output are directly related to movements in employment. When these technology changes are ignored the results in Table 1 can be obtained, where relevant working are presented in Appendix A.

Table 1: Disaggregate algebraic relationships in general equilibrium in the absence of productivity shocks

Disaggregated Consumption		Disaggregated Employment	
$\frac{\partial c_t^{NR}}{\partial y_t} > 0$	(C1)	$\frac{\partial n_t^{NR}}{\partial y_t} \lesseqgtr 0$	(N1)
$\frac{\partial c_t^{R}}{\partial y_t} > 0$	(C2)	$\frac{\partial n_t^{R}}{\partial y_t} > 0$	(N2)
$\frac{\partial c_t^{NR}}{\partial y_t} > \frac{\partial c_t}{\partial y_t} > \frac{\partial c_t^{R}}{\partial y_t}$	(C3)	$\frac{\partial n_t^{NR}}{\partial y_t} < \frac{\partial n_t}{\partial y_t} < \frac{\partial n_t^{R}}{\partial y_t}$	(N3)

Results (C3), (N2) and (N3) require that $\mu < \varphi + \sigma(1-\alpha)$: this is a reasonable restriction. Result (C2) is true providing $\lambda < \lambda^*$. Adverse shocks are defined as those resulting in $y_t < 0$. Note that in aggregate $\partial c_t/\partial y_t = 1$ and $\partial n_t/\partial y_t = 1/(1-\alpha)$ in the absence of exogenous movements in productivity.

An adverse shock leads to a decrease in both non-Ricardian and Ricardian consumption (C1 and C2), however, the fall of the former is bigger than that of the latter (C3). The Ricardian agents have access to capital markets and therefore can substitute future consumption for consumption today, which they do to maximise utility. Rule-of-thumb agents, on the other hand, do not have this access and as a result their consumption is more volatile: the only way they can smooth consumption is through their labour supply decisions. In the presence of an adverse shock, this is performed through increasing their labour supply above that which would prevail were they to have access to credit in order to increase their disposable income: they substitute leisure for consumption. This reaction is confirmed through observing that non-Ricardian employment as a result of an adverse shock is higher than that of their Ricardian neighbours (N2); credit constrained households work more than the unconstrained in order to insulate themselves from the impact of the shock, and at certain calibrations their employment will rise in the presence of reducing output (N1). Ricardian households unambiguously decrease their labour supply and this occurs for two reasons: first, less output is produced in the economy overall, therefore lowering demand for labour; and second, the rule-of-thumb agent's increase in labour supply drives down the real wage which makes Ricardian agents substitute consumption for leisure. The reaction of rule-of-thumb households to increase their labour supply in the presence of adverse shocks appears to insulate Ricardian consumption because its movement with respect to output is less than unity (C3): that which would prevail in a fully-Ricardian economy. However, this result does not take into account that the inclusion of the credit constrained households may amplify the aggregate response to exogenous shocks.

The presence of productivity shocks however provides ambiguous results compared to those discussed above because increases in productivity allow increases in output with no additional increases in employment. Using the method of undetermined coefficients it is possible to produce algebraic results similar to those used to find the properties in Table 1, however the inference is no longer clear. Disposable income for non-Ricardian households can fall in the presence of a shock increasing productivity, at certain calibrations, as a rise in wages (reflecting an increase in the marginal product of labour) is tempered by a fall in labour demand. Note that a shock increasing productivity is an 'adverse' shock as defined above because the presence of sticky prices means that output capacity expands at a greater rate than actual production leading to a negative output gap. Although a productivity increase will lead to a fall in inflation it may also lead to a reduction of output when the demand effect of a reduction in non-Ricardian consumption outweighs the impact of the increase in productive capacity: this will occur at high values of λ and low levels of ρ_a. However, results and intuition from Table 1 still follow through. An adverse technology shock leads to a rise in Ricardian consumption which dominates that of non-Ricardian consumption, which may indeed fall. In order to insulate themselves from this divergence rule-of-thumb agents will supply more labour in order to increase their disposable income: non-Ricardian labour will be greater than Ricardian labour.

Aggregate relationships

The method of undetermined coefficients can be used to derive algebraic results similar to those above for the aggregate economy.[11] This method solves for the linear relationship between output and inflation, notated as Ψ_y and Ψ_π respectively, to each shock in turn and provides the results in Table 2. A positive exogenous movement in the interest rate or the discount factor leads to a reduction in both output (Y1) and inflation (Π1), whereas positive movements in the desired markup lead to a reduction in output (Y2) with a rise in inflation (Π2); these are intuitive results that have been identified before and which are true of a fully Ricardian economy. The ambiguous result between technology shocks and output (Y3) derives from the impact discussed above of an increase in productivity leading to a reduction in labour with ambiguous impacts on non-Ricardian disposable income. In a fully Ricardian economy the increase in productive capacity is only relevant and output unambiguously improves. In an economy with or without non-Ricardian households the presence of a positive productivity shock leads to a fall in inflation as the same level of output can be produced at a lower labour cost.

Table 2: Aggregate algebraic relationships in general equilibrium

	Output		Inflation	
Demand shocks	$\Psi_y < 0$	(Y1)	$\Psi_\pi < 0$	(Π1)
Supply (markup) shocks	$\Psi_y < 0$	(Y2)	$\Psi_\pi > 0$	(Π2)
Technology Shocks	$\Psi_y \lesseqgtr 0$	(Y3)	$\Psi_\pi < 0$	(Π3)

All conditions trivially require that $\lambda < \lambda^*$, $\phi_y \geq 0$ and $\phi_\pi \geq 1$.

Increasing the share of non-Ricardian households in the economy has two effects on these coefficients: first, the impact of the shock is increased due to the non-linear relationship between any stimulant and λ, demonstrated in the λ^* relationship (13); and second, any adverse impact is met with an increase in labour supply from these rule-of-thumb agents who attempt insulate themselves

[11] This analysis makes the unconditional results of Table 1 now conditional on specific shocks through linking the impact of individual shocks to movements in output.

from the impact of the recession. This latter effect also insulates the economy and can be seen through the increasing impact of monetary policy at higher levels of λ.[12] The first effect will always dominate leading to a larger impact from any exogenous shock.

3.2 Dynamic simulations

Calibration

The algebraic properties above can now be observed through dynamic simulations using the calibration outlined below. Each unit of time is a quarter and the discount factor, β, is set at 0.99 corresponding to a steady state rate of return to bonds of 4%. The preference parameters are set such that $\sigma = 1$, which represents log utility with respect to consumption, and $\varphi = 0.2$, the inverse elasticity of work with respect to the real wage.[13] The degree of decreasing returns to labour, α, is set to equal 1/3 and the steady state mark-up, μ, is calibrated to 0.2; this value corresponds to a elasticity of substitution across the intermediate goods, ϵ, of 6. The parameter governing the stickiness of prices, θ, is set at 0.75 leading to the average price duration for a given firm of four quarters. The Taylor rule parameters are set such that $\varphi_\pi = 1.5$ and $\varphi_y = 0.125$. The persistence parameter of all shocks, ρ_i, are set equal to 0.8 to make comparisons across the results for different shocks.

This calibration follows closely that of Galí et al. (2007), which has been criticised for its low value of the wage elasticity of employment.[14] As demonstrated in equation (13), if it is required that $\lambda < \lambda^*$, there are five parameters to calibrate and only four degrees of freedom: the choice of calibration of one parameter needs to be sacrificed to the model and not to the data. Through setting $\varphi = 0.2$ a calibration of $\lambda = 0.5 < \lambda^*$ is achievable ($\lambda = 0.5$ is reflective of empirical observations and is the calibration used in Galí et al. 2007): φ is sacrificed to allow a reasonable calibration of λ. A similar strategy as in Galí et al. (2007) is adopted here by using the same calibration. The value of λ^* with other parameters at reasonable calibrations is too low, which is primarily due to the simplicity of the model. It has been implicitly shown that through including further rigidities such as fixed capital accumulation, habit persistence, Kimball demand curves and firm specific capital, the values of λ^* increase.[15] To demonstrate the proposal that it is a lack of complexity of the model that limits the value of λ^*, if non-Ricardian households are included into the medium scale DSGE model of Smets & Wouters (2003) with reasonable parameter calibrations, a value of $\lambda^* > 1$ is provided. It is a preference to not overcomplicate the model in order to identify algebraic properties and transmission mechanisms. Moreover, it is desired that the share of the two types of households are approximately equal to their empirical size because the paper reviews welfare movements and with appropriate proportions of each household comes appropriate weighted average welfare calculations. Sensitivity of the results to the calibrations of φ and λ parameters, amongst others, will be commented on throughout.

[12] It can be shown that providing $\lambda < \lambda^*$, $\partial\Phi\Theta_A/\partial\lambda > 0$.

[13] Under this specification with log utility of consumption Bilbiie (2008) finds that non-Ricardian employment is constant as the income and substitution effect cancel one another out. However, the introduction of steady state transfers (even at a constant level out of steady state) drive a wedge between the income and substitution effect due to movements in the real wage such that this is not true in our model.

[14] In particular this calibration has been questioned in Furlanetto & Seneca (2009) and Colciago (2011); on the other hand, Galí et al. (2007) cite Rotemberg & Woodford (1997) as evidence to support this calibration.

[15] Although this cause and effect is not explicitly shown, it is highlighted through sensitivity analysis which varies the level of λ. The examples above are shown in Galí et al. (2007) and Furlanetto & Seneca (2009).

Simulations

The aggregate impacts of the three individual shocks are those predicted in Table 2: output falls in response to both a positive interest rate shock and a positive markup shock whereas inflation falls and rises with these two shocks respectively. Output is found to increase from a positive productivity shock in this calibration and, as predicted above, inflation falls. The disaggregate responses to these shocks, which are the main focus of this paper, are presented to Figure 1 for adverse demand (first column), cost push (second column) and productivity (third column) shocks, where the first and second rows presents disaggregated consumption and employment responses respectively, and the third row presents dynamics for the disposable income of Ricardian households.

As is illustrated in the figure, the predictions of Section 3.1 are observed; non-Ricardian households work more and consume less than their Ricardian neighbours, who themselves work less and consume more in the presence of the rule-of-thumb agents, compared to a similar shock in a fully-Ricardian economy ($\lambda = 0$). In the absence of access to credit, rule-of-thumb agents use the labour market as their smoothing resource and in the presence of adverse shocks they do this through increasing their labour supply. This results in additional disposable income for these agents and additional production in the economy however the former is not sufficient to purchase all of the latter and the surplus goes to the Ricardian agents. This is funded by these agents through additional profits which is as a result of cheap labour in the market and also due to the Ricardian agents sharing dividends across only a fraction of the population when $\lambda \neq 0$.[16] The reduction of both Ricardian employment and wages means that these agents' labour incomes fall as a result of the adverse shock, however the rise in profits (dividends) compensates them for this. The third row in Figure 1 shows that as a result of all adverse shocks Ricardian disposable income increases in the presence of rule-of-thumb agents, relative to the fully-Ricardian benchmark. Ricardian income rises whilst non-Ricardian income falls despite the former working less than the latter: the path of rule-of-thumb income is given by their consumption. Not only is the welfare of the Ricardian agents improving as a result of the presence of non-Ricardian agents so too are their incomes which improve relative to their credit constrained neighbours.

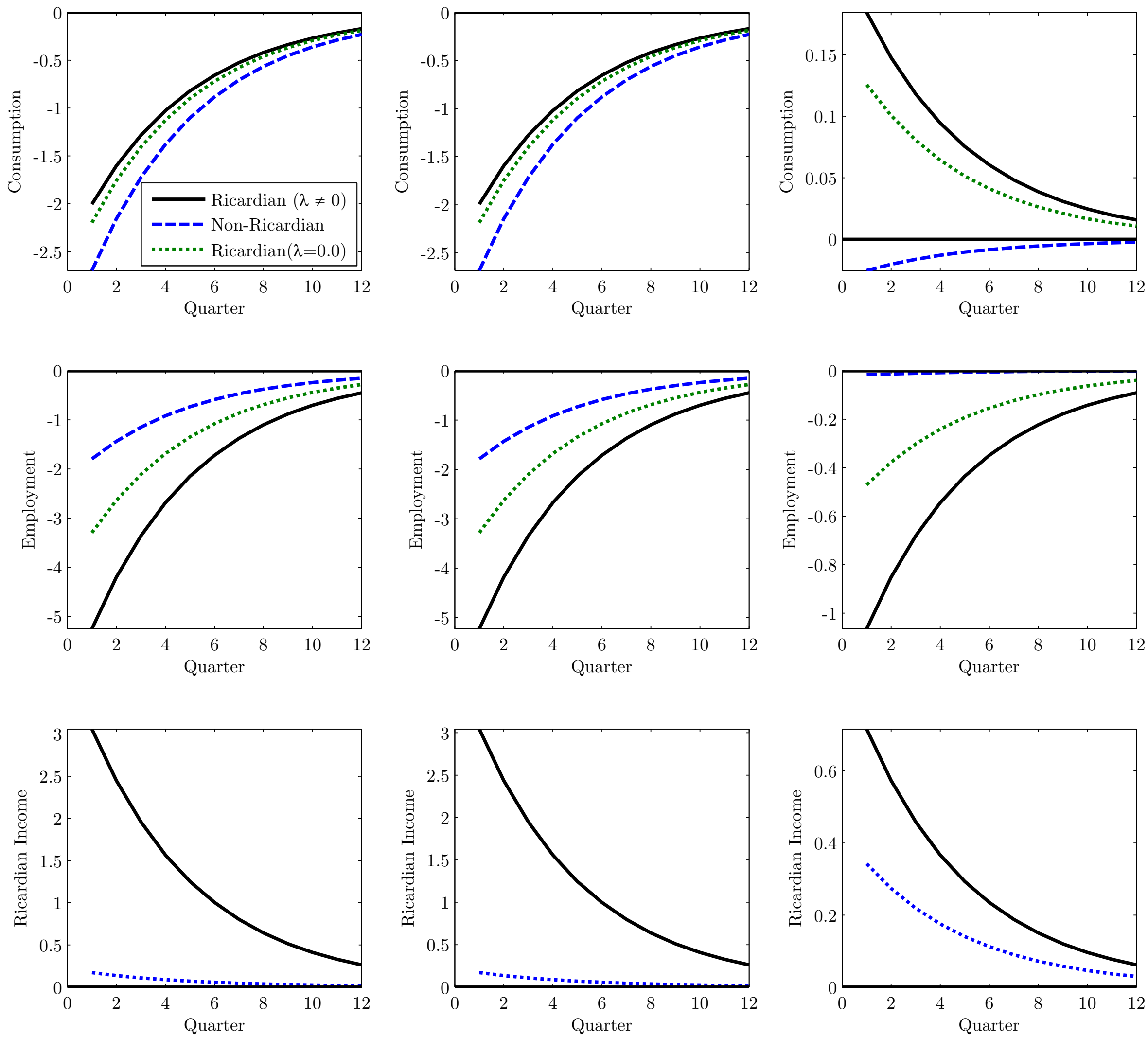
A clear redistribution of welfare is observed from non-Ricardian to Ricardian households as a consequence of the former's employment decisions and the latter's access to capital. This is with respect to the arguments in the utility function but moreover in income streams: a clear inequality of experience is observed in the model as a result of adverse shocks.

3.3 A welfare criterion

The dynamic responses for the heterogeneous households in these example are clear to see, interpret and evaluate; the responses for Ricardian households' variables strictly dominate those for the non-Ricardian households in both the employment and consumption dimensions. Moreover, the presence of the non-Ricardian households benefits the Ricardian households whose dynamics where $\lambda \neq 0$ strictly dominate their dynamics in an economy where $\lambda = 0$. However, a more formal analysis can be performed by determining a welfare criterion based on a second order Taylor series expansion of the utility function around steady state values. This procedure provides a criterion expressed as the equivalent one period consumption loss, as a proportion of steady state consumption, that

[16] Dividends paid out through profits are subject to the relationship $D_t = (1 - \lambda)D_t^R$, as only Ricardian agents receive dividends.

Figure 1: Disaggregate dynamics from adverse shocks



Dynamics achieved through the calibration discussed in Section 3.2 and through setting $\eta_t^r = 1.0$ when $t = 1$ for demand shocks, setting $\eta_t^\mu = 1.4$ when $t = 1$ for cost push shocks and setting $\eta_t^a = 0.4$ when $t = 1$ for productivity shocks. The latter two calibrations were performed to make the welfare loss (gain) equal to that of the interest rate shock in a fully Ricardian economy for the cost push (productivity) shocks. Note only interest rate shocks have been included to represent an aggregate demand shock because, as discussed above, preference shocks enter the aggregate demand condition (10) in the same way and therefore the analysis would be identical. The first row presents results for disaggregated consumption, the second row results for disaggregated employment and the third row dynamics on Ricardian disposable income given by the sum of their employment income and dividends received. The graph also presents results for Ricardian household dynamics in a fully-Ricardian economy ($\lambda = 0$).

leaves the agent indifferent between living through the shock or the one period consumption loss. When the necessary calculations have been performed this criterion takes the following form:

$$W^i = E_0 \sum_{t=0}^{\infty} \beta^t \left(c_t^i + \frac{1-\sigma}{2} \left(c_t^i\right)^2 \right) - \frac{(1-\alpha)}{1+\mu} E_0 \sum_{t=0}^{\infty} \beta^t \left(n_t^i + \frac{1+\varphi}{2} \left(n_t^i\right)^2 \right) \quad (14)$$

Applying the disaggregated dynamics of the three adverse shocks simulated above to this criterion gives the results presented in Table 3. The table confirms the above analysis; Ricardian households experience greater welfare, as a result of the adverse shocks, than non-Ricardian households, and when the latter are present the Ricardian households experience greater levels of welfare than when they are not. However, one result that is not directly predicted through observing the disaggregated dynamics is that these Ricardian households experience a welfare gain as a result of the adverse shocks. In the two shocks that lead to a fall in output, the large fall in employment more than compensates Ricardian households for their small fall in consumption. Moreover, in the presence of the adverse technology shock output rises and all gains accrue to Ricardian agents. The unambiguous fall in lifetime utility is observed for the non-Ricardian households for adverse demand and cost push shocks, and in the presence of adverse productivity shocks, at this calibration, they experience a mild loss of welfare.

Table 3: Disaggregated welfare - perfectly competitive labour markets

	Non-Ricardian	Ricardian	Weighted Average
Demand (interest rate) shock			
$\lambda = 0$	N/A	-0.018	-0.018
$\lambda = 0.5$	-0.081	0.041	-0.020
Cost push (markup) shock			
$\lambda = 0$	N/A	-0.018	-0.014
$\lambda = 0.5$	-0.081	0.041	-0.020
Productivity shock			
$\lambda = 0$	N/A	0.018	0.018
$\lambda = 0.5$	-0.001	0.037	-0.018

The welfare loss is expressed as the equivalent one period consumption loss, as a proportion of steady state consumption, that would leave the household indifferent between living through the shock or the one period sacrifice. Dynamics achieved through the calibration discussed in Section 3.2 and through setting $\eta_t^r = 1.0$ when $t = 1$ for demand shocks, setting $\eta_t^\mu = 1.4$ when $t = 1$ for cost push shocks and setting $\eta_t^a = 0.4$ when $t = 1$ for productivity shocks. The latter two calibrations were performed to make the welfare loss (gain) equal to that of the interest rate shock in a fully Ricardian economy for the cost push (productivity) shocks.

3.4 Discussion

This section has shown that there is a redistribution of welfare as a result of adverse shocks from non-Ricardian to Ricardian households. This has been illustrated through identifying algebraic properties of the model, through dynamic simulation and through deriving a disaggregated welfare criterion for the two households based on a second order Taylor series expansion of their utility functions. This redistribution of welfare comes as a result of the credit constrained agents increasing their labour supply in order to smooth their consumption. This action consequently insulates the economy which indirectly insulates the Ricardian households whose welfare improves as a result of the presence of the credit constrained agents. It has been shown that Ricardian household's welfare is unambiguously better than their non-Ricardian neighbours and moreover, these agents actually gain as a result of adverse shocks under reasonable calibrations. In aggregate, the weighted average welfare movement as a result of the shocks is small: this is in line with the analysis of Lucas (2003). However, the disaggregate movements within this overall average figure are not trivial with a negative covariance between the experiences of the different households. This negative correlation of agents' welfare implies that we are not 'all in this together' and provides support for the anecdotal evidence presented in Section 1.

The results also lend support to the concept of the *squeezed* or *anxious* middle, depending on the beliefs on the identification of the rule-of-thumb agents.[17] We argued that these non-Ricardian households will be predominantly those agents at the lower end of the income distribution. This hypothesis in conjunction with estimates on their relative share implies that the middle class could be within this credit constrained group. Moreover, as the economic downturn has continued, access to credit has become more restrictive, therefore eliminating further agents from this smoothing resource. The combination of the theoretical results and the proposal that rule-of-thumb agents come from the lower end of the income scale is coherent with the literature of Krusell et al. (1999, 2009) and Mukoyama & Şahin (2006) and also with the anecdotal evidence. However, these results have been derived with a stylised labour market which plays an important role in the transmission mechanisms in the economy which is relaxed in the next section.

4 An alternative labour market assumption

It is through the labour market that the non-Ricardian households perform their optimisation and insulate themselves against the impact of the shocks: an important transmission mechanism in the above results. To test the sensitivity of the results to the labour market assumption, the imperfectly competitive labour market of Galí et al. (2007) is used.[18] In this design, a continuum of monopolistically competitive unions add a markup to wages. There is no friction to wage negotiations, and all households are treated indiscriminately receiving the same wages and working the same hours. This is performed by the unions aggregating preferences across households to find a weighted average labour supply function. Each union is assumed to have the same ratio of non-Ricardian to Ricardian members of $\lambda : (1-\lambda)$. With such an assumption there is no heterogeneous supply of labour across households: this will have a significant impact on both the aggregate and disaggregate economy. In the presence of an adverse shock, non-Ricardian households have a preference to increase their labour supply whereas Ricardian households have a preference to decrease theirs; the aggregation process across households results in the former working less and the latter more than if they were allowed to supply labour independently.[19]

[17] This conclusion also requires considerations of the shock process generating the recent recession. As is seen above, an 'adverse' technology shock increases output (but at less than its potential), therefore, if it is believed a negative technology shock produced the recession then the analysis would be inverted. Ireland (2011) use variance decomposition techniques to estimate that the recent recession was mainly as a result of demand shocks and there were positive productivity shocks to compensate for this: this conclusion is reached in other papers.

[18] Only a skeleton of the derivation is presented here: for full details see Galí et al. (2007).

[19] The only change this makes to the analytical properties of the aggregate model is to change the coefficients in the aggregate demand function (10) which now become $\Phi_{TU} = \Gamma_{TU}^{-1}\left[\Gamma_{TU}^{-1} - \lambda(1+\varphi)\right]^{-1}; \Theta_{A,TU} = (1-\lambda)\frac{1}{\sigma}(1+\mu)\Gamma_{TU}; \Gamma_{TU} = [1+\mu-\lambda\sigma(1-\alpha)]^{-1}$ where subscript TU represents that these coefficients now relate to the economy

One important aspect to consider when performing simulations under this different labour market will be that of the calibrated value of λ, because the change in the labour market will have a corresponding change in λ^*. The strategy here is to maintain a constant value for the ratio λ/λ^*, where this constant is determined by the perfectly competitive labour market analysis above; this strategy isolates the impact that the change in the labour market assumption is creating by eliminating the impact of a change in λ^*.[20]

Table 4: Disaggregated welfare - imperfectly competitive labour markets

	Non-Ricardian	Ricardian	Weighted Average
Demand (interest rate) shock			
$\lambda = 0$	N/A	-0.018	-0.018
$\lambda = 0.37$	-0.090	0.018	-0.022
Cost push (markup) shock			
$\lambda = 0$	N/A	-0.018	-0.018
$\lambda = 0.37$	-0.089	0.018	-0.022
Productivity Shock			
$\lambda = 0$	N/A	0.018	0.018
$\lambda = 0.37$	-0.003	0.030	0.018

The welfare loss is expressed as the equivalent one period consumption loss, as a proportion of steady state consumption, that would leave the household indifferent between living through the shock or the one period sacrifice. Dynamics achieved through the calibration discussed in Section 3.2 and through setting $\eta_t^r = 1.0$ when $t = 1$ for demand shocks, setting $\eta_t^\mu = 1.4$ when $t = 1$ for cost push shocks and setting $\eta_t^a = 0.4$ when $t = 1$ for productivity shocks. The model is the same as that presented in Section 2 however the coefficients in the aggregate demand conditions change due to the change in labour market. The calibration of $\lambda = 0.37$ is used to reflect the fact that $\lambda^* = 0.64$ under this labour market assumption.

Comparable results to those in Table 3 are presented in Table 4 for an economy with the new labour market assumption subjected to the three types of shock. These show that the welfare gains of the Ricardian households are reduced in an economy which includes non-Ricardian households compared to the perfectly competitive labour market benchmark. This is an intuitive result; the aggregation process performed by the trade unions in order to derive an aggregate labour supply function results in these households working more than they otherwise would were they to supply labour independently. Moreover, because non-Ricardian households are working less compared to the benchmark of the perfectly competitive labour markets, profits distributed to the Ricardian agents are not rising to a similar degree. Likewise, the welfare penalty for non-Ricardian households is also observed whose utility losses are increased as a result of the new labour market assumption. The redistribution of welfare is still observed and the results are robust to the change in the labour market.

This extension does merit further consideration however. Subjecting both types of household to the same labour market conditions may be inappropriate if it is believed that there is a distinct difference between both types of agent in the model; under this circumstance separate labour

with imperfectly competitive labour markets.

[20] Analytically, this condition now becomes $\lambda^*_{TU} = (1+\mu)/[1+\varphi+\sigma(1-\alpha)]$; maintaining the λ/λ^* ratio from above leads to calibrated value of $\lambda_{TU} = 0.37$.

markets for the two types of household may be deemed more appropriate. For example, the hypothesis we proposed was that the rule-of-thumb agents would predominantly come from the lower end of the income spectrum. These agents would have lower skill levels and it is these who are observed to be most empirically vulnerable to business cycles. The experience of unemployment during recessions is what causes the most pain to individuals; the fact that the above results were obtained in a model that does not include unemployment and where labour can move freely is compelling.

5 An empirical investigation

The model presents clear results with respect to both welfare and income movements of the two types of agents as a result of adverse shocks. Combined with our proposal that the non-Ricardian households come predominantly from the lower end of the income distribution, a working hypothesis is provided that income inequality will increase during periods of recession; this is in line with the anecdotal evidence and can be tested empirically. The method proposed here is to review the correlation between the changes in the Gini coefficient against movements in the cyclical component of growth. The Gini coefficient is a measure of income inequality and if the proposed hypothesis is correct it would be expected that this should increase in times of negative cyclical growth. The following panel regression is applied:

$$\Delta Gini_{i,t} = \alpha_i + \beta CYCGrowth_{i,t} + \theta Z_{i,t} + \varepsilon_{i,t} \tag{15}$$

where $\Delta Gini_{i,t}$ is the change in the Gini coefficient from year $t-1$ to year t in country i, α_i are country specific constants identified using a fixed effects panel, $CYCGrowth_{i,t}$ is the cyclical component of growth for country i in year t identified using the Hodrick-Prescott filter, $Z_{i,t}$ a group of control variables and $\varepsilon_{i,t}$ is an error term. The problems with such an approach is the availability of data on the Gini coefficient and the selection of appropriate control variables. For such a specification to be worthwhile data on inequality needs to be observed for a number of consecutive years, to ensure an appropriate number of observations, and from the same study to ensure consistent methodology for comparable statistics. The selection of control variables is complicated by the short term nature of the annual changes; inequality movements are generally studied over longer horizons.

Table 5 presents results for this analysis. In all, seven countries are used which represents the number of developed nations with more than 10 consecutive observations for Gini coefficients, each year from the same study, in the United Nations World Income Inequality dataset. The results show that not only are the coefficients attached to the $CYCGrowth$ variable always with the expected sign, but moreover, they are always significant to at least 5% confidence levels. This is true across five separate specifications with different combinations of independent variables, controlling for the political stance of ruling parties and a time trend to pick up the universal steady growth of inequality post 1979.[21] Further, the results are not sensitive to regression method used, countries in the sample and the method of detrending GDP data: similar results can be achieved from OLS and random effects panel regression; from dropping any one country in the sample and from dropping both France and Germany which represent the two countries with the

[21] The former was calculated using information on government formation over the period and benefited from sample countries having predominantly a two party system. The latter was included due to the pervasive increases in inequality observed for all countries in the dataset.

Table 5: The inequality of recessions: an empirical investigation

	(1)	(2)	(3)	(4)	(5)
CYCGrowth	-7.285**	-5.180***	-5.217***	-3.907**	-3.944**
	(0.023)	(0.002)	(0.002)	(0.016)	(0.015)
LWing			0.037		0.051
			(0.628)		(0.480)
Time				0.249***	0.251***
				(0.001)	(0.000)
Observations	184	148	148	148	148
R^2	0.029	0.054	0.058	0.131	0.136

Dependent variable is $\Delta Gini_{i,t}$, where $Gini_{i,t}$ is obtained from the United Nations World Income Inequality dataset. The dataset includes the countries of the Australia (41), Canada (19), France (11), Germany (11), Japan (18), United Kingdom (41) and the USA (44) where the numbers in parenthesis represent the number of observations for each country in the total sample. The results are obtained using a fixed effects panel regression where the country fixed effects and constant are not presented. The specification in column (1) represents all data in the sample where all other columns excludes those years where the change in the Gini coefficient is equal to or greater than one in modulus. $LWing$ is a dummy variable that takes the value 1 if the country has a political party in power that is from the centre-left of the political scale for that country, and zero otherwise. $Time$ is a dummy variable which takes the value 1 for any observation between the years 1979 and 2000 inclusive, 0 otherwise. Data used to calculate $CYCGrowth_{i,t}$ was obtained from the IMF International Financial Statistics. P-values of t-statistics for each individual coefficient are presented in parenthesis. The notation * is used to represent t-statistics significant to at least 10% confidence, ** significant to at least 5% and *** significant to at least 1%.

smallest number of observations; and from detrending growth rate data using a polynomial time trend. The results suggest a redistribution of welfare from the poor (non-Ricardian households) to the rich (Ricardian households) occurs during times of below trend growth. Although it is beyond the scope of this paper to provide a fully fledged empirical analysis, results presented in Table 5 are not only supportive of the anecdotal evidence presented in Section 1, but also for the theoretical modelling of divergent experiences during recessions and moreover the hypothesis that non-Ricardian households come from the lower end of the income distribution.

6 Further sensitivity tests

The redistribution of welfare from non-Ricardian to Ricardian households associated with adverse shocks has been shown to be algebraically robust and illustrated through dynamic simulation: the results have also been seen not to be sensitive to the labour market assumption. This section further tests the robustness of the results.

6.1 Non-Adverse shocks

The above analysis has been performed considering only adverse shocks, defined as those leading to a negative output gap. This restriction was made for two main reasons: first, to consider the impact of the current recession; and second, because it was argued that restricting access to capital is more relevant in downturns when agents want to borrow. The model is linear and therefore the impact

Table 6: Consumption variance throughout the business cycle

	Labour Market			
	Perfectly Competitive		Imperfectly Competitive	
	$\varphi = 0.2$	$\varphi = 1.0$	$\varphi = 0.2$	$\varphi = 1.0$
$\text{Var}\left(c_t^{NR}\right)/\text{Var}\left(c_t\right)$	1.26	2.92	2.63	4.96
$\text{Var}\left(c_t^{R}\right)/\text{Var}\left(c_t\right)$	0.92	0.85	0.82	0.82
$\text{Var}\left(c_t^{R}\right)/\text{Var}\left(c_t \mid_{\lambda=0}\right)$	0.95	0.90	0.89	0.89

Results achieved through simulating the model with both perfectly competitive (columns 2 & 3) and imperfectly competitive (columns 4 & 5) labour markets with equal standard deviation weights for demand, cost push and productivity shocks (where demand shocks are shared across both interest rate and preference shocks); the results are not sensitive to this equal weighting. A simulation period of 10,000 quarters is used where the first 200 observations were dropped from the analysis. A calibration of $\lambda_{PC} = 0.3$ is used as this is in line with many estimation results from DSGE analysis and $\lambda_{TU} = 0.22$ to maintain a constant λ/λ^* ratio; results are amplified at higher values of λ. Moreover, a calibration of $\varphi = 1$ is used as a sensitivity check as this is more in line with the literature and moreover, as demonstrated, has a significant impact on the results. $\text{Var}\left(C_t \mid_{\lambda=0}\right)$ is the variance of (Ricardian) consumption in a fully Ricardian economy.

from non-adverse shocks would be the reverse of those presented above: non-Ricardian households would benefit at the expense of Ricardian households in boom periods. This result breaks economic logic, to some extent, as those agents who are more constricted are achieving higher levels of utility than those who are less constricted. However, it could be argued that such behaviour over the course of the business cycle leads to lower levels of welfare as the linear impacts net-off but the variance in consumption will be higher for credit constrained agents, thus leading to lower levels of utility.

To extend the analysis to consider the whole business cycle, simulations of the model are performed and key statistics observed and documented in Table 6. This is performed under both labour market assumptions and also where φ is increased to 1 because the imperfectly competitive labour market and a higher wage elasticity to employment are more coherent with empirical observations. Ricardian households experience a lower variance in consumption than their non-Ricardian neighbours and moreover, than in a fully Ricardian economy; credit constrained households insulate the unconstrained despite the fact that their presence increases the impact of shocks and therefore exposes the aggregate economy more to these exogenous movements. In a model with perfectly competitive labour markets the variance in non-Ricardian consumption is more than a third greater than that in Ricardian consumption; however in the more empirically appropriate assumption of imperfect labour markets this increases to 320% greater as rule-of-agents are less able to smooth consumption through their labour supply. When φ is allowed to increase to more frequently used calibrations (with a corresponding reduction in λ to maintain a constant λ/λ^* ratio) these results are amplified; with imperfect labour markets non-Ricardian consumption variance is more than six times greater than their Ricardian neighbours. This occurs because agents are more adverse to movements in employment and therefore use this resource less to smooth consumption.

Therefore, if we allow the credit constraint assumption to apply throughout the business cycle the predicted results are observed and the magnitude of these are compelling high. The negative correlation between period-by-period utility movements are seen, with Ricardian households benefiting from recessions and non-Ricardian households benefiting in booms. This negative covariance results in inference based on aggregate variables being biased downwards as to the costs of business cycles.

6.2 The degree of asset market non-participation

As was discussed in Section 3.1 the greater the share of non-Ricardian households the greater the impact of a shock on the aggregate economy due to the non-linear feedback mechanism between any exogenous movement and aggregate demand. From a disaggregate perspective two factors are influencing the welfare of individual agents: first, the increased aggregate impact of the shock is leading to a larger labour supply reaction by rule-of-thumb agents; and second, as the proportion of rule-of-thumb households increase, the proportion of Ricardian households is decreasing, and therefore so too are the agents who can take advantage of the additional employment and dividends in the economy. These two factors work in unison such that the losses of non-Ricardian households and the gains of Ricardian households are amplified and so too is the redistribution of welfare. However, movements in λ are having the greatest impact on Ricardian agents: for example, an increase of $\lambda = 0.3$ to $\lambda = 0.5$ leads to a rise in Ricardian gains of 85% with a rise in non-Ricardian losses of only 4% for demand and cost push shocks. This dominance occurs due to the impact of having fewer agents from which to share dividends across. However, these quantitative movements are met with the same intuition and qualitative responses: at all calibrations in the region of $\lambda < \lambda^*$ and in the presence of adverse shocks Ricardian welfare always dominates non-Ricardian welfare.

We can also remove the assumption that both types of agent consume and work the same amounts in steady state. The only impact this would have on the model would be to change the aggregate consumption condition to:[22]

$$c_t = \chi c_t^{NR} + (1-\chi) c_t^R$$

where $\chi = \lambda C^{NR}/C$. Empirical studies on rule-of-thumb consumption behaviour, for which the calibration of λ is based, are finding χ as they are estimating the share of income accruing to rule-of-thumb households. Therefore, if the proposition that these agents will come from the lower end of the income distribution is believed, this will result in more non-Ricardian households than their share of total income; the only impact that this will have on the above results is to change the weighted average welfare figures.

6.3 Other parameters

The calibration used, which follows closely that of Galí et al. (2007), has been criticised for its low value of the wage elasticity of labour (see for example Furlanetto & Seneca 2009). The main reason for having φ set at 0.2 is to allow λ to be set at 0.5, recalling from equation (13) that these two variables cannot be independently calibrated if all other parameters are whilst maintaining $\lambda < \lambda^*$. An increase in this parameter has three main effects: first, as the disutility of labour increases, the labour response of rule-of-thumb agents to adverse shocks will decrease; second, the value of λ^* will decrease leading a bigger impact from shocks for a given value of λ; third, the weight attached to squared movements in employment in the welfare criterion (14) increase. These impacts combine to increase the redistribution of welfare in the presence of adverse shocks as the second of these

[22] Note that the remaining areas where disaggregate steady state levels arise in the derivation of the model (the welfare criterion and the log linear non-Ricardian consumption function) require the ratio of these variables (N^i/C^i) which given the assumed identical period utility function will be equal in steady state.

will dominate. An increase from $\varphi = 0.2$ to $\varphi = 1$ for example will lead to Ricardian gains and non-Ricardian losses approximately trebling and doubling respectively.

The calibration is also criticised for the high level of price stickiness (see again Furlanetto & Seneca 2009) with $\theta = 0.75$ leading to an average price duration of one year. With respect to demand and productivity shocks an increase in θ increases the aggregate impact of the shock thus increasing the redistribution of welfare. With respect to cost push shocks however, the reverse is true because at high levels of θ the shock is not transmitted into the economy, as is clear through inspection of the New Keynesian Phillips curve (11). These impacts are shared across both types of households; for example, an increase from $\theta = 0.5$ to $\theta = 0.75$ leads to an increase in Ricardian gains and non-Ricardian losses equal to approximately 150% for demand shocks and a fall of approximately 55% for cost push shocks.

Finally, the policy parameters are of significance because the two types of agent may have differing preferences over these. In general, more conservative monetary policy leads to a lower impact from shocks and therefore will reduce both gains and losses to Ricardian and non-Ricardian households respectively: these impacts, although quantitatively small, are shared across both types of agent. The only exception to this is with cost push shocks where a higher calibration of φ_π leads to a greater aversion to the inflation these shocks generate and subsequently the impact they have on output: in this scenario higher values lead to a rise in the gains and losses of Ricardian and non-Ricardian households respectively. However, these are only quantitatively small changes and qualitatively the above results are robust.

7 Conclusions

The objective of this paper was to identify whether a simple DSGE model with heterogeneity among households could provide theoretical support for the recent anecdotal evidence of an inequality of sacrifice in the current recession: the results have gone some way to dispelling the notion that ‘we’re all in this together’. This was performed by dividing agents into those with access to capital and those without, which was argued to be empirically appropriate and apt in the current *credit crisis*. It was found that those agents who are credit constrained disproportionately loose in the presence of adverse shocks in the economy. Further, it was observed that this suffering from one set of individuals was to the benefit of the other, whereby the actions of the rule-of-thumb households insulated the economy: under reasonable calibrations the Ricardian households gain as a result of the adverse shocks. These results were shown through algebraic proof, dynamic simulation and through a derivation of disaggregated welfare criteria based on a second order Taylor series expansion of the utility function. The implication of the model was also shown to be supported through an empirical investigation that suggested a negative correlation between cyclical growth and income inequality.

These results are important because it provides a further critique to the Lucas (2003) argument that the welfare consequences of economic fluctuations are trivial. In this respect it contributes and compliments the existing literature that finds similar results suggesting business cycles disproportionately impact the poor, low skilled and unemployed: this is coherent with the proposition behind the identification of these agents. A further contribution of the paper is to provide simple and tractable techniques that can be applied in the growing DSGE literature that includes a fraction of rule-of-thumb agents. Up to date, welfare analysis from DSGE models has been performed on a representative agent, the same assumption used by Lucas (2003) to imply that such studies are not an important priority. This paper illustrates that there are disaggregated and redistributional impacts to exogenous shocks and subsequent policy actions. These impacts by themselves are of importance but moreover provide potential frictions that can commentate on why some policies are followed over others: the model can be used to discuss political motives to policy. The potential extensions that follow this line of argument are vast, certainly with the debate around current policy actions.

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A Appendix: algebraic properties

The assumption that non-Ricardian households consume entirely their disposable income makes it possible to derive functions of their consumption and employment behaviour with respect to output. Log linearising the non-Ricardian consumption function (3) provides:

$$c_t^{NR} = \frac{1-\alpha}{1+\mu}\left[n_t^{NR} + (w_t - p_t)\right] \tag{16}$$

where the steady state ratio has been substituted with $N^{NR}W/C^{NR}P = (1-\alpha)/(1+\mu)$ using the observation that employment gets paid its marginal product net of the markup charged by the monopolistically competitive firms. From the maximisation of both Ricardian and non-Ricardian household utility it is possible to obtain the labour supply condition $W_t/P_t = (N_t^i)^{\varphi}(C_t^i)^{\sigma}$ for $i = R, NR$, which once log-linearised can give the aggregate expression $(w_t - p_t) = \varphi n_t + \sigma c_t$. Substituting this into (16) and deriving an expression for non-Ricardian employment, n_t^{NR}, in terms of aggregate consumption, employment and non-Ricardian consumption using both the aggregate and disaggregate labour supply functions provides (after manipulation):

$$c_t^{NR} = \frac{(1+\varphi)(1-\alpha)}{\varphi(1+\mu)+\sigma(1-\alpha)}\left[\varphi n_t + \sigma c_t\right]$$

Finally, using the aggregate log linear production function (5) and market clearing condition (9) to substitute out for both employment and consumption provides:

$$c_t^{NR} = \frac{(1+\varphi)(\varphi+\sigma(1-\alpha))}{\varphi(1+\mu)+\sigma(1-\alpha)}y_t - \frac{(1+\varphi)\varphi}{\varphi(1+\mu)+\sigma(1-\alpha)}\varepsilon_t^a \tag{17}$$

From this it is possible to observe the complications made through productivity shocks. Setting $\varepsilon_t^a = 0$, condition (C1) from Table 1 is trivial. Moreover, substituting out for non-Ricardian consumption using the log linear version of the market clearing condition (9) provides:

$$c_t^R = \left[\frac{\lambda}{1-\lambda}\frac{(1+\mu)\varphi + \sigma(1-\alpha) - \lambda(1+\varphi)(\varphi+\sigma(1-\alpha))}{1+\mu+\sigma(1-\alpha)}\varphi\right]y_t$$

Where condition (C2) from Table 1 can be obtained, providing $\lambda < \lambda^*$. Moreover, the above two conditions can combine to obtain condition (C3) from Table 1.[23]

Using the log linear expressions for the aggregate and non-Ricardian labour supply functions in conjunction with the production function and goods market clearing condition, one can obtain the result:

$$\varphi n_t^{NR} + \sigma c_t^{NR} = \left[\frac{\varphi + \sigma(1-\alpha)}{1-\alpha}\right]y_t$$

which can be used to substitute out c_t^{NR} from (17) to obtain a relationship between non-Ricardian employment and output (N2):

$$n_t^{NR} = \frac{1}{\varphi}\left[\frac{\varphi+\sigma(1-\alpha)}{1-\alpha} - \frac{\sigma(1+\varphi)(\varphi+\sigma(1-\alpha))}{(1+\mu)\varphi+\sigma(1-\alpha)}\right]y_t$$

[23] Another way of finding (C3) is to show that $\partial c_t^{NR}/\partial y_t > 1$ and $\partial c_t^R/\partial y_t < 1$ using the functions derived above.

Which can be used, once it has been observed that in the absence of technology shocks $\partial n_t/\partial y_t = 1/(1-\alpha)$, to obtain the first half of condition (N3) from Table 1 where the second half and condition (N3) and then logical extensions from this first result.